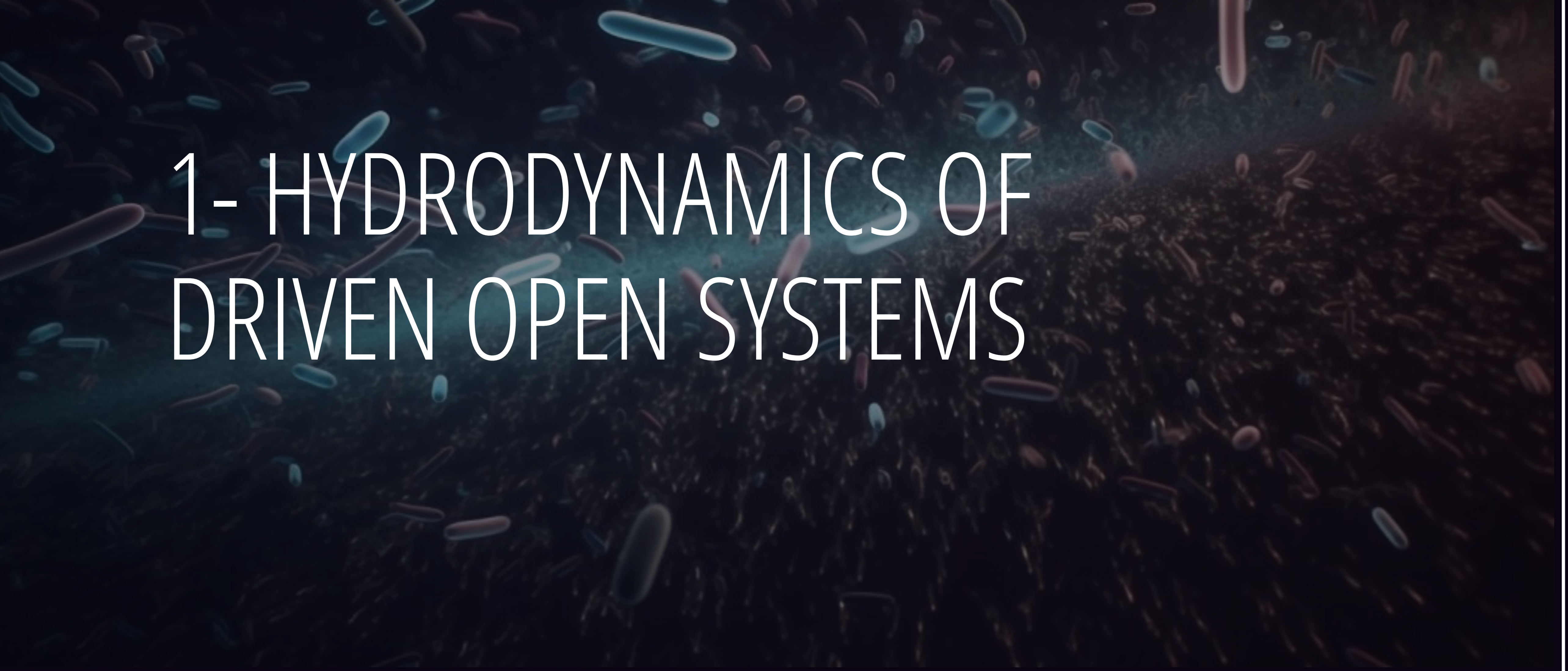
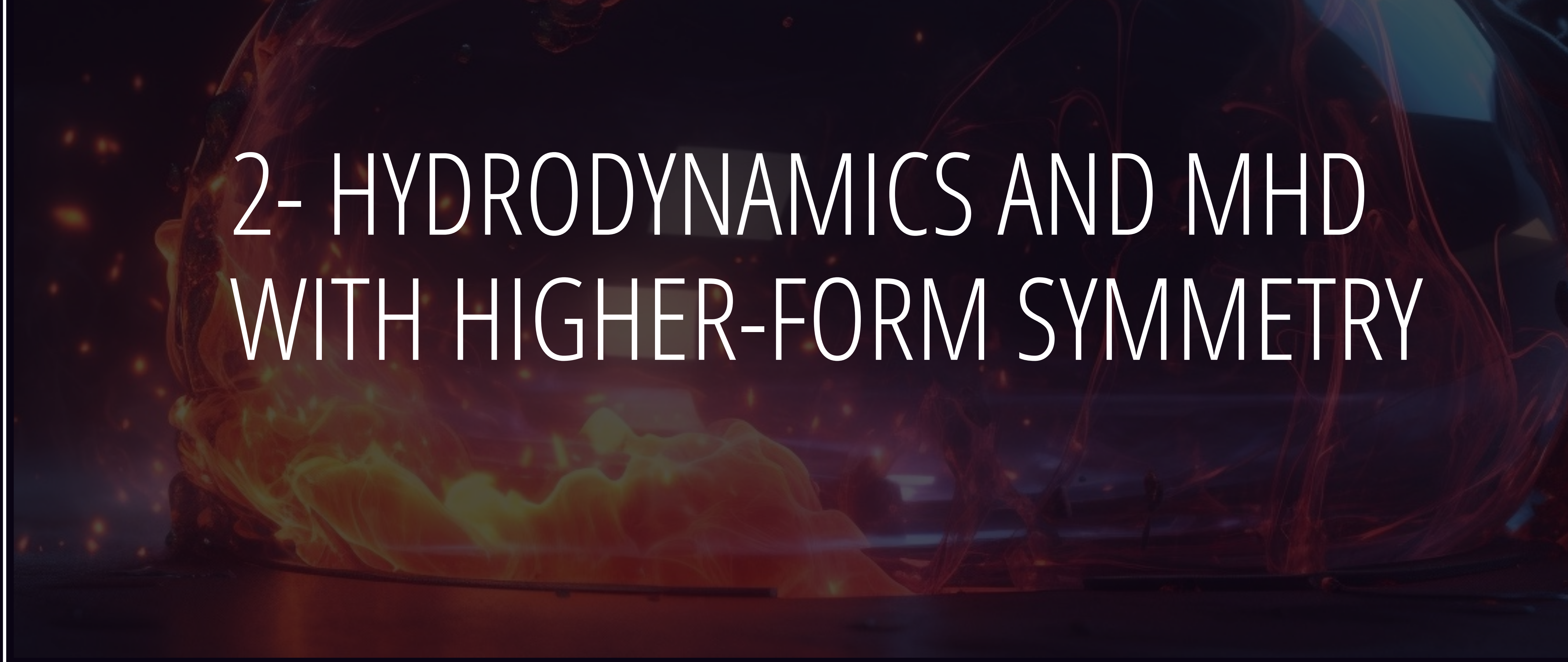


1- HYDRODYNAMICS OF DRIVEN OPEN SYSTEMS

- HOW TO FORMULATE HYDRODYNAMICS AROUND NON-EQUILIBRIUM STEADY STATES
- ACTIVE SYSTEMS (DRIVEN AND OPEN)
- SK FOR DRIVEN OPEN SYSTEMS
- TEMPERATURE EFFECTS IN DRIVEN OPEN SYSTEMS



2- HYDRODYNAMICS AND MHD WITH HIGHER-FORM SYMMETRY

- HOW TO FORMULATE MHD (OVERVIEW)
- HOW TO FORMULATE ONE-FORM SUPERFLUIDS
- CAUSALITY ISSUES AND RESOLUTION IN MHD
- NUMERICAL IMPLEMENTATION OF CAUSAL MHD
- GENERALIZED MELTING***

HYDRODYNAMICS AND MHD WITH HIGHER-FORM SYMMETRY

BASED ON arXiv: 1808.01939 (PRL) & 1811.04913 (JHEP) by JA & A. Jain,
arXiv: 2201.06847 (JCAP) by JA & F. Camilloni,
arXiv: 2301.09628 (PRD), by JA & A. Jain
arXiv: 2501.04638 by R. Lier, A. Jain. JA, O. Porth

Jay Armas
Associate Prof. at University of Amsterdam
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jacomearmas.org

AT THE UNIVERSITY OF AMSTERDAM: A SMALL RESEARCH GROUP ON HYDRODYNAMICS AT ALL SCALES AND COMPLEX SYSTEMS

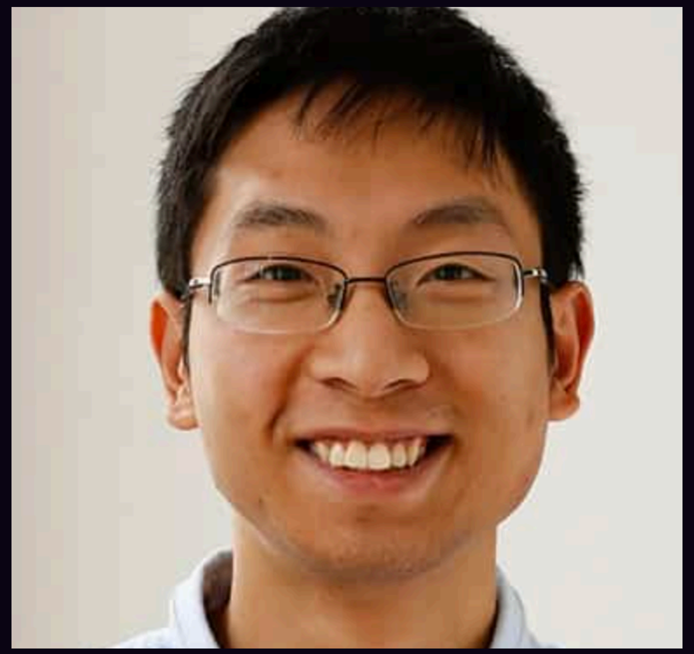

Akash Jain


Ruben Lier


Gian Nicosia


Daniel Jordan


George Batzios


Tuan Pham

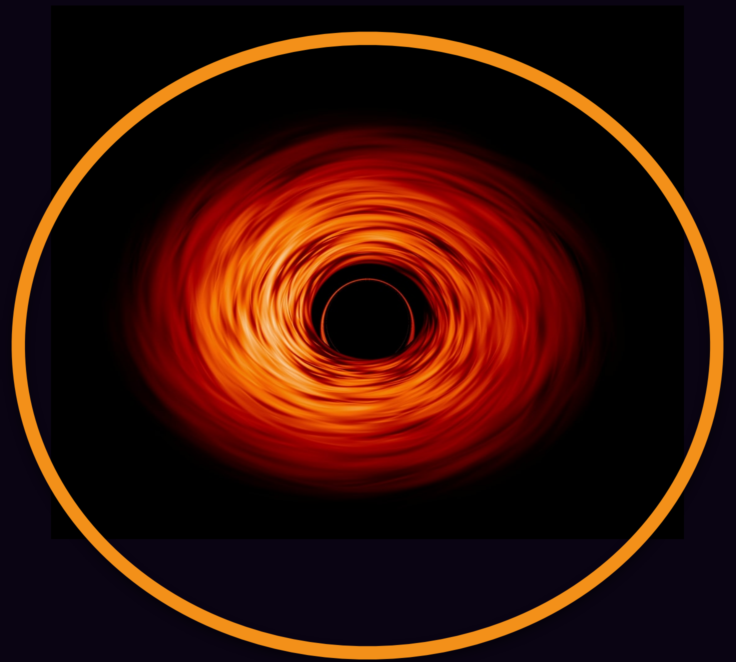
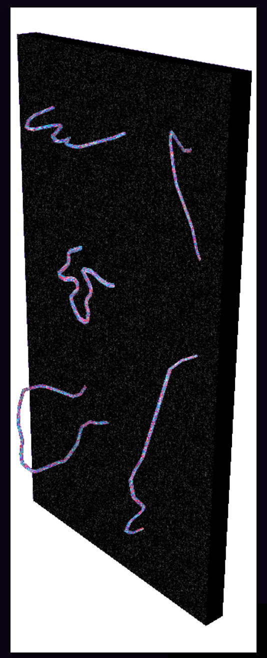
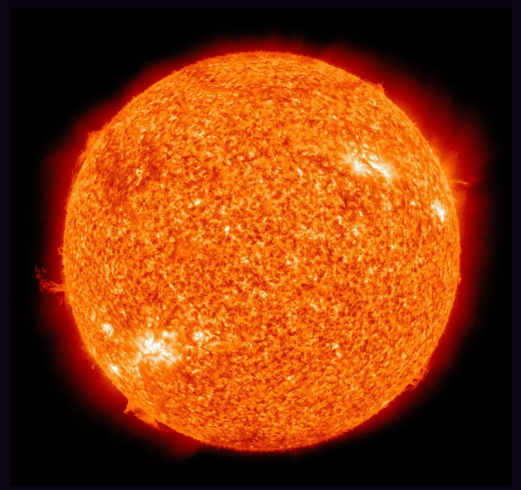
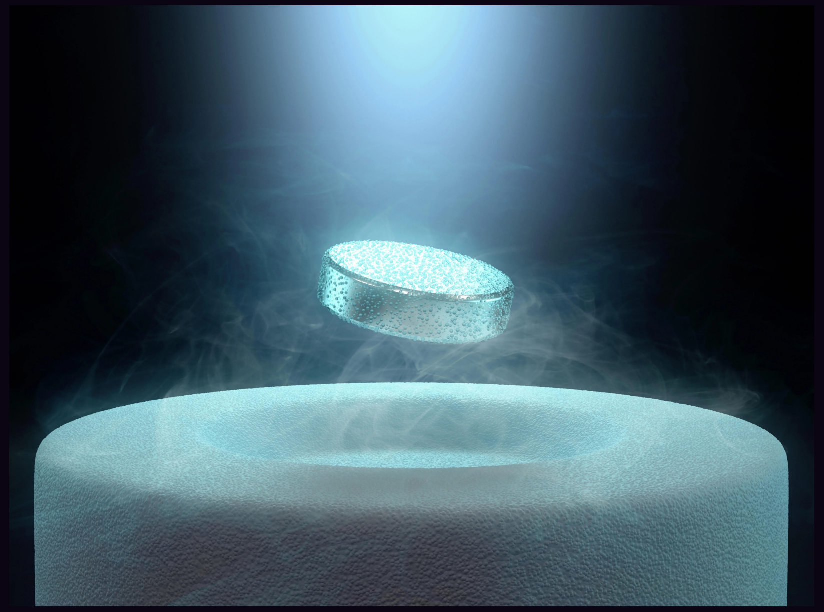
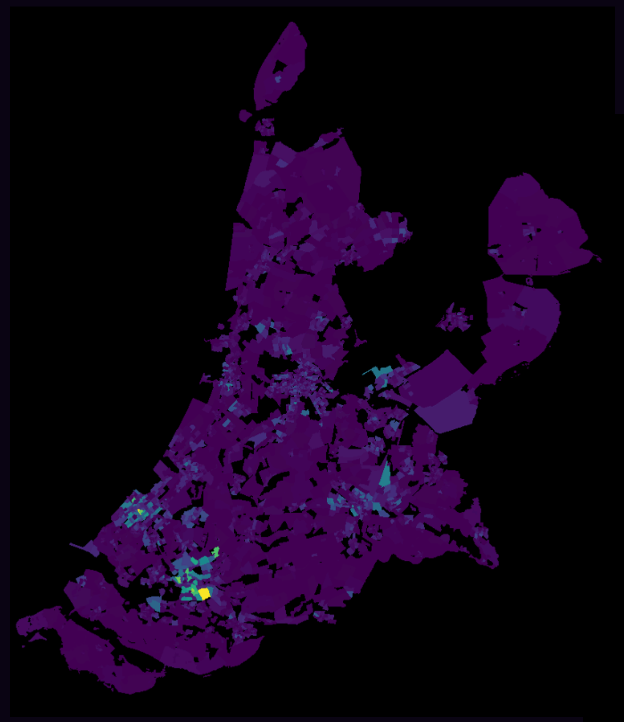

Wout Merbis

COLLABORATORS

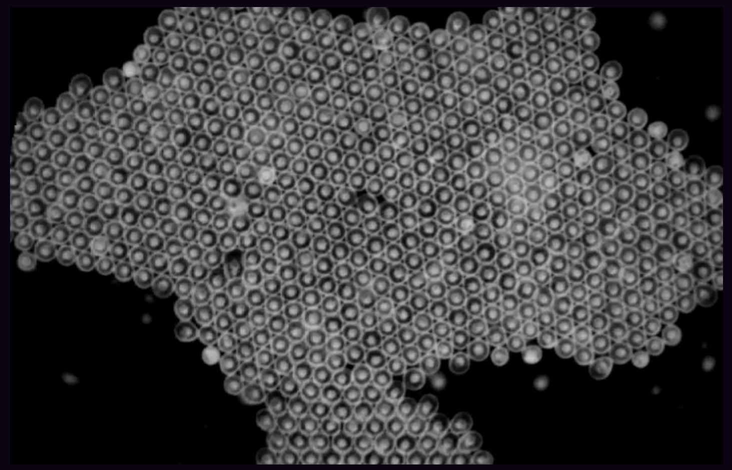
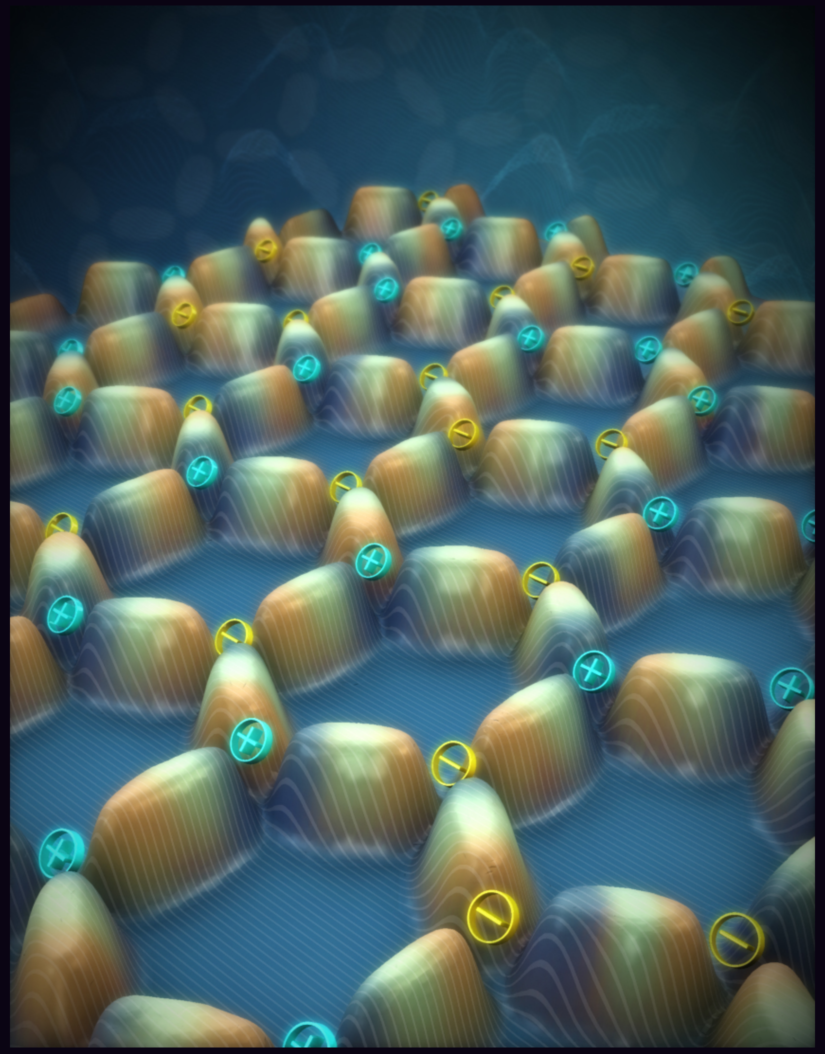
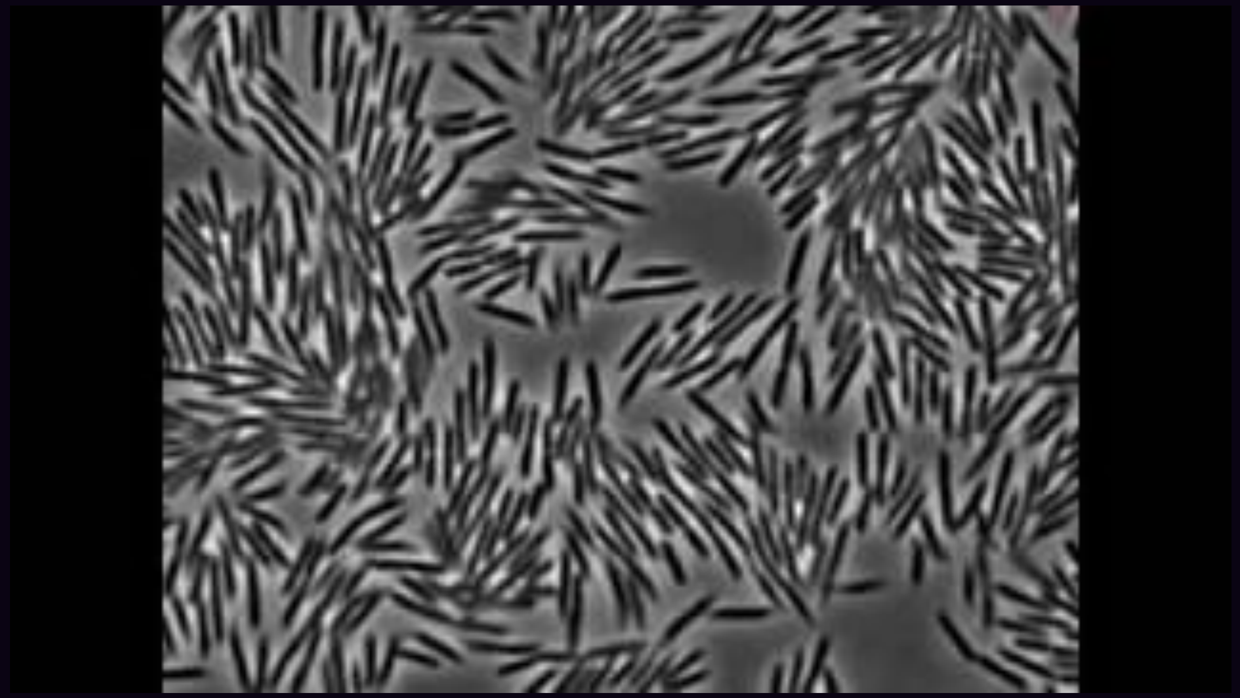
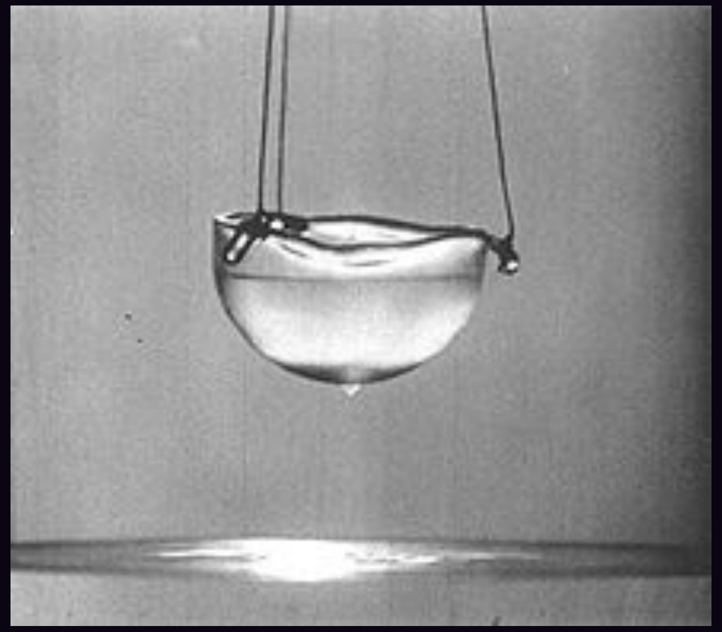
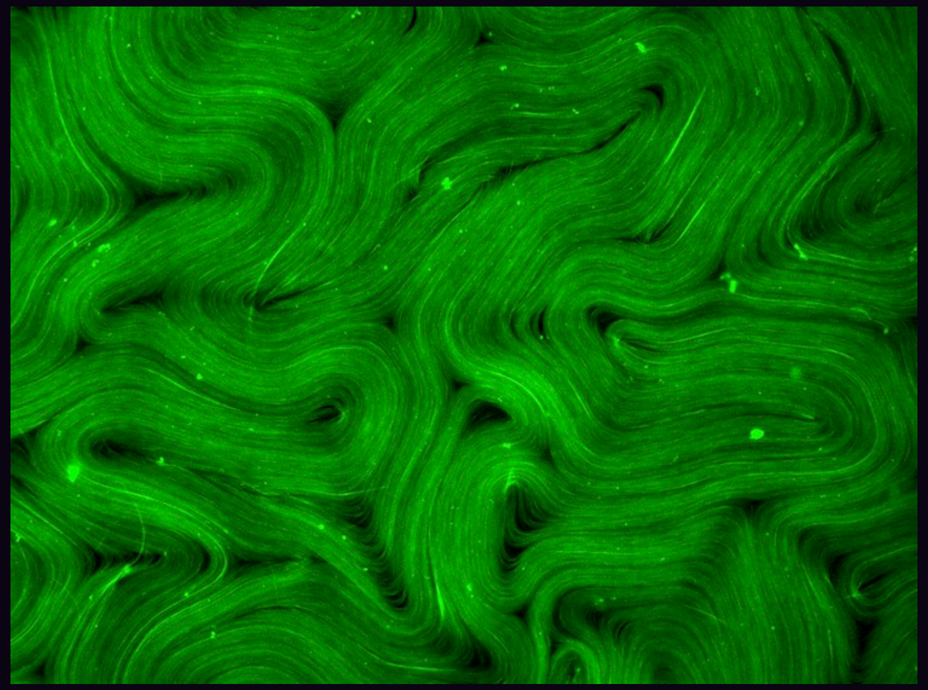
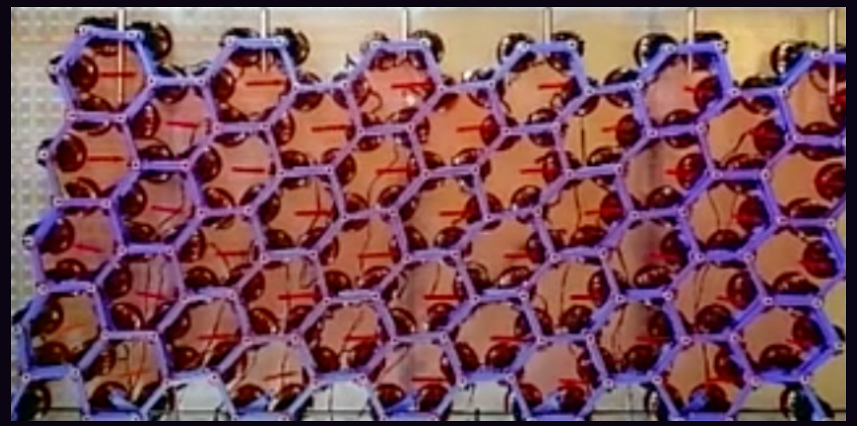
Filippo Camilloni
(Frankfurt U.)

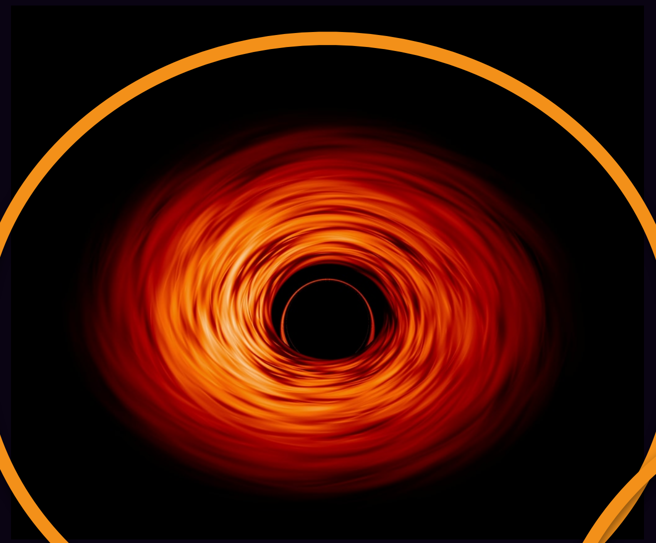
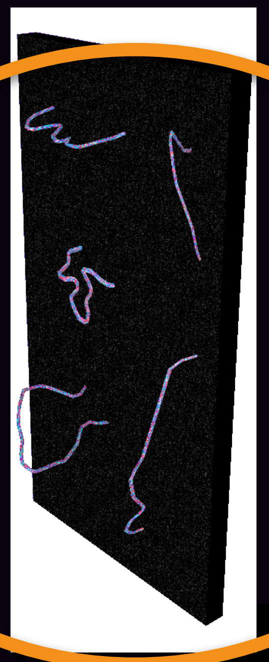
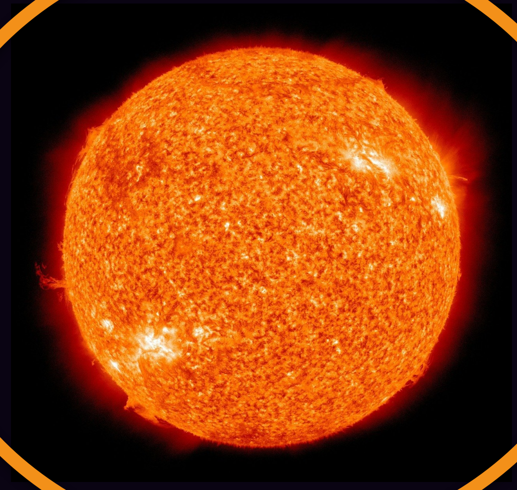
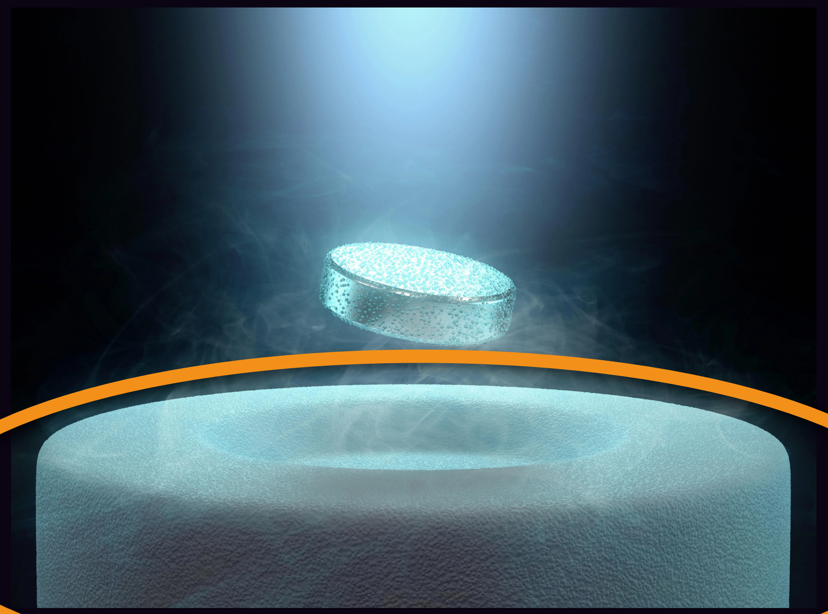
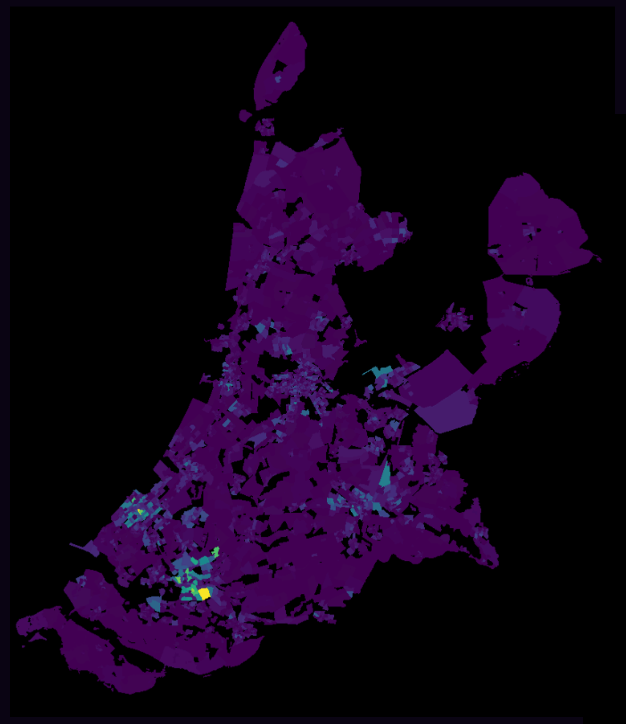
Oliver Porth
(U. Amsterdam)

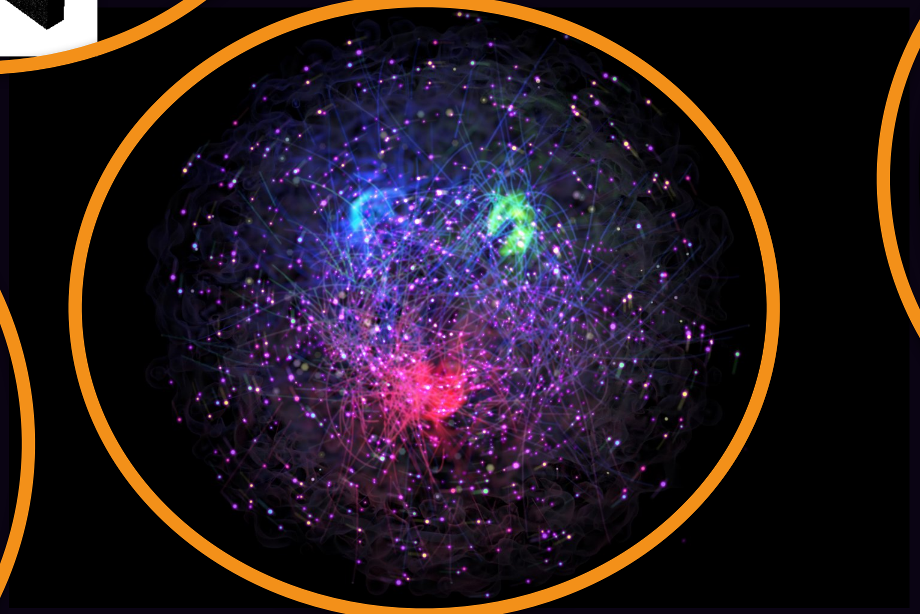
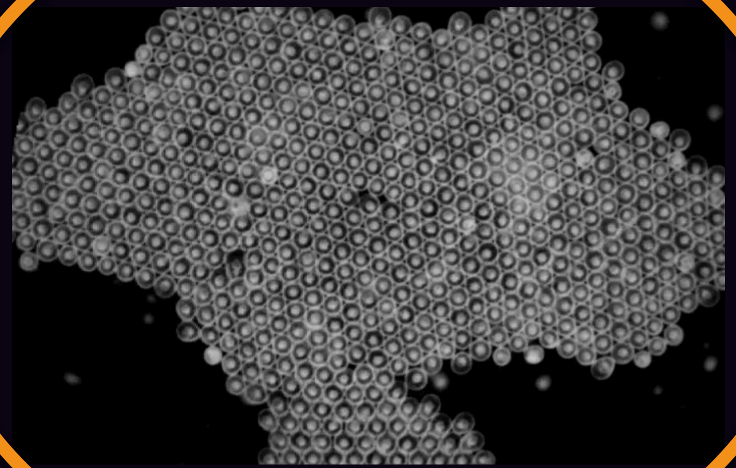
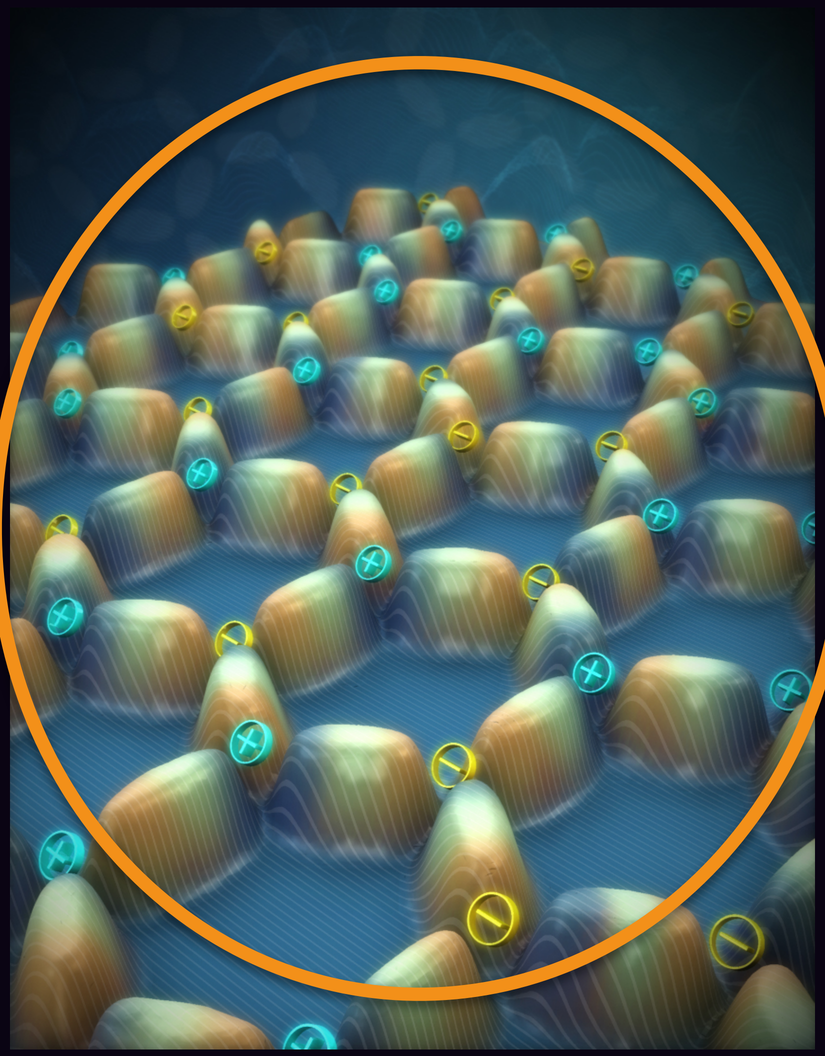
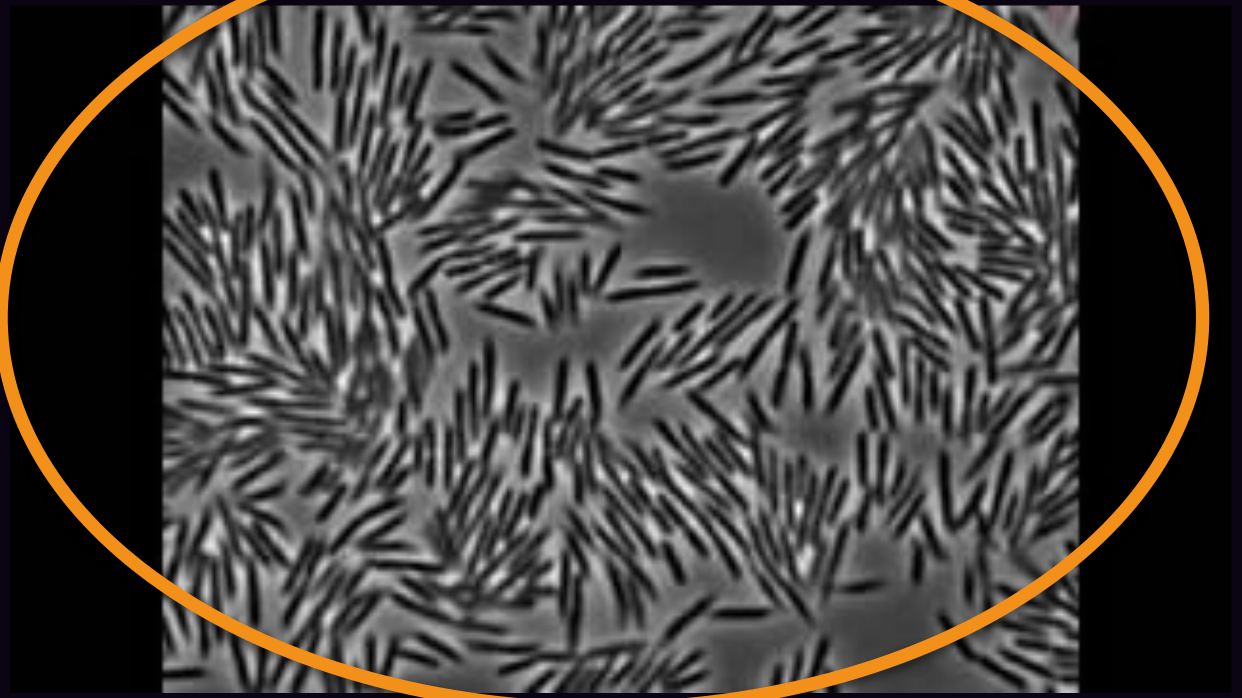
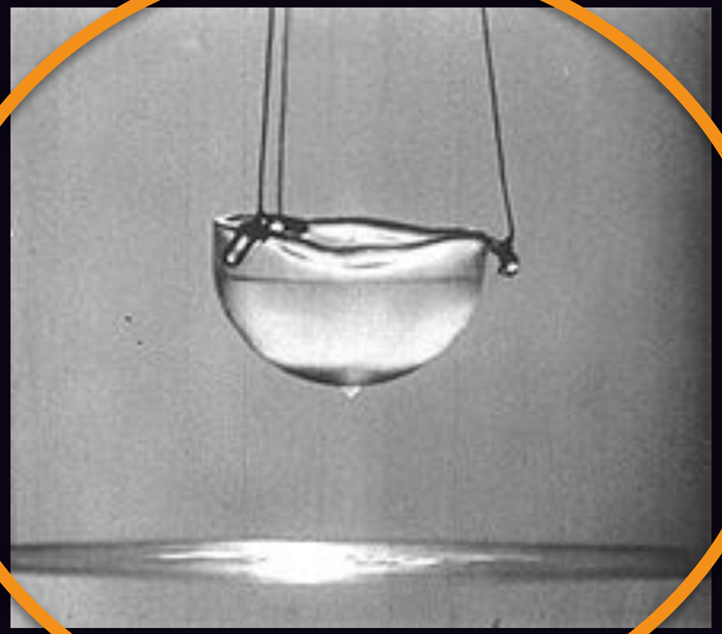
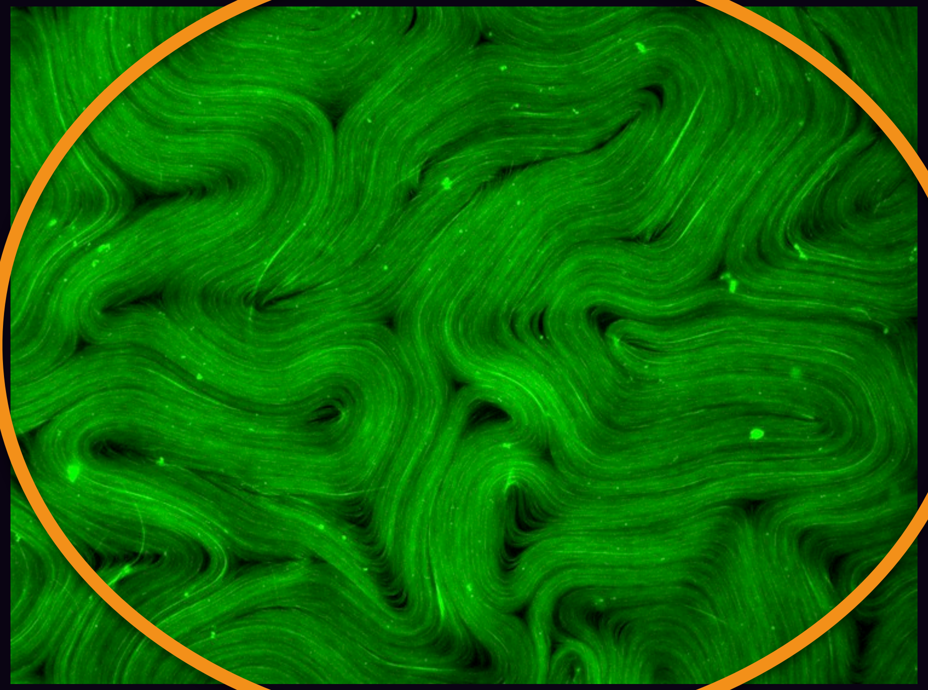
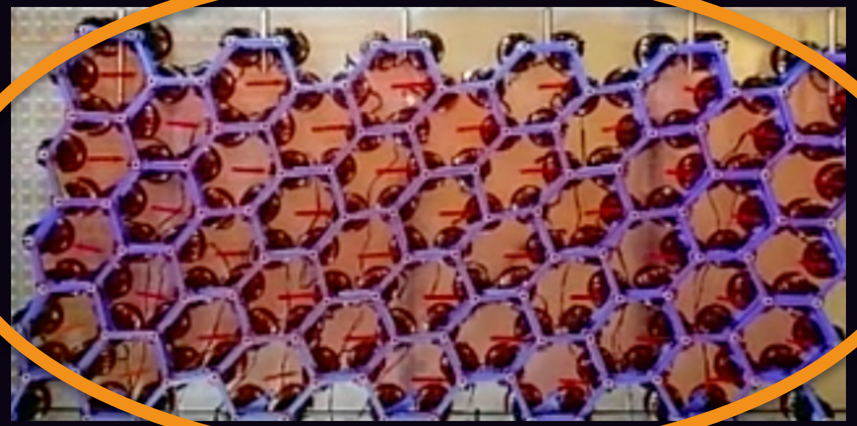
I- MOTIVATIONS



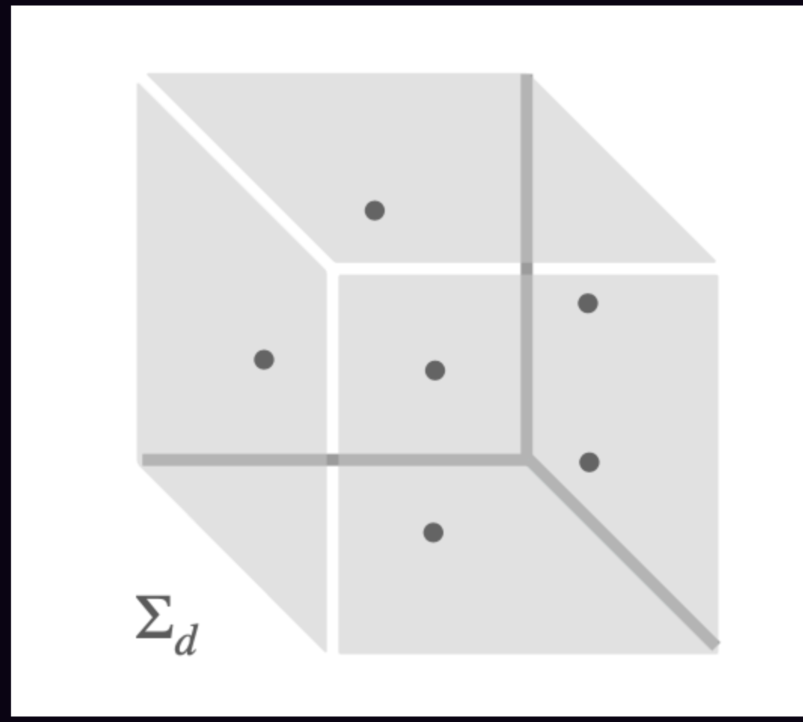







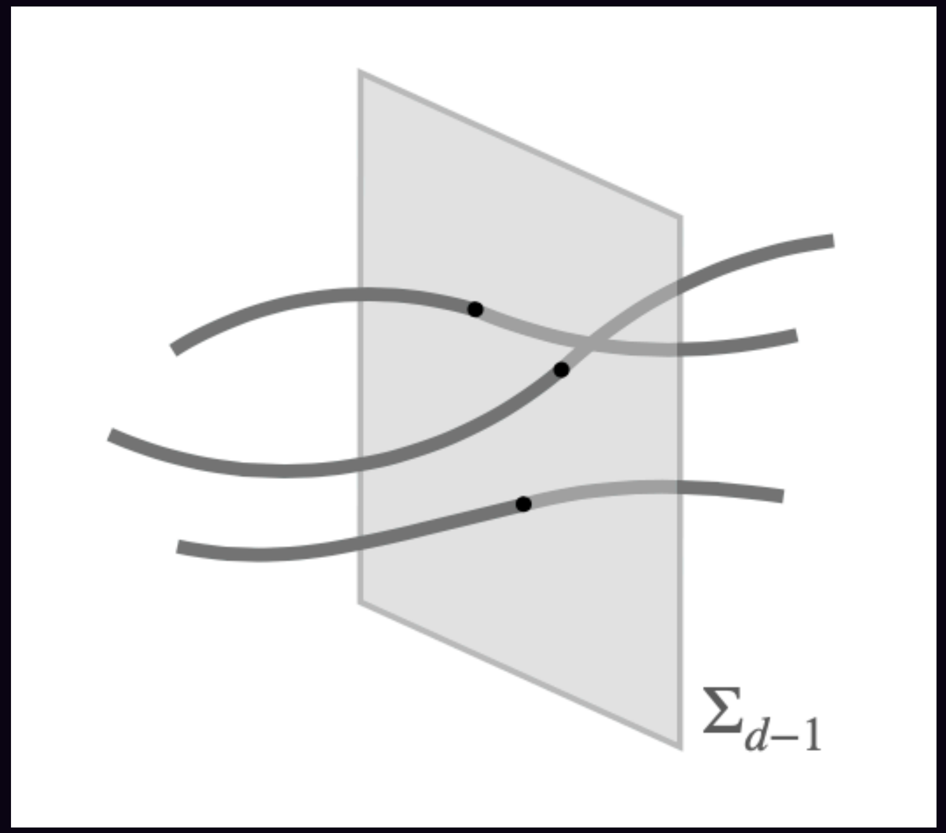
EXACT SYMMETRY

$$\partial_t J^t + \partial_i J^i = 0$$



(conventional symmetry)

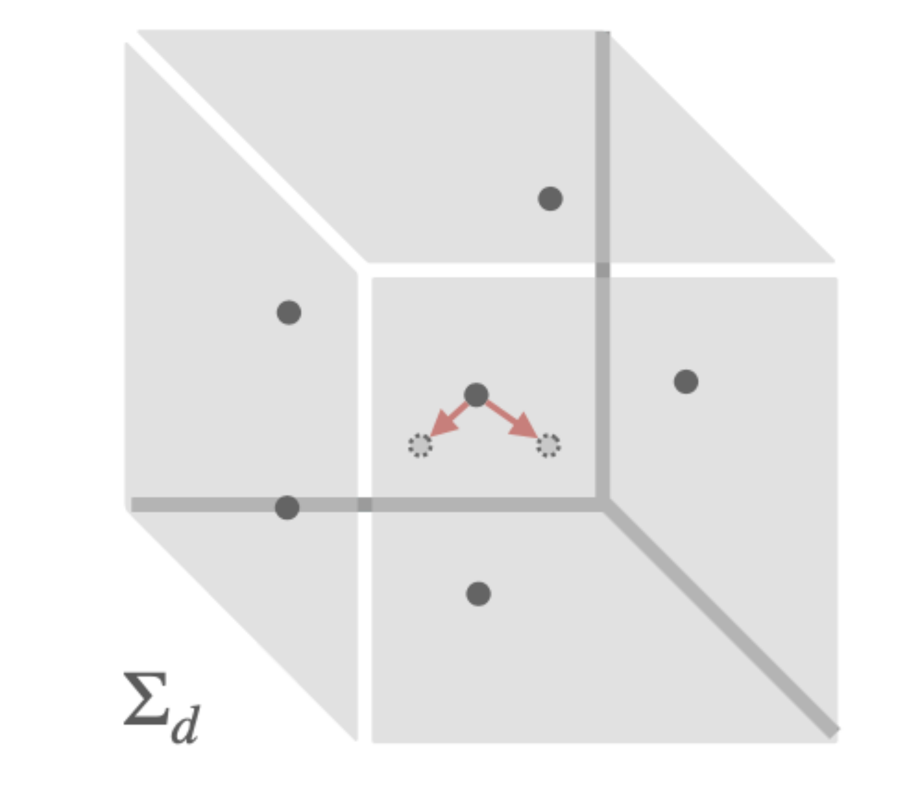
$$\partial_\mu J^{\mu\nu} = 0$$



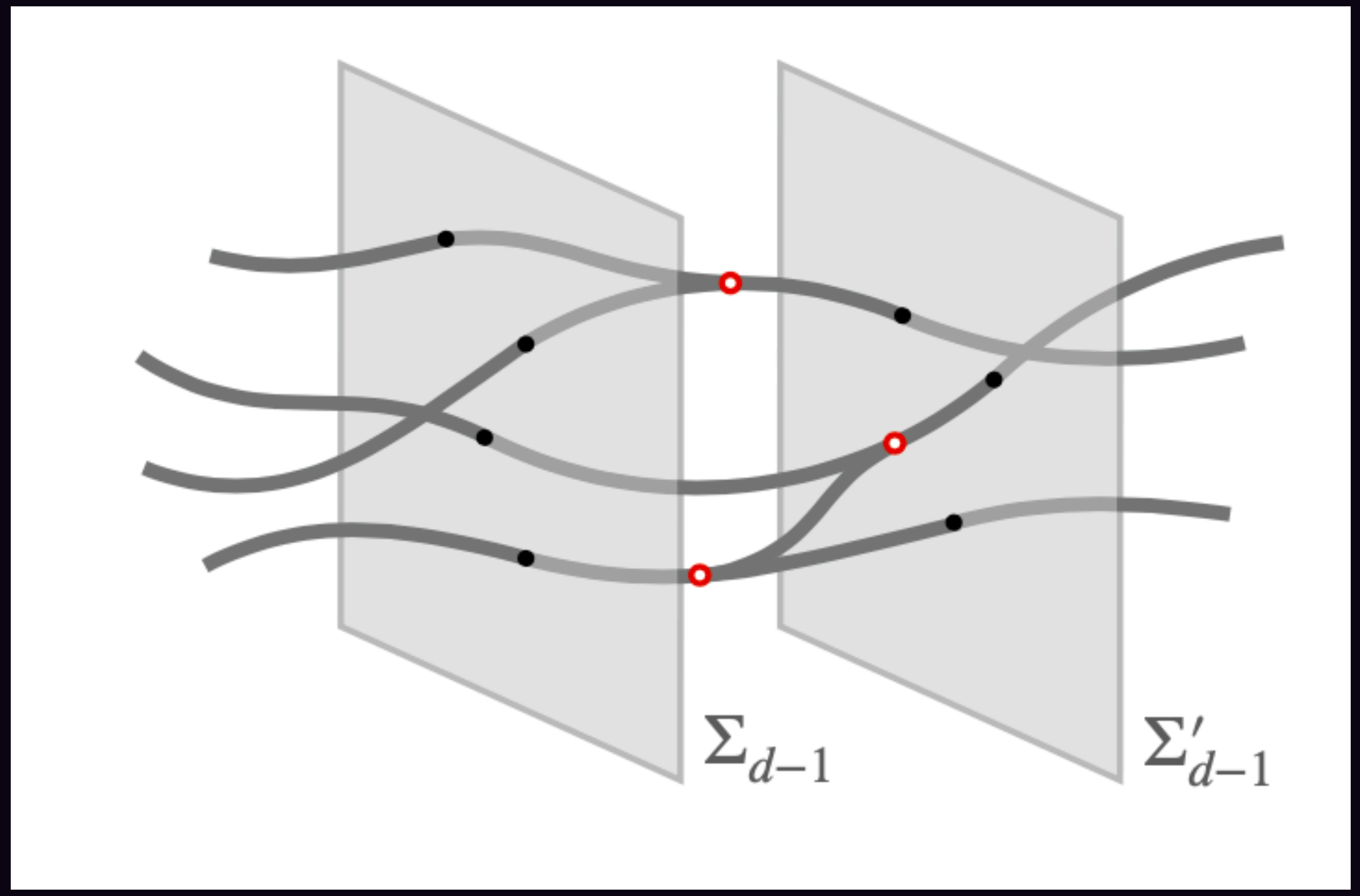
(higher-form symmetry)

APPROXIMATE SYMMETRY

$$\partial_t J^t + \partial_i J^i = -\ell L$$



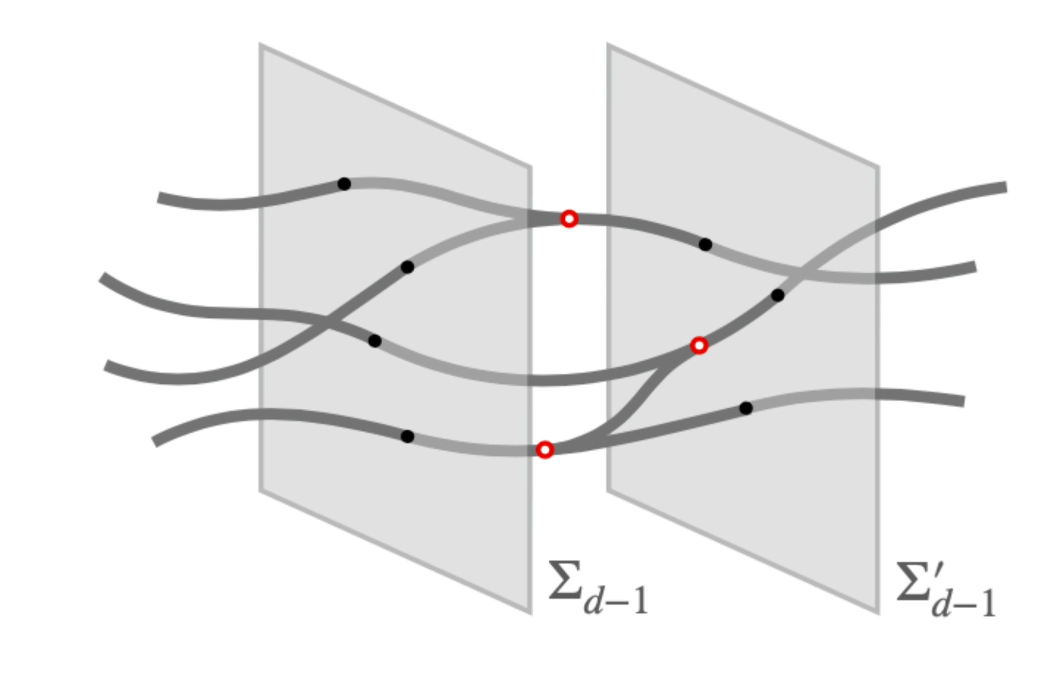
$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$



APPROXIMATE SYMMETRY

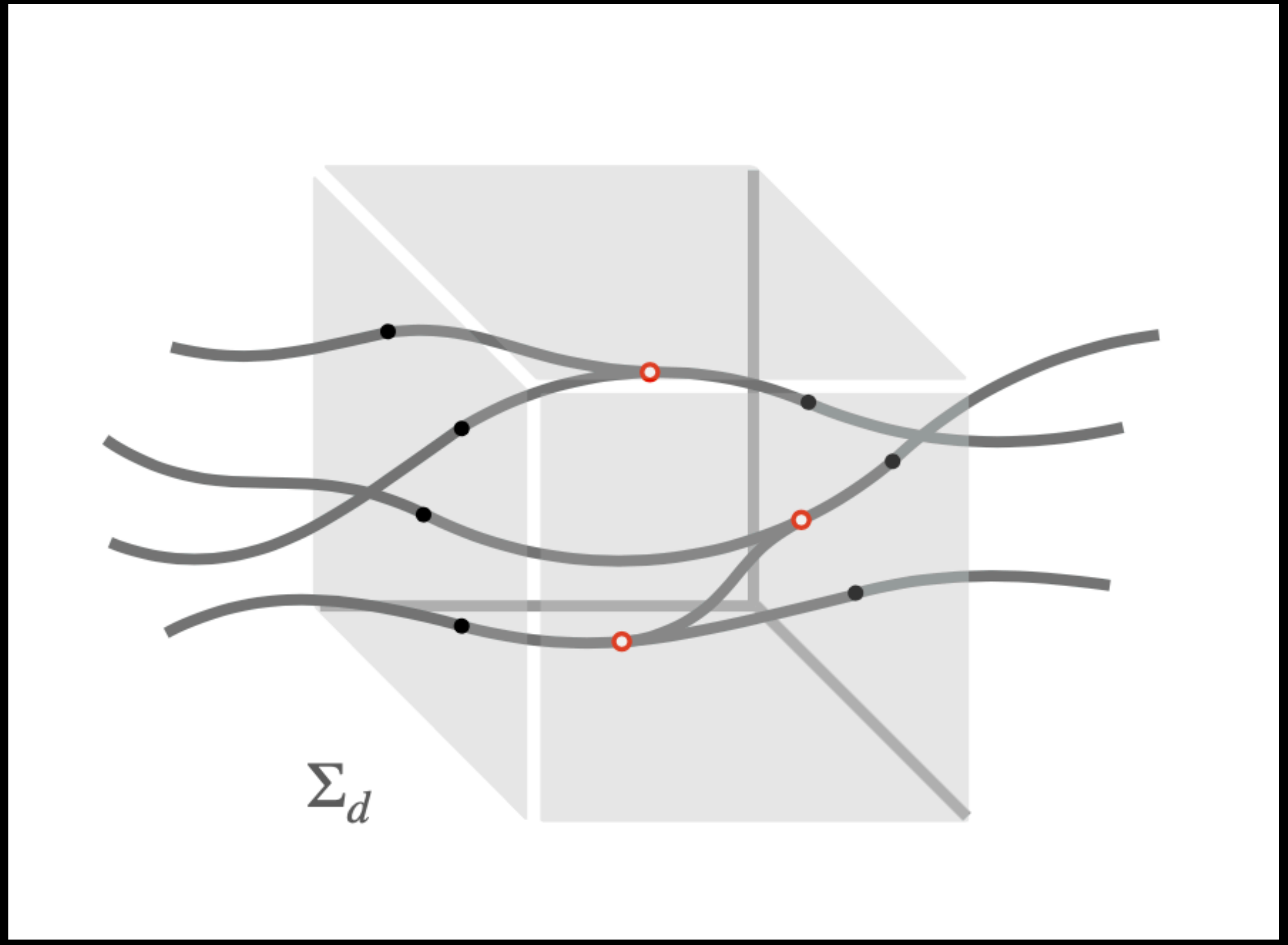
(higher-form symmetry)

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$



(topological symmetry)

$$\partial_\mu L^\mu = 0$$

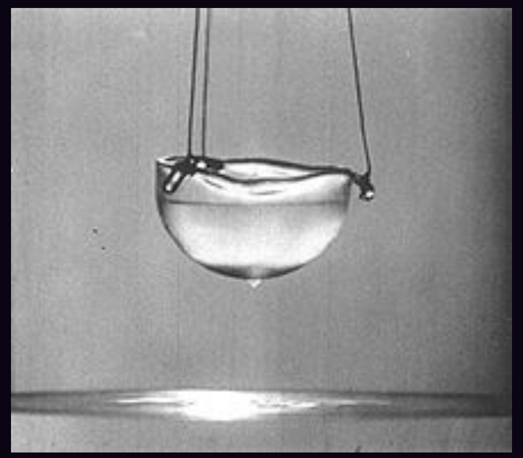


SYMMETRY BREAKING PATTERN	SYMMETRY
UNBROKEN	TIME TRANSLATION
SPONTANEOUSLY BROKEN	SPATIAL TRANSLATION
EXPLICITLY BROKEN	ROTATION
APPROXIMATE	U(1)
PSEUDO-SPONTANEOUS	HIGHER-FORM
ANOMALOUS	HIGHER-GROUP
TEMPORAL - SSB	DIPOLE
	BOOSTS

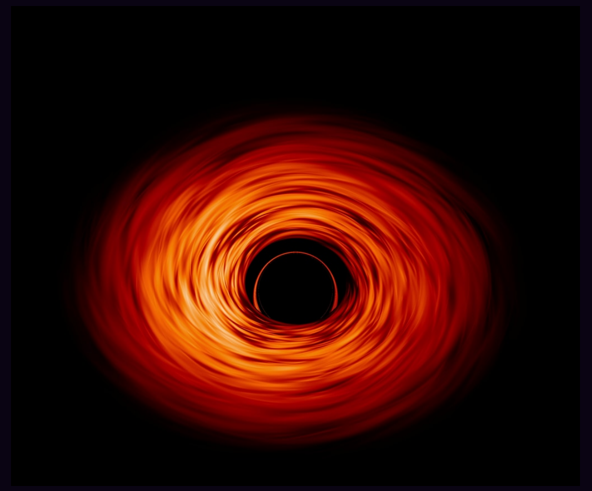
LANDAU PARADIGM: SYMMETRIES

ROTATION AND TRANSLATION INVARIANT

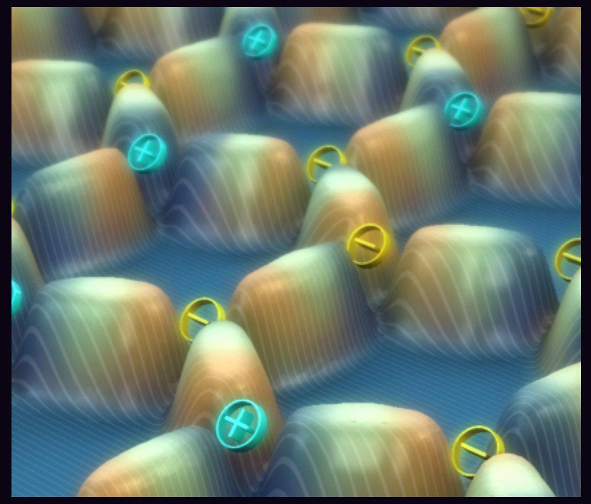
SPONTANEOUSLY BROKEN SPATIAL TRANSLATIONS



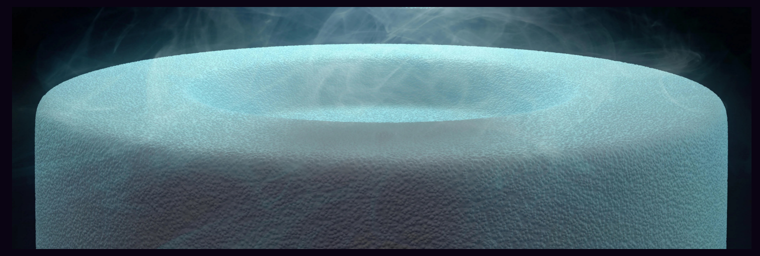
SPONTANEOUSLY BROKEN U(1) SYMMETRY



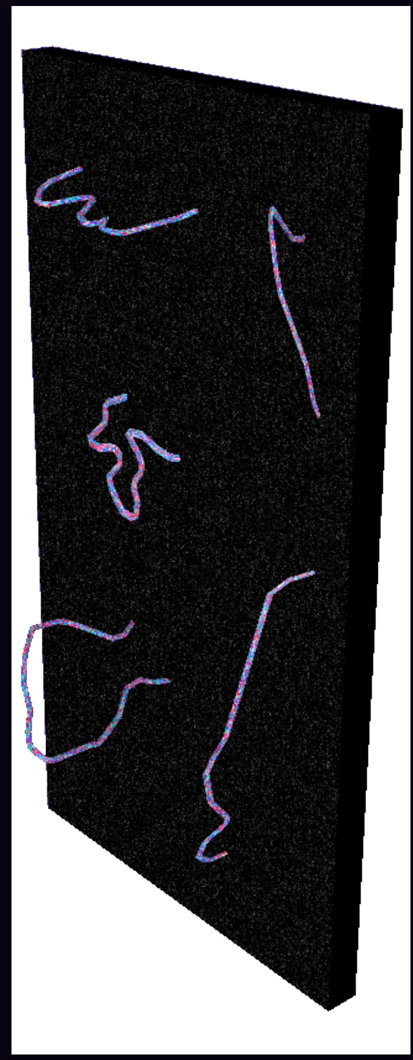
TEMPORAL SSB U(1) 1-FORM SYMMETRY



PSEUDO-SPONTANEOUS U(1) 1-FORM



PSEUDO-SPONTANEOUS U(1) 1-FORM + SSB EMERGENT U(1)



SSB HIGHER-GROUP

MOTIVATIONS

>> How do we consistently formulate hydrodynamics with (approximate) higher-form symmetries? [dissipative effects, gradient expansion, etc]

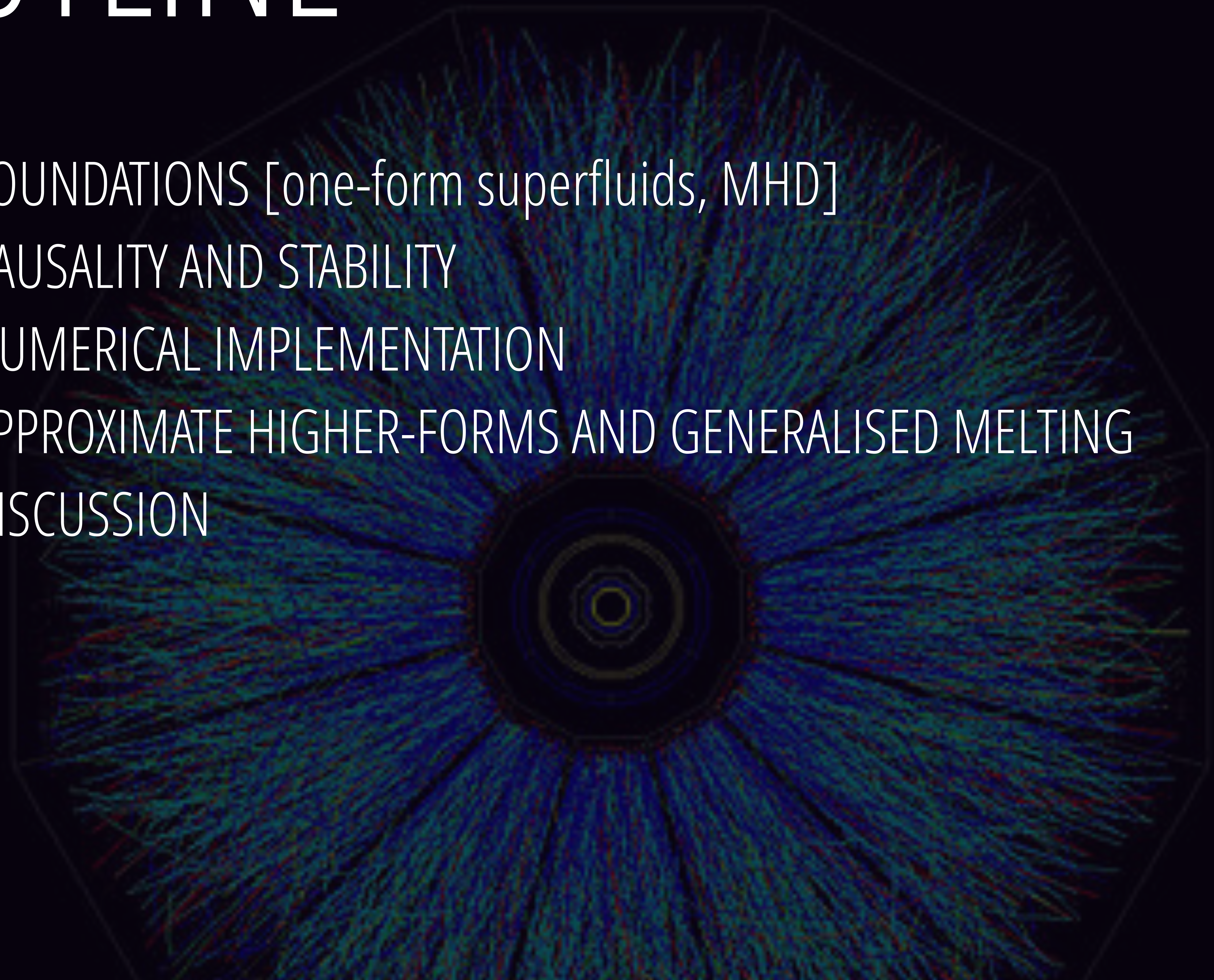
>> How do we describe equilibrium states in theories with higher-forms? [effective action, partition functions, etc]

>> Can stability be dealt in the same way? [MIS, BDNK or density frame completions, etc]

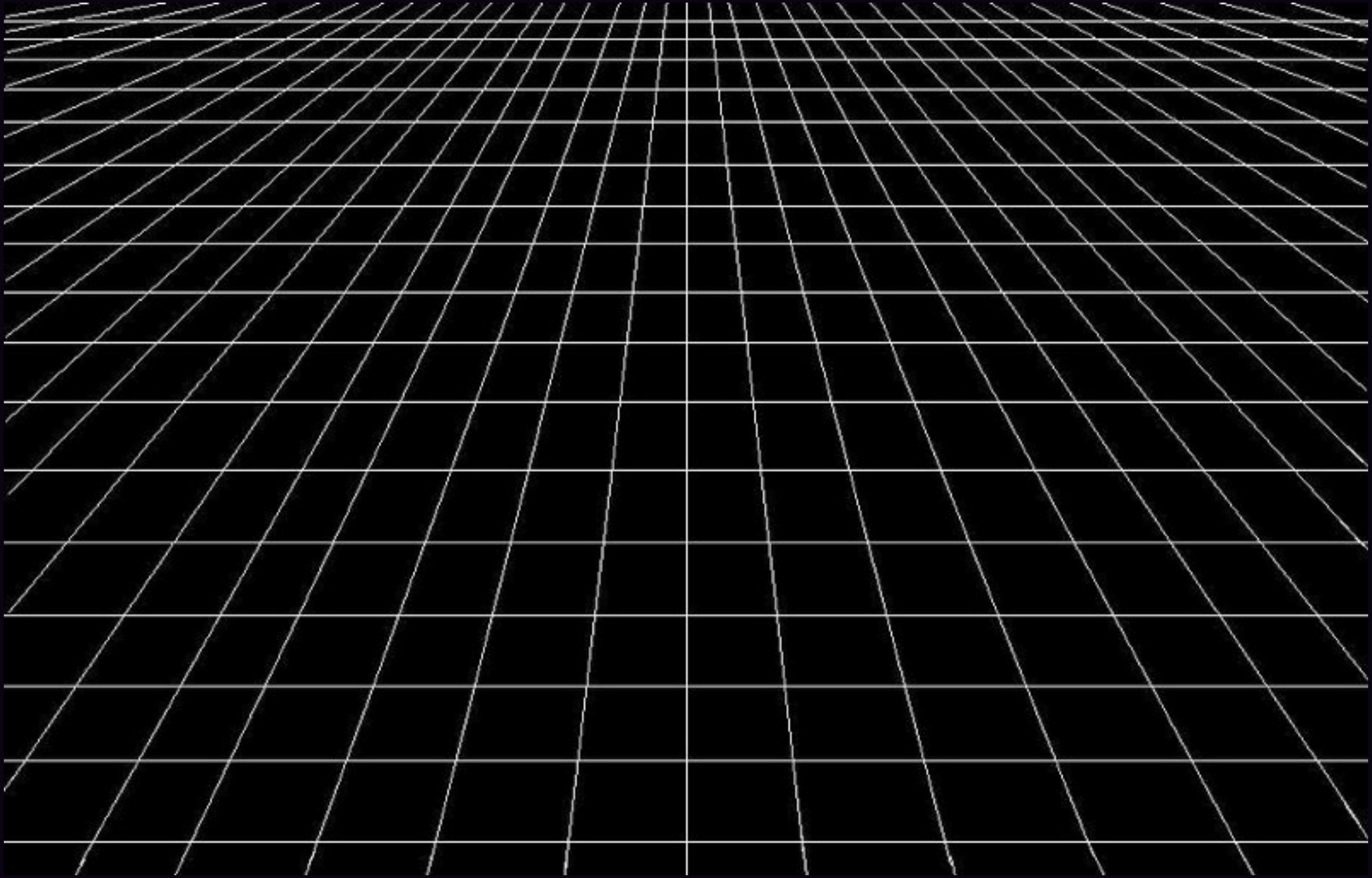
>> What phases and phase transitions are possible? [topological defects, impurities, etc]

OUTLINE

(1) FOUNDATIONS [one-form superfluids, MHD]
(2) CAUSALITY AND STABILITY
(3) NUMERICAL IMPLEMENTATION
(4) APPROXIMATE HIGHER-FORMS AND GENERALISED MELTING
(5) DISCUSSION



I- FOUNDATIONS



$g_{\mu\nu}$ In many cases $g_{\mu\nu} \sim \eta_{\mu\nu}$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu)$$

$$\nabla_\mu T^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0$$

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu)$$

Entropy current:

$$S^\mu = s u^\mu \quad , \quad \epsilon + P = Ts$$

What are the equilibrium solutions?

$$\mathcal{L}_K g_{\mu\nu} = 0$$

The fluid fields are determined:

$$u^\mu = \frac{K^\mu}{|K|} \qquad T = \frac{T_0}{|K|}$$

Free energy functional (purely geometric)

$$F[g_{\mu\nu}] = -\int d^4x\sqrt{-g}P(T)$$

$T = \frac{T_0}{|K|}$

Variation gives

$$\delta F[g_{\mu\nu}] = \frac{1}{2}\int d^4x\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu}$$

Out of equilibrium:

$$T^{\mu\nu} = \epsilon(T)u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu) + T^{\mu\nu}_{(1)} + \mathcal{O}\left(\partial^2\right)$$

Temperature and velocity can be redefined:

$$T \to T + \delta T$$

$$u^\mu \to u^\mu + \delta u^\mu$$

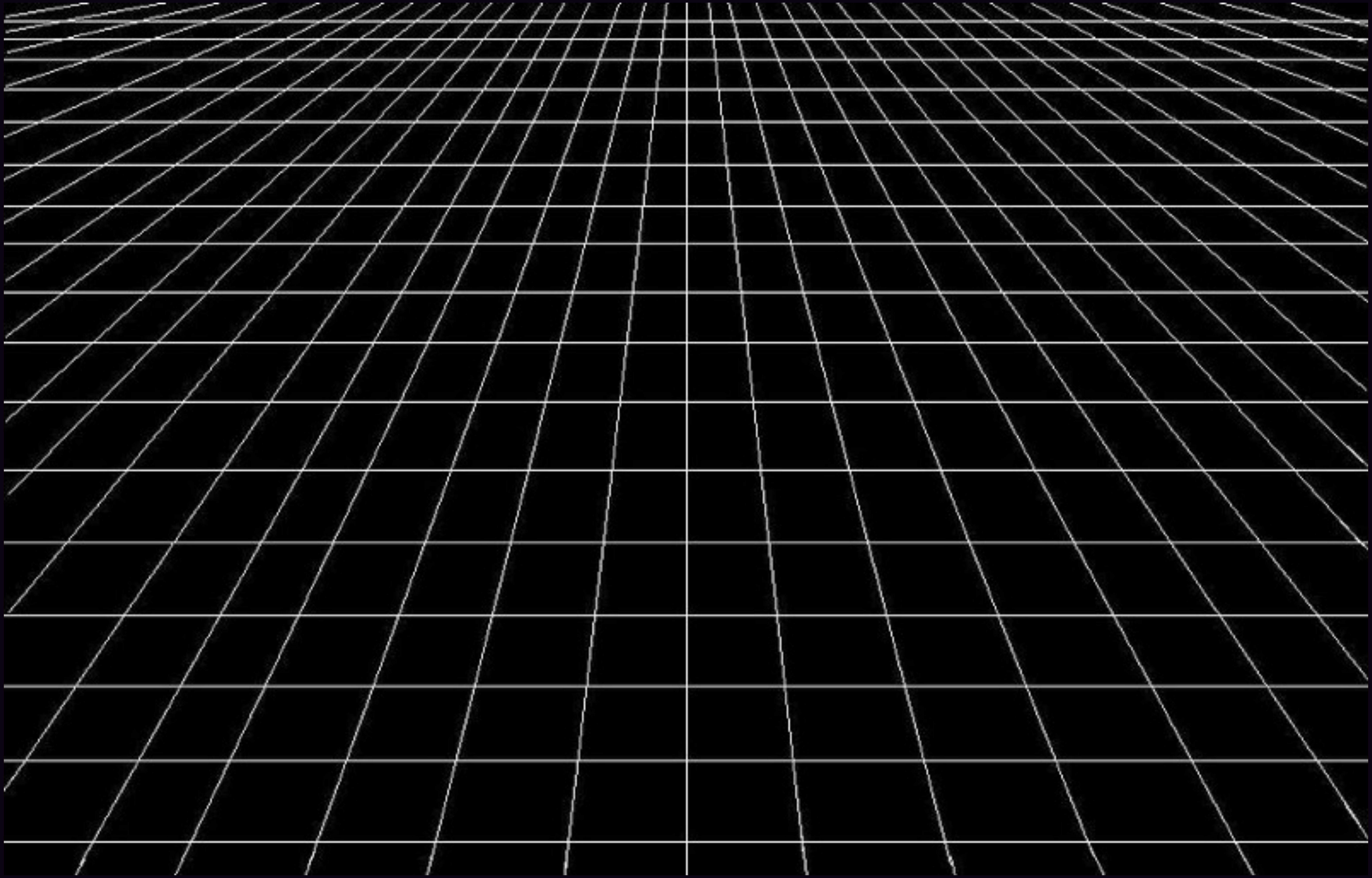
Landau frame:

$$T^{\mu\nu}_{(1)} u_\nu = 0$$

+ second law fixes the form to be:

$$\boxed{T^{\mu\nu}_{(1)} = -\zeta \nabla_\lambda u^\lambda P^{\mu\nu} - \eta \sigma^{\mu\nu}}$$

$P^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$



$g_{\mu\nu}$ A_μ

Charged fluids:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$
$$\nabla_\mu J^\mu = 0$$
$$\nabla_\mu S^\mu \geq 0$$

Global U(1) symmetry

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T,\mu)u^\mu u^\nu + P(T,\mu)(g^{\mu\nu} + u^\mu u^\nu)$$

Ideal current:

$$J^\mu = q(T,\mu)u^\mu$$

Stress tensor correction

$$T^{\mu\nu}_{(1)} = -\zeta \nabla_\lambda u^\lambda P^{\mu\nu} - \eta\sigma^{\mu\nu}$$

Current correction

$$J^\mu_{(1)} = -\sigma P^\mu{}_\nu \left(\partial^\nu \frac{\mu}{T} - E^\nu\right) \qquad \boxed{E^\nu = F^{\mu\nu}u_\mu}$$

Superfluids:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

$$\nabla_\mu J^\mu = 0$$

$$\nabla_\mu S^\mu \geq 0$$

Spontaneously broken U(1) symmetry

There is a new dynamical field:

$$\boxed{\xi_\mu = \partial_\mu \phi + A_\mu}$$

$u^\mu \xi_\mu = \mu$

Stress tensor:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + f \xi^\mu \xi^\nu$$

Current:

$$J^\mu = q u^\mu - f \xi^\mu$$

Thermodynamic quantities are now functions of

$$P(T, \mu, \chi)$$

with $\chi = -\xi^\mu \xi_\mu$

The free energy functional is:

$$F[g_{\mu\nu}, A_\mu; \phi] = -\int d^4x\sqrt{-g}P(T, \mu, \chi)$$

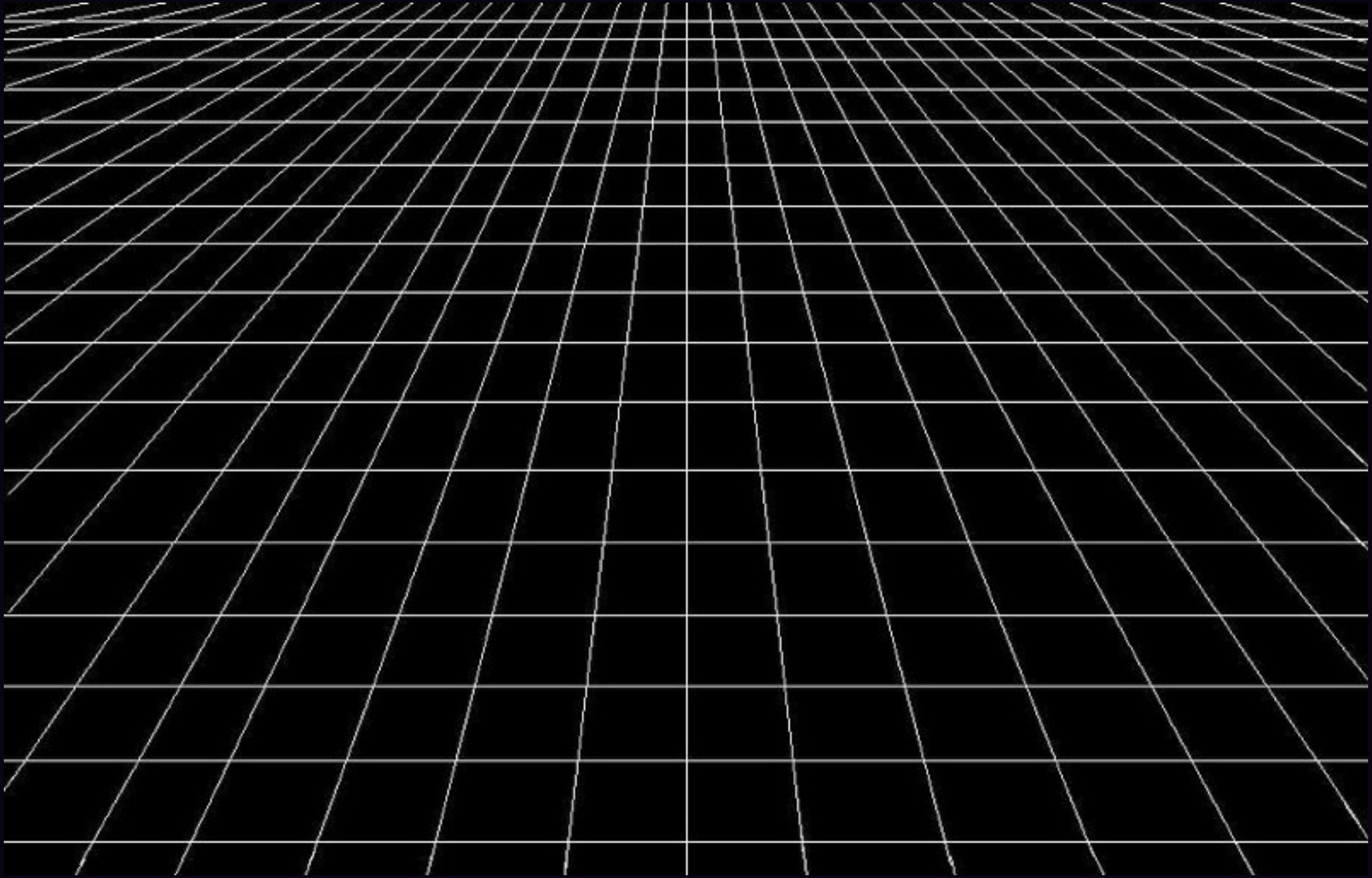
$$T = \frac{T_0}{|K|} \qquad \mu = u^\mu A_\mu$$

Equilibrium states are degenerate

$$\delta_\phi F \;\Rightarrow \boxed{\nabla_\mu \left(f\xi^\mu\right) = 0}$$

WHAT IS MHD?

THE GAUGE FIELD IS NO LONGER A BACKGROUND FIELD BUT A DYNAMICAL FIELD



$g_{\mu\nu}$ J^{μ}_{ext}

MHD EQUATIONS:

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu$$

$$J^\mu + J^\mu_{\text{ext}} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

$$J^\nu = \nabla_\mu F^{\mu\nu} + J^\nu_{\text{matter}}$$

Decompose the field strength as:

$$F_{\mu\nu} = 2u_{[\mu} E_{\nu]} - \epsilon_{\mu\nu\lambda\rho} u^\lambda B^\rho$$

$$E^\mu = F^{\mu\nu} u_\nu \qquad B^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$$

arXiv:1703.08757, Relativistic MHD by J. Hernandez & P. Kovtun

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

- Minimal coupling between fluid and fields

$$T^{\mu\nu} = F^{\mu}{}_{\rho}F^{\nu\rho} - \frac{1}{4}F_{\rho\sigma}F^{\rho\sigma}g^{\mu\nu} + \epsilon(T,\mu)u^{\mu}u^{\nu} + p(T,\mu)\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right)$$

$$J^{\mu} = \nabla_{\nu}F^{\nu\mu} + q(T,\mu)\,u^{\mu} - \sigma(T,\mu)P^{\mu\nu}\left(T\partial_{\nu}\frac{\mu}{T} - E_{\nu}\right) \quad ,$$

Decomposing Maxwell's equations

$$q(T,\mu) = u_{\mu} \text{[illegible]} F^{\nu\mu} + u_{\mu}J^{\mu}_{\text{ext}}$$

$$J^{\mu} + J^{\mu}_{\text{ext}} = 0$$

$$\sigma(T,\mu)E^{\mu} = -P^{\mu} \text{[illegible]} F^{\nu\lambda} - P^{\mu}{}_{\lambda}J^{\lambda}_{\text{ext}} + T\sigma(\text{[illegible]})P^{\mu\nu}\partial_{\nu}\frac{\mu}{T}$$

- Electric fields are Debye screened and the plasma is electrically neutral

$$q(T,\mu) = E^\mu = \mathcal{O}(\partial) \qquad \text{instead of} \quad \sigma(T,\mu) \to \infty$$

Stress tensor becomes:

$$T^{\mu\nu} = (\epsilon(T,\mu) + p(T,\mu))\, u^\mu u^\nu + \left(p(T,\mu) - \frac{1}{2}B^2\right) g^{\mu\nu} + B^2 \mathbb{B}^{\mu\nu} + \mathcal{O}(\partial)$$

$$\mathbb{B}^{\mu\nu} = P^{\mu\nu} - \hat{B}^\mu \hat{B}^\nu, \; P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\hat{B}^\mu = B^\mu / |B|$$

- General equation of state and stress

$$P \equiv P(T, \mu, B^2)$$

$$\mathrm{d}P = s\mathrm{d}T + q\mathrm{d}\mu - \frac{\varpi}{2|B|}\mathrm{d}B^2 \quad , \quad \epsilon + P = sT + q\mu$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \varpi|B|\mathbb{B}^{\mu\nu}$$

$$J^\mu = qu^\mu$$

$$S^\mu = su^\mu$$

Minimal coupling:

$$P(T, \mu, B^2) = -B^2/2 + p(T, \mu)$$

Second law:

$$\nabla_\mu S^\mu \geq 0$$

- Equilibrium partition function

$$u^\mu = \frac{\mathbf{k}^\mu}{\mathbf{k}} \qquad T = \frac{T_0}{\mathbf{k}} \qquad \mu = u^\mu A_\mu$$

$$\mathcal{L}_{\mathbf{k}} g_{\mu\nu} = 0 \qquad \mathcal{L}_{\mathbf{k}} A_\mu = 0$$

$$S = \int_{B_3} P(T, \mu, B^2) + A_\mu J^\mu_{\text{ext}}$$

Currents:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta A_\mu}$$

- Higher-orders (hydrostatic)

$$S = \int_{B_3} P(T, \mu, B^2) + A_\mu J^\mu_{\text{ext}}$$

$$+ M_1 B^\mu \partial_\mu \frac{B^2}{T^4} + M_2 \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \partial_\rho B_\sigma$$

$$- \frac{M_3}{T} B^\mu \partial_\mu T - M_4 \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \partial_\rho u_\sigma + T M_5 B^\mu \partial_\mu \frac{\mu}{T}$$

arXiv:1703.08757, Relativistic MHD by J. Hernandez & P. Kovtun

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

- Higher-orders (non-hydrostatic)

$$T^{\mu\nu}_{\rm nhs} = \delta\mathcal{F}\mathbb{B}^{\mu\nu} + \delta\mathcal{T}\hat{B}^{\mu}\hat{B}^{\nu} + 2\mathcal{L}^{(\mu}\hat{B}^{\nu)} + \mathcal{T}^{\mu\nu}$$
$$J^{\mu}_{\rm nhs} = \delta\mathcal{S}\hat{B}^{\mu} + \mathcal{M}^{\mu} \quad ,$$

$$\begin{pmatrix}\delta\mathcal{F}\\ \delta\mathcal{T}\\ \delta\mathcal{S}\end{pmatrix} = -T\begin{pmatrix}\zeta_{11} & \zeta_{12} & \tilde{\chi}_1\\ \zeta'_{12} & \zeta_{22} & \tilde{\chi}_2\\ \tilde{\chi}'_1 & \tilde{\chi}'_2 & \sigma_{\parallel}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\mathbb{B}^{\mu\nu}\delta_{\mathscr{B}}g_{\mu\nu}\\ \frac{1}{2}\hat{B}^{\mu}\hat{B}^{\nu}\delta_{\mathscr{B}}g_{\mu\nu}\\ \hat{B}^{\mu}\delta_{\mathscr{B}}A_{\mu}\end{pmatrix} \quad ,$$

$$\begin{pmatrix}\mathcal{L}^{\mu}\\ \mathcal{M}^{\mu}\end{pmatrix} = -T\begin{pmatrix}\eta_{11} & \sigma_{\times} & \tilde{\eta}_{11} & \tilde{\sigma}_{\times}\\ \sigma'_{\times} & \sigma_{\perp} & \tilde{\sigma}'_{\times} & \tilde{\sigma}_{\perp}\end{pmatrix}\begin{pmatrix}\mathbb{B}^{\mu\sigma}\hat{B}^{\nu}\delta_{\mathscr{B}}g_{\sigma\nu}\\ \mathbb{B}^{\mu\sigma}\delta_{\mathscr{B}}A_{\sigma}\\ \epsilon^{\mu\alpha\beta\sigma}u_{\alpha}\hat{B}_{\beta}\hat{B}^{\nu}\delta_{\mathscr{B}}g_{\sigma\nu}\\ \epsilon^{\mu\alpha\beta\sigma}u_{\alpha}\hat{B}_{\beta}\delta_{\mathscr{B}}A_{\sigma}\end{pmatrix} \quad ,$$

$$\mathcal{T}^{\mu\nu} = -\eta_{22}T\mathbb{B}^{\rho\langle\mu}\mathbb{B}^{\nu\rangle\sigma}\delta_{\mathscr{B}}g_{\rho\sigma} + \tilde{\eta}_{22}T\epsilon^{\rho\alpha\beta\langle\mu}u_{\alpha}\hat{B}_{\beta}\mathbb{B}^{\nu\rangle\sigma}\delta_{\mathscr{B}}g_{\rho\sigma} \quad .$$

$$\delta_{\mathscr{B}}g_{\mu\nu} = 2\nabla_{(\mu}\left(\frac{u_{\nu)}}{T}\right) \quad , \quad \delta_{\mathscr{B}}A_{\mu} = \partial_{\mu}\frac{\mu}{T} - \frac{1}{T}E_{\mu}$$

- Total transport coefficients:

  >> 5 hydrostatic coefficients

  >> 19 non-hydrostatic coefficients

  >> Onsager reduces to 14

- Need to solve:

$$J^\mu + J^\mu_{\text{ext}} = 0$$

$$\mu = \mu_0(T, B^2) + \frac{1}{\partial q/\partial\mu}\Bigg[u_\mu J^\mu_{\text{ext}} - B^\lambda \partial_\lambda \frac{B^2}{T^4}\frac{\partial M_1}{\partial\mu} - \epsilon^{\lambda\nu\rho\sigma} u_\lambda B_\nu \partial_\rho B_\sigma \frac{\partial M_2}{\partial\mu} + \frac{1}{T} B^\lambda \partial_\lambda T \frac{\partial M_3}{\partial\mu} - \left(\frac{\partial M_4}{\partial\mu} + \frac{\varpi}{|B|}\right)\epsilon^{\lambda\nu\rho\sigma} B_\lambda u_\nu \partial_\rho u_\sigma - T B^\lambda \partial_\lambda \frac{\mu}{T}\frac{\partial M_5}{\partial\mu} + \frac{1}{T}\nabla_\lambda\left(T M_5 B^\lambda\right)\Bigg]_{\mu=\mu_0} + \mathcal{O}(\partial^2) \;,$$

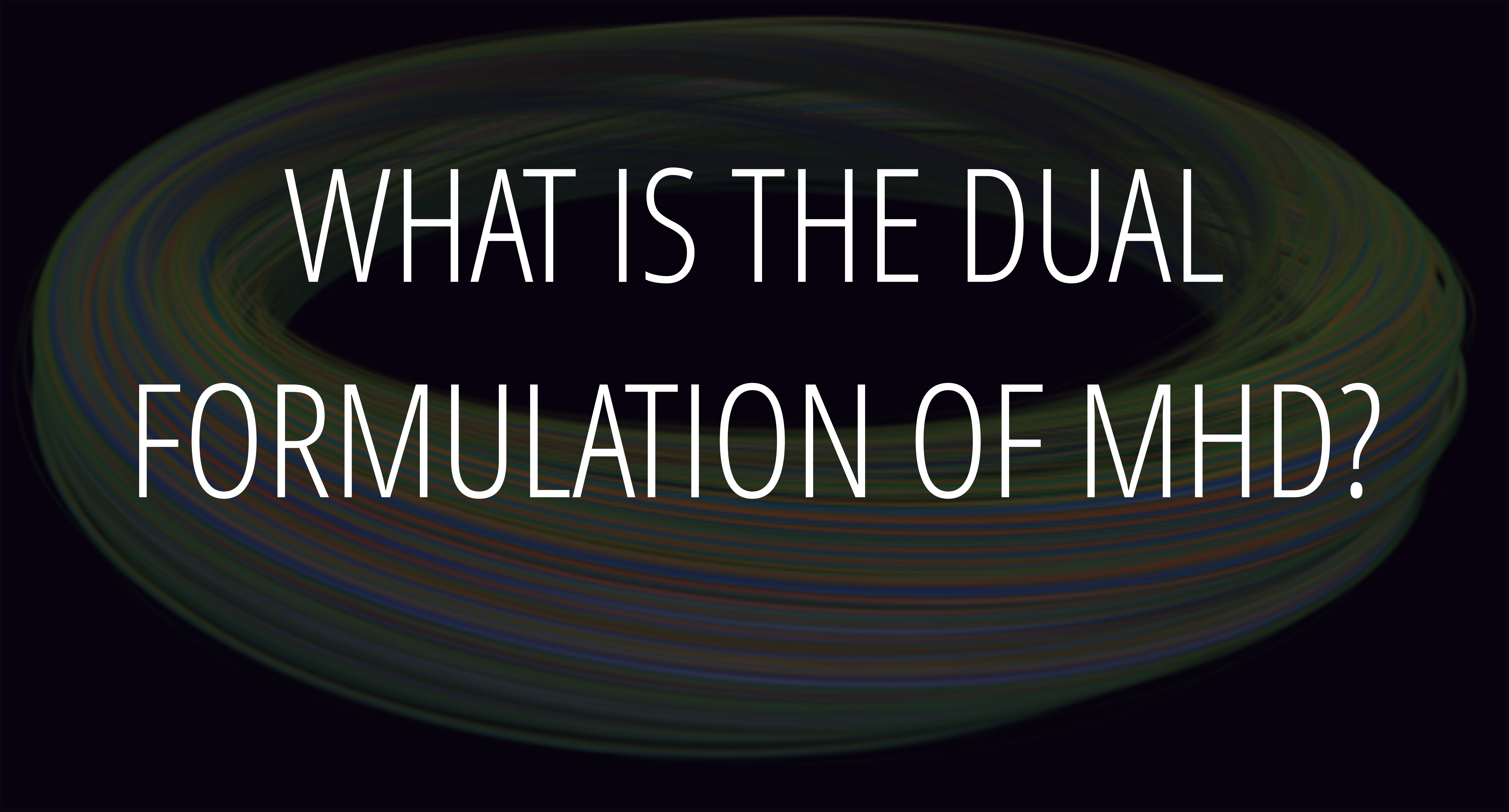
$$E^\mu = T P^{\mu\nu}\partial_\nu \frac{\mu}{T} - \frac{T}{2\sigma_\parallel}\hat{B}^\mu \mathbb{E}^{\rho\sigma} X_{\rho\sigma} + \frac{T}{\sigma_\parallel}\hat{B}^\mu\left(\tilde{\chi}'_1 \mathbb{B}^{\rho\sigma} + \tilde{\chi}'_2 \hat{B}^\rho \hat{B}^\sigma\right)\frac{1}{2}\delta_{\mathcal{B}} g_{\rho\sigma} - T\left(\frac{\epsilon + P}{\epsilon + P + \varpi|B|}\right)\left(\frac{\sigma_\perp}{\sigma_\perp^2 + \tilde{\sigma}_\perp^2}\mathbb{E}^{\mu\rho}\hat{B}^\sigma + \frac{\tilde{\sigma}_\perp}{\sigma_\perp^2 + \tilde{\sigma}_\perp^2}\mathbb{B}^{\mu\rho}\hat{B}^\sigma\right) X_{\rho\sigma} - 2T\left(\frac{\tilde{\sigma}_\perp \sigma'_\times - \sigma_\perp \tilde{\sigma}'_\times}{\sigma_\perp^2 + \tilde{\sigma}_\perp^2}\mathbb{E}^{\mu(\rho}\hat{B}^{\sigma)} - \frac{\tilde{\sigma}_\perp \tilde{\sigma}'_\times + \sigma_\perp \sigma'_\times}{\sigma_\perp^2 + \tilde{\sigma}_\perp^2}\mathbb{B}^{\mu(\rho}\hat{B}^{\sigma)}\right)\frac{1}{2}\delta_{\mathcal{B}} g_{\rho\sigma} + \mathcal{O}(\partial^2)$$

$$X_{\mu\nu} = 2\partial_{[\mu}\left(\frac{\varpi \hat{B}_{\nu]}}{T}\right) + \frac{u^\lambda}{T}\epsilon_{\lambda\mu\nu\rho} J^\rho_{\text{ext}}$$

$$\mathbb{E}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} u_\rho \hat{B}_\sigma$$

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

WHAT IS THE DUAL FORMULATION OF MHD?



HISTORICAL RECORD

>> Higher-dimensional Rotating Charged Black Holes (2011)

M. Caldarelli, R. Emparan & B. Pol

>> Blackfolds in Supergravity and String Theory (2011)

R. Emparan, T. Harmark, V. Niarchos & N. Obers

>> Thermal Giant Gravitons (2012)

JA, T. Harmark, N. Obers, M. Orselli & A. Pedersen

HISTORICAL RECORD

>> Dissipative string fluids (2014)
D. Schubring

>> Generalised global symmetries and dissipative MHD (2016)
S. Grozdanov, D. M. Hofman & N. Iqbal

>> Relativistic magnetohydrodynamics (2017)
J. Hernandez & P. Kovtun

>> Dissipative hydrodynamics with higher-form symmetry (2018)
JA, J. Gath, A. V. Pedersen & A. Jain

>> Magnetohydrodynamics as superfluidity (2018)
JA & A. Jain

>> One-form superfluids and magnetohydrodynamics (2018)
JA & A. Jain

>> EFT of MHD from generalised global symmetries (2018)
P. Glorioso & D.T. Son

>> A causal and stable model of MHD (2022)
JA & F. Camilloni

HISTORICAL RECORD

>> Strong-field magnetohydrodynamics for neutron stars (2022)

S. Vardhan, S. Grozdanov, S. Leutheusser, H. Liu

>> Approximate higher-form symmetries (2023)

JA & A. Jain

>> Effective field theories of dissipative fluids with one-form symmetries (2024)

S. Vardhan, S. Grozdanov, S. Leutheusser & H. Liu

>> Causality in dissipative relativistic magnetohydrodynamics (2024)

R. Hoult & P. Kovtun

>> RMHD without Amperè’s law

R. Lier, A. Jain, JA, O. Porth

ELECTROMAGNETISM

- Maxwell's equations

$$\nabla_\mu F^{\mu\nu} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

$$d \star F = 0 \qquad dF = 0$$

"Dual Maxwell equations"

$$J^{\mu\nu} = \star F^{\mu\nu}$$

$$\nabla_{[\mu} J_{\nu\rho]} = 0 \qquad \nabla_\mu J^{\mu\nu} = 0$$

Hence a 2-form dipole magnetic charge:

$$Q = \int_{M_2} \star J$$

Global U(1) 1-form magnetic symmetry

MHD DUAL

- MHD Equations of motion

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu$$

$$J^\mu + J^\mu_{\text{ext}} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

Given that Maxwell equations can be solved one can formulate the theory simply as:

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} H^{\nu\lambda\rho} J_{\lambda\rho} \qquad \nabla_\mu J^{\mu\nu} = 0$$

$$\boxed{\nabla_\mu J^\mu_{ext} = 0} \longrightarrow J^\mu_{\text{ext}} = \frac{1}{6} \epsilon^{\mu\nu\lambda\rho} H_{\nu\lambda\rho}$$

$$J^{\mu\nu} = \star F^{\mu\nu} \qquad H_{\nu\rho\sigma} = 3\partial_{[\nu} b_{\rho\sigma]}$$

- Breaking the one-form symmetry

1-form Goldstone	0-form Goldstone
$\delta_\chi \varphi_\mu = \pounds_\chi \varphi_\mu - \Lambda^\chi_\mu$	$\delta_\chi \phi = \pounds_\chi \phi - \Lambda^\chi$
2-form superfluid velocity	1-form superfluid velocity
$\xi_{\mu\nu} = 2\partial_{[\mu}\varphi_{\nu]} + b_{\mu\nu}$	$\xi_\mu = \partial_\mu \phi + A_\mu$
1-form Josephson condition	0-form Josephson condition
$\mu^\varphi_\mu = u^\nu \xi_{\nu\mu}$	$\mu = u^\mu \xi_\mu$

$$\frac{\mu_\mu}{T} = \Lambda^\beta_\mu + \beta^\nu b_{\nu\mu} \qquad \mu^\varphi_\mu = \mu_\mu - T\partial_\mu(\beta^\nu \varphi_\nu)$$

FULLY SSB PHASE

$$\xi_{\mu\nu} = 2u_{[\mu}\zeta_{\nu]} - \epsilon_{\mu\nu\rho\sigma}u^{\rho}\bar{\zeta}^{\sigma}$$

$$\zeta_{\mu} = \xi_{\mu\nu}u^{\nu} \quad \bar{\zeta}_{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\xi_{\rho\sigma}$$

$$P \equiv P(T, \zeta^2, \zeta.\bar{\zeta}, \bar{\zeta}^2)$$

(INSULATING PHASE, 166 coeff, 11 electric limit)

TEMPORAL SSB

$$\delta_{\chi}\varphi = \pounds_{\chi}\varphi - \beta^{\mu}\Lambda^{\chi}_{\mu}$$

$$\varpi h_{\mu} = \mu_{\mu} - T\partial_{\mu}\varphi$$

$$\zeta_{\mu} = u^{\nu}\xi_{\nu\mu} = \varpi h_{\mu}$$

$$P \equiv P(T, \zeta^2)$$

MHD

See Fracton story: Glorioso, Lucas, Jensen, Jay +++++

PLASMA PHASE TRANSITION

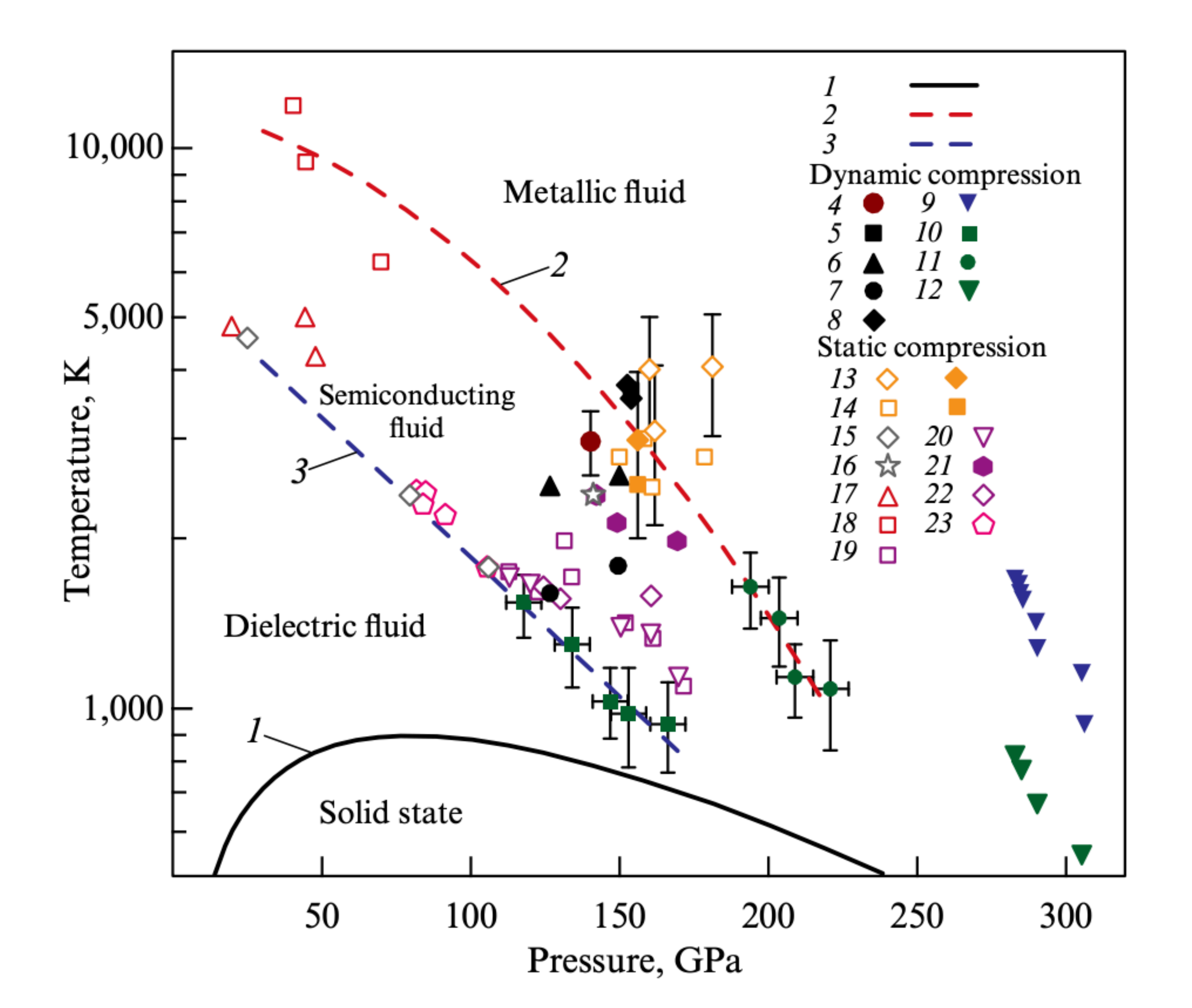


Figure 1. (Color online.) Experimental phase diagram of hydrogen/deuterium fluid at high pressures. *1* — hydrogen melt line [67]. Filled

MHD DUAL

- Can define partition function:

$$F[g_{\mu\nu}, b_{\mu\nu}; \varphi_\mu] = -\int d^4x\sqrt{-g}P(T,\zeta^2)$$

which gives the stress tensor and current:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + pg^{\mu\nu} - \varpi\rho h^\mu h^\nu + \mathcal{O}(\partial)$$
$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]} + \mathcal{O}(\partial)$$

and thermodynamic relations upon identification: $\zeta_\mu = -\varpi h_\mu$

$$dp = s dT + \rho d\varpi \qquad \epsilon + P = Ts + \rho\varpi$$

MHD DUAL

- Variation with respect to $\varphi \equiv \beta^\mu \varphi_\mu$

$$\frac{\delta S}{\delta \varphi} \Rightarrow \nabla_\mu (T \rho h^\mu) = 0$$

$$\partial_i J^{ti} = 0$$

- At ideal order the identification is:

$$B^\mu = \rho(T, \varpi) h^\mu + \mathcal{O}(\partial)$$

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

- Higher-order partition function

$$\mathcal{N} = p - \frac{\alpha}{6}\epsilon^{\mu\nu\rho\sigma}u_\mu H_{\nu\rho\sigma} - \beta\epsilon^{\mu\nu\rho\sigma}u_\mu h_\nu \partial_\rho u_\sigma$$

$$- \tilde{\beta}_1 h^\mu \partial_\mu T - \tilde{\beta}_2 h^\mu \partial_\mu \frac{\varpi}{T} - \tilde{\beta}_3 \epsilon^{\mu\nu\rho\sigma} u_\mu h_\nu \partial_\rho h_\sigma$$

- Non-hydrostatic corrections

$$T^{\mu\nu}_{\text{nhs}} = \delta\epsilon\, u^\mu u^\nu + \delta f \Delta^{\mu\nu} + \delta\tau\, h^\mu h^\nu + 2\ell^{(\mu}h^{\nu)} + 2k^{(\mu}u^{\nu)} + t^{\mu\nu} \quad ,$$

$$J^{\mu\nu}_{\text{nhs}} = 2\delta\rho\, u^{[\mu}h^{\nu]} + 2m^{[\mu}h^{\nu]} + 2n^{[\mu}u^{\nu]} + \delta s\, \epsilon^{\mu\nu} \quad ,$$

- Total transport coefficients:
  >> 5 hydrostatic coefficients, 19 non-hydrostatic coefficients

$$\begin{pmatrix} \delta f \\ \delta\tau \\ \delta s \end{pmatrix} = -\frac{T}{2} \begin{pmatrix} \zeta_\perp & \zeta_\times & \tilde\kappa_1 \\ \zeta'_\times & \zeta_\| & \tilde\kappa_2 \\ \tilde\kappa'_1 & \tilde\kappa'_2 & r_\| \end{pmatrix} \begin{pmatrix} \Delta^{\mu\nu}\delta_{\mathscr{B}} g_{\mu\nu} \\ h^\mu h^\nu \delta_{\mathscr{B}} g_{\mu\nu} \\ \epsilon^{\mu\nu}\delta_{\mathscr{B}}\xi_{\mu\nu} \end{pmatrix} ,$$

$$\begin{pmatrix} \ell^\mu \\ m^\mu \end{pmatrix} = -T \begin{pmatrix} \eta_\| & r_\times & \tilde\eta_\| & \tilde r_\times \\ r'_\times & r_\perp & \tilde r'_\times & \tilde r_\perp \end{pmatrix} \begin{pmatrix} \Delta^{\mu\sigma} h^\nu \delta_{\mathscr{B}} g_{\sigma\nu} \\ \Delta^{\mu\sigma} h^\nu \delta_{\mathscr{B}} \xi_{\sigma\nu} \\ \epsilon^{\mu\sigma} h^\nu \delta_{\mathscr{B}} g_{\sigma\nu} \\ \epsilon^{\mu\sigma} h^\nu \delta_{\mathscr{B}} \xi_{\sigma\nu} \end{pmatrix}$$

$$t^{\mu\nu} = -\eta_\perp T \Delta^{\rho\langle\mu}\Delta^{\nu\rangle\sigma}\delta_{\mathscr{B}} g_{\rho\sigma} + \tilde\eta_\perp T \epsilon^{\rho\langle\mu}\Delta^{\nu\rangle\sigma}\delta_{\mathscr{B}} g_{\rho\sigma}$$

$$\delta_{\mathscr{B}} g_{\mu\nu} = 2\nabla_{(\mu}\left(\frac{u_{\nu)}}{T}\right) \quad , \quad \delta_{\mathscr{B}} b_{\mu\nu} = 2\partial_{[\mu}\left(\frac{\varpi h_{\nu]}}{T}\right) + \frac{u^\sigma}{T} H_{\sigma\mu\nu}$$

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

Map between formulations

$$\zeta_\perp = \zeta_{11} - \frac{\tilde\chi_1\tilde\chi'_1}{\sigma_\parallel}\,, \quad \zeta_\times = \zeta_{12} - \frac{\tilde\chi_1\tilde\chi'_2}{\sigma_\parallel}\,, \quad \zeta'_\times = \zeta'_{12} - \frac{\tilde\chi_2\tilde\chi'_1}{\sigma_\parallel}\,, \quad \zeta_\parallel = \zeta_{22} - \frac{\tilde\chi_2\tilde\chi'_2}{\sigma_\parallel}\,,$$

$$\tilde\kappa_1 = \frac{\tilde\chi_1}{\sigma_\parallel}\,, \quad \tilde\kappa_2 = \frac{\tilde\chi_2}{\sigma_\parallel}\,, \quad \tilde\kappa'_1 = -\frac{\tilde\chi'_1}{\sigma_\parallel}\,, \quad \tilde\kappa'_2 = -\frac{\tilde\chi'_2}{\sigma_\parallel}\,, \quad r_\parallel = \frac{1}{\sigma_\parallel}\,,$$

$$\eta_\parallel = \eta_{11} - \frac{\sigma_\perp(\sigma_\times\sigma'_\times - \tilde\sigma_\times\tilde\sigma'_\times) + \tilde\sigma_\perp(\sigma_\times\tilde\sigma'_\times + \tilde\sigma_\times\sigma'_\times)}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,, \quad r_\perp = \left(\frac{sT}{\epsilon+p}\right)^2 \frac{\sigma_\perp}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,,$$

$$\tilde\eta_\parallel = \tilde\eta_{11} - \frac{\sigma_\perp(\sigma_\times\tilde\sigma'_\times + \tilde\sigma_\times\sigma'_\times) - \tilde\sigma_\perp(\sigma_\times\sigma'_\times - \tilde\sigma_\times\tilde\sigma'_\times)}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,, \quad \tilde r_\perp = \left(\frac{sT}{\epsilon+p}\right)^2 \left(\frac{-\tilde\sigma_\perp}{\sigma_\perp^2 + \tilde\sigma_\perp^2} + \frac{2\alpha\rho}{sT}\right)$$

$$r_\times = \frac{sT}{\epsilon+p}\frac{-\sigma_\perp\tilde\sigma_\times + \tilde\sigma_\perp\sigma_\times}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,, \quad r'_\times = \frac{sT}{\epsilon+p}\frac{-\sigma_\perp\tilde\sigma'_\times + \tilde\sigma_\perp\sigma'_\times}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,,$$

$$\tilde r_\times = \frac{sT}{\epsilon+p}\frac{\sigma_\perp\sigma_\times + \tilde\sigma_\perp\tilde\sigma_\times}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,, \quad \tilde r'_\times = \frac{sT}{\epsilon+p}\frac{\sigma_\perp\sigma'_\times + \tilde\sigma_\perp\tilde\sigma'_\times}{\sigma_\perp^2 + \tilde\sigma_\perp^2}\,,$$

$$\eta_\perp = \eta_{22}, \qquad \tilde\eta_\perp = \tilde\eta_{22}\,.$$

arXiv:1811.04913, One-form superfluids and MHD by JA & A. Jain

II - CAUSALITY AND STABILITY

J. Armas & F. Camilloni, JCAP 2022

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Initial equilibrium state:

$$T = T_0 \qquad u^{\mu}_{(0)} = \frac{1}{\sqrt{1 - v_0^2}}(1, 0, 0, v_0)$$

Perturbations around equilibrium:

$$T = T_0 + \delta T$$
$$u^{\mu} = u^{\mu}_{(0)} + \delta u^{\mu}$$

Modes:

$$w(k) = i\frac{(\epsilon + P)\sqrt{1 - v_0^2}}{\eta v_0^2} + \mathcal{O}(k)$$

⇒ Unstable mode

Hiscock and Lindblom, 1985

Kovtun, 2019; Bemfica, Disconzi, Noronha, 2019

Causality is also violated:

$$1 > \lim_{k \to \infty} |Re(\frac{w(k)}{k})| > 0$$

HOW TO DEAL WITH THIS?

- MIS
- BDNK
- Density frame (Armas and Jain 2020) (see 2024 papers by Bhambure, Singh, Teaney et al)

$$\nabla_\mu T^{\mu\nu} = 0 \quad \nabla_\mu J^{\mu\nu} = 0$$

Equilibrium (ideal order) stress tensor and current: $p(T, \mu)$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - \mu \rho h^\mu h^\nu$$

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]}$$

Out of equilibrium:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \mu\rho h^\mu h^\nu \quad + T^{\mu\nu}_{(1)}$$

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]} \quad + J^{\mu\nu}_{(1)}$$

Temperature, velocity, chemical potential and director can be redefined:

$$T \to T + \delta T \qquad \mu \to \mu + \delta\mu$$

$$u^\mu \to u^\mu + \delta u^\mu \qquad h^\mu \to h^\mu + \delta h^\mu$$

General parametrisation:

$$T^{\mu\nu}_{(1)} = \delta\varepsilon u^\mu u^\nu + \delta f \Delta^{\mu\nu} + \delta\tau h^\mu h^\nu + 2\delta\chi h^{(\mu}u^{\nu)} + 2\ell^{(\mu}h^{\nu)} + 2k^{(\mu}u^{\nu)} + t^{\mu\nu}$$

$$J^{\mu\nu}_{(1)} = 2\delta\varrho u^{[\mu}h^{\nu]} + 2m^{[\mu}h^{\nu]} + 2n^{[\mu}u^{\nu]} + s^{\mu\nu}$$

Fixing frame gives 7 transport coefficients

General frame gives 28 transport coefficients!
(big applause)

Why you don’t want to fix the frame:

$$w = i\frac{\sqrt{1-v_0^2}}{v_0^2}\frac{\rho}{\mu r_{||}} + \mathcal{O}(k)$$

⇒ Instability!

Not fixing a frame gives

$$\delta\varepsilon = -\varepsilon_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \varepsilon_2 h^\mu h^\nu \nabla_\mu u_\nu - \varepsilon_3 u^\mu \nabla_\mu T - \varepsilon_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta f = -f_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - f_2 h^\mu h^\nu \nabla_\mu u_\nu - f_3 u^\mu \nabla_\mu T - f_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta\tau = -\tau_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \tau_2 h^\mu h^\nu \nabla_\mu u_\nu - \tau_3 u^\mu \nabla_\mu T - \tau_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta\chi = -T\chi_1 u^\mu h^\nu \delta_B g_{\mu\nu} - \chi_2 \nabla_\mu (T\rho h^\mu) \quad ,$$

$$\ell^\mu = -T\ell_1 \Delta^{\mu\sigma} h^\nu \delta_B g_{\nu\sigma} - T\ell_2 \Delta^{\mu\sigma} u^\nu \delta_B b_{\sigma\nu} \quad ,$$

$$k^\mu = -Tk_1 \Delta^{\mu\nu} h^\lambda \delta_B b_{\nu\lambda} - Tk_2 \Delta^{\mu\nu} u^\lambda \delta_B g_{\nu\lambda}$$

$$\delta\varrho = -\varrho_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \varrho_2 h^\mu h^\nu \nabla_\mu u_\nu - \varrho_3 u^\mu \nabla_\mu T - \varrho_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$m^\mu = -Tm_1 \Delta^{\mu\nu} h^\lambda \delta_B b_{\nu\lambda} - Tm_2 \Delta^{\mu\nu} u^\lambda \delta_B g_{\nu\lambda} \quad ,$$

$$n^\mu = -Tn_1 \Delta^{\mu\sigma} h^\nu \delta_B g_{\nu\sigma} - Tn_2 \Delta^{\mu\sigma} u^\nu \delta_B b_{\sigma\nu} \quad ,$$

$$t^{\mu\nu} = -T\eta_\perp \left(\Delta^{\mu\rho}\Delta^{\nu\sigma} - \frac{1}{2}\Delta^{\mu\nu}\Delta^{\rho\sigma} \right) \delta_B g_{\rho\sigma} \quad ,$$

$$s^{\mu\nu} = -Tr_{||} \Delta^{\mu\rho}\Delta^{\nu\sigma} \delta_B b_{\rho\sigma} \quad ,$$

TAKE CONSTANT MAGNETIC FIELDS

$$\mathrm{Im}(w(k)) < 0 \qquad 1 > \lim_{k\to\infty} \left| \frac{\mathrm{Re}(w(k))}{k} \right| > 0$$

stability

causality

GENERAL ANALYSIS

- Two channels, 13 modes, 7 gapped
- Many inequalities

$$n_2 < 0 \quad , \quad k_2 < 0 \quad , \quad (\chi_1 + \chi_2 \rho T) < 0 \quad , \quad g > 0 \quad , \quad (\varepsilon_4 \varrho_3 - \varepsilon_3 \varrho_4) < 0$$

$$g = T\lambda(T^2 \varrho_3 + \varepsilon_4 - 2\mu\varrho_4) - T^2 \chi(\varepsilon_3 - \mu\varrho_3) + \mu\chi(\varepsilon_4 - \mu\varrho_4) - cT^2 \varrho_4$$

J. Armas & F. Camilloni, 2022

SOLVING CONSTRAINTS:

choose holographic equation of state and transport coefficients

	weak field ($T/\sqrt{\mathcal{B}} \gg 1$)	strong field ($T/\sqrt{\mathcal{B}} \ll 1$)
ε	$\frac{N_c^2}{2\pi^2}\left(74.1 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.62 \times \mathcal{B}^2\right)$
p	$\frac{N_c^2}{2\pi^2}\left(25.3 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.32 \times \mathcal{B}^2\right)$
s	$\frac{N_c^2}{2\pi^2}\left(99.4 \times T^3\right)$	$\frac{N_c^2}{2\pi^2}\left(7.41 \times \mathcal{B}\, T\right)$
μ	$\frac{N_c^2}{2\pi^2}\left(10.9 \times \mathcal{B}\right)$	$\frac{N_c^2}{2\pi^2}\left(2.88 \times \mathcal{B}\right)$

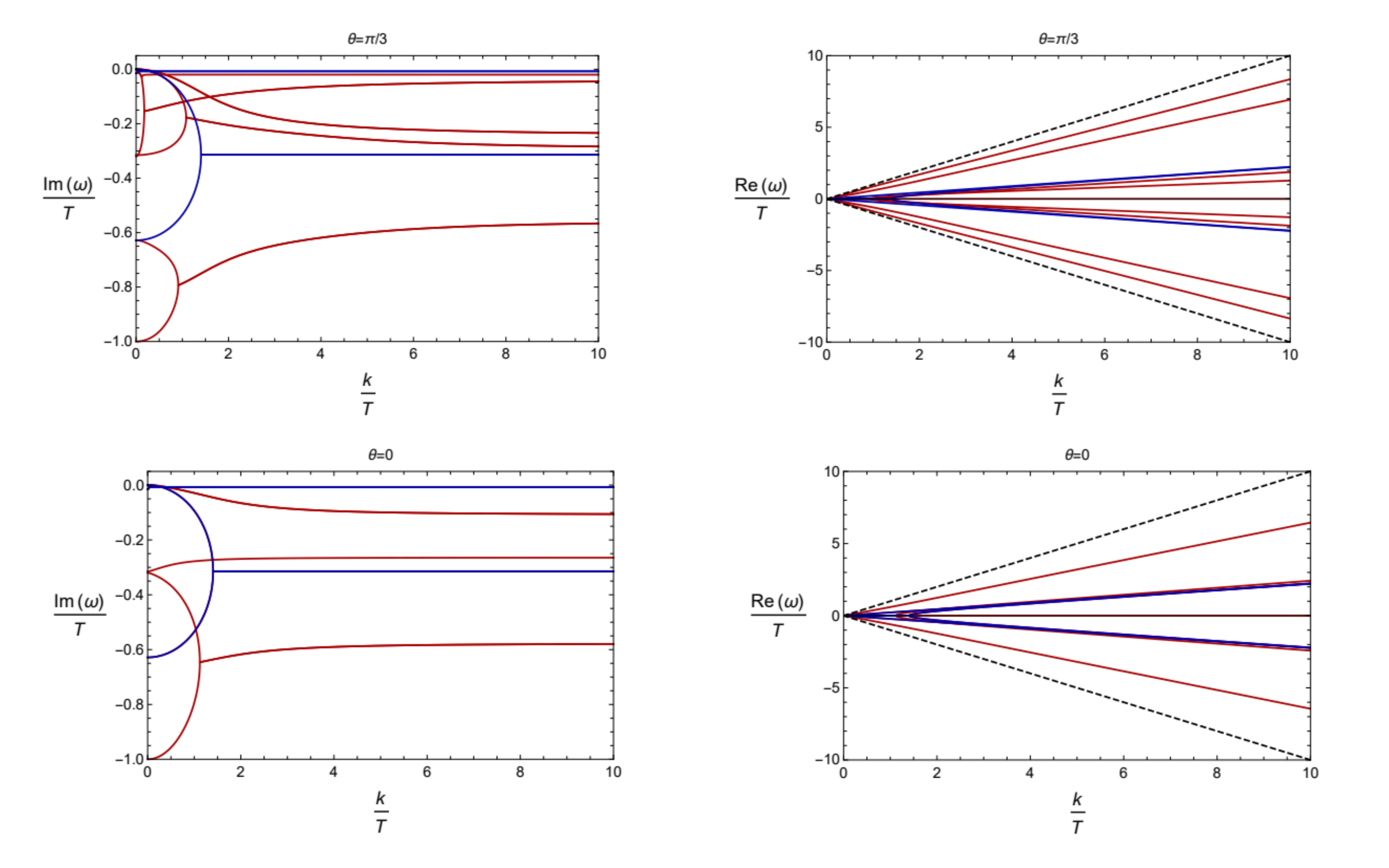
	weak field ($T/\sqrt{\mathcal{B}} \gg 1$)	strong field ($T/\sqrt{\mathcal{B}} \ll 1$)
$\eta_\perp$	$\frac{s}{4\pi}$	$\frac{s}{4\pi}$
$\eta_\parallel$	$1.00 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(21.32 \times \frac{T^2}{\mathcal{B}}\right)$
$\zeta_\perp$	$0.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(16.34 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\parallel$	$1.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(65.37 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\times$	$-0.66 \times \frac{s}{4\pi}$	$-\frac{s}{4\pi}\left(32.69 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\perp$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(4.7 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\parallel$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(62.3 \times \frac{T}{\sqrt{\mathcal{B}}}\right)$

Grozdanov & Poovuttikul 2017

CAUSAL

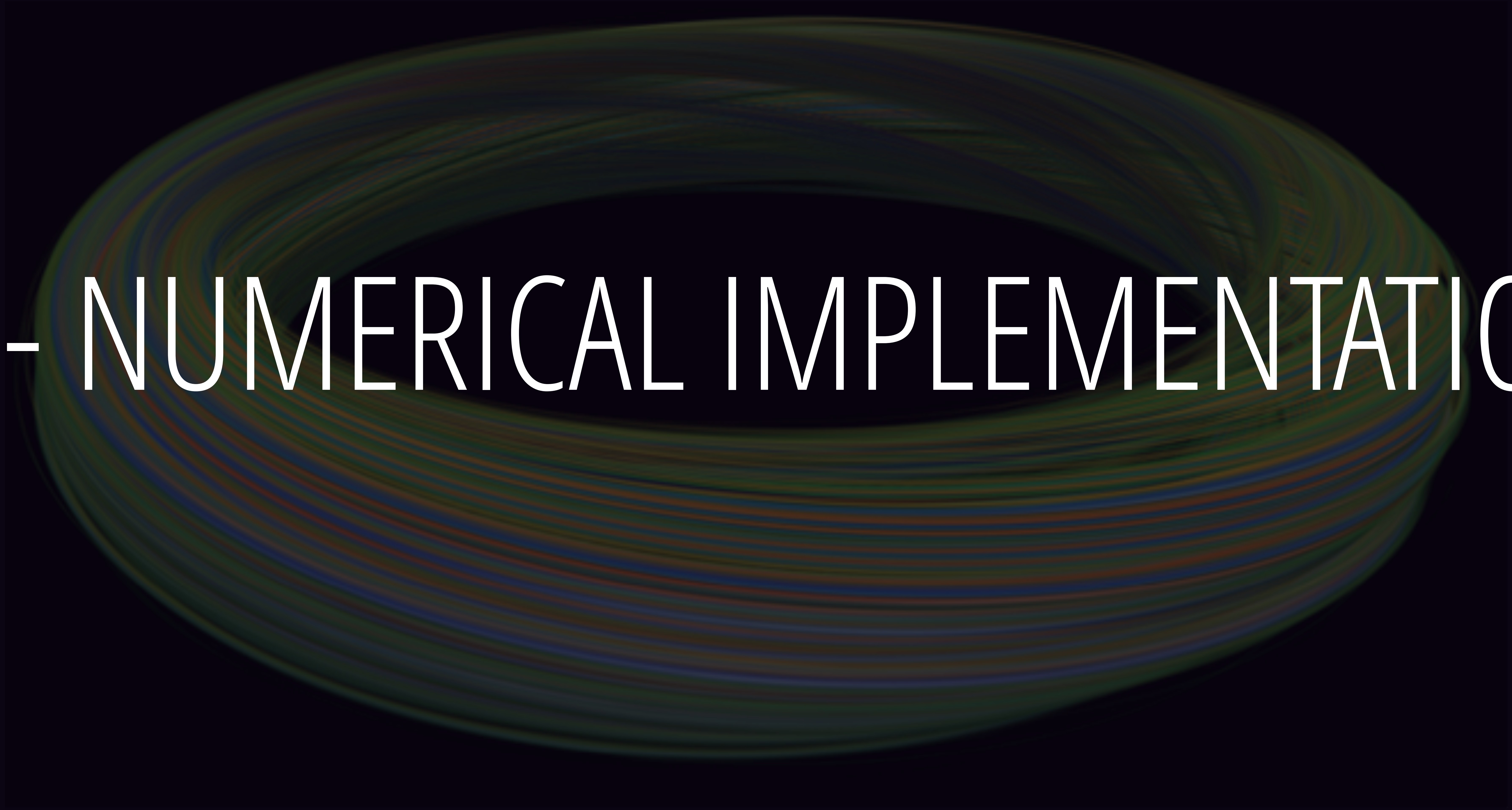
AND

STABLE!



Had to turn on 7 BDNK coefficients

III - NUMERICAL IMPLEMENTATION



R. Lier, A. Jain, JA, O. Porth, 2025

I have 28 coefficients, put that into code

It ain’t gonna happen

RESISTIVE MHD

$T^{\mu\nu}, J^{\mu}_{\text{ext}}, \rho^{\nu}$

$$\nabla_\nu F^{\mu\nu} = q\, u^\mu + \left(\sigma_{\|} \hat{b}^\mu \hat{b}^\nu + \sigma_\perp \mathbb{B}^{\mu\nu}\right) e_\nu,$$

$$e^\mu = \Gamma(\mathbf{E}\cdot\mathbf{v}, \mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$\Gamma = 1/\sqrt{1-\mathbf{v}^2}$

STIFFNESS PROBLEM

(IMEX)

ONE-FORM MHD

$T^{\mu\nu}, J^{\mu\nu}, \rho^{\nu}$

?

$$\nabla_\mu T^{\mu\nu} = 0 \quad \nabla_\mu J^{\mu\nu} = 0 \quad \nabla_\mu \rho^\mu = 0$$

Equilibrium (ideal order) stress tensor and current:

$$T^{\mu\nu} = \left(\epsilon + P + b^2\right) u^\mu u^\nu + \left(p + \frac{1}{2} b^2\right) g^{\mu\nu} - b^\mu b^\nu$$

$$J^{\mu\nu} = 2u^{[\mu} b^{\nu]}$$

$$\rho^\nu = \rho u^\mu$$

RESISTIVE MHD DUAL

(see also Hoult, Kovtun, 2024)

$T^{\mu\nu}$ IDEAL

ρ^{ν} IDEAL

$$J^{\mu\nu} = J^{\mu\nu}_{(0)} + J^{\mu\nu}_{(1)} - 2\tau u^{[\mu} \nabla_\rho J^{\nu]\rho}_{(0)} + \mathcal{O}(\partial^3)$$

$$J^{\mu\nu}_{(1)} = -\left(2r_\perp \mathbb{B}^{\rho[\mu}\hat{b}^{\nu]}\hat{b}^\sigma + r_\parallel \mathbb{B}^{\mu\rho}\mathbb{B}^{\nu\sigma}\right) 2T\partial_{[\rho}\left(\frac{b_{\sigma]}}{T}\right)$$

IDEAL GAS

$$\epsilon = \rho + \frac{p}{\hat{\gamma} - 1}$$

$$T \propto \frac{p}{\rho}$$

SYSTEM IS STABLE AND CAUSAL FOR SUFFICIENTLY LARGE τ

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FLUX CONSERVATIVE FORM

(see also Pandya, Most, Pretorius, 2022)

$$\partial_t \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}$$

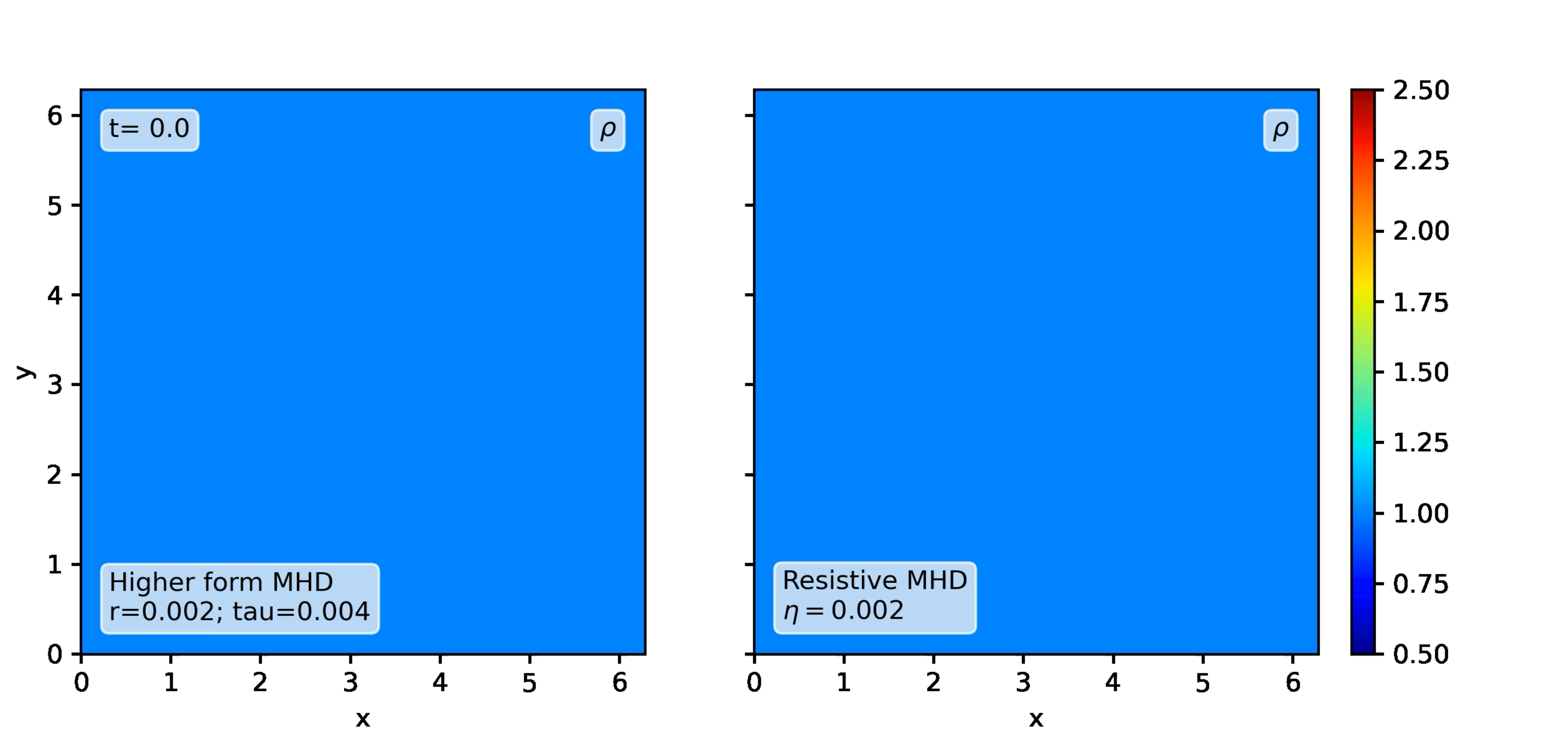
$$\mathbf{U} = \begin{pmatrix} \rho^t \\ T^{tk} \\ T^{tt} \\ J^{tk} \\ b^k \end{pmatrix}, \quad \mathbf{F}^i = \begin{pmatrix} \rho u^i \\ T^{ik} \\ T^{it} \\ J^{ik} \\ 0 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dot{b}^k \end{pmatrix},$$

$$\mathbf{F}^i = \mathbf{F}^i(\mathbf{P}(\mathbf{U})) \qquad \mathbf{P} = \left(\rho\, ,\, u^i\, ,\, p\, ,\, J^{ti}\, ,\, b^i\, ,\, \dot{u}^i\, ,\, \dot{\epsilon}\, ,\, \dot{\rho} \right)$$

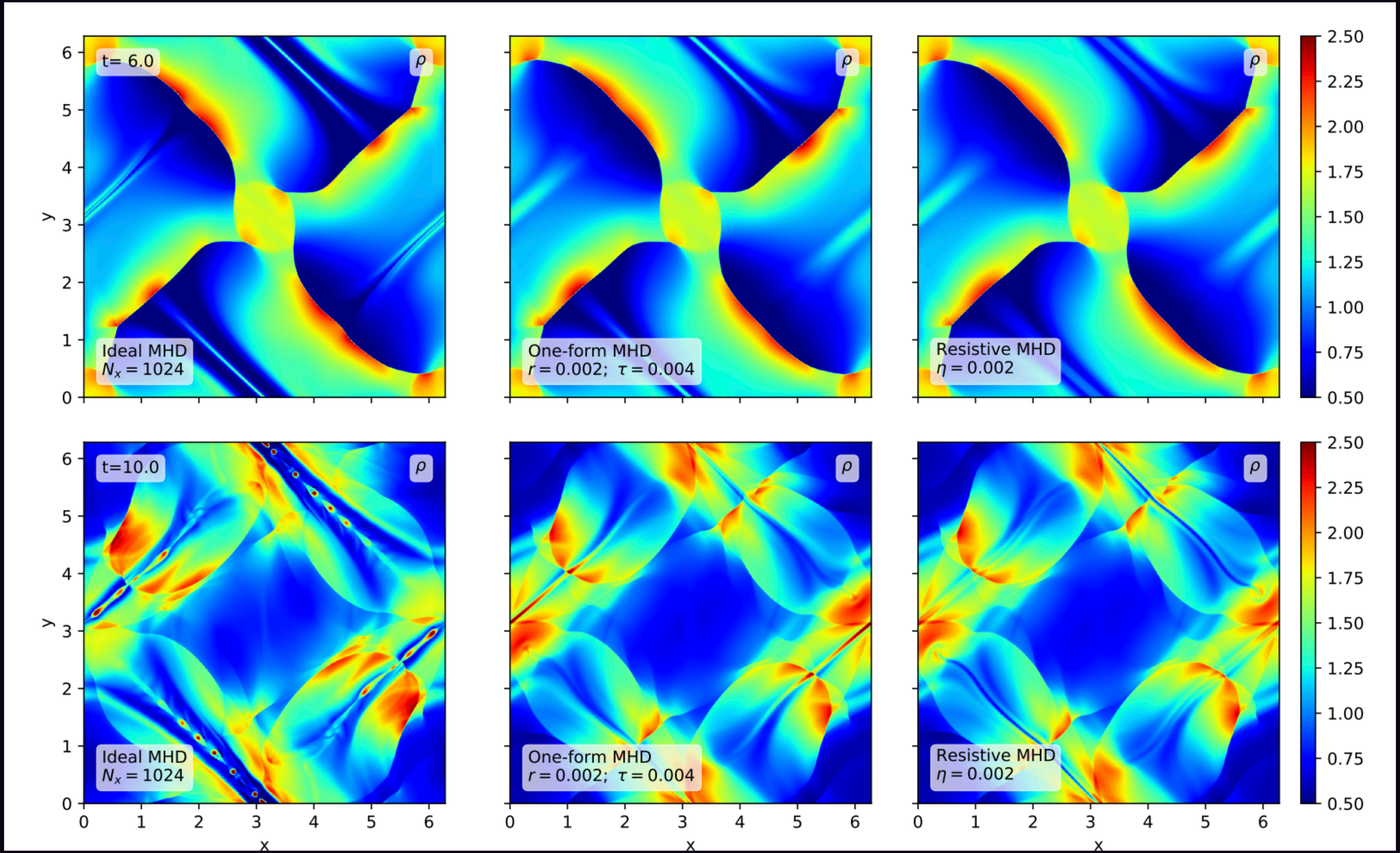
THE ORSZAG-TANG VORTEX

$$(u^x, u^y) = \left(-\Gamma v_{\max} \sin(y), \Gamma v_{\max} \sin(x)\right)$$

$$(J^{tx}, J^{ty}) = \left(-\sin(y), \sin(2x)\right)$$



THE ORSZAG-TANG VORTEX



THE ORSZAG-TANG VORTEX

HOW WELL
DOES IT DO?

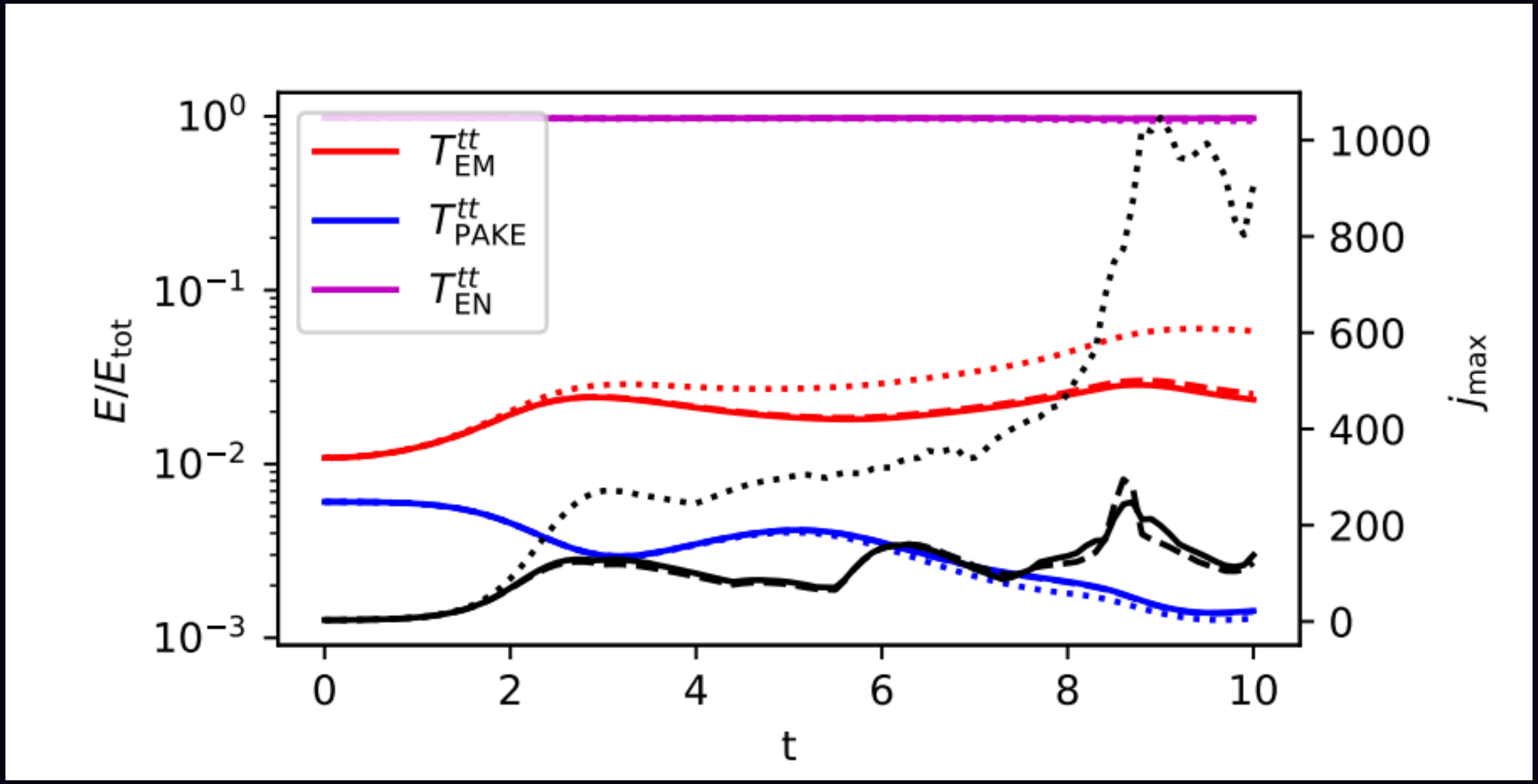
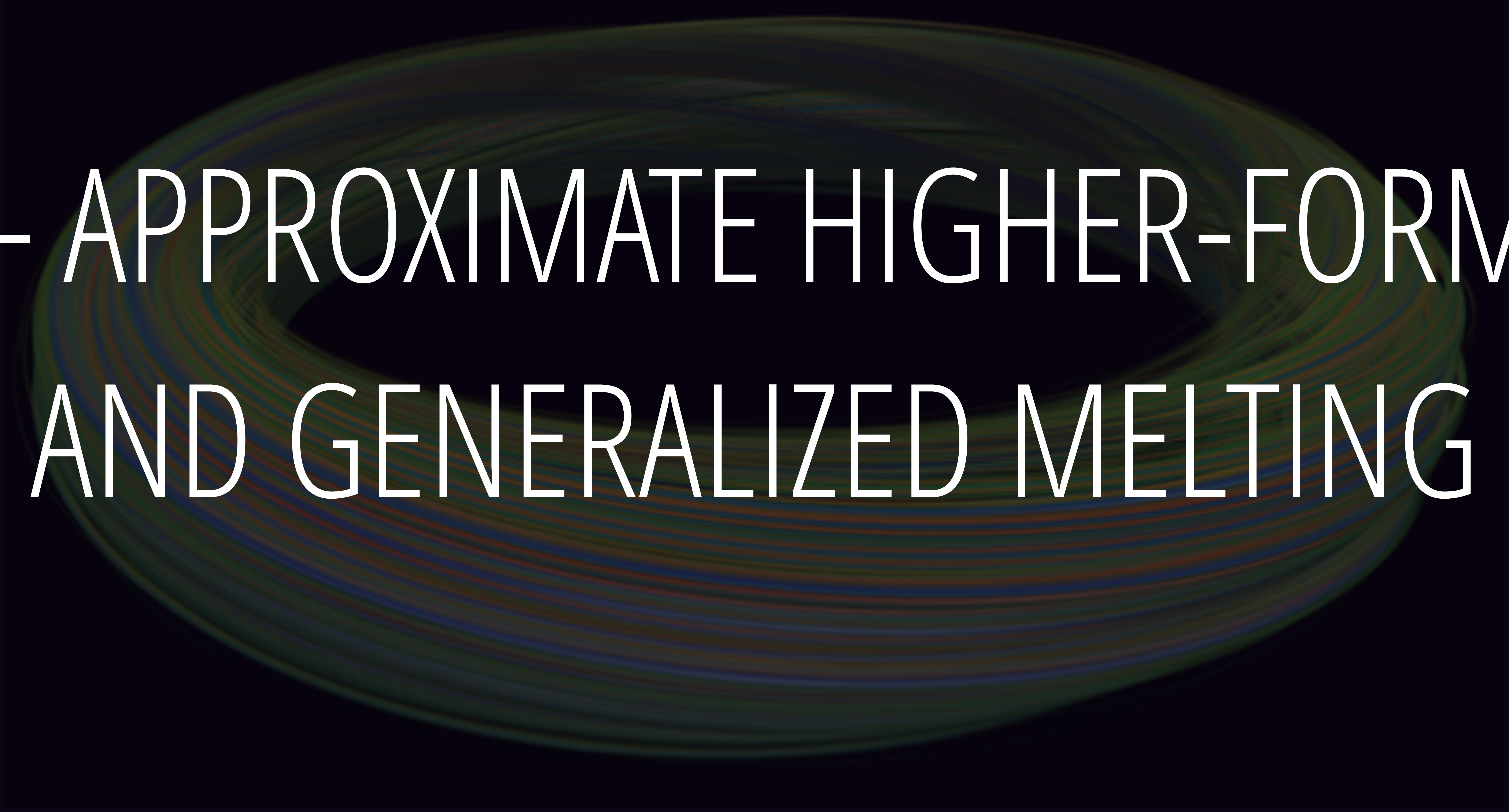


FIG. 2. Energy evolution of the Orszag-Tang vortex problem. Solid lines: one-form MHD, dashed lines: traditional resistive MHD, dotted lines: ideal MHD (at resolution of 1024^2 grid-points). In black (right y-axis), we show the maximum value of the current density magnitude for all three cases.

$$T^{tt} = T^{tt}_{\rm EM} + T^{tt}_{\rm PAKE} + T^{tt}_{\rm EN}$$



IV- APPROXIMATE HIGHER-FORMS AND GENERALIZED MELTING

MAGNETIC MONOPOLES

Gauss constraint:

$$\nabla_\mu J^{\mu t} = 0$$

Magnetic monopoles:

$$\nabla_\mu J^{\mu\nu} = \ell L^\nu$$

$\ell \to \mathcal{O}(\partial)$

JA, A. Jain, Approximate higher-form symmetries, PRD, 2023

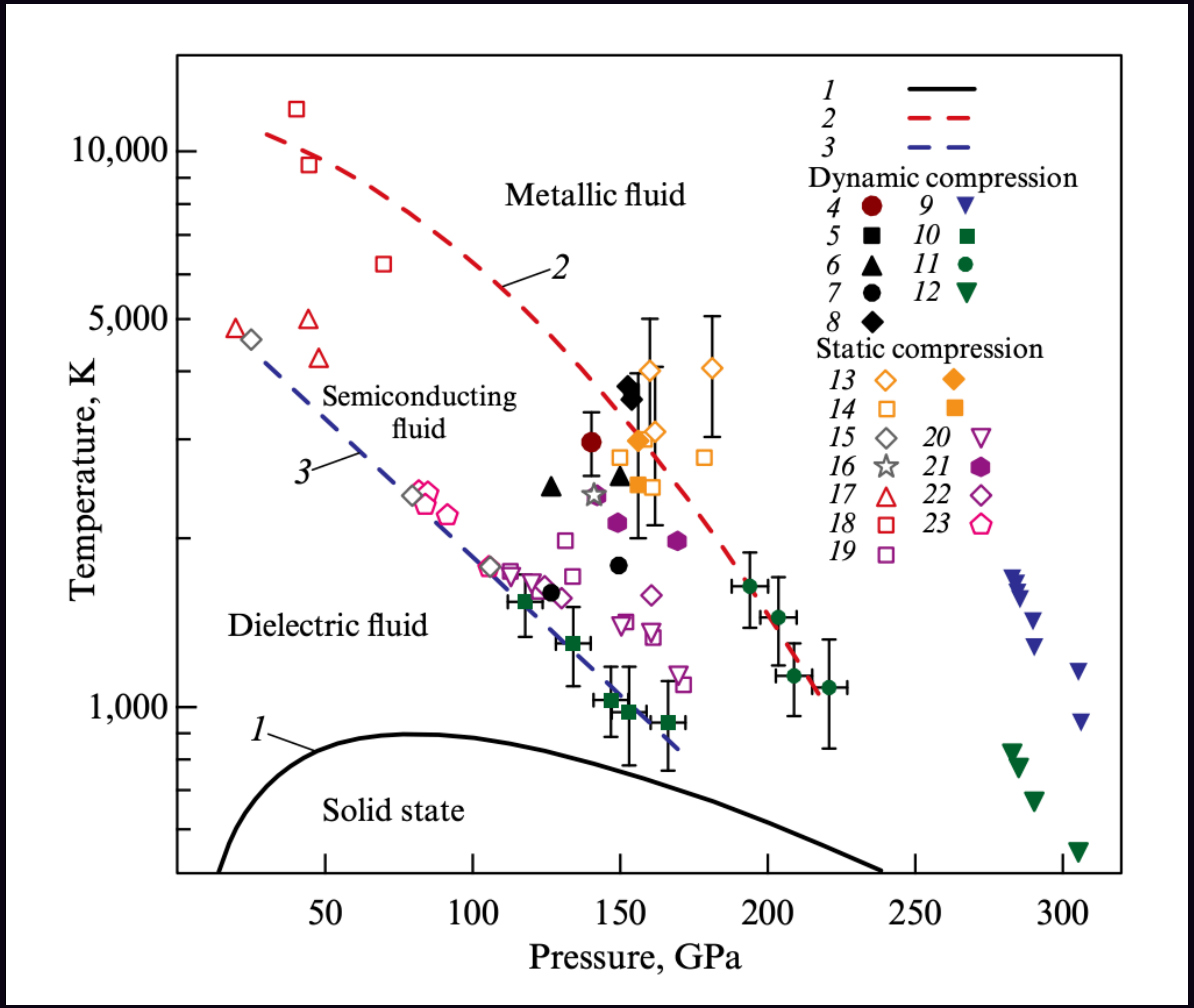
HOW TO INCLUDE VORTICES?

$$\xi_{\mu\nu} = 2\partial_{[\mu}\phi_{\nu]} + b_{\mu\nu}$$

$$2\partial_{[\mu}\phi_{\nu]} = 2\partial_{[\mu}\bar{\phi}_{\nu]} + \ell V_{\mu\nu}.$$

$$\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\xi_{\rho\sigma} = \frac{c_\phi}{6}\epsilon^{\mu\nu\rho\sigma}F_{\nu\rho\sigma} + \ell\tilde{L}^\mu$$

$$\tilde{L}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu V_{\rho\sigma}$$



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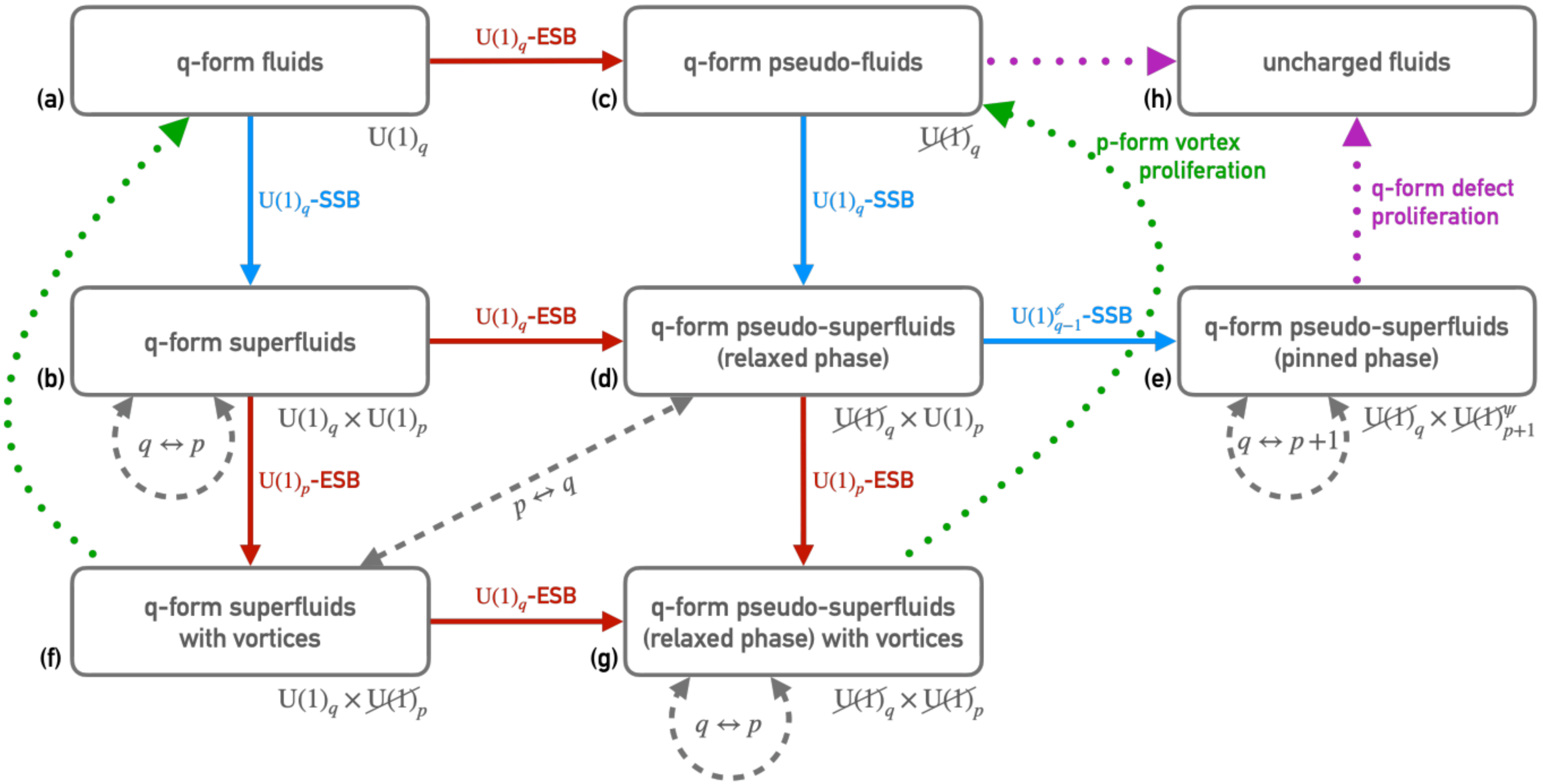

FIG. 1. Phases with approximate higher-form symmetry.

JA, A. Jain, Approximate higher-form symmetries, PRD, 2023

OUTLOOK

(1) Higher-forms are useful
(2) More numerical tests of BDNK
(3) Numerical implementation with viscosity
(4) Does it solve variability issues in EHT?
(5) Understand in more detail the various phase transitions in higher-form fluids

THANKS

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