# Recent numerical developments in relativistic spin hydrodynamics

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*based mainly on*
**SKS**, R. Ryblewski, and W. Florkowski, PRC 107, 085679 (2025)
Sapna, **SKS**, and D. Wagner, arXiv:2503.22552

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# Rotation and Polarization : Barnett effect

An initially unmagnetized body becomes magnetized under rotation, due to spin alignment induced by conservation of total angular momentum.

$$M = \chi\Omega/\gamma$$

where $\chi$ is the magnetic susceptibility, $\Omega$ is the angular velocity and $\gamma$ is the gyromagnetic ratio.

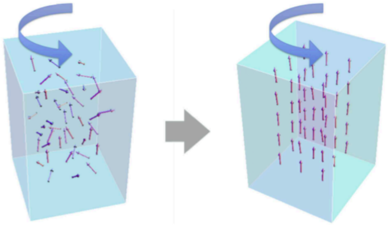


Image source : Front. Phys. 3:54 (2015)

# Polarization in lab. QGP?

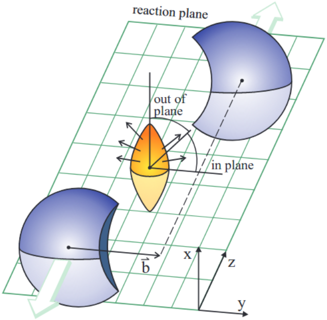
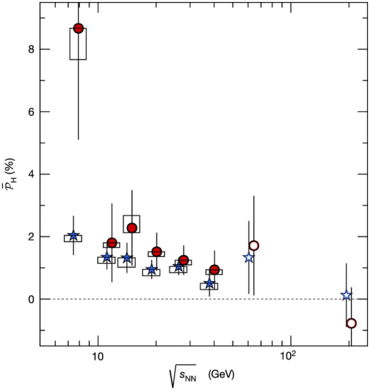



Image source - arXiv:0910.4114

- Nuclei carry a large orbital angular momentum (OAM), $L_0 = pb \simeq A\sqrt{s_{NN}}b/2$.
- e.g. for $\sqrt{s_{NN}} = 200$ GeV and $b = 5$ fm, $L_0 \sim 5 \times 10^5$.
- A fraction of $L_0$ if transferred to QGP fireball will result in polarization of quarks.
- A signature of an OAM would be the polarization of the emitted hadrons

# Experimental observation of Λ-polarization

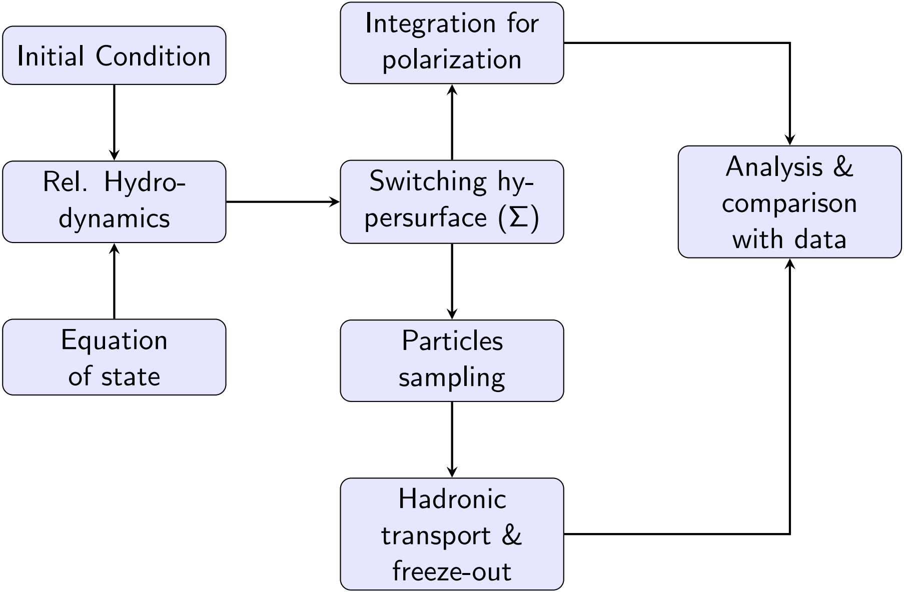



STAR Collaboration, Nature 548, 62-65 (2017)

# Purpose of talk !

- Currently, there are **four** main prescriptions for computing spin polarization, assuming collective rotation of the system.
- The purpose is to demonstrate that *if the temperature is high and the lifetime of the fireball is long enough, all four provide "equivalent" descriptions at freezeout within the experimental uncertainities.*
- Conclusion based on numerical simulations of spin polarization data for $\Lambda$ hyperons produced in Au+Au collisions at $\sqrt{s_{NN}}$ =200 GeV.

# Basic flow chart for simulation



# Relativistic Hydrodynamics

Continuity equations for energy-momentum and charge conservation

$$D_\mu T^{\mu\nu} = 0 \quad , \quad D_\mu N^\mu = 0$$

In the Landau frame ($T^\mu_\nu u^\nu = \varepsilon u^\mu$),

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \qquad , \qquad N^\mu = n u^\mu + V^\mu$$

We consider $V^\mu = 0$ for simplicity. The shear-stress tensor ($\pi^{\mu\nu}$) and bulk pressure ($\Pi$) are treated as dynamical variables, evolution equations are

$$\dot{\Pi} = \frac{\Pi_{NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = \frac{\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\mu\nu}\theta + \frac{\phi_7}{\tau_\pi}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\sigma^{\mu\nu}$$

where

$$\dot{A} = u^\mu D_\mu A \qquad , \qquad A^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}A^{\alpha\beta}$$

# Model description and parameter calibration

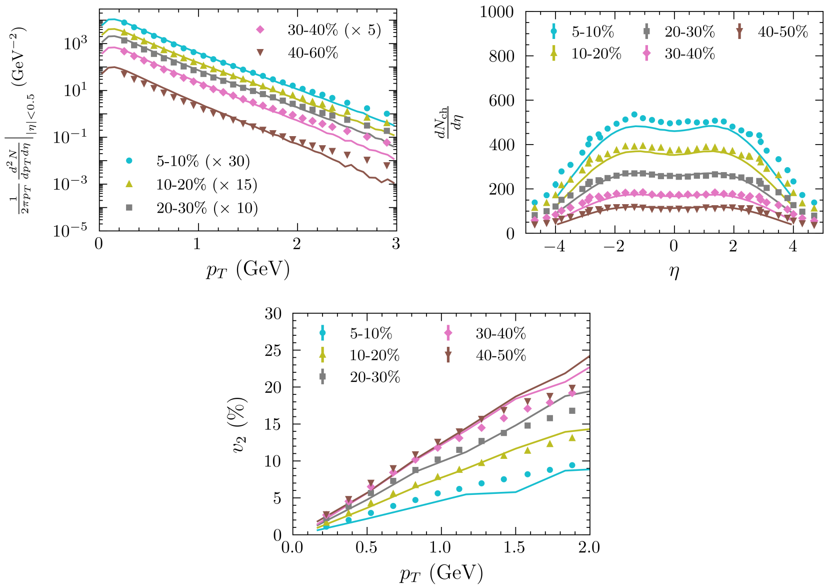
- **Initial condition**: PRC 102, 014909 (2020) and PRC 104, 054908 (2021), **six** parameters.
- **Equation of state**: `NEOS-BQS` (PRC 100, 024907 (2019))
- **Transport coefficients** ($C_\eta$=0.12):

$$\eta = C_\eta \frac{\varepsilon + P}{T}\ , \quad \zeta = 75\eta \left(\frac{1}{3} - c_s^2\right)^2\ ,$$

$$\tau_\pi = \frac{5\eta}{\varepsilon + P}\ , \quad \tau_\Pi = \frac{\zeta}{15\left(\frac{1}{3} - c_s^2\right)^2 (\varepsilon + P)}\ , \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\ ,$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}\ , \quad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}\ , \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}\ , \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}\ , \quad \frac{\varphi_7}{\tau_\pi} = \frac{9}{70 P \tau_\pi}\ .$$

- **Switching hypersurface** : Constant energy density, $\varepsilon = 0.5$ GeV/fm$^3$.




**SKS**, R. Ryblewski, and W. Florkowski, PRC 111, 024907 (2025)

# Quantum statistical approach

AOP 338, 32 (2013); PLB 820, 136519 (2021) and PRL 127, 272302 (2021)

- Compute local equilibrium density operator ($\hat{\rho}_{\text{LE}}$) by maximizing entropy ($S = -\text{tr}[\hat{\rho}\ \log\hat{\rho}]$) subject to constraints

$$n_\mu\ \text{tr}[\hat{\rho}\hat{T}^{\mu\nu}] = n_\mu T^{\mu\nu} \qquad , \qquad n_\mu\ \text{tr}[\hat{\rho}\hat{N}^{\mu}] = n_\mu N^{\mu}$$

  In the Belinfante pseudogauge (spin tensor is 0), the procedure gives:

$$\hat{\rho}_{\text{LE}} = \frac{1}{Z_{\text{LE}}} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\beta_\nu \hat{T}^{\mu\nu} - \zeta \hat{N}^\mu\right)\right] \text{ with } \beta_\nu = \frac{u_\nu}{T}\ , \zeta = \frac{\mu}{T}$$

- Evaluate the mean value of a quantum operator as

$$O(x) = \text{tr}[\hat{\rho}_{\text{LE}}\hat{O}(x)]$$

.

Polarization of spin-1/2 particles in a fluid cell is

$$S_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau + \mathcal{O}(\varpi^2)$$

where $\varpi_{\rho\sigma}$ is thermal vorticity defined as

$$\varpi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \qquad \text{with} \qquad \beta_\rho = \frac{u_\rho}{T}$$

Mean polarization vector is given by

$$S^\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, S^\mu(x,p)n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$

Input $T$, $u^\mu$ and their gradients from the numerical solution of relativistic hydrodynamics to compute the above expression.

# Hydrodynamic simulation for global polarization

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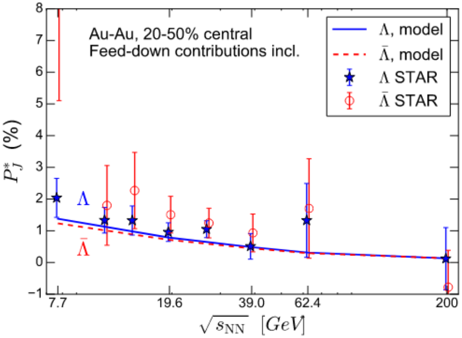
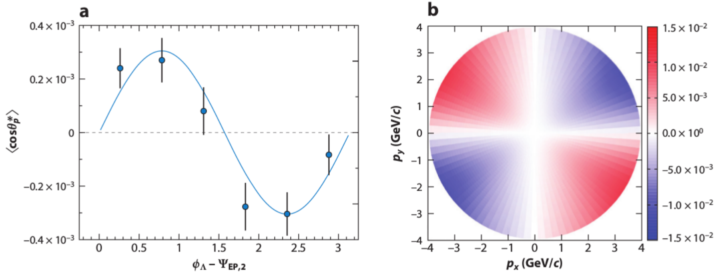
Vorticity in the QGP liquid and Λ polarization at the RHIC Beam Energy Scan

Iurii Karpenko[a,b], Francesco Becattini[a,c]



# Spin sign puzzle

*"Hydrodynamics predict a negative sign of the longitudinal component of the polarization vector."*




Ann. Rev. Nucl. Part. Sci. 70 (2020) 395

Additional contribution from thermal shear, $\xi_{\mu\nu} = \frac{1}{2}(\partial_\sigma\beta_\rho + \partial_\rho\beta_\sigma)$, was derived such that

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

The sign of longitudinal polarization is still negative.

# Isothermal approximation

At high energies $\mu_B \approx 0$. Hence, constant energy density implies constant $T$ on $\Sigma$, so that

$$\hat{\rho}_{\mathrm{LE}} \sim \exp\left[-\frac{1}{T}\int_\Sigma d\Sigma_\mu \ \hat{T}^{\mu\nu} u_\nu\right]$$

This gives Prescription I (PRL 127, 272302 (2021))

$$\boxed{S^\mu(p) = -\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int d\Sigma \cdot p \ n_F(1-n_F)\left[\omega_{\nu\rho} + 2\hat{t}_\nu \frac{p^\lambda}{p\cdot\hat{t}} \Xi_{\lambda\rho}\right]}{8 m_\Lambda T \int d\Sigma \cdot p \ n_F}}$$

where $\hat{t} = (1,0,0,0)$ and

$$\omega_{\nu\rho} = \frac{1}{2}(\partial_\rho u_\nu - \partial_\nu u_\rho)$$
$$\Xi_{\nu\rho} = \frac{1}{2}(\partial_\rho u_\nu + \partial_\nu u_\rho)$$

# Chiral kinetic theory approach

JHEP07(2021)188 and PRL 127, 142301 (2021)

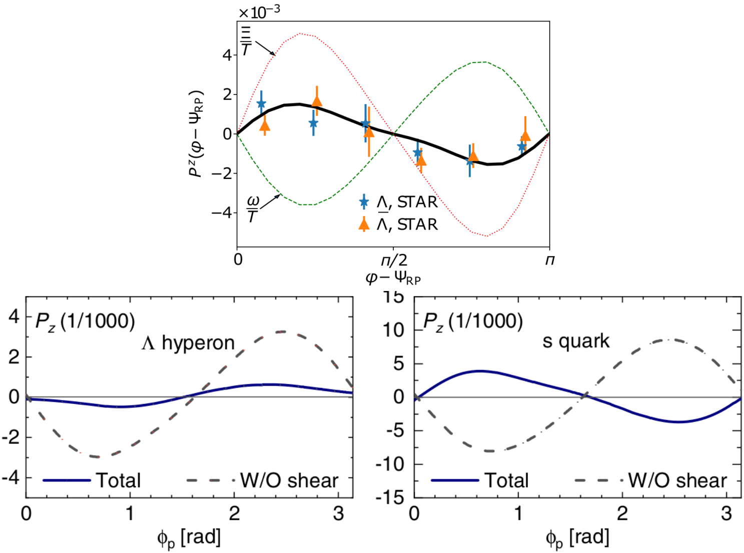
- The expression of axial Wigner function $\mathcal{A}^\mu$ from chiral kinetic theory is

$$\mathcal{A}^\mu = \sum_{\lambda=\pm}\left(\lambda p^\mu f_\lambda + \frac{1}{2}\frac{\epsilon^{\mu\nu\alpha\rho}p_\nu u_\alpha \partial_\rho f_\lambda}{p\cdot u}\right)$$

  Replace $f_\lambda$ with local equilibrium distribution $n_F(\beta(\varepsilon_0 - \Delta\varepsilon_\lambda))$ with $\varepsilon_0 = p\cdot u$, $\Delta\varepsilon_\lambda = -\lambda\omega\cdot p/(2\varepsilon_0)$ and expand to first order in gradients. Finally, averaging over $\Sigma$ gives Prescription II

$$\boxed{\begin{aligned} S^\mu(p) &= S^\mu_\varpi(p) + S^\mu_{\xi,\mathrm{LY}}(p) \\ S^\mu_{\xi,\mathrm{LY}}(p) &= -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m}p_\sigma \frac{\int d\Sigma\cdot p\ \ n_F(1-n_F)\frac{p_\perp^\lambda u_\nu}{p\cdot u}\xi_{\rho\lambda}}{\int d\Sigma\cdot p\ \ n_F} \end{aligned}}$$

- For $m = m_\Lambda$, $S_z(\phi)$ has wrong sign. However, for $m \approx 300$ MeV, the sign is correct. *"The memory of strange quark polarization is preserved in the measured $\Lambda$ polarization"*.




Results from Prescriptions I (top) and II (bottom). Image source: PRL 127, 272302 (2021) and PRL 127, 142301 (2021)

# Relativistic spin hydrodynamics

- Prescriptions I & II assume instantaneous equilibration of spin degrees of freedom.

- Several studies suggest that the spin relaxation time is comparable to the lifetime of the fireball. This implies that spin dynamics cannot be neglected.

- Spin dynamics is introduced by demanding conservation of total angular momentum
$$D_\mu J^{\mu,\alpha\beta} = 0$$
in addition to conservation of energy-momentum and charge
$$D_\mu T^{\mu\nu}(x) = 0 \quad , \quad D_\mu N^\mu(x) = 0.$$

- We have $J^{\mu,\alpha\beta} = L^{\mu,\alpha\beta} + S^{\mu,\alpha\beta}$, where $L^{\mu,\alpha\beta} = T^{\mu[\beta}x^{\alpha]}$. This gives
$$D_\mu S^{\mu,\alpha\beta} = T^{[\beta\alpha]}$$
where $T^{[\beta\alpha]}$ is the antisymmetric part of energy-momentum tensor.

# Kinetic theory approach with local collisions only

- In the GLW pseudogauge, the energy-momentum tensor is symmetric, so that total spin is conserved (PRC 97 (2018) 4, 041901)

$$D_\mu S^{\mu,\alpha\beta} = 0$$

- Spin tensor is given by (PRC 98, 044906 (2018))

$$S^{\alpha,\beta\gamma} = A_1 u^\alpha \omega^{\beta\gamma} + A_2 u^\alpha u^{[\beta}\kappa_0^{\gamma]} + A_3 \left( u^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\kappa_0^{\gamma]} \right)$$

where $\kappa_{0,\mu} = \omega_{\mu\alpha}u^\alpha$ and

$$A_1 = \mathcal{C}\frac{T^3}{\pi^2}\left[\left(4 + \frac{m^2}{2T^2}\right)K_2\left(\frac{m}{T}\right) + \frac{m}{T}K_1\left(\frac{m}{T}\right)\right]$$

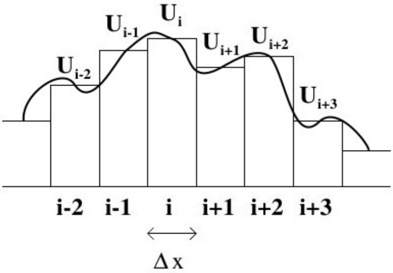
$$A_2 = 2\mathcal{C}\frac{T^3}{\pi^2}\left[\left(12 + \frac{m^2}{2T^2}\right)K_2\left(\frac{m}{T}\right) + 3\frac{m}{T}K_1\left(\frac{m}{T}\right)\right]$$

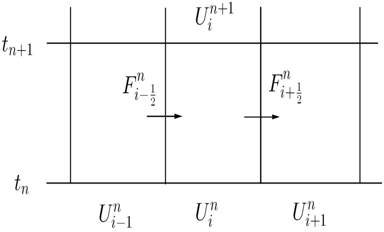
$$A_3 = -\mathcal{C}\frac{T^3}{\pi^2}\left[4K_2\left(\frac{m}{T}\right) + \frac{m}{T}K_1\left(\frac{m}{T}\right)\right]$$

- Spin evolution decouples from the background. Numerical solution of spin hydrodynamics requires $\mu_B$, $T$ and $u^\mu$ from the background.

# Algorithm

- Convert the original problem to solving Riemann problem at each cell boundary.




- Approximate the numerical flux of a Riemann problem using relativistic HLLE prescription. Compute the net flux entering each fluid cell and update.

- The scheme is conservative. The loss in total spin is between 3-6%.

# Numerical solution of spin conservation equations

- The spin conservation equations are numerically solved in **SKS**, R. Ryblewski and W. Florkowski, PRC 111, 024907 (2025).

- Similar to traditional hydrodynamics, we need an initial condition and information about the spin thermalization time ($\tau_0^s$). We treat the initial time as a parameter, with two parameters in total: $m$ and $\tau_0^s$.

- We use the following initial condition for the spin polarization tensor

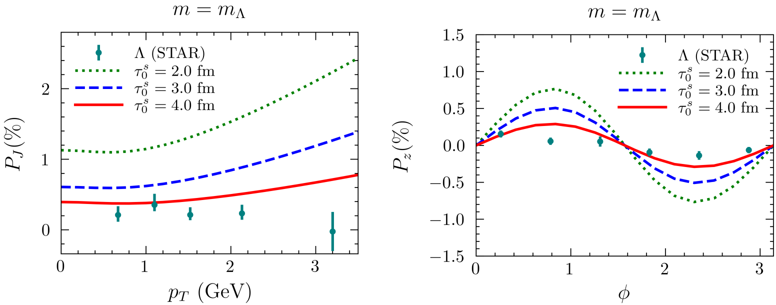
$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu}^{\text{iso}} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}^{\text{iso}} u^\rho,$$

  $\hat{\tau} = (1, 0, 0, 0)$ being the unit normal to constant $\tau$ hypersurface.

- The spin polarization is obatined using Prescription III (see also PRC 105 (2022), 044907)

$$\boxed{S^\mu(p) = -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int d\Sigma \cdot p \; n_F(1-n_F)\omega_{\nu\rho}}{\int d\Sigma \cdot p \; n_F}}$$

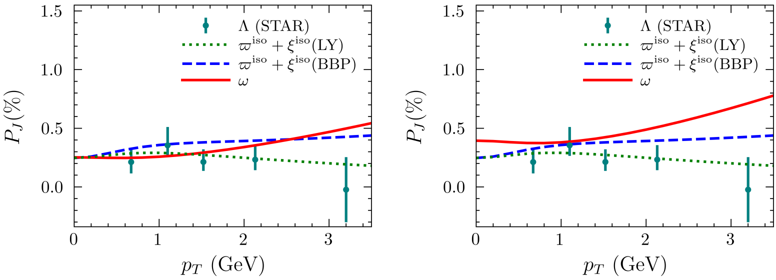
- Note that the initial time for background hydrodynamics is 1 fm. Also, $m_\Lambda = 1.115$ GeV.




Simulation results for a fixed $m$ but different $\tau_0^s$ PRC 111, 024907 (2025)

- "Good" fit is obtained if the initial spin time is 4 fm. What happens between 1 and 4 fm? *Dissipative processes are important at early times, while spin-conserving processes dominate at later stages.*

- We fix $\tau_0^s = 4$ fm,




Simulation results for a fixed $\tau_0^s$ but different mass (left) $m = 300$ MeV, (right) $m = 1.115$ GeV PRC 111, 024907 (2025)

- "Good" fit is obtained for small $m$. Relaxation seems to be faster for less massive particles.

# Kinetic theory approach with local and non-local collisions

- Boltzmann equation with spin DOF (PRD 106 (2022) 11, 116021)

$$k.\partial f(x, k, \mathfrak{s}) = C_l[f] + C_{nl}[f]$$

- Non-local collisions provide an exchange mechanism between spin and orbital angular momentum, facilitating spin equilibration.

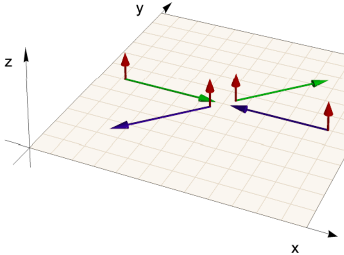



Image source:Prog. Part. Nucl. Phys. 108 (2019) 103709

- Local equilibrium distribution, $f_{\text{eq}}$, is defined as

$$C_l[f_{\text{eq}}] = 0$$

- Due to non-local collisions, energy-momentum tensor has an anti-symmetric contribution even in equilibrium.

$$T_0^{[\mu\nu]} = \frac{\hbar\sigma}{2}\int d\Gamma\ \Sigma_{\mathfrak{s}}^{\mu\nu}\ C_{nl}[f_{\text{eq}}] \quad \text{where } \Sigma_{\mathfrak{s}}^{\mu\nu} = -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta$$

- The equations of ideal spin hydrodynamics are

$$D_\mu T_0^{(\mu\nu)} = 0 + \mathcal{O}(\hbar^2) \qquad , \qquad D_\lambda S_0^{\lambda,\mu\nu} = \frac{1}{\hbar}T_0^{[\nu\mu]} + \mathcal{O}(\hbar^2)$$

- Equations of dissipative spin hydrodynamics were obtained in D. Wagner, PRD 111, 016008 (2025) using the moment expansion method. The equations involve spin degrees of freedom $\omega_0^\mu$, $\kappa_0^\mu$, and the spin-shear stress ($\mathfrak{t}^{\mu\nu}$) where

$$\Omega_0^{\mu\nu} = u^{[\mu}\kappa_0^{\nu]} + \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_{0,\beta}$$

- The spin-shear stress, $\mathfrak{t}^{\mu\nu}$, has a dissipative effect and plays a role analogous to thermal shear $\xi_{\mu\nu}$ in equilibrium approach.

The evolution equations for spin degrees of freedom that we solve numerically in Sapna, **SKS** and D. Wagner, arXiv:2503.22552 are

$$
\begin{aligned}
\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu &= -\frac{\omega_{\mathrm{K}}^\mu}{T} + \delta_{\omega\omega}\omega_0^\mu\theta + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\ell_{\omega\kappa}\nabla_\alpha\kappa_{0,\beta} - \tau_\omega\dot{u}_\alpha\kappa_{0,\beta}\right) \\
&\quad + \lambda_{\omega\omega}\sigma^{\mu\nu}\omega_{0,\nu} + \lambda_{\omega\mathfrak{t}}\mathfrak{t}^\mu{}_\nu\omega_{\mathrm{K}}^\nu\ , \\
\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu &= -\frac{\dot{u}^\mu}{T} + \delta_{\kappa\kappa}\kappa_0^\mu\theta + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\frac{\tau_\kappa}{2}\nabla_\alpha\omega_{0,\beta} + \tau_\kappa\dot{u}_\alpha\omega_{0,\beta}\right) \\
&\quad + \ell_{\kappa\mathfrak{t}}\Delta^\mu_\lambda\nabla_\nu\mathfrak{t}^{\nu\lambda} + \tau_{\kappa\mathfrak{t}}\mathfrak{t}^{\mu\nu}\dot{u}_\nu + \left(\lambda_{\kappa\kappa}\sigma^{\mu\nu} + \frac{\tau_\kappa}{2}\omega_{\mathrm{K}}^{\mu\nu}\right)\kappa_{0,\nu}, \\
\tau_{\mathfrak{t}}\dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} &= \frac{\mathfrak{d}}{T}\sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_{\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\omega_{\mathrm{K}}^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_0^{\nu\rangle} \\
&\quad + \tau_{\mathfrak{t}\omega}\omega_{\mathrm{K}}^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega}\sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\alpha\omega_{0,\beta}.
\end{aligned}
$$

In above equations, the symbols are as follows:

$$
A^{\langle\mu\rangle} = \Delta^{\mu\nu}A_\nu, \quad \dot{A} = u\cdot\partial A, \quad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu, \quad \sigma^{\mu\nu} = D^{\langle\mu}u^{\nu\rangle}
$$

$$
\omega_K^{\mu\nu} = \frac{1}{2}\Delta^\mu_\alpha\Delta^\nu_\beta D^{[\alpha}u^{\beta]} = \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_{K,\beta}
$$

# Algorithm

The equations for spin are of the form

$$\gamma \left( \frac{\partial}{\partial t} + v^i \frac{\partial}{\partial x^i} \right) \mathcal{W} = -\frac{\mathcal{W} - \mathcal{W}_{NS}}{\tau_{\mathcal{W}}} - I_{\mathcal{W}}$$

Use Strang-Splitting method to obtain

$$\frac{\partial \mathcal{W}}{\partial t} = -\frac{\mathcal{W} - \mathcal{W}_{NS}}{\overline{\tau}_{\mathcal{W}}} - S_{\mathcal{W}} \qquad , \qquad \frac{\partial \mathcal{W}}{\partial t} + v^i \frac{\partial \mathcal{W}}{\partial x^i} = 0$$

First solve relaxation equation using PES method. If $\Delta t > \overline{\tau}_{\mathcal{W}}$,

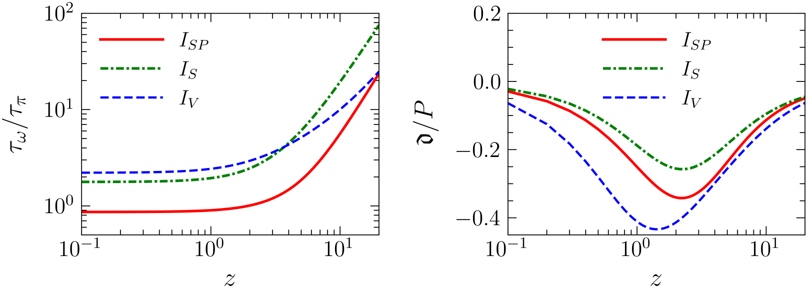
$$\mathcal{W}_i^{n+1} = \mathcal{W}_i^n - \Delta t \frac{\mathcal{W}_i^{n+\frac{1}{2}} - \mathcal{W}_{i,NS}^{n+\frac{1}{2}}}{\overline{\tau}_{\mathcal{W}}} - S_{i,\mathcal{W}}^{n+\frac{1}{2}}$$

If $\Delta t < \overline{\tau}_{\mathcal{W}}$

$$\mathcal{W}_i^{n+1} = \mathcal{W}_{i,NS}^{n+\frac{1}{2}} + \exp\left[ -\frac{\Delta t}{\overline{\tau}_{\mathcal{W}}} \left( \mathcal{W}_i^n - \mathcal{W}_{i,NS}^{n+\frac{1}{2}} \right) \right] - S_{i,\mathcal{W}}^{n+\frac{1}{2}}$$

Use upwind scheme for advection.

# Numerical solution of dissipative spin hydrodynamics




- The transport coefficients are computed for three interactions (i) Scalar ($I_S$): $\mathcal{L}_{\text{int,S}} = G(\bar{\psi}\psi)^2$, (ii) Scalar+Pseudoscalar ($I_{SP}$): $\mathcal{L}_{\text{int,SP}} = G[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$, and (iii) Vector ($I_V$): $\mathcal{L}_{\text{int,V}} = -G(\bar{\psi}\gamma^\mu\psi)^2$

- The transport coefficients are independent of $G$ when computed as ratios.

- The only parameter of the model is the mass of particles, which we fix at 300 MeV. For Au+Au collision at 200 GeV with $b = 8.4$ fm, $z$ increases from 1.2 to approximately 1.9.

- The spin polarization is given by Prescription IV

$$\boxed{S^{\mu}(p) = S^{\mu}_{\omega}(p) + S^{\mu}_{\kappa}(p) + S^{\mu}_{\mathfrak{t}}(p)}$$

where

$$S^{\mu}_{\omega}(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \, \frac{u^{\mu}(\omega_0 \cdot p) - \omega_0^{\mu}(p \cdot u)}{2m_{\Lambda}} f_0 \widetilde{f_0},$$

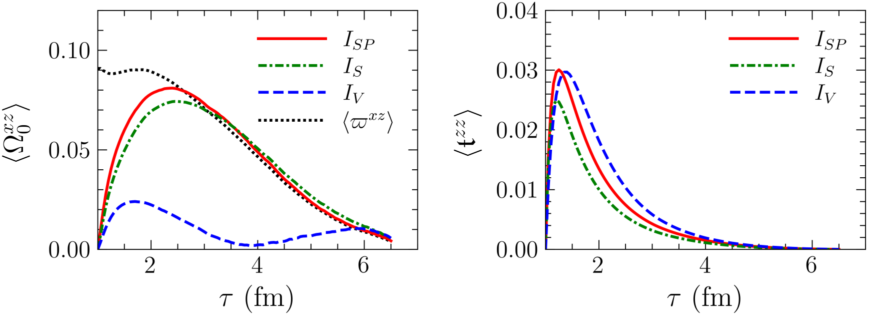
$$S^{\mu}_{\kappa}(p) = -\frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \, \frac{\epsilon^{\mu\nu\rho\sigma} u_{\nu} p_{\sigma}}{2m_{\Lambda}} \kappa_{0,\rho} f_0 \widetilde{f_0} \ ,$$

$$S^{\mu}_{\mathfrak{t}}(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \, \frac{\epsilon^{\mu\nu\rho\sigma} u_{\nu} p^{\lambda} p_{\sigma}}{3T^2(\varepsilon + P)} \mathfrak{t}_{\rho\lambda} f_0 \widetilde{f_0} \ ,$$

and $f_0$ denotes the Fermi-Dirac distribution, $\widetilde{f_0} = 1 - f_0$, and $N(p) = 2 \int \mathrm{d}\Sigma \cdot p \, f_0$.

- Note that $S^{\mu}_{\mathfrak{t}}(p)$ is a dissipative correction to spin polarization, in absence of which Prescription IV reduces to Prescription III.

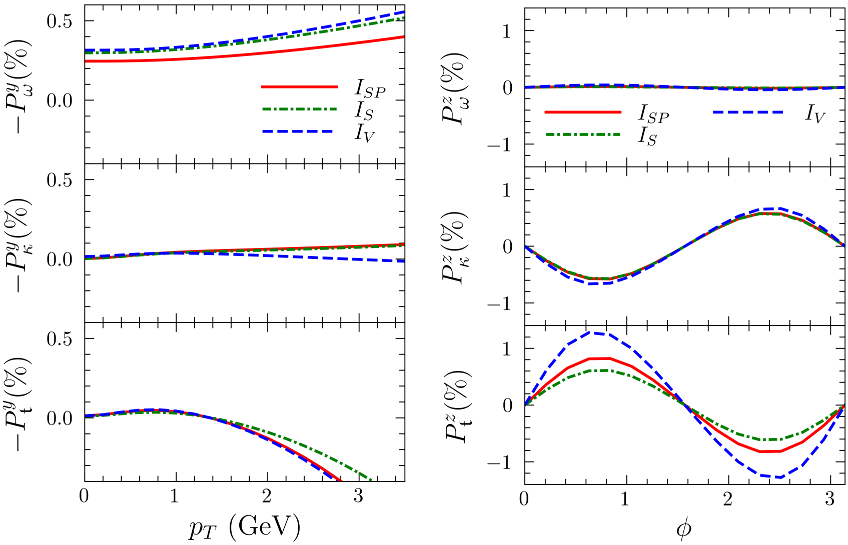
# Results: Time evolution of spin potential and spin-shear stress




arXiv:2503.22552

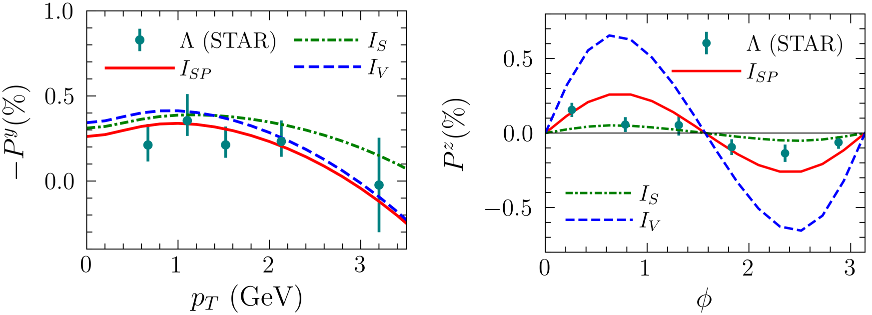
$$\langle A \rangle(\tau) = \frac{\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}\eta_s\ A(\tau, x, y, \eta_s)\varepsilon(\tau, x, y, \eta_s)\Theta(\varepsilon - \varepsilon_{\text{cut}})}{\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}\eta_s\ \varepsilon(\tau, x, y, \eta_s)\Theta(\varepsilon - \varepsilon_{\text{cut}})}$$

# Results: Individual contributions



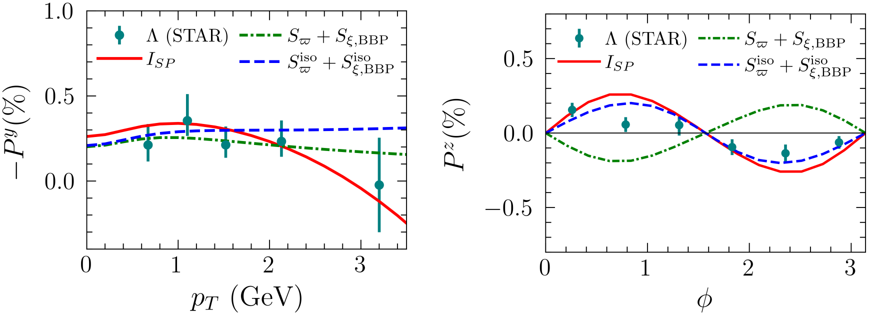

arXiv:2503.22552

# Results: Spin Polarization

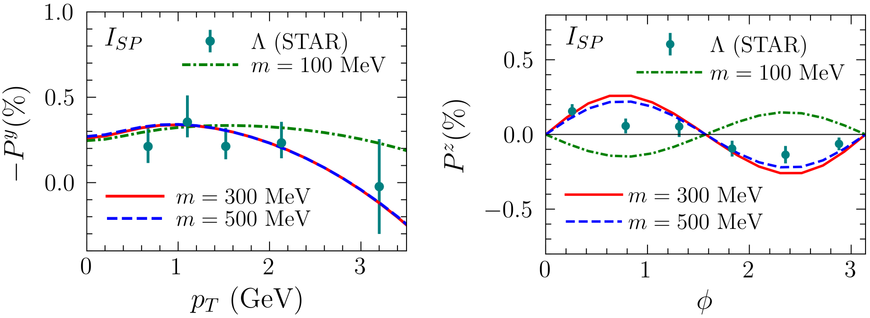



Effects due to interaction type. arXiv:2503.22552

# Results: Spin Polarization




Comparison with Prescription I. arXiv:2503.22552




Spin polarization for different values of $m$. arXiv:2503.22552

- Dissipative effects are necessary to describe the polarization measurements.

# Summary

- We study spin dynamics in the fireball produced in heavy-ion collisions, guided by polarization measurements from Au+Au collisions at 200 GeV.
- Results suggest that at high temperatures and with long fireball lifetimes, the four prescriptions yield "equivalent" descriptions at the switching hypersurface
- The results also highlight that a consistent treatment of dissipative effects is crucial in microscopic approaches for matching experimental data.
- The real test of the approaches will be at lower collision energies or in small systems.