Strong coupling estimates of bulk viscosity in compact stars

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Foundations and Applications of Relativistic Hydrodynamics

Galileo Galilei Institute

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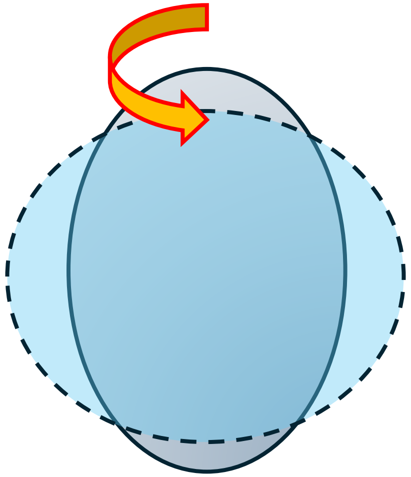
Neutron stars: ultradense fluid coupled to dynamical gravity

We can measure properties of QCD at extreme conditions from astrophysical observations!

Oscillations, tidal deformations



Gravitational waves

Detector

Neutron stars contain one of the most extreme forms of matter, with densities above atomic nuclei and relatively low temperatures
(compared to the QGP of RHIC and the LHC)

Some heavy ion collision experiments have been designed to study similar regions of the phase diagram
(FAIR in Darmstadt, NICA in Dubna, J-PARC in Tokai)

Outer layers of the star can be described with nuclear matter models and chiral effective theory, the interior however is poorly understood

First principles methods cannot be used, neither perturbative QCD (strongly coupled matter) nor lattice QCD (sign problem at large densities)

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Holographic models avoid these issues

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Holographic models avoid these issues

Complementary to other phenomenological models

Holographic models

Strongly coupled gauge theory D-dimensions	⟺	Weakly coupled gravity D+1 - dimensions
Deconfined phase (Quark-gluon plasma)		Black hole geometry
Confined phase (Hadrons)		“Soliton” geometry
Baryon density (Global current)		Electric charge density (Gauge field)

Holographic models

Strongly coupled gauge theory D-dimensions	⟺	Weakly coupled gravity D+1 - dimensions
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Most relevant property of matter: equation of state

Determines mass vs radius curve, tidal deformability, moment of inertia, etc

Equation of State from holographic models

D3-D7 model Karch, Katz, hep-th/0205236

Hoyos, Rodriguez-Fernandez, Jokela, Vuorinen,1603.02943

Annala, Ecker, Hoyos, Jokela, Rodriguez-Fernandez, Vuorinen, 1711.06244

Fadafan, Cruz Rojas, Evans, 1911.12705, 2009.14079

V-QCD model Jarvinen, Kiritsis, 1112.1261

Jokela, Jarvinen, Remes, 1809.07770, 2111.12101

Bartolini, Gudnason, Jarvinen, 2504.01758

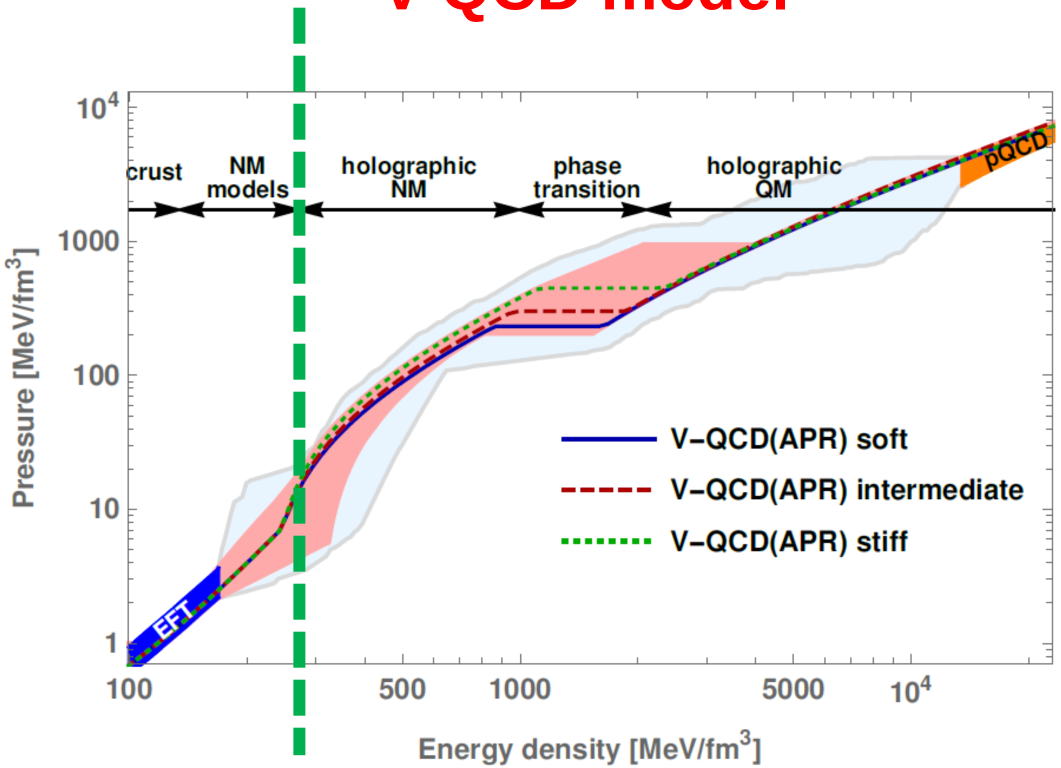
WSS model Sakai, Sugimoto, hep-th/0412141

Kovensky, Poole, Schmitt, 2111.03374

Papadopoulos, Schmitt, 2411.08023

Equation of State from holographic models

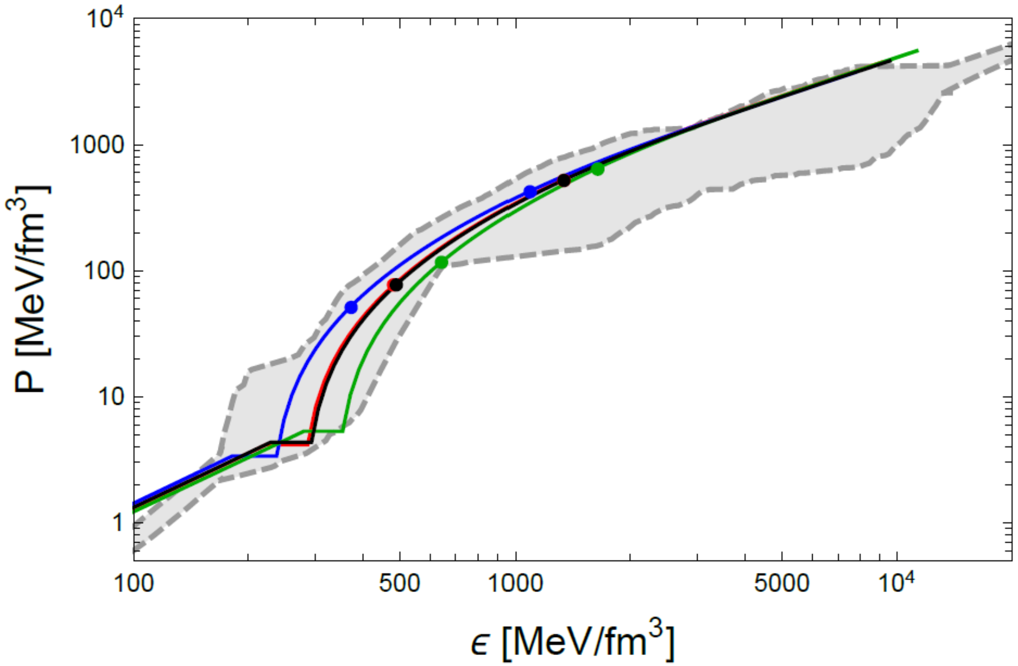
V-QCD model



Hybrid model

Jarvinen, 2110.08281

WSS model



Kovensky, Poole, Schmitt, 2111.03374

The EoS determines the static properties of neutron stars

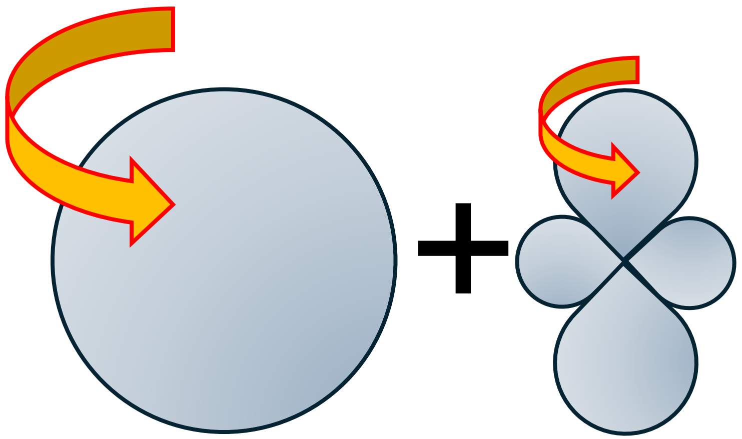
However, when interested in dynamical evolution other properties become relevant: transport, emission rates, etc

Holography is well-suited for this task: previous success in description of quark-gluon plasma

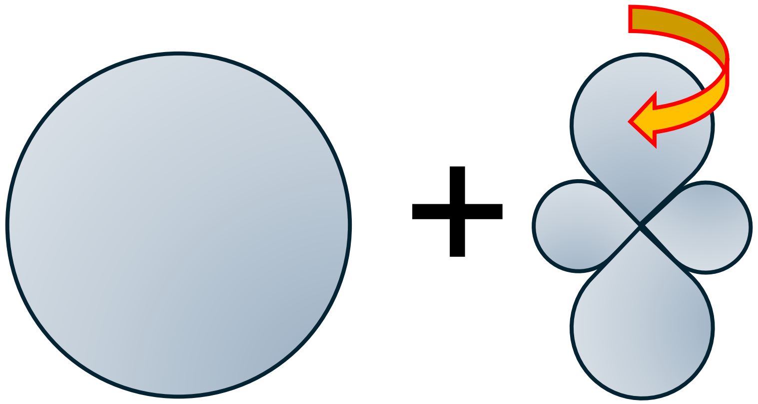
e.g. KSS estimate of shear viscosity
Kovtun, Son. Starinets, hep-th/0405231

r-mode instability

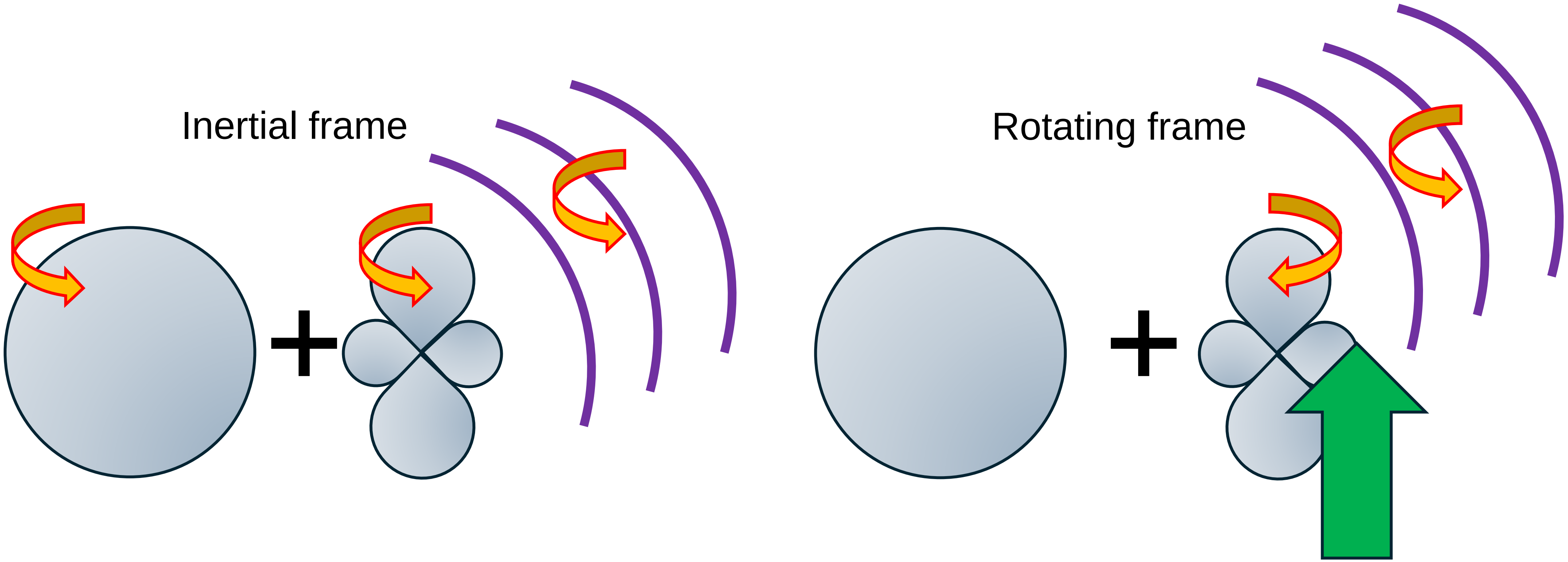
Inertial frame



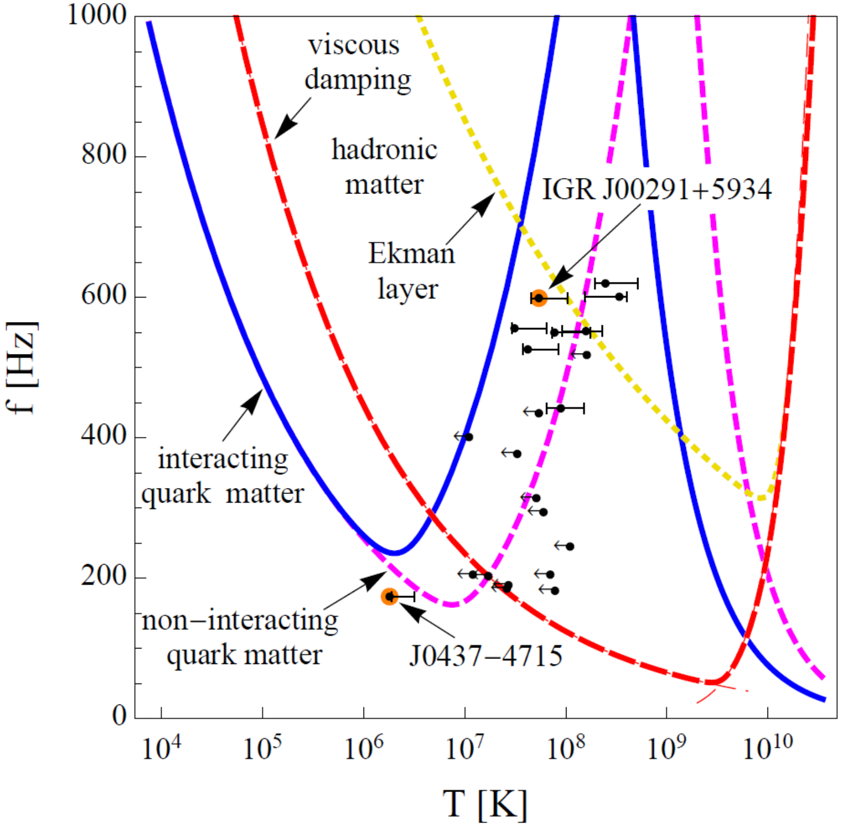
Rotating frame



r-mode instability

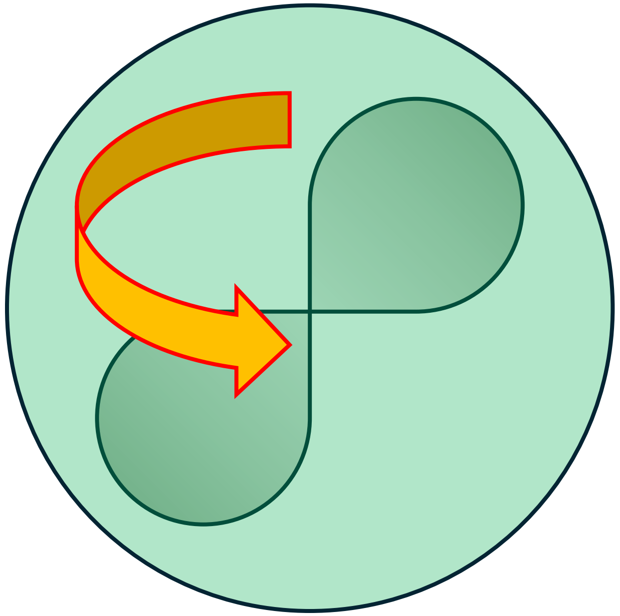


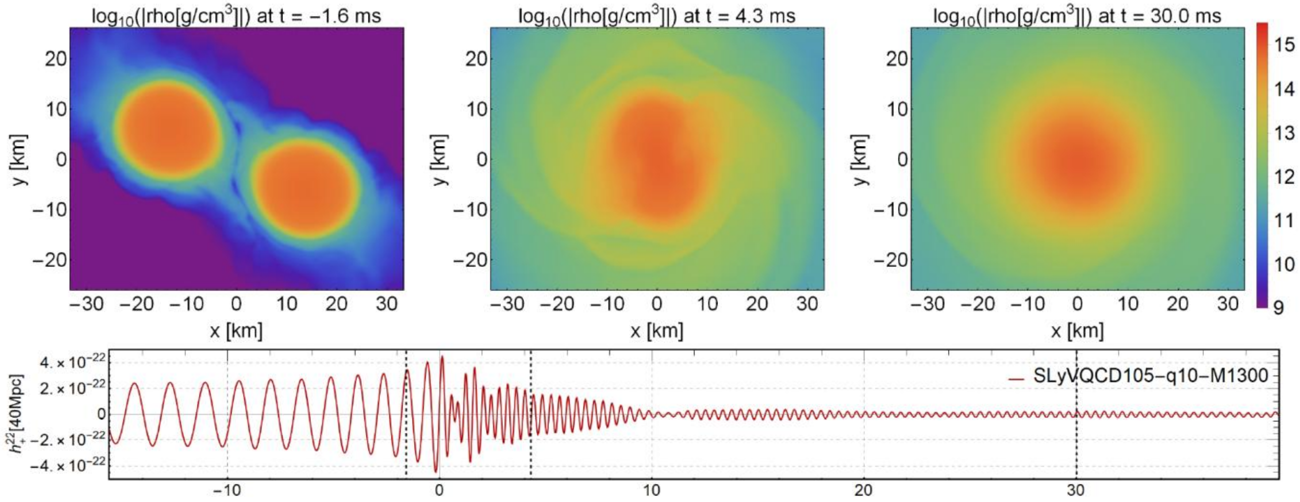
Damping of oscillations: viscosity

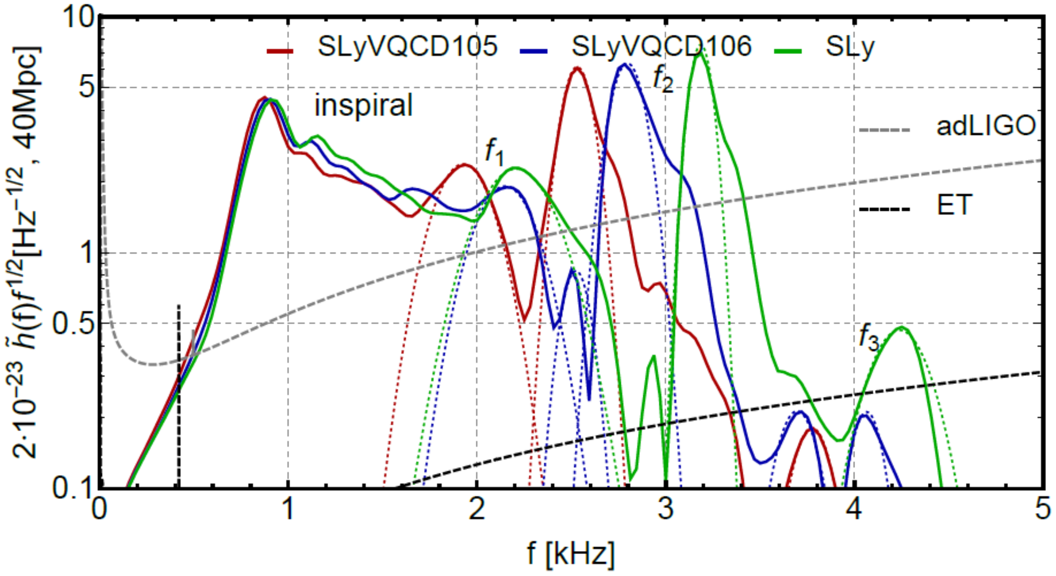


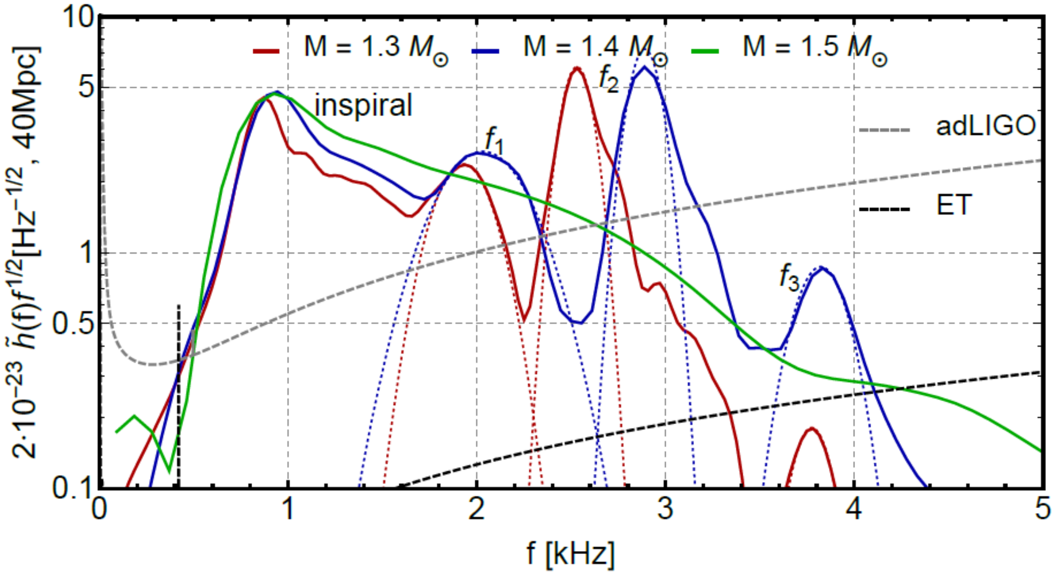
Alford, Schwenzer 1310.3524

Mergers









Ecker, Jarvinen, Nijs, van der Schee, 1908.03213

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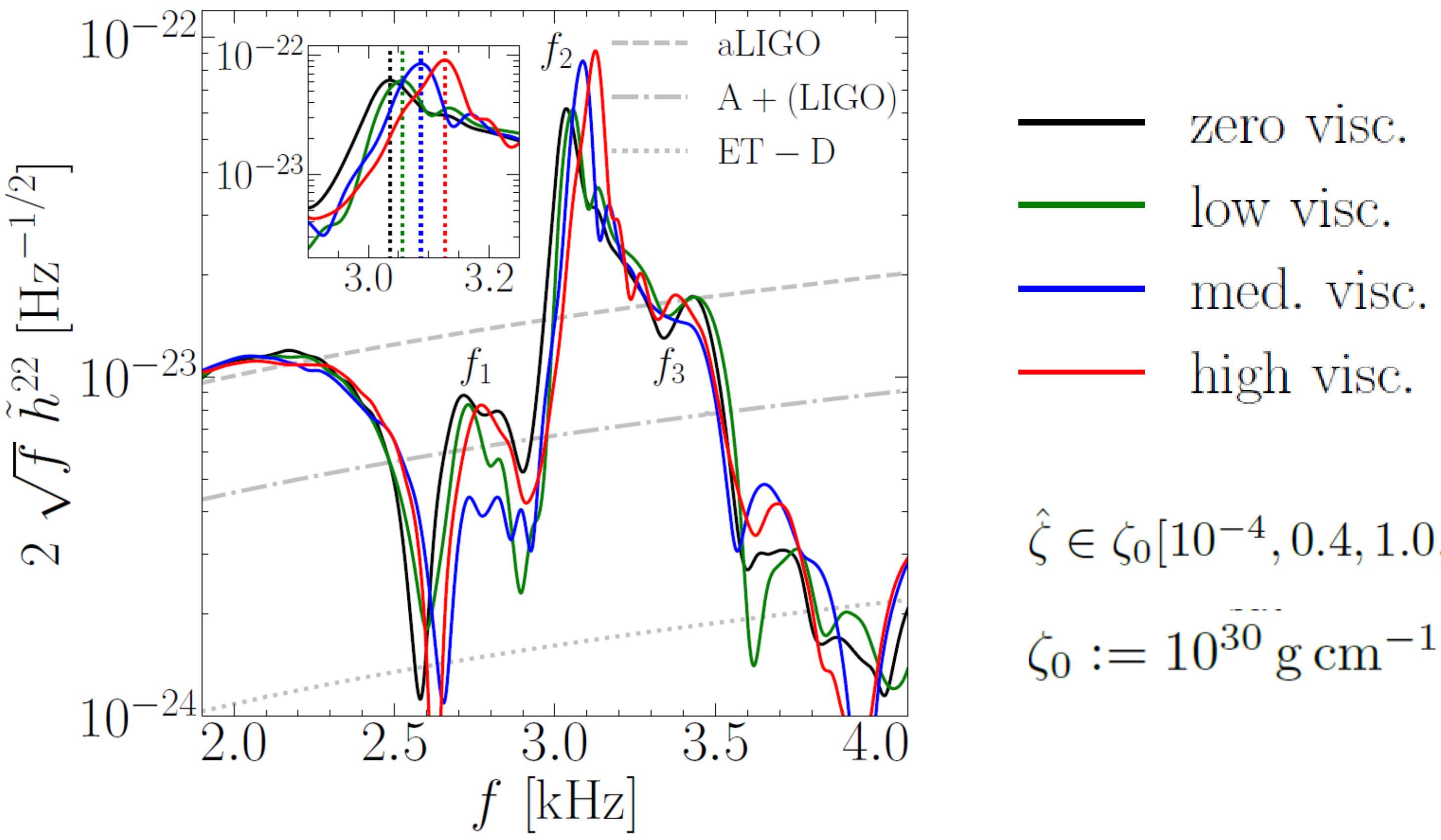


$\hat{\zeta} \in \zeta_0[10^{-4}, 0.4, 1.0, 2.4]$

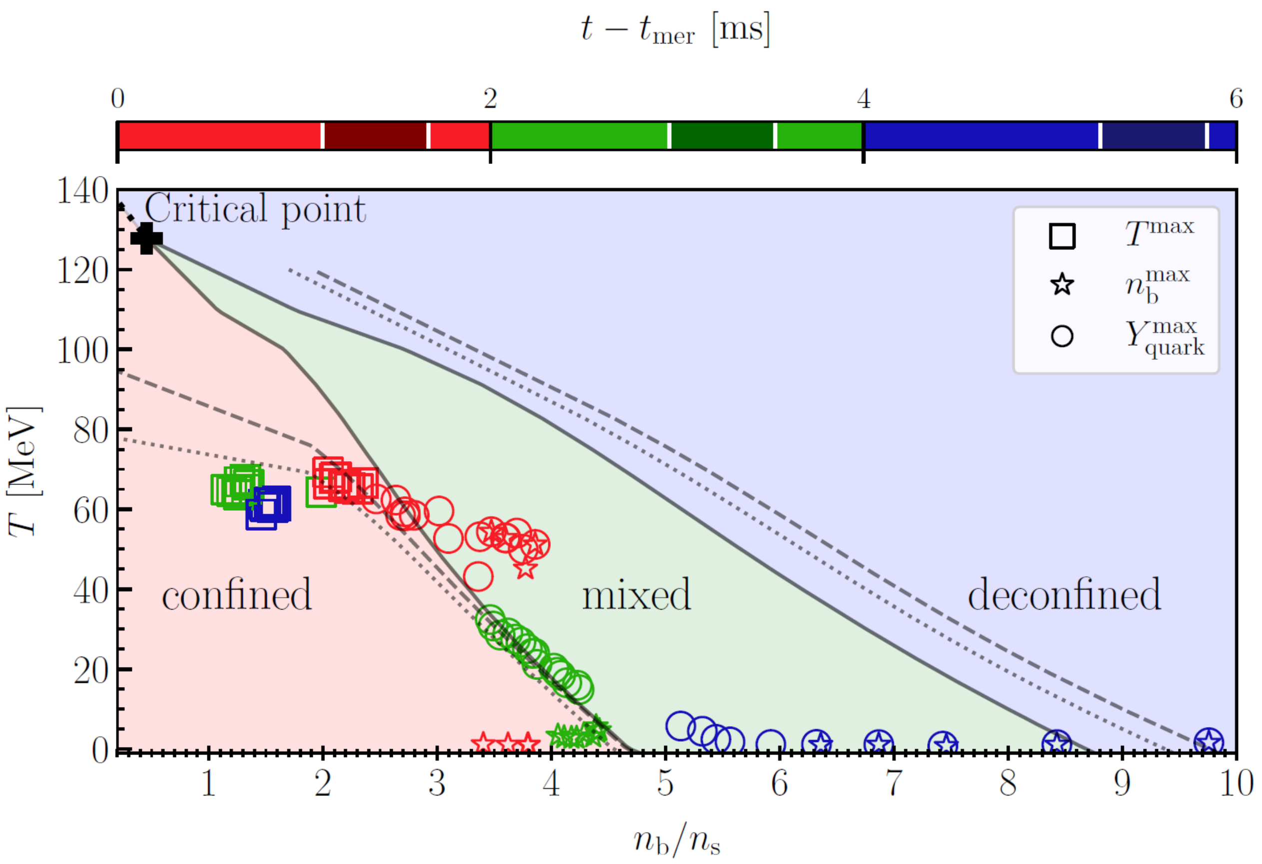
$\zeta_0 := 10^{30}\,\mathrm{g\,cm^{-1}\,s^{-1}}$

Chabanov, Rezzolla 2307.10464

In the following we will be interested in estimating the viscosity of (unpaired) quark matter at large baryon density

Holographic dual description = charged black hole

Quark matter in mergers and core of neutron stars



Tootle, Ecker, Topolski, Demircik, Jarvinen 2205.05691

Other hybrid models e.g. Blacker, Bauswein 2406.14669

Hints of a quark core in massive stars from model-independent estimates of EoS

Annala, Gorda, Kurkela, Nattila, Vuorinen 1903.09121

Textbook definition of viscosities

Hydrodynamic constitutive relations (Landau frame)

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}\partial_\alpha u^\alpha$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu, \quad u^\mu u_\mu = -1$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\partial_{(\alpha}u_{\beta)} - \frac{1}{3}P_{\alpha\beta}\partial_\sigma u^\sigma\right)$$

Kubo formulas

$$\eta = -\lim_{\omega\to 0}\frac{\mathrm{Im}G^R_\eta(\omega)}{\omega}, \qquad \zeta = -\lim_{\omega\to 0}\frac{\mathrm{Im}G^R_\zeta(\omega)}{\omega}$$

$$G^R_\eta(\omega) = -i\int dt d^3\vec{x} e^{i\omega t}\theta(t)\langle T_{xy}(t,\vec{x})T_{xy}(0,\vec{0})\rangle$$

$$G^R_\zeta(\omega) = -\frac{i}{9}\int dt d^3\vec{x} e^{i\omega t}\theta(t)\langle T_j{}^j(t,\vec{x})T_k{}^k(0,\vec{0})\rangle$$

Microscopic (QCD) contributions

perturbative QCD

$$\eta \approx 4.4 \times 10^{-3} \frac{\mu^2 m_{\mathrm{D}}^{2/3}}{\alpha_s^2 T^{5/3}}$$

Heiselberg, Pethick, Phys.Rev.D 48 (1993)

Holographic (D3-D7) model at T=0

$$\eta_{\mathrm{f}} = \frac{s_{\mathrm{f}}}{4\pi} = \frac{N_{\mathrm{f}} N_{\mathrm{c}} \gamma_*}{8\pi\sqrt{\lambda}}\, \mu(\mu^2 - M_{\mathrm{q}}^2)$$

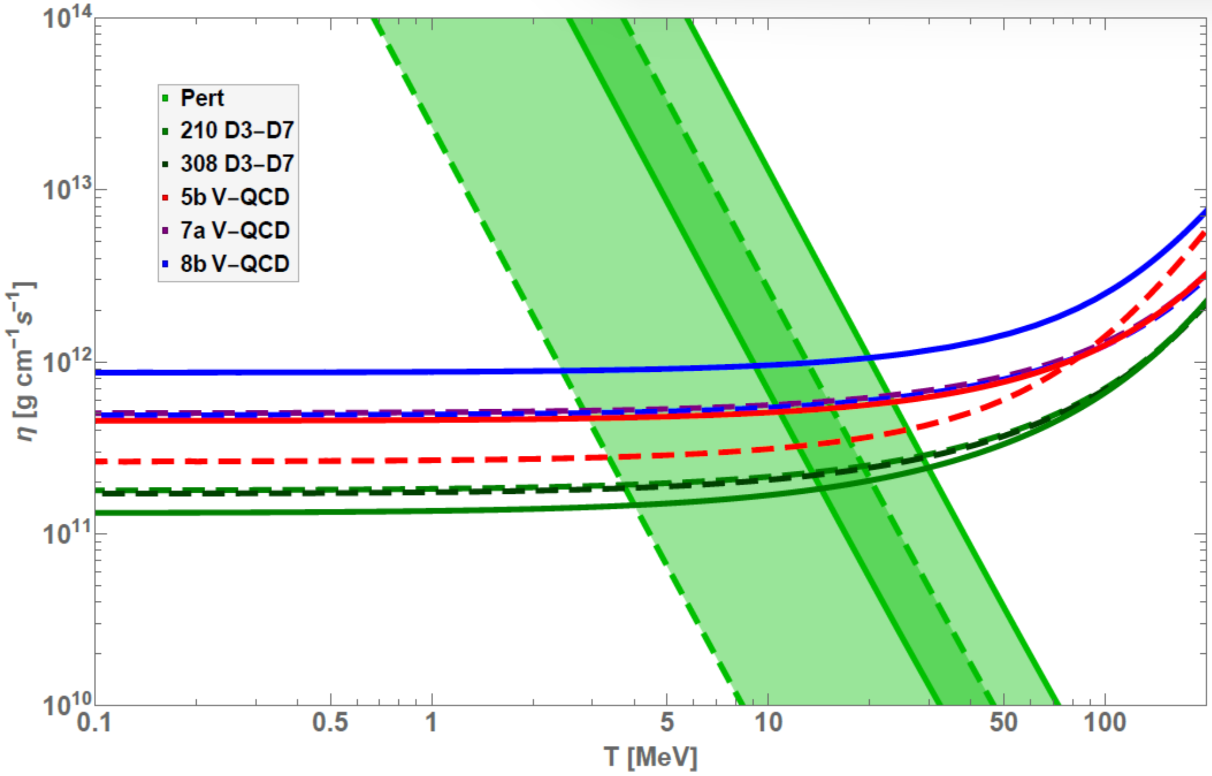
$$\zeta = N_{\mathrm{c}} N_{\mathrm{f}} \frac{\gamma_*}{2\pi} \frac{M_{\mathrm{q}}^2 \mu}{\sqrt{\lambda}} \frac{(\mu^2 - M_{\mathrm{q}}^2)^2}{(3\mu^2 - M_{\mathrm{q}}^2)^2}$$

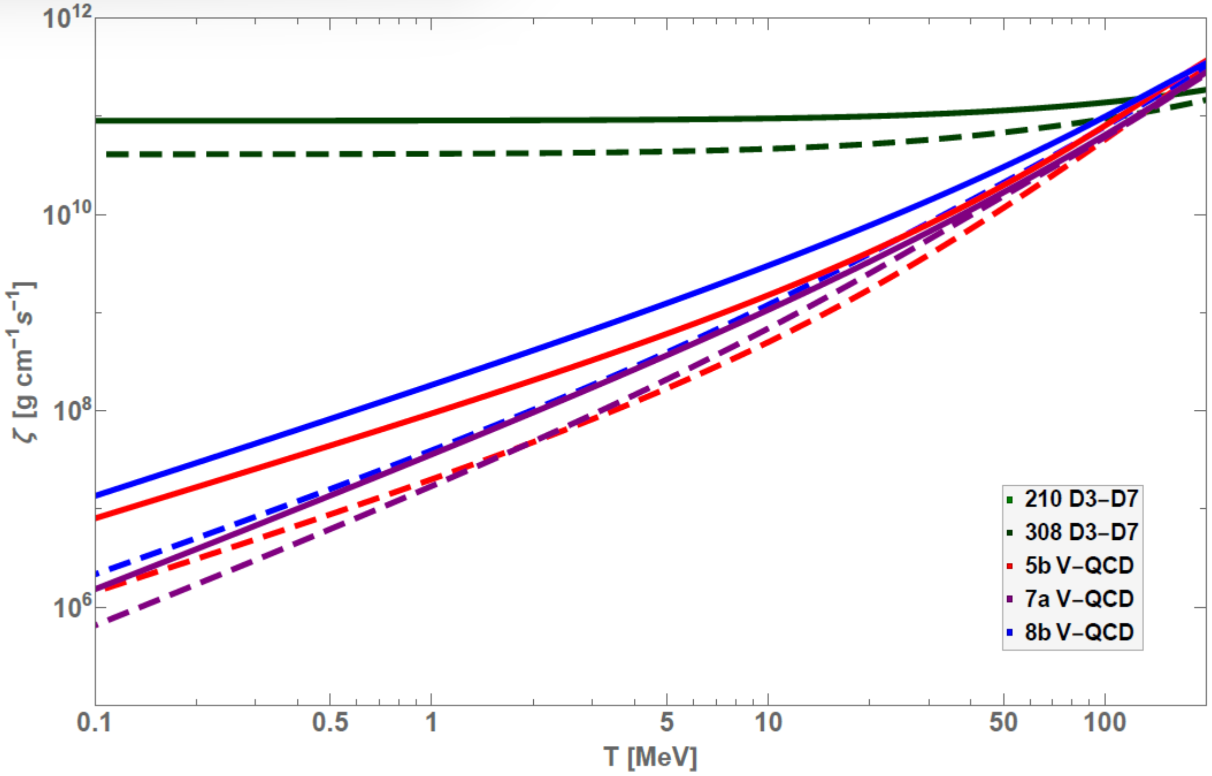
Hoyos, Jokela, Jarvinen, Subils, Tarrio, Vuorinen 2109.12122

Microscopic (QCD) contributions

Shear viscosity

Bulk viscosity





$\mu =$ 450 MeV (dashed lines) 600 MeV (solid lines)

Hoyos, Jokela, Jarvinen, Subils, Tarrio, Vuorinen
2005.14205

In addition to the microscopic viscosity there is an effective bulk viscosity due to chemical re-equilibration

Due to a coincidence in time scales, the effective bulk viscosity contribution is the most relevant one for density oscillations

The effective viscosity can be implemented in Müller-Israel-Stewart (MIS) formulation of hydrodynamics or in reacting fluid mixtures

Some references studying bulk viscosity effects in binary mergers

Perego, Bernuzzi, Radice 1903.07898

Chabanov, Rezzolla, Rischke 2102.10419

Most, Harris, Plumberg, Alford, Noronha, Noronha-Hostler, Pretorius, Witek, Yunes 2107.05094

Hammond, Hawke, Andersson 2108.08649

Radice, Bernuzzi, Perego, Haas 2111.14858

Camelio, Gavassino, Antonelli, Bernuzzi, Haskell 2204.11809, 2204.11810

Zappa, Bernuzzi, Radice, Perego 2210.11491

Chabanov, Rezzolla 2307.10464, 2311.13027

Yang, Hippert, Speranza, Noronha 2309.01864

NEUTRINO TRAPPING
Espino, Hammond, Radice, Bernuzzi, Gamba, Zappa, Longo Micchi, Perego 2311.00031

BOUNDS, TIDAL DEFORMABILITY
Ripley, Hegade, Chandramouli, Yunes 2312.11659
Hegade, Ripley, Yunes 2407.02584

OTHER CALCULATION OF BULK VISCOSITY
Hernandez, Manuel,Tolos 2402.06595

Multi-component reacting fluid mixtures

Camelio, Gavassino, Antonelli, Bernuzzi, Haskell
2204.11809

$$\nabla_\mu(\rho u^\mu) = 0,$$
$$\nabla_\mu(T^{\mu\nu}) = -\mathcal{Q}u^\nu,$$
$$\nabla_\mu(\rho Y_i u^\mu) = m_{\rm n}\mathcal{R}_i,$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu},$$

$$\mathcal{Q} = \sum_i \mathcal{Q}_j$$

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MIS hydrodynamics: Hiscock-Lindblom

$$\nabla_\mu(\rho u^\mu) = 0,$$
$$\nabla_\mu(T^{\mu\nu}) = -\mathcal{Q}_{\mathrm{bv}}u^\nu$$
$$\nabla_\mu(\Pi u^\mu) = -\frac{\Pi}{\tau} - \left(\frac{1}{\chi} - \frac{\Pi}{2}\right)\nabla_\mu u^\mu - \frac{\Pi}{2}u^\mu\nabla_\mu\left(\log\frac{\chi}{T^{\mathrm{eq}}}\right).$$

$$T^{\mu\nu} = \left(\epsilon + p^{\mathrm{eq}}(\rho,\epsilon) + \Pi\right)u^\mu u^\nu + \left(p^{\mathrm{eq}}(\rho,\epsilon) + \Pi\right)g^{\mu\nu}.$$
$$\mathcal{Q}_{\mathrm{bv}}(\rho, s, \Pi) = \mathcal{Q}^{\mathrm{eq}}(\rho, s^{\mathrm{eq}}) + \frac{\partial\mathcal{Q}}{\partial\Pi}\Pi + \mathcal{O}(\Pi^2)$$

$$\Pi = -\zeta\left[\nabla_\mu u^\mu + \chi u^\mu\nabla_\mu\Pi + \frac{\Pi}{2}T^{\mathrm{eq}}\nabla_\mu\left(\frac{\chi u^\mu}{T^{\mathrm{eq}}}\right)\right]$$

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MIS Maxwell-Cattaneo:

$$\nabla_\mu(\Pi u^\mu) = -\frac{\Pi}{\tau} - \left(\frac{1}{\chi} - \Pi\right)\nabla_\mu u^\mu \qquad \Pi = -\zeta\nabla_\mu u^\mu - \tau u^\mu\nabla_\mu\Pi$$

Mathematical map from multicomponent reacting fluids to MIS

Gavassino, Antonelli, Haskell, 2003.04609

$$\zeta = n^4 \Xi^{ab} \frac{\partial Y_a^{\mathrm{eq}}}{\partial n}\bigg|_s \frac{\partial Y_b^{\mathrm{eq}}}{\partial n}\bigg|_s, \qquad n = \rho/m_{\mathrm{n}}$$

$$\Xi_{ab} = \frac{\partial \mathcal{R}_a(\{\mathbb{A}^j = 0\}_{\forall j})}{\partial \mathbb{A}^b}\bigg|_{\rho, s, \{\mathbb{A}^i\}_{i \neq b}},$$

$$\mathrm{d}u(\rho, s, \{Y_i\}_i) = \frac{p}{\rho^2}\mathrm{d}\rho + \frac{T}{m_{\mathrm{n}}}\mathrm{d}s - \sum_i \frac{\mathbb{A}^i}{m_{\mathrm{n}}}\mathrm{d}Y_i,$$

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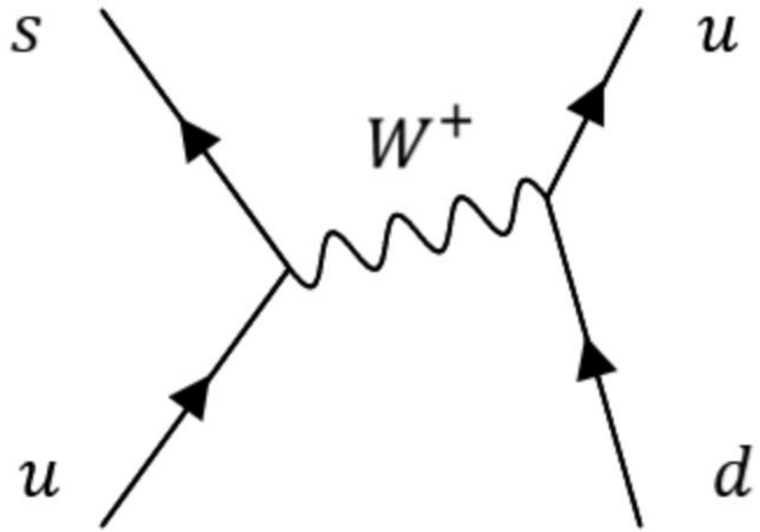
Let us see how the effective viscosity emerges

beta equilibrium



$$u + d \rightleftharpoons u + s, \quad u + \ell^- \to d, s + \nu_\ell,$$

$$d, s \to u + \ell^- + \bar{\nu}_\ell, \quad \ell_1^- \to \ell_2^- + \nu_{\ell_1} + \bar{\nu}_{\ell_2}$$

$$\mu_u + \mu_\ell = \mu_d = \mu_s, \qquad \mu_\ell = \mu_e = \mu_\mu$$

Weak reaction rates

$$
\begin{aligned}
\frac{dn_u}{dt} &= \sum_{\ell=e,\mu} \left(\Gamma_{d\to u\ell\bar{\nu}} - \Gamma_{u\ell\to d\nu} + \Gamma_{s\to u\ell\bar{\nu}} - \Gamma_{u\ell\to s\nu}\right) \ , \\
\frac{dn_d}{dt} &= \Gamma_{su\to ud} - \Gamma_{ud\to su} + \sum_{\ell=e,\mu} \left(\Gamma_{u\ell\to d\nu} - \Gamma_{d\to u\ell\bar{\nu}}\right) \ , \\
\frac{dn_s}{dt} &= \Gamma_{ud\to su} - \Gamma_{su\to ud} + \sum_{\ell=e,\mu} \left(\Gamma_{u\ell\to s\nu} - \Gamma_{s\to u\ell\bar{\nu}}\right) \ , \\
\frac{dn_{\ell_1}}{dt} &= \Gamma_{d\to u\ell_1\bar{\nu}} - \Gamma_{u\ell_1\to d\nu} + \Gamma_{s\to u\ell_1\bar{\nu}} - \Gamma_{u\ell_1\to s\nu} + \Gamma_{\ell_2\to\ell_1\nu_2\bar{\nu}_1} - \Gamma_{\ell_1\to\ell_2\nu_1\bar{\nu}_2}
\end{aligned}
$$

EW processes conserve baryon number and electric charge

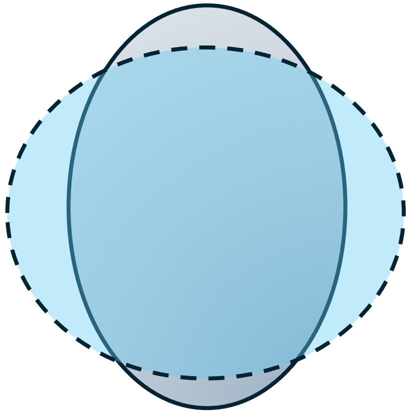
$$\frac{dn_B}{dt} = \frac{dn_Q}{dt} = 0$$

$$n_B = \frac{1}{3}(n_u + n_d + n_s) \qquad n_Q = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu$$

Small deviations from equilibrium

$$
\begin{aligned}
\Gamma_{ud\to su} - \Gamma_{su\to ud} &\approx \lambda_{ds}(\delta\mu_s - \delta\mu_d), \\
\Gamma_{u\ell\to d\nu} - \Gamma_{d\to u\ell\bar{\nu}} &\approx \lambda^{\ell}_{ud}(\delta\mu_d - \delta\mu_u - \delta\mu_\ell), \\
\Gamma_{u\ell\to s\nu} - \Gamma_{s\to u\ell\bar{\nu}} &\approx \lambda^{\ell}_{us}(\delta\mu_s - \delta\mu_u - \delta\mu_\ell), \\
\Gamma_{e\to\mu\nu_2\bar{\nu}_1} - \Gamma_{\mu\to e\nu_1\bar{\nu}_2} &\approx \lambda_{e\mu}(\delta\mu_\mu - \delta\mu_e).
\end{aligned}
$$

Effective bulk viscosity



$$n_B \approx n_B^{\mathrm{eq}} + \Delta n_B \sin(\omega t)$$

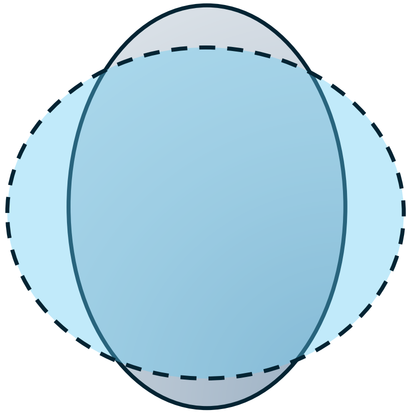
$$\overline{\partial_t \varepsilon} \approx -\zeta \overline{(\boldsymbol{\nabla} \cdot \boldsymbol{v})^2}$$

$$\overline{\partial_t \varepsilon} \approx -n_B^{eq} \overline{P \frac{dV_B}{dt}} \qquad V_B = \frac{1}{n_B}$$

Sawyer, Phys. Rev. D 39 (1989)

Haensel, Levensh, Yakovlev, astro-ph/0004183

Effective bulk viscosity



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$$\overline{\partial_t \varepsilon} \approx -n_B^{eq} \overline{P \frac{dV_B}{dt}} \qquad V_B = \frac{1}{n_B}$$

$$\partial_t n_B + \boldsymbol{\nabla}(n_B \boldsymbol{v}) = 0 \quad \Rightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{v} \approx -\frac{\partial_t \delta n_B}{n_B^{eq}}$$

$$P \approx P^{eq} + \sum_f \frac{\partial P}{\partial \mu_f} \delta \mu_f \qquad \delta \mu_f = \left(\frac{\partial n_f}{\partial \mu_f} \right)^{-1} \delta n_f = \chi_{ff}^{-1} \delta n_f$$

Sawyer, Phys. Rev. D 39 (1989)

Haensel, Levensh, Yakovlev, astro-ph/0004183

Leading contribution

$$u + d \rightleftharpoons u + s$$

Effective bulk viscosity

$$\zeta = \frac{\lambda_1 A_1^2}{\omega^2 + \lambda_1^2 C_1^2}$$

$$\lambda_1 = \lambda_{ds}$$

$$C_1 = \frac{1}{\chi_{dd}} + \frac{1}{\chi_{ss}} \,, \quad A_1 = \frac{n_d}{\chi_{dd}} - \frac{n_s}{\chi_{ss}}$$

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Equation of State

Leading contribution

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Effective bulk viscosity

$$\zeta = \frac{\lambda_1 A_1^2}{\omega^2 + \lambda_1^2 C_1^2}$$

Scattering amplitudes

$$\lambda_1 = \lambda_{ds}$$

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Equation of State

Leading contribution

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Effective bulk viscosity

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Scattering amplitudes

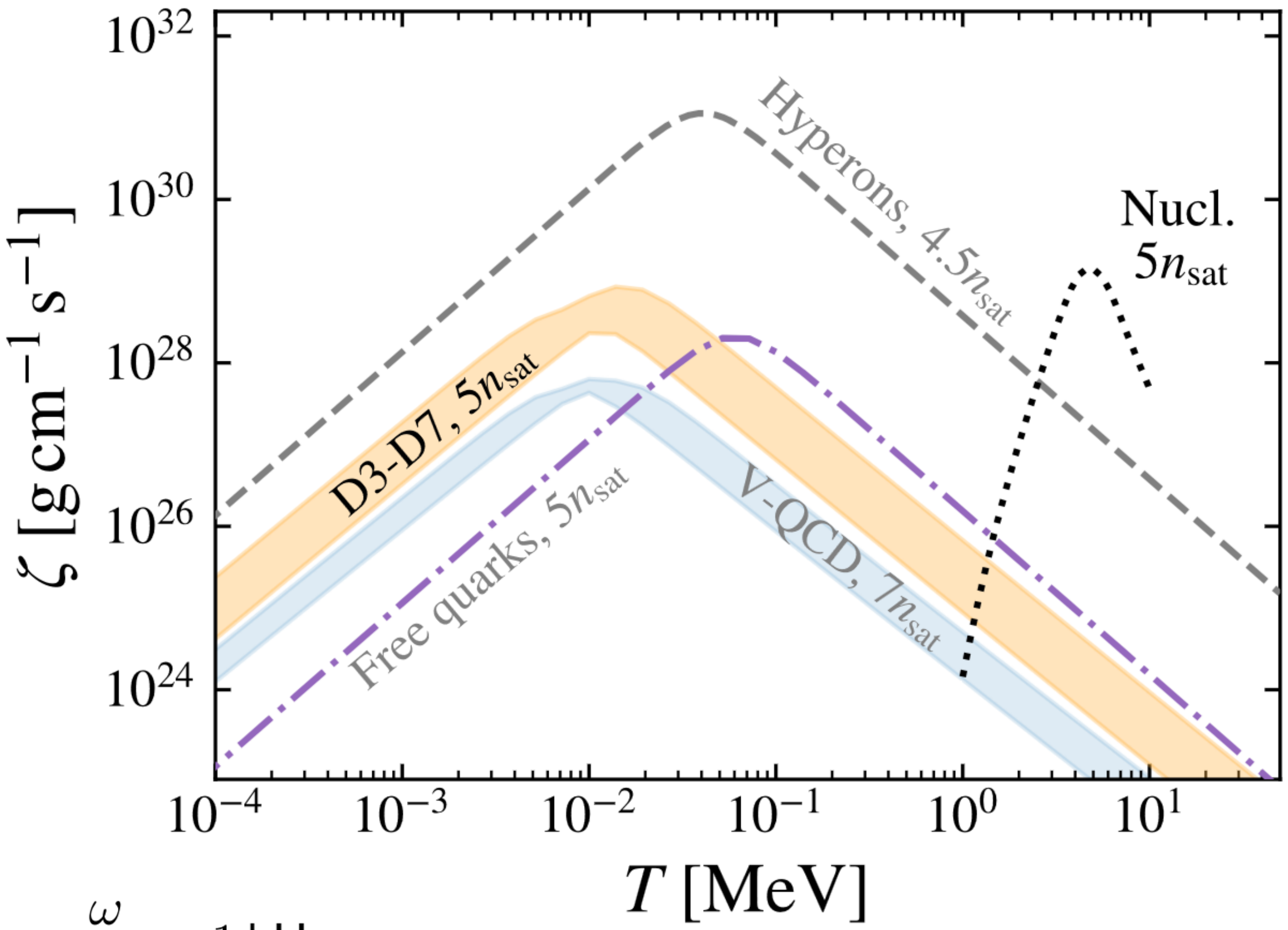
$$\lambda_1 = \lambda_{ds}$$

Take pQCD value

$$C_1 = \frac{1}{\chi_{dd}} + \frac{1}{\chi_{ss}}\,, \quad A_1 = \frac{n_d}{\chi_{dd}} - \frac{n_s}{\chi_{ss}}$$

Equation of State

Effective bulk viscosity from holography



$$f = \frac{\omega}{2\pi} = 1\,\mathrm{kHz}$$

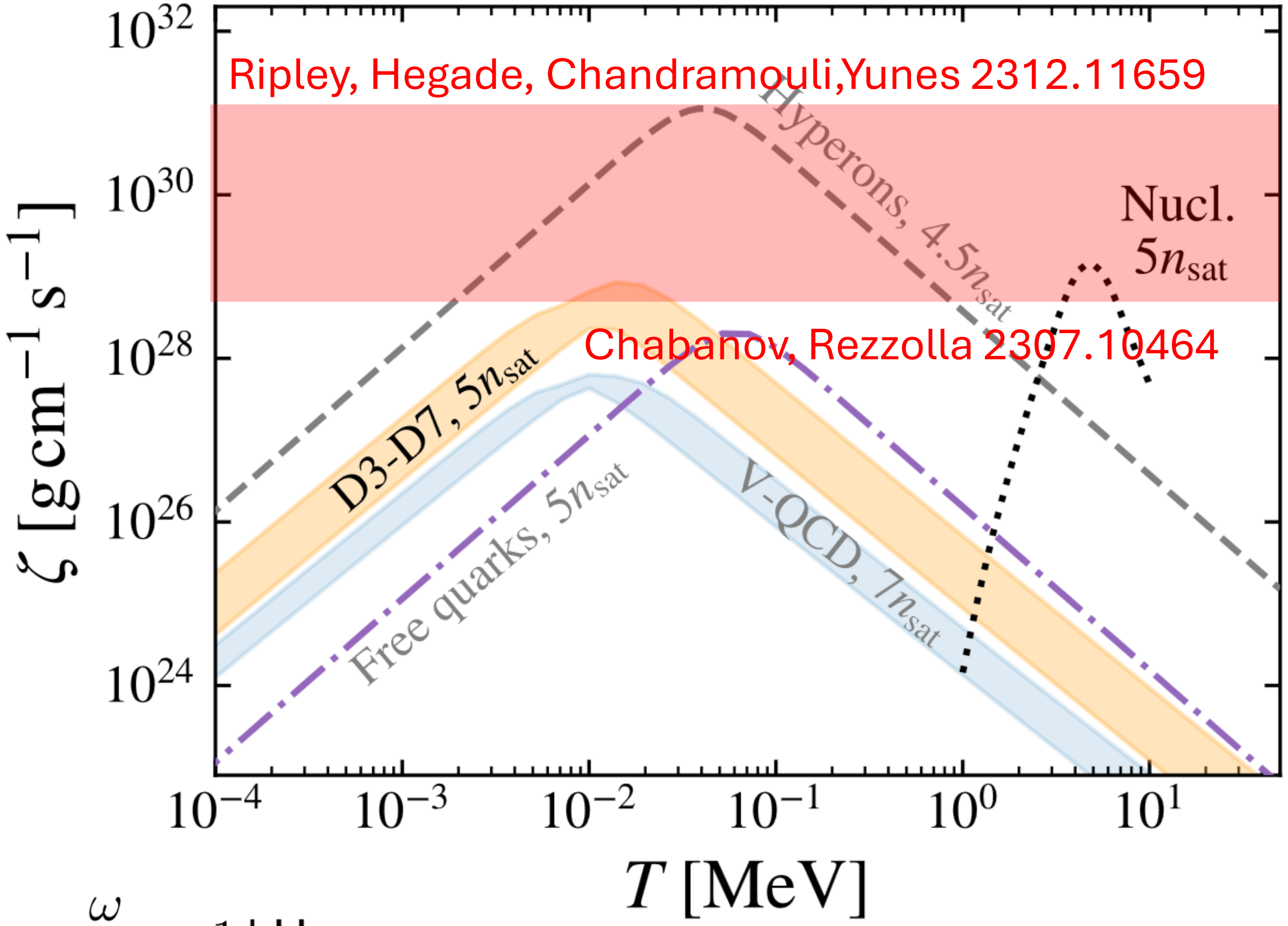
EoS D3-D7 and V-QCD computed holographically

D3-D7 matched to pQCD at very large density

V-QCD fitted to lattice data at small density

Cruz Rojas, Gorda, Hoyos, Jokela, Jarvinen, Kurkela, Paatelainen, Sappi, Vuorinen 2402.00621

Effective bulk viscosity from holography



EoS D3-D7 and V-QCD computed holographically

D3-D7 matched to pQCD at very large density

V-QCD fitted to lattice data at small density

$$f = \frac{\omega}{2\pi} = 1\,\text{kHz}$$

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Analytic formula for D3-D7 with zero temperature EoS

$$\zeta = \frac{4\lambda_1\,\mu_d^6\,(M_s^2 - M_d^2)^2}{K_d^2 K_s^2 \omega^2 + \pi^4 \lambda_1^2\,(K_d + K_s)^2}$$

$$K_i \equiv 3\mu_d^2 - M_i^2$$

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Temperature dependence determined by reaction rate

Formula simplifies for massless quarks and small temperature

Perturbative calculation + non-Fermi liquid correction:

$$\lambda_1 = \left(1 + \sigma \log \frac{\Lambda}{T}\right)^4 \frac{64}{5\pi^3} G_F^2 \sin^2\theta_c \cos^2\theta_c \mu_d^5 T^2$$

$$\Lambda \approx 0.158\sqrt{\alpha_s}\sqrt{\mu_u^2 + \mu_d^2 + \mu_s^2}$$

$$\sigma \equiv 4\alpha_s/(9\pi)$$

Heiselberg, Madsen, Riisager, Phys. Scripta 34 (1986)

Heiselberg, Phys. Scripta 46 (1992)

Madsen, Phys. Rev. D47 (1993)

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Madsen, Phys. Rev. D47 (1993)

Strong coupling calculation? Holography has access only to gauge-invariant objects

Hoyos, Olzi, Rodriguez-Fernandez 2407.21643

Breaking of flavor symmetry by Fermi's interaction

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F (J^\mu_{ch})^\dagger J_{ch\,\mu}$$

$$J^\mu_{ch} = \overline{\nu}_{e\,L}\gamma^\mu e_L + \overline{\nu}_{\mu\,L}\gamma^\mu \mu_L + \cos\theta_C\,\overline{u}_L\gamma^\mu d_L + \sin\theta_C\,\overline{u}_L\gamma^\mu s_L$$

Not invariant under flavor rotations

$$(J^\mu_{f\,\chi})^a{}_b = \bar{f}_{\chi\,b}\gamma^\mu f^a_\chi$$

Fermion species

Chirality

$$\delta_{\theta_{f\,\chi}} J^\mu_{f\,\chi} = i\theta^A_{f\,\chi}[J^\mu_{f\,\chi}, T^A_{f\,\chi}]$$

Ward identities

$$
\begin{aligned}
\partial_\mu \left\langle (J_q^\mu)^u{}_u \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \sum_{\ell=e,\mu} \Big[\cos\theta_C \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^d{}_u - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad + \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] , \\
\partial_\mu \left\langle (J_q^\mu)^d{}_d \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \sin\theta_C \eta_{\mu\nu} \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{q\,L})^d{}_u - (J^\mu_{q\,L})^s{}_u (J^\nu_{q\,L})^u{}_d \right\rangle \\
&\quad - \cos\theta_C \sum_{\ell=e,\mu} \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] , \\
\partial_\mu \left\langle (J_q^\mu)^s{}_s \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[- \cos\theta_C \sin\theta_C \eta_{\mu\nu} \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{q\,L})^d{}_u - (J^\mu_{q\,L})^s{}_u (J^\nu_{q\,L})^u{}_d \right\rangle \\
&\quad - \sin\theta_C \sum_{\ell=e,\mu} \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] , \\
\partial_\mu \left\langle (J_l^\mu)^{\nu_\ell}{}_{\nu_\ell} \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[- \cos\theta_C \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^\ell{}_{\nu_\ell} \right\rangle \\
&\quad - \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad - \left\langle (J^\mu_{l\,L})^{\nu_{\ell'}}{}_{\ell'} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{l\,L})^{\ell'}{}_{\nu_{\ell'}} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} \right\rangle \Big] , \\
\partial_\mu \left\langle (J_l^\mu)^\ell{}_\ell \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad + \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad + \left\langle (J^\mu_{l\,L})^{\nu_{\ell'}}{}_{\ell'} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{l\,L})^{\ell'}{}_{\nu_{\ell'}} (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] .
\end{aligned}
$$

Ward identities

$$
\begin{aligned}
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&\quad + \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] , \\
\partial_\mu \left\langle (J_q^\mu)^d{}_d \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \sin\theta_C \eta_{\mu\nu} \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{q\,L})^d{}_u - (J^\mu_{q\,L})^s{}_u (J^\nu_{q\,L})^u{}_d \right\rangle
\end{aligned}
$$

$$
\boxed{\partial_\mu J^\mu \sim G_F \eta_{\mu\nu} \left\langle J^\mu J^\nu \right\rangle}
$$

$$
\begin{aligned}
\partial_\mu \left\langle (J_l^\mu)^{\nu_\ell}{}_{\nu_\ell} \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[-\cos\theta_C \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^\ell{}_{\nu_\ell} \right\rangle \\
&\quad - \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad - \left\langle (J^\mu_{l\,L})^{\nu_{\ell'}}{}_{\ell'} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{l\,L})^{\ell'}{}_{\nu_{\ell'}} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} \right\rangle \Big] , \\
\partial_\mu \left\langle (J_l^\mu)^\ell{}_\ell \right\rangle &= i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \left\langle (J^\mu_{q\,L})^u{}_d (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^d{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad + \sin\theta_C \left\langle (J^\mu_{q\,L})^u{}_s (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{q\,L})^s{}_u (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \\
&\quad + \left\langle (J^\mu_{l\,L})^{\nu_{\ell'}}{}_{\ell'} (J^\nu_{l\,L})^\ell{}_{\nu_\ell} - (J^\mu_{l\,L})^{\ell'}{}_{\nu_{\ell'}} (J^\nu_{l\,L})^{\nu_\ell}{}_\ell \right\rangle \Big] .
\end{aligned}
$$

Conservation of electric charge and baryon and lepton numbers

$$\partial_\mu \left\langle (J_q^\mu)^u{}_u + (J_q^\mu)^d{}_d + (J_q^\mu)^s{}_s \right\rangle = 0, \quad \partial_\mu \left\langle (J_l^\mu)^{\nu_\ell}{}_{\nu_\ell} + (J_l^\mu)^\ell{}_\ell \right\rangle = 0,$$

$$\partial_\mu \left\langle \frac{2}{3}(J_q^\mu)^u{}_u - \frac{1}{3}(J_q^\mu)^d{}_d - \frac{1}{3}(J_q^\mu)^s{}_s - \sum_{\ell=e,\mu} (J_l^\mu)^\ell{}_\ell \right\rangle = 0.$$

Leading contributions in the Fermi coupling

$$\langle \mathcal{O}(x)\rangle_{G_F} = \left\langle T_C \left[\mathcal{O}(x) e^{i\int_C \mathcal{L}_{\rm Fermi}}\right]\right\rangle_0 \approx \langle \mathcal{O}(x)\rangle_0 + i\int_C d^4x' \, \langle T_C \left[\mathcal{O}(x)\mathcal{L}_{\rm Fermi}(x')\right]\rangle_0 \,.$$

Factorization of the current four-point function

$$\left\langle \left[(J_f^\mu)^a{}_b(x)(J_f^\nu)^c{}_d(x)\right]\left[(J_{f'}^\alpha)^{a'}{}_{b'}(x')(J_{f'}^\beta)^{c'}{}_{d'}(x')\right]\right\rangle_0 \approx$$
$$\delta_{ff'}\left[\left\langle (J_f^\mu)^a{}_b(x)(J_f^\alpha)^{a'}{}_{b'}(x')\right\rangle_0 \left\langle (J_f^\nu)^c{}_d(x)(J_f^\beta)^{c'}{}_{d'}(x')\right\rangle_0 + \left\langle (J_f^\mu)^a{}_b(x)(J_f^\beta)^{c'}{}_{d'}(x')\right\rangle_0 \left\langle (J_f^\nu)^c{}_d(x)(J_f^\alpha)^{a'}{}_{b'}(x')\right\rangle_0\right] .$$

Expected at large-N

Non-factorization from effective eight-fermion interactions

Non-conservation equations

$$
\begin{aligned}
\partial_\mu \left\langle (J_q^\mu)^u{}_u \right\rangle &\approx -4G_F^2 \sum_{\ell=e,\mu} \left(\cos^2\theta_C \gamma_{u\ell\to d\nu} + \sin^2\theta_C \gamma_{u\ell\to s\nu}\right), \\
\partial_\mu \left\langle (J_q^\mu)^d{}_d \right\rangle &\approx 4G_F^2 \left(-\sin\theta_C^2 \cos^2\theta_C \gamma_{ud\to su} + \cos^2\theta_C \sum_{\ell=e,\mu} \gamma_{u\ell\to d\nu} \right), \\
\partial_\mu \left\langle (J_q^\mu)^s{}_s \right\rangle &\approx 4G_F^2 \left(\sin\theta_C^2 \cos^2\theta_C \gamma_{ud\to su} + \sin^2\theta_C \sum_{\ell=e,\mu} \gamma_{u\ell\to s\nu} \right), \\
\partial_\mu \left\langle (J_l^\mu)^{\nu_\ell}{}_{\nu_\ell} \right\rangle &\approx 4G_F^2 \left(\cos^2\theta_C \gamma_{u\ell\to d\nu} + \sin^2\theta_C \gamma_{u\ell\to s\nu} + \gamma_{\ell\nu'\to\nu\ell'}\right), \\
\partial_\mu \left\langle (J_l^\mu)^{\ell}{}_{\ell} \right\rangle &\approx -4G_F^2 \left(\cos^2\theta_C \gamma_{u\ell\to d\nu} + \sin^2\theta_C \gamma_{u\ell\to s\nu} + \gamma_{\ell\nu'\to\nu\ell'}\right).
\end{aligned}
$$

Gauge-invariant formula for the rates
(assuming time-reversal invariance and local thermal equilibrium)

$$\gamma_{f_1^a f_2^b \to f_1^c f_2^d} = \gamma^<_{f_1^a f_2^b \to f_1^c f_2^d} = \eta_{\mu\nu}\eta_{\alpha\beta}\int \frac{d^4k}{(2\pi)^4}\left[(n_{f_1;ac}(k_0) - n_{f_2;db}(k_0))\rho^{\mu\alpha}_{f_1;ac}(k_0,\boldsymbol{k})\rho^{\beta\nu}_{f_2;db}(k_0,\boldsymbol{k})\right]$$

$$n_{f;ab}(k_0) = \frac{1}{e^{\beta(k_0+\mu_{f^a}-\mu_{f^b})} - 1}$$

Vanishes at beta equilibrium

$$\gamma_{f_1^a f_2^b \to f_1^c f_2^d} \approx \left(\delta\mu_{f_2^d} - \delta\mu_{f_2^b} - (\delta\mu_{f_1^a} - \delta\mu_{f_1^c})\right)\boxed{\Lambda_{f_1^a f_2^b \to f_1^c f_2^d}}$$

Holographic calculation

Holographic QCD model

$$S_f = \int d^5x\,\sqrt{-g}\,\mathrm{Tr}\left[g_X^2\left(-|DX|^2 + \frac{3}{L^2}|X|^2\right) - \frac{1}{4g_5^2}\left(F_{(R)}^2 + F_{(L)}^2\right)\right]$$

$$D_N X = \partial_N X - iL_N X + iXR_N\,, \qquad D_N X^\dagger = \partial_N X^\dagger + iX^\dagger L_N - iR_N X^\dagger$$

$$F_{(A)MN} = \partial_M A_N - \partial_N A_M - i\,[A_M, A_N]$$

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$$F_{(A)MN} = \partial_M A_N - \partial_N A_M - i\,[A_M, A_N]$$

Charged black hole geometry

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2+\frac{dz^2}{f(z)}+d\vec{x}^{\,2}\right), \qquad f(z) = 1 - M\frac{z^4}{z_h^4} + Q^2\frac{z^6}{z_h^6}$$

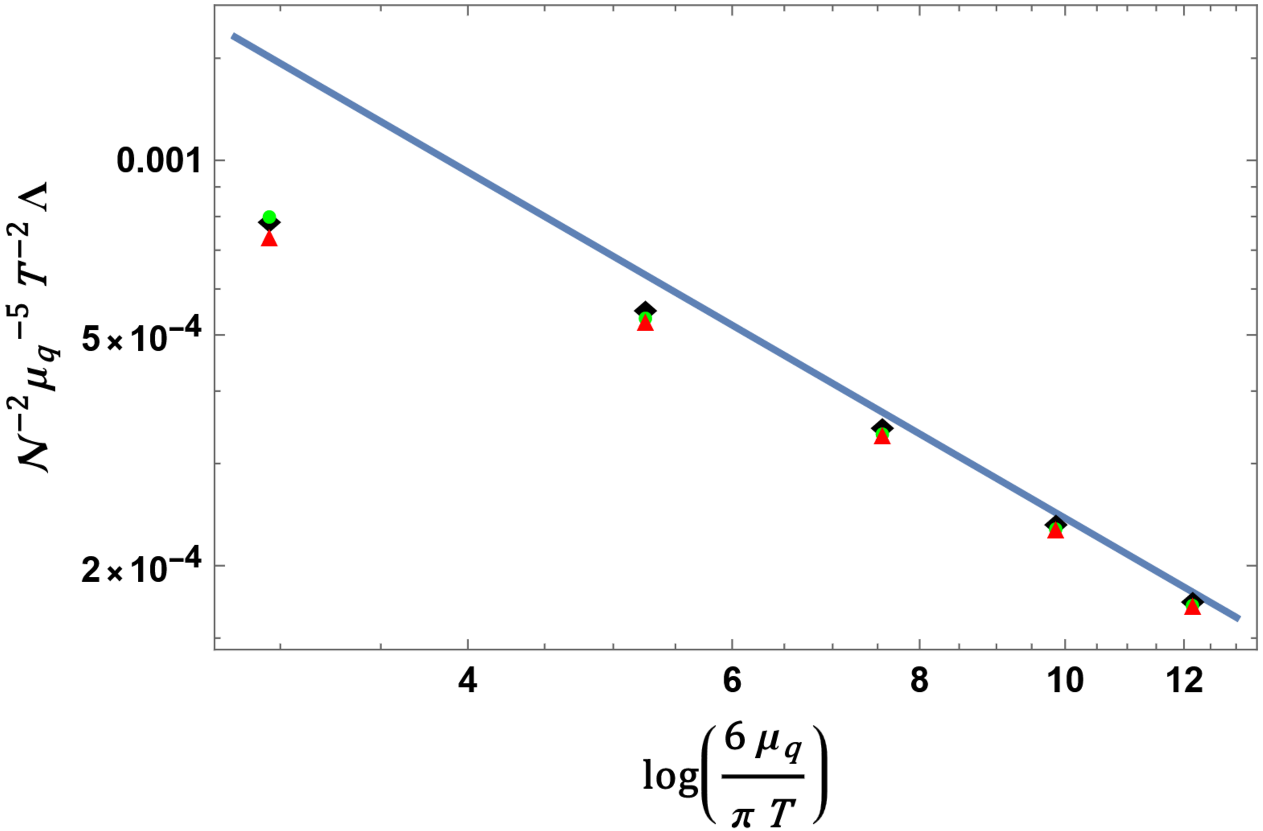
$$(L_0^{\mathrm{bkg}})^a{}_b = (R_0^{\mathrm{bkg}})^a{}_b = \frac{\mu_q}{2}\left(1-\frac{z^2}{z_h^2}\right)\delta^a{}_b$$

Zero quark masses

Fluctuations of the gauge fields => Current correlators => Spectral functions

$$\Lambda_{ud\to us} \approx \frac{\mathcal{N}^2}{128}\sqrt{\frac{3}{\pi}}\mu_q^5 T^2 \left(\log\frac{6\mu_q}{\pi T}\right)^{-3/2}$$

$$\mathcal{N} = \frac{2L}{g_5^2} = \frac{N_c}{6\pi^2}$$



$$T/\mu_q = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$$

Comparison with pQCD: non-Fermi liquid

$$\lambda_{ds} \approx \boxed{G_F^2 \sin^2\theta_C \cos^2\theta_C} \frac{\mathcal{N}^2}{32}\sqrt{\frac{3}{\pi}}\,\boxed{\mu_q^5 T^2}\,\boxed{\left(\log\frac{6\mu_q}{\pi T}\right)^{-3/2}}$$

$$\lambda_{ds} \approx \boxed{G_F^2 \sin^2\theta_C \cos^2\theta_C} \frac{64}{4\pi^2}\,\boxed{\mu_q^5 T^2}\,\boxed{\left(1 + \frac{4\alpha_s}{9\pi}\log\frac{\kappa\,\alpha_s\mu_q}{T}\right)^{4}}$$

Comparison with pQCD: non-Fermi liquid

Suppression

$$\lambda_{ds} \approx \boxed{G_F^2 \sin^2\theta_C \cos^2\theta_C} \frac{\mathcal{N}^2}{32}\sqrt{\frac{3}{\pi}} \boxed{\mu_q^5 T^2} \boxed{\left(\log\frac{6\mu_q}{\pi T}\right)^{-3/2}}$$

$$\lambda_{ds} \approx \boxed{G_F^2 \sin^2\theta_C \cos^2\theta_C} \frac{64}{4\pi^2} \boxed{\mu_q^5 T^2} \boxed{\left(1 + \frac{4\alpha_s}{9\pi}\log\frac{\kappa\,\alpha_s\mu_q}{T}\right)^4}$$

Enhancement

Outlook

Turn on nonzero quark masses

Compute rate in models with EoS fitted to QCD: D3-D7, V-QCD, others

Full dependence on chemical potentials: large deviations from beta equilibrium

Implementation of flavor non-conservation equations in hydrodynamics

Other weak rates: neutrino emission   Jarvinen, Kiritsis, Nitti, Preau 2306.00192

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Thanks!