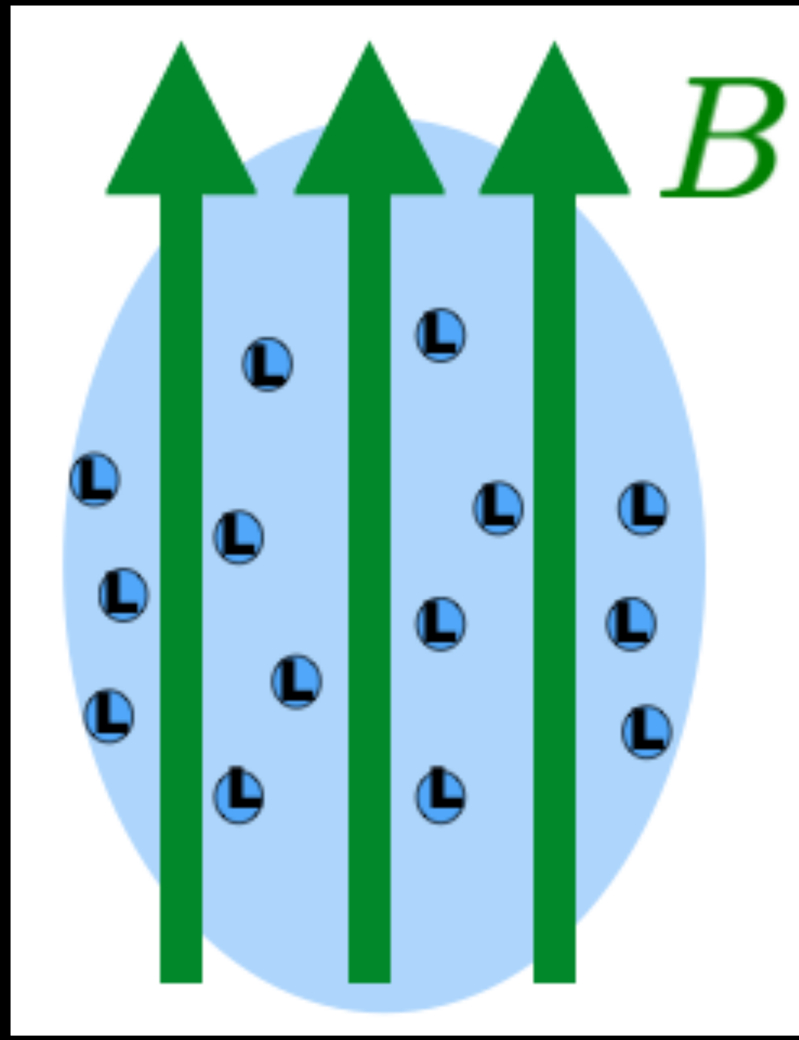
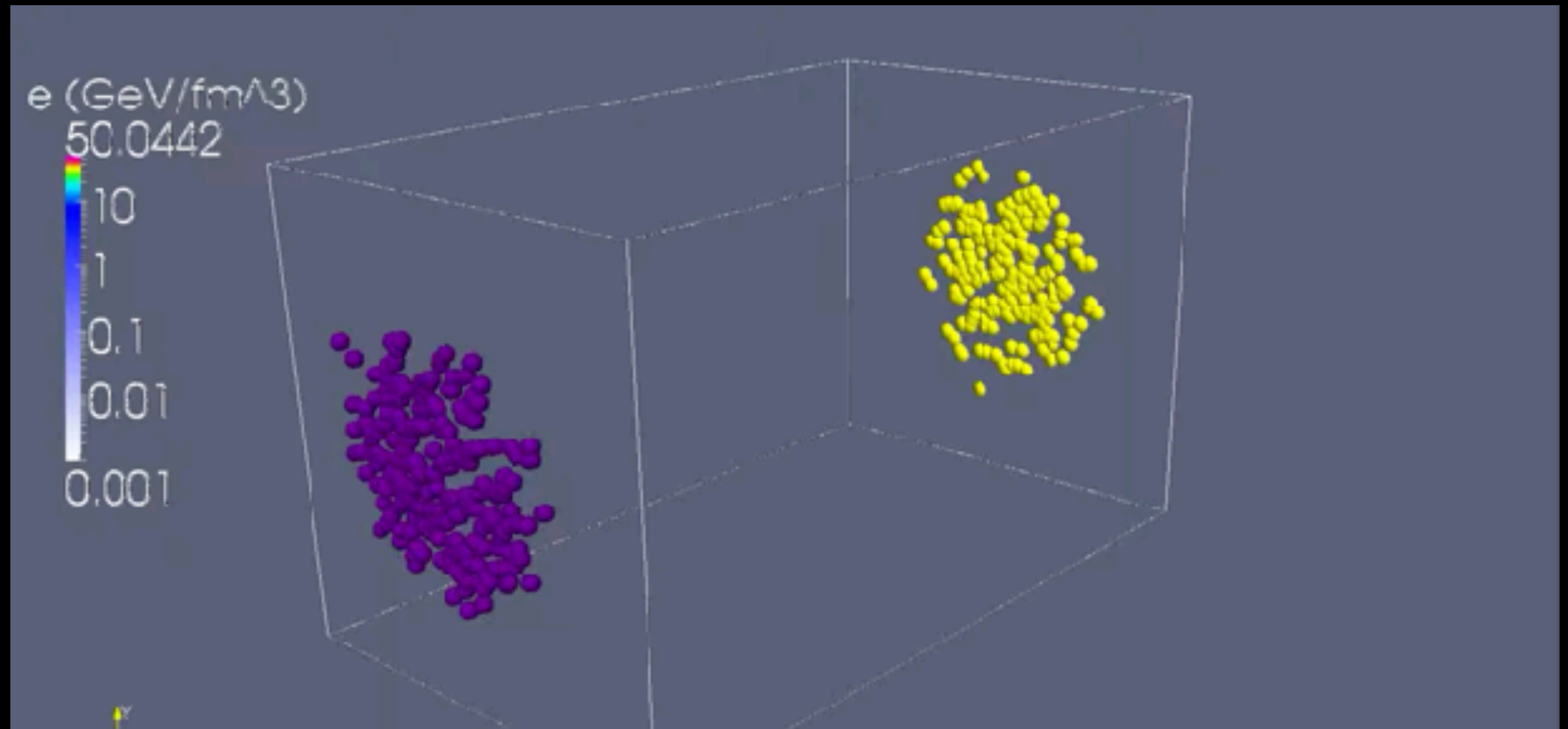
Hydrodynamics in Anisotropic Systems

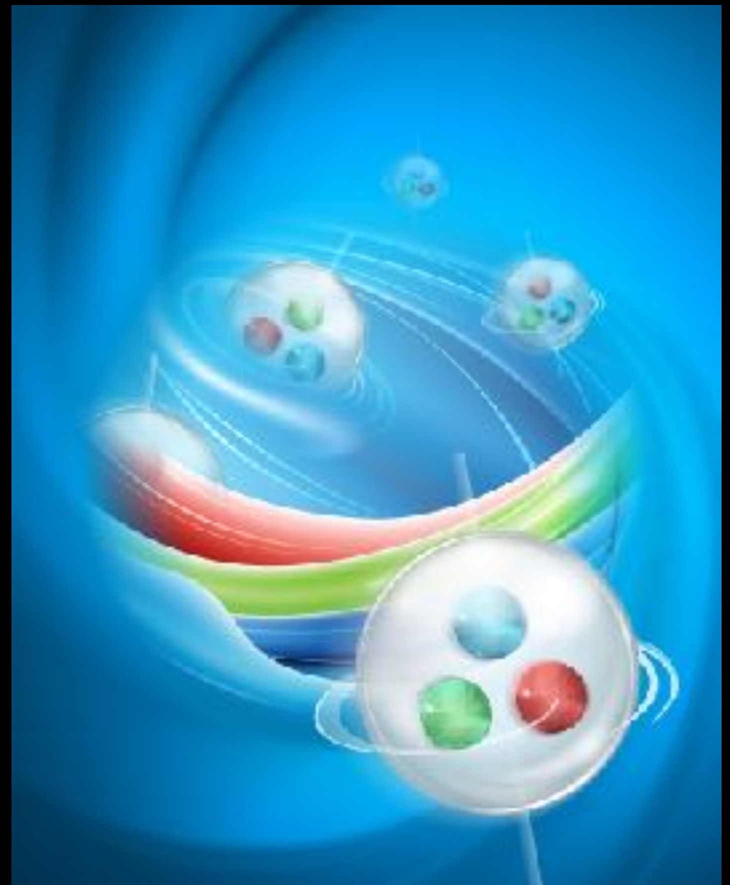
Workshop "Foundations and Applications of Relativistic Hydrodynamics", GGI Florence, Italy

May 7th, 2025





hydrodynamic simulation of heavy-ion collision by [OSU Group]



[STAR; Nature (2017)]

Matthias Kaminski
University of Alabama

U.S. DEPARTMENT OF ENERGY

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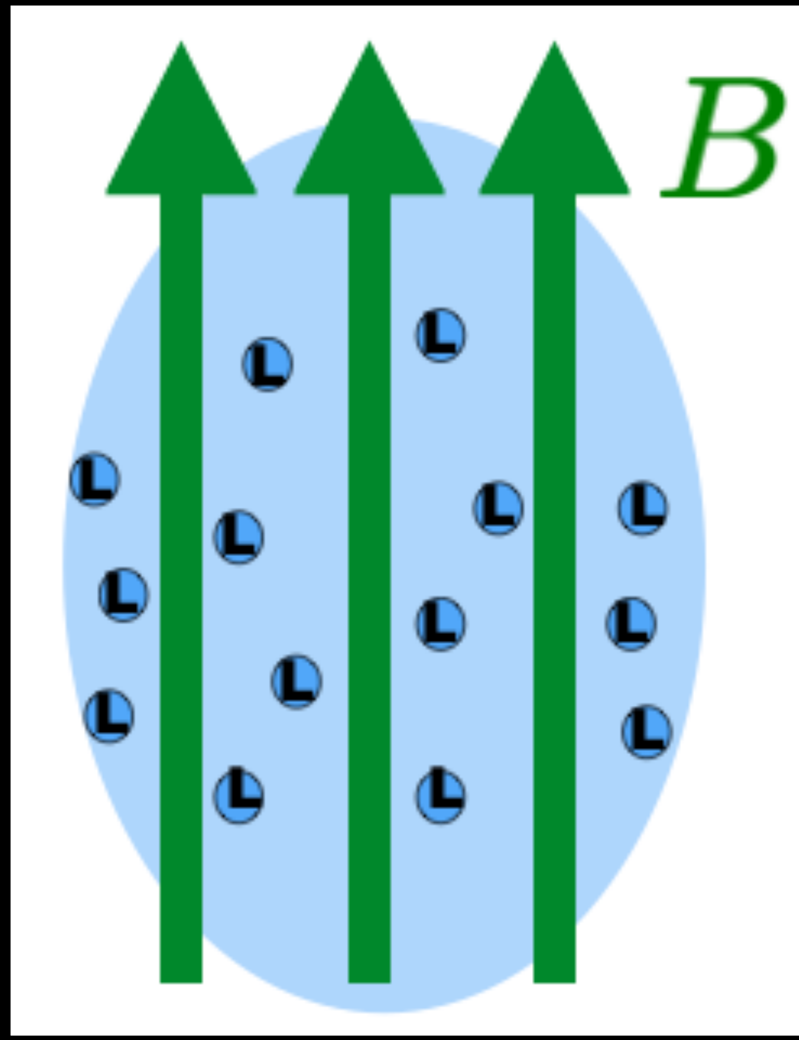

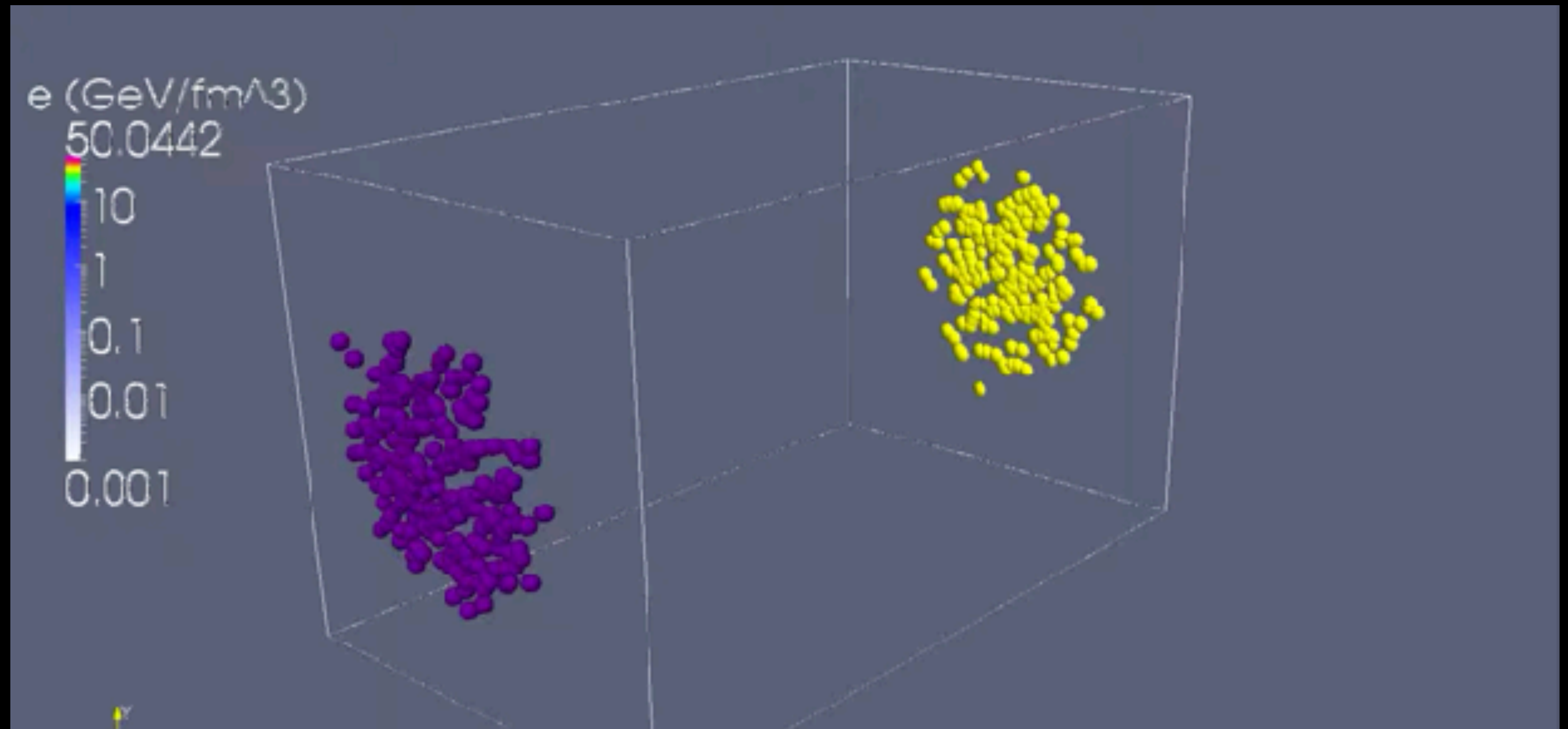

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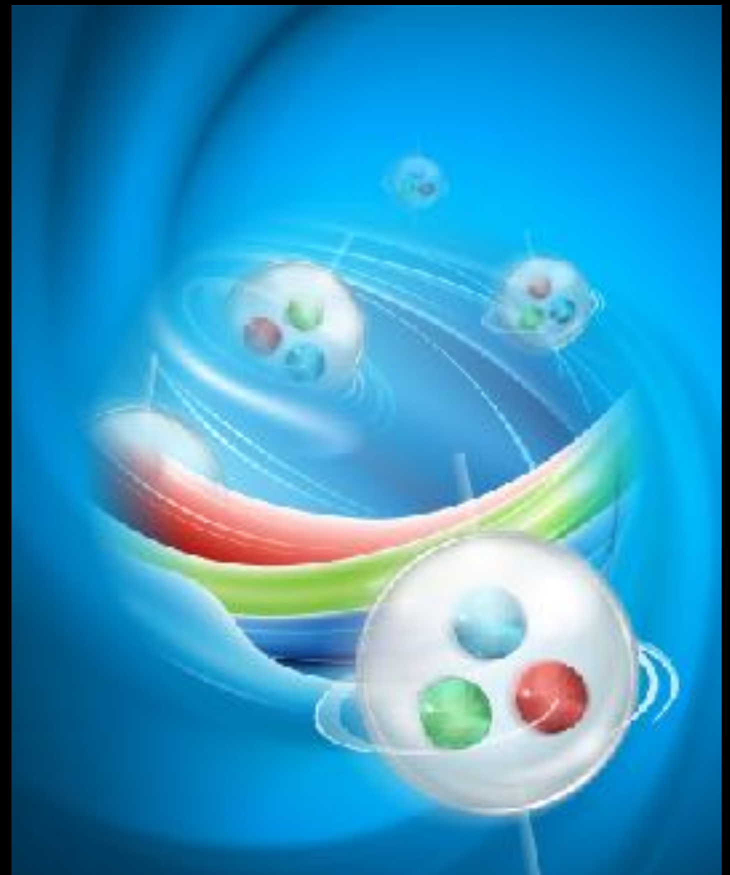

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Anisotropies at GGI in 2018

Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions 2018

Mar 19 – 22, 2018
Galileo Galilei Institute
Europe/Rome timezone

Enter your search term

The workshop is concluded, we thanks all participants and collaborators.

The theme 'standardshort' does not exist.

- Overview
- Committees
- Timetable
- Program
- First Circular
- Second Circular
- Third Circular
- Important Dates
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The 4th **Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions** will be held at the Galileo Galilei Institute in Florence from March 19 through March 22, 2018. The workshop will cover recent theoretical developments and experimental measurements related to these topics.

AdS/CMT at GGI in 2010

Event at Galileo Galilei Institute

Workshop

AdS4/CFT3 and the Holographic States of Matter

Aug 30, 2010 - Nov 05, 2010

Abstract
An exciting and largely unexpected consequence of Holography is that String and M-theory can provide useful information for transport phenomena of strongly interacting theories in low dimensions, fluid mechanics and non-relativistic systems. Physical systems that may have dual holographic descriptions include quantum critical points in 2+1 dimensions, high-Tc superconductors, quantum Hall systems, systems that exhibit parity breaking, non-relativistic critical systems as well as fluid mechanics and turbulence. Such systems – the Holographic States of Matter - have the potential to radically alter the perception of string theory and its relevance for physics. A basic theoretic setup for the holographic study of such systems is AdS4/CFT3 correspondence. This is also the main framework for holographic studies of the mysterious M-theory. The subject has experienced great formal growth, driven by the discovery of various field theoretical models for M2-branes. It is a fortunate and intriguing that progress in the more applied directions coincides with enhancement in the understanding of more formal aspects of M-theory. By bringing together experts in both the applied and formal directions we aim to create a fertile environment where future developments regarding the Holographic States of Matter in connection with our understanding of M-theory can be studied.

Topics
- AdS4/CFT3 Correspondence
- M2 and M5 branes
- The holographic description of high-Tc superconductivity, superfluidity, Quantum-Hall systems.
- Gravity and fluid dynamics
- Gravitational description of non-relativistic systems.

Hydrodynamics is based on symmetries

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- construct **constitutive equations** out of all (pseudo)scalars, (pseudo)vectors and (pseudo)tensors under Lorentz group

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$$< j^\mu > = n u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots$$

e.g. charge gradient
(covariant derivative) $\nabla^\mu n$

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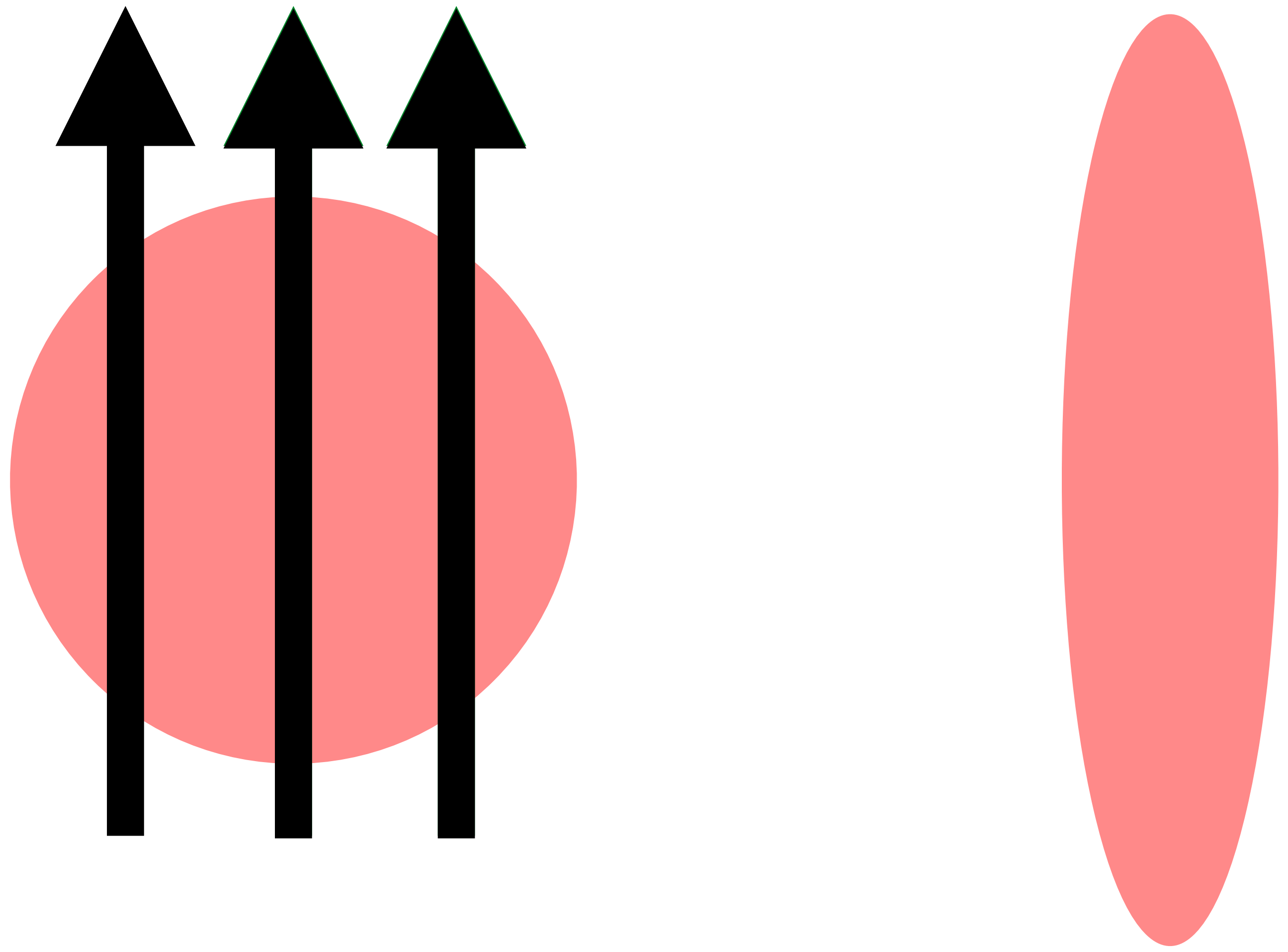

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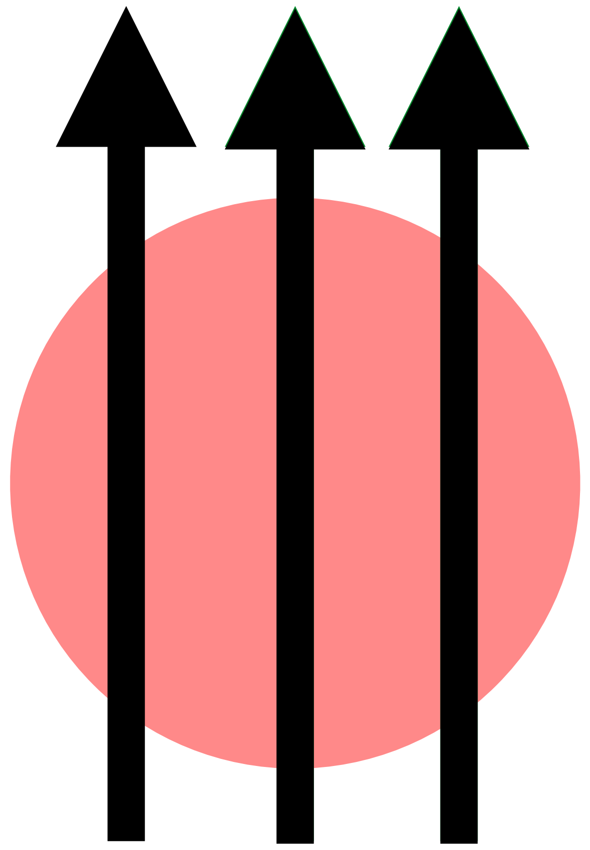

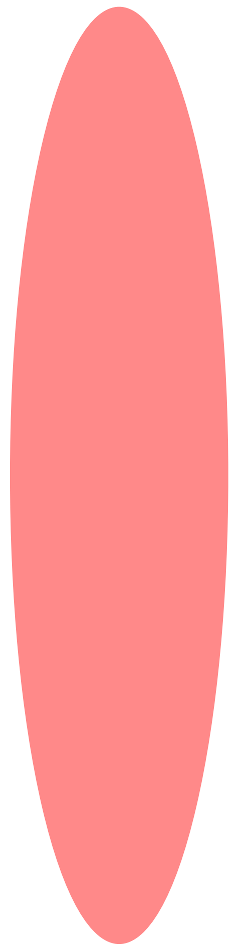

additional Lorentz vector, e.g.

$$b^\mu = \frac{B^\mu}{B}$$

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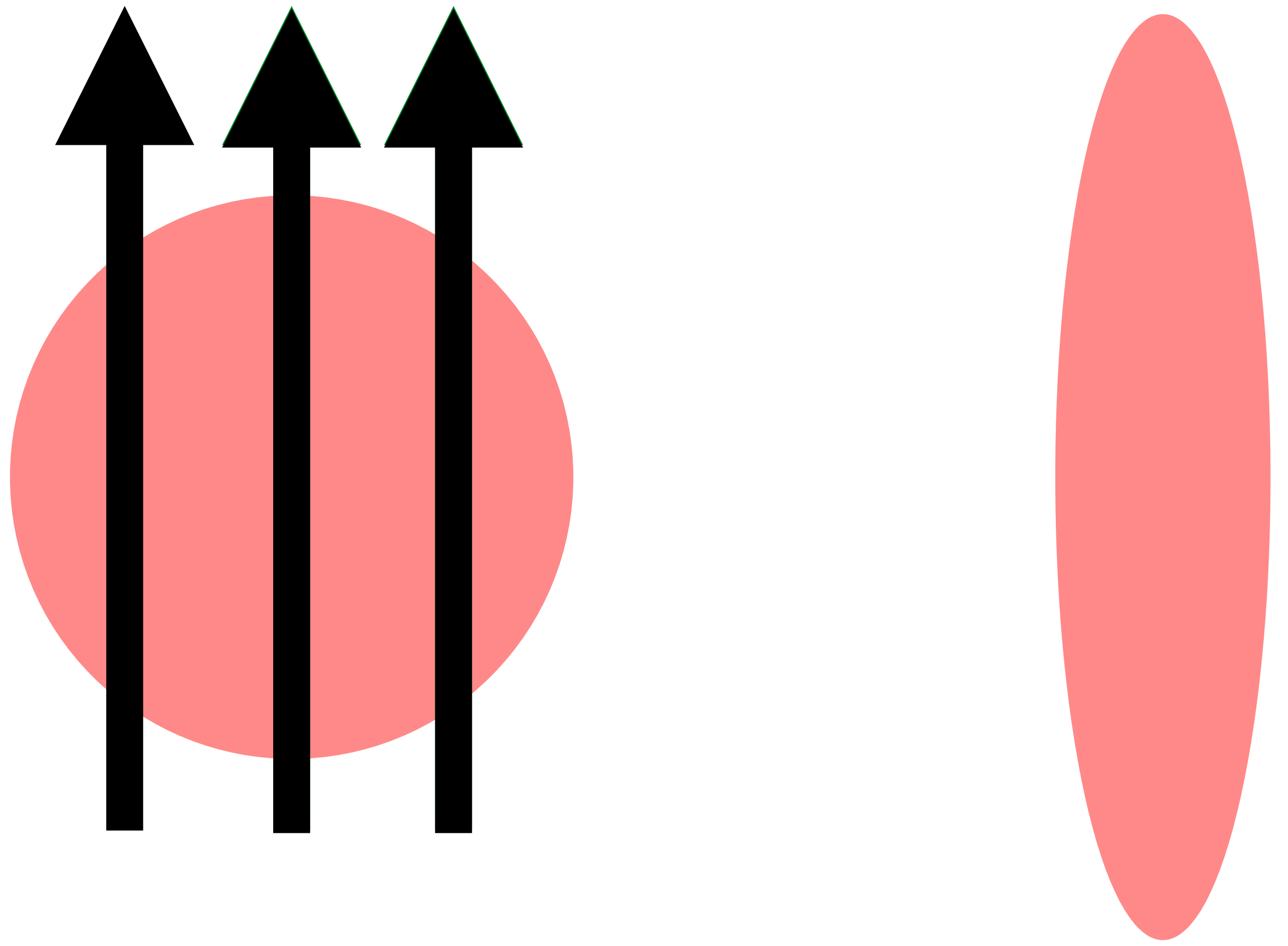
➡ more terms in constitutive equations

➡ novel transport effects



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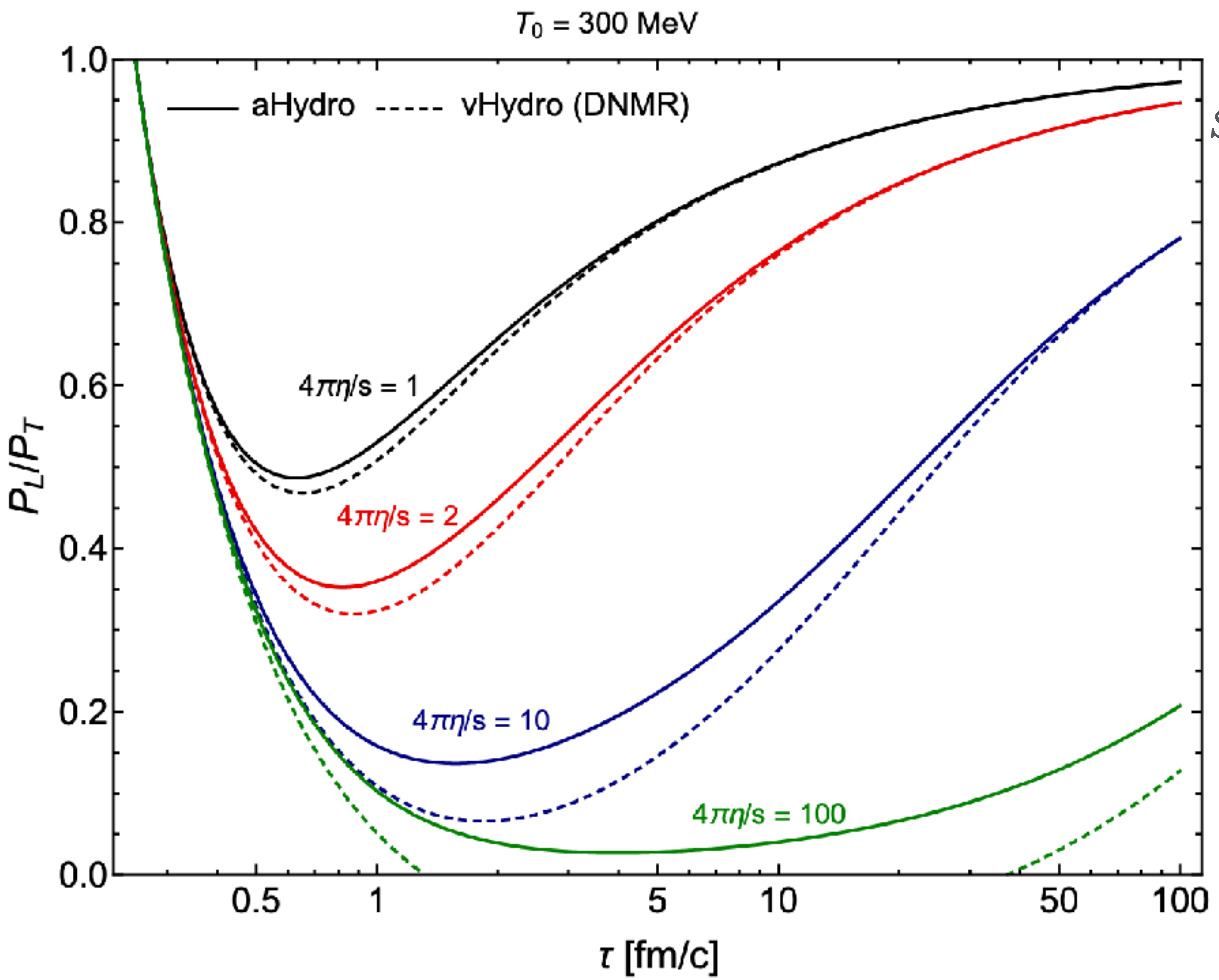
Anisotropic Hydrodynamics by Martinez/Strickland, Florkowski, Ryblewski

- **anisotropy breaks rotation symmetry, consider pressure anisotropy**

[Florkowski, Ryblewski; PRC (2010)]
[Martinez, Strickland; Nucl.Phys.A (2010)]



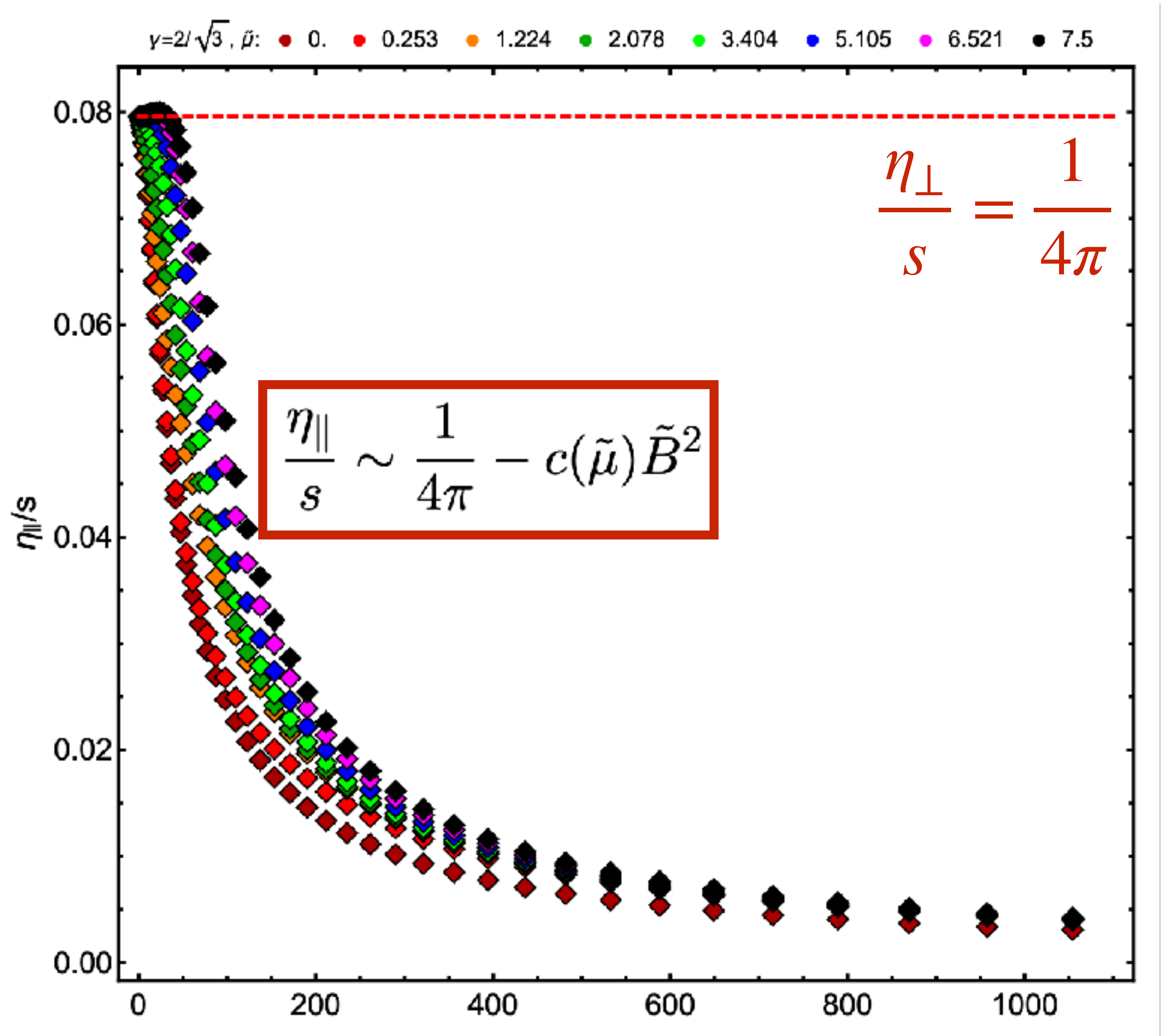
[Alqahtania, Nopoush, Strickland; PPNP (2018)]

➡ difference large enough to matter for heavy-ion collisions?

Longitudinal versus transverse shear viscosities in $\mathcal{N}$=4 SYM

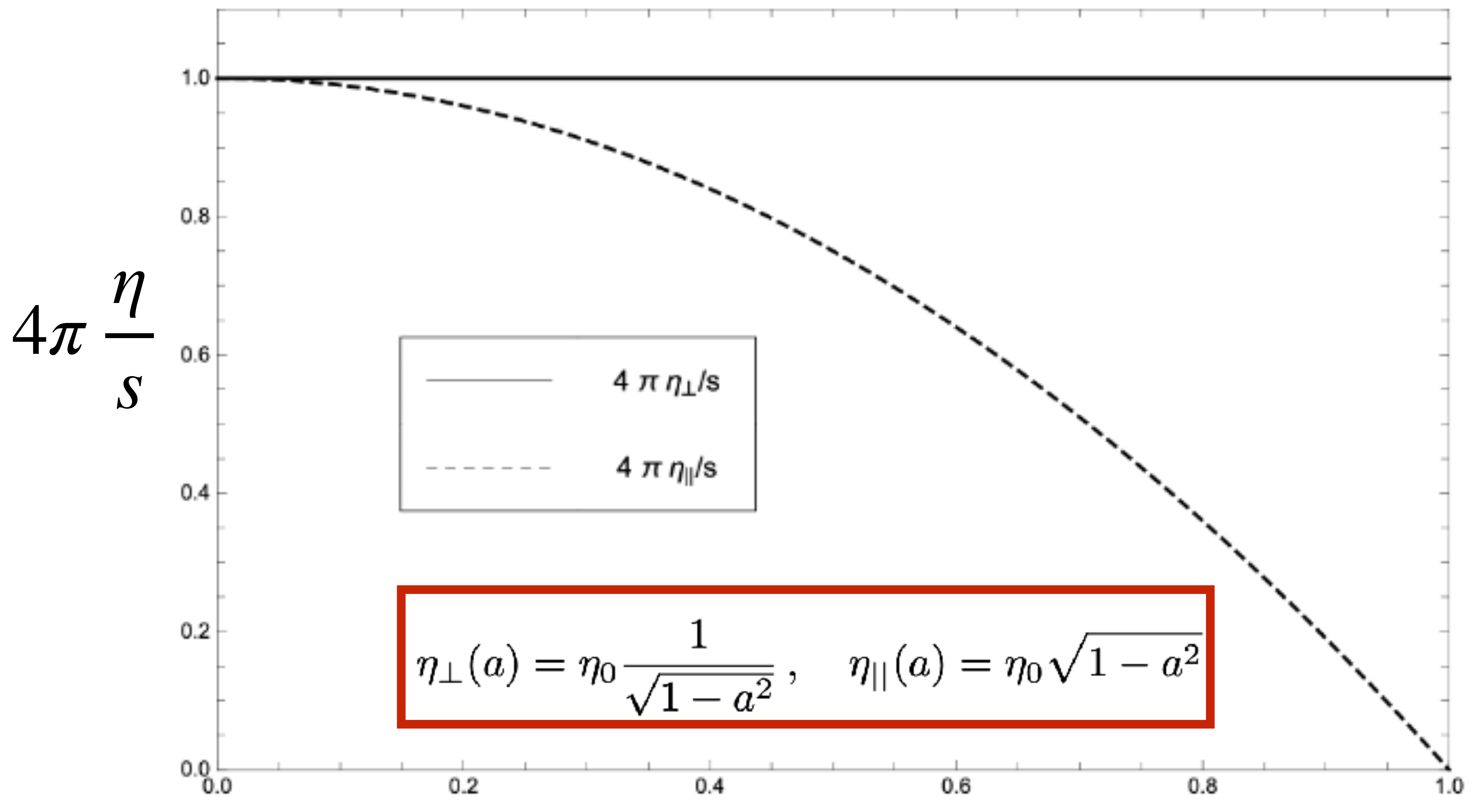
Anisotropy from magnetic field

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]



strong magnetic field $\tilde{B}$

Anisotropy from rotation



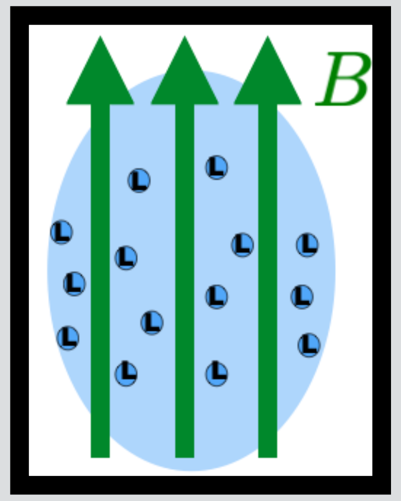
angular momentum a

➡ **qualitative differences**

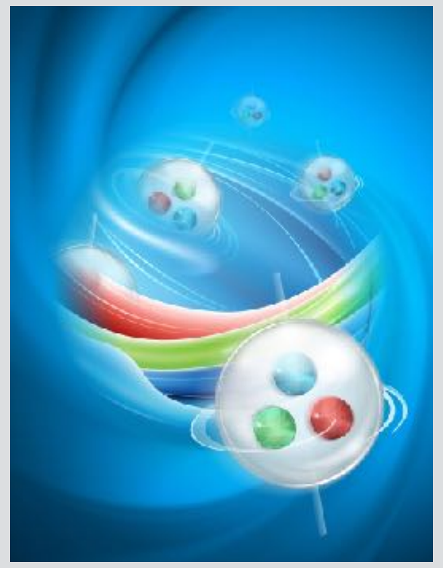
➡ **large quantitative differences**

Outline

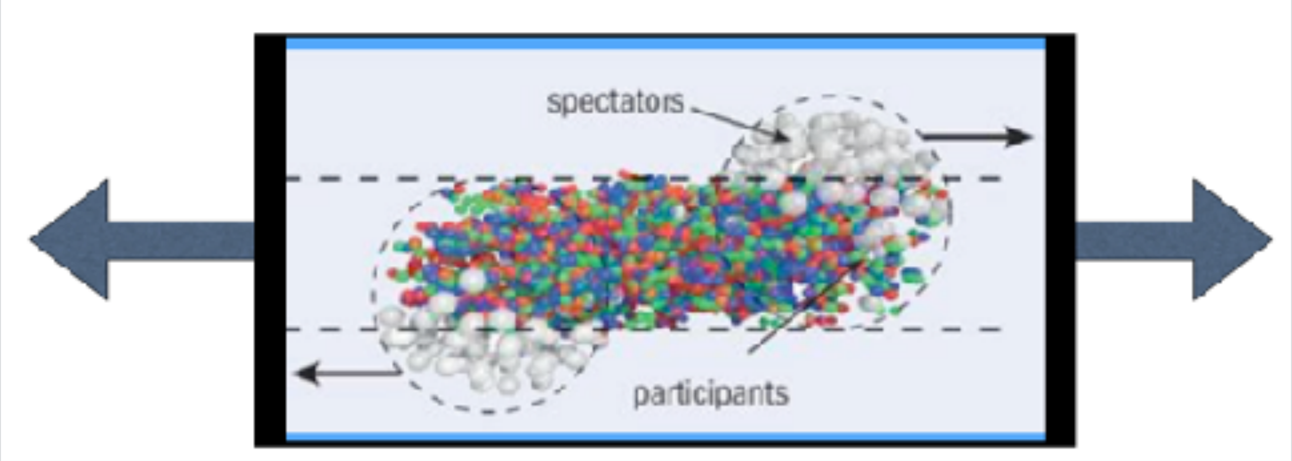
1. **Strong external magnetic field**



2. **Large vorticity**

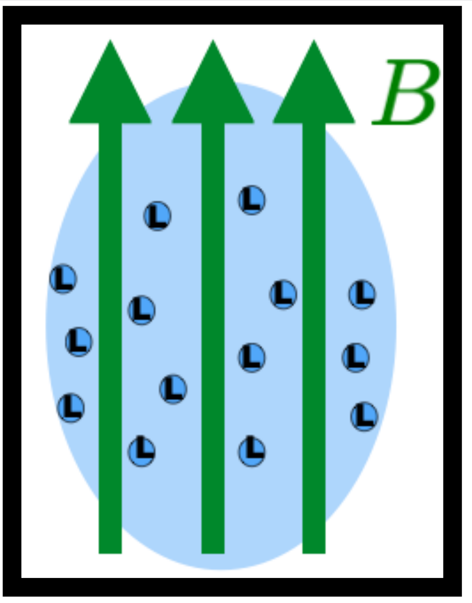


3. **Bjorken expansion**



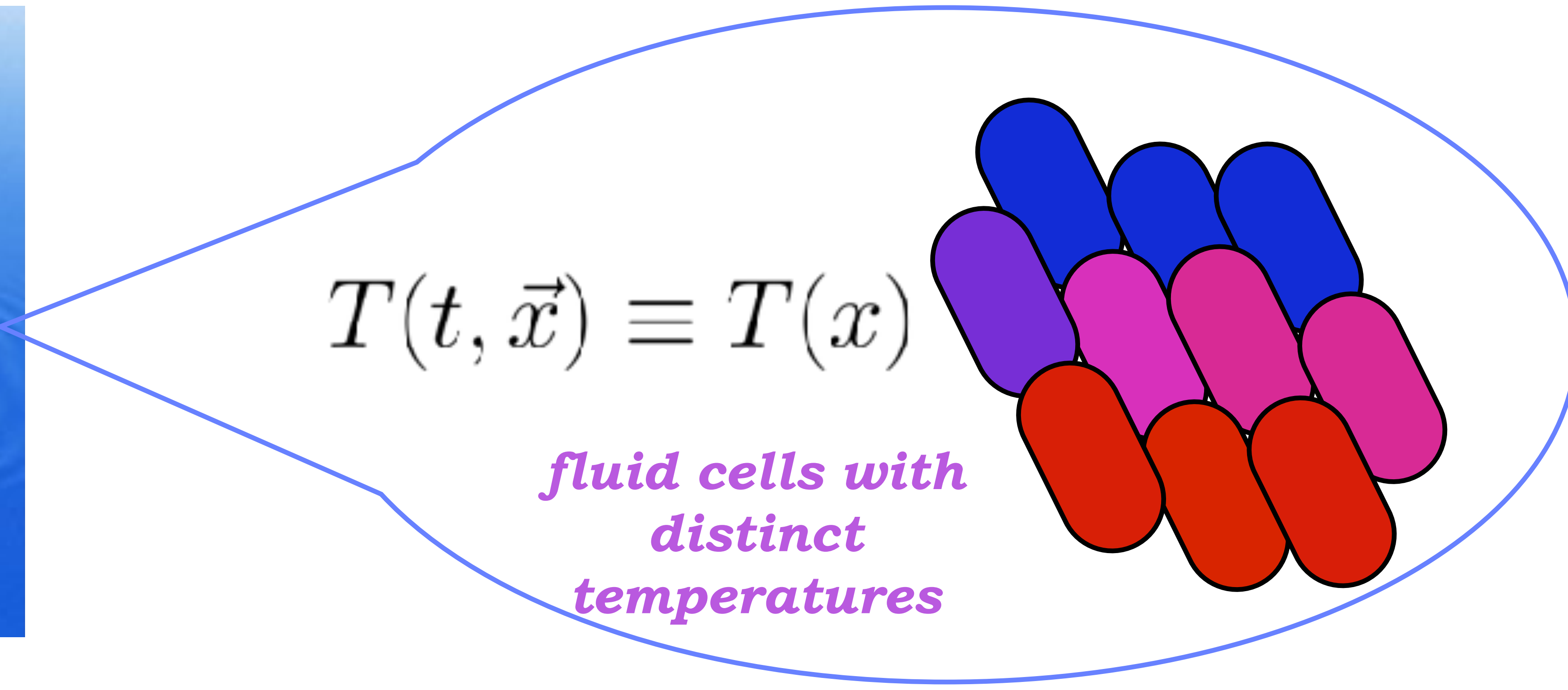
4. **Discussion**

Chiral hydrodynamics - Concepts



Hydrodynamics

- **effective field theory**
- expansion in small gradients
- large temperature
- conserved quantities survive



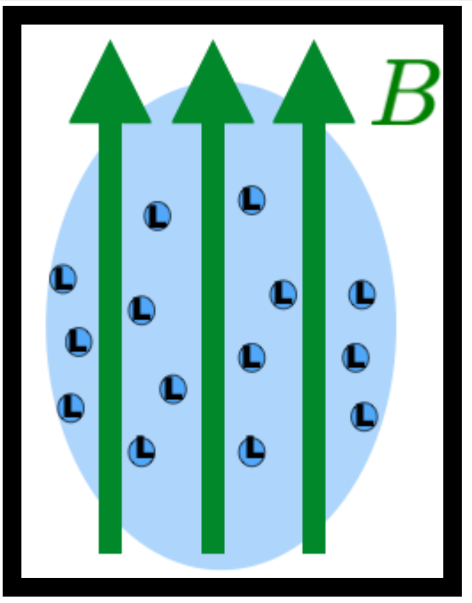
Constitutive equations

$$\langle T^{\mu\nu} \rangle = \epsilon\, u^\mu u^\nu + P\,\Delta^{\mu\nu} + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots$$
$$\langle j^\mu_{\text{vector}} \rangle = n u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots$$
$$\langle j^\mu_{\text{axial}} \rangle = n_a u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots$$

Conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} j_\mu$$
$$\nabla_\mu j^\mu_{\text{vector}} = 0$$
$$\nabla_\mu j^\mu_{\text{axial}} = C\, \vec{E} \cdot \vec{B}$$

Chiral hydrodynamics - Concepts



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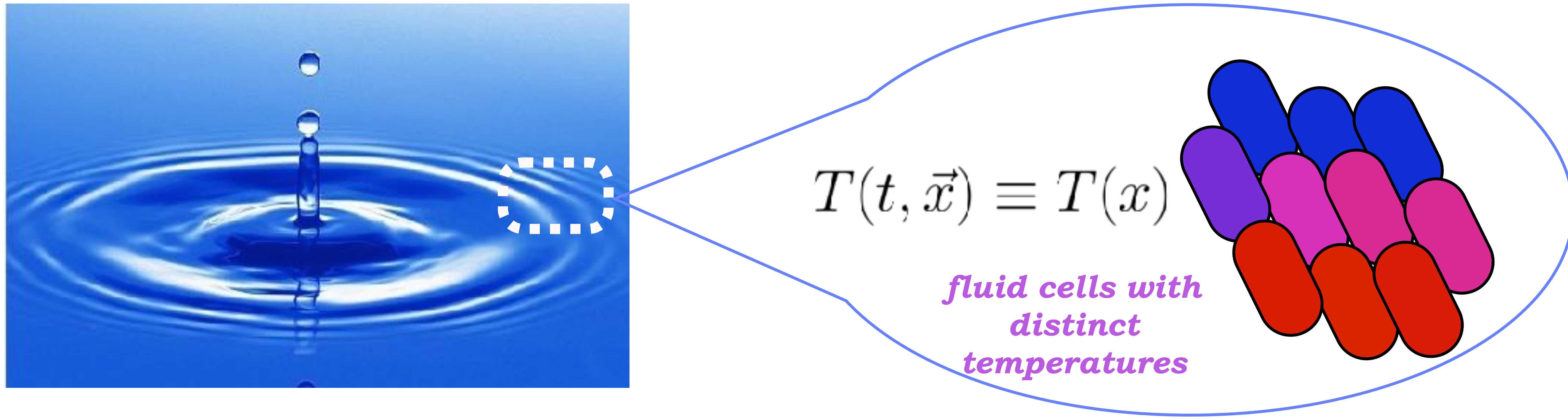

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Fourier transform hydro fields, e.g. T(x):

$$\partial_t\, e^{-i\omega t} = -i\omega\, e^{-i\omega t}$$

$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$

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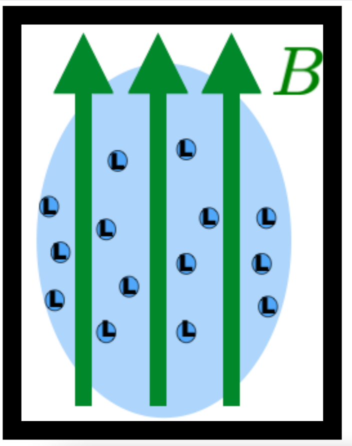

1. Construct **constitutive equations or generating functional**: all (pseudo)scalars, (pseudo)vectors and (pseudo)tensors under Lorentz group

[Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
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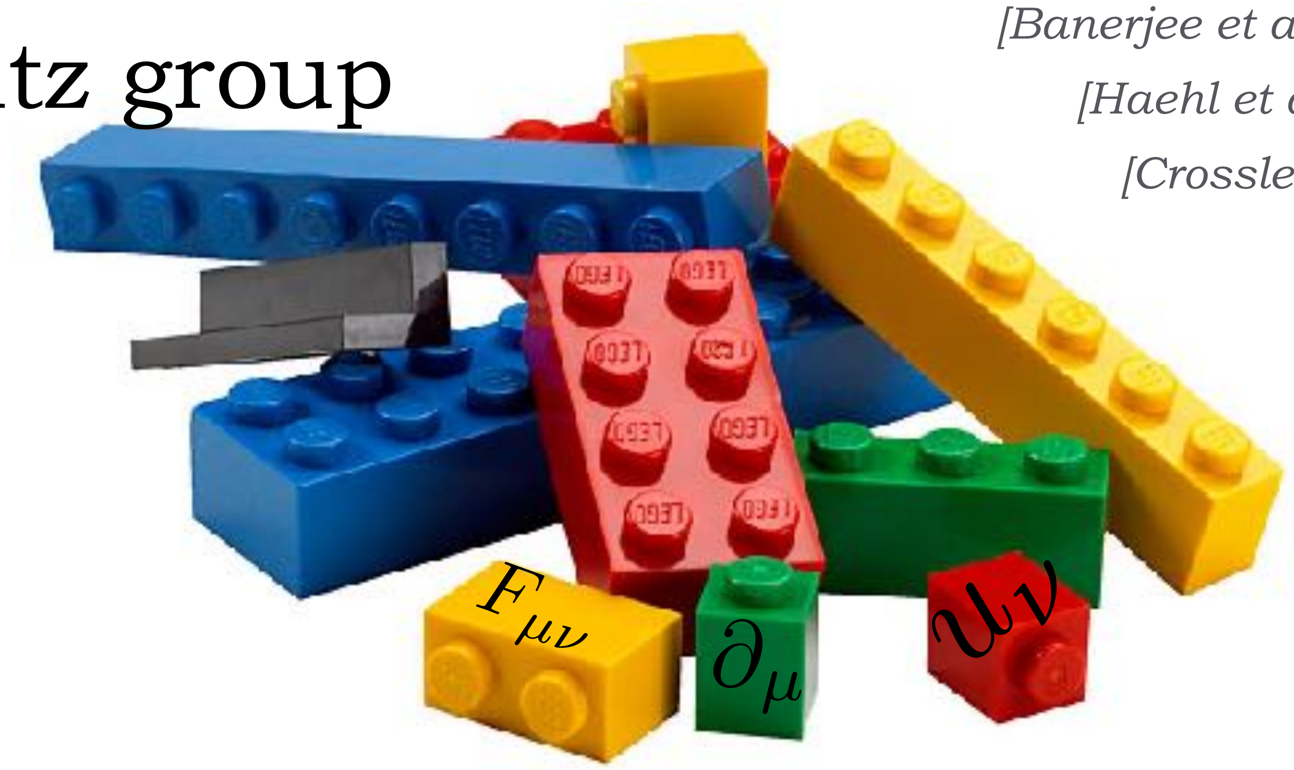

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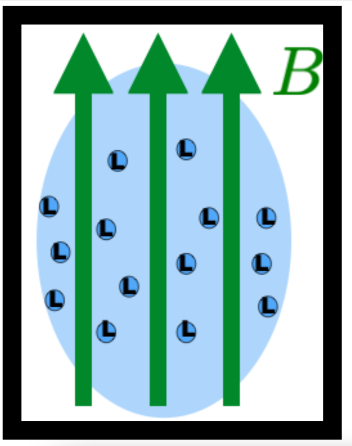
Examples at $\mathcal{O}(\partial)$: $\nabla^\mu n$ *charge gradient (covariant derivative)*

vorticity $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho$

Idea: generating functional $W[T,\mu]$

$$\langle T^{\mu\nu}\rangle \sim \frac{\delta W[T,\mu]}{\delta g_{\mu\nu}}$$

$$\langle T^{\mu\nu} T^{\alpha\beta}\rangle \sim \frac{\delta^2 W}{\delta g_{\mu\nu}\delta\delta g_{\alpha\beta}}$$



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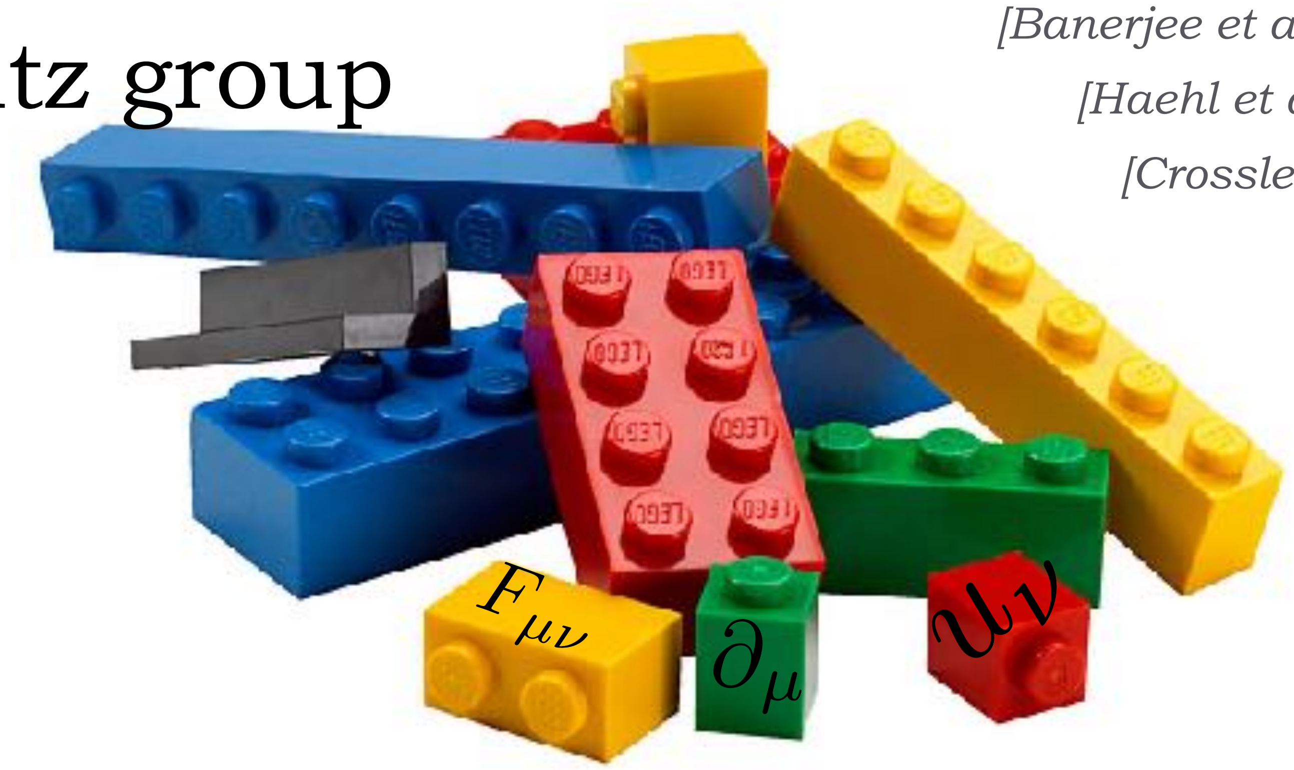

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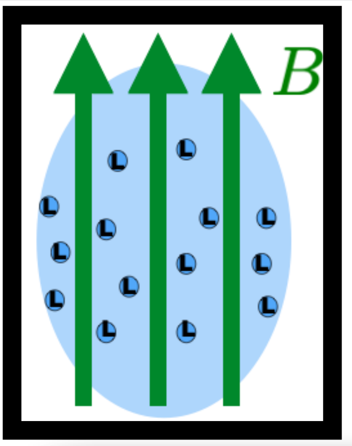
2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

Chiral hydrodynamics - Construction



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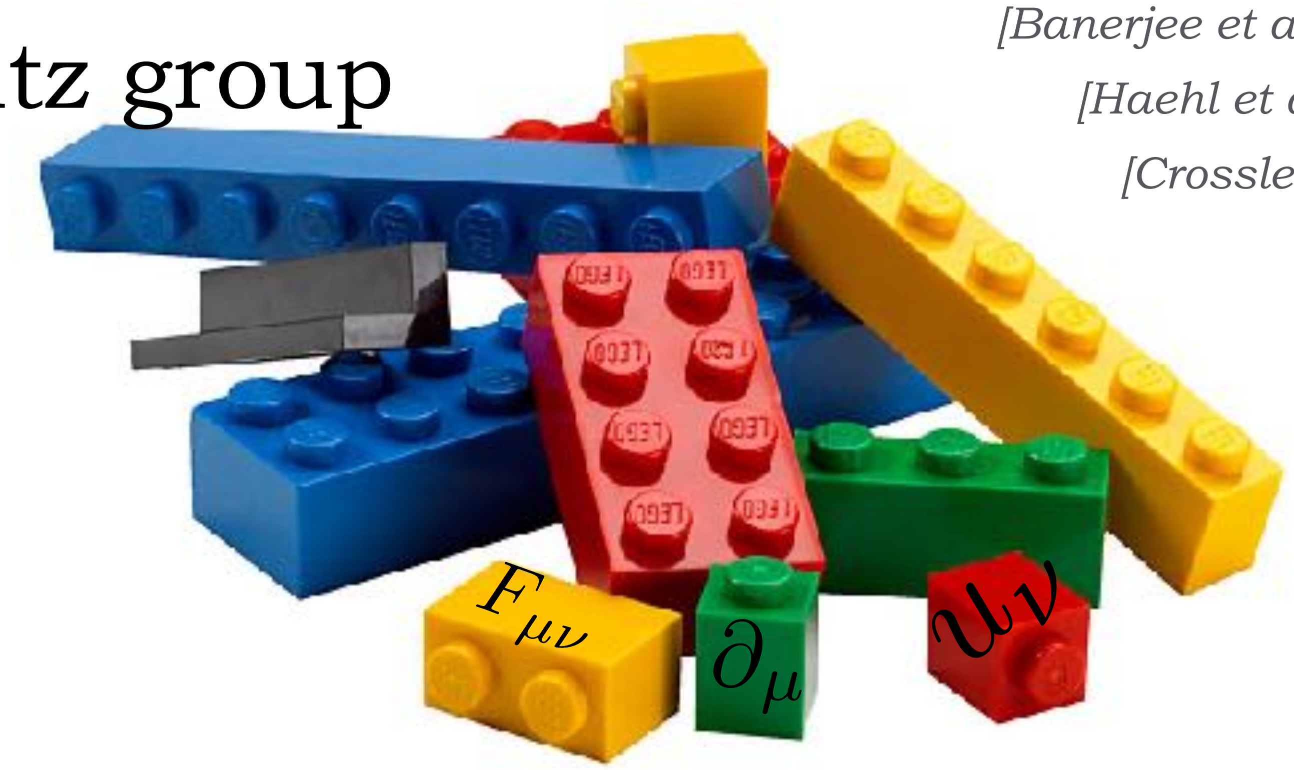

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3. Further restricted by positivity of local entropy production:

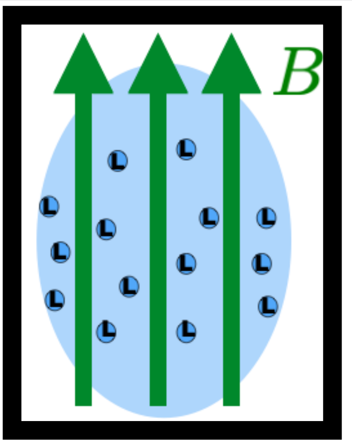
$$\nabla_\mu J^\mu_s \geq 0$$

[Landau, Lifshitz]

➡ Most general hydrodynamic 1-point functions for chiral charged fluid in strong magnetic field [Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

1. Kubo-formula derivation example: hydrodynamic correlators in 2+1



Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1,0,0)$$

sources $\quad A_t,\ A_x \propto e^{-i\omega t + ikx}$

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx} \quad$ (fix T and u)

one point functions $\quad$ (use $\nabla_\mu j^\mu = 0$)

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

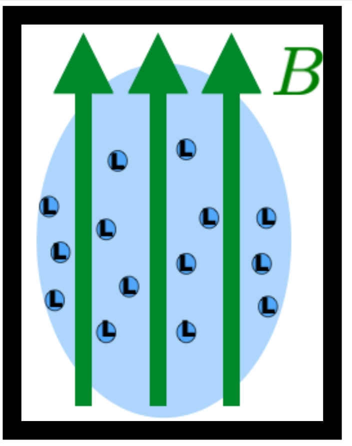
susceptibility: $\quad \chi = \frac{\partial n}{\partial \mu}$

Einstein relation: $\quad D = \frac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

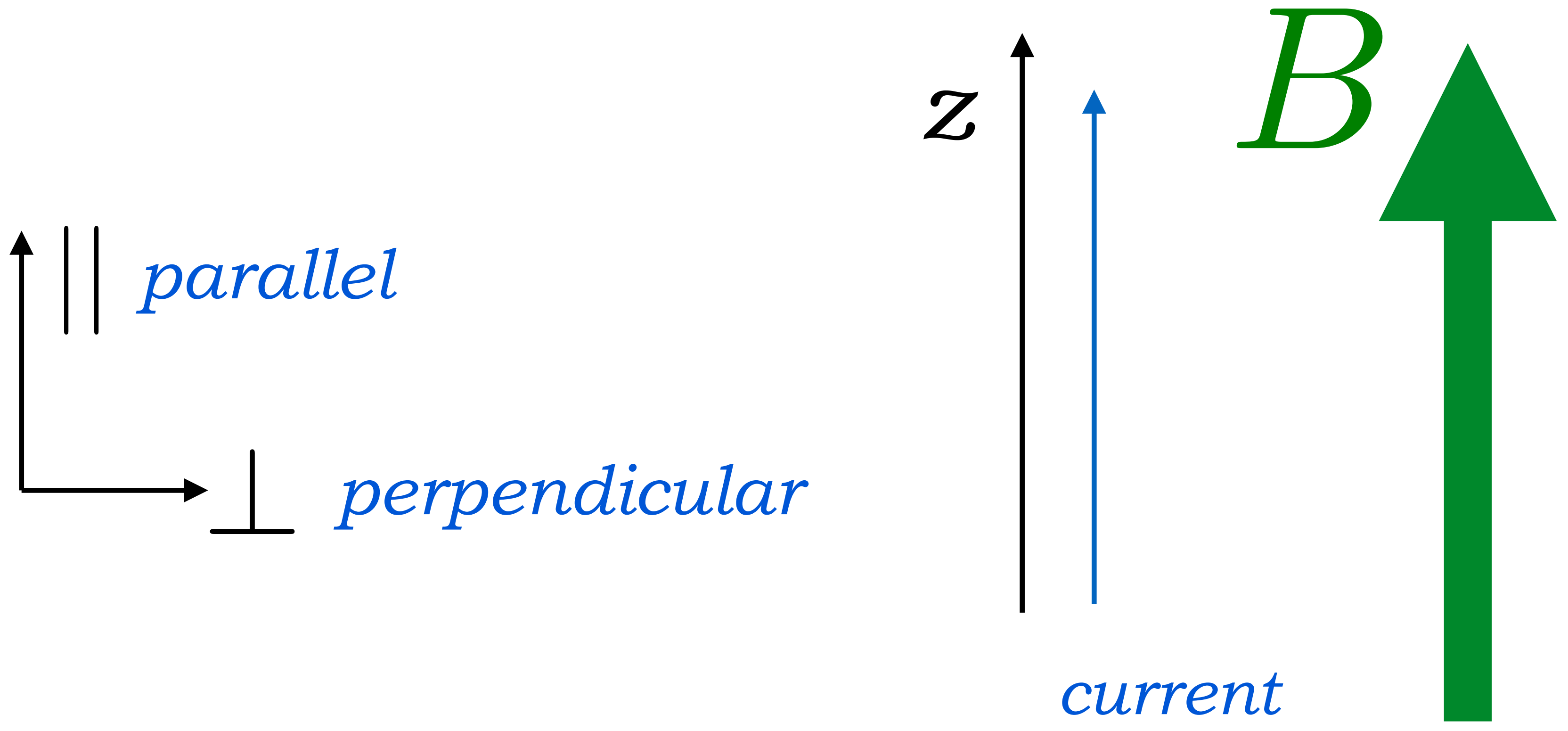
$\Longrightarrow$ **Kubo formula:** $\quad \sigma = \lim_{\omega \to 0} \frac{1}{i\omega} \langle j^x j^x \rangle (\omega, k=0)$

Chiral hydrodynamics - conductivity Kubo formulae



Parallel **conductivity**

$$\lim_{\omega\to 0} \frac{1}{\omega}\text{Im}\langle J^z J^z\rangle(\omega,\mathbf{k}=0) = \sigma_{||} + \cdots$$



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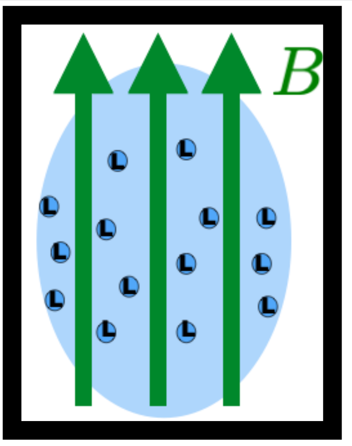

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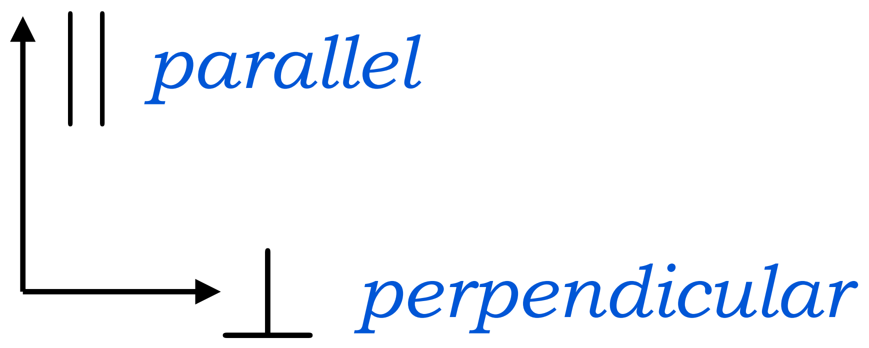
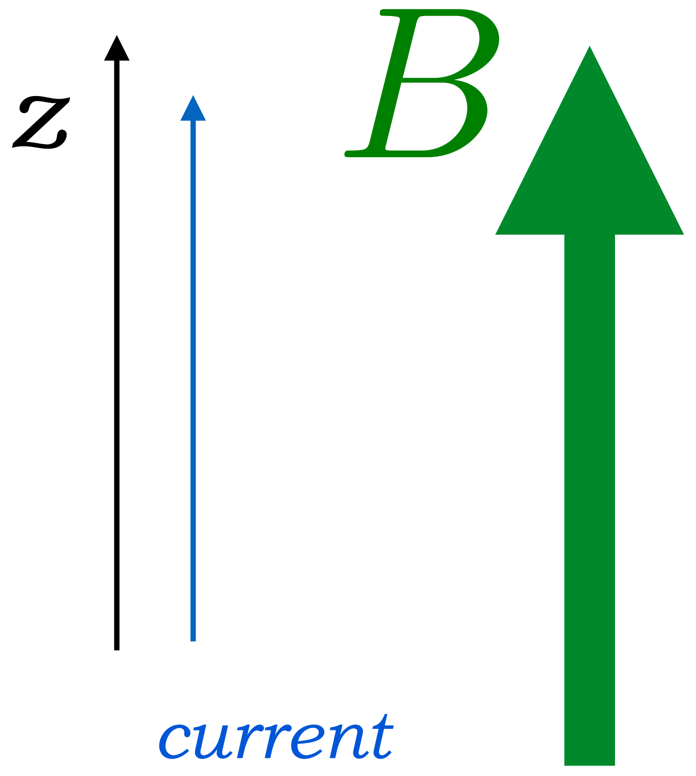
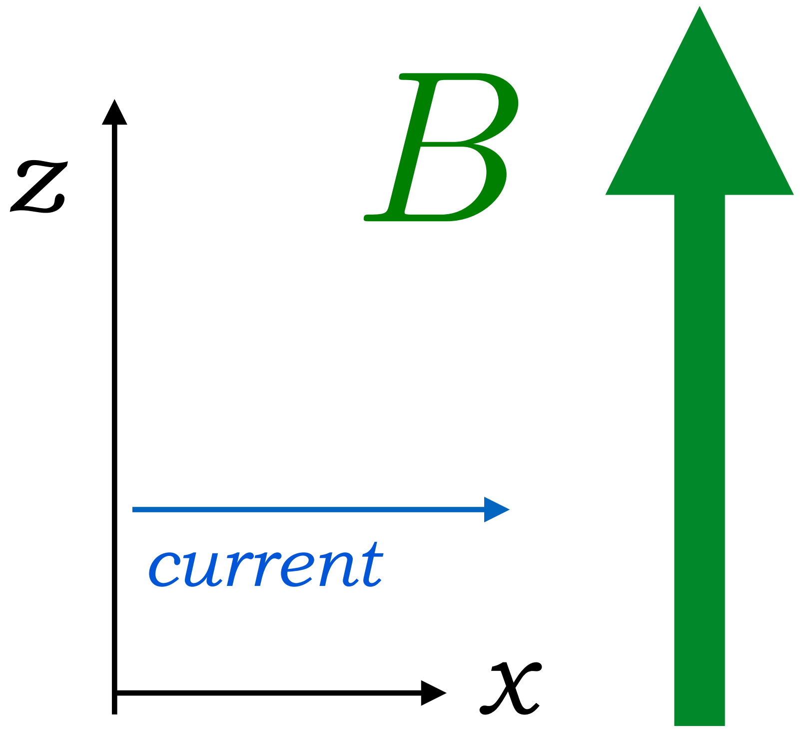




Perpendicular **resistivity**

$$\lim_{\omega\to 0} \frac{1}{\omega}\mathrm{Im}\,\langle J^x J^x\rangle(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$



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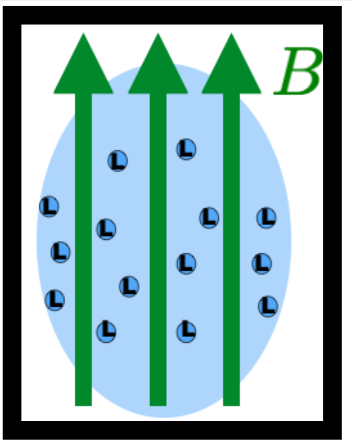

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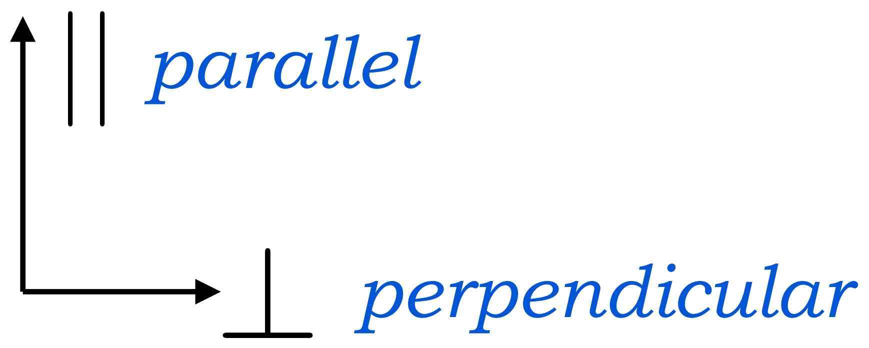

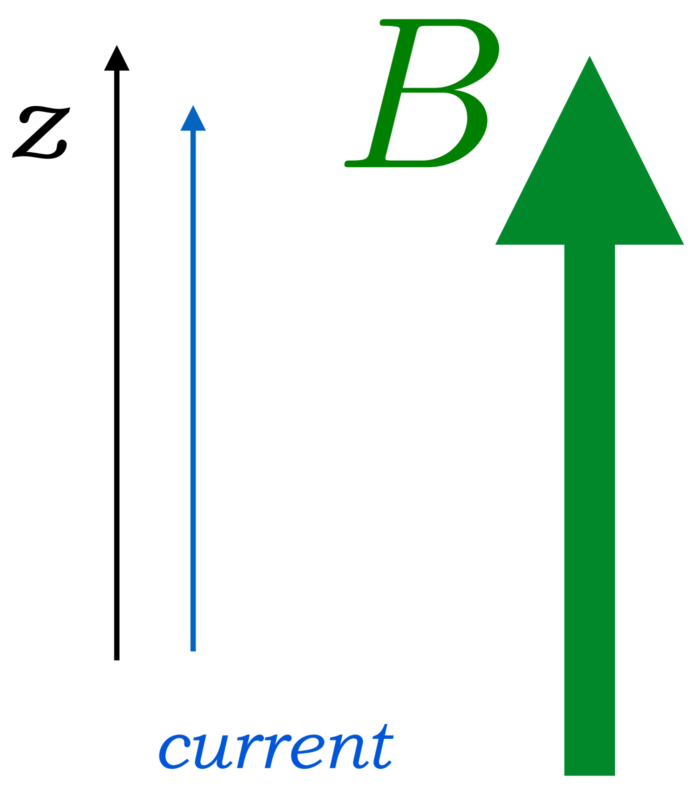

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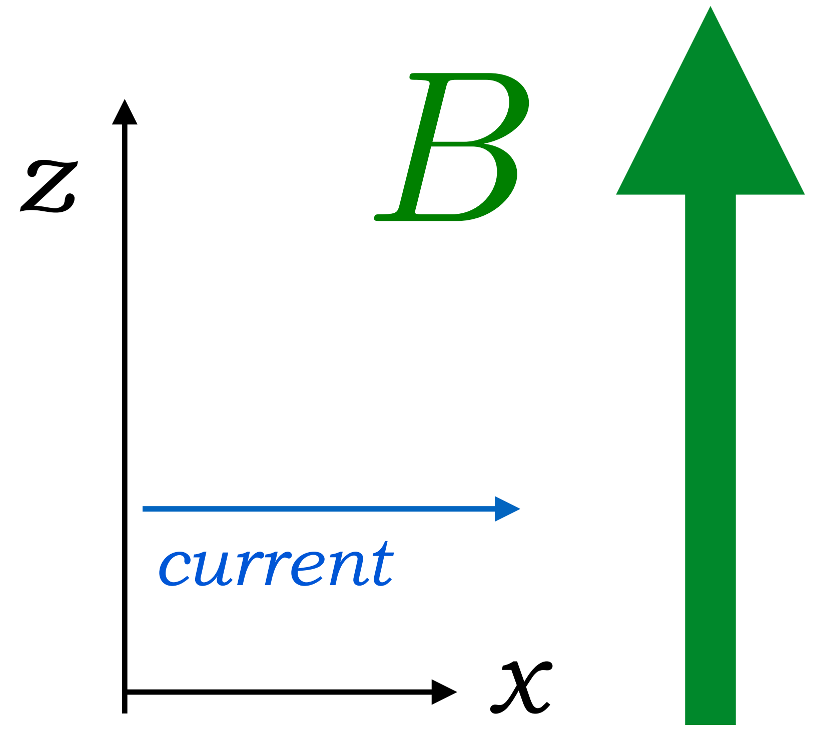
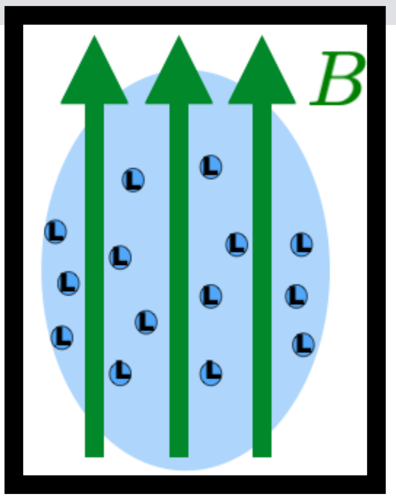


Very different parallel versus perpendicular

$$\langle J^z J^z\rangle(\omega, \mathbf{k} = 0) \sim \sigma_{||}$$

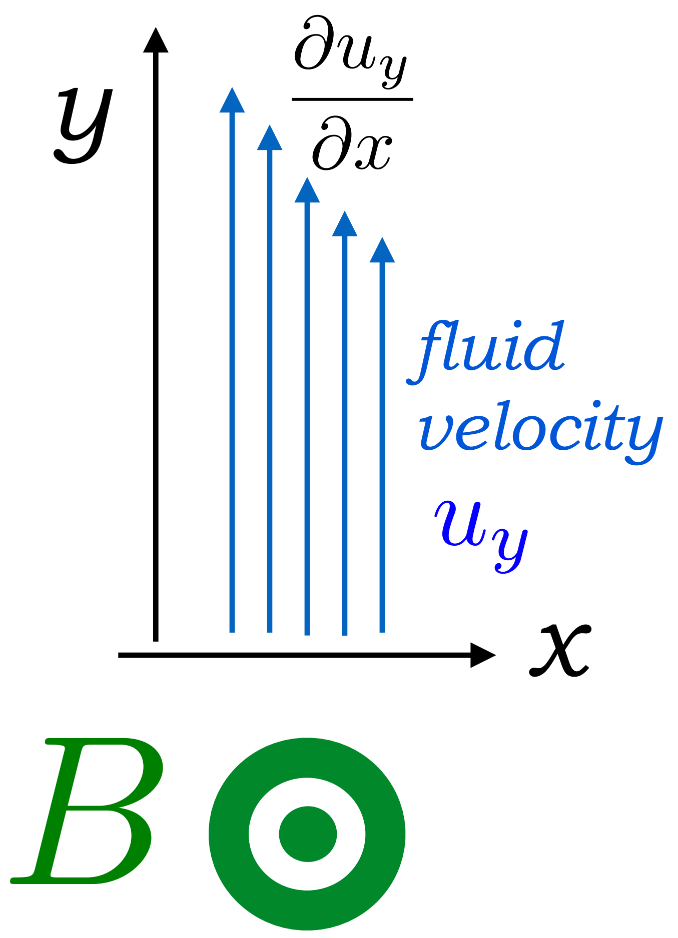
$$\langle J^x J^x\rangle(\omega, \mathbf{k} = 0) \sim \rho_\perp$$

Two shear viscosities



Shear viscosity perpendicular

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$

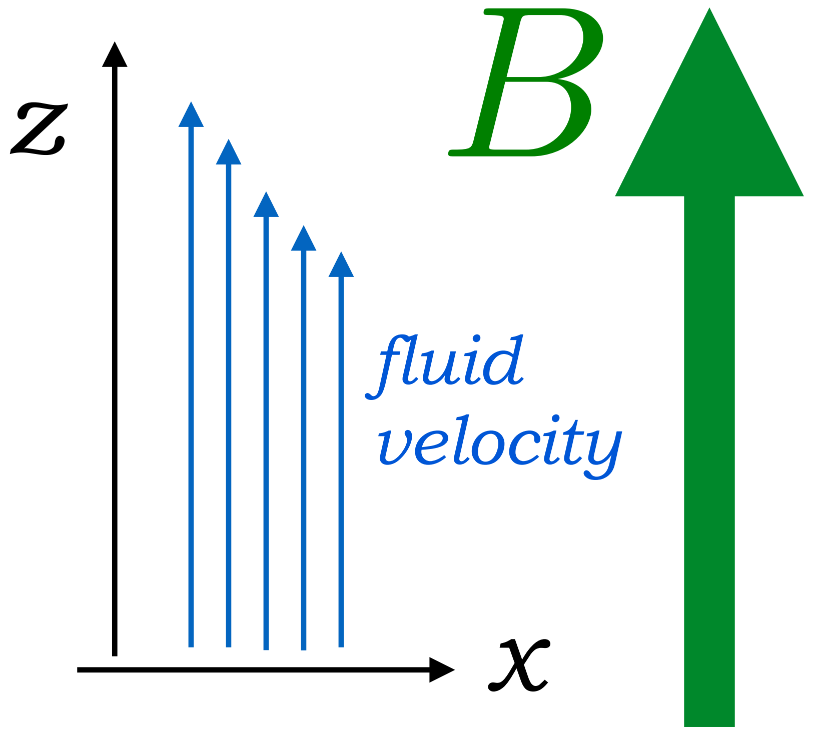


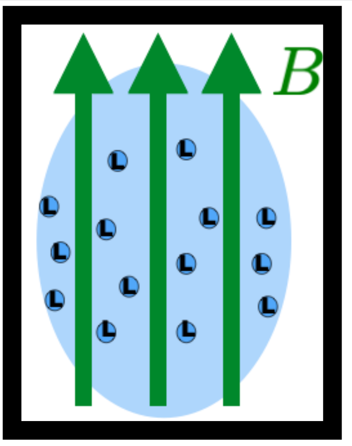
Shear viscosity parallel

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xz}T^{xz}}(\omega, \mathbf{k}{=}0) = \eta_{||} + (\bar{c}_8 c_{15} - c_{10}\bar{c}_{17})\rho_\perp - (\bar{c}_8\bar{c}_{17} + c_{10}c_{15})\tilde{\rho}_\perp$$

perpendicular resistivity ($\rho_\perp$) *Hall resistivity* ($\tilde{\rho}_\perp$)

➡ Value of shear viscosity depends on direction of magnetic field
➡ Can lead to creation of flow at early times



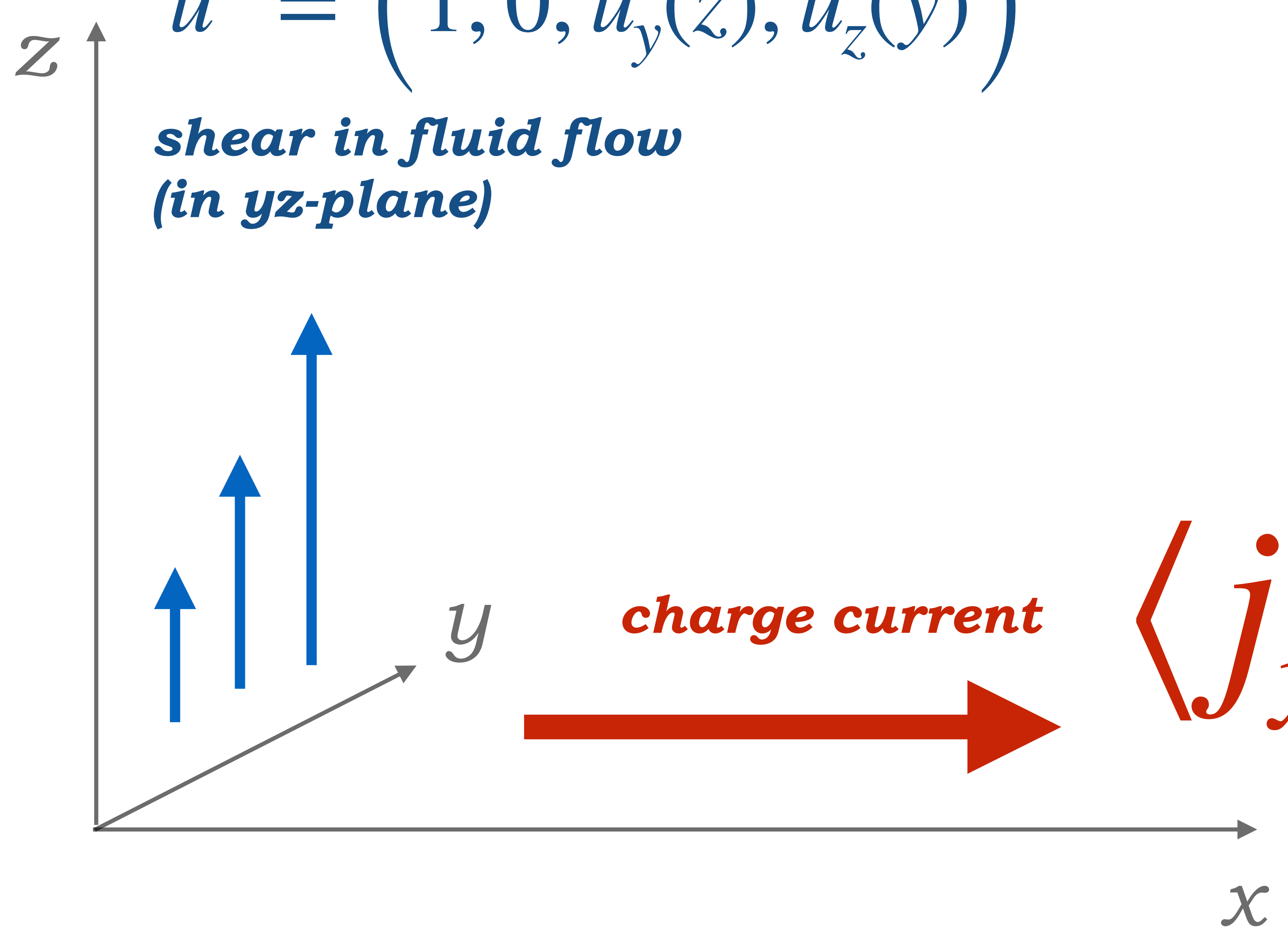


[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Shear-induced Hall conductivity c_{10}

$$c_{10} \sim \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle T^{tx} T^{yz} \rangle (\omega, \vec{k} = 0)$$

$$u^\nu = \left(1, 0, u_y(z), u_z(y)\right)$$

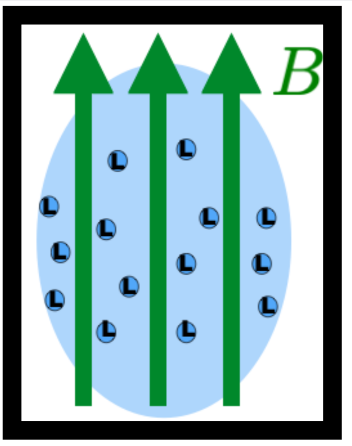


charge current

$$\langle j_x \rangle \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

- ➡ **novel Hall response**
- ➡ **non-dissipative**
- ➡ **interplay: shear-charge**

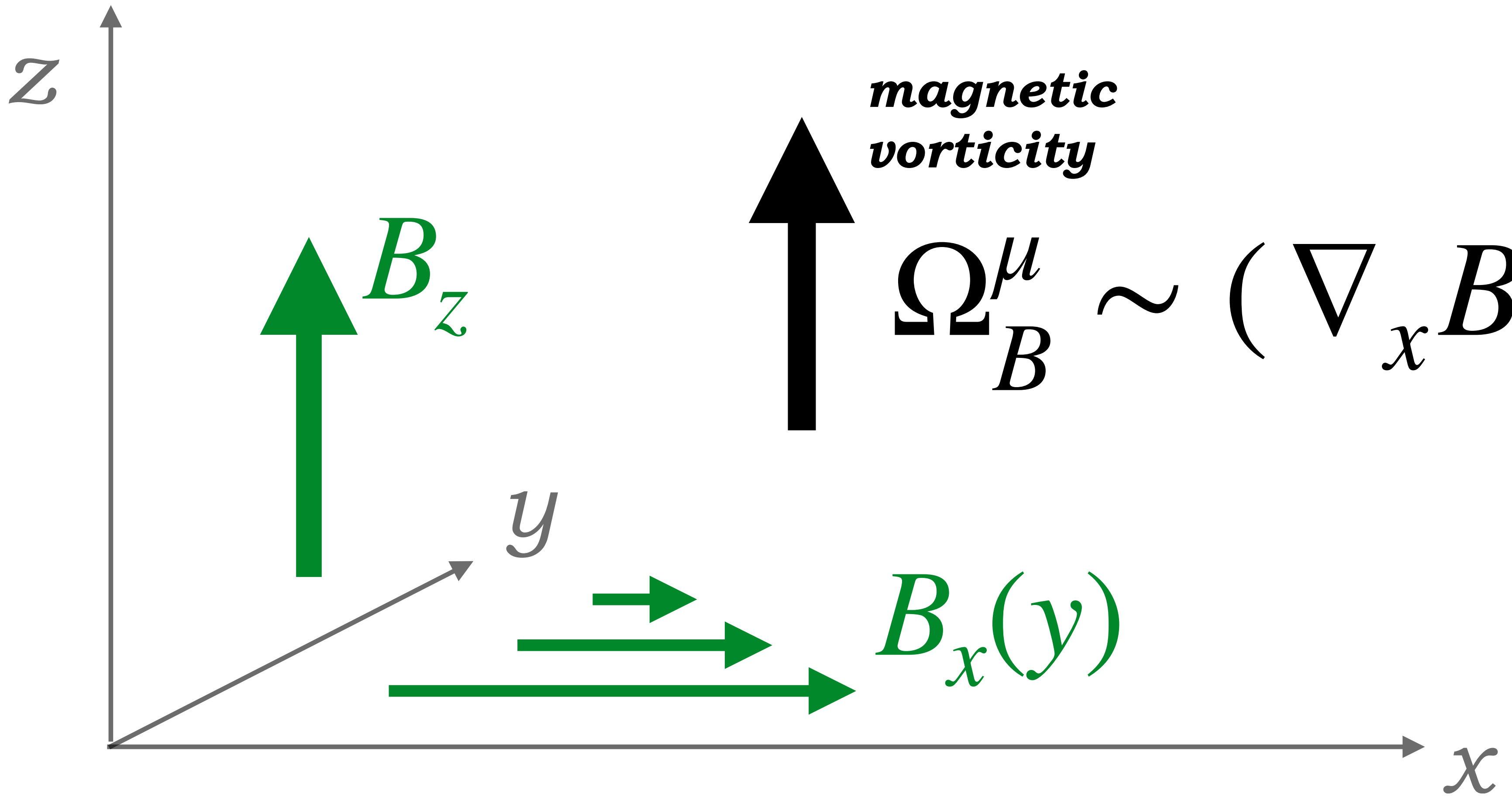
Chiral hydrodynamics - novel equilibrium coefficient M_2



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Perpendicular magnetic vorticity susceptibility M_2

$$M_2 = -\lim_{k_z \to 0} \frac{1}{2k_z B_0^2} \mathrm{Im}\langle T^{xz} T^{yz} \rangle (\omega = 0, k_z)$$



response in energy/pressure :

$$\langle T^{tt} \rangle = \mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_2 B \cdot \Omega_B$$

magnetic vorticity : $\Omega_B^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$

➡ **Can be computed on lattice** [Adhikari et al.; to appear in PPNP (2025)]

➡ **Test these Kubo formulae and constitutive relations now?**

Holographic model for chiral hydrodynamics

➡ **Construct holographic dual to charged plasma in strong *B***

➡ **Compute values for novel transport coefficients (*N*=4 SYM) from quasi normal modes and correlation functions**

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

cf. [Son,Surowka; PRL (2009)]
[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes

[D'Hoker, Kraus; JHEP (2010)]

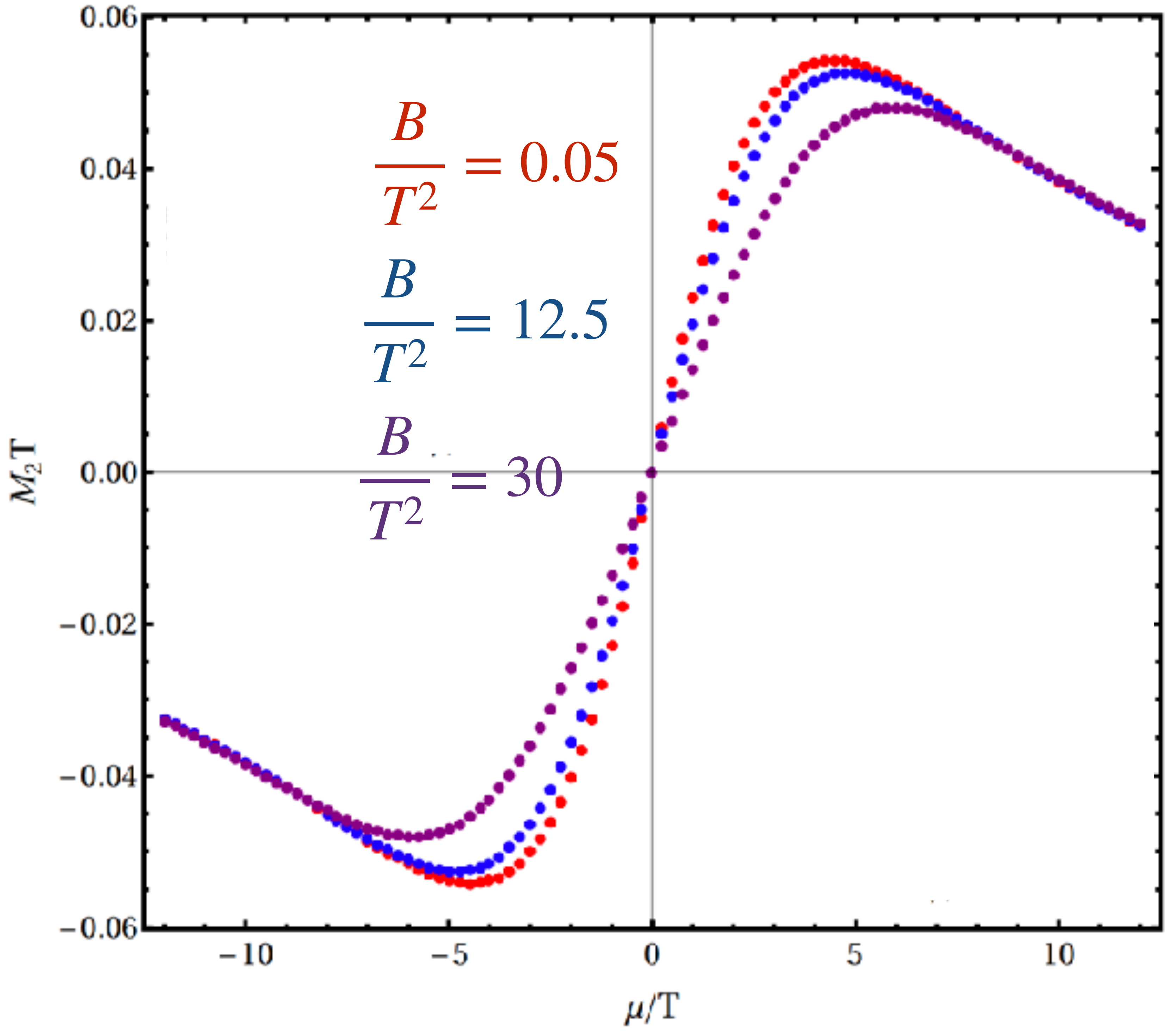
- **charged magnetic** analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

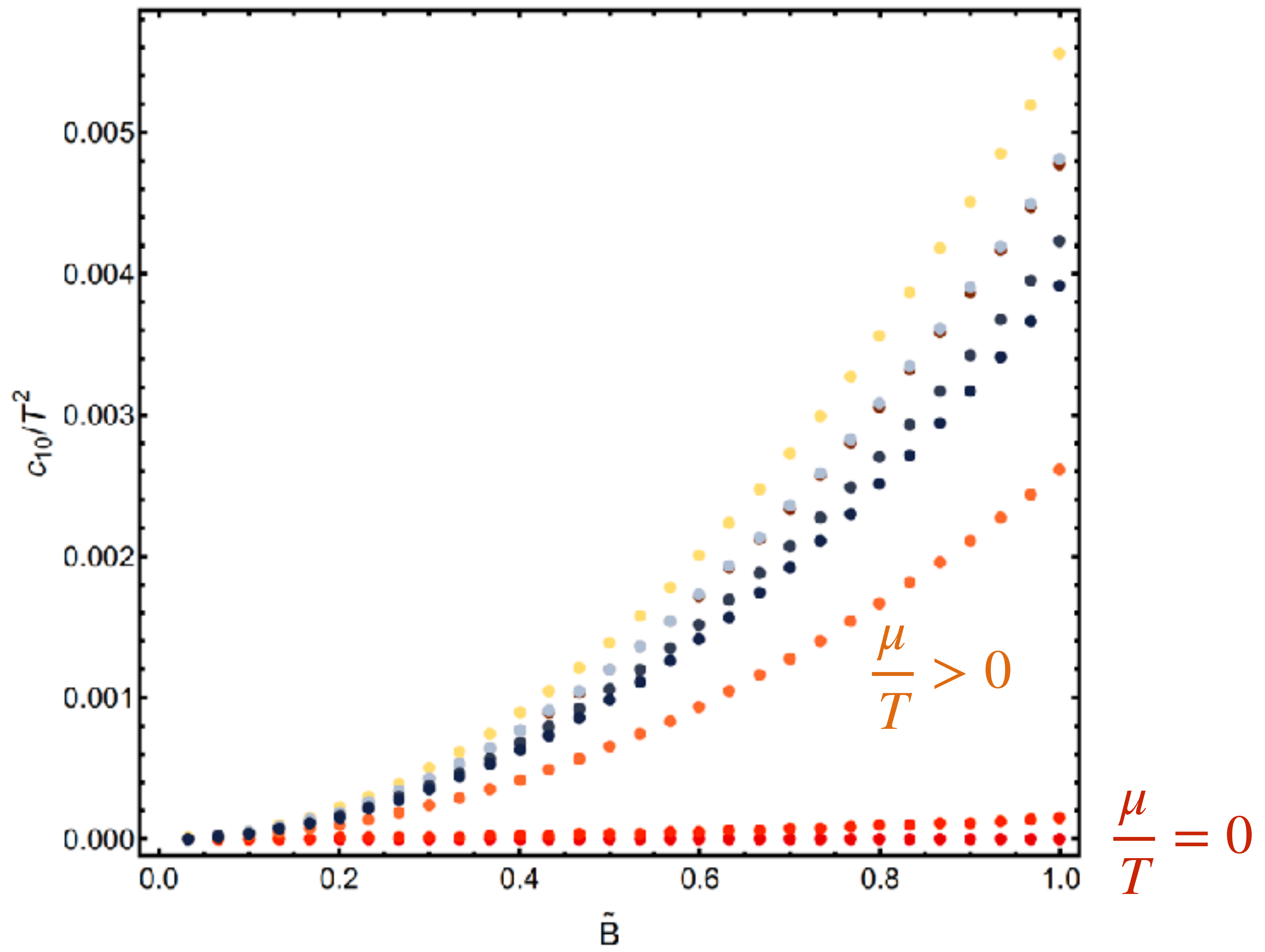
2. Holographic model for chiral hydrodynamics - Results

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Perpendicular magnetic vorticity susceptibility M_2



Shear-induced Hall conductivity c_{10}



➡ **not zero, finite, Onsager satisfied**
➡ ***all*** **Kubo formulae consistent**

Chiral hydrodynamics - all coefficients

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

cf. [Hernandez, Kovtun; JHEP (2017)]

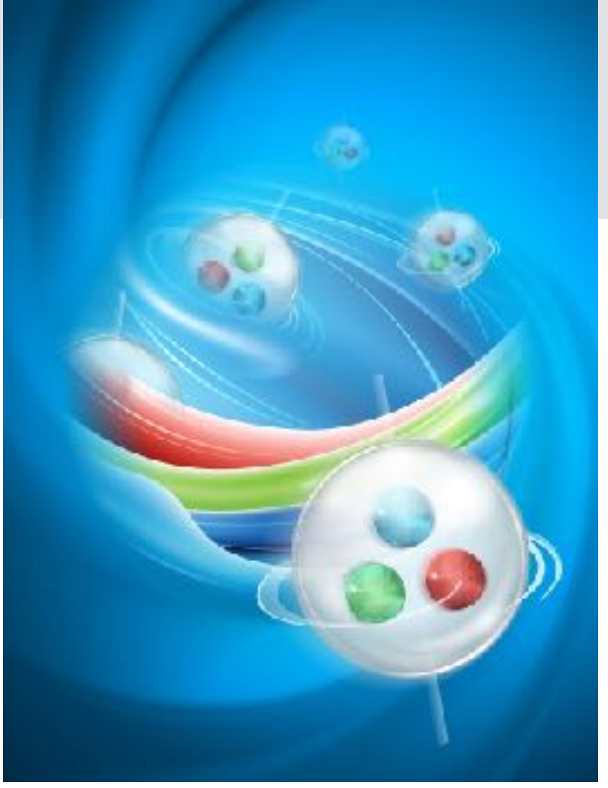
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim_{\mathbf{k}\to 0}\lim_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{xz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	J_xT_{ty} (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	J^xJ^y (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	J^tT^{xx} (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	J^tT^{tt} (2.32)	+	+	-
M_4	magneto-electric susceptibility	J^tJ^t (2.32)	+	-	-

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Non-dissipative Hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx}-T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}, T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	J^xJ^x, J^xJ^y (2.54,2.53b,2.53c)	+	-	+

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
dissipative, hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_{\|}$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_{\|}$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	J^xJ^x (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	J^xJ^y (2.55e)	+	+	-
$\sigma_{\|}$	long. conductivity	J^zJ^z (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp\delta_{ab} + \tilde{\rho}_\perp\,\epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	J_zT_{xx} (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	J_zT_{zz} (2.57)	+	-	-
c_3		$c_5 = -3(c_3+c_4)$	+	-	-

➡ **relevant for QGP or cond-mat?**

[Cartwright, Kaminski, Schenke; PRC (2022)]



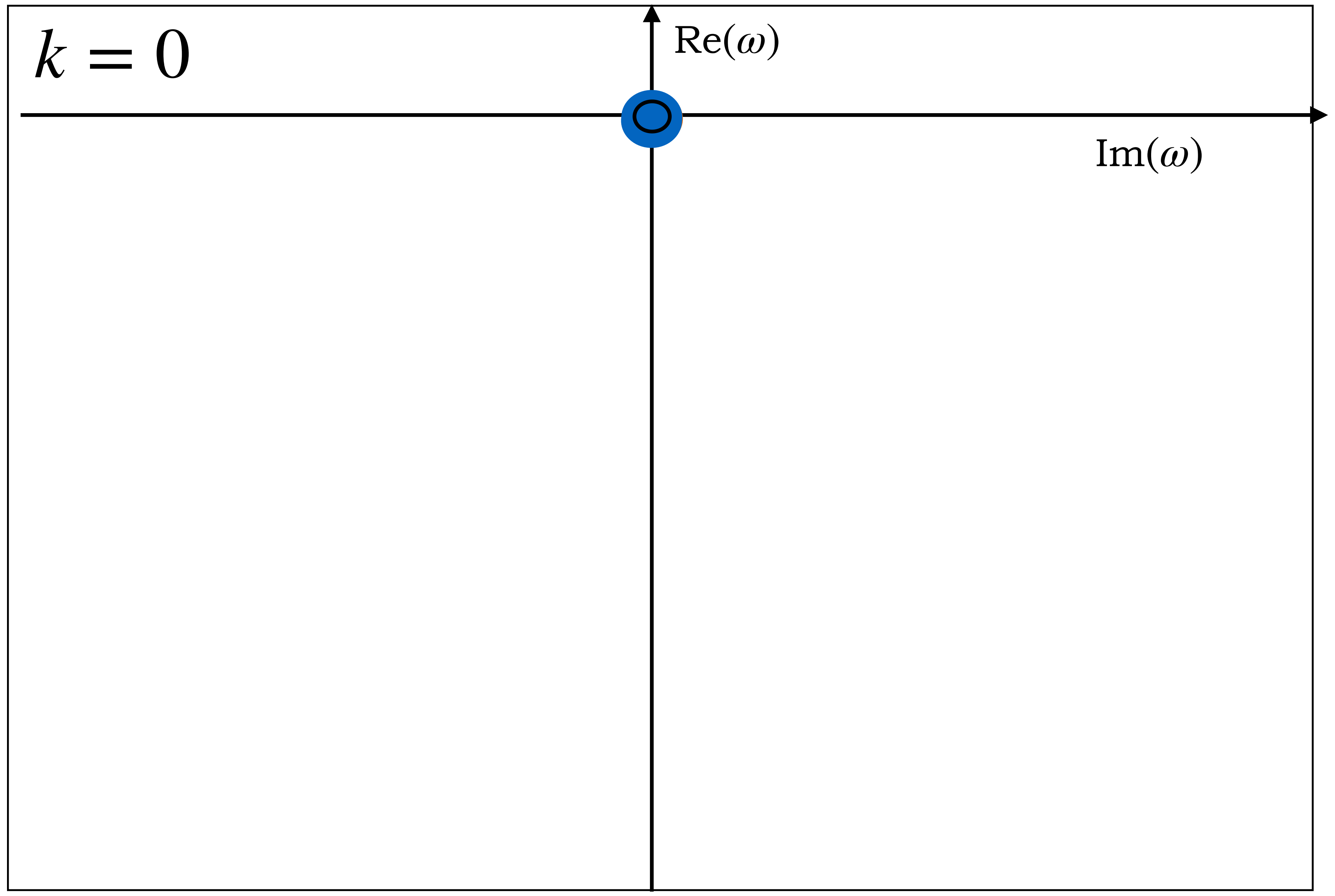
Hydrodynamic modes from holography

Interacting many-body systems at large temperature T have collective excitations, damped **eigenmodes**, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

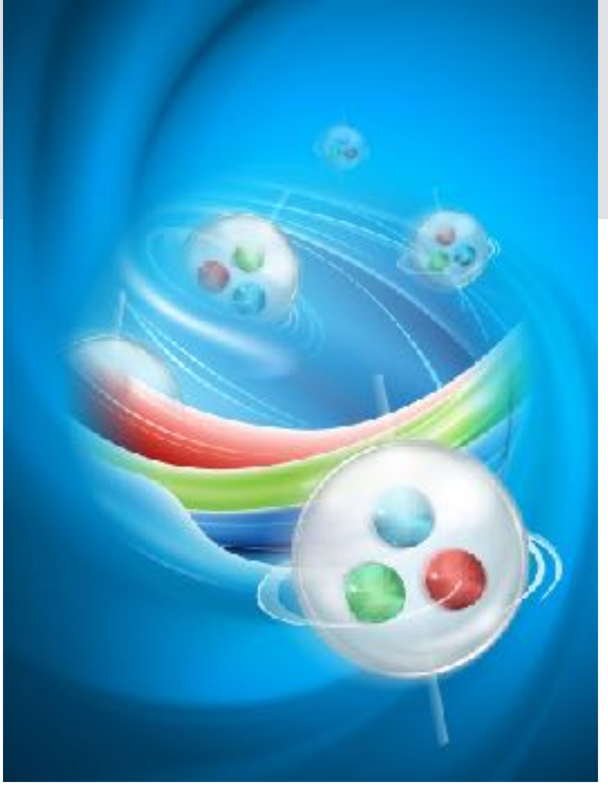
Sound modes

Momentum diffusion mode



Complex frequency plane

Hydrodynamic modes from holography



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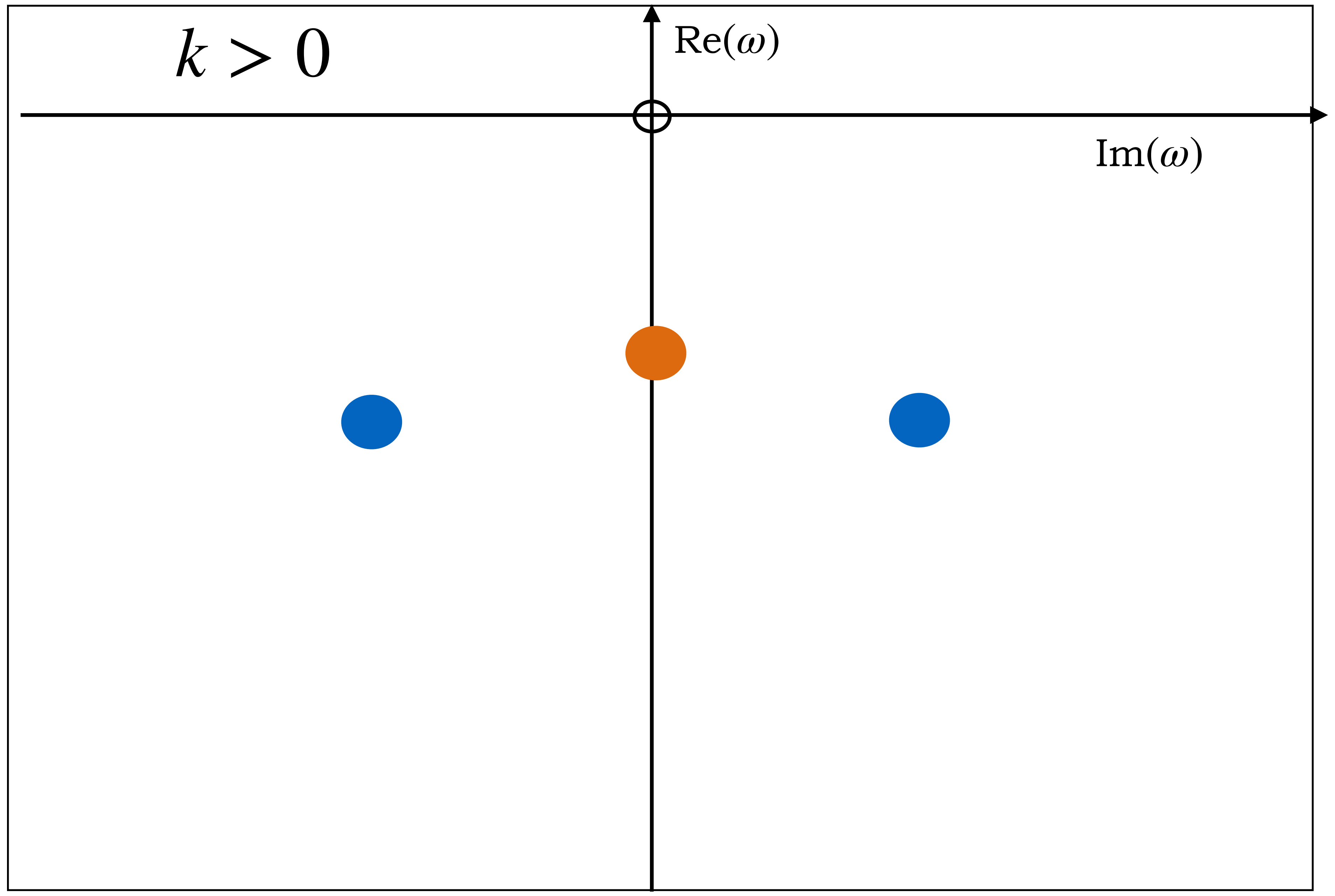

Complex frequency plane

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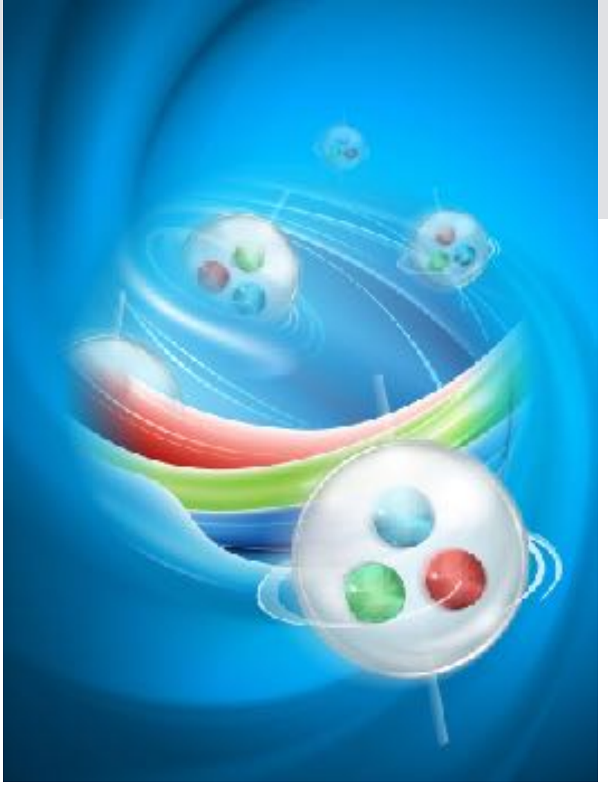

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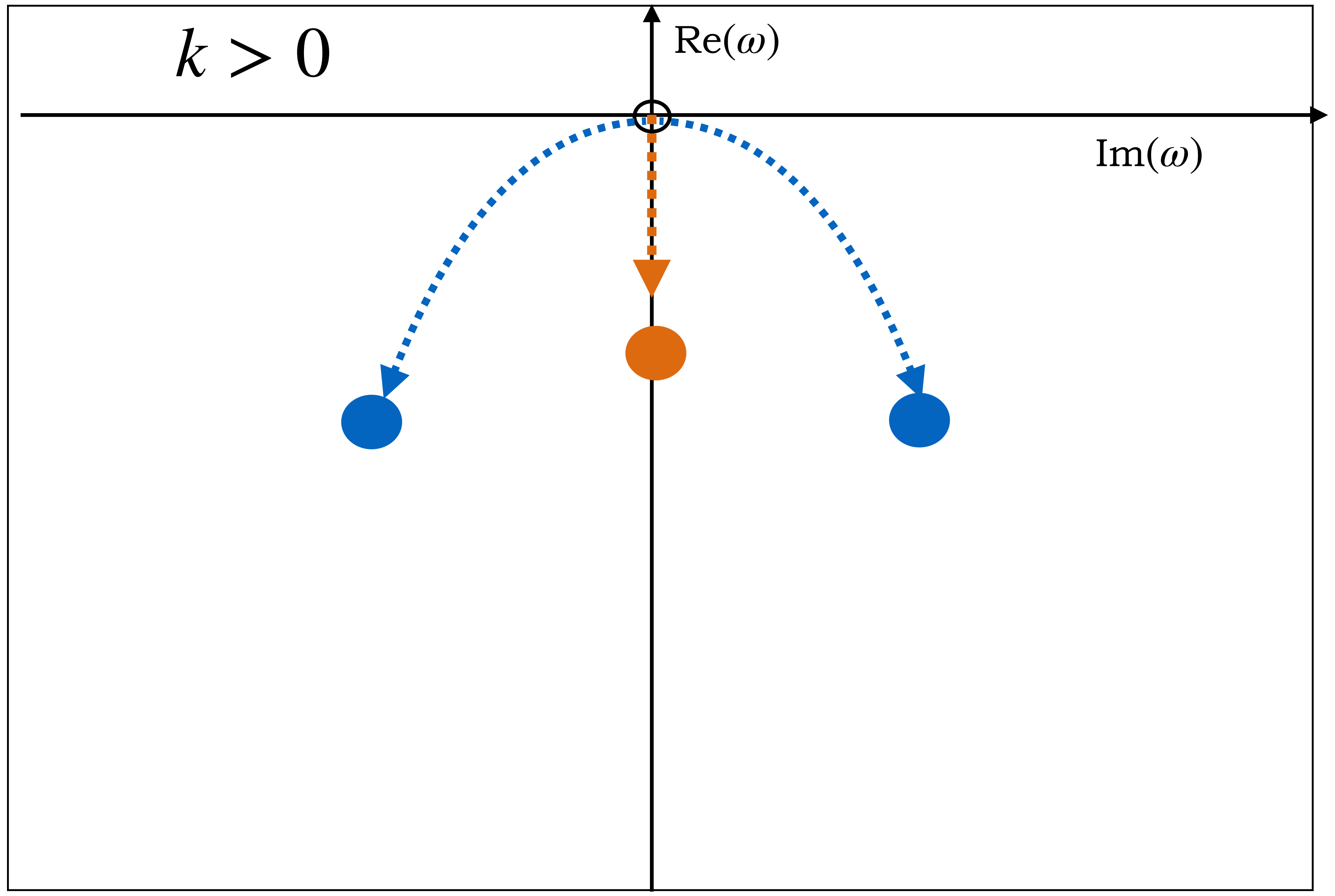

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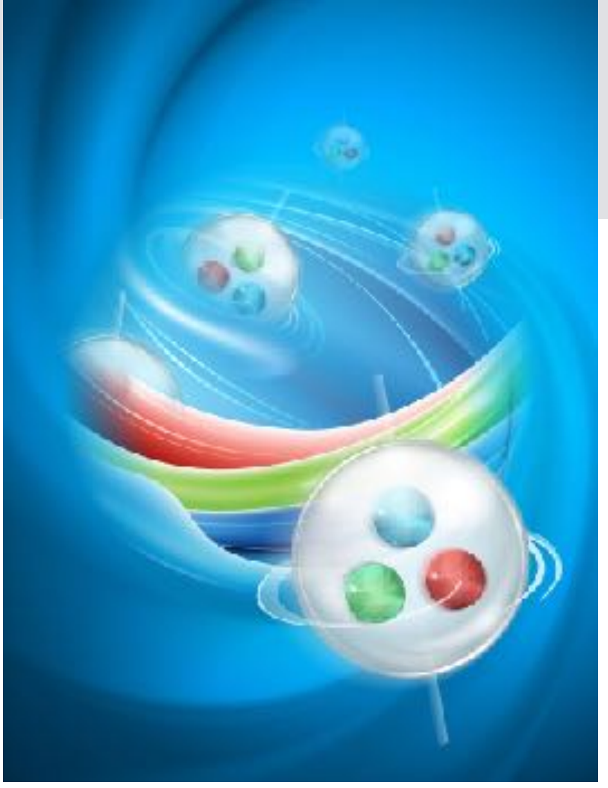

Hydrodynamic modes from holography

Interacting many-body systems at large temperature T have collective excitations, damped **eigenmodes**, with specific dispersion relations :

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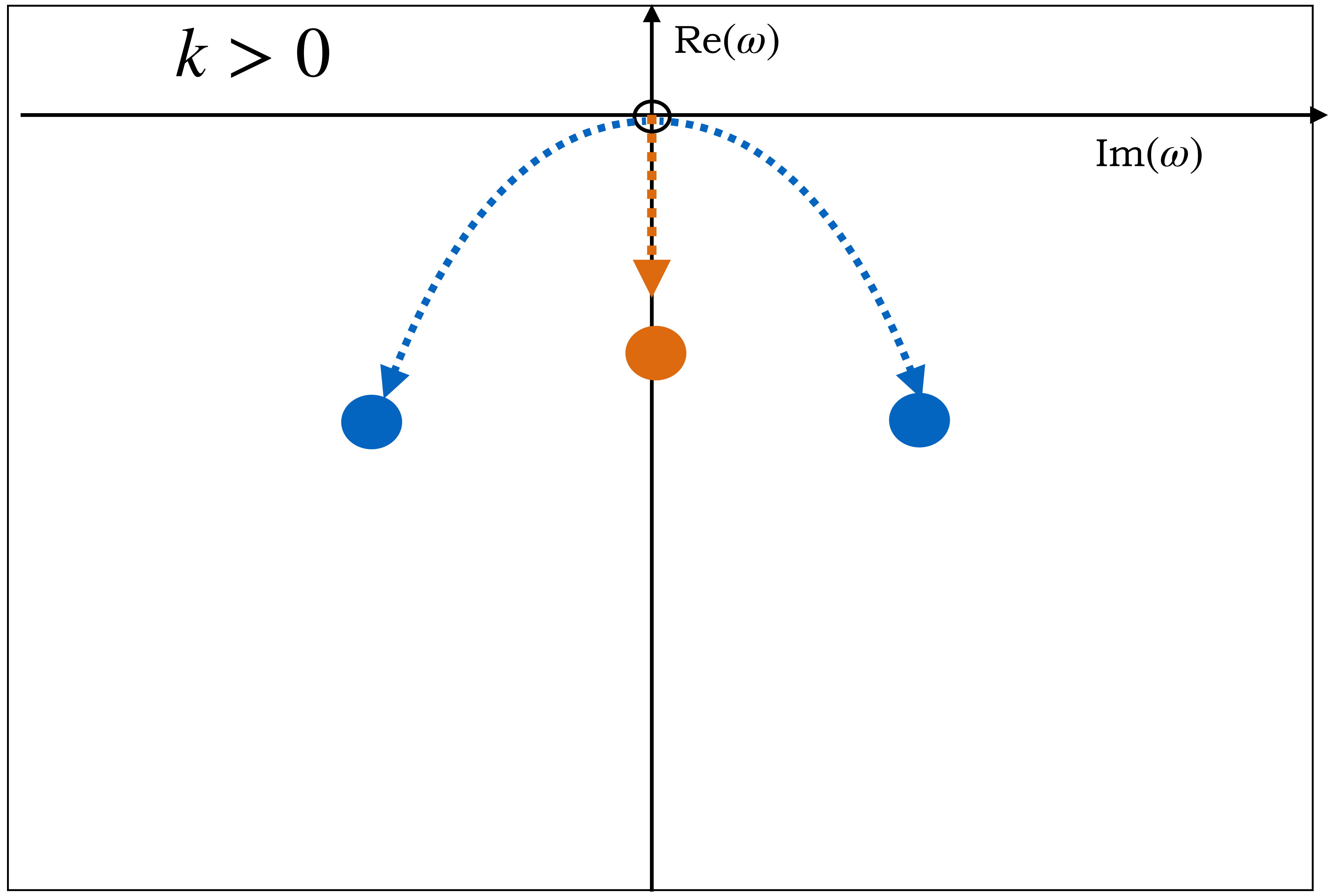
Sound modes

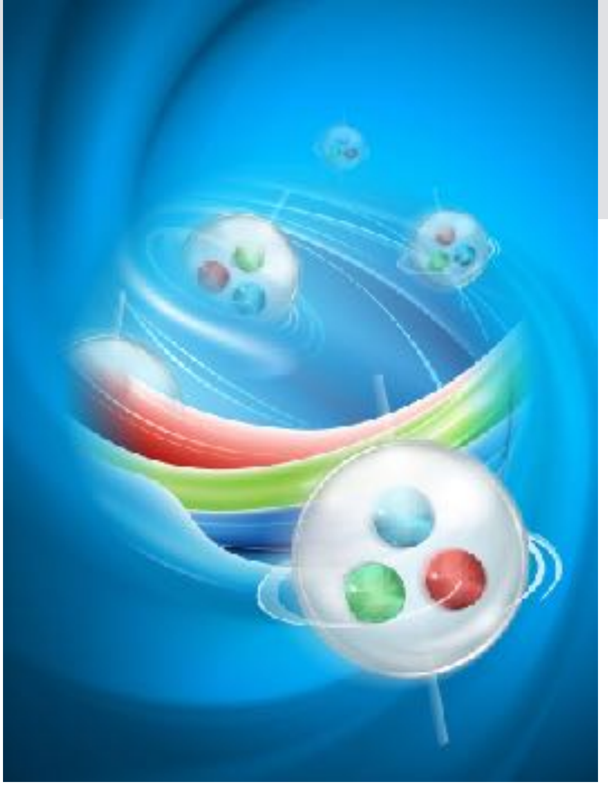
$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Momentum diffusion mode

$$\omega(k) = - iDk^2 + \mathcal{O}(3)$$



Complex frequency plane



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Momentum diffusion mode

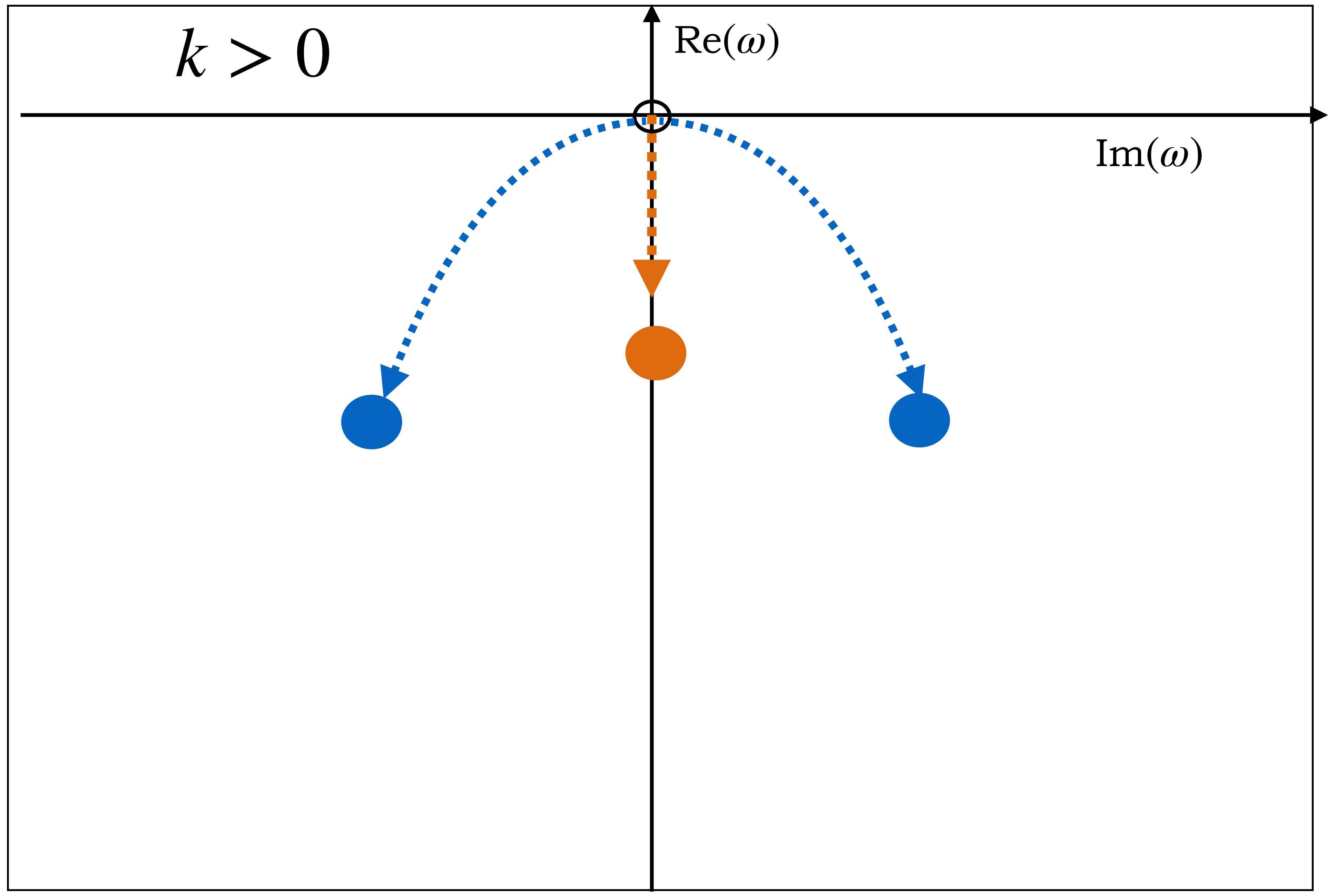
$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

$$\mathscr{P}\phi = 0$$

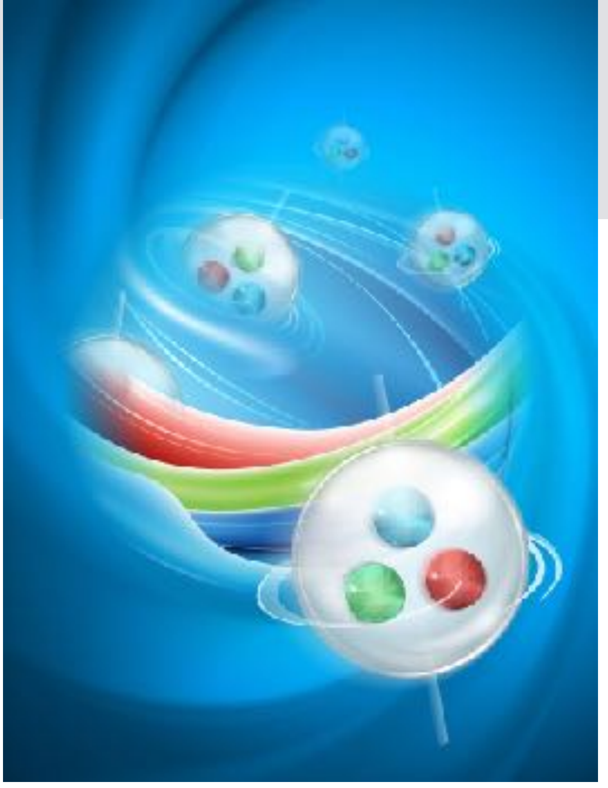
linear equation of motion for conserved quantity

$$\mathscr{P}\, G^R = \delta$$

$$G^R_{diffusion} \propto \mathscr{P}^{-1}_{diffusion} \propto \frac{1}{\partial_t - D\partial_x^2 + \mathcal{O}(3)} \propto \frac{1}{\omega + iDk^2 + \mathcal{O}(3)}$$



Complex frequency plane



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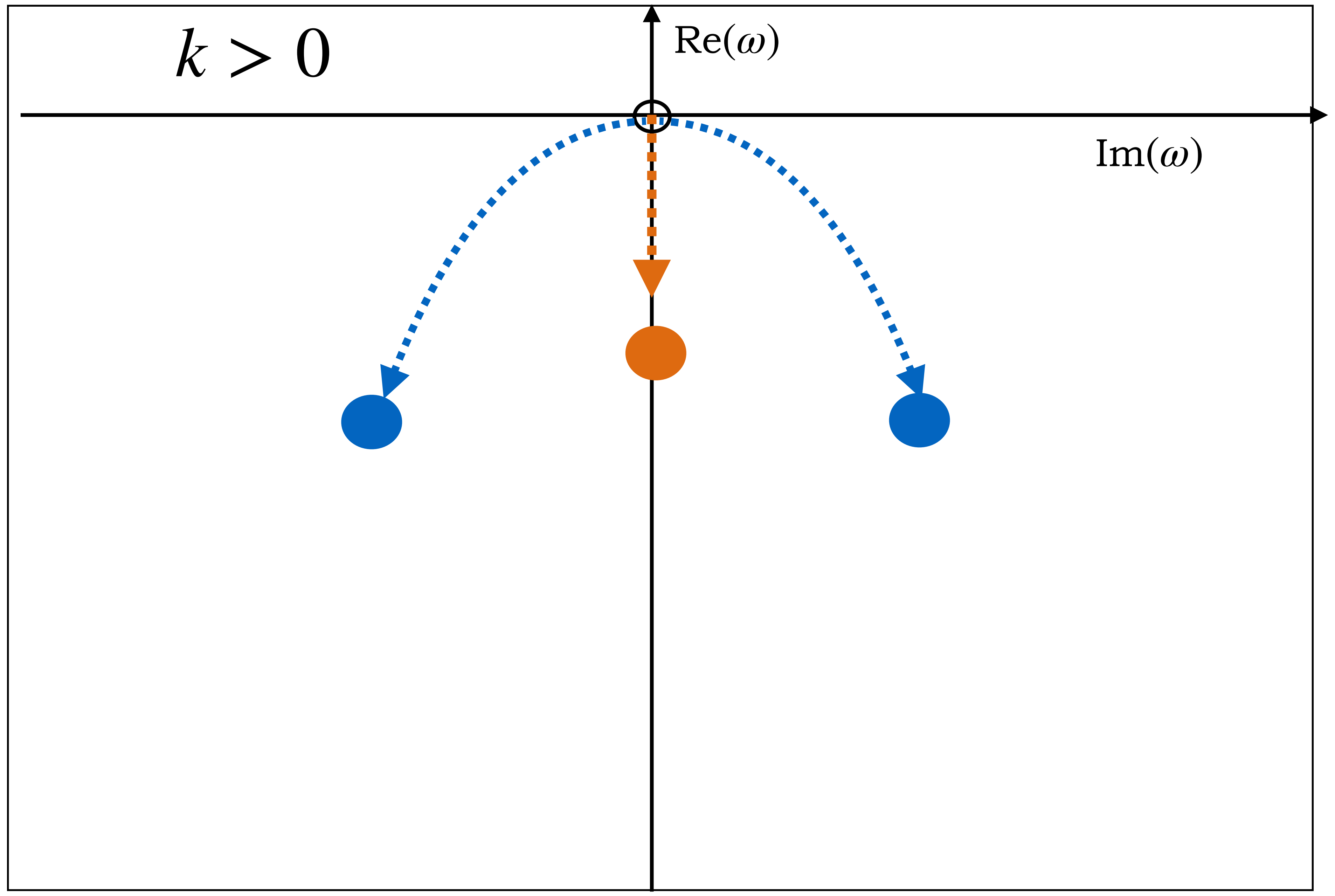
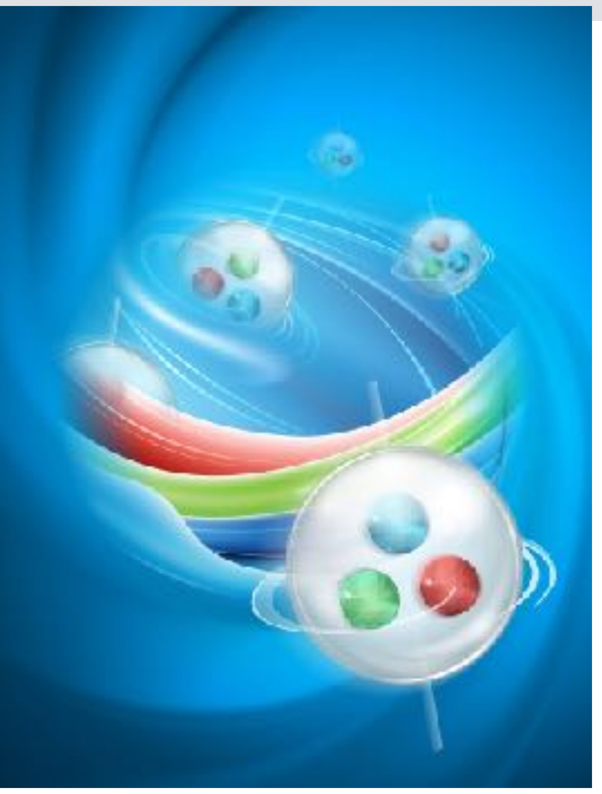


Complex frequency plane

➡Compute $\mathscr{P}(\omega, k) = 0$ from holography: $\mathscr{P} \sim |\delta g_{\mu\nu}|_{\text{boundary}}$

Holographic model exhibits hydrodynamic modes under rotation



Fluctuations

- **Einstein gravity** dual to *N*=4 SYM theory
- metric of a **rotating asymptotically AdS5 black hole** (solution to Einstein equations) dual to a rotating thermal SYM state
- **black hole thermodynamics** "determines" thermodynamics of the rotating SYM state
- poles of the SYM Green's functions dual to quasi normal mode (QNM) frequencies of black hole: **QNMs encode SYM dispersion relations**

Example: rotation-invariant fluid from QNMs of metric fluctuations *[Kovtun/Starinets; JHEP (2005)]*

Momentum diffusion mode

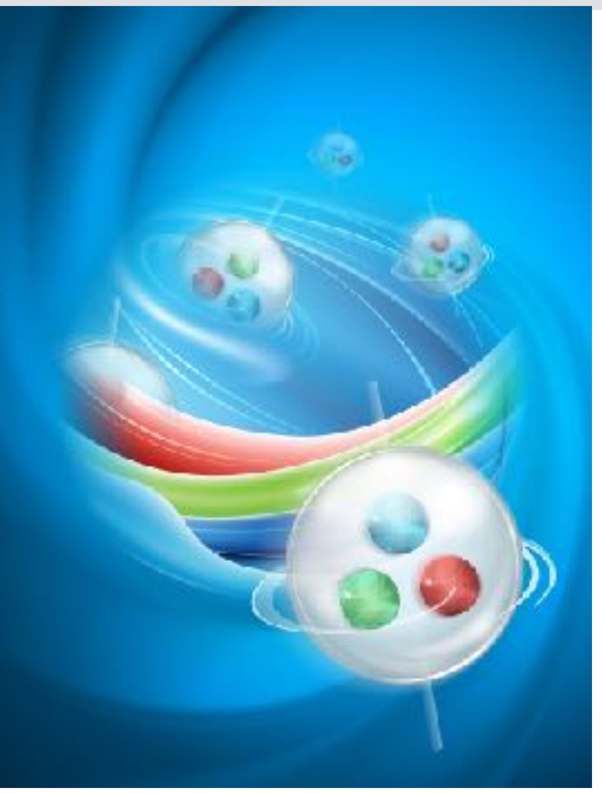
$\omega(k) = -iDk^2 + \mathcal{O}(3)$ ⟷ $\delta g_{tx}, \delta g_{zx}, \ldots$ **(vector)**

Sound modes

$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$ ⟷ $\delta g_{tt}, \delta g_{tz}, \delta g_{zz}$ **(scalar)**

➡Compute the QNM frequencies around rotating black hole as function of momentum.

Holographic model exhibits hydrodynamic modes under rotation



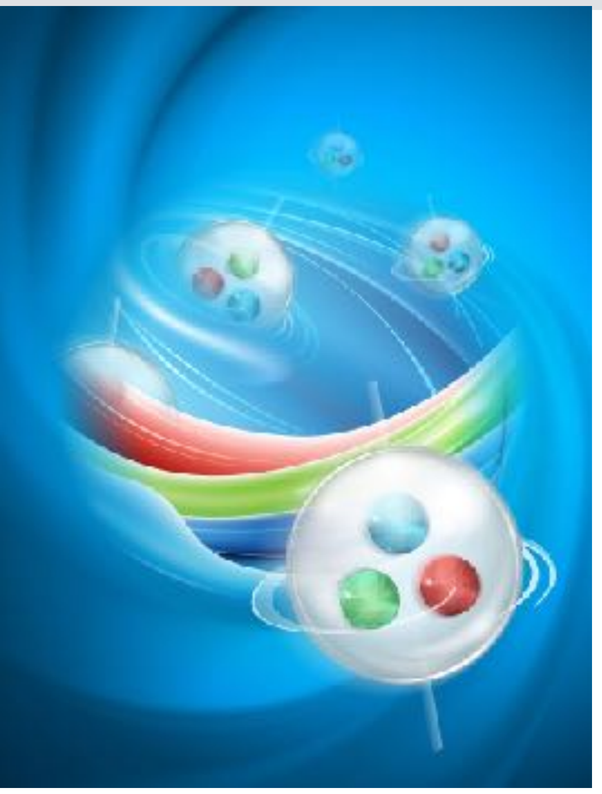
Rotating AdS5 black hole

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2$$

$$G(r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(1-a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4}\,,$$

$$\mu = \frac{r_+^4\left(L^2 + r_+^2\right)}{2L^2 r_+^2 - 2a^2\left(L^2 + r_+^2\right)}\,,$$

Holographic model exhibits hydrodynamic modes under rotation



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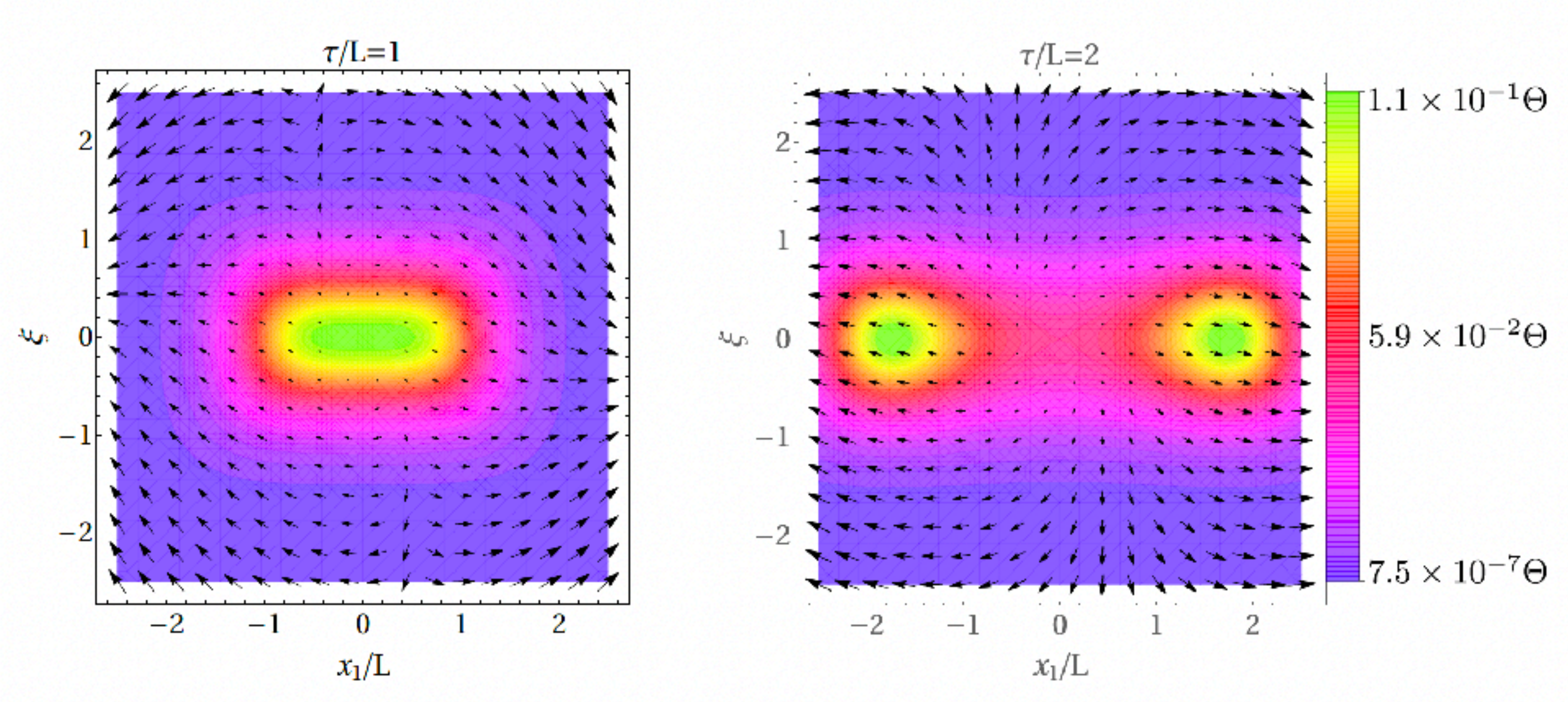


Rotating thermal SYM state

analytic fluid flow (cf. Gubser flow)

$$u^\tau = \lambda\left[\cosh\xi\left(L^2+\tau^2+x_\perp^2\right) + 2\Omega(Lx_1\sinh\xi + \tau x_2)\right]$$

$$u^1 = \lambda\left[2(L\tau\Omega\sinh\xi + \tau x_1\cosh\xi + x_1x_2\Omega)\right]\,,$$

$$u^2 = \lambda\left[\Omega\left(L^2+\tau^2-x_1^2+x_2^2\right) + 2\tau x_2\cosh\xi\right]\,,$$

$$u^\xi = -\tau^{-1}\lambda\left[-\sinh\xi\left(L^2-\tau^2+x_\perp^2\right) - 2Lx_1\Omega\cosh\xi\right]$$

$$\epsilon = (16L^8\Theta^4)\left(1-\Omega^2\right)^{-2}\times\left(2L^2\tau^2\cosh 2\xi + \left(L^2+x_\perp^2\right)^2 + \tau^4 - 2\tau^2x_\perp^2\right)^{-2}\,,$$

$$\lambda = \left(\frac{\epsilon}{16L^8\Theta^4}\right)^{1/4}, \quad \Theta = \left(\frac{3(1-\Omega^2)\mu}{8\pi G_5L^3}\right)^{1/4}\,,$$

Large black holes: large T

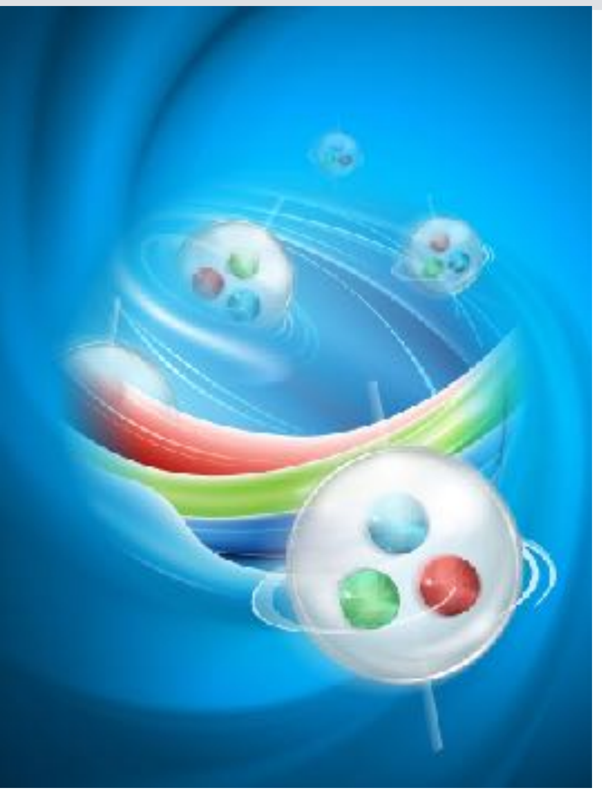
$$r_+ \to \alpha r_+\,, \quad r \to \alpha r\,, \quad \alpha \to \infty$$

[Bantilan, Ishii, Romatschke; PLB (2018)]

Milne coordinates $(\tau, x_1, x_2, \xi; r)$

$$\xi = \tfrac{1}{2}\ln[(t+x_3)/(t-x_3)] \qquad \tau = \sqrt{t^2 - x_3^2}$$

High temperature: dispersion relations of rotating black hole look like boosted fluid



[Garbiso-Amano, Kaminski; JHEP (2019)]
[Garbiso-Amano, Cartwright, Kaminski, Wu; PPNP (2024)]
cf. [Hoult,Kovtun (2020)] [Kovtun (2019)]

Dispersion relations:

$$\nu(j) = -aj - i\frac{1}{2}(1-a^2)^{3/2}j^2 + \mathcal{O}(j^3)$$

$$\nu(j) = \frac{\pm 1 - \sqrt{3}a}{\sqrt{3} \mp a} j - i\sqrt{3}\frac{(1-a^2)^{3/2}}{(\sqrt{3}-a)^3} j^2 + \mathcal{O}(j^3)$$

Boost transformation:

$$\mathfrak{q}^2 = \frac{(a\nu + j)^2}{1-a^2}\,, \qquad \mathfrak{w}^2 = \frac{(\nu + aj)^2}{1-a^2}$$

"Speeds of diffusion":

$$v_{||} = a,$$

Speeds of sound:

$$v_{s,\pm} = v_{s,0}\frac{\sqrt{3}a \pm 1}{1 \pm \frac{a}{\sqrt{3}}}\,,$$

Corresponding damping:

$$\mathcal{D}_{||} = \mathcal{D}_0(1-a^2)^{3/2}, \qquad \Gamma_{s,\pm} = \Gamma_0\frac{\left(1-a^2\right)^{3/2}}{\left(1 \pm \frac{a}{\sqrt{3}}\right)^3}\,,$$

Einstein relations:

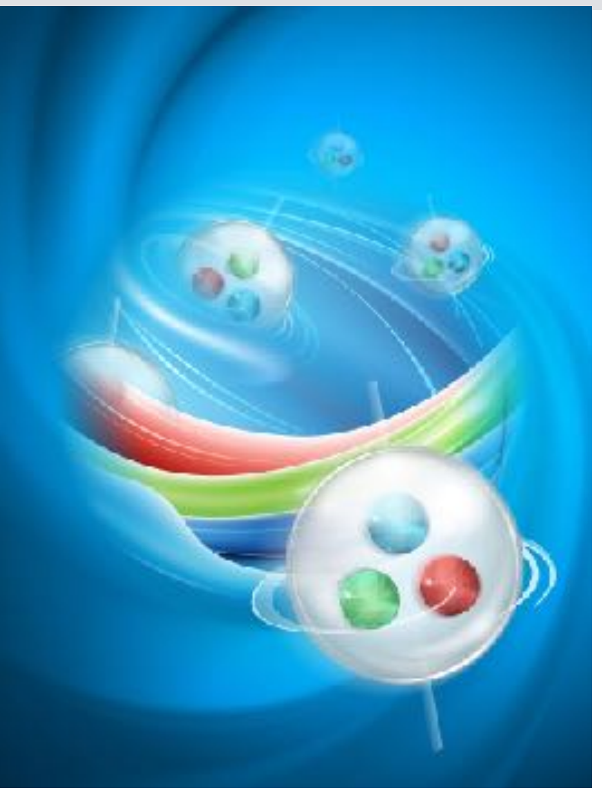
$$\mathcal{D}_{||}(a) = 2\pi T_0 \frac{\eta_{||}(a)}{\epsilon(a) + P_\perp(a)}\,,$$

$$\Gamma_\pm(a) = \frac{2\eta_{||}(a)}{3(\epsilon(a)+P_\perp(a))}\frac{1}{(1 \pm a/\sqrt{3})^3}\,.$$

Shear viscosities:

$$\eta_\perp(a) = \eta_0\frac{1}{\sqrt{1-a^2}}\,, \qquad \eta_{||}(a) = \eta_0\sqrt{1-a^2}\,,$$

➡If transport coefficients known at rest, then they are known in high *T* rotating fluid (boosted fluid).

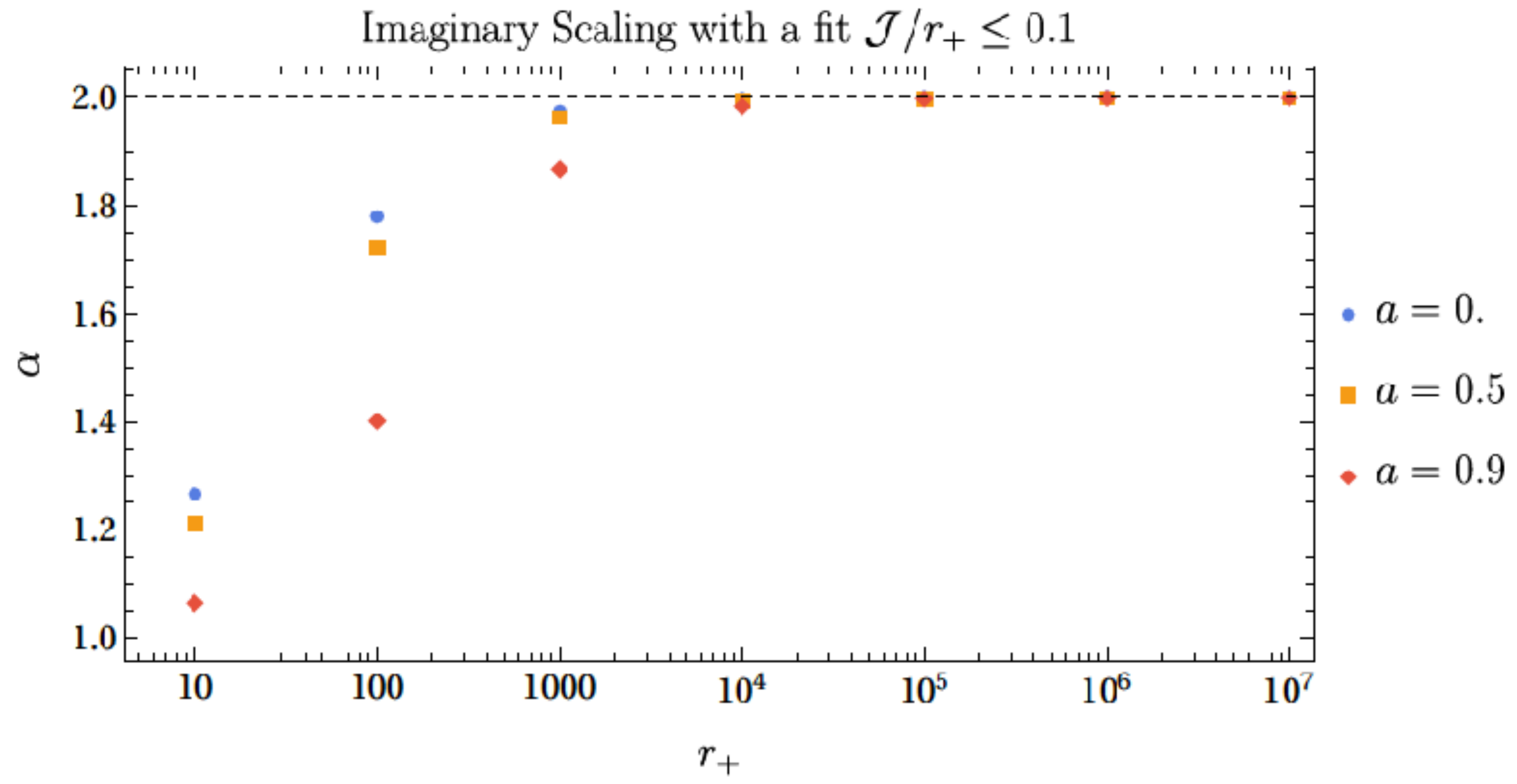


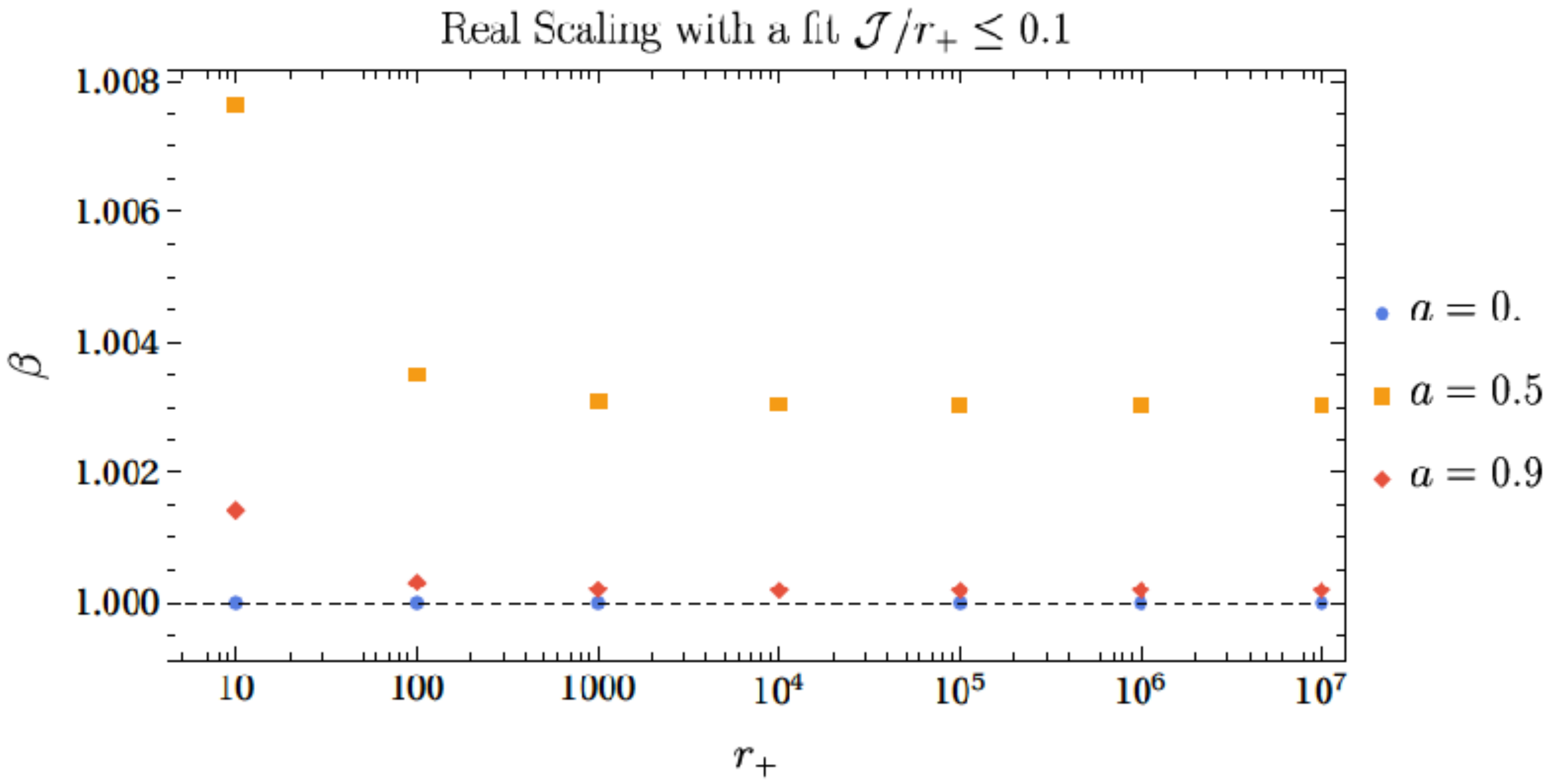
[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

- **validity of the constitutive relations and transport coefficients**

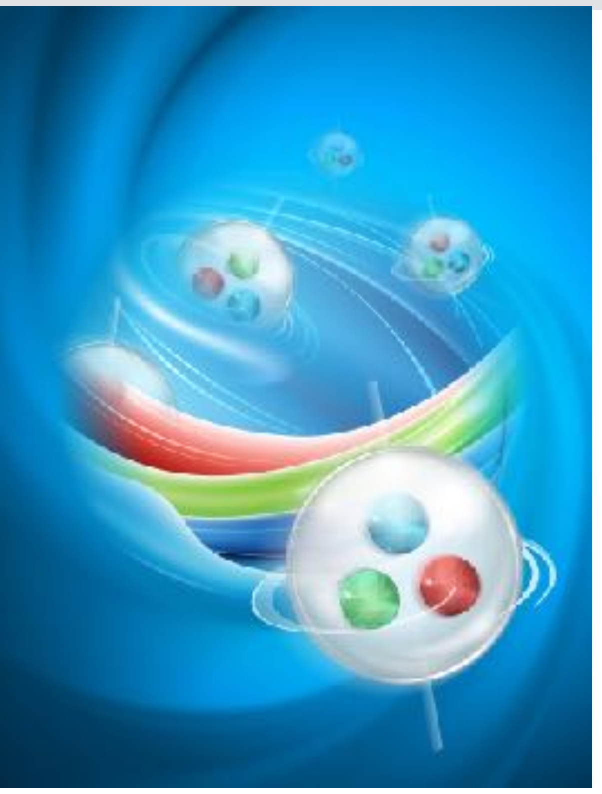
Momentum diffusion

$$\omega = v\mathcal{J}^{\beta} - iD\mathcal{J}^{\alpha}$$



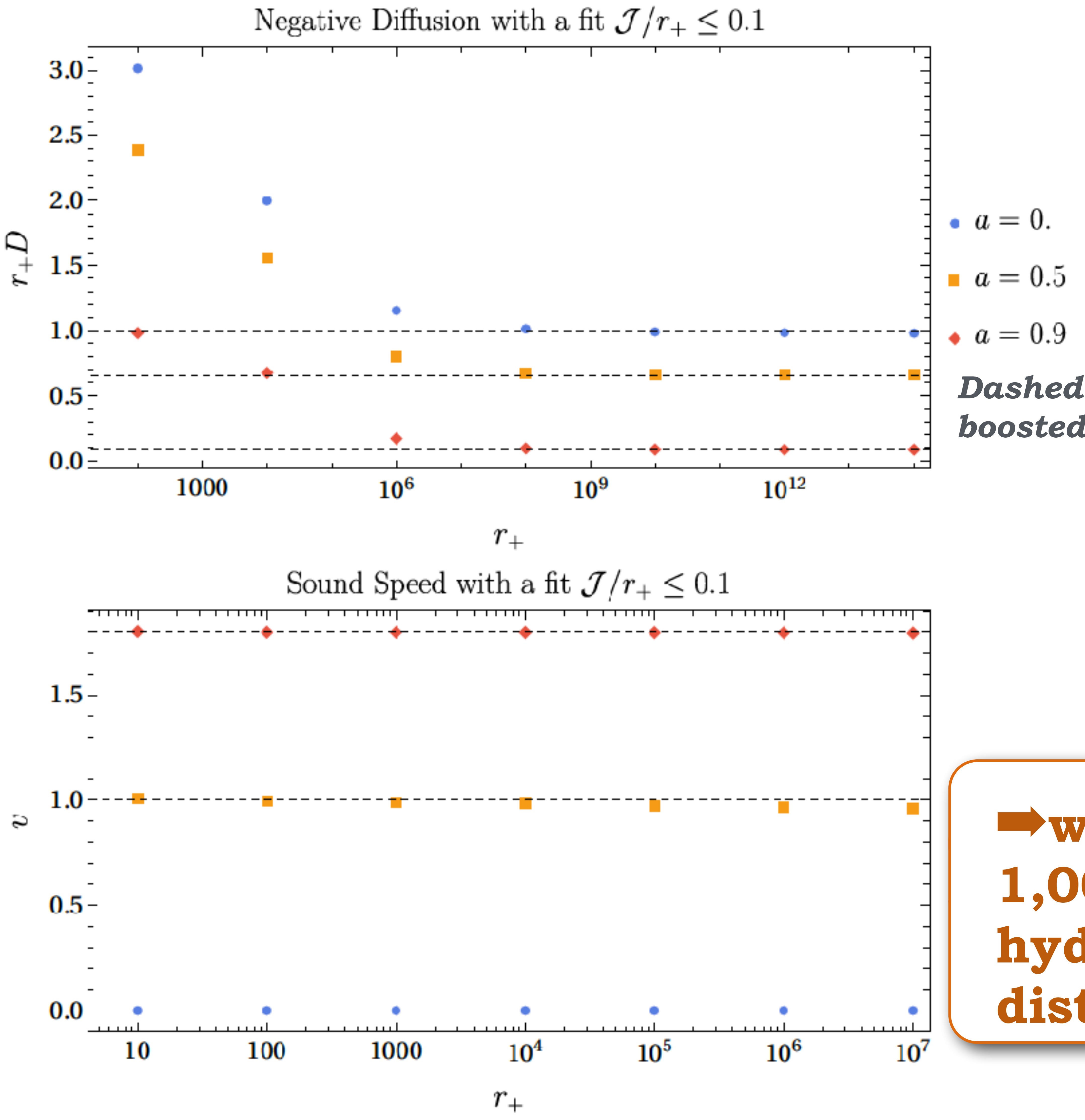


Is hydrodynamics valid? - Transport coefficients



[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

- **validity of the <u>constitutive relations and transport coefficients</u>**



Momentum diffusion

$$\omega = v\mathcal{J}^{\beta} - iD\mathcal{J}^{\alpha}$$

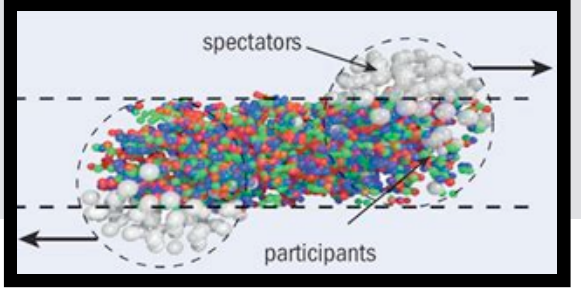
Dashed horizontal lines: boosted fluid values

$$v_{||} = a,$$

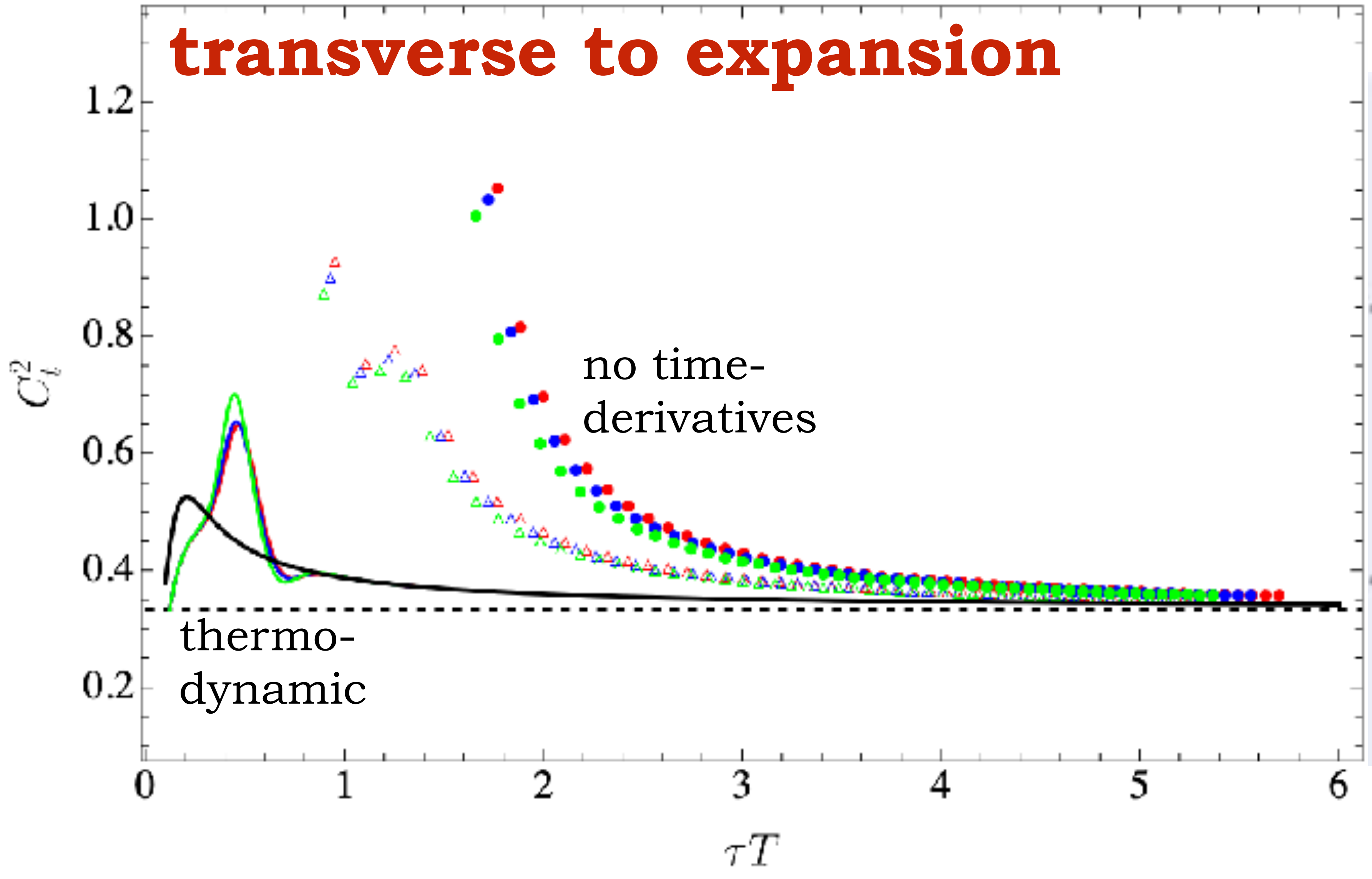
$$\mathcal{D}_{||} = \mathcal{D}_0(1 - a^2)^{3/2}$$

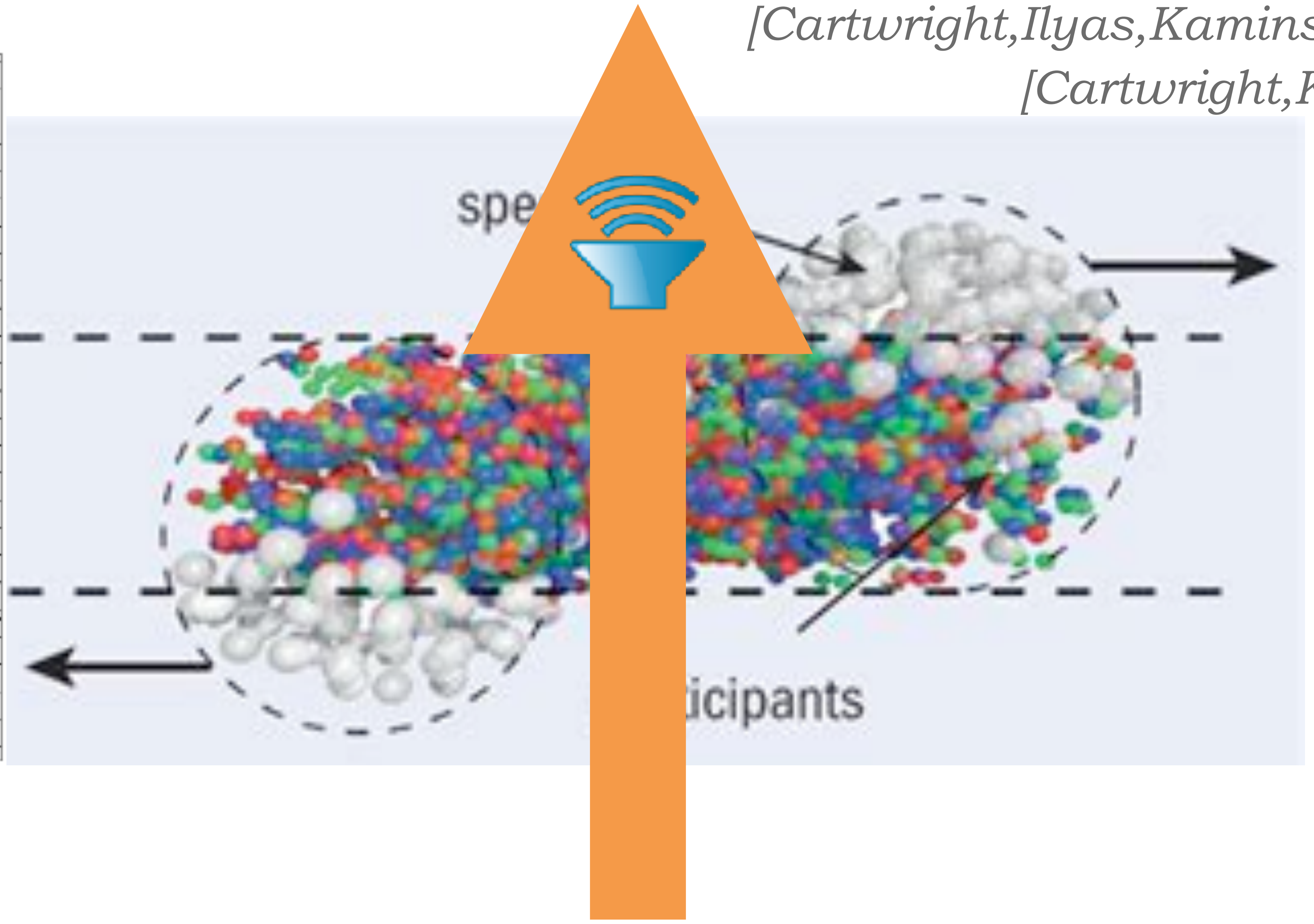
➡window of horizon values 1,000 < r+ < 10^7: hydrodynamic behavior distinct from a boosted fluid

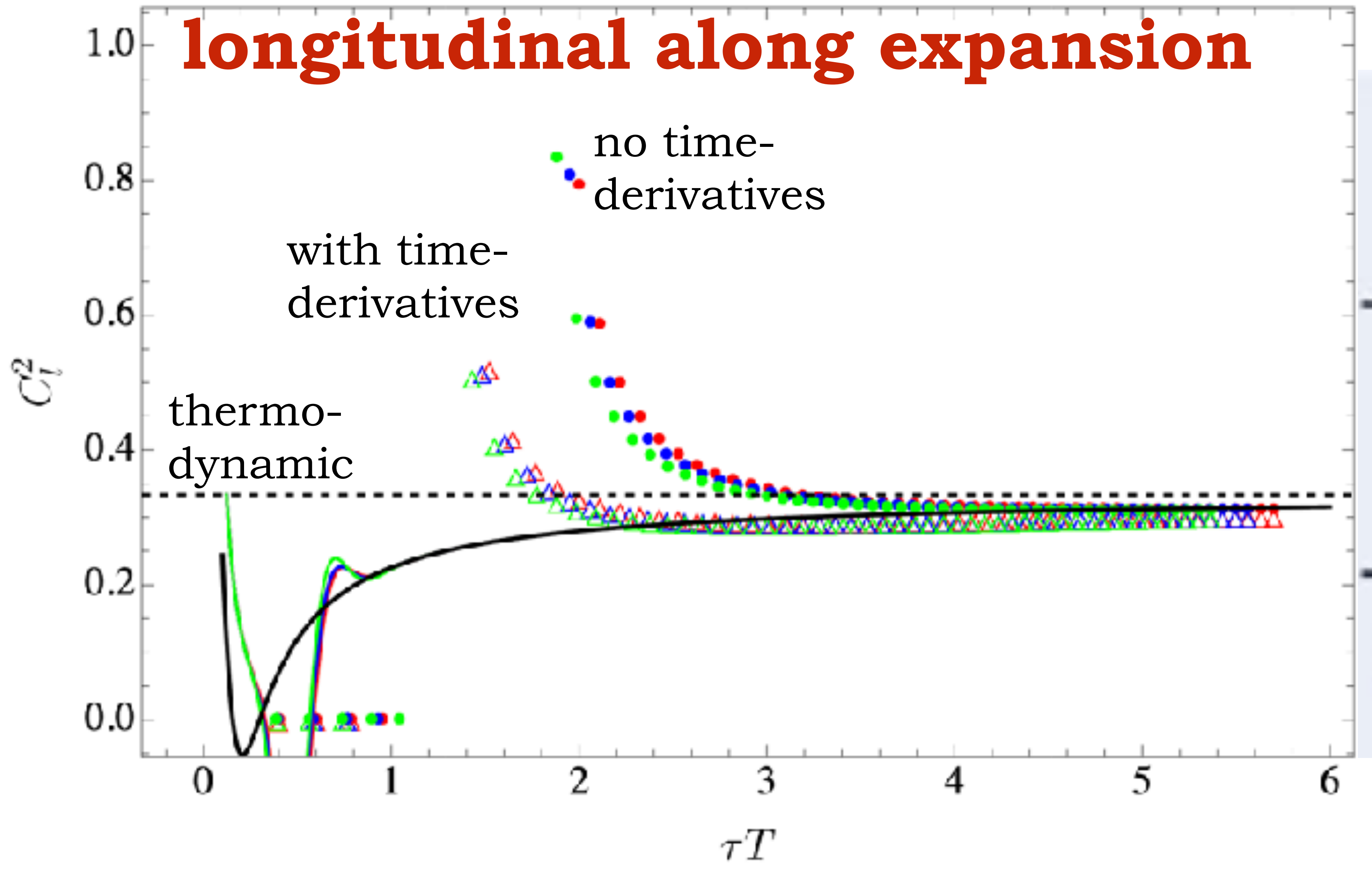
Two Speeds of Sound in Bjorken-Expanding $\mathcal{N}$=4 SYM QGP

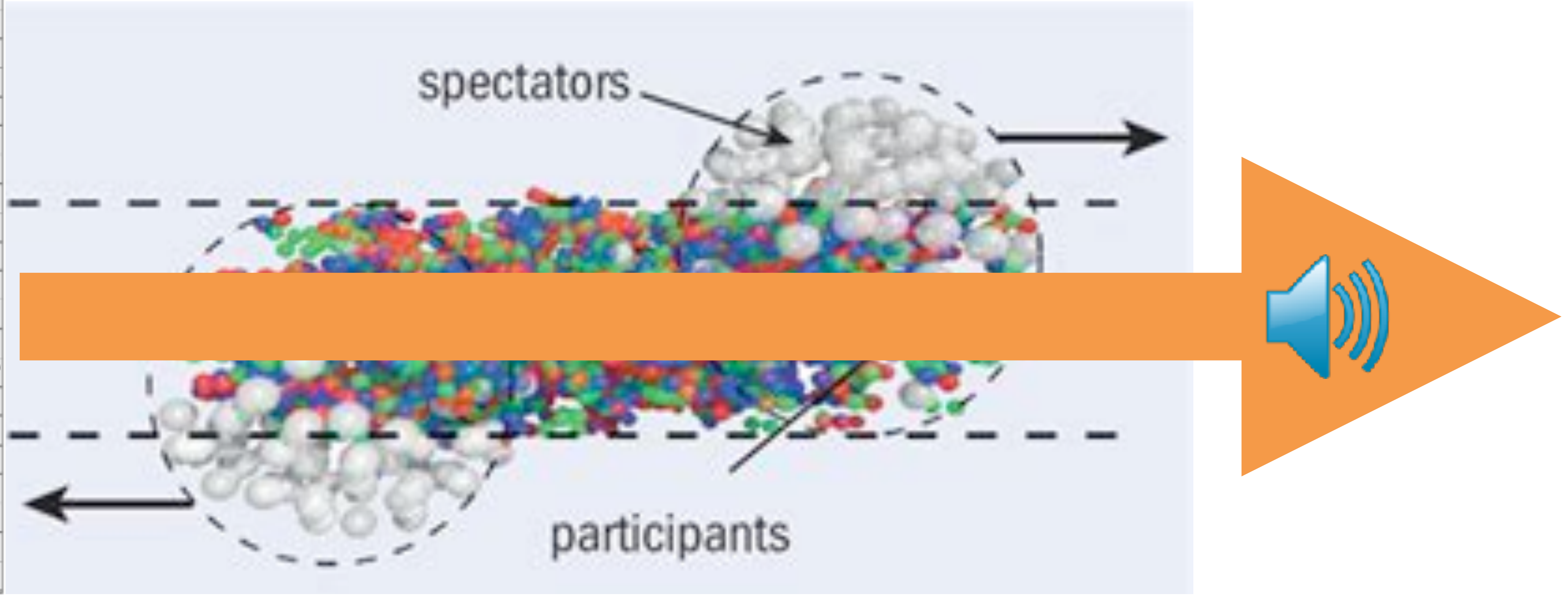


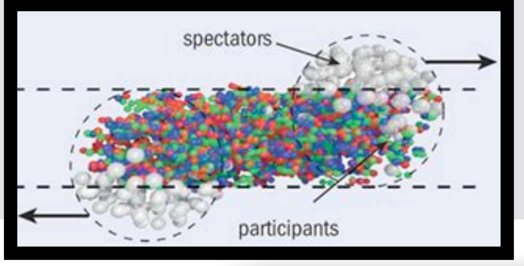
[Cartwright,Ilyas,Kaminski,Knipfer,Zhang; in progress]
[Cartwright,Kaminski,Knipfer; PRD (2023)]







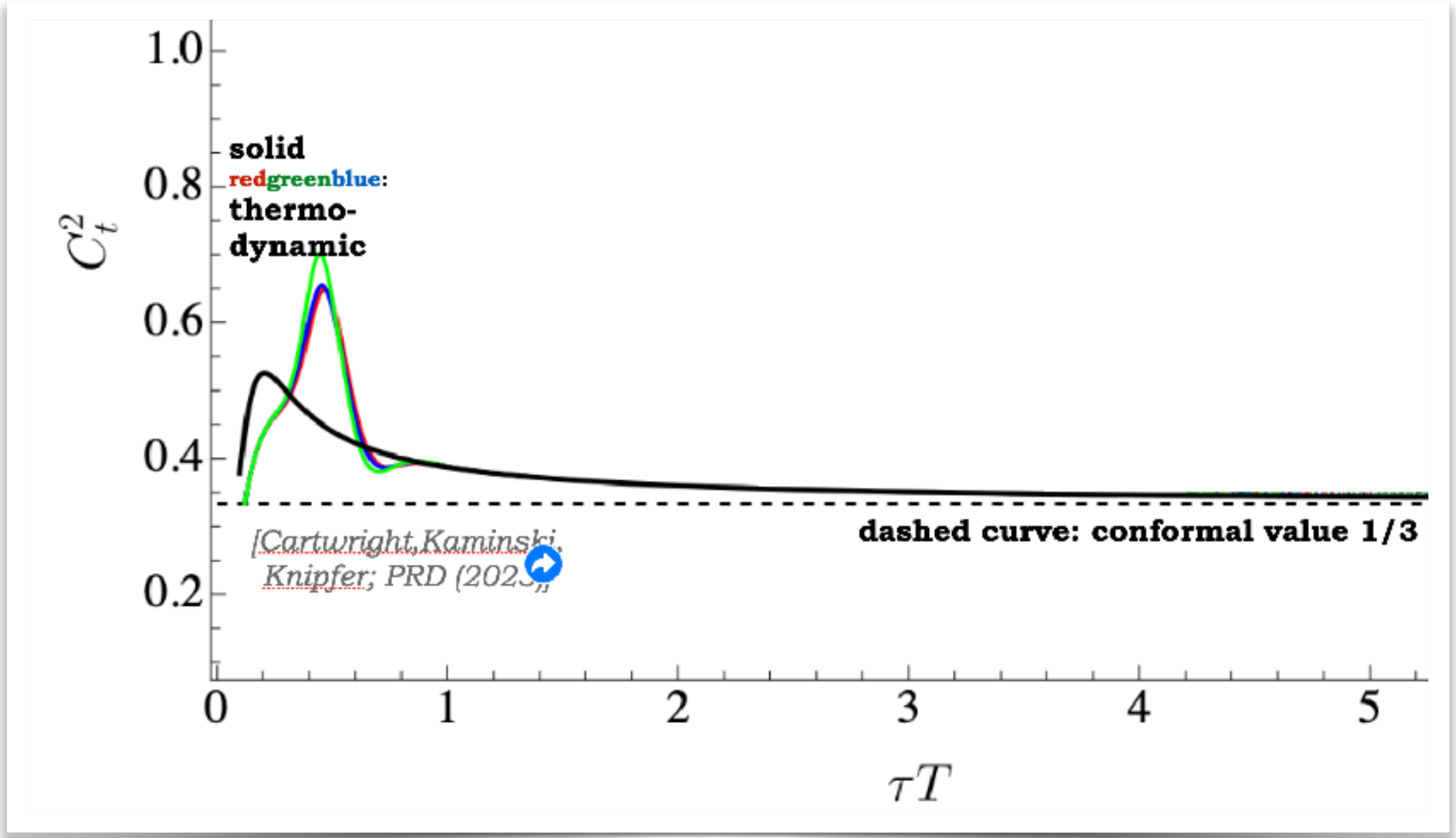




Methods: Thermodynamic Definition

Equilibrium speed of sound

$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s$$



Transverse/longitudinal speeds of sound out of equilibrium

[Cartwright,Kaminski,Knipfer; PRD (2023)]

$$c_\perp^2 = -\frac{\partial \langle T^{x_1}_{x_1}\rangle}{\partial \langle T^0_0\rangle}\,, \qquad c_\parallel^2 = -\frac{\partial \langle T^{\xi}_{\xi}\rangle}{\partial \langle T^0_0\rangle}$$

Metric near-boundary expansion

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$g_{\mu\nu} \sim g^{(0)}_{\mu\nu} + \langle T_{\mu\nu}\rangle z^4 + \ldots$$

metric ***source*** ***one-point function***

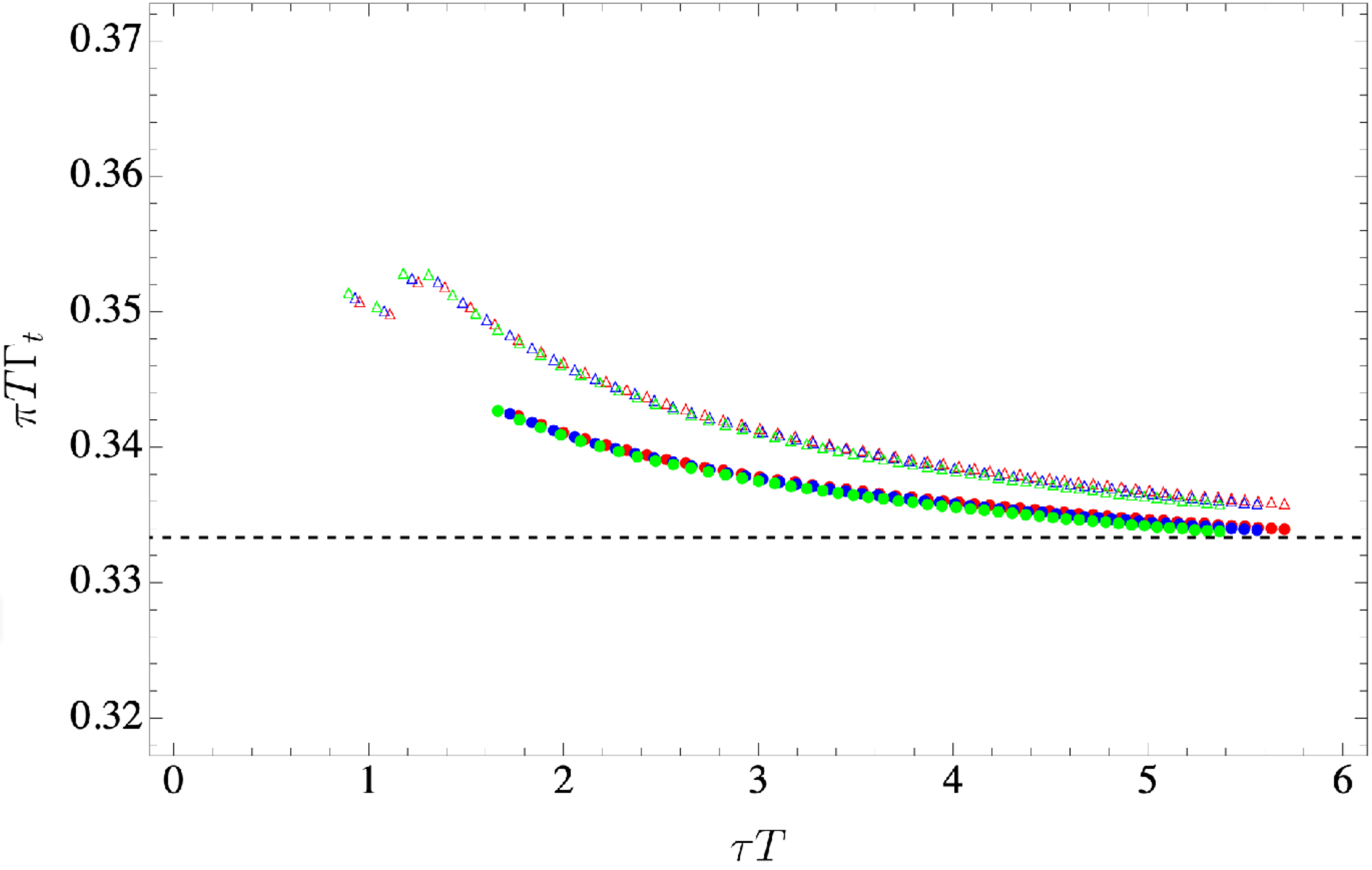
➡ verify with perturbative calculation

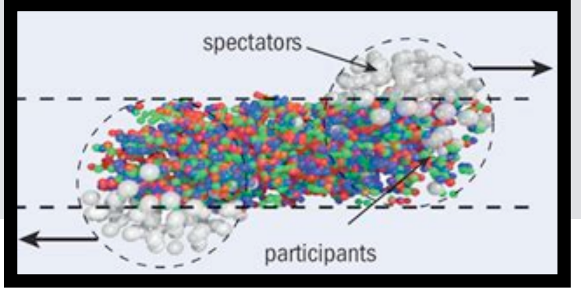
$$g_{\mu\nu}(\tau) + h^{(\text{sound})}_{\mu\nu}$$

Using technique from [Wondrak, Kaminski, Bleicher; Phys.Lett.B (2020)]

Sound Attenuation

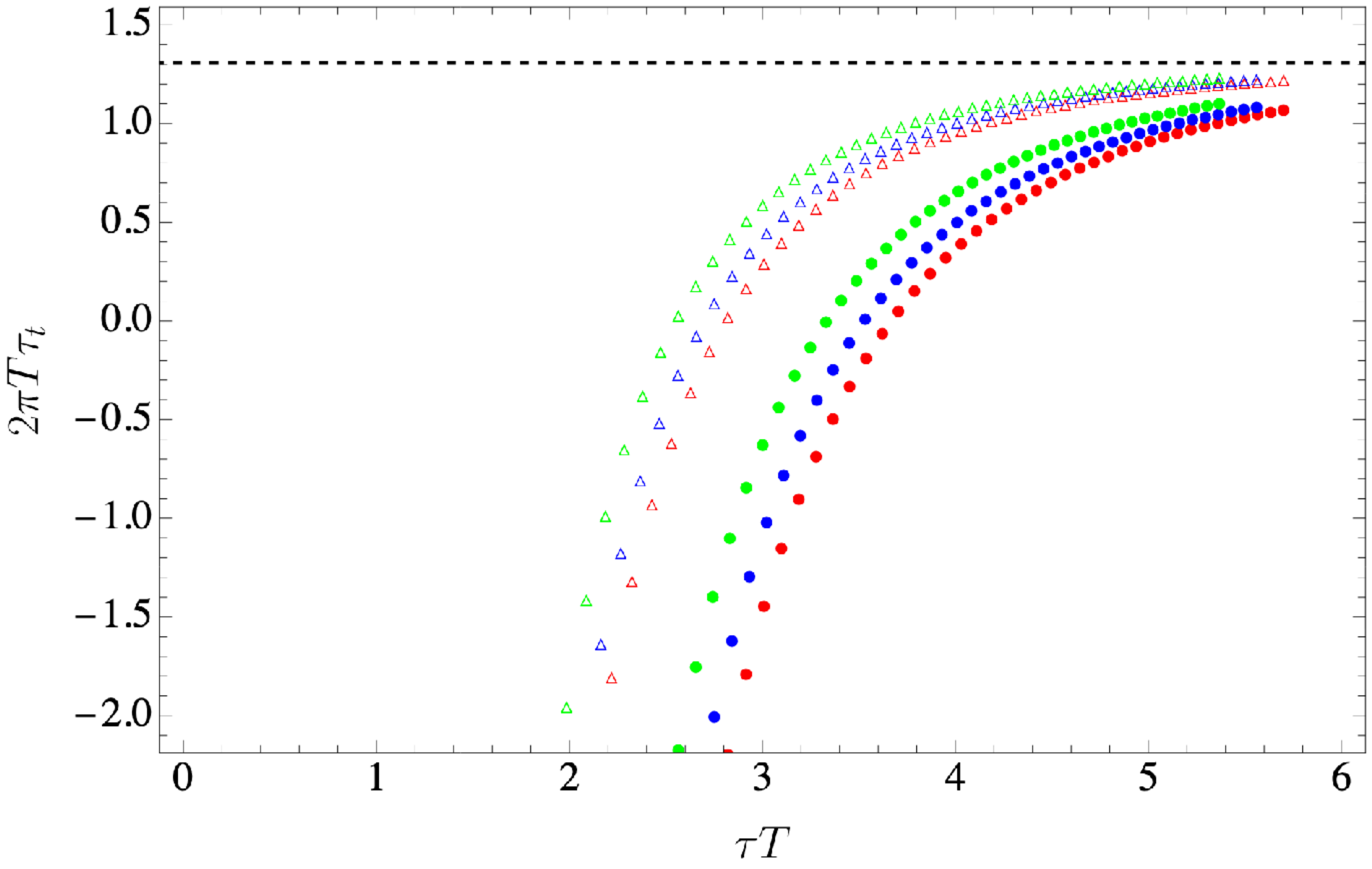
[Cartwright,Ilyas,Kaminski,Knipfer,Zhang; in progress]

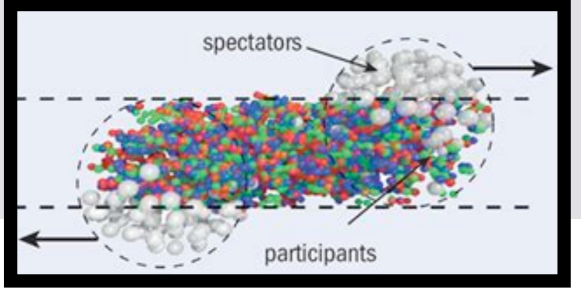




Relaxation Time

[Cartwright,Ilyas,Kaminski,Knipfer,Zhang; in progress]





Methods: Perturbative Definition

[Cartwright,Ilyas,Kaminski,Knipfer,Zhang; in progress]

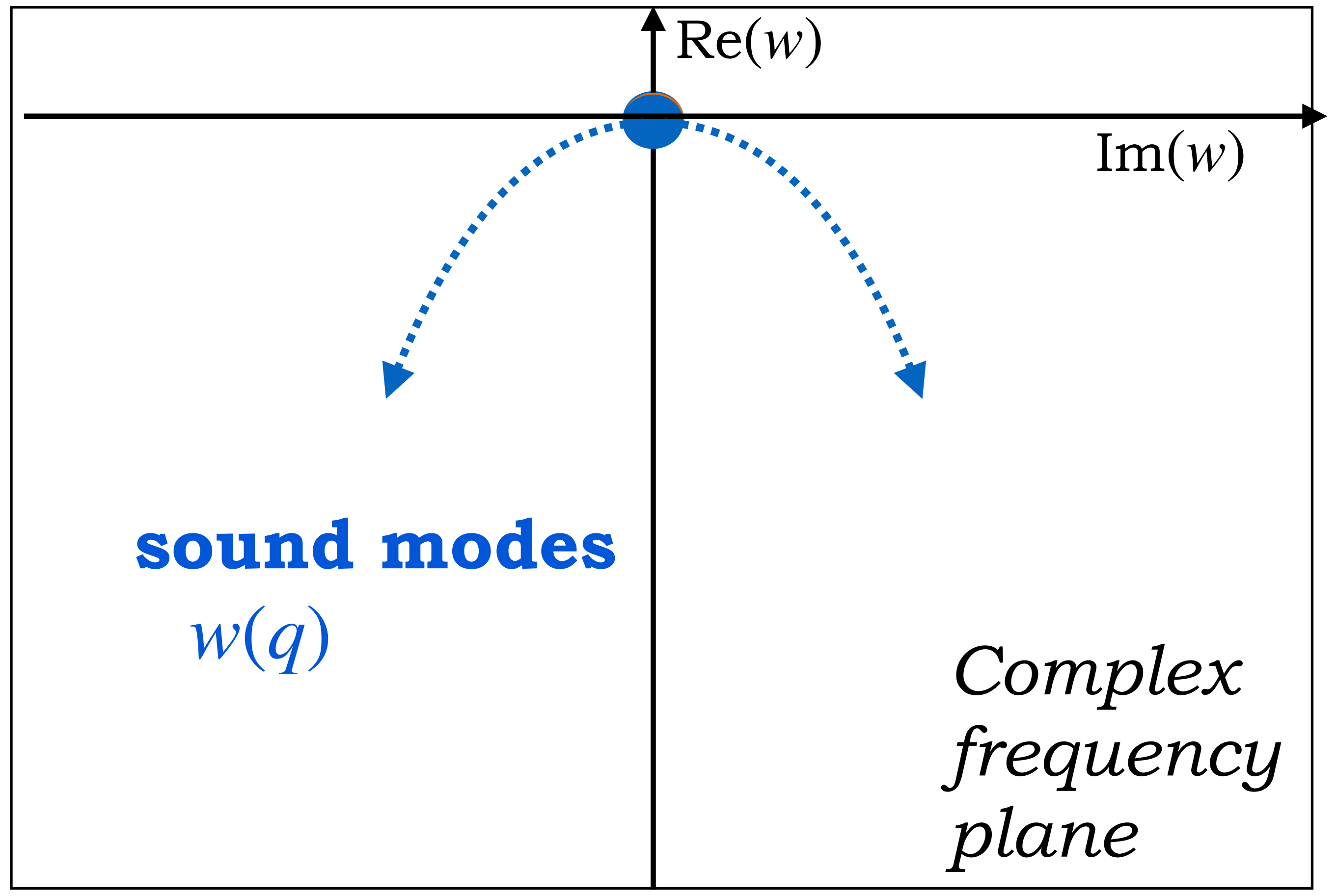
Dispersion relation in *sound sector*

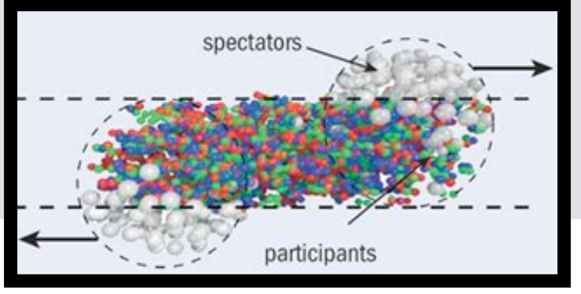
$$w(q) = \pm C_s q - i\frac{\Gamma_s}{2} q^2 \pm \frac{\Gamma_s}{2C_s}\left(C_s^2 \tau_\pi - \frac{\Gamma_s}{4}\right) q^3 + O(q^4)$$

speed of sound (C_s) — **sound attenuation** (Γ_s) — **relaxation time** (τ_π)

… extracted from *quasinormal modes* in *spin-0 sector* of metric perturbations

1. **background metric** dual to Bjorken-expanding plasma $g_{\mu\nu}^{(\text{Bjorken})}(\tau)$
2. add **perturbation** $g_{\mu\nu}^{(\text{Bjorken})}(\tau) + h_{\mu\nu}^{(\text{sound})}$
3. **quasi-static:** on fixed time slice τ : Fourier-transform $h_{\mu\nu}^{(\text{sound})}$
4. calculate quasinormal mode **frequency *w*** at **momentum *q***





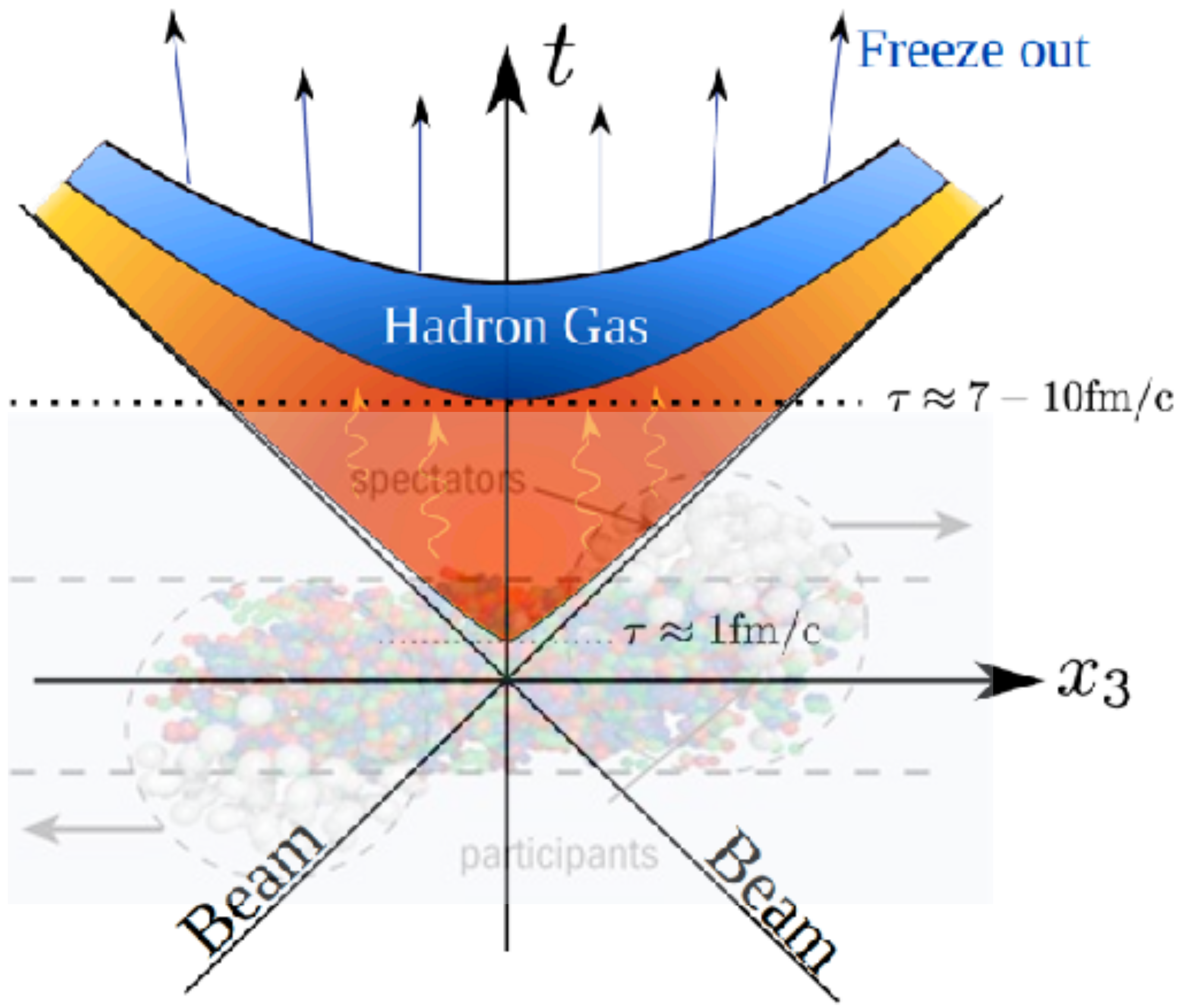
Bjorken-Expanding Plasma

[Cartwright,Kaminski,Knipfer; PRD (2023)]

- **far away from equilibrium thermodynamic quantities are not well-defined**
- **plasma is approximately boost invariant along the beam-line**
- **initially large anisotropy between that direction and the transverse plane**

proper time $\tau = \sqrt{t^2 - x_3^2}$

rapidity $\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$



Gravity dual: Einstein Gravity, anisotropic metric ***AdS radial coordinate*** $r = 1/z$

$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + e^{B(v,r)}S(v,r)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S(v,r)^2 e^{-2B(v,r)}\mathrm{d}\xi^2$$

boundary at $r = \infty$ ***has Milne metric:*** $\lim_{r\to\infty} \frac{1}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$

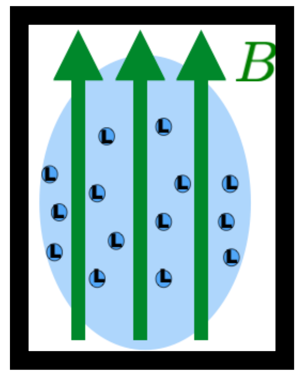
➡late times: system still expanding but approximately isotropic

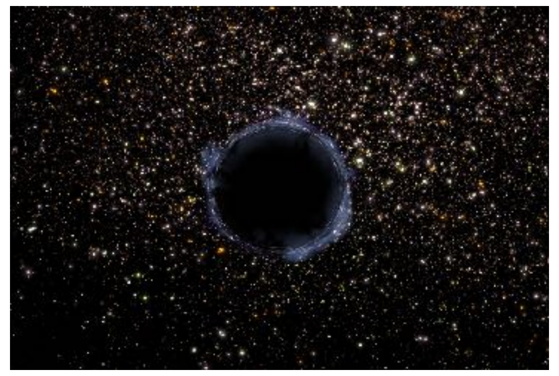
➡early times: far from equilibrium

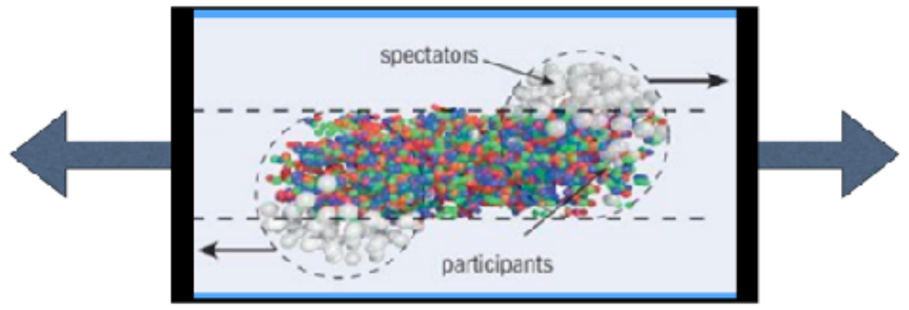
Discussion

Summary

- considered three examples of **anisotropic systems**
  - ➡ external magnetic field
  - ➡ rotation
  - ➡ Bjorken expansion
- **novel transport coefficients, changed Kubo formulae**
- **drastic differences, e.g. specific shear viscosity** drops to zero (below $1/(4\pi)$)
- anisotropic hydrodynamics **needed**

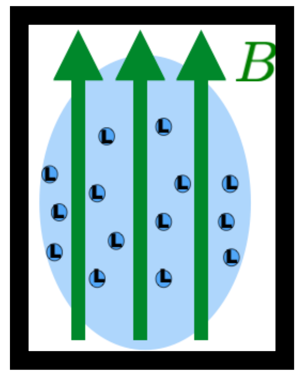


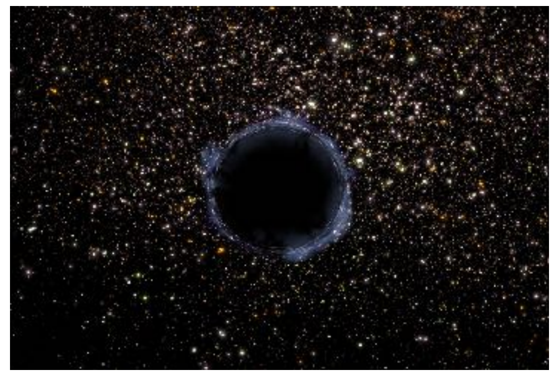




Discussion

Summary





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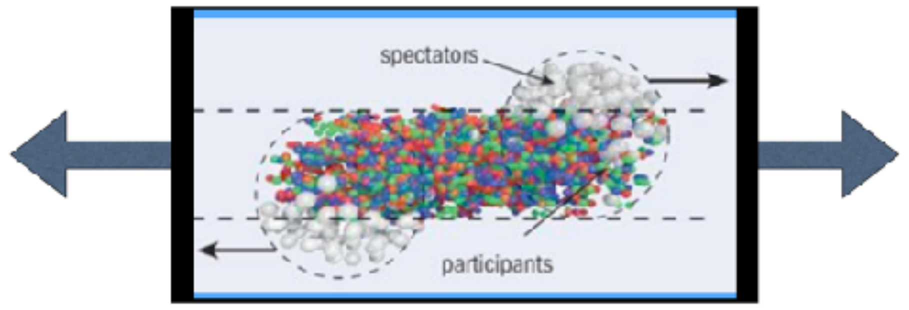
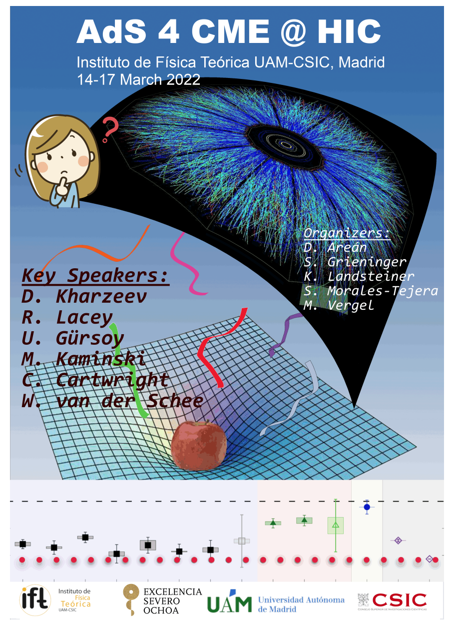


Outlook

[Adhikari et al.; to appear in PPNP (2025)] *[Ghosh, Shovkovy, Eur.Phys.J.C. (2024)]*

- calculate novel transport coefficients **on the lattice** and **perturbative QCD**
- effect of anisotropies on **(elliptic) flow** v_n? *[Bernhard et al., Nature Physics (2019)]*
- construct **holographic heavy ion collisions to model QGP** (dynamical magnetic field and dynamically created axial imbalance)
- use holographic collisions to **test formulations of hydrodynamics**



[AdS4CME Collaboration]

Thanks to my collaborators (since 2012) and Thank You!

University of Washington, Seattle, USA
Prof. Dr. Laurence Yaffe

University of Victoria, Canada
Prof. Dr. Pavel Kovt
Prof. Dr. Adam Ritz
Prof. Dr. Kristan Jensen

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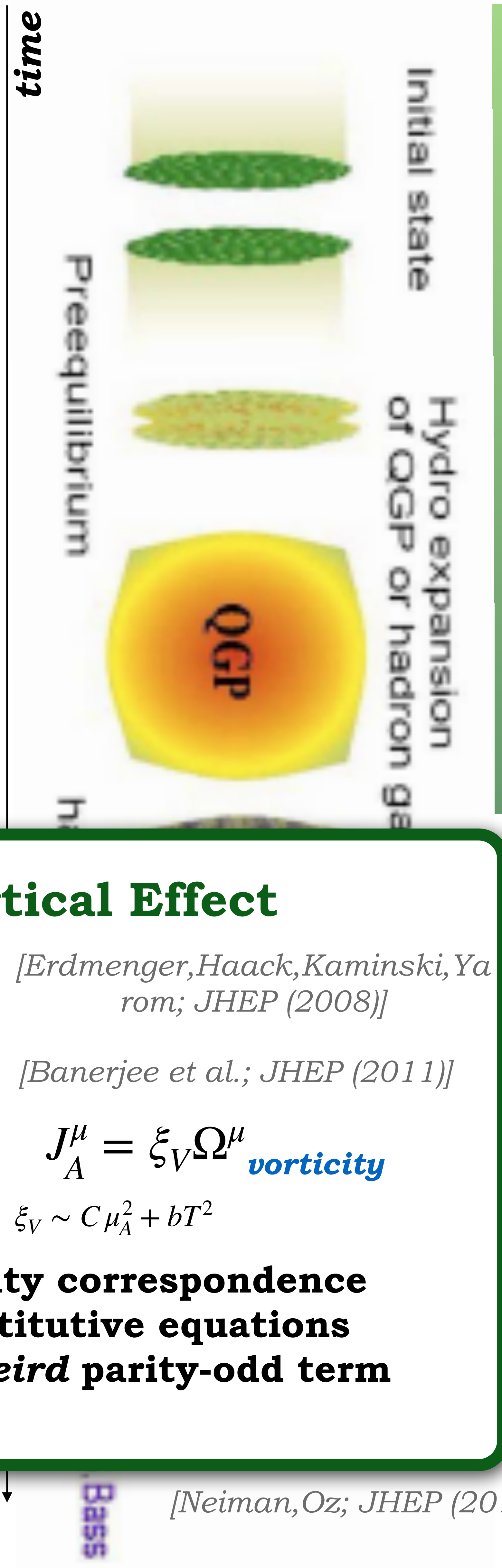
Utrecht University, Netherlands
Prof. Dr. Gerard 't Hooft (Nobel Laureate)

APPENDIX

A winning team: hydrodynamics and holography in parallel

More balanced review in my Section 5.2 on Hydrodynamics in White Paper [Sorensen et al.; Prog.Part.Nucl.Phys. (2024)]



HYDRODYNAMICS & THERMODYNAMICS

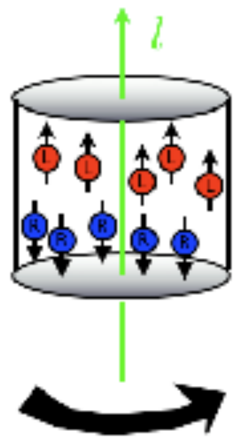
Chiral Magnetic Effect (CME) from chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J^\mu_A = \xi_B B$$

hydro and holo in parallel

Chiral Vortical Effect



[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

$$J^\mu_A = \xi_V \Omega^\mu$$ ***vorticity***

$$\xi_V \sim C\mu_A^2 + bT^2$$

- **fluid/gravity correspondence**
- **gives constitutive equations**
- **contain *weird* parity-odd term**

[Neiman,Oz; JHEP (2010)]

HOLOGRAPHY

CME far from equilibrium, strong *B*

- **non-expanding plasma**
  [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]
- **expanding plasma**
  [Cartwright,Kaminski,Schenke; PRC (2022)]

Frequency dependence of CME

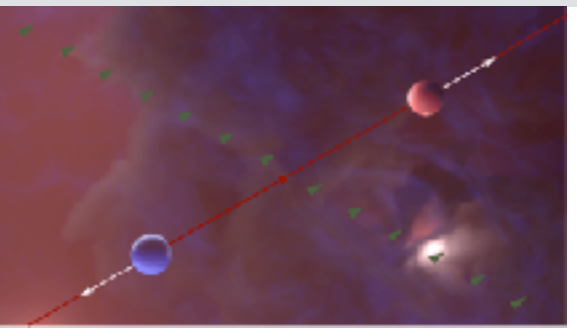
[Amado,Landsteiner,Pena_Benitez; JHEP (2011)]
[Li,Yee; PRD (2018)]
[Koirala; PhD thesis (2020)]

CME near equilibrium (+hydro)

- **weak magnetic field *B***
  [Son,Surowka; PRL (2009)]
  [Kharzeev,Yee; PRD (2011)]
  [Ammon, Kaminski et al.; JHEP (2017)]
- **strong *B*** *[Ammon,Leiber,Macedo; JHEP (2016)]*
  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

APPENDIX - CME far from equilibrium - case I

[Cartwright,Kaminski,Schenke; PRC (2022)]



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

Initial state:
constant B,
pressure anisotropy

time-dependent μ_5,
plasma expanding along beam line

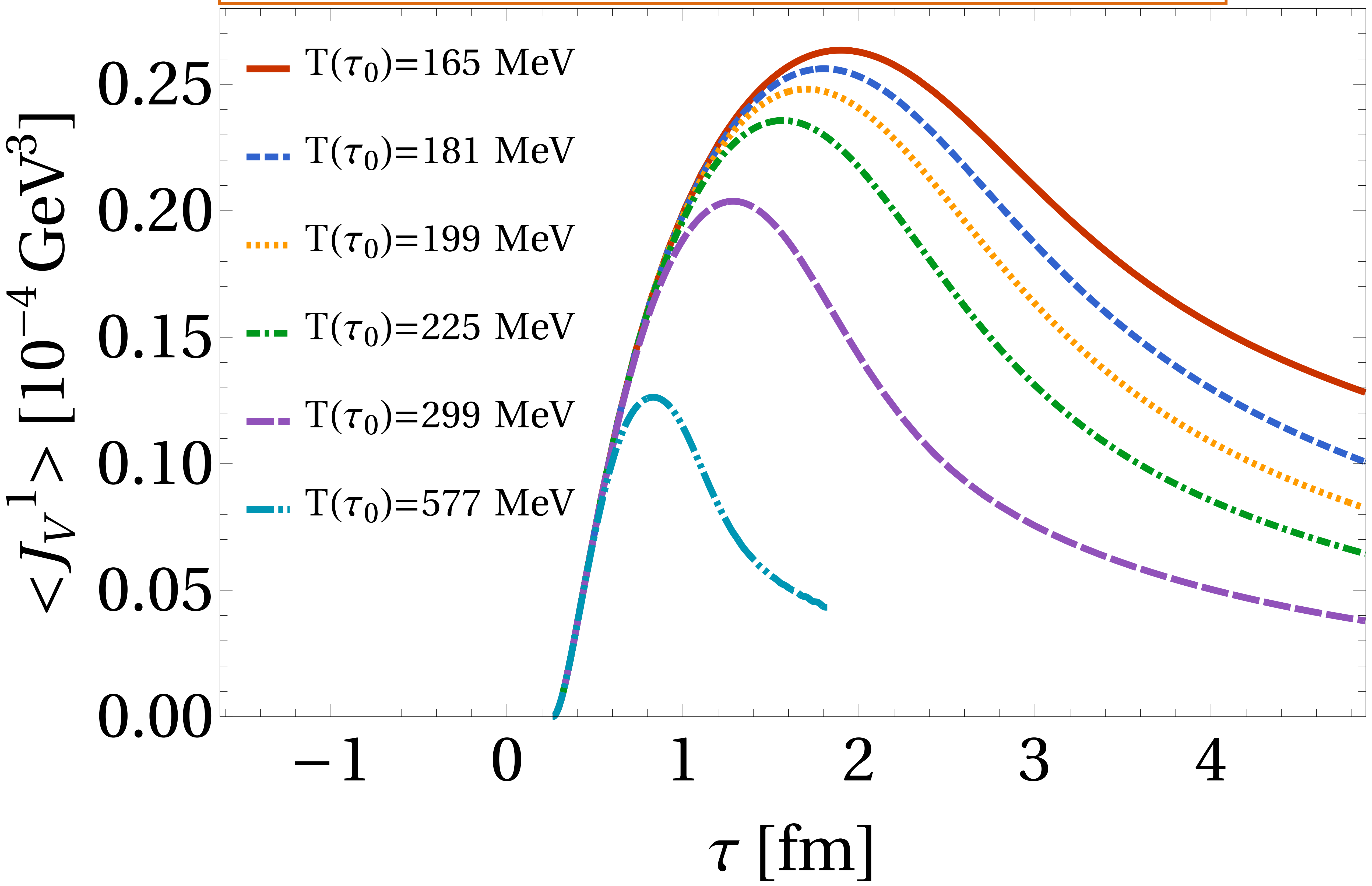
Matching to QCD:
SUSY value for α
L=1fm fixes κ

Near-equilibrium CME

$$J_V^\mu = \xi_\chi B \qquad \xi_\chi = C\,\mu_A$$

[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]

Fixed initial $eB \approx m_\pi^2$, $n_A = 0.00032\,\mathrm{GeV}^3$



➡CME-current more likely to be seen at lower energies!

agrees with non-expanding holographic model:
[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

APPENDIX - Far from equilibrium shear: Results

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

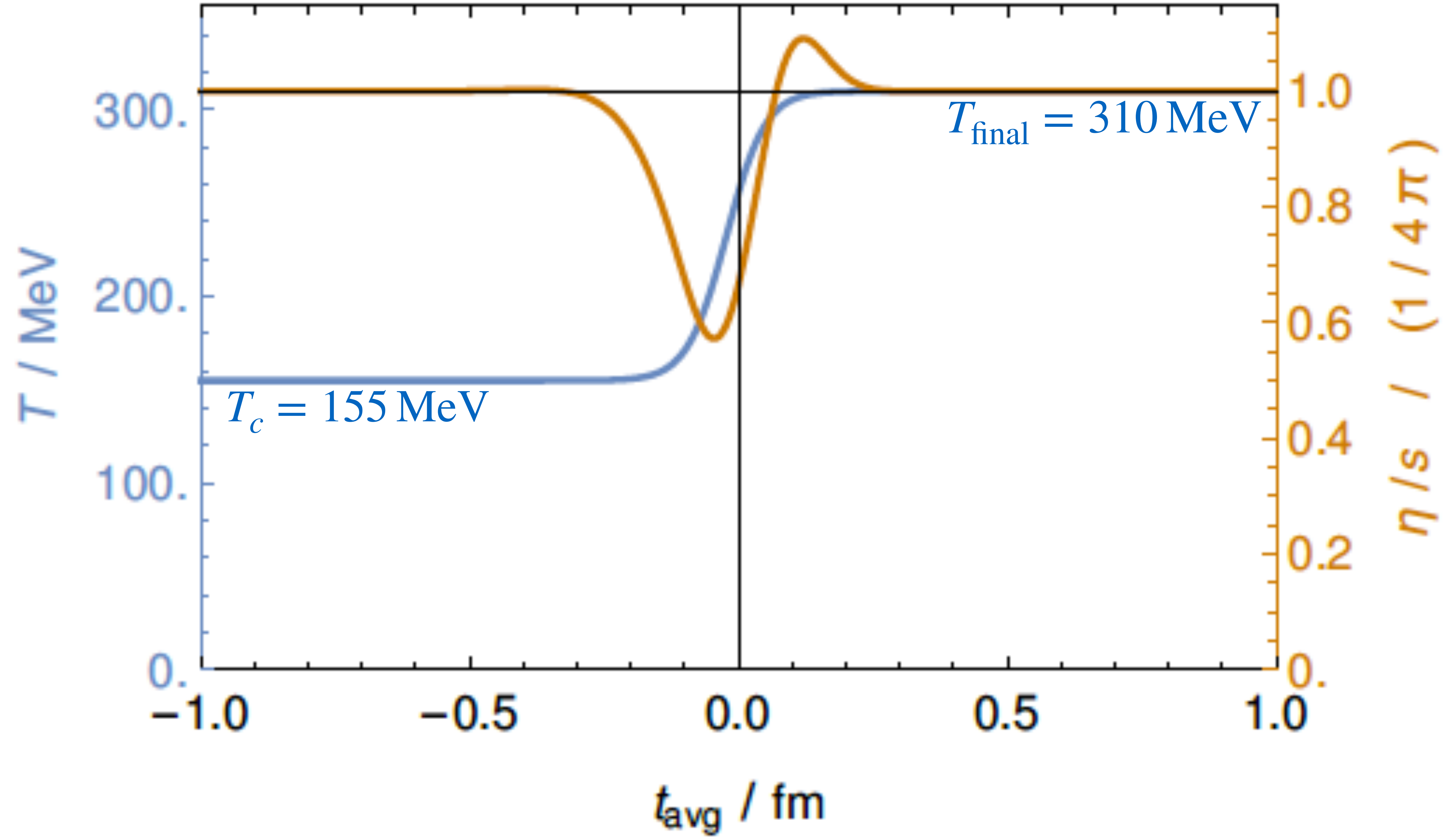
RHIC parameters: $\sqrt{s_{NN}} = 200\,\text{GeV}$ $\Delta t = 0.3\text{ fm}$

Temperature

$$T = T_{\text{Hawking}}$$

Entropy density from generating functional

$$s \sim \frac{\partial S^{\text{on-shell}}}{\partial T}$$



KSS equilibrium result

[Kovtun,Son, Starinets; PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

No universal bound

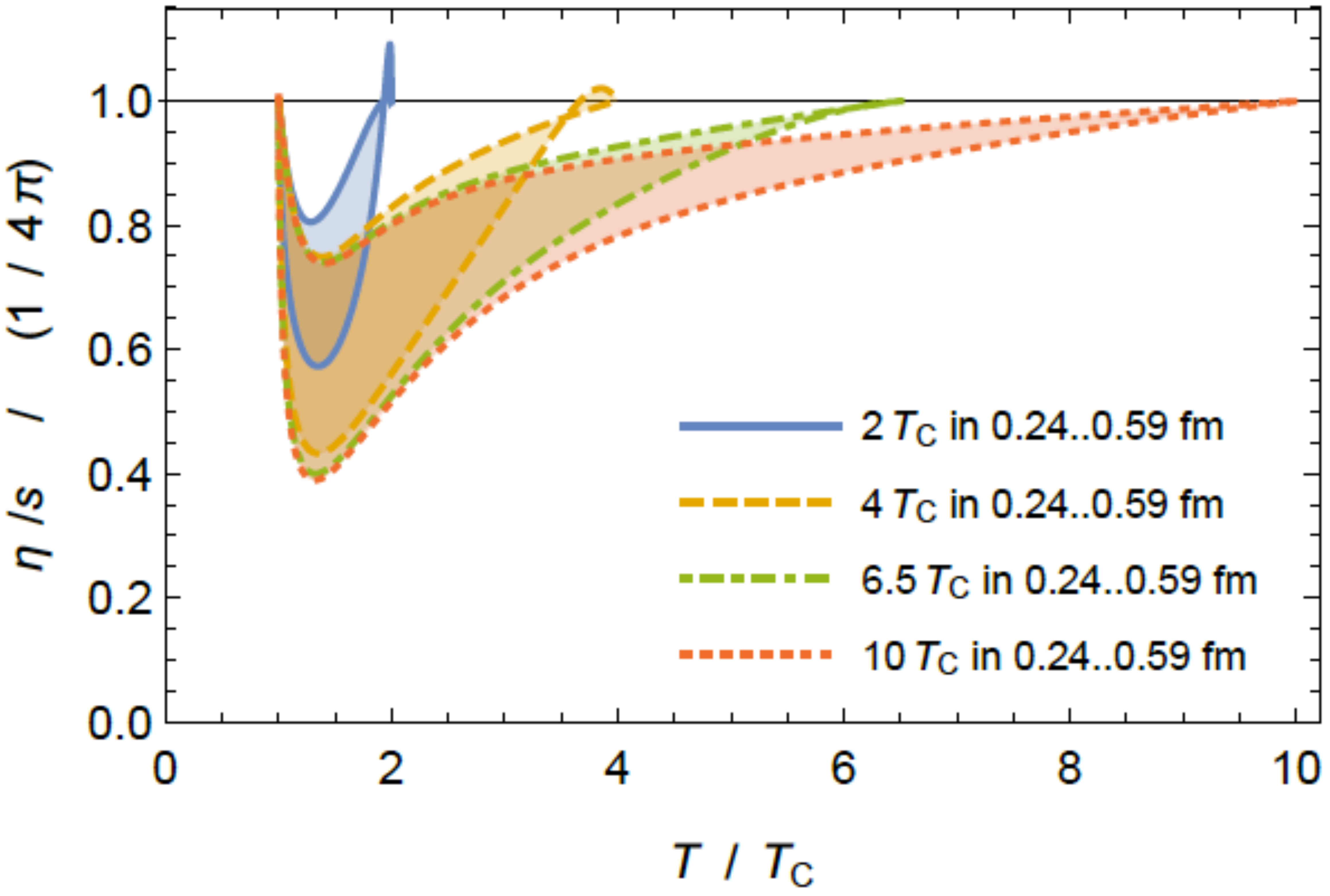
[Buchel, Myers, Sindha; JHEP (2008)]

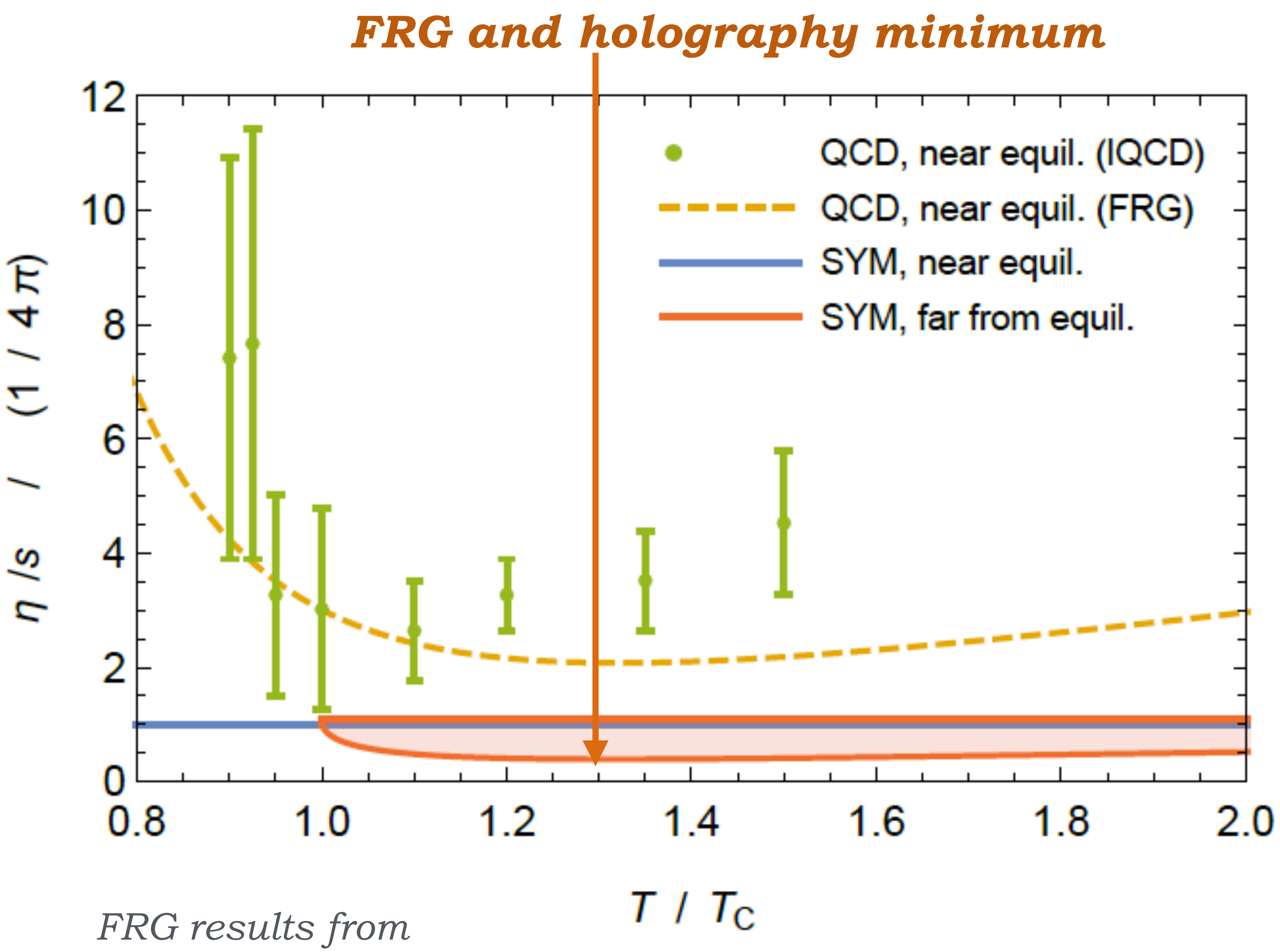
➡ Shear transport ratio first drops below 60%, then rises above 110% of KSS value $1/(4\pi)$

$\frac{\eta}{s}$

APPENDIX - Far from equilibrium shear: Results

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]





FRG results from
[Christiansen, Haas, Pawlowski, Strodthoff; PRL (2015)]
Lattice QCD data from
[Astrakhantsev, Braguta, Kotov; JHEP (2017)]

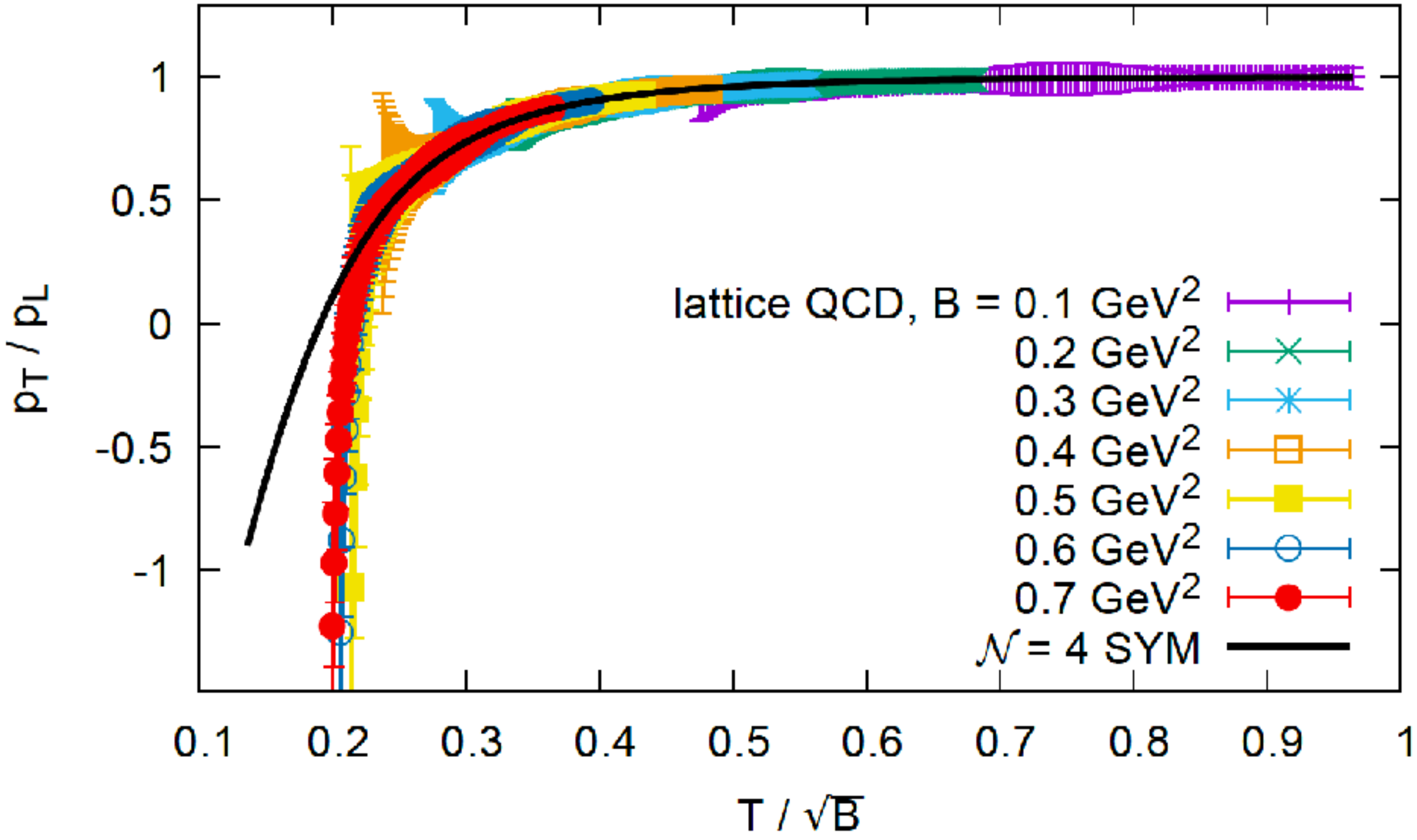
➡ stark contrast: near equilibrium lattice QCD / FRG suggest $\eta/s > 1/(4\pi)$

whereas far from equilibrium Super-Yang-Mills (SYM) plasma suggests $\eta/s < 1/(4\pi)$

➡ currently underestimating flow generated at early times *[Bernhard, Moreland, Bass, Nature (2019)]*

APPENDIX: Same magneto response in LQCD and N=4 SYM with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$

longitudinal pressure: $p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

APPENDIX: Strong B thermodynamics

$$\boxed{B \sim \mathcal{O}(1)}$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly : $\langle T^{\alpha\beta}\rangle = \epsilon\, u^\alpha u^\beta + p\Delta^{\alpha\beta} + \tau^{\alpha\beta}$

Energy momentum tensor:

equilibrium heat current

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^\mu\rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ new contributions to thermodynamic equilibrium observables

previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

APPENDIX: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $\omega = \pm \frac{B_0 n_0}{w_0} - \frac{i B_0^2}{w_0}(\sigma_\perp \pm i\tilde{\sigma}) - i D_c k^2$

parity-odd

weak B: $\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$

Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $\omega = \pm k v_s - i \frac{\Gamma_{s,\parallel}}{2} k^2 ,$

$\omega = -i D_\parallel k^2 ,$

Anisotropic transport coefficients

$$D_\parallel = \frac{\sigma_\parallel w_0^2}{n_0^2 \chi_{11} + w_0^2 \chi_{33} - 2 n_0 w_0 \chi_{13}}$$

parity-odd

weak B: $\omega_0 = v_0 k - i D_0 k^2 + \mathcal{O}(\partial^3)$

$\omega_+ = v_+ k - i \Gamma_+ k^2 + \mathcal{O}(\partial^3)$

$\omega_- = v_- k - i \Gamma_- k^2 + \mathcal{O}(\partial^3)$

Agreement in form

$$D_0 = \frac{w_0^2 \sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2 B T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3 C \mathfrak{s}_0^2\right)$$

$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

APPENDIX: weak B hydrodynamics comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients
— take B=0 values of this model instead

except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]

weak B hydro prediction:

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

APPENDIX: Dispersion relations: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions : $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle,\ \langle T^{\mu\nu}\, J^{\alpha}\rangle,\ \langle J^{\mu}\, T^{\alpha\beta}\rangle,\ \langle J^{\mu}\, J^{\alpha}\rangle$

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0 / n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$
$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**

[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$
$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

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$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$
former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial \epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6 w_0} + c_s^2 \frac{w_0\,\sigma}{2 n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3 C T_0 \mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1 - c_s^2}{w_0}\,\xi_V^{(0)}\,, \qquad v_0 = \frac{2 B\, T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

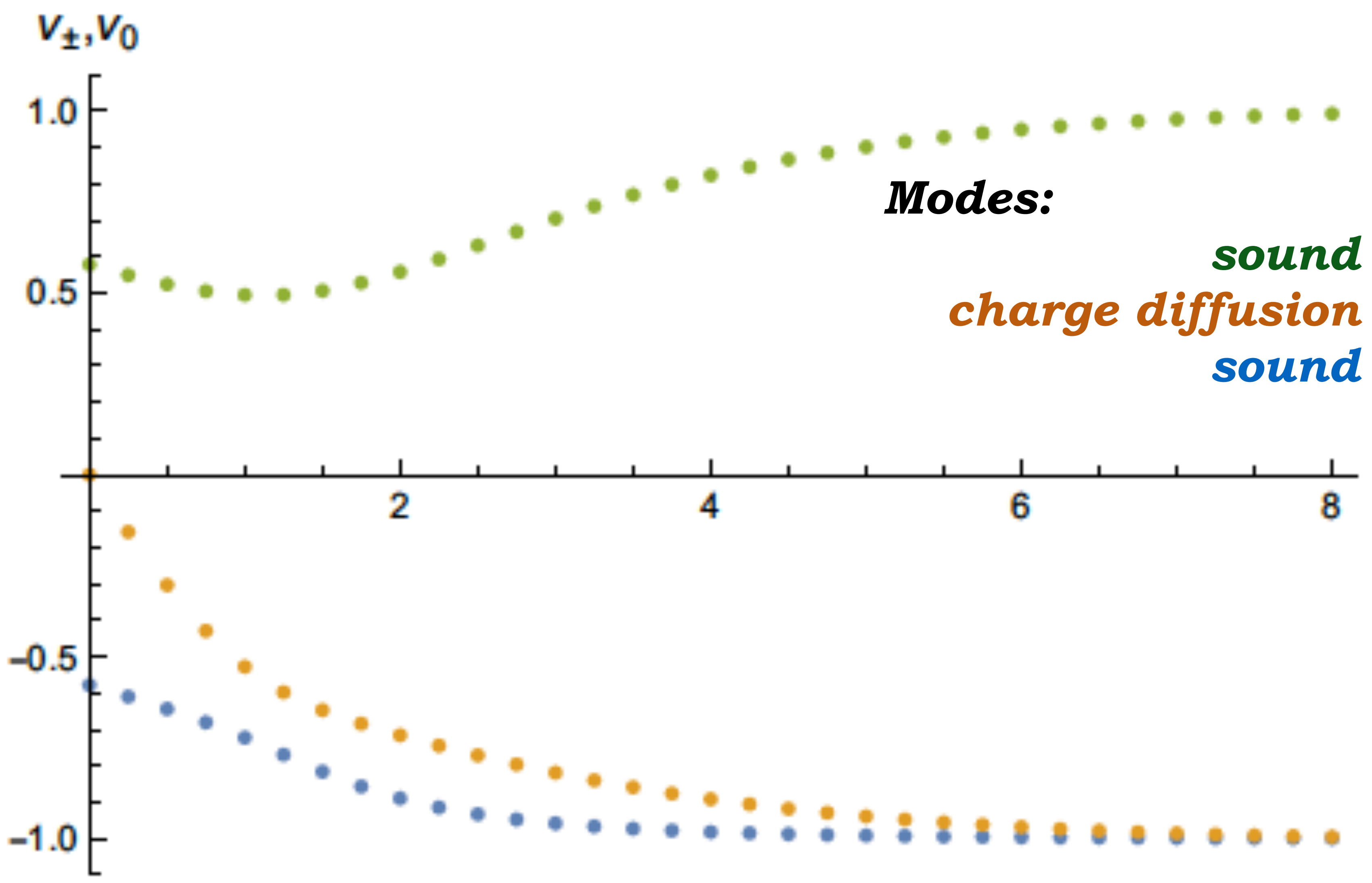
APPENDIX: Holographic result: hydrodynamic poles

[Ammon, Kaminski et al.; JHEP (2017)]

Fluctuations around charged magnetic black branes (QNMs)

- Weak *B*: **holographic results are in "agreement" with hydrodynamics.**
- Strong *B*: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate:**

(i) at the speed of light **and (ii) without attenuation**



RECALL: weak B hydrodynamic poles

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$ *former sound modes*

confirming conjectures and results in probe brane approach [Kharzeev, Yee; PRD (2011)]

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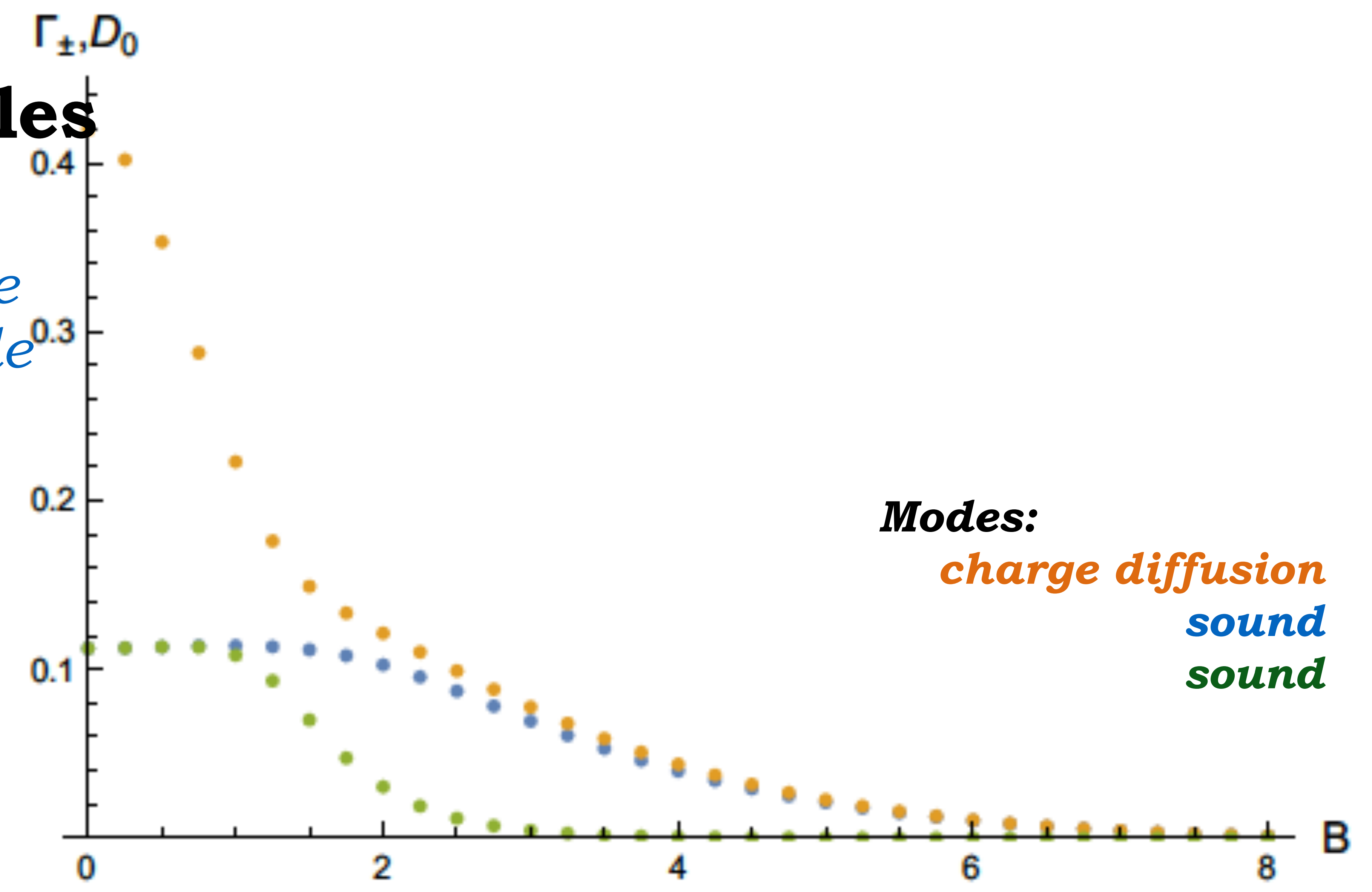

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APPENDIX: More thermodynamic transport coefficients

Magneto-thermal susceptibility M_1 **:**

$$\mathcal{E}_{\text{eq}} \sim M_1 B^\mu \partial_\mu \left(\frac{B^2}{T^4} \right)$$

Magneto-acceleration susceptibility M_3 **:**

$$\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} \sim M_{3,B^2}\, B\cdot a$$

Magneto-electric susceptibility M_4 **:**

$$\mathcal{E}_{\text{eq}} \sim M_{4,T}\, B\cdot E\,, \qquad \mathcal{P}_{\text{eq}} \sim M_{4,B^2}\, B\cdot E$$

Magneto-vortical susceptibility M_5 **:**

$$\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} \sim M_5 B\cdot\Omega$$