FIRST ORDER TRANSPORT COEFFICIENTS OF SPIN POLARIZATION

[based on MB, 2502.15520]

Matteo Buzzegoli

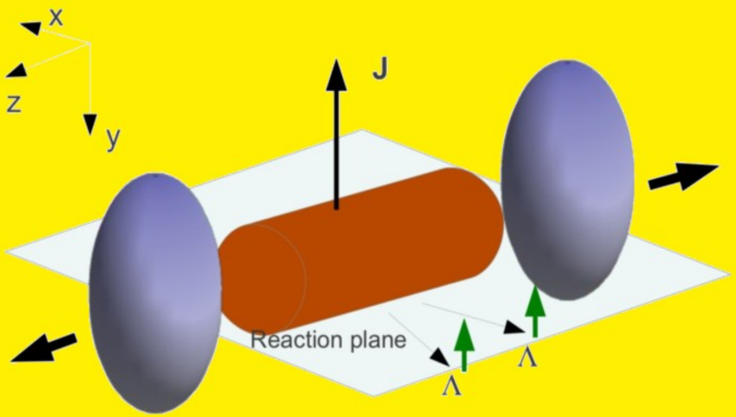
West University of Timişoara

Foundations and Applications of Relativistic Hydrodynamics Workshop
GGI

09/05/25

Peripheral collisions: large angular momentum

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t. reaction plane

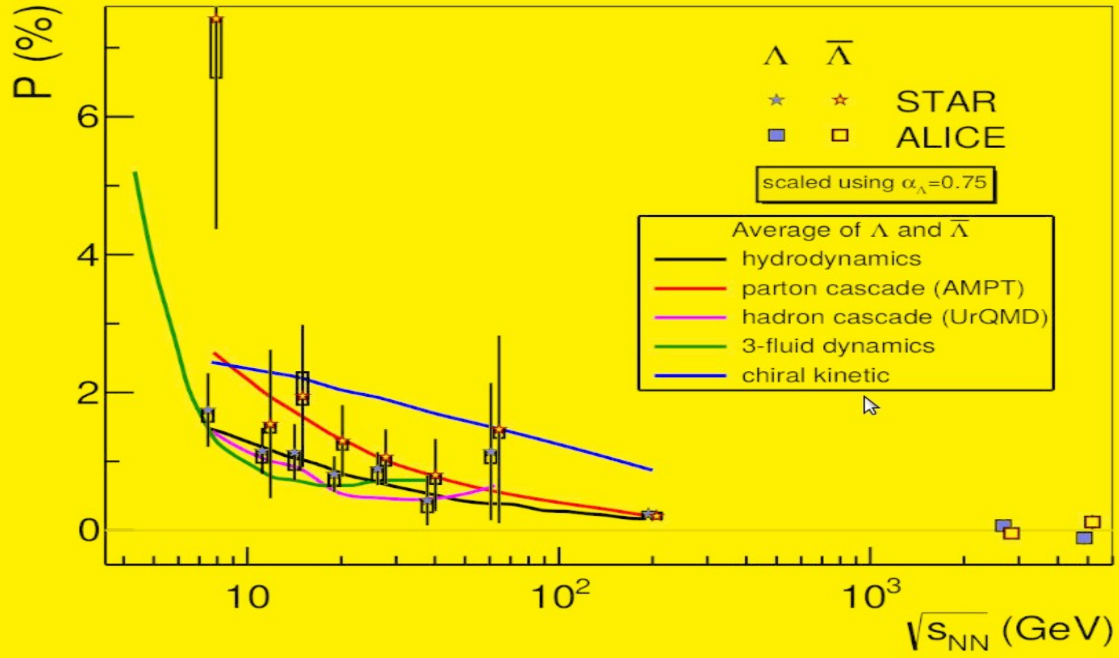


- Polarization estimated at quark level by spin-orbit coupling
  Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301
- By local thermodynamic equilibrium of the spin degrees of freedom
  F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452;
  F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin $\propto$(thermal) vorticity

Agreement between hydrodynamic **predictions** and the data

Global Spin Polarization



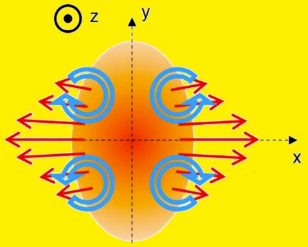
F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)

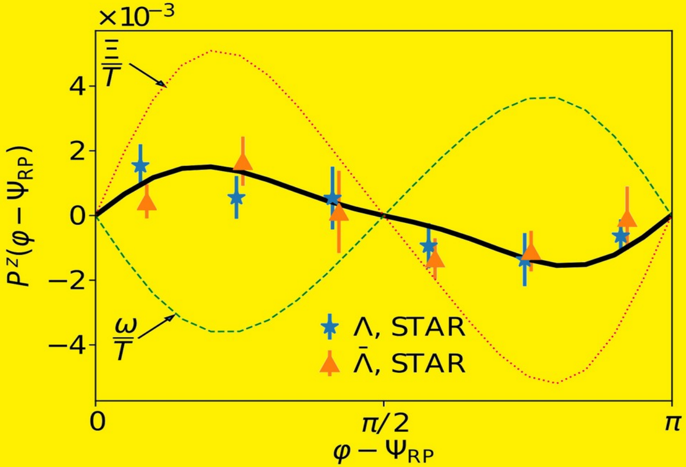
$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\partial_\rho\beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$n_F = \left(\mathrm{e}^{\beta\cdot p-\zeta}+1\right)^{-1}$$

Different models of the collision, same formula for polarization

Local spin polarization





F. Becattini, MB, G. Inghirami, I. Karpenko, and A. Palermo, PRL 127, 272302 (2021)

B. Fu, S. Y. F. Liu, L. Pang, H. Song and Y. Yin, PRL 127 (2021)

- "Local": Momentum dependent polarization (along beam direction)
- Explained by incorporating shear effects

However, the picture of equilibrated spins might not be complete

J.I. Kapusta, E. Rrapaj and S. Rudaz, PRCC 101 (2020)
A. Ayala, D. De La Cruz, S. Hernández-Ortíz, L.A. Hernández and J. Salinas, PLB, 801 (2020)
M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov and H.-U. Yee, JHEP 08 (2022) 263
D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)

→ Develop Spin hydrodynamic and include a Spin potential

See also Sushant Singh Talk 30/04

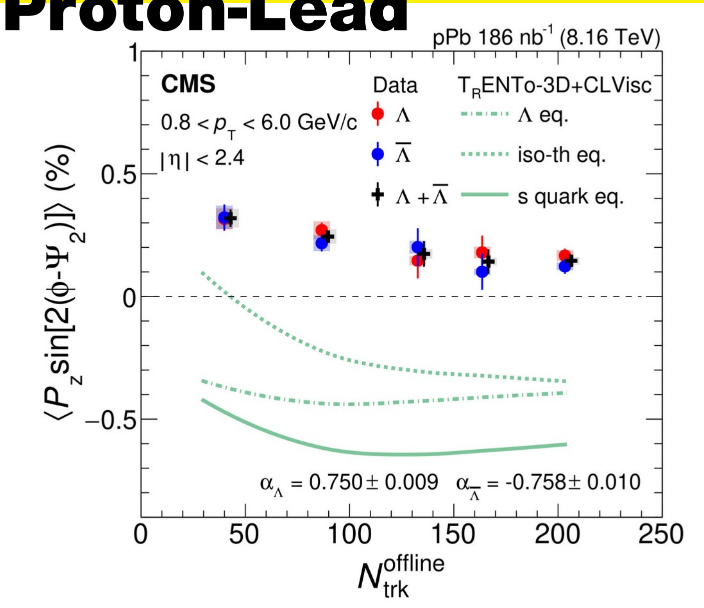
Dissipative contributions

This talk goal: extend the spin polarization formula to dissipative effects

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\partial_\rho \beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F} + \text{Local (out-of-equilibrium) effects}$$ **+ DISSIPATIVE EFFECTS**

Where dissipative effects might be relevant?

Proton-Lead

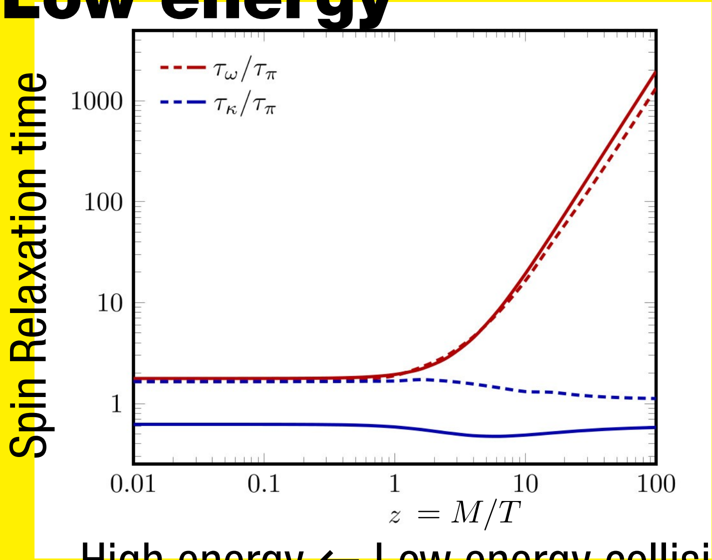


Data: CMS, 2502.07898

Prediction: Yi, Wu, Zhu, Pu, Qin, PRC (2025)

(No spin hydro and no dissipative contributions)

Low energy



High energy ← Low energy collision

D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)

Earlier works on dissipative effects

- Quantum kinetic theory for massless field

S. Shi, C. Gale, S. Jeon, Phys. Rev. C 103 (2021) 4, 044906

$$S^\mu(p) = \frac{1}{2m_H}\Big\{\Big[\int_\Sigma f_{V,0}\Big] + \int_\Sigma f_{V,0}(1-f_{V,0})(\lambda_\nu \nu^\alpha p_\alpha + \lambda_\pi \pi^{\alpha\beta} p_\alpha p_\beta)\Big\}^{-1}$$
$$\times \Big\{\Big[-\frac{\hbar}{4}\epsilon^{\mu\nu\rho\sigma}\int_\Sigma p_\nu \varpi_{\rho\sigma} f_{V,0}(1-f_{V,0})\Big] + \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\frac{\mu_A}{T}$$
$$+ \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\Big(\frac{\lambda_\nu}{2}\nu_A^\alpha p_\alpha + \frac{\lambda_\nu^+ - \lambda_\nu^-}{2}\nu^\alpha p_\alpha + \frac{\lambda_\pi^+ - \lambda_\pi^-}{2}\pi^{\alpha\beta}p_\alpha p_\beta\Big)\Big\} + \mathcal{O}(\hbar^2)\,,$$

- Quantum kinetic theory with non-local collisions

N. Weickgenannt, D. Wagner, E. Speranza and D. H. Rischke, PRD 106 (2022)

$$S^\mu_{\rm GLW}(p) = \int d\Sigma\cdot p\,\frac{f_{0p}}{2\mathcal{N}}\Big\{-\frac{1}{2m}\tilde{\mathfrak{S}}^{\mu\nu}p_\nu - \chi_{\mathfrak{q}}\mathfrak{d}\eta^\mu_\nu\beta_0\sigma_\rho{}^{\langle\alpha}\epsilon^{\beta\rangle\nu\sigma\rho}u_\sigma p_{\langle\alpha}p_{\beta\rangle} + \mathfrak{e}\chi_{\mathfrak{p}}\left(\eta^\mu_\nu - \frac{u^\mu p_{\langle\nu\rangle}}{E_p}\right)\left(\tilde{\mathfrak{S}}^{\nu\rho} - \tilde{\varpi}^{\nu\rho}\right)u_\rho\Big\}$$

$$\mathcal{N} = \int d\Sigma\cdot p\,dS(p)f(x,p,\mathfrak{s}), \quad f_{0p} = (2\pi\hbar)^{-3}e^{-\beta_0 E_p+\zeta_0}, \quad A^{\langle\mu_1\cdots\mu_\ell\rangle} = \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell}A^{\nu_1\cdots\nu_\ell}$$

Is this dissipative?
Caveat: Different terminology

Earlier works on dissipative effects

- Quantum kinetic theory with non-local collisions

D. Wagner, PRD 111 (2025)

$$S^\mu(p) = S^\mu_\omega(p) + S^\mu_\kappa(p) + S^\mu_{\mathfrak{t}}(p)$$

$$S^\mu_\omega(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{u^\mu(\omega_0 \cdot p) - \omega_0^\mu (p \cdot u)}{2m_\Lambda} f_0(1 - f_0)$$

$$N(p) = 2 \int \mathrm{d}\Sigma \cdot p f_0$$

$$S^\mu_\kappa(p) = -\frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p_\sigma}{2m_\Lambda} \kappa_{0,\rho}\, f_0(1 - f_0)$$

$$S^\mu_{\mathfrak{t}}(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p^\lambda p_\sigma}{3T^2(\epsilon + P)} \mathfrak{t}_{\rho\lambda}\, f_0(1 - f_0)$$

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\frac{\omega_{\mathrm{K}}^\mu}{T} + \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\ell_{\omega\kappa} \nabla_\alpha \kappa_{0,\beta} - \tau_\omega \dot{u}_\alpha \kappa_{0,\beta}\right) + \delta_{\omega\omega} \omega_0^\mu \theta + \lambda_{\omega\omega} \sigma^{\mu\nu} \omega_{0,\nu} + \lambda_{\omega\mathfrak{t}} \mathfrak{t}^\mu{}_\nu \omega_{\mathrm{K}}^\nu \,,$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = -\frac{\dot{u}^\mu}{T} + \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\frac{\tau_\kappa}{2} \nabla_\alpha \omega_{0,\beta} + \tau_\kappa \dot{u}_\alpha \omega_{0,\beta}\right) + \delta_{\kappa\kappa} \kappa_0^\mu \theta + \left(\lambda_{\kappa\kappa} \sigma^{\mu\nu} + \frac{\tau_\kappa}{2} \omega_{\mathrm{K}}^{\mu\nu}\right) \kappa_{0,\nu}$$
$$+ \tau_{\kappa\mathfrak{t}} \mathfrak{t}^{\mu\nu} \dot{u}_\nu + \ell_{\kappa\mathfrak{t}} \Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda} \,,$$

$$\tau_{\mathfrak{t}} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \frac{\mathfrak{d}}{T} \sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}^{\mu\nu} \theta + \lambda_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}_\lambda{}^{\langle\mu} \sigma^{\nu\rangle\lambda} + \frac{5}{3} \tau_{\mathfrak{t}} \mathfrak{t}_\lambda{}^{\langle\mu} \omega_{\mathrm{K}}^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa} \nabla^{\langle\mu} \kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega} \omega_{\mathrm{K}}^{\langle\mu} \omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega} \sigma_\lambda{}^{\langle\mu} \epsilon^{\nu\rangle\lambda\alpha\beta} u_\alpha \omega_{0,\beta}$$

Polarization from Wigner function

F. Becattini, MB, T. Niida, S. Pu, A. H. Tang and Q. Wang, Int. J. Mod. Phys. E 33 (2024) no.06, 2430006

The covariant Wigner function:

$$W(x,k)_{AB} = \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-\mathrm{i}k\cdot y}\langle :\bar{\Psi}_B(x+y/2)\Psi_A(x-y/2):\rangle$$

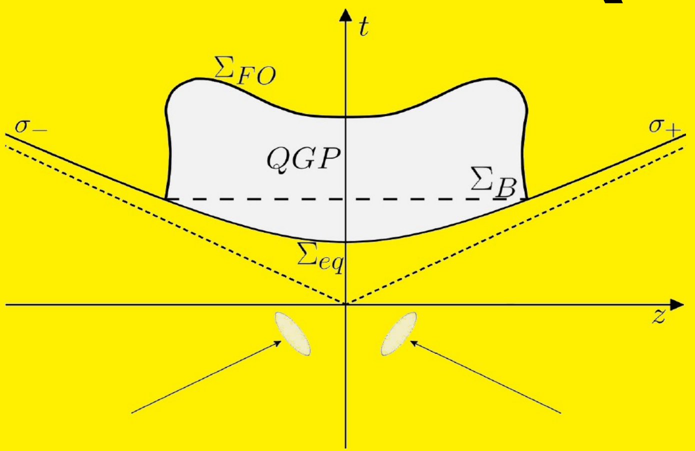
where:

$$\langle \widehat{X}\rangle = \mathrm{tr}\left(\widehat{\rho}\,\widehat{X}\right)$$

It allows to calculate the mean spin vector:

$$S^\mu(k) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot k\ \mathrm{tr}_4\left(\gamma^\mu\gamma^5 W_+(x,k)\right)}{\int \mathrm{d}\Sigma\cdot k\ \mathrm{tr}_4 W_+(x,k)} = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot k\ \mathcal{A}^\mu_+(x,k)}{\int \mathrm{d}\Sigma\cdot k\ \mathcal{F}_+(x,k)}$$

Non equilibrium statistical operator (Zubarev theory)



D.N. Zubarev, et al, Theor. Math. Phys. 1979, 40, 821
C.G. van-Weert, Ann. Phys. 1982, 140, 133
T. Hayata, Y. Hidaka, T. Noumi and M. Hongo, PRD 92 (2015)
F. Becattini, MB, E. Grossi, Particles 2 (2019) 2, 197-207;
MB, Lect. Notes Phys. 987 (2021) 53-93.

$$\beta^\nu = \frac{u^\nu}{T}$$

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{eq}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)\right]$$

With the Gauss's theorem:

$$\widehat{\rho} = \frac{1}{Z}\exp\Bigg[\underbrace{-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)}_{\text{Local thermal equilibrium}} + \underbrace{\int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)}_{\text{Dissipative}}\Bigg]$$

Local equilibrium: non-dissipative contribution

D.N. Zubarev, A.V. Prozorkevich, S.A. Smolyanskii, Theor. Math. Phys. 1979, 40, 821
C.G. van-Weert, Ann. Phys. 1982, 140, 133
F. Becattini, M. B., E. Grossi, Particles 2 (2019) 2, 197-207; 1902.01089

$$\widehat{\rho}_{\rm LE}(\tau) = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_B \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Entropy:

$$S(\tau) = -{\rm tr}\left(\widehat{\rho}_{\rm LE}(\tau) \log \widehat{\rho}_{\rm LE}(\tau) \right) = \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu s^\mu$$

$$= \log Z_{\rm LE} + \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \left(\langle \widehat{T}^{\mu\nu}_B \rangle_{\rm LE} \beta_\nu - \langle \widehat{j}^\mu \rangle_{\rm LE} \zeta \right)$$

$$\nabla \cdot s = \left(T^{\mu\nu} - \langle \widehat{T}^{\mu\nu}_B \rangle_{\rm LE} \right) \nabla_\mu \beta_\nu - \left(j^\mu - \langle \widehat{j}^\mu \rangle_{\rm LE} \right) \nabla_\mu \zeta$$

Hydrodynamic Limit

$$W(x,k) = \mathrm{tr}\left(\widehat{\rho}\,\widehat{W}(x,k)\right)$$

Expand the β, ζ and all the hydrodynamic fields from the point x where the Wigner operator is to be evaluated. For instance:

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda \beta_\nu(x)(y-x)^\lambda + \cdots$$

This gives at leading order

$$\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y)\beta_\nu = \beta_\nu(x)\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

And the local thermal equilibrium (LTE) part is approximated as

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

$$\widehat{J}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right] \qquad \widehat{Q}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right]$$

We expand at first order in gradients all the hydrodynamic fields $\mathcal{U}_{(\alpha)}(y)$

Linear response theory

In general, we obtain

$$\widehat{\rho} \simeq \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + b_{\mathcal{U}}\,\mathcal{U}_{(\alpha)}(x)\,\widehat{\mathcal{B}}^{(\alpha)}_{\mathcal{U}} + c_{\mathcal{U}} \int_\Omega \mathrm{d}\Omega\, \mathcal{U}_{(\alpha)}(x_2)\,\widehat{\mathcal{C}}^{(\alpha)}_{\mathcal{U}}(x_2) + \cdots\right]$$

Where $\mathcal{U}_{(\alpha)}(x)$ is a generic hydrodynamic field with its LTE and dissipative contributions respectively.

Using linear response theory, thermal averages reduce to the equilibrium ones: $\widehat{\rho}_{\mathrm{Eq}} = \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q}\right]$

$$W(x,k) = \langle \widehat{W}(x,k)\rangle_{\beta(x)} + \Delta_{\mathcal{U},\mathrm{LTE}} W(x,k) + \Delta_{\mathcal{U},\mathrm{D}} W(x,k) + \cdots$$

$$\Delta_{\mathcal{U},\,\mathrm{LTE}} W(x) = \mathcal{U}_{(\alpha)}(x)\, b_{\mathcal{U}} \left(\widehat{W},\, \widehat{\mathcal{B}}^{(\alpha)}_{\mathcal{U}}\right)_{\mathrm{LTE}} \qquad \Delta_{\mathcal{U},\,\mathrm{D}} W(x,k) = \mathcal{U}_{(\alpha)}(x)\, c_{\mathcal{U}} \left(\widehat{W},\, \widehat{\mathcal{C}}^{(\alpha)}_{\mathcal{U}}\right)_{\mathrm{D}}$$

$$\left(\widehat{X},\, \widehat{Y}\right)_{\mathrm{LTE}} = \int_0^{|\beta|} \frac{\mathrm{d}\tau}{|\beta(x)|} \langle \widehat{Y}_{[\tau/|\beta|]}\widehat{X}(x)\rangle_{\beta(x),\,\mathrm{c}}$$

$$\left(\widehat{X},\, \widehat{Y}\right)_{\mathrm{D}} = \frac{\mathrm{i}}{|\beta(x)|} \int_{-\infty}^{t} \mathrm{d}^4 x_2 \int_{-\infty}^{t_2} \mathrm{d}s \left\langle \left[\widehat{X}(x),\, \widehat{Y}(s,\, x_2)\right]\right\rangle_{\beta(x)}$$

$$\widehat{Y}_{[\tau/|\beta|]} = \mathrm{e}^{\frac{\tau}{|\beta|}\left(\beta(x)\cdot\widehat{P} - \zeta(x)\widehat{Q}\right)}\,\widehat{Y}\,\mathrm{e}^{-\frac{\tau}{|\beta|}\left(\beta(x)\cdot\widehat{P} - \zeta(x)\widehat{Q}\right)}$$

Spin hydrodynamics

We consider a general case without specifying the underlying QFT

[A. Daher, X. L. Sheng, D. Wagner and F. Becattini, 2503.03713]
[MB, 2502.15520]

$$\widehat{\rho} = \frac{1}{Z} \exp\left\{ -\int_\Sigma \mathrm{d}\Sigma_\mu(y) \left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y) - \zeta_A(y)\,\widehat{j}^\mu_A(y) - \frac{1}{2}\mathfrak{S}_{\lambda\nu}(y)\widehat{S}^{\mu\lambda\nu}(y) \right) + \int_\Omega \mathrm{d}\Omega \left[\widehat{T}^{\mu\nu}_S \xi_{\mu\nu} + \widehat{T}^{\mu\nu}_A \left(\mathfrak{S}_{\mu\nu} - \varpi_{\mu\nu}\right) - \widehat{j}^\mu \nabla_\mu \zeta - \nabla_\mu \left(\zeta_A \widehat{j}^\mu_A\right) - \frac{1}{2}\widehat{S}^{\mu\lambda\nu}\nabla_\mu \mathfrak{S}_{\lambda\nu} \right] \right\},$$

Where:

Thermal vorticity $\varpi_{\rho\sigma}(x) = -\frac{1}{2}\left[\partial_\rho\beta_\sigma(x) - \partial_\sigma\beta_\rho(x)\right]$

Thermal shear $\xi_{\rho\sigma}(x) = \frac{1}{2}\left[\partial_\rho\beta_\sigma(x) + \partial_\sigma\beta_\rho(x)\right]$

Spin potential $\mathfrak{S}_{\mu\nu}$

Axial chemical potential $\zeta_A = \mu_A/T$

$$T^{\mu\nu}_S = \frac{1}{2}\left(T^{\mu\nu} + T^{\nu\mu}\right), \quad T^{\mu\nu}_A = \frac{1}{2}\left(T^{\mu\nu} - T^{\nu\mu}\right)$$

At local equilibrium see also
F. Becattini, A. Daher and X. L. Sheng, PLB 850 (2024)

Spin hydrodynamics

We consider a general case without specifying the underlying QFT

[MB, 2502.15520]

$$\widehat{\rho} = \frac{1}{Z}\exp\left\{-\int_\Sigma \mathrm{d}\Sigma_\mu(y)\left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y) - \zeta_A(y)\,\widehat{j}^\mu_A(y) - \frac{1}{2}\mathfrak{S}_{\lambda\nu}(y)\widehat{S}^{\mu\lambda\nu}(y)\right)\right.$$
$$\left. + \int_\Omega \mathrm{d}\Omega\left[\widehat{T}^{\mu\nu}_S\xi_{\mu\nu} + \widehat{T}^{\mu\nu}_A\left(\mathfrak{S}_{\mu\nu} - \varpi_{\mu\nu}\right) - \widehat{j}^\mu\nabla_\mu\zeta - \nabla_\mu\left(\zeta_A\widehat{j}^\mu_A\right) - \frac{1}{2}\widehat{S}^{\mu\lambda\nu}\nabla_\mu\mathfrak{S}_{\lambda\nu}\right]\right\},$$

- Hydrodynamic limit: expand the hydrodynamic fields
- The equilibrium has a residual SO(3) symmetry

$$\widehat{\rho}_{\mathrm{Eq}} = \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q}\right]$$

- Decompose the hydrodynamic fields into irreducible components
- Write down all the possible first order contribution to the Wigner function

$$\zeta_A \sim \mathcal{O}(\partial^0), \quad \varpi \sim \varpi - \mathfrak{S} \sim \xi \sim \partial\zeta \sim \partial\zeta_A \sim \partial\mathfrak{S} \sim \mathcal{O}(\partial^1)$$

- Use linear response theory to obtain the first order LTE and dissipative correction to Wigner function
- Match the linear response with the first order expression

  → Obtain the thermal and transport coefficients as Kubo formulas

Same method as
[S. Liu, Y. Yin, JHEP 07 (2021) 188]

Pseudo-gauge ambiguity

The definition of spin vector is based on the Pauli-Lubanski vector and it is pseudo gauge independent

$$\widehat{S}^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\widehat{J}_{\nu\rho}\widehat{P}_\sigma$$

The spin-tensor, which is pseudo gauge dependent, is NOT the Pauli-Lubanski vector

$$\widehat{S}^{\mu\nu}(\Sigma) = \int_\Sigma \mathrm{d}\Sigma_\lambda \widehat{S}^{\lambda,\mu\nu}$$

The statistical operator depends on the pseudo-gauge, → hence also the mean spin vector is pseudo-gauge dependent

F. Becattini, W. Florkowski, and E. Speranza, PLB 789 (2019)
W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

$$S^\mu = \langle \widehat{S}^\mu \rangle = \mathrm{tr}\left[\widehat{\rho}\,\widehat{S}^\mu\right]$$

MB, PRC 105 (2022)

The following results are obtained for a generic choice of the spin tensor. The expressions and the Kubo formulas are the same, but the actual values depend on the pseudo-gauge.

Decomposition of the hydro fields

Gradients of chemical potential

$$\partial^\rho \zeta = u^\rho D\zeta + r^\rho, \quad r^\rho = \partial^{\langle\rho\rangle}\zeta$$

Gradients of axial chemical potential

$$\partial^\rho \zeta_A = u^\rho D\zeta_A + r^\rho_A, \quad r^\rho_A = \partial^{\langle\rho\rangle}\zeta_A$$

Thermal vorticity

Acceleration — Rotation

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) = \alpha_\mu u_\nu - \alpha_\nu u_\mu + \epsilon_{\mu\nu\rho\sigma} w^\rho u^\sigma$$

Thermal shear

Shear tensor

$$\xi_{\rho\sigma} = u_\rho u_\sigma D\beta + \frac{\Delta_{\rho\sigma}}{3}\beta\theta + \frac{1}{2}\left(u_\rho \Delta^\tau_\sigma + u_\sigma \Delta^\tau_\rho\right)\left(\beta D u_\tau + \partial_\tau\beta\right) + \Delta^{\lambda\tau}_{\rho\sigma}\beta\sigma_{\lambda\tau}$$

Spin potential

$$\mathfrak{S}_{\mu\nu} = \mathfrak{a}_\mu u_\nu - \mathfrak{a}_\nu u_\mu + \epsilon_{\mu\nu\rho\sigma}\mathfrak{w}^\rho u^\sigma$$

Gradients of spin potential

$$\partial^\lambda \mathfrak{S}^{\mu\nu} = u^\lambda\left(f^\mu u^\nu - f^\nu u^\mu\right) + \epsilon^{\lambda\mu\nu\rho}\Upsilon_\rho + \left(\Delta^{\lambda\mu}u^\nu - \Delta^{\lambda\nu}u^\mu\right) I + \left(I_S^{\lambda\mu}u^\nu - I_S^{\lambda\nu}u^\mu\right)$$
$$+ \left(\epsilon^{\lambda\mu\alpha\beta}u^\nu - \epsilon^{\lambda\nu\alpha\beta}u^\mu\right) u_\alpha (I_\beta - \Upsilon_\beta) + \varphi\,\epsilon^{\lambda\mu\nu\rho}u_\rho + \Phi^{\lambda,\mu\nu}_{S12} + \Phi^{\lambda,\mu\nu}_{S13}$$

Notation:

$$\Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu, \quad V^{\langle\rho\rangle} = V^\rho_\perp = \Delta^\rho_\lambda V^\lambda, \quad D = u^\mu\partial_\mu, \quad \theta = \partial_\mu u^\mu,$$

$$\Delta_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Delta_{\mu\rho}\Delta_{\nu\sigma} + \Delta_{\mu\sigma}\Delta_{\nu\rho}\right) - \frac{1}{3}\Delta_{\mu\nu}\Delta_{\rho\sigma} \qquad Q^{\mu\nu} = \frac{\Delta^{\mu\nu}}{3} - \frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp}$$

Example: shear tensor

$$\sigma_{\mu\nu} = \Delta_{\mu\nu\rho\sigma}\partial^\rho u^\sigma = \left[\frac{1}{2}\left(\Delta_{\mu\rho}\Delta_{\nu\sigma} + \Delta_{\mu\sigma}\Delta_{\nu\rho}\right) - \frac{1}{3}\Delta_{\mu\nu}\Delta_{\rho\sigma}\right]\partial^\rho u^\sigma$$

The first order in the statistical operator is

$$\widehat{\rho} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + \widehat{B}_\sigma + \widehat{C}_\sigma + \cdots\right]$$

where

$$\widehat{B}_\sigma = -\frac{1}{2}\beta(x)\sigma_{\rho\sigma}(x)\widehat{\pi}^{\rho\sigma}_\Xi \qquad \widehat{C}_\sigma = \beta(x)\sigma_{\rho\sigma}(x)\int_\Omega \mathrm{d}\Omega\,\widehat{\pi}^{\rho\sigma}(x_2)$$

$$\widehat{\pi}^{\rho\sigma}_\Xi = \Delta^{\rho\sigma}_{\alpha\beta}\int_\Sigma \mathrm{d}\Sigma_\lambda(y)\left[(y-x)^\alpha\widehat{T}^{\lambda\beta}(y) + (y-x)^\beta\widehat{T}^{\lambda\alpha}(y)\right] \qquad \widehat{\pi}^{\rho\sigma}(x_2) = \Delta^{\rho\sigma}_{\alpha\beta}\widehat{T}^{\alpha\beta}(x_2)$$

The linear response in the axial Wigner function is

$$\Delta_\sigma\mathcal{A}^\mu(x,k) = \Delta_{\mathcal{U},\mathrm{LTE}}\mathcal{A}^\mu(x,k) + \Delta_{\mathcal{U},\mathrm{D}}\mathcal{A}^\mu(x,k)$$

$$= \beta(x)\sigma_{\rho\sigma}(x)\left[\left(\widehat{\mathcal{A}}^\mu,\,\widehat{\pi}^{\rho\sigma}\right)_\mathrm{D} - \frac{1}{2}\left(\widehat{\mathcal{A}}^\mu,\,\widehat{\pi}^{\rho\sigma}_\Xi\right)_\mathrm{LTE}\right]$$

Example: shear tensor

All possible vector terms that are linear in the shear tensor are:

$$\Delta_\sigma \mathcal{A}^\mu = \bar{\mathfrak{a}}_{\sigma u}\mathfrak{A}^\mu_{\sigma u} + \bar{\mathfrak{a}}_{\sigma\Delta}\mathfrak{A}^\mu_{\sigma\Delta} + \bar{\mathfrak{a}}_{\sigma k}\mathfrak{A}^\mu_{\sigma k} + a_{\sigma\epsilon}A^\mu_{\sigma\epsilon}$$

where

$$A^\mu_{\sigma\epsilon} = \epsilon^{\mu\nu\alpha\rho}k^\sigma_\perp \frac{u_\nu k_\alpha}{(k\cdot u)}\beta\sigma_{\rho\sigma}, \qquad \mathfrak{A}^\mu_{\sigma u} = (k\cdot u)\frac{k^\rho_\perp k^\sigma_\perp}{k^2_\perp}u^\mu\,\beta\sigma_{\rho\sigma},$$

$$\mathfrak{A}^\mu_{\sigma\Delta} = \Delta^{\mu\rho}k^\sigma_\perp\beta\sigma_{\rho\sigma}, \qquad \mathfrak{A}^\mu_{\sigma k} = Q^{\mu\rho}k^\sigma_\perp\,\beta\sigma_{\rho\sigma},$$

$$Q^{\mu\nu} = \frac{\Delta^{\mu\nu}}{3} - \frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp}$$

We then study the properties under discrete transformations:

	β	$D\beta$	θ	$Du^{\langle\mu\rangle}$, $\partial^{\langle\mu\rangle}\beta$	$\mathcal{A}^\mu$, $A^\mu_{\sigma\epsilon}$	a^μ, $\mathfrak{A}^\mu_{\sigma X}$	$\widehat{\varepsilon}_\Xi$, $\widehat{p}_\Xi$	$\widehat{\varepsilon}$, $\widehat{p}$	$\widehat{q}^{\langle\mu\rangle}_\Xi$	$\widehat{q}^{\langle\mu\rangle}$	$\widehat{\pi}^{\rho\sigma}_\Xi$	$\widehat{\pi}^{\rho\sigma}$
P	$+$	$+$	$+$	$-$	$(-,+)$	$(+,-)$	$+$	$+$	$-$	$-$	$+$	$+$
T	$+$	$-$	$-$	$+$	$(+,-)$	$(-,+)$	$-$	$+$	$+$	$-$	$-$	$+$
C	$+$	$+$	$+$	$+$	$(+,+)$	$(+,+)$	$+$	$+$	$+$	$+$	$+$	$+$

From which follows that

	$a_{\sigma\epsilon}$	$\bar{\mathfrak{a}}_{\sigma u}$ $\bar{\mathfrak{a}}_{\sigma\Delta}$ $\bar{\mathfrak{a}}_{\sigma k}$
P	$+$	$-$
T	$+$	$-$
C	$+$	$+$

Example: shear tensor

We can tell which one are vanishing

$$a_{\sigma\epsilon} = -\frac{(k\cdot u)^2}{4(k_\perp^2)^2}\left(\widehat{\mathcal{A}}^\mu,\, u^\lambda \epsilon_{\lambda\mu\tau\rho} k^\perp_\sigma k^\tau_\perp \widehat{\pi}^{\rho\sigma}_{\Xi}\right)_{\rm LTE} - \frac{(k\cdot u)^2}{4(k_\perp^2)^2}\left(\widehat{\mathcal{A}}^\mu,\, u^\lambda \epsilon_{\lambda\mu\tau\rho} k^\perp_\sigma k^\tau_\perp \widehat{\pi}^{\rho\sigma}\right)_{\rm D}$$

	LTE: $\widehat{\mathcal{A}}^\mu$	LTE: $u^\lambda \epsilon_{\lambda\mu\tau\rho} k^\perp_\sigma k^\tau_\perp \widehat{\pi}^{\rho\sigma}_{\Xi}$	D: $\widehat{\mathcal{A}}^\mu$	D: $u^\lambda \epsilon_{\lambda\mu\tau\rho} k^\perp_\sigma k^\tau_\perp \widehat{\pi}^{\rho\sigma}$
P	-	-	-	+
T	+	+	+	-
C	+	+	+	+

This is vanishing!

This term can only come from the LTE part:

$$\Delta_{\sigma \rm LTE}\mathcal{A}^\mu = a_{\sigma\epsilon}\epsilon^{\mu\nu\alpha\rho} k^\sigma_\perp \frac{u_\nu k_\alpha}{(k\cdot u)}\beta\sigma_{\rho\sigma}$$

$$a_{\sigma\epsilon} = -\frac{(k\cdot u)^2}{4(k_\perp^2)^2}\left(\widehat{\mathcal{A}}^\mu,\, u^\lambda \epsilon_{\lambda\mu\tau\rho} k^\perp_\sigma k^\tau_\perp \widehat{\pi}^{\rho\sigma}_{\Xi}\right)_{\rm LTE}$$

We study all the terms in the same fashion.

We always find that

- LTE only contribute to T-symmetric coefficients
- Dissipative part only contribute to T-odd coefficients

Axial Wigner function

[MB, 2502.15520]

The non-dissipative part:

Chiral imbalance

Spin Hall Effect

Thermal vorticity

Spin potential

Acceleration and grad. of temperature

Shear tensor

$$\Delta_{\rm LTE}\mathcal{A}^\mu_+ = \left[a_{\zeta_A u} u^\mu + a_{\zeta_A k}\frac{k_\perp^\mu}{(k\cdot u)}\right]\zeta_A + a^c_{r\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\partial_{\langle\rho\rangle}\zeta$$

$$-a_\varpi \frac{2\widetilde{\varpi}^{\mu\nu}k_\nu}{(2\pi)^3} - (a_{wu} - a_{w\Delta})\frac{(k\cdot w)}{(k\cdot u)}u^\mu + (a_{\alpha\epsilon} - a_{w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\alpha_\rho + a_{wk}Q^{\mu\rho}w_\rho$$

$$-a_{\mathfrak{S}-\varpi}\frac{2(\widetilde{\mathfrak{S}}^{\mu\nu} - \widetilde{\varpi}^{\mu\nu})k_\nu}{(2\pi)^3} - (a_{\mathfrak{w}-wu} - a_{\mathfrak{w}-w\Delta})\frac{k_\perp^\rho u^\mu}{(k\cdot u)}(\mathfrak{w} - w)_\rho$$

$$+(a_{\mathfrak{a}-\alpha\epsilon} - a_{\mathfrak{w}-w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\mathfrak{a} - \alpha)_\rho + a_{\mathfrak{w}-wk}Q^{\mu\rho}(\mathfrak{w} - w)_\rho$$

$$+ a_{q\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\beta D u_\rho + \partial_\rho\beta) + a_{\sigma\epsilon}\epsilon^{\mu\nu\alpha\rho}k_\perp^\sigma\frac{u_\nu k_\alpha}{(k\cdot u)}\beta\sigma_{\rho\sigma}$$

Remind that spin polarization is $S^\mu(k) = \frac{1}{8m}\frac{\int \mathrm{d}\Sigma\cdot k\,\mathcal{A}^\mu_+(x,k)}{\int \mathrm{d}\Sigma\cdot k\,n_f(\beta(x)\cdot k)}$

Axial Wigner function

[MB, 2502.15520]

Chiral imbalance

Spin Hall Effect

Thermal vorticity

Spin potential

Acceleration

Shear tensor

$$
\begin{aligned}
\Delta_{\rm LTE}\mathcal{A}^\mu_+ = & \left[a_{\zeta_A u}u^\mu + a_{\zeta_A k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\zeta_A + a^c_{r\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\partial_{\langle\rho\rangle}\zeta \\
& - a_\varpi\frac{2\widetilde{\varpi}^{\mu\nu}k_\nu}{(2\pi)^3} - (a_{wu}-a_{w\Delta})\frac{(k\cdot w)}{(k\cdot u)}u^\mu + (a_{\alpha\epsilon}-a_{w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\alpha_\rho + a_{wk}Q^{\mu\rho}w_\rho \\
& - a_{\mathfrak{S}-\varpi}\frac{2(\widetilde{\mathfrak{S}}^{\mu\nu}-\widetilde{\varpi}^{\mu\nu})k_\nu}{(2\pi)^3} - (a_{\mathfrak{w}-wu}-a_{\mathfrak{w}-w\Delta})\frac{k^\rho_\perp u^\mu}{(k\cdot u)}(\mathfrak{w}-w)_\rho \\
& + (a_{\mathfrak{a}-\alpha\epsilon}-a_{\mathfrak{w}-w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\mathfrak{a}-\alpha)_\rho + a_{\mathfrak{w}-wk}Q^{\mu\rho}(\mathfrak{w}-w)_\rho \\
& + a_{q\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\beta Du_\rho + \partial_\rho\beta) + a_{\sigma\epsilon}\epsilon^{\mu\nu\alpha\rho}k^\sigma_\perp\frac{u_\nu k_\alpha}{(k\cdot u)}\beta\sigma_{\rho\sigma}
\end{aligned}
$$

Thermal vorticity → main effect for global spin polarization

[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)]

Thermal shear → needed to explain local spin polarization

[F. Becattini, MB, A. Palermo, PLB 820 (2021)]

[S. Liu, Y. Yin, JHEP 07 (2021) 188]

Chiral imbalance → Can be used to probe topological charge in alternative to CME

[F. Becattini, MB, A. Palermo and G. Prokhorov, PLB 822 (2021)]

Spin potential → When different from thermal vorticity gives an additional contribution

[MB, PRC 105 (2022)]

Axial Wigner function

NEW IN THIS WORK The dissipative part:

[MB, 2502.15520]

$$\Delta_{\rm D}\mathcal{A}^\mu_+ = \left[\bar a_{D\zeta_A u} u^\mu + \bar a_{D\zeta_A k}\frac{k^\mu_\perp}{(k\cdot u)}\right] D\zeta_A + \left[\bar a_{r_A u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar a_{r_A\Delta}\Delta^{\mu\rho} + \bar a_{r_A k} Q^{\mu\rho}\right]\partial_{\langle\rho\rangle}\zeta_A$$

Gradients of chiral imbalance

$$+ \bar a_{f\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)} f_\rho + \left[\bar a_{\Upsilon u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar a_{\Upsilon\Delta}\Delta^{\mu\rho} + \bar a_{\Upsilon k}Q^{\mu\rho}\right]\Upsilon_\rho$$

Gradients of spin potential

$$+ \left[\bar a_{I-\Upsilon u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar a_{I-\Upsilon\Delta}\Delta^{\mu\rho} + \bar a_{I-\Upsilon k}Q^{\mu\rho}\right](I_\rho - \Upsilon_\rho) + \left[\bar a_{\varphi u}u^\mu + \bar a_{\varphi k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\varphi + \bar a_{I_S\epsilon}\,\epsilon^{\mu\nu\alpha\rho}k^\sigma_\perp\frac{u_\nu k^\perp_\alpha}{(k\cdot u)^2} I_{S\,\rho\sigma}$$

$$+ \left[\bar a_{{\rm S}12\Delta\epsilon}\Delta^{\mu\tau}\epsilon^{\lambda\nu\rho\sigma}\frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar a_{{\rm S}12\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right]\Phi^{S12}_{\tau,\rho\sigma} + \left[\bar a_{{\rm S}13\Delta\epsilon}\Delta^{\mu\tau}\epsilon^{\lambda\nu\rho\sigma}\frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar a_{{\rm S}13\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right]\Phi^{S13}_{\tau,\rho\sigma}$$

$$\partial^\lambda\mathfrak{S}^{\mu\nu} = u^\lambda\left(f^\mu u^\nu - f^\nu u^\mu\right) + \epsilon^{\lambda\mu\nu\rho}\Upsilon_\rho + \left(\Delta^{\lambda\mu}u^\nu - \Delta^{\lambda\nu}u^\mu\right)I + \left(I_S^{\lambda\mu}u^\nu - I_S^{\lambda\nu}u^\mu\right)$$
$$+ \left(\epsilon^{\lambda\mu\alpha\beta}u^\nu - \epsilon^{\lambda\nu\alpha\beta}u^\mu\right)u_\alpha(I_\beta - \Upsilon_\beta) + \varphi\,\epsilon^{\lambda\mu\nu\rho}u_\rho + \Phi^{\lambda,\mu\nu}_{S12} + \Phi^{\lambda,\mu\nu}_{S13}$$

Remind that spin polarization is $S^\mu(k) = \dfrac{1}{8m}\dfrac{\int {\rm d}\Sigma\cdot k\,\mathcal{A}^\mu_+(x,k)}{\int {\rm d}\Sigma\cdot k\, n_f(\beta(x)\cdot k)}$

Axial Wigner function ⟶ (Local) Spin polarization

- All dissipative coefficients are odd under time-reversal
- No dissipative contribution from: shear tensor, rate of expansion or gradients of temperature because they break parity symmetry
- The only dissipative contributions at first order are given by the chiral imbalance and by gradients of the spin potential

- For interacting fields, there could be more contributions proportional to thermal vorticity at LTE!

$$\Delta_{\varpi,\mathrm{LTE}}\mathcal{A}^\mu_+ = -a_\varpi \frac{2\tilde{\varpi}^{\mu\nu}k_\nu}{(2\pi)^3} \underbrace{-(a_{wu}-a_{w\Delta})\frac{(k\cdot w)}{(k\cdot u)}u^\mu + (a_{\alpha\epsilon}-a_{w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\alpha_\rho + a_{wk}Q^{\mu\rho}w_\rho}_{=0 \text{ for free Dirac field}}$$

$$a^{\mathrm{Free}}_\varpi = \delta(k^2-m^2)\theta(k\cdot u)n_F(\beta\cdot k)\left(1-n_F(\beta\cdot k)\right)$$

⟶ Radiative corrections: [MB, D. Kharzeev, PRD 103 (2021)][S. Fang, S. Pu and D. L. Yang, 2503.13320]

The transport and thermal coefficients

Examples, LTE thermal-vorticity:

$$a_{\varpi} = -\frac{(2\pi)^3}{6(k \cdot u)} \left(\widehat{\mathcal{A}}^{\mu}_{+},\, \widehat{J}_{x\,\mu} \right)_{\mathrm{LTE}} \qquad \widehat{J}^{\rho}_{x} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\rho} u_{\alpha} \widehat{J}_{x\,\beta\gamma}$$

LTE shear tensor:

$$a_{\sigma\epsilon} = -\frac{(k \cdot u)^2}{4(k_{\perp}^2)^2} \left(\widehat{\mathcal{A}}^{\mu}_{+},\, u^{\lambda} \epsilon_{\lambda\mu\tau\rho} k^{\perp}_{\sigma} k^{\tau}_{\perp} \widehat{\pi}^{\rho\sigma}_{\Xi} \right)_{\mathrm{LTE}}$$

$$\widehat{\pi}^{\mu\nu}_{\Xi} = \Delta^{\mu\nu}_{\alpha\beta} \int_{\Sigma} \mathrm{d}\Sigma_{\lambda}(y) \left[(y-x)^{\alpha} \widehat{T}^{\lambda\beta}(y) + (y-x)^{\beta} \widehat{T}^{\lambda\alpha}(y) \right]$$

Dissipative spin potential:

$$\bar{a}_{\Upsilon u} = \frac{(k \cdot u)}{k_{\perp}^2} \left(u_{\mu} \widehat{\mathcal{A}}^{\mu},\, k^{\perp}_{\rho} \widehat{\mathcal{C}}^{\rho}_{\Upsilon} \right)_{\mathrm{D}}$$

$$\widehat{\mathcal{C}}_{\partial\mathfrak{S}}(x_2) = \widehat{S}^{\tau,\rho\sigma}(x_2) - 2(x_2 - x)^{\tau} \widehat{T}^{\rho\sigma}_{A}(x_2) \to \widehat{\mathcal{C}}^{\rho}_{\Upsilon}$$

$$\left(\widehat{X},\, \widehat{Y} \right)_{\mathrm{D}} = \frac{\mathrm{i}}{|\beta(x)|} \int_{-\infty}^{t} \mathrm{d}^4 x_2 \int_{-\infty}^{t_2} \mathrm{d}s \left\langle \left[\widehat{X}(x),\, \widehat{Y}(s,\, x_2) \right] \right\rangle_{\beta(x)}$$

$$\left(\widehat{X},\, \widehat{Y} \right)_{\mathrm{LTE}} = \int_{0}^{|\beta|} \frac{\mathrm{d}\tau}{|\beta(x)|} \langle \widehat{Y}_{[\tau/|\beta|]} \widehat{X}(x) \rangle_{\beta(x),\,\mathrm{c}}$$

Classification of coefficients

The coefficient have been classified according to their properties under discrete transformations:
P parity conjugation, T time reversal and C charge conjugation

	$a_{\mathcal{U}}$	$a^c_{\mathcal{U}}$	$\bar{a}_{\mathcal{U}}$	$\mathfrak{a}_{\mathcal{U}}$	$\bar{a}^c_{\mathcal{U}}$	$\mathfrak{a}^c_{\mathcal{U}}$	$\bar{\mathfrak{a}}_{\mathcal{U}}$	$\bar{\mathfrak{a}}^c_{\mathcal{U}}$
P	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$
T	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$-$
C	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$

Example, a coefficient is chiral if its parity under charge conjugation is odd, i.e.

$$\widehat{\mathsf{P}}\,\widehat{O}\,\widehat{\mathsf{P}}^{-1} = \eta_O \widehat{O}, \quad \widehat{\mathsf{P}}\,\widehat{B}\,\widehat{\mathsf{P}}^{-1} = \eta_B \widehat{B}, \quad \mathfrak{a} = \langle \widehat{O}\,\widehat{B} \rangle_\beta \text{ if } \eta_O\,\eta_B = -1$$

To have a non-vanishing chiral coefficient, a chiral imbalance or parity violating interactions are needed!
-We could treat the axial imbalance as leading order:

$$\widehat{\rho}_{\mathrm{Eq}} = \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + \zeta_A(x)\widehat{Q}_A\right]$$

-or we could treat the axial imbalance as higher order correction:

$$\left(\widehat{B}_X,\, \widehat{O}(x)\right) \simeq 2\zeta_A(x)\left(\widehat{Q}_A,\, \widehat{B}_X,\, \widehat{O}(x)\right)$$

Axial Wigner function with parity breaking

[MB, 2502.15520]

NEW IN THIS WORK The chiral non-dissipative part:

Chiral spin Hall effect

$$\Delta_{\mathrm{LTE},\chi}\mathcal{A}^\mu_+ = \mathfrak{a}_{r_A\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\partial_{\langle\rho\rangle}\zeta_A$$

Gradients of spin potential

$$+\left[\mathfrak{a}_{fu}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \mathfrak{a}_{f\Delta}\Delta^{\mu\rho} + \mathfrak{a}_{fk}Q^{\mu\rho}\right] f_\rho + \mathfrak{a}_{\Upsilon\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\Upsilon_\rho$$

$$+\left[\mathfrak{a}_{Iu}u^\mu + \mathfrak{a}_{Ik}\frac{k^\mu_\perp}{(k\cdot u)}\right] I + \mathfrak{a}_{I-\Upsilon\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(I_\rho - \Upsilon_\rho) + \left[\mathfrak{a}_{I_S u}\frac{k^\rho_\perp k^\sigma_\perp}{k^2_\perp}u^\mu + \mathfrak{a}_{I_S\Delta}\frac{\Delta^{\mu\rho}k^\sigma_\perp}{(k\cdot u)} + \mathfrak{a}_{I_S k}\frac{Q^{\mu\rho}k^\sigma_\perp}{(k\cdot u)}\right] I_{S\,\rho\sigma}$$

$$+\left[\mathfrak{a}_{\mathrm{S12}\Delta}\Delta^{\tau\rho}\Delta^{\mu\sigma} + \mathfrak{a}_{\mathrm{S12k}}Q^{\tau\rho}\Delta^{\mu\sigma}\right]\Phi^{S12}_{\tau,\rho\sigma} + \left[\mathfrak{a}_{\mathrm{S13}\Delta}\Delta^{\tau\sigma}\Delta^{\mu\rho} + \mathfrak{a}_{\mathrm{S13k}}Q^{\tau\sigma}\Delta^{\mu\rho}\right]\Phi^{S13}_{\tau,\rho\sigma}.$$

NEW IN THIS WORK The chiral dissipative part:

Gradients of chemical potential

$$\Delta_{\mathrm{D},\chi}\mathcal{A}^\mu_+ = \left[\bar{\mathfrak{a}}^c_{D\zeta u}u^\mu + \bar{\mathfrak{a}}^c_{D\zeta k}\frac{k^\mu_\perp}{(k\cdot u)}\right] D\zeta + \left[\bar{\mathfrak{a}}^c_{ru}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{\mathfrak{a}}^c_{r\Delta}\Delta^{\mu\rho} + \bar{\mathfrak{a}}^c_{rk}Q^{\mu\rho}\right]\partial_{\langle\rho\rangle}\zeta$$

Spin potential

$$+\bar{\mathfrak{a}}_{\mathfrak{a}-\alpha\Delta}\Delta^{\mu\rho}(\mathfrak{a}-\alpha)_\rho - \bar{\mathfrak{a}}_{\mathfrak{a}-\alpha u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)}(\mathfrak{a}-\alpha)_\rho + \bar{\mathfrak{a}}_{\mathfrak{w}-w\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\mathfrak{w}-w)_\rho$$

$$+\bar{\mathfrak{a}}_{\mathfrak{a}-\alpha k}Q^{\mu\rho}(\mathfrak{a}-\alpha)_\rho + \left[\bar{\mathfrak{a}}_{D\beta u}u^\mu + \bar{\mathfrak{a}}_{D\beta k}\frac{k^\mu_\perp}{(k\cdot u)}\right] D\beta + \left[\bar{\mathfrak{a}}_{\theta u}u^\mu + \bar{\mathfrak{a}}_{\theta k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\beta\theta$$

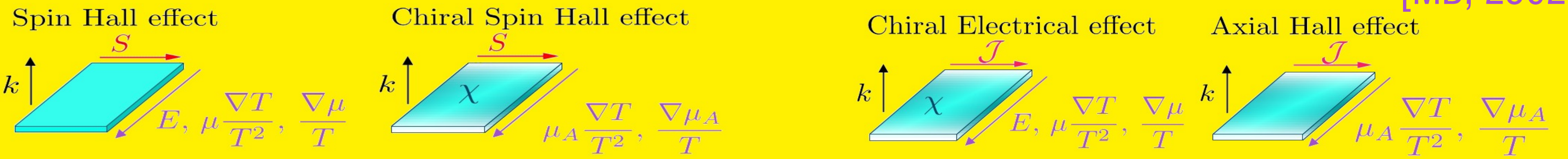
Shear tensor

$$+\left[\bar{\mathfrak{a}}_{qu}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{\mathfrak{a}}_{q\Delta}\Delta^{\mu\rho} + \bar{\mathfrak{a}}_{qk}Q^{\mu\rho}\right](\beta Du_\rho + \partial_\rho\beta) + \bar{\mathfrak{a}}_{\sigma u}(k\cdot u)\frac{k^\rho_\perp k^\sigma_\perp}{k^2_\perp}u^\mu\,\beta\sigma_{\rho\sigma}$$

$$+\bar{\mathfrak{a}}_{\sigma\Delta}\Delta^{\mu\rho}k^\sigma_\perp\beta\sigma_{\rho\sigma} + \bar{\mathfrak{a}}_{\sigma k}Q^{\mu\rho}k^\sigma_\perp\,\beta\sigma_{\rho\sigma}$$

Chiral Spin Hall Effects

[MB, 2502.15520]



Axial part of Wigner function: SHE, Chiral SHE

$$\Delta_{\rm SHE}\mathcal{A}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[a^c_{r\epsilon}(k)\partial_\rho\zeta + \mathfrak{a}_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

Vector part of Wigner function: Chiral electrical effect, Axial Hall effect

$$\Delta_{\rm SHE}\mathcal{V}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[\mathfrak{v}_{r\epsilon}(k)\partial_\rho\zeta + v^c_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

The currents are vanishing:

$$j^\mu_{\rm SHE} = \int {\rm d}^4k\,\Delta_{\rm SHE}\mathcal{V}^\mu(x,k) = 0, \quad j^\mu_{\rm A,SHE} = \int {\rm d}^4k\,\Delta_{\rm SHE}\mathcal{A}^\mu(x,k) = 0$$

But the spin vector is

$$\mathbf{S}(k) = \mathbf{k}\times\left\langle\left\langle a^c_{r\epsilon}\left(\frac{\boldsymbol{\nabla}\mu}{T} + \mu\boldsymbol{\nabla}\frac{1}{T}\right)\right\rangle\right\rangle + \mathbf{k}\times\left\langle\left\langle \mathfrak{a}_{r_A\epsilon}\left(\frac{\boldsymbol{\nabla}\mu_A}{T} + \mu_A\boldsymbol{\nabla}\frac{1}{T}\right)\right\rangle\right\rangle$$

It can be used to probe anisotropies in the topological charge

Chiral Spin Hall Effect: free field

[MB, 2502.15520]

Axial part of Wigner function:

SHE, Chiral SHE

$$\Delta_{\rm SHE}\mathcal{A}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[a^c_{r\epsilon}(k)\partial_\rho\zeta + \mathfrak{a}_{r\,A\,\epsilon}(k)\partial_\rho\zeta_A\right]$$

Vector part of Wigner function:

Chiral electrical effect, Axial Hall effect

$$\Delta_{\rm SHE}\mathcal{V}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[\mathfrak{v}_{r\epsilon}(k)\partial_\rho\zeta + v^c_{r\,A\,\epsilon}(k)\partial_\rho\zeta_A\right]$$

$$a^c_{r\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n^R_F(x,k)(1-n^R_F(x,k)) + n^L_F(x,k)(1-n^L_F(x,k))\right],$$

$$\mathfrak{a}_{r\,A\,\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n^R_F(x,k)(1-n^R_F(x,k)) - n^L_F(x,k)(1-n^L_F(x,k))\right],$$

$$\mathfrak{v}_{r\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n^R_F(x,k)(1-n^R_F(x,k)) - n^L_F(x,k)(1-n^L_F(x,k))\right],$$

$$v^c_{r\,A\,\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n^R_F(x,k)(1-n^R_F(x,k)) + n^L_F(x,k)(1-n^L_F(x,k))\right],$$

$$n^\chi_F(x,k) = \frac{1}{\mathrm{e}^{\beta(x)\cdot k - \zeta(x) - \chi\zeta_A(x)}+1}, \qquad \chi = \begin{cases} +1 & R \\ -1 & L \end{cases}$$

Conclusions

- All possible first order dissipative effects on spin polarization have been classified
- Only the gradients of spin potential contribute without breaking the parity symmetry
- Outlook: estimate the phenomenological impact,
  for instance, compute the transport coefficients

- With interaction there could be additional contribution even at LTE
- Chiral Spin Hall Effect is a LTE effect contributing to local spin polarization

Thank you for the attention!

BACKUP SLIDES

Kubo formulas in momentum space

$$\left(\widehat{X},\,\widehat{Y}\right)_{\mathrm{D}} = \frac{\mathrm{i}}{|\beta(x)|}\int_{-\infty}^{t}\mathrm{d}^4x_2\int_{-\infty}^{t_2}\mathrm{d}s\,\left\langle\left[\widehat{X}(x),\,\widehat{Y}(s,\,x_2)\right]\right\rangle_{\beta(x)}$$

$$G^R_{\widehat{X}\widehat{Y}}(x-x_2) = -\,\mathrm{i}\theta(x-x_2)\left\langle\left[\widehat{X}(x),\,\widehat{Y}(x_2)\right]\right\rangle_{\beta(x)},$$

$$G^R_{\widehat{X}\widehat{Y}}(x) = \int\frac{\mathrm{d}^4p}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}p\cdot x}G^R_{\widehat{X}\widehat{Y}}(p).$$

$$\left(\widehat{X},\,\widehat{Y}\right)_{\mathrm{D}} = -\frac{1}{|\beta(x)|}u^\lambda\lim_{p\cdot u\to 0}\lim_{p_\perp\to 0}\frac{\partial}{\partial p^\lambda}\mathrm{Im}\,G^R_{\widehat{X}\widehat{Y}}(p)$$

See also: F. Becattini, MB and E. Grossi, Particles 2 (2019) 197

A. Hosoya, M.-a. Sakagami and M. Takao, Annals Phys. 154 (1984) 229

X.-G. Huang, A. Sedrakian and D.H. Rischke, Annals Phys. 326 (2011) 3075

A. Harutyunyan, A. Sedrakian and D.H. Rischke, Particles 1 (2018) 155

A. Harutyunyan, A. Sedrakian and D.H. Rischke, Annals Phys. 438 (2022) 168755

Comparison with D. Wagner, PRD 111 (2025)

- Quantum kinetic theory with non-local collisions

$$N(p) = 2 \int \mathrm{d}\Sigma \cdot p f_0$$

$$S^\mu(p) = S^\mu_\omega(p) + S^\mu_\kappa(p) + S^\mu_{\mathfrak{t}}(p)$$

$$S^\mu_\omega(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{u^\mu(\omega_0 \cdot p) - \omega_0^\mu (p \cdot u)}{2m_\Lambda} f_0(1 - f_0)$$

$$\omega_0^\mu \to \mathfrak{w}^\mu, \quad \dot{\omega}_0^{<\mu>} \to \Upsilon^\mu, \quad \omega_K^\mu \to w^\mu,$$
$$\epsilon^{\mu\nu\alpha\beta} u_\nu \dot{u}_\alpha \kappa_{0,\beta} \to \Upsilon^\mu, \quad \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta} \to I^\mu,$$

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\frac{\omega_{\mathrm{K}}^\mu}{T} + \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\ell_{\omega\kappa} \nabla_\alpha \kappa_{0,\beta} - \tau_\omega \dot{u}_\alpha \kappa_{0,\beta}\right) + \delta_{\omega\omega} \omega_0^\mu \theta + \lambda_{\omega\omega} \sigma^{\mu\nu} \omega_{0,\nu} + \lambda_{\omega\mathfrak{t}} \mathfrak{t}^\mu{}_\nu \omega_{\mathrm{K}}^\nu ,$$

$\mathcal{O}\left(\partial^2\right)$

$$S^\mu_\kappa(p) = -\frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p_\sigma}{2m_\Lambda} \kappa_{0,\rho} f_0(1 - f_0)$$

$$\kappa_0^\mu \to \mathfrak{a}^\mu, \quad \dot{\kappa}_0^{<\mu>} \to f^\mu$$
$$\epsilon^{\mu\nu\alpha\beta} u_\nu \dot{u}_\alpha \omega_{0,\beta} \to f^\mu, \quad \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} \to \Phi^{\tau,\rho\sigma},$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = -\frac{\dot{u}^\mu}{T} + \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\frac{\tau_\kappa}{2} \nabla_\alpha \omega_{0,\beta} + \tau_\kappa \dot{u}_\alpha \omega_{0,\beta}\right) + \delta_{\kappa\kappa} \kappa_0^\mu \theta + \left(\lambda_{\kappa\kappa} \sigma^{\mu\nu} + \frac{\tau_\kappa}{2} \omega_{\mathrm{K}}^{\mu\nu}\right) \kappa_{0,\nu}$$
$$+ \tau_{\kappa\mathfrak{t}} \mathfrak{t}^{\mu\nu} \dot{u}_\nu + \ell_{\kappa\mathfrak{t}} \Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda} ,$$

$\mathcal{O}\left(\partial^2\right)$

$$S^\mu_{\mathfrak{t}}(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p^\lambda p_\sigma}{3T^2(\epsilon + P)} \mathfrak{t}_{\rho\lambda} f_0(1 - f_0)$$

$$\mathfrak{t}^{\mu\nu} \to ?$$

$$\tau_{\mathfrak{t}} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \frac{\mathfrak{d}}{T} \sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}^{\mu\nu} \theta + \lambda_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}_\lambda{}^{\langle\mu} \sigma^{\nu\rangle\lambda} + \frac{5}{3} \tau_{\mathfrak{t}} \mathfrak{t}_\lambda{}^{\langle\mu} \omega_{\mathrm{K}}^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa} \nabla^{\langle\mu} \kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega} \omega_{\mathrm{K}}^{\langle\mu} \omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega} \sigma_\lambda{}^{\langle\mu} \epsilon^{\nu\rangle\lambda\alpha\beta} u_\alpha \omega_{0,\beta}$$

$\mathcal{O}\left(\partial^2\right)$