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Quantum corrections to the AdS-Schwarzschild black brane and viscosities of the quark-gluon plasma

ROLDÃO DA ROCHA

Foundations and Applications of Relativistic Hydrodynamics Focus Week.
12 May 2025

The Galileo Galilei Institute
For Theoretical Physics
Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare
Arcetri, Firenze

Outline

- **Deformations of AdS_5-Schwarzschild black branes**.

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- **Response and transport coefficients of the dual hydrodynamics**.
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- **Response and transport coefficients of the dual hydrodynamics**.
- η/s and ζ/s.
- Deformations are consistent with **2-loop quantum corrections** to dual gravity.
- **Constraints by QCD and QGP**.

AdS_5–Schwarzschild black brane

- Einstein–Hilbert action in 5D:

$$S = \int d^5x \sqrt{-g}\,(R - \Lambda_5).$$

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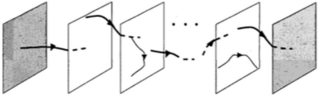
- Emparán, Reall, Living Rev. Rel., **11** (2008) 6.

  "*The AdS–Schwarzschild black brane is the unique static, asymptotically AdS, solution in the vacuum*".

- Stack of N D_3-branes in AdS/CFT.

  - Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.
  - Witten, Adv. Theor. Math. Phys. **2** (1998) 253.
  - Gubser, Klebanov, Polyakov, Phys. Lett. B **428** (1998) 105.



AdS/CFT

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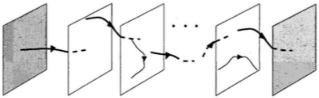

- Open strings ending on N branes $\Leftrightarrow$ SU(N) gauge fields, for $N \gg 1$.
  - t' Hooft, Nucl. Phys. B **72** (1974) 461

  S-matrix for string scatterings $\sim$ S-matrix in SU(N) Yang-Mills theory, $N \gg 1$.

D_3-branes

- Stack of N D_3-branes metric:
  - Horowitz, Strominger, Nucl. Phys. B **360** (1991) 197.

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-1/2}(-dt^2 + dx^i dx_i) + \left(1 + \frac{R^4}{r^4}\right)^{1/2}(dr^2 + r^2 d\Omega_5^2),$$

where $R^4 = \frac{N}{2\pi^2 T_3}$, and T_3 is the D_3-brane tension.

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where $R^4 = \frac{N}{2\pi^2 T_3}$, and T_3 is the D_3-brane tension.

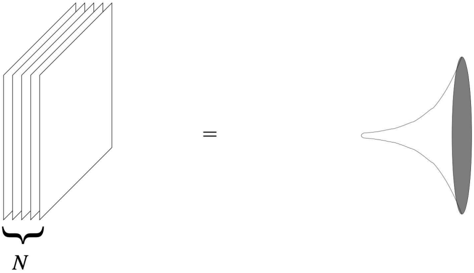
- **Near-horizon**:

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^i dx_i) + \frac{R^2}{r^2} dr^2 + \frac{R^2}{\not{r}^2} \not{r}^2 d\Omega_5^2.$$

$$\mathrm{AdS}_5 \times S^5.$$

Stack of N D_3-branes: AdS/CFT

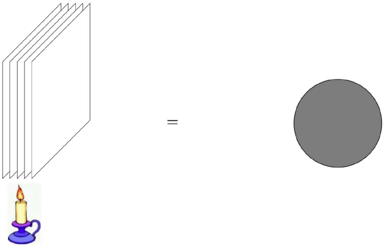
Effective geometry AdS_5 at low energies.



Son, Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

Finite temperature: AdS/CFT

- Finite temperature: effective geometry AdS$_5$–Schwarzschild.



*"...near-extremal D_3-**brane** is **dual to finite-temperature** $\mathcal{N} = 4$ supersymmetric SU(N_c) **Yang–Mills theory**, in the limit of large N_c and large 't Hooft coupling..."*

- Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.
- Witten, Adv. Theor. Math. Phys. **2** (1998) 253.
- Gubser, Klebanov, Polyakov, Phys. Lett. B **428** (1998) 105.

AdS_5–Schwarzschild black brane

- Janik, Peschanski, Phys. Rev. D **73** (2006) 045013.
- Herzog, Karch, Kovtun, Kozcaz, Yaffe, JHEP **07** (2006) 013.

Strongly-coupled 4D CFT on the AdS boundary dual to the **AdS_5–Schwarzschild black brane**, at finite temperature.

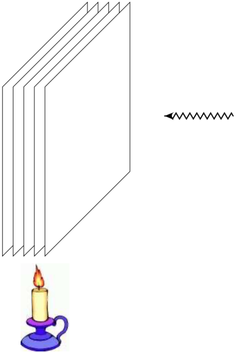
$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + \sum_{i=1}^{3} dx_i^2\right) + \frac{L^2}{r^2 f(r)}dr^2,$$

where

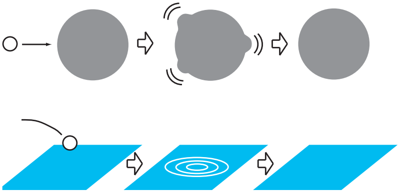
$$f(r) = 1 - \frac{r_0^4}{r^4}.$$

Viscosity and duality

- Interaction between the graviton and the stack of N D_3-branes:
  - Romatschke, Son, Phys. Rev. D **80** (2009) 065021.



Son, Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

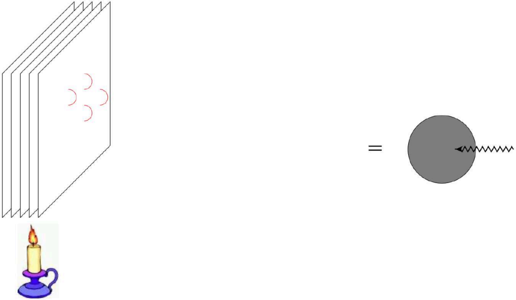


Bulk	Boundary
⋆ Response properties at the horizon	⋆ transport coefficients [Kovtun, Son, Starinets (KSS)]
Einstein's equations	Navier-Stokes equations

Natsuume, Lect. Notes Phys. **903** (2015).

Perturbations $\boxed{g_{\mu\nu} \mapsto g_{\mu\nu} + h_{\mu\nu}}$

Viscosity and duality



- Viscosity: absorption cross-section for gravitons at low energy $\propto$ black brane horizon area.
- Kovtun, Son, Starinets, JHEP **10** (2003) 064:

$$\eta = \lim_{\omega \to 0} \sigma_{\rm abs}(\omega) \quad = \quad - \lim_{\omega \to 0} \frac{1}{\omega} \int dt\, d\vec{x} e^{i\omega t} \left\langle \left[T_{xy}(t, \vec{x}), T_{xy}(0, \vec{0}) \right] \right\rangle .$$

Kovtun–Son–Starinets (KSS) result

Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601:

$$\boxed{\frac{\textbf{Shear viscosity}}{\textbf{Entropy density}} = \frac{\eta}{\mathbf{s}} = \frac{1}{4\,\pi}\left[1 + \frac{135\,\zeta(3)}{8(2g^2 N_c)^{3/2}} + \cdots\right]}.$$

$$\boxed{\lim_{g^2 N_c \gg 1} \frac{\eta}{s} = \frac{1}{4\,\pi} \simeq 6.08 \times 10^{-13}\ \mathrm{K\,s}}$$

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- Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601:

  "*For any isotropic* **holographic model** *with an effective* **gravitational action** *with* **at most two derivatives**, *the shear viscosity satisfies* $\eta/s \gtrsim 1/4\pi$".

Fluid response + dissipation

- Energy-momentum tensor: $0^{\rm th}$-order = perfect fluid:

$$\left\langle T^{\mu\nu}_{(0)} \right\rangle = (\epsilon + P)\, u^\mu\, u^\nu + p\, g^{\mu\nu}\,.$$

• Son, Starinets, JHEP **0603** (2006) 052.

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• Son, Starinets, JHEP **0603** (2006) 052.

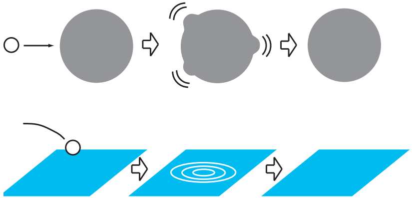
- $\Rightarrow$ $1^{\rm st}$-order (dissipation):

$$\left\langle T^{\mu\nu}_{(1)} \right\rangle = -P^{\mu\alpha} P^{\nu\beta} \left[\eta \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \nabla_\lambda u^\lambda\right) + \zeta\, g_{\alpha\beta} \nabla_\lambda u^\lambda\right]\;,$$

η: Shear viscosity,
ζ: Bulk viscosity,

$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$: projection.

Measuring the viscosity



Perturb the system by gravitational waves
(Natsuume, Lect. Notes Phys. **903** (2015))

$$g^{(0)}_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & h_{xy}(t) & 0 \\ 0 & h_{xy}(t) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

= perturbation on the boundary metric.

Shear viscosity

- **Viscous fluids**:

$$\langle T^{\mu\nu}\rangle = \overbrace{(\varepsilon + p)u^{\mu}u^{\nu} + pg^{\mu\nu}}^{\left\langle T^{\mu\nu}_{(0)}\right\rangle} + \left\langle T^{\mu\nu}_{(1)}\right\rangle .$$

$$\left\langle T^{\mu\nu}_{(1)}\right\rangle = -P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\nabla_{\alpha}u_{\beta} + \nabla_{\beta}u_{\alpha} - \frac{2}{3}\,g_{\alpha\beta}\,\nabla_{\lambda}u^{\lambda}\right) + \zeta\,g_{\alpha\beta}\nabla_{\lambda}u^{\lambda}\right] ,$$

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- $1^{\rm st}$ order in $h_{\mu\nu}$:

$$\delta\left\langle T^{\mu\nu}_{(1)}\right\rangle \sim -\eta(\nabla_x u_y + \nabla_y u_x) .$$

Shear viscosity

- Therefore

$$\delta \left\langle T^{\mu\nu}_{(1)} \right\rangle = -2\eta\Gamma^0_{xy} = -\eta\partial_0 h_{xy} \ .$$

- Fourier transform

$$\boxed{\delta \left\langle T^{\mu\nu}_{(1)}(\omega, \vec{q} = 0) \right\rangle = i\omega\eta h_{xy}.}$$

Shear viscosity

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$$\delta \left\langle T^{\mu\nu}_{(1)} \right\rangle = -2\eta\Gamma^0_{xy} = -\eta\partial_0 h_{xy} \ .$$

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$$\boxed{\delta \left\langle T^{\mu\nu}_{(1)}(\omega, \vec{q} = 0) \right\rangle = i\omega\eta h_{xy}.}$$

- Comparing to

$$\delta \left\langle T^{\mu\nu}_{(1)} \right\rangle = -G_R^{xy,xy} h_{xy},$$

one obtains the **Kubo formula** for the **shear viscosity**:

$$\eta = -\lim_{\omega\to 0} \frac{1}{\omega} \Im\, G_R^{xy,xy}(\omega, \vec{q} = 0) \ .$$

- **Shear viscosity**: **Kubo formula**

$$\eta = -\lim_{\substack{\omega\to 0\\ q\to 0}} \frac{\Im\, G_R^{xy,xy}(\omega,\vec{q})}{\omega}$$

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$$\eta = -\lim_{\substack{\omega\to 0\\ q\to 0}} \frac{\Im\, G_R^{xy,xy}(\omega,\vec{q})}{\omega}$$

- **Bulk viscosity**: **Kubo formula**

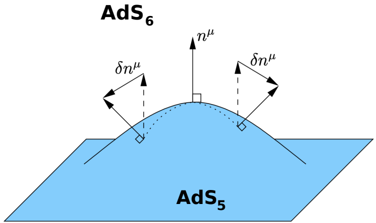
$$\zeta = \lim_{\substack{\omega\to 0\\ q\to 0}} \frac{1}{\omega} \Im G_R^{PP}(\omega,\vec{q})$$

where

$$G_R^{PP}(\omega,\vec{q}) \quad = \quad \frac{k_i k_j k_m k_n}{k^4}\left[G_R^{ij,mn}(\omega,\vec{q}) + \frac{1}{3}\delta_{ab}T^{ab}\left(\delta^{im}\delta^{jn} + \delta^{in}\delta^{jm} - \delta^{ij}\delta^{mn}\right)\right] + \frac{1}{3}\delta_{ij}T^{ij} - \frac{4}{3}G_R^{xy,xy}(\omega,\vec{q}).$$

is the response to longitudinal fluctuations.

(M. Natsuume, Lect. Notes Phys. **903** (2015)).



- **AdS$_5$–Schwarzschild black brane**

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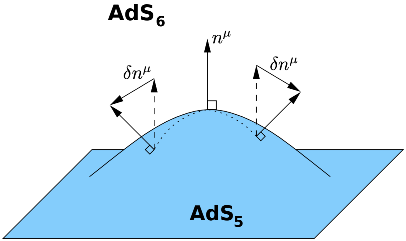
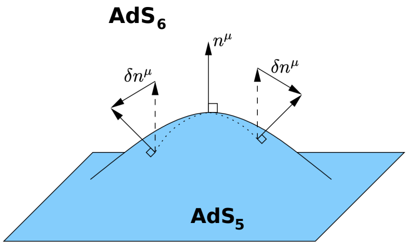


- **Deforming the AdS$_5$-Schwarzschild black brane by embedding**.



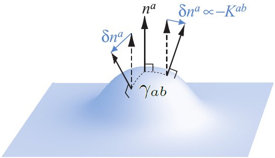
- **Deforming the AdS$_5$-Schwarzschild black brane by embedding**.
- $\gamma_{\mu\nu}$ = AdS$_6$ metric;
- $g_{\mu\nu}$ = AdS$_5$ **metric** induced by $\gamma_{\mu\nu}$:

$$g_{\mu\nu} = \gamma_{\mu\nu} + n_\mu n_\nu .$$

Embedding

- Extrinsic curvature:

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_n g_{\mu\nu} = -g_\mu{}^\rho g_\nu{}^\sigma \nabla_\rho n_\sigma .$$



- **Weyl tensor**:

$$C_{\mu\nu\sigma\rho} = R_{\mu\nu\sigma\rho} - \frac{1}{4}(g_{[\mu\sigma}R_{\nu]\rho} + g_{[\nu\rho}R_{\mu]\sigma}) + \frac{1}{20}R\,g_{\mu[\sigma}g_{\nu\rho]},$$

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- **Weyl tensor electric component** (ADM):

$$E_{\mu\nu} = C_{\mu\nu\sigma\rho}n^\sigma n^\rho = -\frac{\Lambda}{30}\gamma_{\mu\nu} - \partial_z K_{\mu\nu} + K_\mu{}^\rho K_{\rho\nu}.$$

Maeda, Sasaki, Shiromizu, Phys. Rev. D **62** (2000) 024012.

- **Weyl tensor**:

$$C_{\mu\nu\sigma\rho} = R_{\mu\nu\sigma\rho} - \frac{1}{4}(g_{[\mu\sigma}R_{\nu]\rho} + g_{[\nu\rho}R_{\mu]\sigma}) + \frac{1}{20}R\, g_{\mu[\sigma}g_{\nu\rho]},$$

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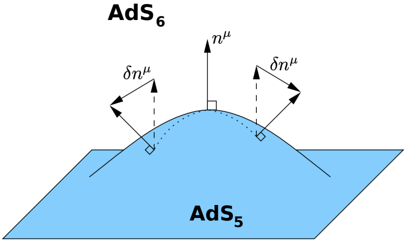
Maeda, Sasaki, Shiromizu, Phys. Rev. D **62** (2000) 024012.

- **Weyl tensor**: part of the curvature that **is not locally** determined by matter:

Einstein's equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda_5 g_{\mu\nu} = T_{\mu\nu} + E_{\mu\nu}}$$

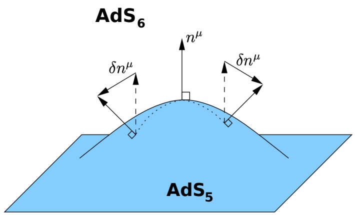


- **Gauss equation**:

$$ {}^{(6)}R^{\mu}{}_{\nu\rho\sigma} = {}^{(5)}R^{\mu}{}_{\nu\rho\sigma} - K^{\mu}{}_{\rho}K_{\nu\sigma} + K^{\mu}{}_{\sigma}K_{\nu\rho}. $$

- Contracting with the induced metric $g_{\mu\nu}$ of AdS$_5$ and using Einstein's equations: **Hamiltonian constraint**.

$$ \boxed{\mathcal{H} \equiv {}^{(5)}R + K^2 - K_{\mu\nu}K^{\mu\nu} - 16\pi n^\mu n^\nu T_{\mu\nu} = 0} $$



- **Codazzi equations**:

$$^{(6)}R_{\mu\nu\rho\sigma}n^{\sigma} = D_{\nu}K_{\mu\rho} - D_{\mu}K_{\nu\rho}$$

- Contracting with the induced metric $g_{\mu\nu}$ of AdS$_5$: **momentum constraint**.

$$\boxed{\mathcal{M}^{\mu} \equiv D_{\nu}K^{\nu\mu} - D^{\mu}K - 8\pi g^{\mu\rho}n^{\sigma}T_{\rho\sigma} = 0}$$

Deformed black branes

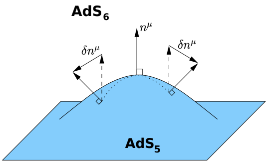
- Coordinate change $\boxed{u = r_0/r}$, AdS_5–Schwarzschild black brane:

$$ds^2 \quad = \quad -\frac{r_0^2}{u^2}\left(1-u^4\right)\mathrm{d}t^2 + \frac{1}{u^2\left(1-u^4\right)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j.$$

- RdR, Phys. Rev. D **105** (2022) 026014
- Martins, Meert, RdR, Nucl. Phys. B **957** (2020) 115087
- Casadio, Cavalcanti, RdR, Eur. Phys. J .C **76** (2016) 556

- **Deformed black branes**:

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$



Deformed black branes

- **AdS$_5$ deformed black branes**

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

$\Rightarrow$ **Hamiltonian constraint** + **momentum constraint**:

$$2\frac{N''}{N} - \frac{N'^2}{N^2} + 2\frac{A''}{A} + \frac{A'^2}{A^2} - \frac{N'A'}{NA} + \frac{4}{r}\left(\frac{N'}{N} - \frac{A'}{A}\right) - 4\frac{A}{r^2} - k(r, r_0, \alpha) = 0,$$

for $\alpha \in \mathbb{R}$.

- Martins, Meert, RdR, Nucl. Phys. B **957** (2020) 115087

Deformed black brane metric:

$$ds^2 = -\frac{r_0^2}{u^2} N(u) \mathrm{d}t^2 + \frac{1}{u^2 A(u)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j,$$

for

$$N(u) = 1 - u^4 + (\alpha - 1) u^6,$$
$$A(u) = \left(1 - u^4\right) \left(\frac{2 - 3u^4}{2 - (4\alpha - 1) u^4} \right),$$

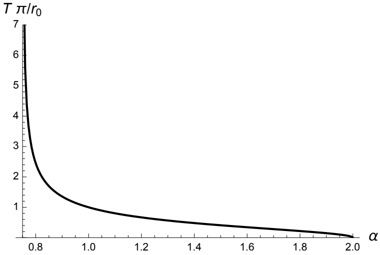
- $\alpha \to 1$ limit: AdS_5–Schwarzschild black brane:

$$ds^2 = -\frac{r_0^2}{u^2} \left(1 - u^4\right) \mathrm{d}t^2 + \frac{1}{u^2 \left(1 - u^4\right)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j.$$

New black brane solutions

- **Hawking temperature** at the horizon:

$$T = \frac{1}{4\pi}\sqrt{\lim_{u\to 1}\frac{g'_{tt}(u)}{g'_{rr}(u)}} = \frac{r_0}{\pi}\sqrt{\frac{\alpha - 2}{3 - 4\alpha}}.$$



Deformed black brane temperature $\times$ α.

$\boxed{T > 0 \Rightarrow \alpha \in (0.75, 2)}$.

New black brane solutions

- Expand the action (near-horizon)

$$S_E = -\frac{1}{16\pi G}\overbrace{\int d^5x\sqrt{g}\,(R-2\Lambda_5)}^{I_{\rm bulk}} - \frac{1}{8\pi G}\overbrace{\lim_{u\to 0}\int d^4x\sqrt{h}K}^{I_{\rm Gibbons–Hawking}} + I_{\rm c.t}\,,$$

- Martins, Meert, RdR, Nucl. Phys. B **957** (2020) 115087
- Casadio, Cavalcanti, RdR, Eur. Phys. J .C **76** (2016) 556

$$S_E = \frac{Vbr_0^4}{8\pi G}\left(\frac{11 - 15\alpha + 3\alpha^2}{2}\right)$$

is the partition function in the dual field theory on the AdS_5 boundary (GKPW)

- Gubser, Klebanov, Polyakov, Phys. Lett. B **428** (1998) 105.
- Witten, Adv. Theor. Math. Phys. **2** (1998) 253.

- $S_E = bF$, where F = free energy.

New black brane solutions: thermodynamics

- **Free energy**:

$$F = \frac{\pi^3 V}{8G}\left(\frac{11-15\alpha+3\alpha^2}{2}\right)\left(\frac{3-4\alpha}{\alpha-2}\right)^2 T^4,$$

- Entropy density:

$$s = -\frac{1}{V}\frac{\partial F}{\partial T} = -\frac{\pi^3}{2G}\left(\frac{11-15\alpha+3\alpha^2}{2}\right)\left(\frac{3-4\alpha}{\alpha-2}\right)^2 T^3, .$$

- Pressure:

$$P = -\frac{\partial F}{\partial V} = -\frac{\pi^3}{8G}\left(\frac{11-15\alpha+3\alpha^2}{2}\right)\left(\frac{3-4\alpha}{\alpha-2}\right)^2 T^4 ,$$

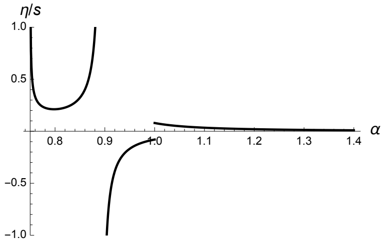
- Energy density:

$$\varepsilon = \frac{F}{V} - Ts = \frac{5\pi^3}{8G}\left(\frac{11-15\alpha+3\alpha^2}{2}\right)\left(\frac{3-4\alpha}{\alpha-2}\right)^2 T^4$$

New black brane solutions

- **Shear viscosity-to-entropy density**:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(\frac{16}{176 - 180\alpha + 27\alpha^2}\right)\sqrt{\frac{5}{12(\alpha - 1)} - \frac{1}{4}}$$



$\frac{\eta}{s}$ as a function of α.

New black brane solutions

- Deformed black brane metric:

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

for

$$N(u) = 1 - u^4 + (\alpha - 1)\,u^6,$$
$$A(u) = \left(1 - u^4\right)\left(\frac{2 - 3u^4}{2 - (4\alpha - 1)\,u^4}\right),$$

- KSS result is reobtained when $\alpha \to 1$:

$$\boxed{\lim_{\alpha \to 1}\frac{\eta}{s} = \frac{1}{4\pi}}$$

- Kuntz, RdR, Nucl. Phys. B **993** (2023) 116258
- RdR, Phys. Rev. D **105** (2022) 026014
- Martins, Meert, RdR, Nucl. Phys. B **957** (2020) 115087 .

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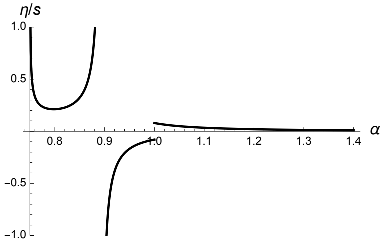
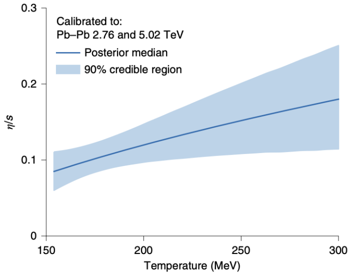
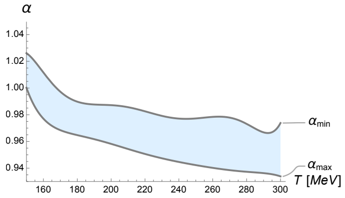


$\frac{\eta}{s}$ as a function of α.

QGP and η/s: Duke group

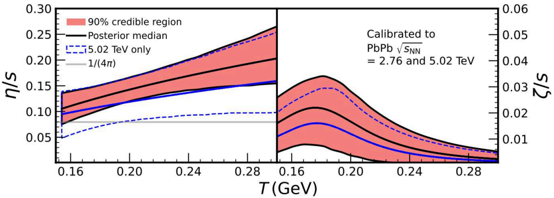


Duke group (Bernhard, Moreland, Bass, Nature Phys. **15** (2019) 1113).

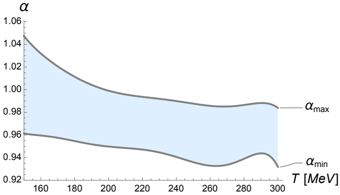


QGP: Duke group (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and η/s: Jyväskylä-Helsinki-Munich

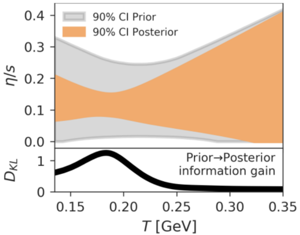


QGP: Jyväskylä-Helsinki-Munich group (Parkkila, Onnerstad, Taghavi, Mordasini, Bilandzic, Virta, Kim, Phys. Lett. B **835** (2022) 137485) .

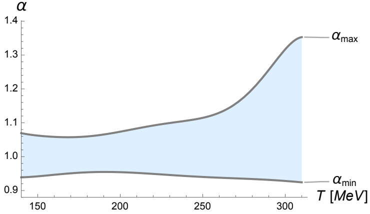


QGP at LHC: Jyväskylä-Helsinki-Munich group (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and η/s: JETSCAPE Bayesian model

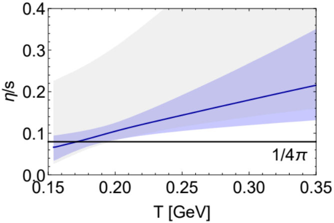


JETSCAPE Bayesian model (Everett *et al.* [JETSCAPE], Phys. Rev. Lett. **126** (2021) 242301).

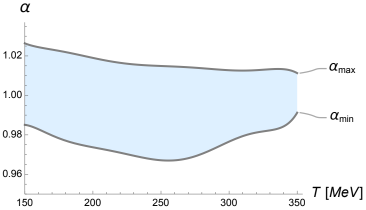


RHIC + LHC; JETSCAPE Bayesian model (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and η/s: MIT-Utrecht-Genève



MIT-Utrecht-Genève (Nijs, van der Schee, Gürsoy, Snellings, Phys. Rev. Lett. **126**, 202301 (2021).



MIT-Utrecht-Genève (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

New black brane solutions

- Deformed black brane metric in AdS_5, from embedding protocol:

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

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QGP and η/s $\Rightarrow$ **black brane deformation parameter**: $\boxed{1 \lesssim \alpha \lesssim 1.05}$

- Remember that the limit $\alpha \to 1$ implies the **AdS_5–Schwarzschild black brane**:

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- Consistent with Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601:

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- **Without embeddings**, we must consider **actions beyond the Einstein–Hilbert one**:

$$S = \int d^5x \sqrt{-g}\,[R - 2\Lambda_5]\,.$$

Consistency with 2-loop quantum corrections to gravity

- $2^{\rm nd}$ construction: **General relativity** + **Lee–Wick** + **Ricci cubic gravity** + **Einstein cubic gravity** + Gibbons–Hawking (GB) + counterterm (c.t).

$$
\begin{aligned}
S \;=\; & \int d^5x\sqrt{-g}\,\Big[R-2\Lambda_5 \\
& +\alpha_1 G_{\mu\nu}\Box R^{\mu\nu} \\
& +\alpha_2\Big(-\frac{65}{324}R^3+\frac{29}{27}RR_{\mu\nu}R^{\mu\nu}-\frac{59}{81}R^{\mu}_{\nu}R^{\nu}_{\rho}R^{\rho}_{\mu}+14R_{\mu\nu}^{\ \ \rho\sigma}R_{\rho\sigma}^{\ \ \alpha\beta}R_{\alpha\beta}^{\ \ \rho\sigma} \\
& \quad -4R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}_{\ \ \ \ \alpha}R^{\sigma\alpha}-\frac{7}{108}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R+4R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}\Big) \\
& +\alpha_3\Big(\nabla_\mu R_{\rho\sigma}\nabla^\mu R^{\rho\sigma}+\nabla_\mu R_{\rho\sigma}\nabla^\sigma R^{\mu\rho}+\nabla_\mu R\nabla^\mu R+\nabla_\mu R_{\rho\sigma\tau\xi}\nabla^\mu R^{\rho\sigma\tau\xi} \\
& \quad -R^{\mu\nu}\Box R_{\mu\nu}+\frac{3}{8}R_{\mu\nu}\nabla^\mu\nabla^\nu R+\frac{7}{18}R\nabla^\mu\nabla^\nu R_{\mu\nu}\Big)\Big] \\
& +\overbrace{\lim_{u\to 0}\int d^4x\sqrt{g}K}^{S_{GH}}+S_{\rm c.t},
\end{aligned}
$$

(RdR, Eur. Phys. J. Plus **139** (2024) 1006)

Exact solution:

$$ds^2 = -\frac{r_0^2}{u^2} N(u) \mathrm{d}t^2 + \frac{1}{u^2 A(u)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j,$$

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where α is some polynomial function of α_1, α_2, and α_3.

- Consistent with **2-loop quantum corrections to dual gravity**:

• Goroff, Sagnotti, Nucl. Phys. B **266** (1986) 709.

Quantum corrections to η/s

- **Shear viscosity**:

$$\boxed{\eta = -\lim_{\substack{\omega\to 0\\ q\to 0}} \frac{\Im\, G_R^{xy,xy}(\omega,\vec{q})}{\omega}}$$

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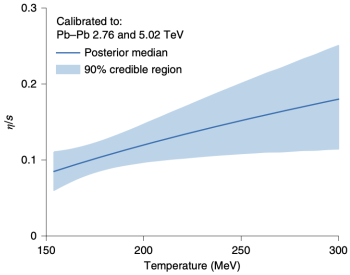
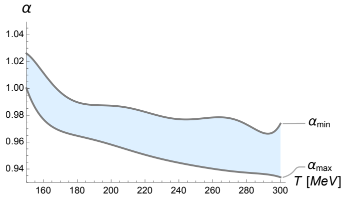
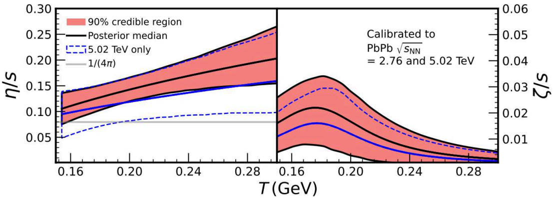


Duke group (Bernhard, Moreland, Bass, Nature Phys. **15** (2019) 1113).



QGP: Duke group (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and η/s: Jyväskylä-Helsinki-Munich



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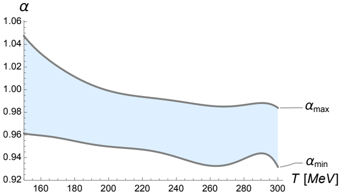
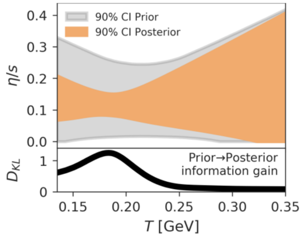
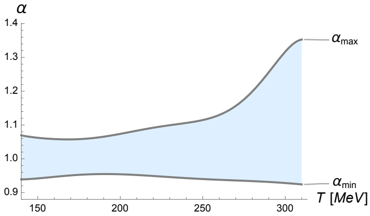


QGP at LHC: Jyväskylä-Helsinki-Munich group (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and η/s: JETSCAPE Bayesian model



JETSCAPE Bayesian model (Everett *et al.* [JETSCAPE], Phys. Rev. Lett. **126** (2021) 242301).



RHIC + LHC; JETSCAPE Bayesian model (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

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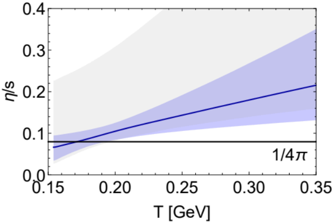

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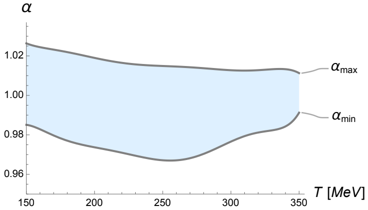

MIT-Utrecht-Genève (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

- **Shear viscosity**: **Kubo formula**

$$\eta = -\lim_{\substack{\omega \to 0 \\ q \to 0}} \frac{\Im\, G_R^{xy,xy}(\omega, \vec{q})}{\omega}$$

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- **Bulk viscosity**: **Kubo formula**

$$\zeta = \lim_{\substack{\omega\to 0\\ q\to 0}} \frac{1}{\omega}\Im G_R^{PP}(\omega,\vec{q})$$

where

$$G_R^{PP}(\omega,\vec{q}) = \frac{k_i k_j k_m k_n}{k^4}\left[G_R^{ij,mn}(\omega,\vec{q}) + \frac{1}{3}\delta_{ab}T^{ab}\left(\delta^{im}\delta^{jn} + \delta^{in}\delta^{jm} - \delta^{ij}\delta^{mn}\right)\right] + \frac{1}{3}\delta_{ij}T^{ij} - \frac{4}{3}G_R^{xy,xy}(\omega,\vec{q}).$$

is the response to longitudinal fluctuations.

(M. Natsuume, Lect. Notes Phys. **903** (2015)).

Deformed black branes

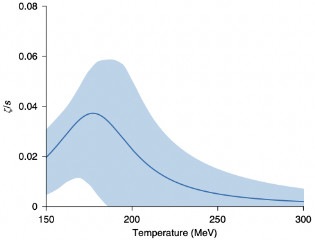
- **Bulk viscosity-to-entropy density ratio**:

$$\frac{\zeta}{s} = \alpha^4(12\alpha^2 - 2\alpha + 7)\Pi\left(\frac{(12\alpha^2 - \alpha^3 + 9)}{(6-5\alpha)^2}; \tanh^{-1}(\alpha^2 - 3)\middle|\alpha^2 - 1\right)$$
$$+(12\alpha^2 - 2\alpha + 7)F\left(\tanh^{-1}\left((\alpha^2 - 3)\right), \frac{14\alpha^2 - 6\alpha + 9}{(\alpha+1)^2}\right),$$

where Π and F are incomplete elliptic integrals.

- Kuntz, RdR, Nucl. Phys. B **993** (2023) 116258
- RdR, Phys. Rev. D **105** (2022) 026014
- Martins, Meert, RdR, Nucl. Phys. B **957** (2020) 115087.

QGP and experiments: Duke group



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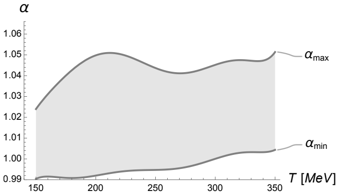

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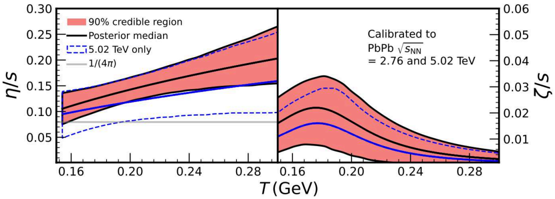
QGP and experiments: Jyväskylä-Helsinki-Munich



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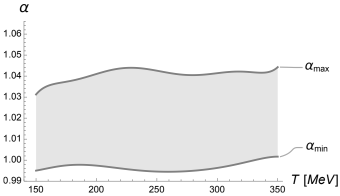
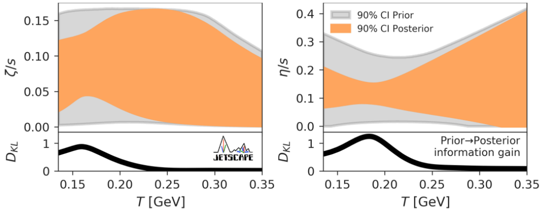


QGP at LHC: Jyväskylä-Helsinki-Munich group (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

QGP and experiments: JETSCAPE Bayesian model



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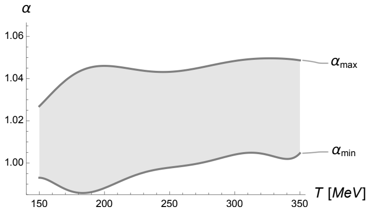

RHIC + LHC; JETSCAPE Bayesian model (RdR, Eur. Phys. J. Plus **139** (2024) 1006).

- AdS/QCD

  Baier, Romatschke, Son, Starinets, Stephanov, JHEP **0804** (2008) 100.

Applications from QCD

- AdS/QCD

  Baier, Romatschke, Son, Starinets, Stephanov, JHEP **0804** (2008) 100.

- RdR, Phys. Rev. D **105** (2022) 026014.

Deformed black branes in Poincaré-like coordinates:

$$ds^2 = \frac{R^2 e^{cz^2/2}}{z^2}\left(-N(z)\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \frac{1}{A(z)}\mathrm{d}z^2\right),$$

where

$$\begin{aligned} N(z) &= 1 - \frac{z^4}{z_0^4} + (\alpha - 1)\,\frac{z^6}{z_0^6}, \\ A(z) &= \left(1 - \frac{z^4}{z_0^4}\right)\left(\frac{2 - \frac{3z^4}{z_0^4}}{2 - (4\alpha - 1)\,\frac{z^4}{z_0^4}}\right). \end{aligned}$$

with event horizon z_0.

Applications to QCD

- RdR, Phys. Rev. D **105** (2022) 026014 [arXiv:2111.01244 [hep-th]]:

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Hagedorn temperature = **cross-over temperature from the hadronic to the deconfined QGP phase**:

$$T_{\rm HAG} = \frac{1}{\pi}\sqrt{\frac{c(\alpha-2)}{2(3-4\alpha)}}.$$

Data from HotQCD Collaboration:

$T_{\bf HAG} = 156.5 \pm 1.5$ **MeV** [Bazavov *et al.*, Phys. Lett. B **795** (2019) 15] $\Rightarrow \alpha = 1.025$
$T_{\bf HAG} = 158.0 \pm 0.6$ **MeV** [Borsanyi *et al.*, Phys. Rev. Lett. **125** (2020) 052001] $\Rightarrow \alpha = 1.021$.

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$$T_{\rm HAG} = \frac{1}{\pi}\sqrt{\frac{c(\alpha-2)}{2(3-4\alpha)}}.$$

Data from HotQCD Collaboration:

$T_{\rm HAG} = 156.5 \pm 1.5$ **MeV** [Bazavov *et al.*, Phys. Lett. B **795** (2019) 15] $\Rightarrow \alpha = 1.025$
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- **Conclusion 1**: the usual AdS_5-Schwarzschild black brane ($\alpha \to 1$) may not be the most suitable one for describing $T_{\rm HAG}$ of the QGP.

Applications to QCD

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- **Conclusion 1**: the usual AdS_5-Schwarzschild black brane ($\alpha \to 1$) may not be the most suitable one for describing $T_{\rm HAG}$ of the QGP.
- **Conclusion 2**: quantum corrections to dual gravity may set in.

- **Conclusion 3**:

  Experimental data (η/s and ζ/s) from the QGP constrain quantum corrections to the AdS_5-Schwarzschild black brane.

Conclusions

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  Experimental data (η/s and ζ/s) from the QGP constrain quantum corrections to the AdS_5-Schwarzschild black brane.

- **Conclusion 4**:

  The **AdS_5-Schwarzschild black brane is robust in AdS/CFT and AdS/QCD**, **but mild quantum corrections deform them, up to $\sim 2.5\%$, to match results from QCD and QGP**.

The end

Thanks!

Universidade Federal do ABC

CNPq
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FAPESP

The Galileo Galilei Institute For Theoretical Physics
Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare
Arcetri, Firenze

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- For $z \to 0$,

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4D Minkowski = boundary of **$\mathbf{AdS}_5$**.

$$
\ldots k(r, r_0, \alpha) = -\frac{1}{r^{10}}\Bigg\{ -\left(10(\alpha-1)+r^6-3r^2r_0^4\right)\left(\alpha+r^6-r^2r_0^4-1\right)+\frac{4r^8\left(-2\alpha+r^6+r^2r_0^4+2\right)^2}{\left(\alpha+r^6-r^2r_0^4-1\right)^2}
$$

$$
+\frac{4r^8\left(4r^{12}+8(2-3\alpha)r^8r_0^4+(20\alpha-23)r^4r_0^8+3(4\alpha-1)r_0^{12}\right)^2}{\left(2r^8-5r^4r_0^4+3r_0^8\right)^2\left(2r^4+(1-4\alpha)r_0^4\right)^2}
$$

$$
-\frac{2r^8\left(8r^{16}-60r^{12}r_0^4+6(40\alpha(2\alpha-3)+67)r^8r_0^8+(4\alpha-1)(20\alpha+43)r^4r_0^{12}-9(1-4\alpha)^2r_0^{16}\right.}{\left(2r^8-5r^4r_0^4+3r_0^8\right)\left(2r^4+(1-4\alpha)r_0^4\right)^2}
$$

$$
+\frac{1}{2r^4+(1-4\alpha)r_0^4}[r^2\left(2r^8+2r^6-5r^4r_0^4+(1-4\alpha)r^2r_0^4+3r_0^8\right)\left(\alpha+r^6-r^4-r^2r_0^4-1\right)]
$$

$$
+\frac{4r^8\left(r^6+r^2r_0^4+2-2\alpha\right)\left(4r^{12}+8(2-3\alpha)r^8r_0^4+3(4\alpha-1)r_0^{12}\right)}{\left(2r^4-3r_0^4\right)\left(r^4-r_0^4\right)\left(2r^4+(1-4\alpha)r_0^4\right)\left(\alpha+r^6-r^2r_0^4-1\right)}
$$

$$
+2r^8\left(\frac{2r^8+5r^4r_0^4-9r_0^8}{2r^8-5r^4r_0^4+3r_0^8}-\frac{4r^4}{2r^4+(1-4\alpha)r_0^4}+\frac{r^2\left(3r^4-r_0^4\right)}{\alpha+r^6-r^2r_0^4-1}\right)\Bigg\}
$$

Lee–Wick + cubic gravity: renormalizability

- Lee–Wick: renormalizability and finite in $D = 5$
  - L. Modesto, Nuc. Phys. B **909** (2016) 584.
  - I. L. Shapiro, Phys. Lett. B **744** (2015) 67 [arXiv:1502.00106 [hep-th]].

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- Here $g_{tt} = -g_{rr}$ and it is (seems to be) ghost-free em 5D:

  - Y. Z. Li, H. Lu and J. B. Wu, *Causality and a-theorem Constraints on Ricci Polynomial and Riemann Cubic Gravities*, Phys. Rev. D **97** (2018) 024023 [arXiv:1711.03650 [hep-th]].

Should we worry about extra dimensions?

- $\mathrm{AdS}_5 \times S^5$ metric:

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx^i dx_i\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2. \tag{1}$$

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- Since

$$E = i\hbar\frac{\partial}{\partial t},$$

then (1) implies

$$\frac{\partial}{\partial t} \mapsto \frac{R}{r}\frac{\partial}{\partial t} \Rightarrow \boxed{E \mapsto \frac{r}{R}E}$$

(Page 325, H. Nastase, *String Theory Methods for Condensed Matter Physics*, Cambridge, 2017).

- Additional dimension in AdS_5 = 4D energy scale. (AdS/QCD)

Violation of the KSS limit

- R. Cai, Z. Nie, N. Ohta, Y. W. Sun, Phys. Rev. D **79** (2009) 066004.
  M. Brigante, H. Liu, R. Myers, S. Shenker, Phys. Rev. D **77** (2008) 126006.
- Gauss–Bonnet + dilaton

$$S = -\frac{1}{16\pi G}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 - 2\Lambda_5\right).$$

- Violation:

$$\frac{\eta}{s} = \frac{16}{25}\frac{1}{4\pi} < \frac{1}{4\pi}.$$

g(*tt*) and g(*rr*)

- T. Jacobson, *When is g(tt) g(rr) = -1*," Class. Quant. Grav. **24** (2007) 5717 [arXiv:0707.3222 [gr-qc]].
  M. Salgado, *A Simple theorem to generate exact black hole solutions*, Class. Quant. Grav. **20** (2003) 4551 [arXiv:gr-qc/0304010 [gr-qc]].
- Metrics with $g_{tt}g_{rr} = -1$ have Ricci tensors (and stress-energy tensor) with vanishing radial null-null components (or, equivalently, if the restriction of $R_{\mu\nu}|_{t\,-\,r\ \text{subspace}} \propto g_{\mu\nu}$ (which implies that the radial pressure is equal to minus the energy density).
- $g_{tt}g_{rr} \neq -1$ the Morris–Thorne traversable wormhole, the Damour–Solodukhin wormhole, the Joshi-Malafarina-Narayan singularity, the naked singularity surrounded by a thin shell of matter, the BH in Clifton-Barrow f(R) gravity, the Sen BH, the Einstein–Maxwell–dilaton-1 BH, the BH in Loop Quantum Gravity, the DST BH, the BH in bumblebee gravity, and the Casimir wormhole (footnote 8, S. Vagnozzi, R. Roy, Y. D. Tsai, L. Visinelli, M. Afrin, A. Allahyari, P. Bambhaniya, D. Dey, S. G. Ghosh and P. S. Joshi, *et al.* Class. Quant. Grav. **40** (2023) 165007 [arXiv:2205.07787 [gr-qc]].)

Consistent with 2-loop quantum corrections to gravity

- Deformed black branes:

$$
\begin{aligned}
ds^2 &= -\frac{r_0^2}{u^2} N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \\
N(u) &= 1 - u^4 + (\alpha - 1)\, u^6, \\
A(u) &= \left(1 - u^4\right)\left(\frac{2 - 3u^4}{2 - (4\alpha - 1)\, u^4}\right),
\end{aligned}
$$

- **Gibbons–Hawking** term:

$$
\begin{aligned}
&-\frac{4}{u^4\left((1-4\alpha)u^4+2\right)^2}\sqrt{-\frac{\left(3u^8-5u^4+2\right)\left((\alpha-1)u^6-u^4+1\right)}{(4\alpha-1)u^4-2}} \\
&\times u^4\Big[-32\alpha + u^2\Big(-4\alpha + u^2\Big(56\alpha + 9(\alpha-1)(4\alpha-1)u^{10} + (6-24\alpha)u^8 \\
&-5\left(4\alpha^2+\alpha-5\right)u^6 + 24u^4 - 8(\alpha-4)(\alpha-1)u^2 - 46\Big) + 4\Big) + 8\Big].
\end{aligned}
$$

- **Counterterm**: $\sim u^{-4}\sqrt{N(u)A(u)}$.

Consider a bulk perturbation h_{xy} such that:

$$ds^2 = ds^2_{\mathrm{AdS}_5-SD} + 2h_{xy}\mathrm{d}x\mathrm{d}y \;, \tag{48}$$

where $ds^2_{\mathrm{AdS}_5-SD}$ denotes the AdS_5–Schwarzschild deformed black brane metric, Eq. (23). In

Recall Eq. (5), for $h_{xy}^{(0)}$ being the perturbation added to the boundary theory, which is asymptotically related to h_{xy}, the bulk perturbation, by[2]

$$g^{xx}h_{xy} \sim h_{xy}^{(0)}\left(1 + h_{xy}^{(1)}u^4\right) , \tag{49}$$

according to Eq. (12). Notice that one can directly use the results for a massless scalar field, as $g^{xx}h_{xy}$ obeys the EOM for a massless scalar field [26,33]. Besides, the deformed AdS_5–Schwarzschild black brane has the same asymptotic behavior of the AdS_5–Schwarzschild black brane (namely, Eq. (10)). One can identify $g^{xx}h_{xy}$ as the bulk field, φ, which plays the role of an external source of a boundary operator, in this case τ^{xy}. Therefore, one can directly obtain the response $\delta\langle\tau^{xy}\rangle$, from Eq. (13),

$$\delta\left\langle\tau^{xy}\right\rangle = \frac{r_0^4}{16\pi G}4h_{xy}^{(1)}h_{xy}^{(0)} , \tag{50}$$

where it is now convenient to reintroduce the $1/16\pi G$ factor. Comparing Eqs. (5) and (50) yields

$$i\omega\eta = \frac{r_0^4}{4\pi G}h_{xy}^{(1)}. \tag{51}$$

Taking the ratio between Eq. (51) and the entropy (47) we find

$$\frac{\eta}{s} = -\frac{r_0}{\pi}\left[\left(\frac{1}{11-15\beta+3\beta^2}\right)\left(\frac{\beta-2}{3-4\beta}\right)^{1/2}\right]\frac{h_{xy}^{(1)}}{i\omega}, \tag{52}$$

where $h_{xy}^{(1)}$ is the solution of the EOM for the perturbation $g^{xx}h_{xy} \equiv \varphi$, which is that of a massless scalar field [26,33]

$$\nabla_M\left(\sqrt{-g}\,g^{MN}\nabla_N\varphi\right) = 0 . \tag{53}$$

Considering a stationary perturbation, given by the form $\varphi(u,t) = \phi(u)e^{-i\omega t}$, the perturbation equation reduces to a second-order ODE for $\phi(u)$,

$$\ddot{\phi} + \frac{1}{2}\left(\frac{\dot{N}A}{2N} + \frac{N\dot{A}}{A} - \frac{3}{u}\right)\dot{\phi} + \frac{1}{NA}\frac{\omega^2}{r_0^2}\phi = 0\,. \tag{54}$$

To derive the solution of Eq. (54), two boundary conditions are imposed: the incoming wave boundary condition in the near-horizon region, corresponding to $u \to 1$, and a Dirichlet boundary condition at the AdS boundary, $\phi(u \to 0) = \phi^{(0)}$, where $h_{xy}^{(0)} = \phi^{(0)}e^{-i\omega t}$.

The incoming wave boundary condition near the horizon is obtained by solving Eq. (54) in the limit $u \to 1$. After a straightforward computation one finds the following

$$\phi \propto \exp\left(\pm i\,\frac{\omega}{r_0}\sqrt{\frac{4\beta - 3}{\beta - 1}}\sqrt{1-u}\right). \tag{55}$$

This solution has a natural interpretation using tortoise coordinates, allowing one to identify it as a plane wave [27]. The positive exponent represents an outgoing wave, whereas the negative one describes the wave incoming to the horizon, which, according to the near-horizon boundary condition, allows us to fix

[2] We are now using the u coordinate, instead of r.