Weak reaction rates from holography

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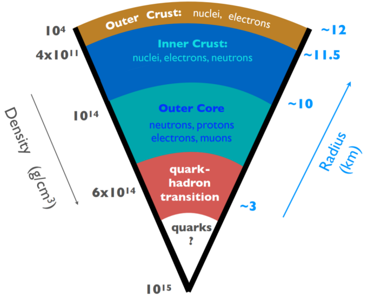
Motivations

Neutron stars are the remnants of old massive stars that undergo a supernova explosion and a subsequent gravitational collapse.

⇒ unique setting for exploring QCD in extreme conditions!



Hints of a quark matter core in massive neutron stars [Annala et al. 1903.09121]

Weak interactions: affect the properties of neutron stars and their binary mergers.

Why are weak reactions important?

The system is in chemical equilibrium via weak processes

Perturbation: density oscillation $\Rightarrow$ drives system out of equilibrium.

Response: weak processes work to restore the balance and dampen the perturbation

$\Downarrow$

Effective bulk viscosity

Stability window of rotating stars, emission of gravitational waves...

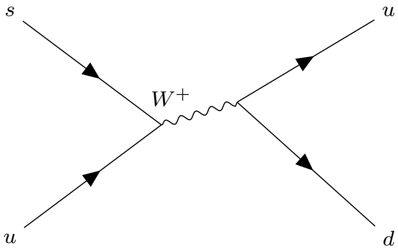
Goal

Holography $\Rightarrow$ weak reaction rates for strongly coupled cold quark matter.

Weak processes

The relevant weak processes, or reactions, are produced by a W-boson exchange or emission:

$$\boxed{u + d \rightleftharpoons u + s}\,, \quad u + \ell^- \to d, s + \nu_\ell\,, \quad d, s \to u + \ell^- + \bar{\nu}_\ell\,, \quad \ell_1^- \to \ell_2^- + \nu_{\ell_1} + \bar{\nu}_{\ell_2}\ .$$



beta-equilibrium condition: $\mu_u + \mu_\ell = \mu_d = \mu_s\,, \qquad \mu_\ell = \mu_e = \mu_\mu\ .$

Weak reaction rates

If a perturbation takes the chemical potentials out of their equilibrium values, the weak reactions will not be in balance, and the densities will change according to the weak rates Γ:

$$\frac{dn_u}{dt} = \sum_{\ell=e,\mu} \left(\Gamma_{d\to u\ell\bar{\nu}} - \Gamma_{u\ell\to d\nu} + \Gamma_{s\to u\ell\bar{\nu}} - \Gamma_{u\ell\to s\nu}\right) ,$$

$$\frac{dn_d}{dt} = \Gamma_{su\to ud} - \Gamma_{ud\to su} + \sum_{\ell=e,\mu} \left(\Gamma_{u\ell\to d\nu} - \Gamma_{d\to u\ell\bar{\nu}}\right) ,$$

$$\frac{dn_s}{dt} = \Gamma_{ud\to su} - \Gamma_{su\to ud} + \sum_{\ell=e,\mu} \left(\Gamma_{u\ell\to s\nu} - \Gamma_{s\to u\ell\bar{\nu}}\right) ,$$

$$\frac{dn_{\ell_1}}{dt} = \Gamma_{d\to u\ell_1\bar{\nu}} - \Gamma_{u\ell_1\to d\nu} + \Gamma_{s\to u\ell_1\bar{\nu}} - \Gamma_{u\ell_1\to s\nu} + \Gamma_{\ell_2\to\ell_1\nu_2\bar{\nu}_1} - \Gamma_{\ell_1\to\ell_2\nu_1\bar{\nu}_2} .$$

Small deviation from beta-equilibrium ($\delta\mu_{f^a}/\mu_B \ll 1$):

$$\Gamma_{ud\to su} - \Gamma_{su\to ud} \approx \lambda_{ds}(\delta\mu_s - \delta\mu_d),$$

$$\Gamma_{u\ell\to d\nu} - \Gamma_{d\to u\ell\bar{\nu}} \approx \lambda^{\ell}_{ud}(\delta\mu_d - \delta\mu_u - \delta\mu_\ell),$$

$$\Gamma_{u\ell\to s\nu} - \Gamma_{s\to u\ell\bar{\nu}} \approx \lambda^{\ell}_{us}(\delta\mu_s - \delta\mu_u - \delta\mu_\ell),$$

$$\Gamma_{e\to\mu\nu_2\bar{\nu}_1} - \Gamma_{\mu\to e\nu_1\bar{\nu}_2} \approx \lambda_{e\mu}(\delta\mu_\mu - \delta\mu_e).$$

Fermi's interaction

QCD preserves flavor symmetry.

Non-Abelian flavor symmetry: $SU(3)_L \times SU(3)_R$ for quarks and $SU(4)_L \times SU(4)_R$ for leptons

Conserved current $J^{\mu}_{f\,\chi}$ with $f = l, q$ and $\chi = L, R$.

Current transformation: $$\delta_{\theta_{f\,\chi}} J^{\mu}_{f\,\chi} = i\theta^{A}_{f\,\chi}[J^{\mu}_{f\,\chi}, T^{A}_{f\,\chi}]$$

Weak interactions can break flavor symmetry!

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}\,G_F\,\left(J^{\mu}_{\text{ch.}}\right)^{\dagger}\,J_{\text{ch.}\,\mu}$$

$$J^{\mu}_{\text{ch.}} = \overline{\nu}_{e\,L}\gamma^{\mu}e_L + \overline{\nu}_{\mu\,L}\gamma^{\mu}\mu_L + \cos\theta_C\,\overline{u}_L\gamma^{\mu}d_L + \sin\theta_C\,\overline{u}_L\gamma^{\mu}s_L,$$

$\mathcal{L}_{\text{Fermi}} \Rightarrow$ non-conservation of flavor currents $J^{\mu}_{f\,\chi}$

Flavor Ward identities

$$\partial_\mu \langle (J_q^\mu)^u{}_u \rangle = i\sqrt{2} G_F \eta_{\mu\nu} \sum_{\ell=e,\mu} \Big[\cos\theta_C \langle (J_{q\,L}^\mu)^u{}_d (J_{q\,L}^\nu)^d{}_u - (J_{q\,L}^\mu)^d{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle$$
$$+ \sin\theta_C \langle (J_{q\,L}^\mu)^u{}_s (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^s{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle \Big],$$

$$\partial_\mu \langle (J_q^\mu)^d{}_d \rangle = i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \sin\theta_C \eta_{\mu\nu} \langle (J_{q\,L}^\mu)^u{}_s (J_{q\,L}^\nu)^d{}_u - (J_{q\,L}^\mu)^s{}_u (J_{q\,L}^\nu)^u{}_d \rangle$$
$$- \cos\theta_C \sum_{\ell=e,\mu} \langle (J_{q\,L}^\mu)^u{}_d (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^d{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle \Big],$$

$$\partial_\mu \langle (J_q^\mu)^s{}_s \rangle = i\sqrt{2} G_F \eta_{\mu\nu} \Big[-\cos\theta_C \sin\theta_C \eta_{\mu\nu} \langle (J_{q\,L}^\mu)^u{}_s (J_{q\,L}^\nu)^d{}_u - (J_{q\,L}^\mu)^s{}_u (J_{q\,L}^\nu)^u{}_d \rangle$$
$$- \sin\theta_C \sum_{\ell=e,\mu} \langle (J_{q\,L}^\mu)^u{}_s (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^s{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle \Big],$$

$$\partial_\mu \langle (J_l^\mu)^{\nu_\ell}{}_{\nu_\ell} \rangle = i\sqrt{2} G_F \eta_{\mu\nu} \Big[-\cos\theta_C \langle (J_{q\,L}^\mu)^u{}_d (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^d{}_u (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} \rangle$$
$$- \sin\theta_C \langle (J_{q\,L}^\mu)^u{}_s (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^s{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle - \langle (J_{l\,L}^\mu)^{\nu_{\ell'}}{}_{\ell'} (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{l\,L}^\mu)^{\ell'}{}_{\nu_{\ell'}} (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} \rangle \Big],$$

$$\partial_\mu \langle (J_l^\mu)^\ell{}_\ell \rangle = i\sqrt{2} G_F \eta_{\mu\nu} \Big[\cos\theta_C \langle (J_{q\,L}^\mu)^u{}_d (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^d{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle$$
$$+ \sin\theta_C \langle (J_{q\,L}^\mu)^u{}_s (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{q\,L}^\mu)^s{}_u (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle + \langle (J_{l\,L}^\mu)^{\nu_{\ell'}}{}_{\ell'} (J_{l\,L}^\nu)^\ell{}_{\nu_\ell} - (J_{l\,L}^\mu)^{\ell'}{}_{\nu_{\ell'}} (J_{l\,L}^\nu)^{\nu_\ell}{}_\ell \rangle \Big].$$

$$\partial_\mu \langle J^\mu \rangle \sim G_F\, \eta_{\mu\nu} \langle J^\mu J^\nu \rangle$$ (non-diagonal operator)

Flavor Ward identites

Finite temperature theory $T = 1/\beta \Rightarrow$ thermal correlation functions

$$\langle T_C[\mathcal{O}(x_1)\mathcal{O}(x_2)]\rangle$$

Effect of Fermi's interaction in the thermal expectation value of an operator $\mathcal{O}(x)$

$$\langle \mathcal{O}(x)\rangle_{G_F} = \langle T_C\left[\mathcal{O}(x) e^{i\int_C \mathcal{L}_{\mathrm{Fermi}}}\right]\rangle_0 \approx \langle \mathcal{O}(x)\rangle_0 + i\int_C d^4x' \langle T_C\left[\mathcal{O}(x)\mathcal{L}_{\mathrm{Fermi}}(x')\right]\rangle_0.$$

Facorization of the current four-point function $\Rightarrow$ happens naturally at large N_c (holographic models)

$$\langle\left[(J^\mu_f)^a{}_b(x)(J^\nu_f)^c{}_d(x)\right]\left[(J^\alpha_{f'})^{a'}{}_{b'}(x')(J^\beta_{f'})^{c'}{}_{d'}(x')\right]\rangle_0 \approx$$

$$\delta_{ff'}\Big[\langle(J^\mu_f)^a{}_b(x)(J^\alpha_f)^{a'}{}_{b'}(x')\rangle_0\langle(J^\nu_f)^c{}_d(x)(J^\beta_f)^{c'}{}_{d'}(x')\rangle_0 +$$

$$\langle(J^\mu_f)^a{}_b(x)(J^\beta_f)^{c'}{}_{d'}(x')\rangle_0\langle(J^\nu_f)^c{}_d(x)(J^\alpha_f)^{a'}{}_{b'}(x')\rangle_0\Big].$$

$\Rightarrow$ leading contribution $\mathcal{O}(G_F)$ is proportional to the **correlators**

Gauge invariant formula for the rates

Quark and lepton densities:

$$n_u = (J_q^0)^u{}_u,\ n_d = (J_q^0)^d{}_d,\ n_s = (J_q^0)^s{}_s,\ n_\ell = (J_l^0)^\ell{}_\ell.$$

Finally, the **rates** are given by

$$\begin{aligned}
\lambda_{ds} &\approx 4G_F^2 \sin^2\theta_C \cos^2\theta_C\ \Lambda_{ud\to su},\\
\lambda_{ud}^\ell &\approx 4G_F^2 \cos^2\theta_C\ \Lambda_{u\ell\to d\nu},\\
\lambda_{us}^\ell &\approx 4G_F^2 \sin^2\theta_C\ \Lambda_{u\ell\to s\nu},\\
\lambda_{e\mu} &\approx 4G_F^2\ \Lambda_{\mu\nu_e\to\nu_\mu e}.
\end{aligned}$$

$$\Lambda_{f_1^a f_2^b\to f_1^c f_2^d} \approx \eta_{\mu\nu}\eta_{\alpha\beta}\int \frac{d^4k}{(2\pi)^4}\frac{\rho^{\mu\alpha}_{f_1;ac}(k_0,\mathbf{k})\rho^{\beta\nu}_{f_2;db}(k_0,\mathbf{k})}{4T\sinh^2\left(\frac{k_0}{T}\right)}.$$

$$\rho^{\mu\nu}_{ab} = -2\mathrm{Im}\left[G^{\mathrm{Ret.}\,;\,\mu\nu}_{ab,ba}\right]$$

The spectral function ρ can be computed using holographic methods!

Why Holography?

Neutron stars involve one of the most extreme forms of matter ($T_{\text{core}} \approx 100$ keV, $\mu_B \approx 1$ GeV)

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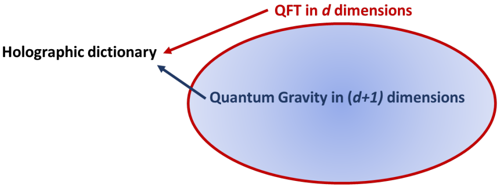
At the densities involved, QCD cannot be treated perturbatively, and lattice QCD suffers from the sign problem $\Rightarrow$ other non-perturbative approaches

Holography can be used as a tool to study non-perturbative aspects of QCD-like theories.



Strongly coupled QFT in d dim. $\Longleftrightarrow$ Classical gravity theory in $(d+1)$ dim.

The holographic model

Bottom-up holographic QCD model dual to a $SU(N_c)$ gauge theory with N_f flavors.
Gravity and matter content:

$$S_c = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,(R-2\Lambda)\,, \quad \Lambda = -\frac{6}{L^2}$$

$$S_f = \int d^5x\,\sqrt{-g}\,\mathrm{Tr}\left[g_X^2\left(-|DX|^2+\frac{3}{L^2}|X|^2\right)-\frac{1}{4g_5^2}\left(F_{(R)}^2+F_{(L)}^2\right)\right]$$

where

$$D_N X = \partial_N X - iA_N^{(L)}X + iXA_N^{(R)} \qquad D_N X^\dagger = \partial_N X^\dagger - iX^\dagger A_N^{(L)} + iA_N^{(R)}X^\dagger$$

$$F_{MN}^{(L,R)} = \partial_M A_N^{(L,R)} - \partial_N A_M^{(L,R)} + i[A_M^{(L,R)}, A_N^{(L,R)}]$$

$$\frac{L}{g_5^2} = \frac{N_c}{12\pi^2}\,, \qquad g_X^2 L^3 = \frac{N_c}{4\pi^2}$$

The holographic model

We consider **zero quark mass** $\Rightarrow X^{\text{bkg}} = 0$ (consistent with the EOMs)

$\Downarrow$

Anti-de-Sitter Reissner-Nordstrom black hole geometry

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \qquad f(z) = 1 - M\frac{z^4}{z_h^4} + Q^2\frac{z^6}{z_h^6}$$

$$\left((A_L)_0^{\text{bkg}}\right)^a_{\ b} = \left((A_R)_0^{\text{bkg}}\right)^a_{\ b} = \frac{\mu_q}{2}\left(1 - \frac{z^2}{z_h^2}\right)\delta^a_{\ b}$$

$$M = 1 + Q^2 \qquad Q^2 = \frac{z_h^2 {\mu_q}^2}{2} \qquad z_h = \frac{2}{\mu_q}\left(\sqrt{1 + \left(\frac{\pi T}{\mu_q}\right)^2} - \frac{\pi T}{\mu_q}\right)$$

Computation of the spectral function

Fluctuations of the background gauge field

$$A_\mu^{(L,R)}(x^\mu, z) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-ik^0x^0 + i\mathbf{k}\cdot\mathbf{x}} A_\mu^{(L,R)}(k^0, \mathbf{k}, z) \; .$$

$$\Downarrow$$

Equations of motion

$$\partial_z^2 A_i^{(L,R)}(z) + \left(\frac{f'(z)}{f(z)} - \frac{1}{z} \right) \partial_z A_i^{(L,R)}(z) + \left(\frac{k_0^2}{f^2(z)} - \frac{k^2}{f(z)} \right) A_i^{(L,R)}(z) = 0$$

$$\partial_z \left(\frac{f(z)}{z} \frac{\partial_z E_{(L,R)}(z)}{k_0^2 - k^2 f(z)} \right) + \frac{1}{f(z)z} E_{(L,R)}(z) = 0, \qquad E_{(L,R)} = k A_0^{(L,R)} + k^0 \frac{k^i}{k} A_i^{(L,R)}$$

Holographic prescription for computing the retarded correlator of an operator $\mathcal{O}$:

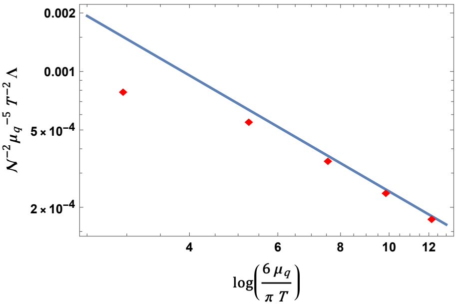
$$G^{\text{Ret.}}(k^0, \mathbf{k}) = \left. \frac{\partial_z \phi(k^0, \mathbf{k}, z)}{\phi(k^0, \mathbf{k}, z)} \right|_{z=0} \qquad \Rightarrow \qquad \rho(k^0, \mathbf{k}) = -2\text{Im}\left[G^{\text{Ret.}}(k^0, \mathbf{k}) \right] .$$

Results

Analytic estimate

$$\boxed{\Lambda_{ud\to ds} \approx \frac{\mathcal{N}^2}{128}\sqrt{\frac{3}{\pi}}\mu_q^5 T^2 \left(\log\frac{6\mu_q}{\pi T}\right)^{-3/2}} \qquad \mathcal{N} = \frac{N_c}{6\pi^2}$$

Numerical result $\left(\frac{T}{\mu_q} = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\right)$



Comparison with the pQCD formula

[H. Heiselberg, J. Madsen, and K. Riisager,Phys. Scripta 34 (1986) 556–560; H. Heiselberg, Phys. Scripta 46 (1992) 485–488; J. Madsen, Phys. Rev. D 47 (1993) 325–330.]

We have obtained the leading order low-T behavior of the weak rate in a large-density state of strongly coupled matter at beta-equilibrium.

$$\lambda_{ds} \approx \underbrace{G_F^2 \sin^2\theta_C \cos^2\theta_C}_{\text{flavor symmetry breaking}} \frac{\mathcal{N}^2}{32}\sqrt{\frac{3}{\pi}} \underbrace{\mu_q^5 T^2}_{\text{Fermi liquid}} \underbrace{\left(\log\frac{6\mu_q}{\pi T}\right)^{-3/2}}_{\text{non-Fermi liquid Log-correction}}$$

We can compare this result to the perturbative QCD result $(\kappa \approx 0.13)$

$$\lambda_{ds} \approx \underbrace{G_F^2 \sin^2\theta_C \cos^2\theta_C}_{\text{flavor symmetry breaking}} \frac{16}{\pi^2} \underbrace{\mu_q^5 T^2}_{\text{Fermi liquid}} \underbrace{\left(1 + \frac{4\alpha_{\text{strong}}}{9\pi}\log\frac{\kappa\,\alpha_{\text{strong}}\mu_q}{T}\right)^4}_{\text{non-Fermi liquid Log-correction}}$$

Comparison with the pQCD formula

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Future directions

- Compute the rates in other models used to study neutron stars: D3/D7, VQCD,...
  [C. Hoyos, N. Jokela, A.O., ongoing work]
- Non-zero quark masses [C. Hoyos, N. Jokela, A.O., ongoing work]
- Quark pairing

Thank you for your time!