Distinguishing between hydro-like and dilute-like dynamics in heavy and light ion collisions

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in collaboration with:
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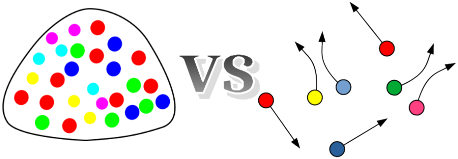
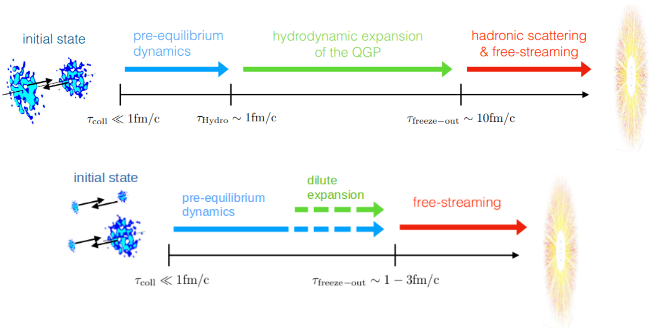
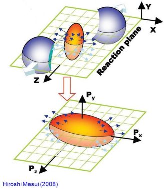
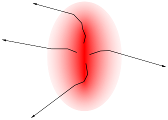
based on:
PRD 111, 054024
PRD 111, 054025



Dynamical modelling in small vs. large systems







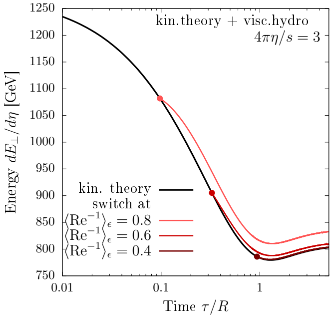
- large systems: dominated by hydrodynamic QGP, leaves imprints of thermalization and collectivity in final state observables
- small systems: might not fully equilibrate $\Rightarrow$ applicability of hydro unclear
- kinetic theory can describe off-equilibrium systems, applicable to free-streaming <u>and</u> hydrodynamic systems
  $\Rightarrow$ in comparison to hydrodynamics, can discern where it is accurate

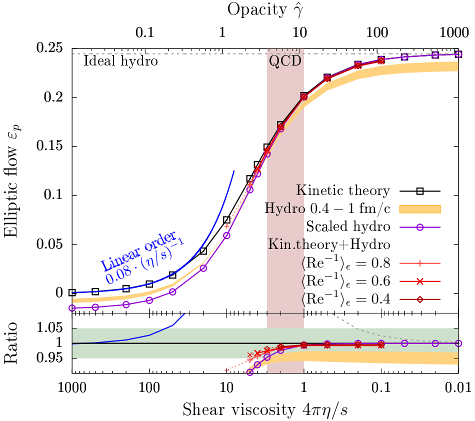
Previous study

Compared hydro and hybrid to full kinetic theory simulations based on $\mathrm{d}E_\perp/\mathrm{d}\eta$, ε_p, $\langle u_\perp \rangle_\epsilon$ and $\langle \mathrm{Re}^{-1} \rangle_\epsilon$ as fct. of opacity $\hat{\gamma}$ for averaged initial state profiles

Ambruș, Schlichting, Werthmann PRD 107 (2023) 094013 and PRL 130 (2023) 152301

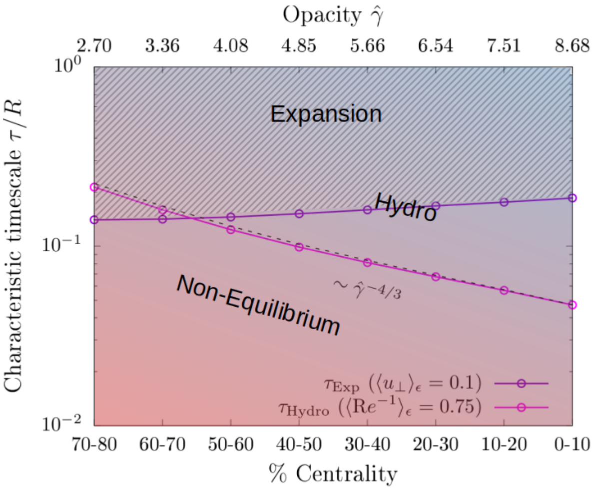
- can expect fixed accuracy when switching based on $\mathrm{Re}^{-1} = \sqrt{\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{e^2}}$: accurate on 5% level for $\mathrm{Re}^{-1}_{\mathrm{switch}} = 0.75$
- pure hydro problematic even at large opacity, hybrid works in a certain range





Regime of Applicability of Hydrodynamics

Tracking timescales of hydrodynamization and onset of transverse expansion:
Hydro applicable for opacities $\hat{\gamma} \gtrsim 3$



Pb+Pb
$\eta/s = 0.12$

Applicability of hydrodynamics in real collisions

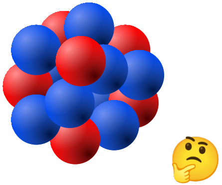
in practice: using an initial state model, can estimate $\hat{\gamma} \gtrsim 3 \hat{=}$ central O+O

problem: Parameters not experimentally accessible, different models give different predictions!

"But hydro works in small system simulations":

Flow results from dynamical response to initial state geometry, which is poorly constrained in small systems

$$v_n = \kappa_{n,n} \cdot \epsilon_n$$



New Aim

- find observables that untangle effects of response and geometry on flow
- look for model-independent quantification of hydrodynamicity
- verify these in event-by-event simulations

Model and Setup: Kinetic Theory

- microscopic description in terms of phase-space distribution

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\, \mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

- time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\text{eq}}) \ , \quad \tau_R = 5\frac{\eta}{s} T^{-1}$$

  results will depend only on initial state and opacity

- dimensionless parameter: opacity $\sim$"total number of interaction"

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi} R \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

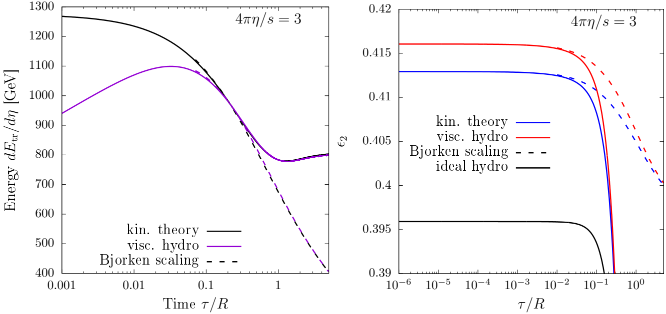
  - encodes dependencies on viscosity, transverse size and energy scale

Model and Setup: Hydrodynamics

- 2nd order Müller-Israel-Steward type hydrodynamics (vHLLE) with RTA transport coefficients

Karpenko, Huovinen, Bleicher Comput. Phys. Commun. 185, 3016 (2014)

- How to define initial state? Hydro deviates at early times!



- solution: hydro initial condition <u>scaled</u> according to attractor curve prediction of early time behaviour

Ambruș, Schlichting, Werthmann PRD 107 (2023) 094013

Initial conditions and Observables

- initial conditions with event-by-event fluctuations (T$_{\mathrm{R}}$ENTo model)

  Moreland, Bernhard, Bass PRC 92 (2015) 011901(R)

- pre-generated nucleon positions to account for correlations like α-clustering
- reasons for O+O:
  - intermediate system size ($\hat{\gamma} \sim 3$)
  - same collision system ran at RHIC and LHC for the first time!

This time we focus on elliptic flow given by

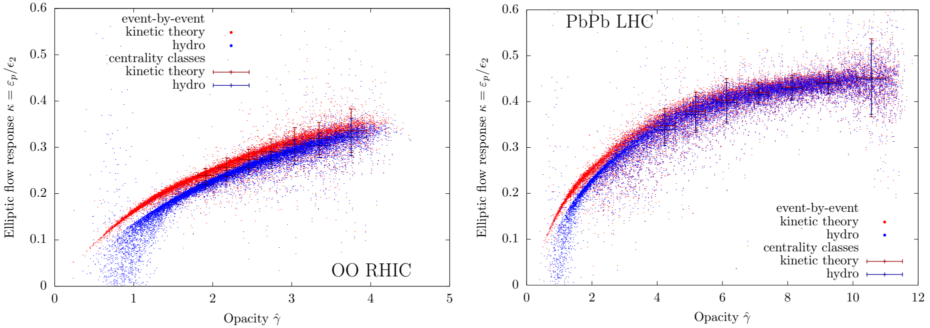
$$\varepsilon_p \, e^{2i\Psi_p} = \frac{\int_{x_\perp} T^{xx} - T^{yy} + 2iT^{xy}}{\int_{x_\perp} T^{xx} + T^{yy}}$$

- measures flow of energy: less sensitive to particlization?
- directly accessible in hydro, no freezeout
- main difference to v_2 is a $\sqrt{s_{NN}}$-dependent conversion factor

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 11, 965

Event-by-event flow responses

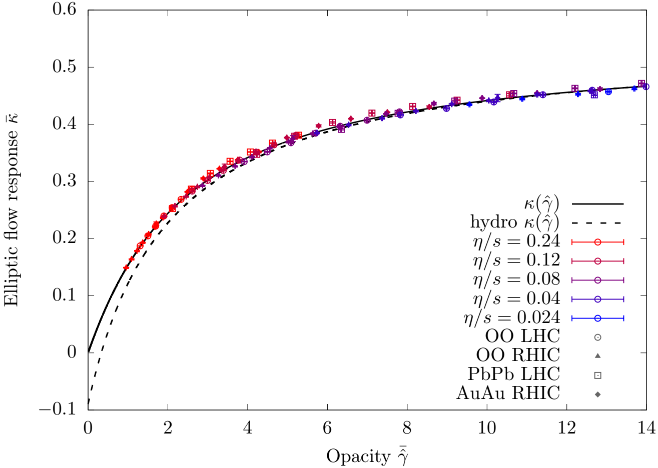
$$\eta/s = 0.12$$



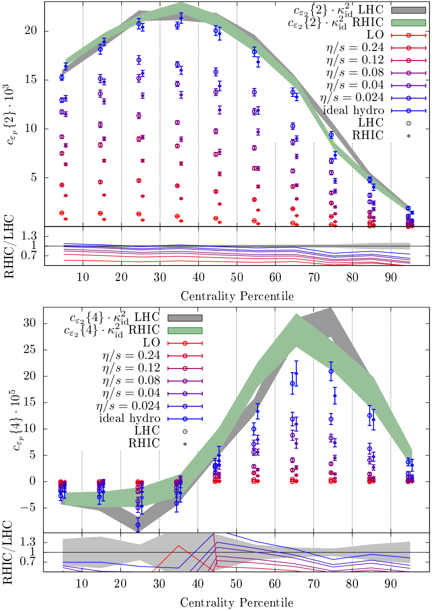
- main difference between different systems is opacity scale
- variation in geometry introduces spread of flow response
- still mostly depends on $\hat{\gamma}$ with $\varepsilon_p^{\text{hydro}} \nearrow \varepsilon_p^{\text{RTA}}$ as before

Universal flow response curve

- mean flow responses in different systems perfectly line up along common curve $\kappa(\hat{\gamma})$
- limiting behaviour: ideal hydrodynamics (constant) and opacity-linearized description



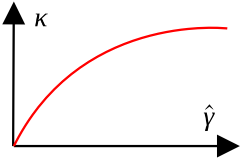
Flow cumulants in O+O



- flow statistics quantified by cumulants

$$c_{\varepsilon_p}\{2\} = \langle \varepsilon_p^2 \rangle\ , \quad c_{\varepsilon_p}\{4\} = \langle \varepsilon_p^4 \rangle - 2\langle \varepsilon_p^2 \rangle^2$$

- ideal hydro follows initial state ellipticity
- centrality dependence of $\kappa(\hat{\gamma})$ introduces modulation



Observation:

flow fluctuations dominated by average response to geometry fluctuations

$$\langle (\epsilon_p)^n \rangle = \langle (\kappa \epsilon_2)^n \rangle = \bar{\kappa}^n \langle (\epsilon_2)^n \rangle + \mathcal{O}(\delta\kappa)$$

If $\langle(\epsilon_p)^n\rangle \approx \bar{\kappa}^n \langle(\epsilon_2)^n\rangle$, then $\bar{\kappa}$ cancels in ratios:

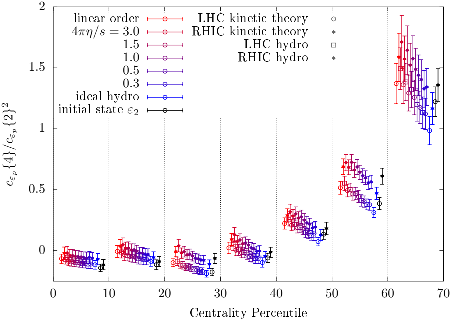
$$\frac{c_{\epsilon_p}\{4\}}{c_{\epsilon_p}\{2\}^2} = \frac{\langle(\epsilon_p)^4\rangle - 2\langle(\epsilon_p)^2\rangle^2}{\langle(\epsilon_p)^2\rangle^2} \approx \frac{\langle(\epsilon_2)^4\rangle - 2\langle(\epsilon_2)^2\rangle^2}{\langle(\epsilon_2)^2\rangle^2} = \frac{c_{\epsilon_2}\{4\}}{c_{\epsilon_2}\{2\}^2}$$

$\Rightarrow$ ratio sensitive mostly to geometry

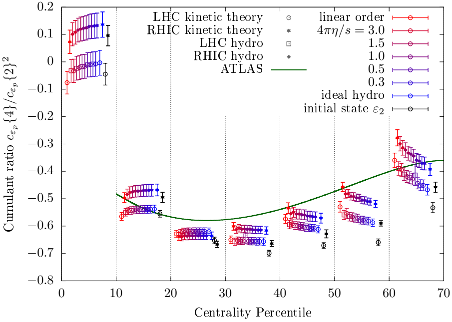
Bhalerao, Luzum, Ollitrault PRC 84 (2011) 034910

simulations in agreement with fit to LHC data for cumulants

O+O



Pb+Pb/Au+Au



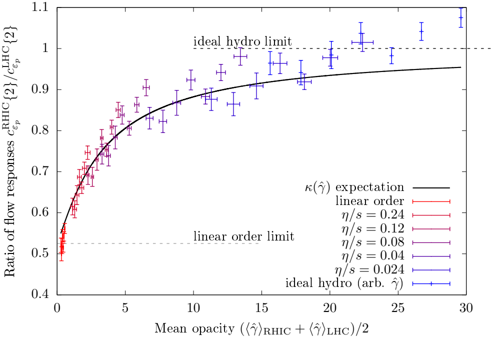
How to extract flow response strength?

If $\langle(\epsilon_p)^n\rangle \approx \bar{\kappa}^n \langle(\epsilon_2)^n\rangle$, then geometry cancels in ratio of cumulants between systems with similar geometry (O+O at RHIC and LHC?)

$$\frac{c_2^{\mathrm{RHIC}}\{2\}}{c_2^{\mathrm{LHC}}\{2\}} \approx \frac{\bar{\kappa}^2_{\mathrm{RHIC}}}{\bar{\kappa}^2_{\mathrm{LHC}}}$$

In our data: response is a smooth transition between limiting cases:

$$\text{ideal hydro}: \quad \kappa(\hat{\gamma} \gg 1) = \kappa_{\mathrm{id}}$$
$$\text{dilute regime}: \quad \kappa(\hat{\gamma} \ll 1) \propto \hat{\gamma}$$



but how do we get at the absolute response coefficient?

Hydrodynamization observable: definition

Idea: set change in κ in relation to change in $\hat{\gamma}$

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{R}{a\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

- comparing systems with same geometry (and same η/s):

$$\frac{\bar{\kappa}^{2k}_{\mathrm{RHIC}}}{\bar{\kappa}^{2k}_{\mathrm{LHC}}} \approx \frac{c_2^{\mathrm{RHIC}}\{2k\}}{c_2^{\mathrm{LHC}}\{2k\}}$$

$$\frac{\hat{\gamma}_{\mathrm{RHIC}}}{\hat{\gamma}_{\mathrm{LHC}}} \approx \left(\frac{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{RHIC}}}{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{LHC}}}\right)^{1/4}$$

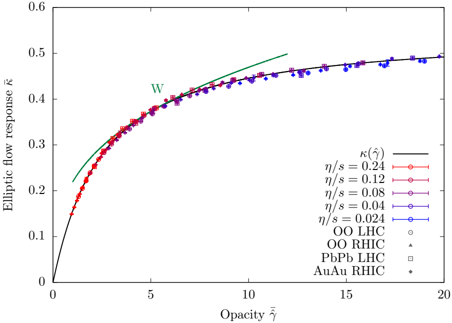
- logarithm turns ratios into differences

  $\rightarrow$ finite difference derivative
- small $\hat{\gamma}$: linear buildup $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \lesssim 1$

  large $\hat{\gamma}$: saturation $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \rightarrow 0$

hydrodynamization observable

$$\mathrm{W} = \frac{2}{k}\frac{\log\left(\frac{c_2\{2k\}_{\mathrm{RHIC}}}{c_2\{2k\}_{\mathrm{LHC}}}\right)}{\log\left(\frac{\mathrm{d}E_\perp/\mathrm{d}\eta|_{\mathrm{RHIC}}}{\mathrm{d}E_\perp/\mathrm{d}\eta|_{\mathrm{LHC}}}\right)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$

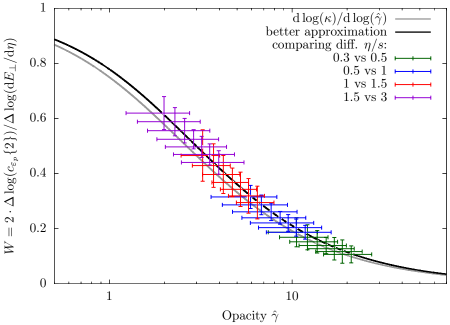


Hydrodynamization observable: Proof of principle

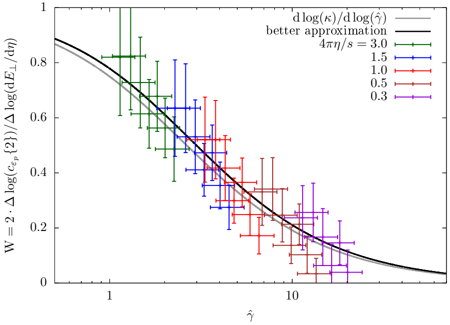
crosscheck of W-observable:

- compute $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$ from extracted $\kappa(\hat{\gamma})$: smooth monotonous transition
  $\Rightarrow$ one-to-one correspondence between W and $\hat{\gamma}$!
- compare with simulation data for proposed observable: agreement!

O+O LHC combining diff. η/s

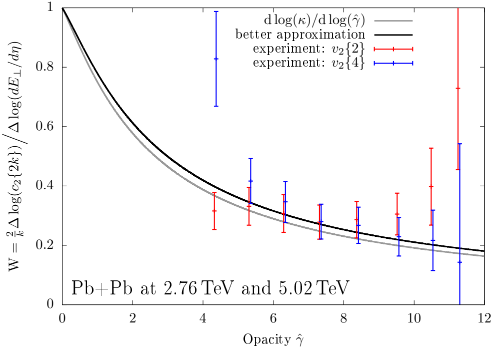


O+O combining RHIC and LHC



Hydrodynamization observable: real data

- first test with LHC data: results agree with theory ($\hat{\gamma}$ from Trento initial conditions, η/s chosen s.t. $\mathrm{d}E_\perp/\mathrm{d}\eta$ matches)



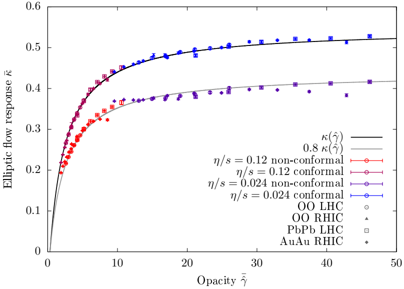
- centrality dependence off for $v_2\{2\}$ (nonflow?), but accurate for $v_2\{4\}$

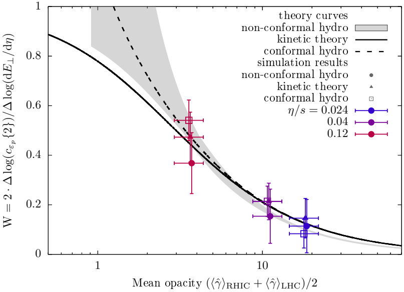
previous hydrodynamization criterion

$\hat{\gamma} \sim 3$ corresponds to $\mathrm{W} \sim 0.5$

Non-conformal effects

- probing non-conformal effects in hydro simulations with chiral eos
  - losing theoretical control
  - mostly relative factor ~ 0.8, but centrality dependence at RHIC
- predictions deviate from conformal case, but still captured by adjusted calibration curve





Summary

- applicability of hydrodynamics can be assessed by comparing to kinetic theory, but uncertainties in initial state obscure results

- can untangle effects of initial state and dynamical response on flow using appropriate observables:
  - cumulant ratios for initial state geometry
  - W-observable for hydrodynamization via variation of strength of flow response

$$\mathrm{W} = 2\frac{\Delta \log(c_{\varepsilon_p}\{2\})}{\Delta \log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$

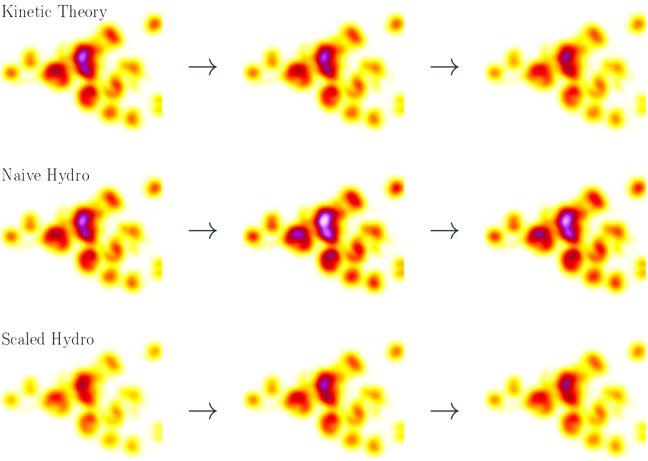
- verified discriminative power in event-by-event simulations

- criterion for hydrodynamic behaviour in experiment: $\mathrm{W} \lesssim 0.5$

Backup

Early time longitudinal cooling and scaled hydro

evolution of τe:



$\tau = 3 \cdot 10^{-6}$fm $\quad$ $\tau = 8 \cdot 10^{-4}$fm $\quad$ $\tau = 3 \cdot 10^{-3}$fm

(times for $4\pi\eta/s = 0.05$)

Hydrodynamics in real collision systems

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\mathrm{Pb+Pb}: \hat{\gamma} \sim \overset{30-40\%}{5.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

$$\mathrm{O+O}: \hat{\gamma} \sim \overset{30-40\%}{2.2} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{1.13\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{55\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{1.4} - \overset{0-5\%}{3.1}$$

probes transition region to hydrodynamic behaviour

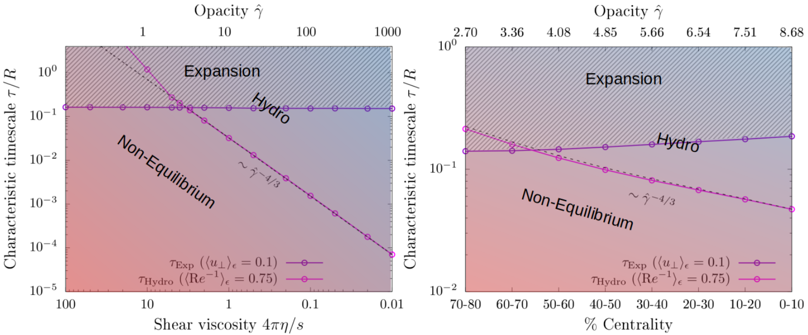
$$\mathrm{p+Pb}: \hat{\gamma} \sim \overset{\mathrm{min.bias}}{1.5} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \overset{\mathrm{high\ mult.}}{\lesssim 2.7}$$

very high multiplicity events approach regime of applicability, but do not reach it

$$\mathrm{p+p}: \hat{\gamma} \sim \overset{\mathrm{min.bias}}{0.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4}$$

far from hydrodynamic behaviour

Hydrodynamization in viscosity and centrality dependence



- transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro appicable when $\mathrm{Re}^{-1} < \mathrm{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\mathrm{Hydro}}/R \approx 1.53\ \hat{\gamma}^{-4/3}\ \left[(\mathrm{Re}_c^{-1})^{-3/2} - 1.21(\mathrm{Re}_c^{-1})^{0.7}\right]$$

- hydrodynamization before transv. Expansion for $\hat{\gamma} \gtrsim 3$

Universal work functions

$$f_{\mathrm{work}}(\hat{\gamma}) = \left\langle \frac{\mathrm{d}E_\perp/\mathrm{d}\eta(\tau_f)}{\mathrm{d}E_\perp/\mathrm{d}\eta(\tau_0)} \right\rangle$$