Electric conductivity and flavor diffusion in a viscous, resistive quark-gluon plasma for weak and strong magnetic field

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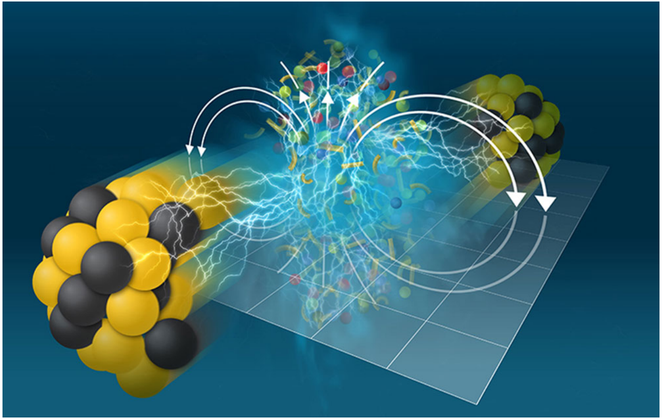
Foundations and Applications of Relativistic Hydrodynamics,
GGI-Florence, 12-16 May 2025

Motivation



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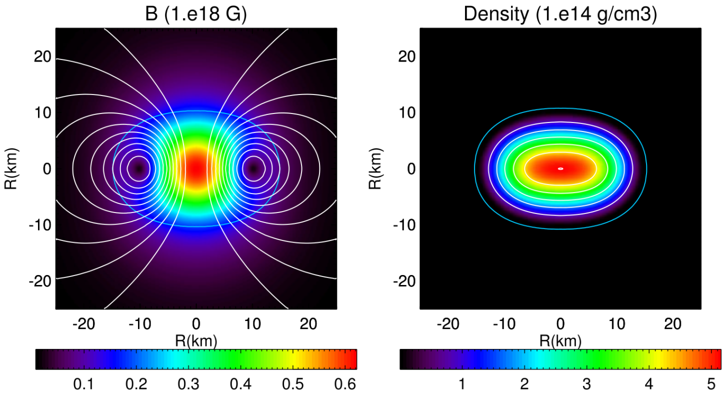
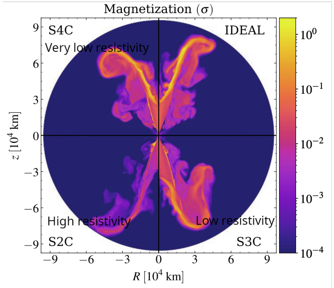
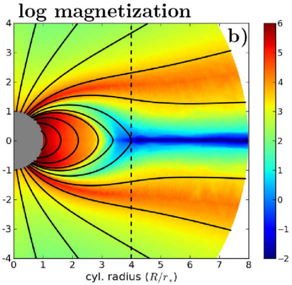
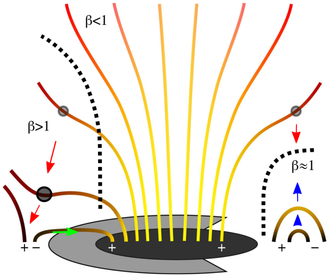


- BNL: "Super Strong Magnetic Fields Leave Imprint on Nuclear Matter"
- $B_{\max} \sim 10^{18}$G even larger than in Magnetars (A. G. Pili, N. Bucciantini and L. Del Zanna, MNRAS 439, 3541–3563 (2014))

Why resistive? Strong wrt what?





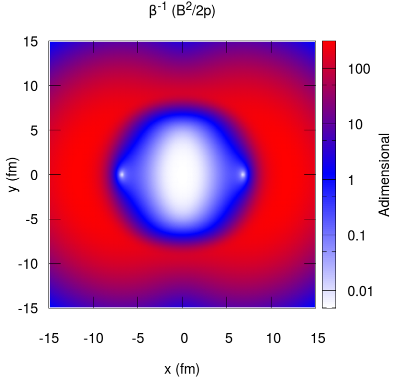


In plasma physics magnetic vs matter dominance usually quantified by

$$\underbrace{\beta_V^{-1} \equiv \frac{B^2/2}{P}}_{\text{Inverse Beta-value}} \quad , \quad \underbrace{\sigma \equiv \frac{B^2}{\varepsilon + P}}_{\text{magnetization}}$$

Strongly magntized plasmas very common in astrophysics: post BNS-merger jets (Mattia, G., et al.: A&A, 691, A105 (2024)), pulsar magnetosphere (M. A. Belyaev, MNRAS 449, 2759–2767 (2015)), solar atmosphere (Ph.-A. Bourdin, ApJL 850:L29, 2017)...

What about HIC's?



Peripheral ($b = 10$ fm) Au-Au collision at $\sqrt{s_{\rm NN}} = 200$ Gev: strongly magnetized plasma at early times where fireball is very rarefied (G. Inghirami et al., EPJC 76 (2016) 12, 659)

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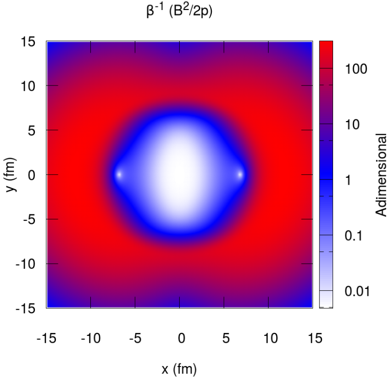

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- *B*-field evolution affected by transport coefficients
- Can transport coefficients be affected by strong *B*-field?

(GR)MHD description of HIC's

Self-consistent solution of conservation laws

$$\nabla_\mu J_f^\mu = 0 \qquad (f = u, d, s)$$
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \text{with} \quad T^{\mu\nu} \equiv T_{\rm m}^{\mu\nu} + T_{\rm em}^{\mu\nu}$$

and Maxwell equations

$$\nabla_\mu F^{\mu\nu} = -J_Q^\nu \quad \text{where} \quad F^{\mu\nu} \equiv u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa \quad (e^\mu = F^{\mu\nu} u_\nu)$$
$$\nabla_\mu {}^\star F^{\mu\nu} = 0 \quad \text{where} \quad {}^\star F^{\mu\nu} \equiv u^\mu b^\nu - u^\nu b^\mu - \epsilon^{\mu\nu\lambda\kappa} e_\lambda u_\kappa \quad (b^\mu = {}^\star F^{\mu\nu} u_\nu)$$

Energy-momentum transfer between fields and matter:

$$\nabla_\mu T_{\rm m}^{\mu\nu} = -(J_Q)_\mu F^{\mu\nu}$$

Macroscopic conserved currents $(q = \mathcal{B}, Q, S)$ connected to flavor ones by

$$J_q^\mu \equiv \mathcal{M}_{qf} J_f^\mu \qquad \text{with} \qquad \mathcal{M} \equiv \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3}|e| & -\frac{1}{3}|e| & -\frac{1}{3}|e| \\ 0 & 0 & -1 \end{pmatrix}$$

Ideal MHD: not a good idea

In the absence of any source of dissipation ($\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$)

$$T_{\rm m}^{\mu\nu} = \varepsilon\, u^\mu u^\nu + P\Delta^{\mu\nu}\,, \qquad J_f^\mu = n_f\, u^\mu\,, \qquad e^\mu = 0$$

Evolution of matter energy-momentum tensor unaffected by e.m. field:

$$\nabla_\mu T_{\rm m}^{\mu\nu} = -(J_Q)_\mu\, F^{\mu\nu} = -n_Q u_\mu\, F^{\mu\nu} = n_Q e^\nu = 0$$

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Vanishing of electric field in LRF equivalent to requiring infinite electric conductivity

$$e^\mu = j^\mu_{Q,\rm cond}/\sigma_Q = 0$$

Slow decay of magnetic fields due to the absence of magnetic diffusion:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma_Q}\nabla^2 \vec{B}$$

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However, from kinetic theory one would get (more details in the following)

$$\underbrace{\eta = \frac{1}{5}\tau_R\,(\varepsilon + P)}_{\text{shear viscosity}}, \qquad \underbrace{\sigma_Q \sim \tau_R\, e^2\, T^2}_{\text{electric conductivity}}$$

One cannot have at the same time an unviscid fluid and an ideal conductor

Dissipative GRMHD

Dissipative corrections (Landau frame $u_\nu T^{\mu\nu}_{\mathrm{m}} = -\varepsilon u^\mu$):

$$J^\mu_f = n_f\, u^\mu + j^\mu_f\,, \qquad T^{\mu\nu}_{\mathrm{m}} = \varepsilon\, u^\mu u^\nu + (P + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu} \quad \text{with} \quad u_\mu j^\mu_f = u_\mu \pi^{\mu\nu} = 0$$

Conservation laws ($D \equiv u^\mu \nabla_\mu$ and $\Delta_\mu \equiv \Delta_{\mu\nu} \nabla^\nu$) for flavor number

$$D n_f + n_f \Theta + \nabla_\mu j^\mu_f = 0$$

energy

$$D\varepsilon + (\varepsilon + P + \Pi)\, \Theta + \pi^{\mu\nu} \sigma_{\mu\nu} = e^\mu\, (j_Q)_\mu$$

and momentum

$$(\varepsilon + P + \Pi)\, a_\mu + \Delta_\mu (P + \Pi) + \Delta_{\mu\beta}\, \Delta_\alpha \pi^{\alpha\beta} + a^\nu\, \pi_{\nu\mu} = n_Q\, e_\mu + \epsilon_{\mu\nu\lambda\rho}\, j^\nu_Q\, u^\lambda\, b^\rho$$

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Constitutive relations for the dissipative corrections:

- Macroscopic approach, $\nabla_\mu \mathcal{S}^\mu \geq 0$, as a starter (generalized Ohm's law)
- Boltzmann-Vlasov equation, *extended to strongly magnetized plasmas*

Generalized Ohm's law for multiple conserved charges

Non-negative entropy production rate

$$\nabla_\mu \mathcal{S}^\mu \geq 0\,, \quad \text{where} \quad \mathcal{S}^\mu = s\, u^\mu - \sum_f^{u,d,s} \alpha_f\, j_f^\mu$$

One gets

$$T\nabla_\mu \mathcal{S}^\mu = -\Pi\,\Theta - \pi^{\mu\nu}\sigma_{\mu\nu} + \sum_f^{u,d,s} (\mu_f - \alpha_f T)\, \nabla_\mu j_f^\mu + \sum_f^{u,d,s} j_f^\mu \left[\mathbb{Q}_f\, |e|\, e_\mu - T\Delta_\mu \left(\frac{\mu_f}{T}\right)\right] \geq 0$$

which is satisfied if $\alpha_f = \mu_f/T$ and

$$\Pi \equiv -\zeta\Theta\,, \quad \pi^{\mu\nu} \equiv -2\eta\, \sigma^{\mu\nu}\,, \quad j_f^\mu \equiv \kappa'_{ff'}\, (\mathbb{Q}_{f'}\, |e|\, e^\mu - T\Delta^\mu \alpha_{f'})$$

so that

$$T\nabla_\mu \mathcal{S}^\mu = \frac{\Pi^2}{\zeta} + \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta} + j_f^\mu\, \left(\kappa'^{-1}\right)_{ff'}\, (j_{f'})_\mu \geq 0$$

A closer look at the dissipative currents

Dissipative quark-flavor currents (off-diagonal response possible!)

$$j_f^\mu \equiv \kappa'_{ff'} \Big(\underbrace{\mathbb{Q}_{f'}\,|e|\,e^\mu}_{\text{conduction}} \underbrace{- T\Delta^\mu \alpha_{f'}}_{\text{diffusion}} \Big)$$

In the macroscopic-charge basis ($q = \mathcal{B}, Q, S$):

$$j_q^\mu = -T\,\underbrace{\mathcal{M}_{qf}\,\kappa'_{ff'}\,(\mathcal{M}^{\mathrm{T}})_{f'q'}}_{\equiv \kappa'_{qq'}}\,\Delta^\mu \alpha_{q'} + \underbrace{\mathcal{M}_{qf}\,\kappa'_{ff'}\,\mathbb{Q}_{f'}\,|e|}_{\equiv \sigma_q}\,e^\mu$$

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Setting $q = Q \longrightarrow$ generalized Wiedemann-Franz law:

$$\boxed{\sigma_Q = \sum_{f,f'} |e|^2 \mathbb{Q}_f\, \kappa'_{ff'}\, \mathbb{Q}_{f'} \equiv \sum_{f,f'} |e|^2 \frac{\mathbb{Q}_f\, \kappa_{ff'}\, \mathbb{Q}_{f'}}{T}}$$

- Ideal electric conductor and $e^\mu = 0 \Longleftrightarrow$ infinite quark diffusion and $\Delta^\mu \alpha_f = 0$

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- Ideal electric conductor and $e^\mu = 0 \Longleftrightarrow$ infinite quark diffusion and $\Delta^\mu \alpha_f = 0$
- "Good" electric conductor: $e^\mu \neq 0$ but small, of first order in the gradients. However, no constraints on b^μ

The Boltzmann-Vlasov equation

BV equation for the (anti-)particle distribution $f_f^\pm$ (flat spacetime for simplicity):

$$\left[p^\mu \partial_\mu + Q_f^\pm \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu}\right] f_f^\pm = \frac{p \cdot u}{\tau_R}(f_f^\pm - f_{0f}^\pm)$$

Recast it into the form

$$f_f^\pm = f_{0f}^\pm + \frac{\tau_R}{p \cdot u} \left[p^\mu \partial_\mu + Q_f^\pm \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu}\right] f_f^\pm$$

which can be seen as the resummation of the following expansion

$$f_f^\pm = \sum_{n=0}^{\infty} \left[\frac{\tau_R}{p \cdot u} \left(p^\mu \partial_\mu + Q_f^\pm \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu}\right)\right]^n f_{0f}^\pm \, .$$

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Conditions to fulfill in order for truncated ($\sim$ Chapman-Enskog) expansion to be meaningful

- $\mathrm{Kn} \equiv \lambda_{\mathrm{mfp}}/L \sim \tau_R \partial \ll 1$: gradient expansion
- $\xi \equiv \tau_R |e| E/T \ll 1$: negligible energy gain between two collisions
- $\chi \equiv \tau_R |e| B/T \sim \lambda_{\mathrm{mfp}}/r_{\mathrm{Larm}} \ll 1$: negligible bending between two collisions

HIC's: weakly or strongly-magnetized QGP?

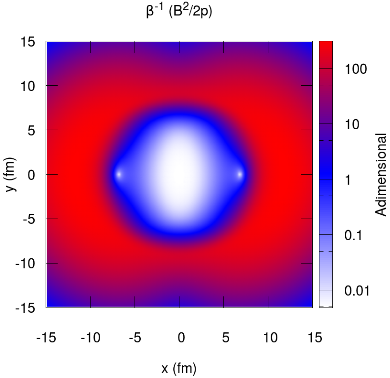
Back-of-the envelope estimates for a conformal plasma of classical particles:

$$\tau_R = 5\,(\eta/s)\,\frac{1}{T}\,, \qquad P = \frac{g_{\rm dof}}{\pi^2}\,T^4\,, \qquad \beta_V \equiv \frac{P}{B^2/2}$$

Hence, for $\eta/s \approx 0.2$ and $g_{\rm dof} \approx 50$

$$\chi^2 = \frac{50\, g_{\rm dof}\,(\eta/s)^2\, 4\pi\,\alpha_{\rm em}}{\pi^2}\,\beta_V^{-1} \approx \beta_V^{-1}$$

- Bulk of the fireball weakly magnetized
- Peripheral regions may require self-consistent resummation of magnetic effects



Dissipative current: microscopic derivation

Quark-flavor currents expressed in terms of off-equilibrium distributions

$$j_f^\mu \equiv g_f\, \Delta^\mu{}_\nu \int d\chi\, p^\nu\, \left[\delta f_f^+ - \delta f_f^-\right], \quad \text{with} \quad d\chi \equiv \frac{d^3p}{(2\pi)^3}\frac{1}{\epsilon_p}$$

where

$$\delta f_f^\pm = \frac{\tau_R}{p\cdot u}\left(p^\mu \partial_\mu + Q_f^\pm\, |e|\, F^{\mu\nu}\, p_\nu \frac{\partial}{\partial p^\mu}\right) f_f^\pm$$

We consider a plasma of gluons and 3 light quark flavors:

$$P = \beta^{-4}\left[\frac{(4g_g + 7\sum_f g_f)\pi^2}{360} + \sum_f \left(\frac{g_f}{12}\,\alpha_f^2 + \frac{g_f}{24\pi^2}\,\alpha_f^4\right)\right]$$

At *first order* in Kn and ξ (but to all orders in χ) one has

$$D\,\alpha_f = 0$$

$$D\,\beta = \frac{1}{3}\,\beta\,\Theta$$

$$\Delta^\mu\beta = \beta\, a^\mu + \sum_f \frac{n_f}{\varepsilon + P}\left(\Delta^\mu \alpha_f - \beta\,\mathbb{Q}_f\,|e|\,e^\mu\right) - \frac{\beta}{\varepsilon + P}\sum_f \mathbb{Q}_f\,|e|\,\epsilon^{\mu\nu\lambda\rho}\,(j_f)_\nu\, u_\lambda\, b_\rho$$

Diffusion and conduction in a weakly-magnetized plasma

Truncating the expansion at first-order one replaces

$$f_f^\pm \to f_{0f}^\pm \equiv f_0(x_f^\pm) = \frac{1}{e^{x_f^\pm}+1}\,, \quad \text{where} \quad x_f^\pm \equiv -\beta\,(p\cdot u) - \alpha_f^\pm \quad \text{and} \quad \alpha_f^\pm \equiv \pm\alpha_f$$

The off-equilibrum correction is then given by ($\widetilde{f_{0f}^\pm} \equiv 1 - f_{0f}^\pm$)

$$\begin{aligned} \delta f_f^\pm &= \frac{\tau_R}{p\cdot u}\left(p^\mu \partial_\mu + Q_f^\pm\,|e|\,F^{\mu\nu}\,p_\nu \frac{\partial}{\partial p^\mu}\right) f_{0f}^\pm \\ &= \frac{\tau_R}{p\cdot u}\left(-f_{0f}^\pm\,\widetilde{f_{0f}^\pm}\right)\Big(\underbrace{p^\mu \partial_\mu x_f^\pm}_{I} \underbrace{+\,\beta\,Q_f^\pm\,|e|\,p^\nu\,e_\nu}_{II}\Big) \end{aligned}$$

Writing only the terms providing a non-zero contribution to j_f^μ

$$p^\mu \partial_\mu x_f^\pm = \underbrace{-\,p^\rho \Delta_\rho \alpha_f^\pm}_{Ia} \underbrace{-\,(p\cdot u)\,p^\rho \sum_{f'} \frac{n'_f}{\varepsilon + P}\left(\Delta_\rho \alpha_{f'} - \beta\,\mathbb{Q}_{f'}\,|e|\,e_\rho\right)}_{Ib} + \ldots$$

Diffusion in a weakly magnetized plasma

Off-equilibrium correction to the (anti-)quark distributions:

$$\delta f_f^{\pm} = p^{\rho} \underbrace{\frac{\tau_R}{(-p\cdot u)} \left[f_{0f}^{\pm} \widetilde{f_{0f}^{\pm}}\right] \left(\pm\delta_{ff'} - \frac{n_{f'}\,(-p\cdot u)}{\epsilon + P}\right)}_{\equiv A_{ff'}^{\pm}} g_{\rho\sigma} \underbrace{\left(\beta\,\mathbb{Q}_{f'}|e|\,e^{\sigma} - \Delta^{\sigma}\alpha_{f'}\right)}_{\equiv \widetilde{e}_{f'}^{\sigma}}$$

leading to the dissipative current flavor current

$$j_f^{\mu} \equiv \sum_{f'} \kappa_{ff'}\,\widetilde{e}_{f'}^{\mu} = \sum_{f'} \tau_R \left[\frac{g_f\,T^3}{18}\left(1 + \frac{3}{\pi^2}\alpha_f^2\right)\delta_{ff'} - \frac{n_f\,n_{f'}\,T}{\varepsilon + P}\right]\widetilde{e}_{f'}^{\mu}$$

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Non-diagonal, symmetric flavor-diffusion matrix. Matrix elements in flavor space are scalars, given by the integral

$$\boxed{\kappa_{ff'} = g_f \, \frac{1}{3} \int d\chi \, \vec{p}^{\,2} \, \left(A_{ff'}^{+} - A_{ff'}^{-} \right)}$$

Electric conductivity: numerical estimates for $\alpha_f = 0$

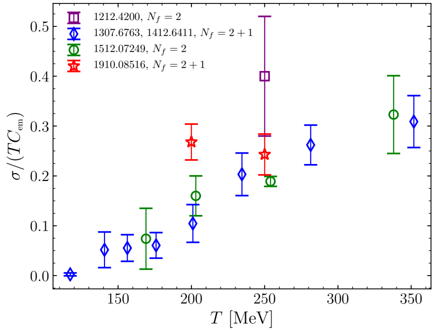
For zero quark density the diffusion matrix is diagonal. From the WF law one gets ($g_f = 6$)

$$\sigma_Q = \sum_{f,f'} |e|^2 \frac{\mathbb{Q}_f \, \kappa_{ff'} \, \mathbb{Q}_{f'}}{T} = \frac{\tau_R}{T} \frac{T^3}{3} |e|^2 \sum_f \mathbb{Q}_f^2 \equiv \tau_R \frac{T^2}{3} C_{\rm em}$$

From $\tau_R = 5(\eta/s)\frac{1}{T}$ one gets

$$\boxed{\frac{\sigma}{T\,C_{\rm em}} = \frac{5}{3}(\eta/s)}$$

comparable, for $0.1 \lesssim \eta/s \lesssim 0.2$, with lattice-QCD calculations (G. Aarts and A. Nikolaev, Eur.Phys.J.A 57 (2021) 4, 118)



Consider the formal expansion

$$f_f^{\pm} = \sum_{n=0}^{\infty} \left[\frac{\tau_R}{p \cdot u} \left(p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n f_{0f}^{\pm} \, .$$

When $\chi \sim 1$ all terms of the form

$$\left[\left(\frac{\tau_R}{p \cdot u} \right) \left(Q_f^{\pm} \, |e| \, b^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n \left[\left(\frac{\tau_R}{p \cdot u} \right) \left(p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right] f_{0f}^{\pm}$$

provide contributions of first order in Kn and ξ.

Diffusion and conductivity in a strongly-magnetized QGP

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provide contributions of first order in Kn and ξ. Truncating the expansion is meaningless. One should rather implicitly resum all these term through a proper ansatz for the off-equilibrium fluctuations.

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Consider the formal expansion

$$f_f^{\pm} = \sum_{n=0}^{\infty} \left[\frac{\tau_R}{p \cdot u} \left(p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n f_{0f}^{\pm} \, .$$

When $\chi \sim 1$ all terms of the form

$$\left[\left(\frac{\tau_R}{p \cdot u} \right) \left(Q_f^{\pm} \, |e| \, b^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n \left[\left(\frac{\tau_R}{p \cdot u} \right) \left(p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right] f_{0f}^{\pm}$$

provide contributions of first order in Kn and ξ. Truncating the expansion is meaningless. One should rather implicitly resum all these term through a proper ansatz for the off-equilibrium fluctuations. Start from the BV equation:

$$\frac{\tau_R}{p \cdot u} \left[p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, (e^\nu p_\nu) \, u^\mu \frac{\partial}{\partial p^\mu} \right] f_{0f}^{\pm} + \frac{\tau_R}{p \cdot u} \, Q_f^{\pm} \, |e| \, b^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \delta f_f^{\pm} = \delta f_f^{\pm}$$

Deriving the diffusion tensor (I)

Ansatz for off-equilibrium fluctuation (LRF for simplicity):

$$\delta f_f^{\pm}\big|_{\mathrm{LRF}} = p^i \left[E^{\pm}_{ff'} \underbrace{(\delta^{ij} - \hat{b}^i \hat{b}^j)}_{\Xi^{ij}} + L^{\pm}_{ff'} \hat{b}^i \hat{b}^j + H^{\pm}_{ff'} \epsilon^{ijk} \hat{b}^k \right] \tilde{e}^j_{f'}$$

Substitute into the previous equation

Deriving the diffusion tensor (I)

Ansatz for off-equilibrium fluctuation (LRF for simplicity):

$$\delta f_f^{\pm}\big|_{\mathrm{LRF}} = p^i \Big[E^{\pm}_{ff'} \underbrace{(\delta^{ij} - \hat{b}^i \hat{b}^j)}_{\Xi^{ij}} + L^{\pm}_{ff'} \, \hat{b}^i \hat{b}^j + H^{\pm}_{ff'} \, \epsilon^{ijk} \hat{b}^k \Big] \tilde{e}^j_{f'}$$

Substitute into the previous equation and use also to get the induced current

$$j_f^i = g_f \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{\epsilon_p} p^l \left[(E^+_{ff'} - E^-_{ff'}) \, \Xi^{lj} + (L^+_{ff'} - L^-_{ff'}) \, \hat{b}^l \hat{b}^j + (H^+_{ff'} - H^-_{ff'}) \, \epsilon^{ljk} \hat{b}^k \right] \tilde{e}^j_{f'} \equiv \kappa^{ij}_{ff'} \tilde{e}^j_{f'}$$

Flavor-diffusion tensor

$$\kappa^{ij}_{ff'} = \kappa^{\perp}_{ff'} \, \Xi^{ij} + \kappa^{||}_{ff'} \, \hat{b}^i \hat{b}^j + \kappa^{\times}_{ff'} \, \epsilon^{ijk} \hat{b}^k$$

where

$$\kappa^{||}_{ff'} = \frac{g_f}{3} \int d\chi \, \vec{p}^{\,2} (L^+_{ff'} - L^-_{ff'}) \quad \kappa^{\perp}_{ff'} = \frac{g_f}{3} \int d\chi \, \vec{p}^{\,2} (E^+_{ff'} - E^-_{ff'}) \quad \kappa^{\times}_{ff'} = \frac{g_f}{3} \int d\chi \, \vec{p}^{\,2} (H^+_{ff'} - H^-_{ff'})$$

Deriving the diffusion tensor (I)

Concerning the first term, it receive a correction from the magnetic term in the Euler equation:

$$\frac{\tau_R}{p\cdot u}\left[p^\mu \partial_\mu + Q_f^\pm\,|e|\,(e^\nu p_\nu)\,u^\mu \frac{\partial}{\partial p^\mu}\right] f_{0f}^\pm = \underbrace{p^i\,A_{ff'}^\pm\,\delta^{ij}\;\tilde{e}_{f'}^j}_{\text{weak}-\text{field result}} +$$

$$+\frac{\tau_R}{p\cdot u}\left[-f_{0f}^\pm \widetilde{f_{0f}^\pm}\right]\frac{\beta\,(p\cdot u)}{\epsilon+P}\,p^i\sum_{f''}\mathbb{Q}_{f''}\,|e|\,B\,(-1)\,\epsilon^{ilk}\,\hat{b}^k j_{f''}^l$$

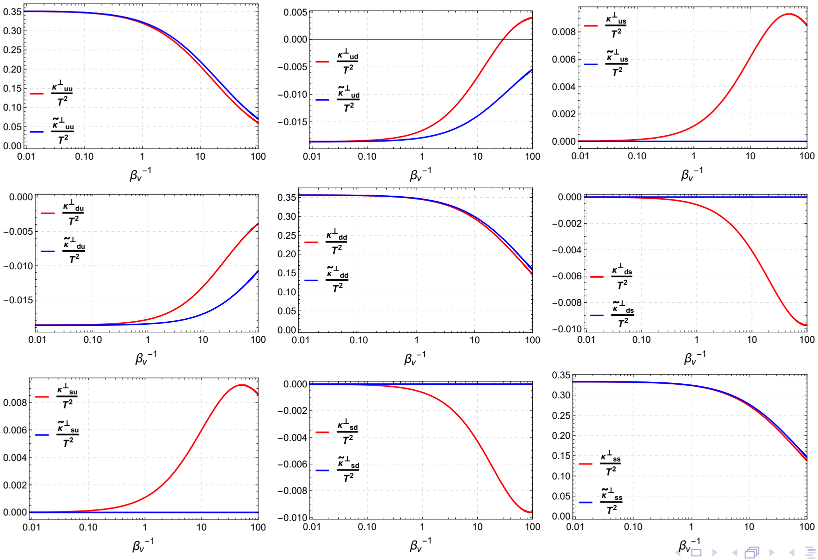
where one has to substitute the previous decomposition

$$j_{f''}^l = \left[\kappa_{f''f'}^{\perp}\,\Xi^{lj} + \kappa_{f''f'}^{||}\,\hat{b}^l\,\hat{b}^j + \kappa_{f''f'}^{\times}\,\epsilon^{ljk}\,\hat{b}^k\right]\,\tilde{e}_{f'}^j$$

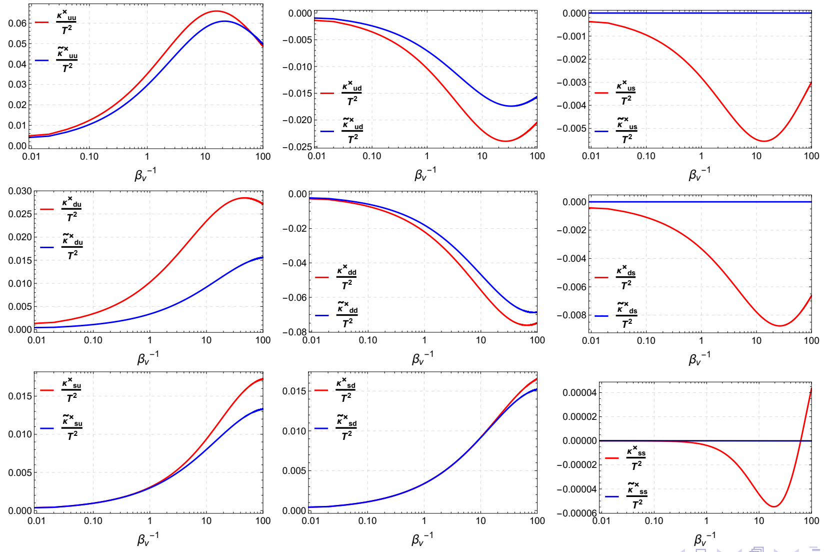
Numerical results

- Results are presented at fixed s/n_B and for $n_Q/n_B = 0.4|e|$ and zero net-strangeness
- For a conformal plasma dimensionless diffusion components κ/T^2 only functions of the the scaling variable β_V^{-1} (no need to specify neither temperature nor the magnetic field)

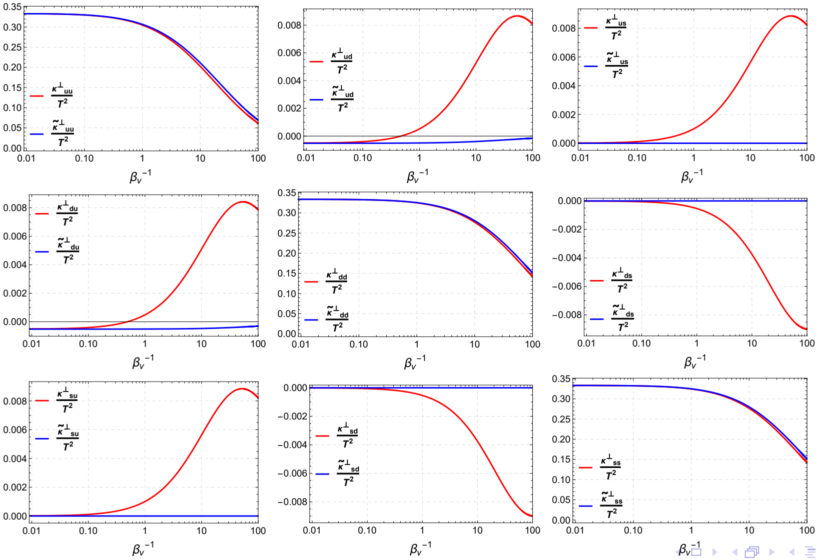
Numerical results: transverse diffusion $s/n_B = 50$



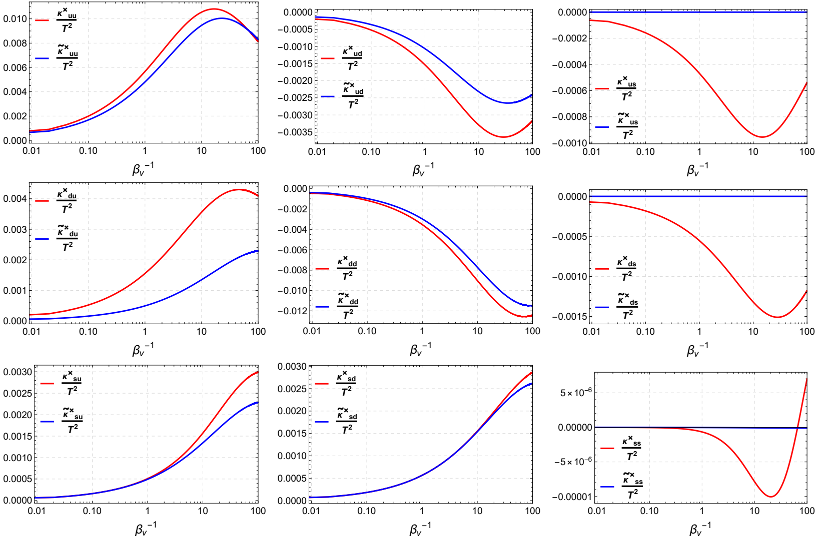
Numerical results: Hall diffusion $s/n_B = 50$



Numerical results: transverse diffusion $s/n_B = 300$



Numerical results: Hall diffusion $s/n_B = 300$



Conclusions

- Work in progress (for details see F. Frascà poster), from $\kappa_{ff'}$ get σ_Q;
- In HIC's for most of the fireball 4-volume $\kappa_{ff'}$ just a scalar and σ_Q from RTA-BV in agreement with l-QCD, but in some domain B-induced breaking of isotropy has to be accounted for;
- In astrophysics strongly-magnetized plasmas much more common and hopefully RTA-BV approach can be a good guidance;
- Deriving the same tensorial constitutive relation $j^i_f = \kappa^{ij}_{ff'} \tilde{e}^j_{f'}$ in the large-β_V^{-1} regime from $\nabla_\mu \mathcal{S}^\mu \geq 0$ non trivial: separate treatment of (non-conserved) particle and antiparticle currents;
- Extension to second order in Kn and ξ a big challenge if β_V^{-1} is large.