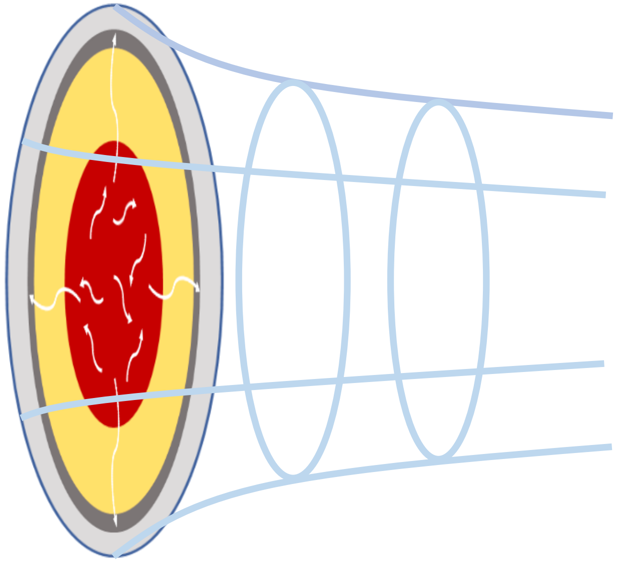
Neutrino Transport in Holography



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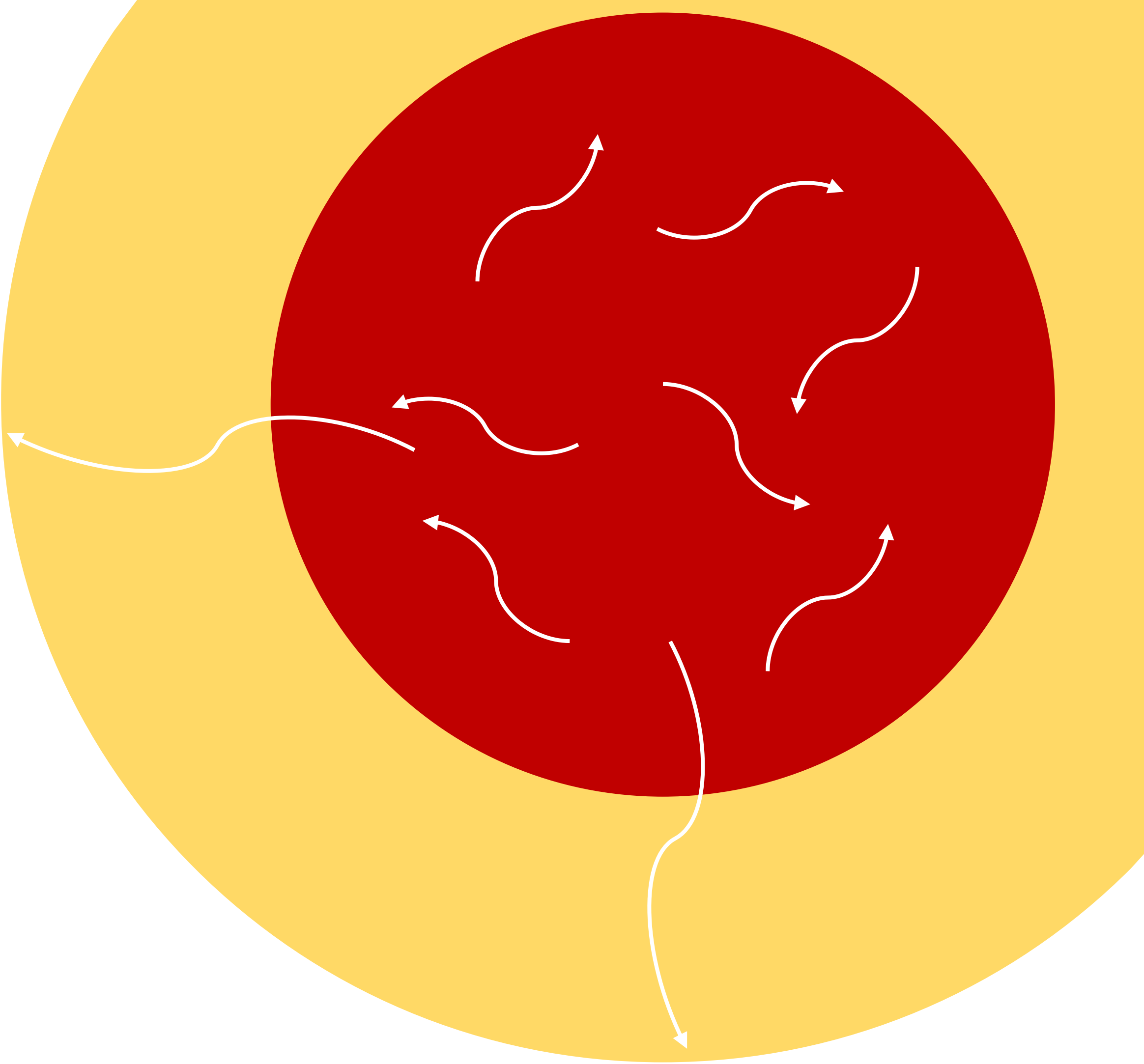
Outline

1) Motivation
2) Introduction : Formalism for neutrino transport
3) Holographic Set-up
4) Holographic calculation of the chiral current correlators
5) Towards isospin asymmetry
6) Summary

Motivation

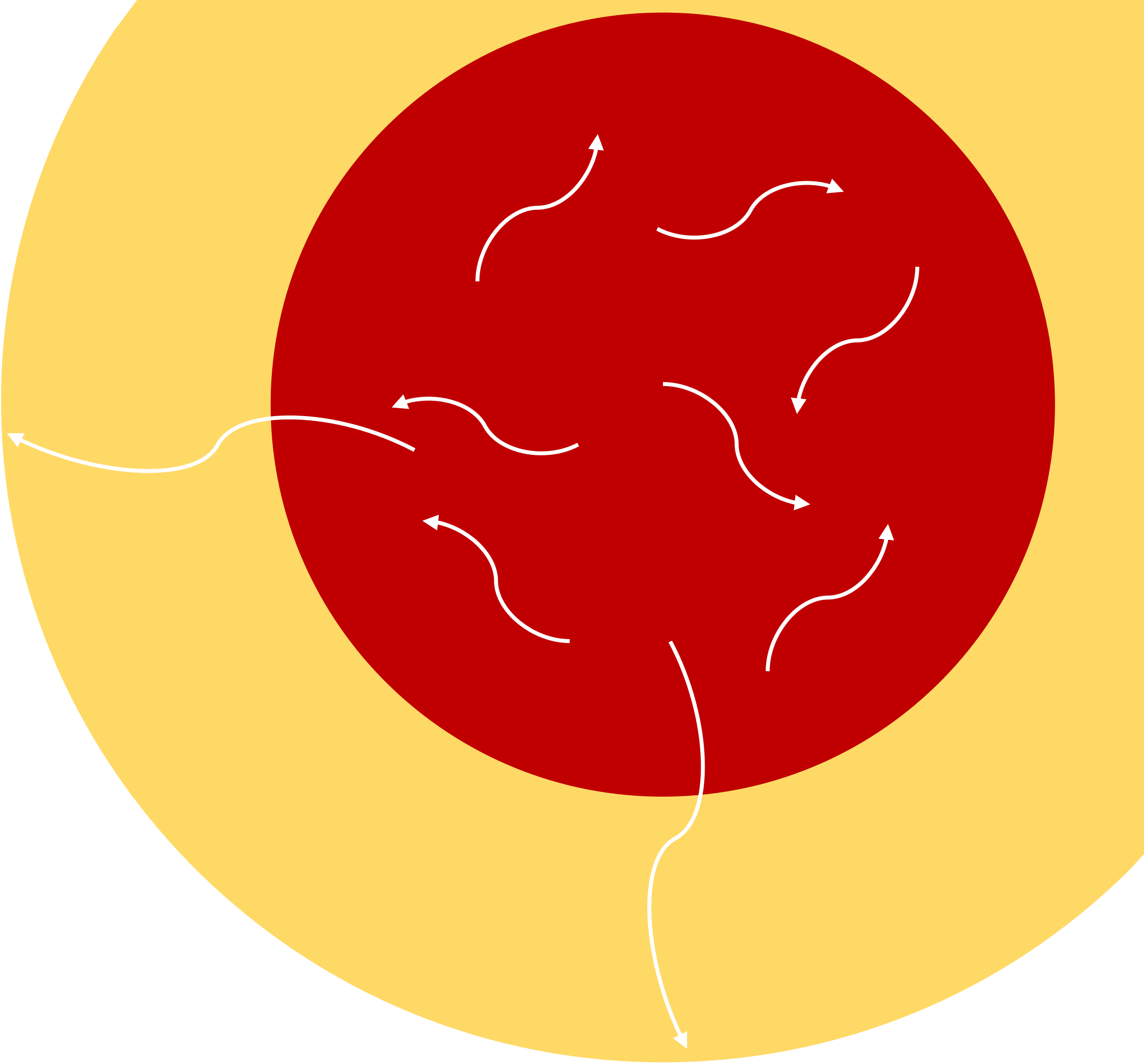
- **Neutrino (ν)** radiation is the main mechanism for **Neutron Star (NS) cooling**
- Requires the knowledge of ν interaction with **dense QCD matter** in the core
- **Simulations** need an **input** from particle physics : $\boldsymbol{j}$ & $\alpha \leftrightarrow \langle J_{L/R} J_{L/R} \rangle^R$



Motivation

- Computing $\langle J_{L/R} J_{L/R} \rangle^R$ in the **dense strongly-coupled** QCD matter is a difficult problem
- We consider the **holographic** approach

Problem : compute $\langle J_{L/R} J_{L/R} \rangle^R$ in **holographic QCD** at finite T and n_B
→ This work : **simplest** toy model (conformal quark matter)



Formalism for neutrino transport

Exercice : compute the **(exact) propagator $\boldsymbol{G_\nu(\vec{x}_1, t_1; \vec{x}_2, t_2)}$** of ν's in a **dense QCD medium**

Quasi-particle approximation :

$\boldsymbol{G_\nu}$ is described by the **$\boldsymbol{\nu}$ distribution function $\boldsymbol{f_\nu(\vec{x}, t; k_\nu)}$**

The transport of neutrinos is described by the **Boltzmann equation** obeyed by f_ν

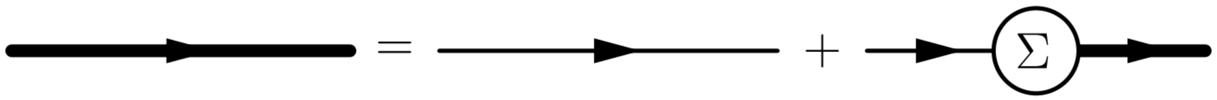
$$(k_\nu \cdot \partial) f_\nu \equiv j(E_\nu)(1 - f_\nu) - \alpha(E_\nu) f_\nu \equiv j(E_\nu) - \kappa(E_\nu) f_\nu$$

Emissivity **Absorptivity** **Opacity**

$\boldsymbol{\kappa = j + \alpha}$

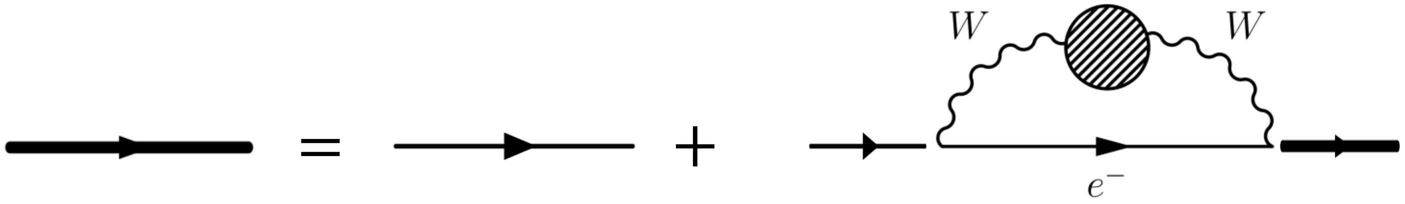
Schwinger-Dyson equation

The kinetic equation can be derived from the finite temperature **Schwinger-Dyson equation**

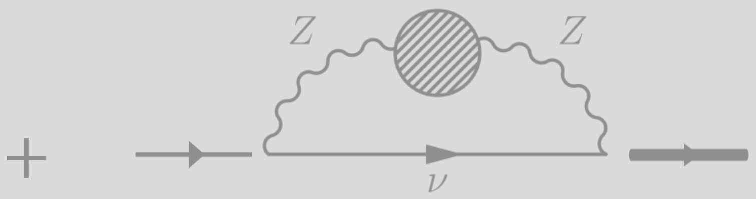


The self-energy Σ **is expanded at order $\mathcal{O}(G_F^2)$** in the weak interaction

$\boldsymbol{\nu + n \leftrightarrow e^- + p}$



$\boldsymbol{\nu + n/p \leftrightarrow \nu + n/p}$

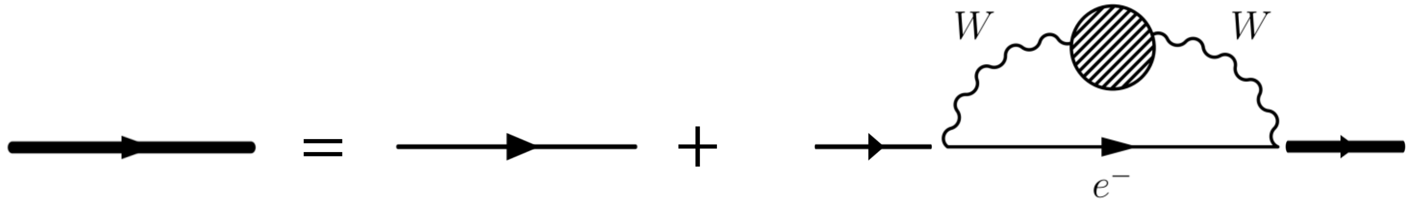


It is fully **non-perturbative** in the **strong** interaction

Schwinger-Dyson equation

The kinetic equation can be derived from the finite temperature **Schwinger-Dyson equation**

$$\boldsymbol{\nu + n \leftrightarrow e^- + p}$$



Dirac equation

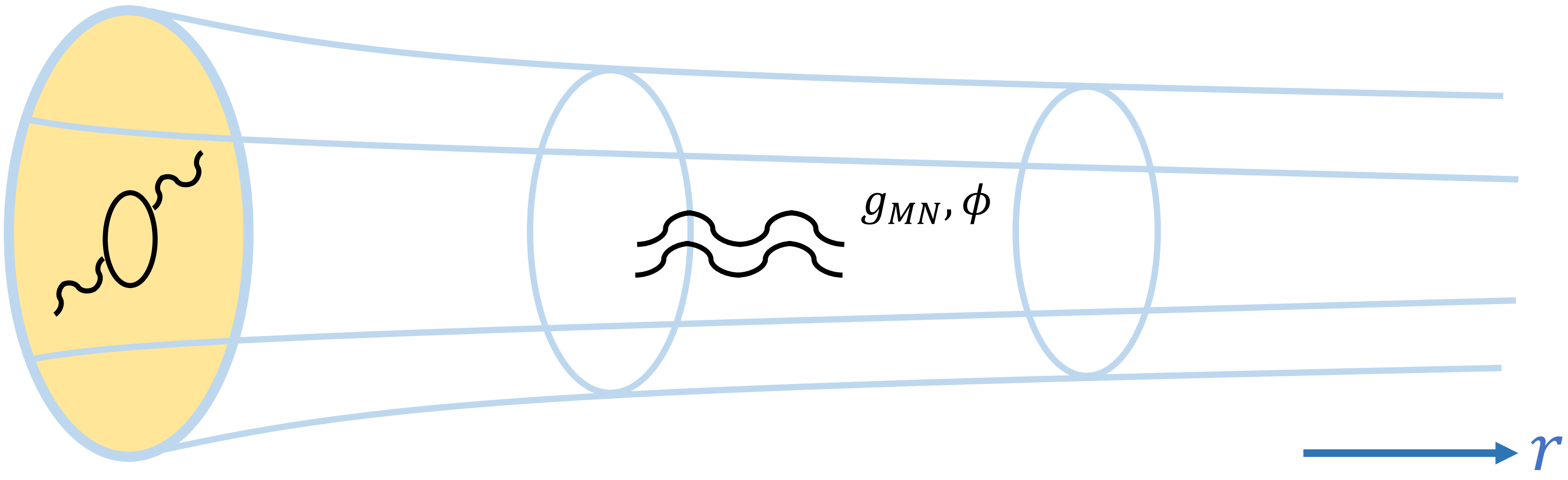
$$j(E_\nu) = G_F^2 \int \frac{d\vec{k}_e^3}{(2\pi)^3} \underbrace{(\text{kins})^{\lambda\,\sigma}}_{\vec{k}_e,\,\vec{k}_\nu} \times \underbrace{(\text{stats})}_{f_e,\,f_W} \times \text{Im}(i\langle J_\lambda^- J_\sigma^+\rangle^R)\,,$$

Dense QCD

$$\sim \langle J_\lambda^L J_\sigma^L \rangle^R$$

The holographic set-up

The holographic correspondence



Strongly-coupled quantum field theory in 4D

Weakly-curved classical gravitational theory in 5D

- The **boundary** of the 5D space (**bulk**) is the **4D space-time** on which the quantum theory is defined
- The additional **dimension *r*** is called the **holographic coordinate** and identified with the **length scale**

The Holographic Set-up

Simplest holographic toy model with **chiral currents $J^{\mu}_{L/R}$**

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
$U(N_f)_L \times U(N_f)_R : \partial_\mu J^\mu_{L/R} = 0$	$\leftrightarrow$	$U(N_f)_L \times U(N_f)_R : A^M_{L/R}$

$N_c \to \infty \, , \frac{N_f}{N_c}$ finite

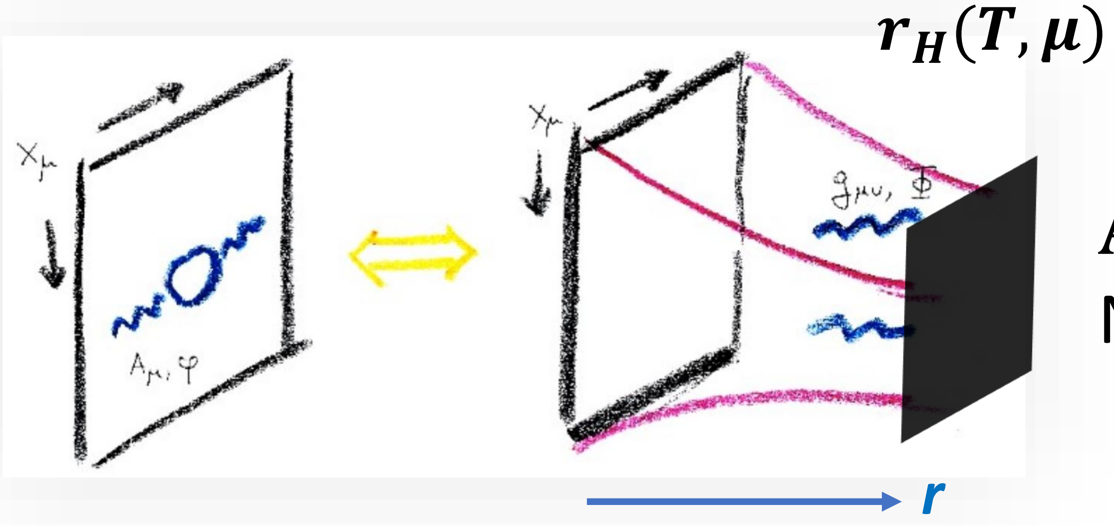
$$S = M_{Pl}^3 N_c^2 \int dx^5 \sqrt{-g} \left(R + \frac{12}{\ell^2} - \frac{\kappa}{N_c} \mathrm{Tr} \left\{ F^{(L)}_{MN} F^{MN}_{(L)} + F^{(R)}_{MN} F^{MN}_{(R)} \right\} \right) ,$$

Background solution

We want to compute $\langle J_\lambda^- J_\sigma^+ \rangle^R$ in an equilibrium state at **finite** (T, μ_q) = dense strongly-coupled **quark matter**

→ Charged AdS **black hole**, with charge $Q \propto \mu_q$

(T, μ_q)



AdS^5-Reisnner-Nordström

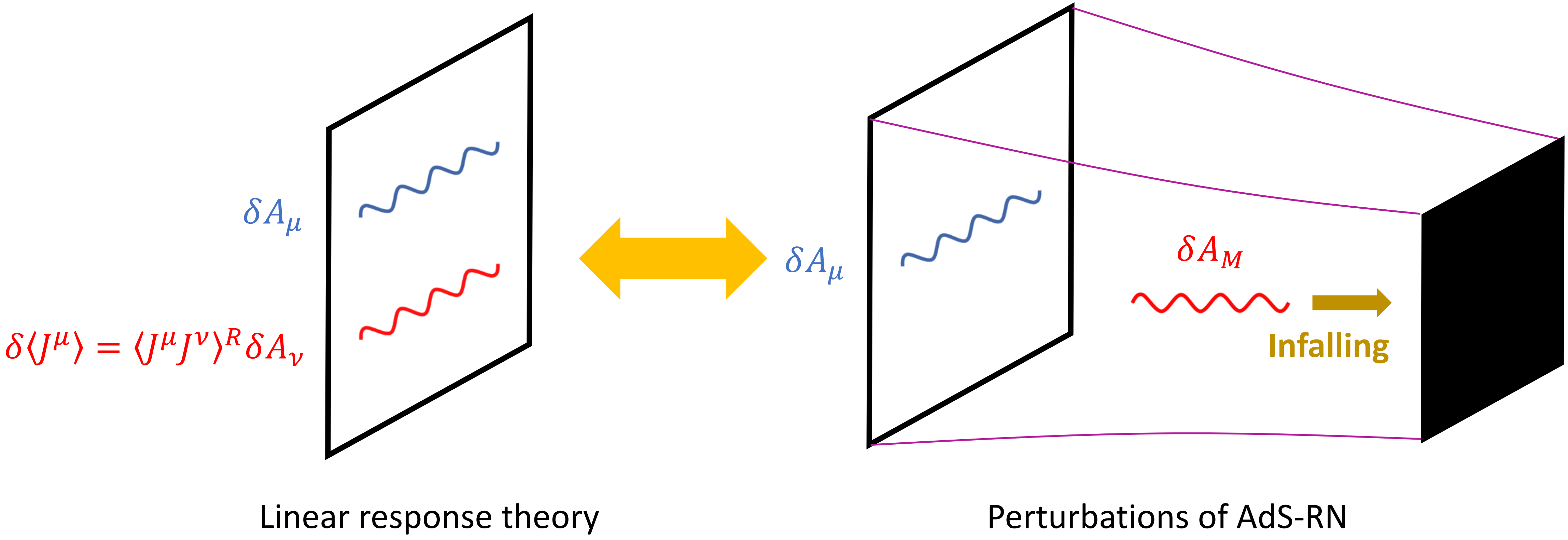
Summary of parameters

Parameters of the **model**	$M_{Pl}\ell$	Fitted to lattice quark-gluon thermodynamics
	κ	
Environmental parameters	$\frac{\mu_q}{T}$	Varied
Neutrino properties	$\frac{E_\nu}{T}$	Varied

Holographic calculation of the chiral current 2-point function

Perturbations of AdS-RN

[Son & Starinets '02]
[Skenderis & van Rees '08]



The boundary plasma has an **SO(3) rotational invariance**

$$\langle J_\lambda J_\sigma\rangle^R(\omega,\vec{k}) = P^{\perp}_{\lambda\sigma}(\omega,\vec{k})\, i\boldsymbol{\Pi}^{\perp}(\boldsymbol{\omega},\vec{\boldsymbol{k}}) + P^{\parallel}_{\lambda\sigma}(\omega,\vec{k})\, i\boldsymbol{\Pi}^{\parallel}(\boldsymbol{\omega},\vec{\boldsymbol{k}})$$

Hydrodynamic approximation

The **long-range** behavior of a system **near equilibrium** is described by **hydrodynamics**

→ Equilibrium **correlators** follow a **universal** long-range structure :

- **Expansion** in $(\omega/T, k/T)$, with **transport coefficients**
- The **hydro modes** appear as **poles** at leading order

$$\langle J_\lambda J_\sigma \rangle^R(\omega, \vec{k}) = \boldsymbol{\sigma} \left(P^{\perp}_{\lambda\sigma}\, \omega + P^{\parallel}_{\lambda\sigma} \frac{\omega^2 - k^2}{\omega + i\boldsymbol{D}k^2} \right) \left(1 + \mathcal{O}\left(\frac{\omega}{T}, \frac{k^2}{T^2} \right) \right),$$

Conductivity

$\partial_t J^0 = D \Delta J^0$

Hydrodynamic approximation at $\mu_q \gg T$

$$\langle J_\lambda J_\sigma \rangle^R(\omega, \vec{k}) = \boldsymbol{\sigma} \left(P^\perp_{\lambda\sigma}\, \omega + P^\parallel_{\lambda\sigma} \frac{\omega^2 - k^2}{\omega + i\boldsymbol{D}k^2} \right) \left(1 + \mathcal{O}\left(\frac{\omega}{T}, \frac{k^2}{T^2} \right) \right),$$

Hydro a priori **breaks down at $\boldsymbol{\omega}, \boldsymbol{k} \gg \boldsymbol{T}$**

AdS-RN : the LO **hydro** approximation remains valid as long as $\boldsymbol{\omega}, \boldsymbol{k} \ll \boldsymbol{\mu_q}$

→ **$\boldsymbol{\nu}$ transport** in a NS: $\boldsymbol{E_\nu}, \boldsymbol{\mu_e}, \boldsymbol{\mu_\nu} \ll \boldsymbol{\mu_q}$

[Davison & Parnachev '13]
[Moitra, Sake & Trivedi '21]

At $\boldsymbol{\mu_q} \gg \boldsymbol{T}$, we have $\mu_e, \mu_\nu \simeq 0.7\, \mu_q$

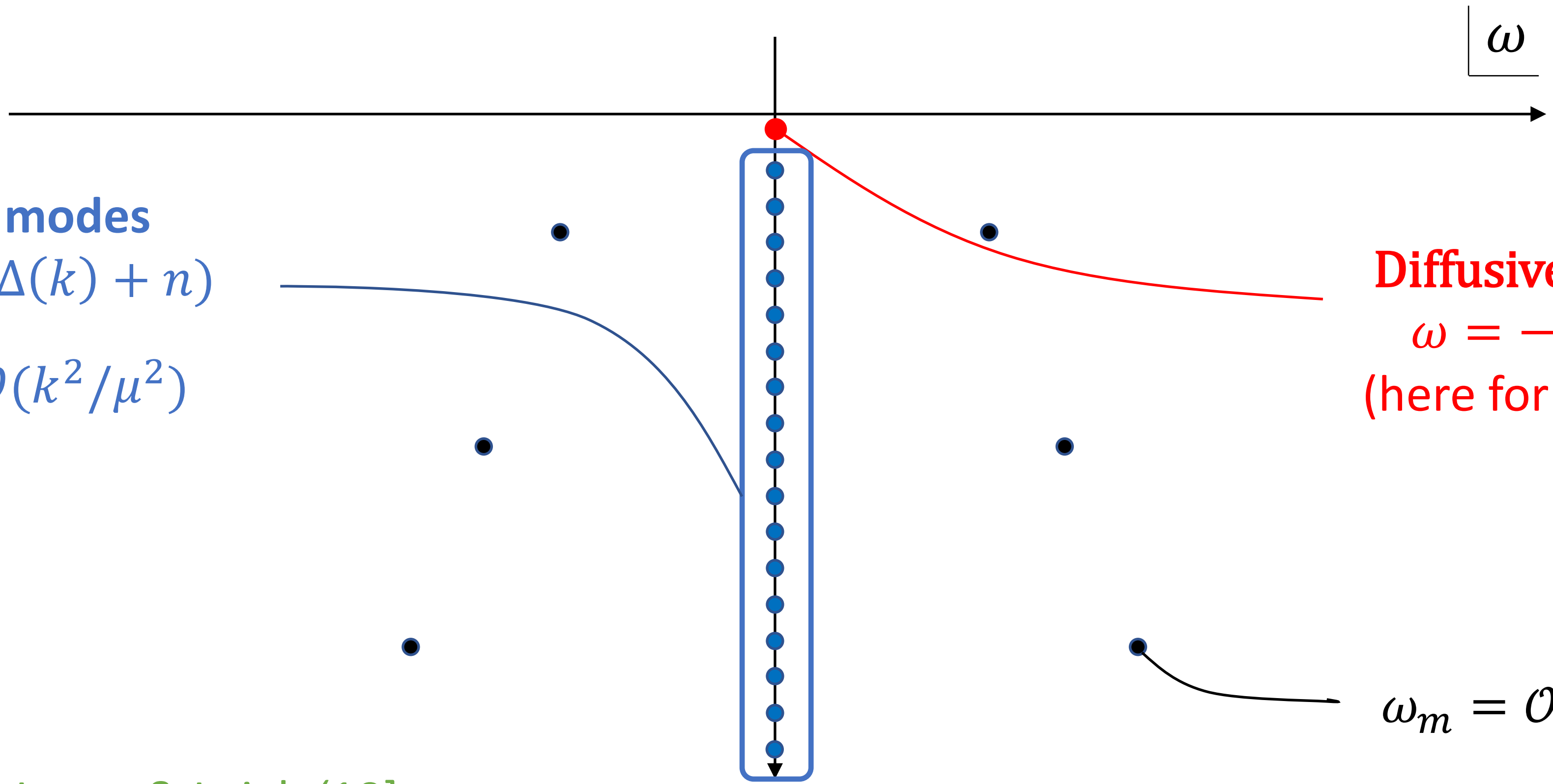
Hydrodynamic approximation at $\mu_q \gg T$

- The scale where **hydrodynamics breaks down** is identified by analyzing the **poles** of the correlator ↔ **QNM's** of AdS-RN

Matsubara modes

$\omega_n = -i2\pi T(\Delta(k) + n)$

$\Delta(k) = 1 + \mathcal{O}(k^2/\mu^2)$



Diffusive mode

$\omega = -iDk^2$

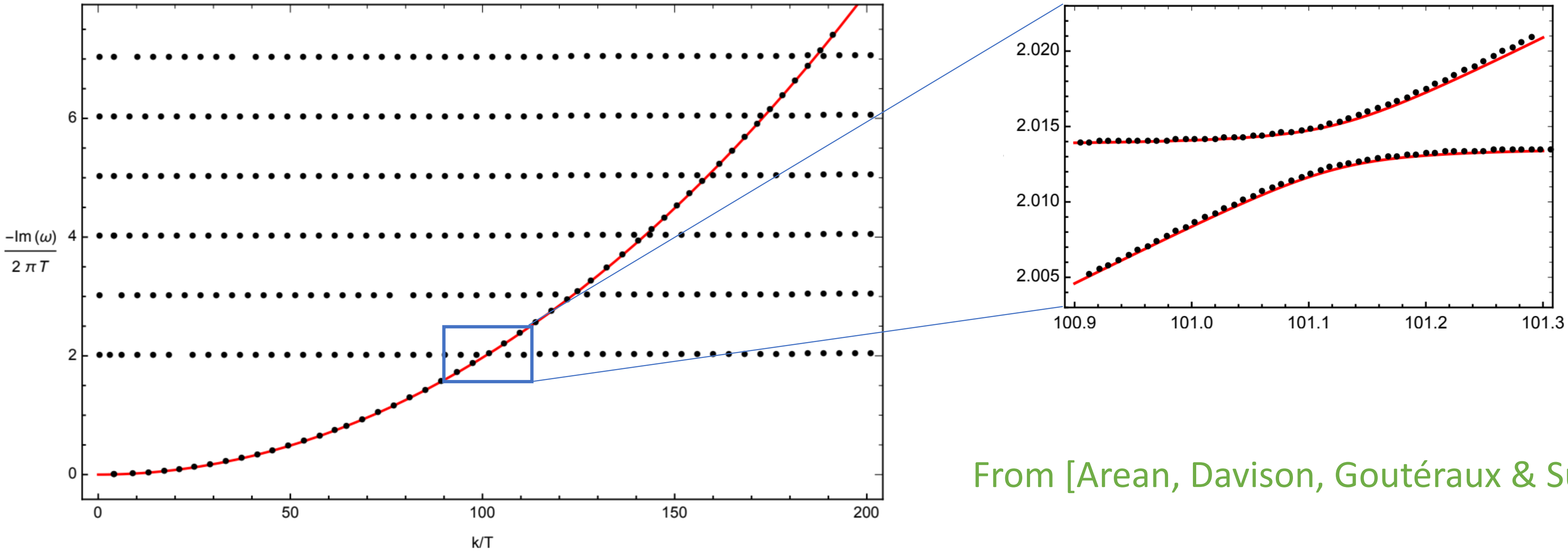
(here for $k \ll T$)

$\omega_m = \mathcal{O}(\mu)$

[e.g. Edalati, Jottar & Leigh '10]

Hydrodynamic approximation at $\mu_q \gg T$

- When **k is increased**, the poles **collide**, but a **diffusive pole** effectively remains
- For **hydrodynamics** to remain valid : $\boldsymbol{Res(\omega_n) \to 0}$ as $\mathrm{T}/\mu \to 0$

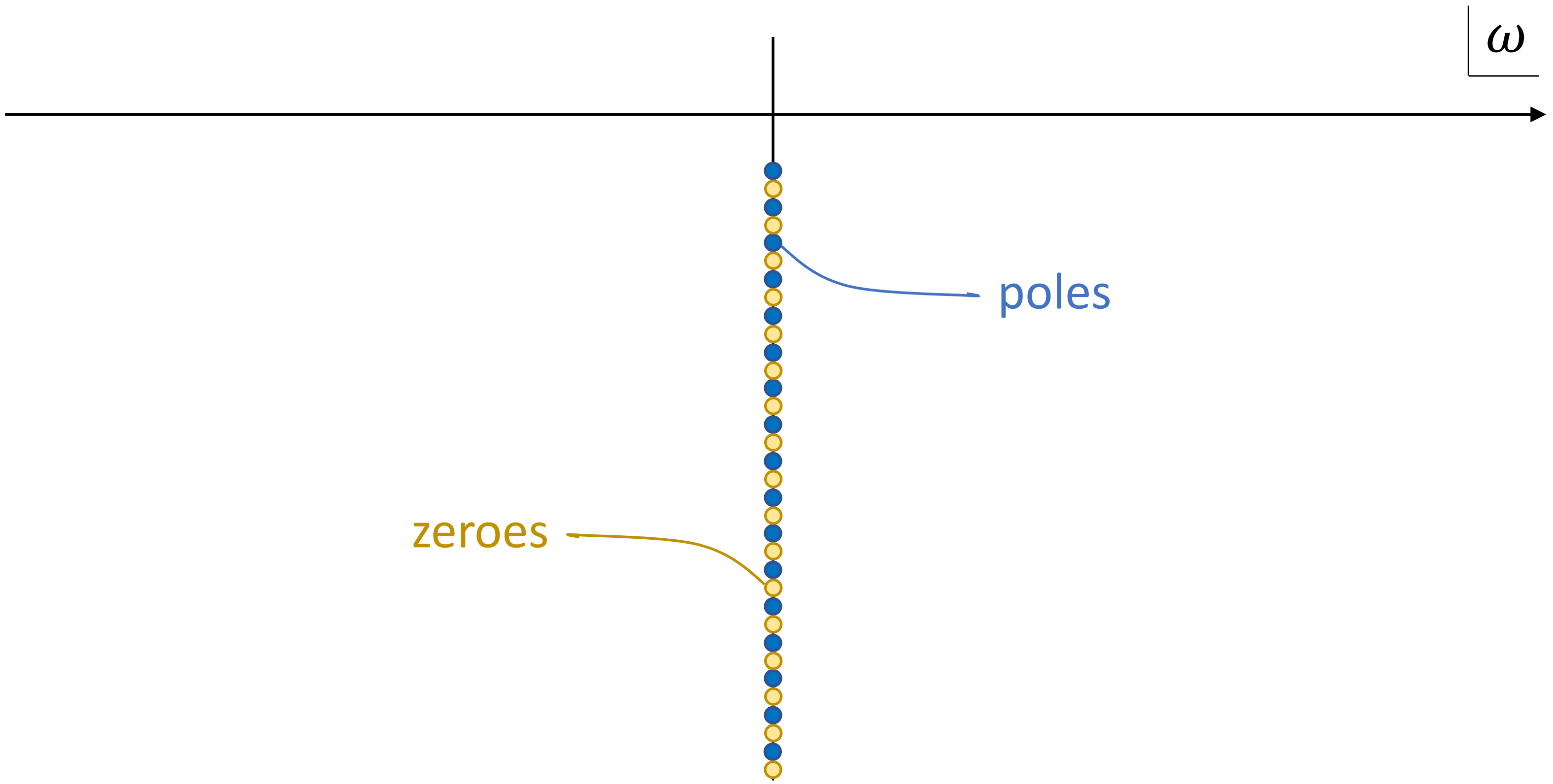


From [Arean, Davison, Goutéraux & Suzuki '21]

Hydrodynamic approximation at $\mu_q \gg T$

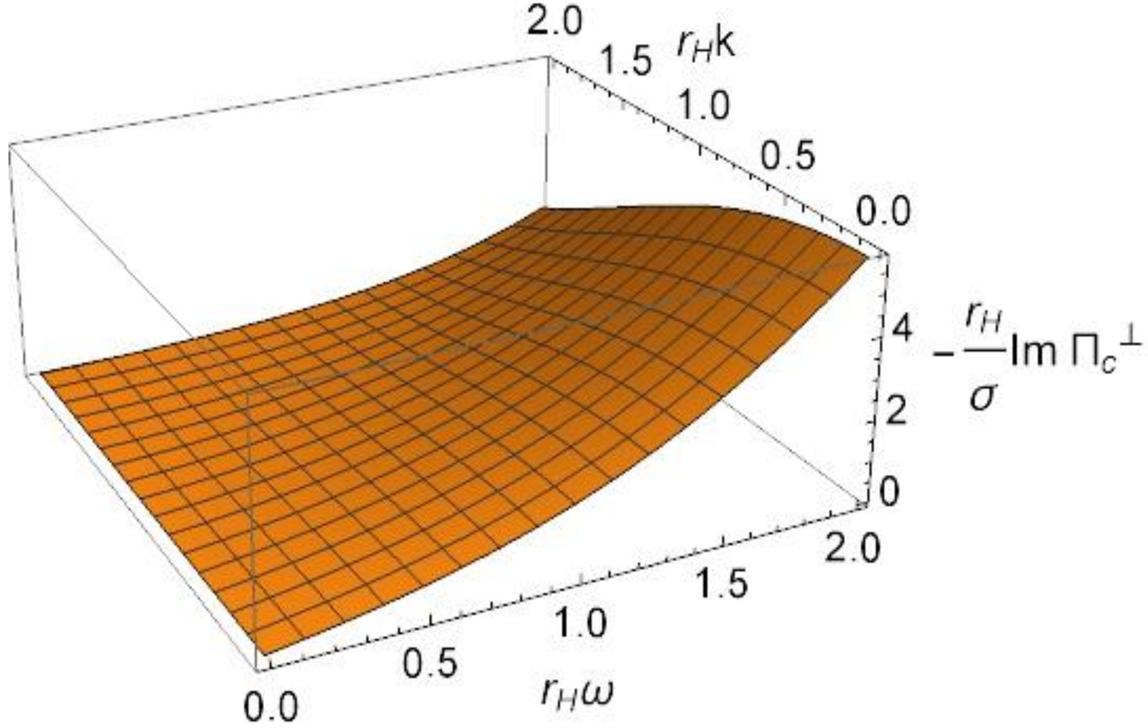
- The line of poles coalesces with a line of **zeroes**

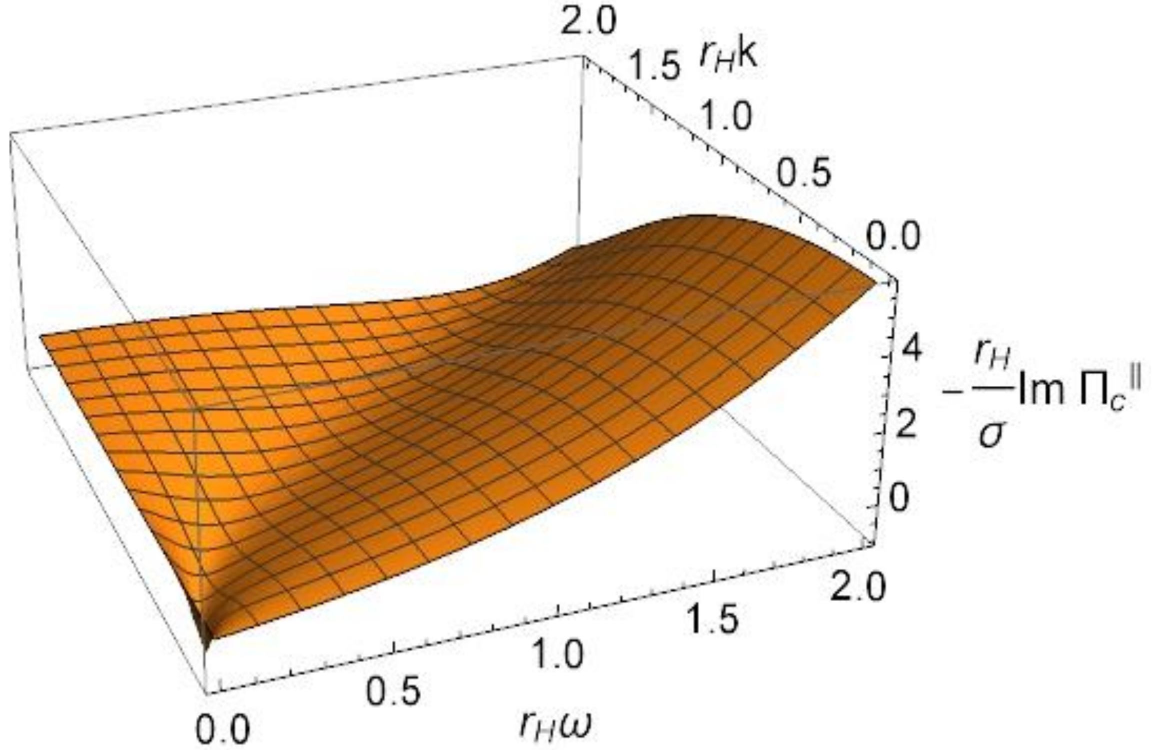
[Arean, Davison, Goutéraux & Suzuki '21]



Numerical results

Charged current correlators

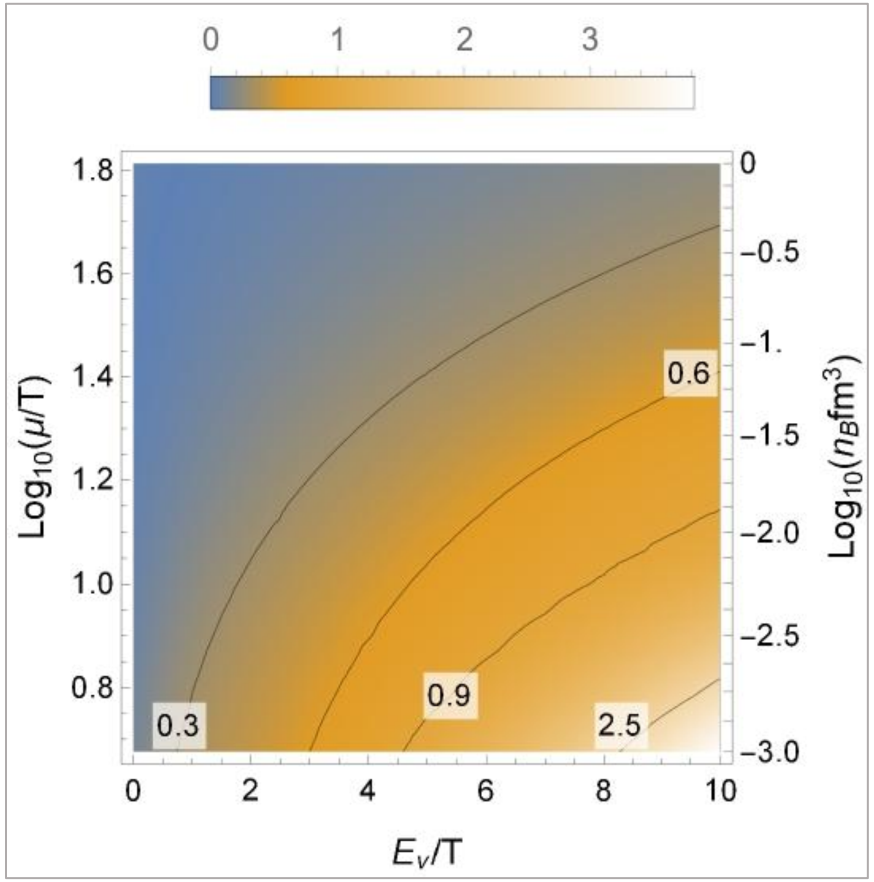




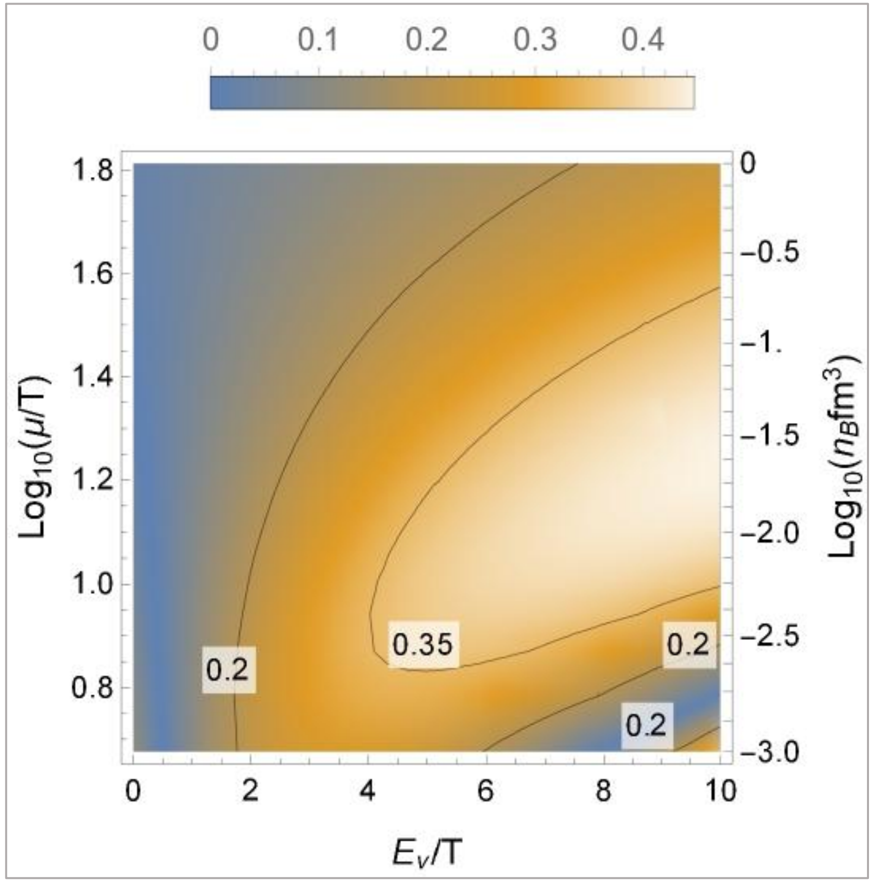
$$\frac{\mu_q}{T} \simeq 65$$

Opacities : comparison with hydro

$T = 10\ \mathrm{MeV}$



$\boldsymbol{\nu}$

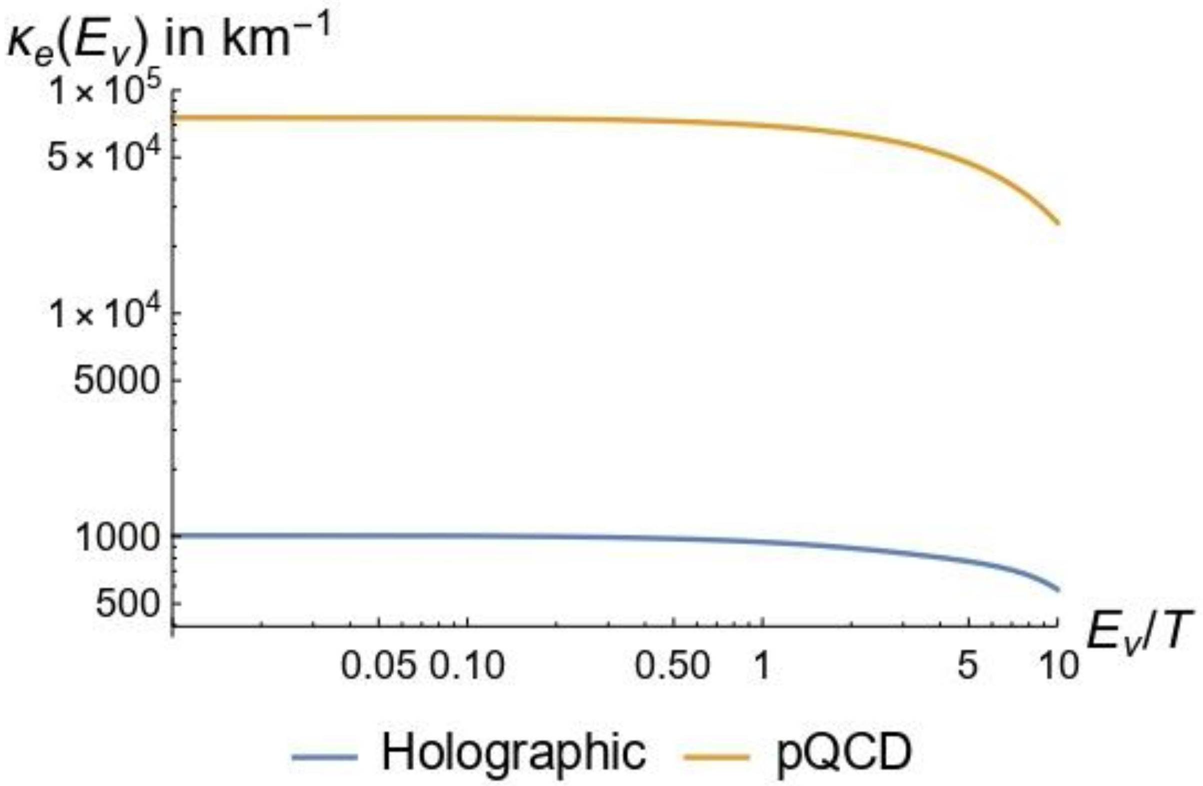


$\bar{\boldsymbol{\nu}}$

$$\kappa(E_\nu) = j(E_\nu) + \frac{1}{\lambda(E_\nu)}$$

Comparison with weak coupling

[Iwamoto ‘82]



$T = 10$ MeV,
$n_B = n_s = 0.11$ fm^{-3}

Different parameter dependence: e.g. at $T \to 0$:

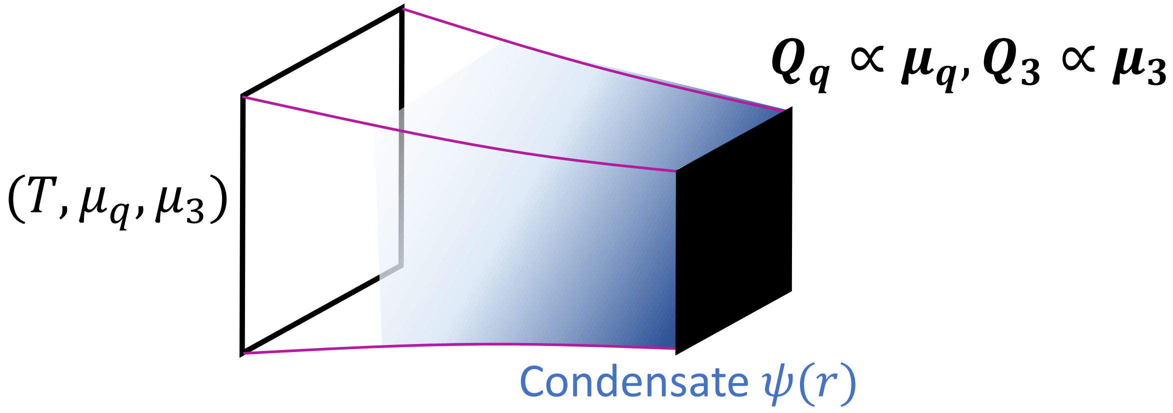
$$\kappa_e^{pQCD}(0) \sim G_F^2 \mu^2 \mu_\nu^3 \ , \qquad \kappa_e^{holo}(0) \sim G_F^2 \mu \mu_\nu^4 \, .$$

Towards isospin asymmetry

Phase diagram at finite μ_3

- μ_3 is introduced as a **source** for $L_0^3 + R_0^3$

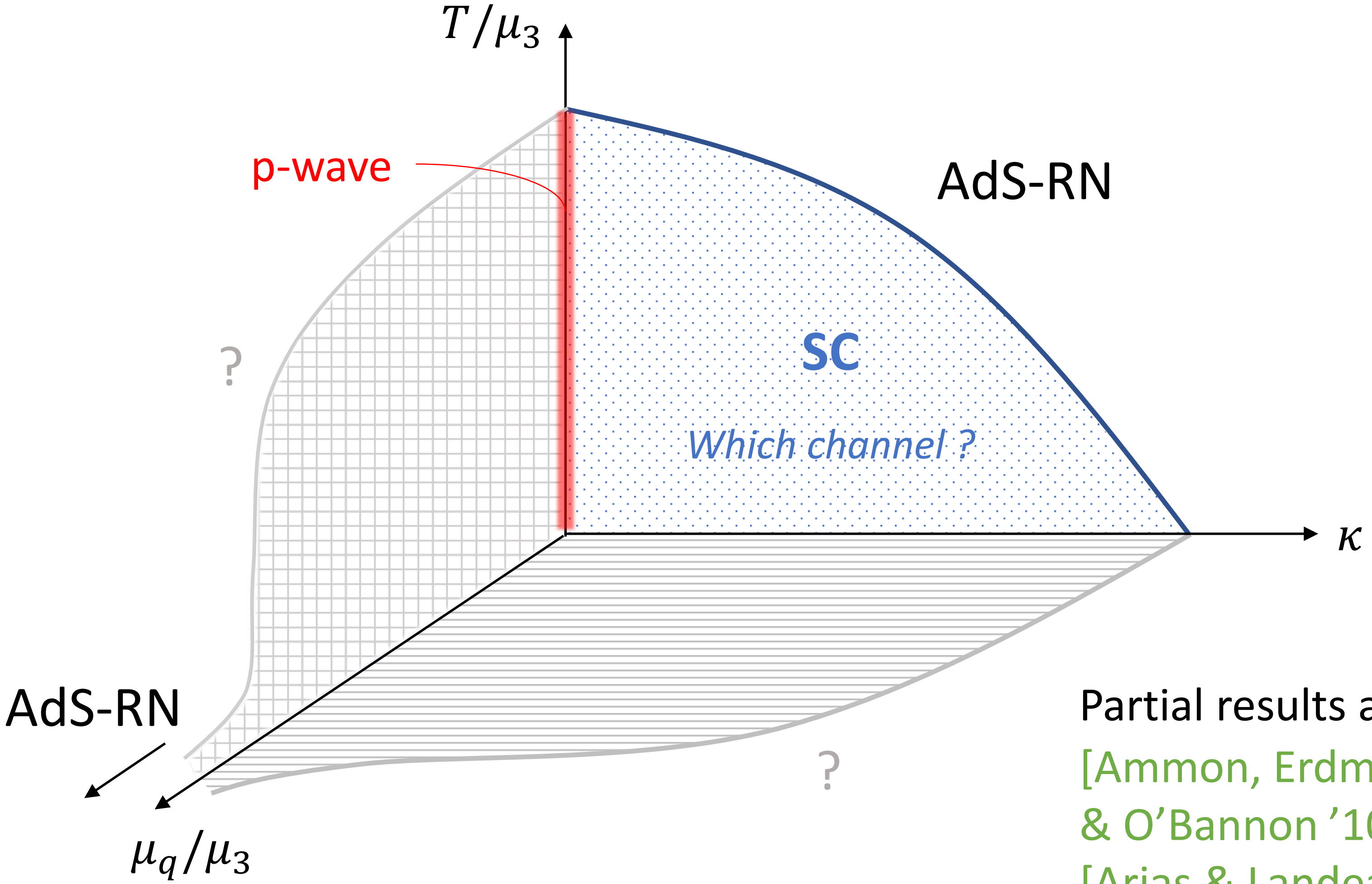
- **AdS RN** is still a solution of the Einstein-Yang-Mills equations (with $Q_3 \propto \mu_3$), but other solutions exist : **p-wave superconductors (SC)**, with **$U(1)_3$ spontaeously broken** [Gubser '08], [Gubser, Pufu, '08]



- Gubser and Pufu considered specific cases :
  - **General ansatz** but **probe** ($\kappa \to 0$)
  - **Back-reacted** but for a **specific ansatz** (κ finite)

- First step for our purpose: derive the **full 3-dimensional phase diagram $(\kappa, \mu_3/T, \mu_q/T)$**

Phase diagram at finite μ_3



Partial results at $\mu_q = 0$ in
[Ammon, Erdmenger, Grass, Kerner & O'Bannon '10]
[Arias & Landea '13]

General ansatz for SC solutions

- At $T \neq 0$ and $\mu_3 \neq 0$, the theory has **$SO(3) \times U(1)_3$** symmetry ($d = 3 + 1$)

→The **most general SC ansatz** for the gauge fields is

$$L = R = \frac{1}{2}\Phi(r)dt\, \mathbb{I}_2 + \frac{1}{2}\Phi_3(r)dt\, \sigma^3 + \frac{1}{2}A_z^1(r)dz\, \sigma^1 + \frac{1}{2}A_x^2(r)dx\, \sigma^2\ ,$$

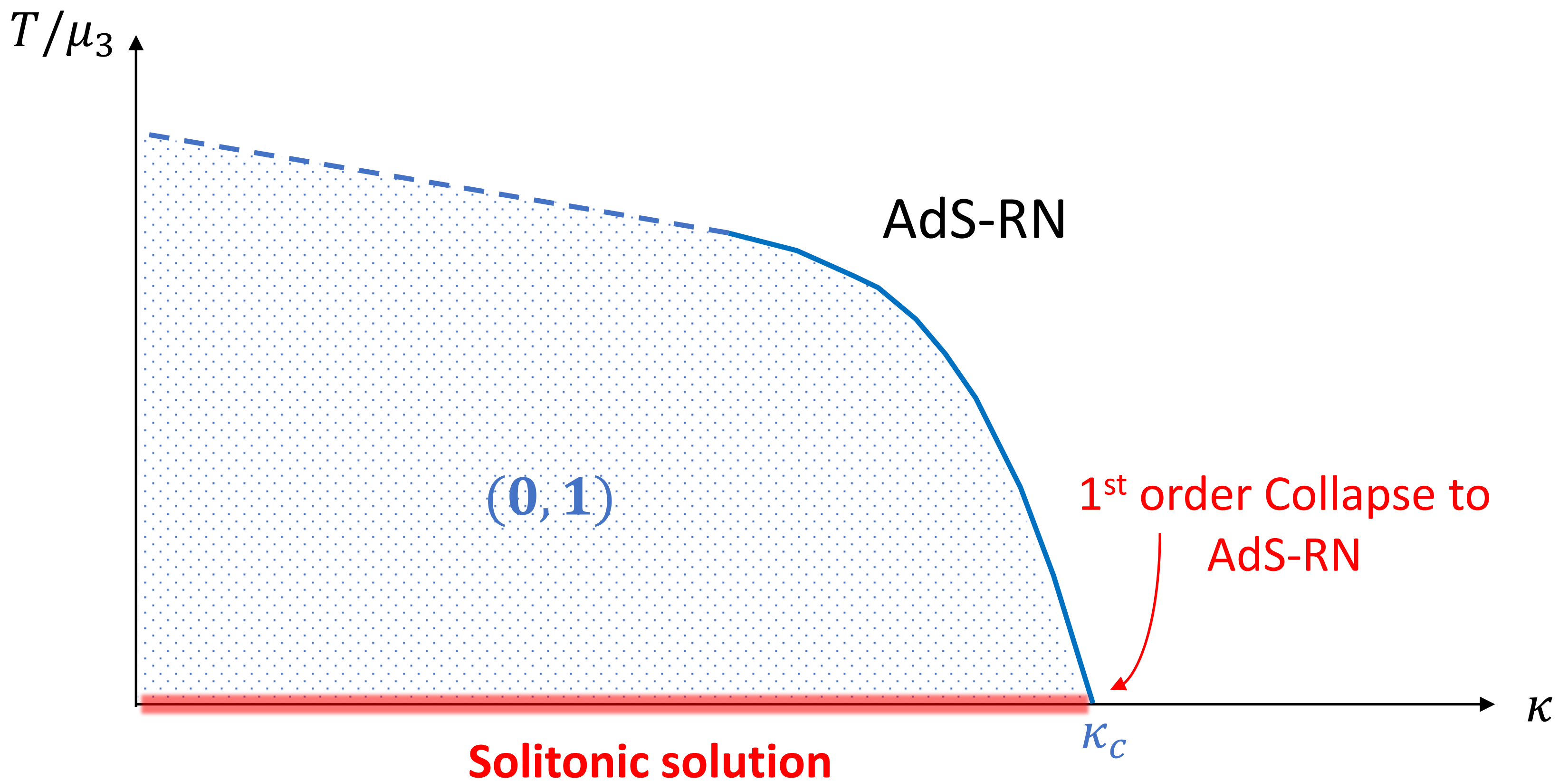
- We expect a **discrete set of solutions**, labeled by the **number of nodes (n, m)**

- The **dominant solutions** should be the **nodeless** solutions $(0,1)$ and $(1,1)$:
  - $(0,1) : A_x^2(r) = 0$, **p-wave** [Gubser, Pufu, '08]
  - $(1,1) : A_z^1(r) = A_x^2(r) \neq 0$, **(p+ip)-wave** [Gubser '08]

- Both **preserve $U(1)$** $\subset SO(3) \times U(1)$, so the **metric ansatz** can be chosen as

$$ds^2 = e^{2A(r)}(-f(r)dt^2 + f(r)^{-1}dr^2 + dx^2 + dy^2 + h(r)dz^2)$$

The plane $\mu_q = 0$

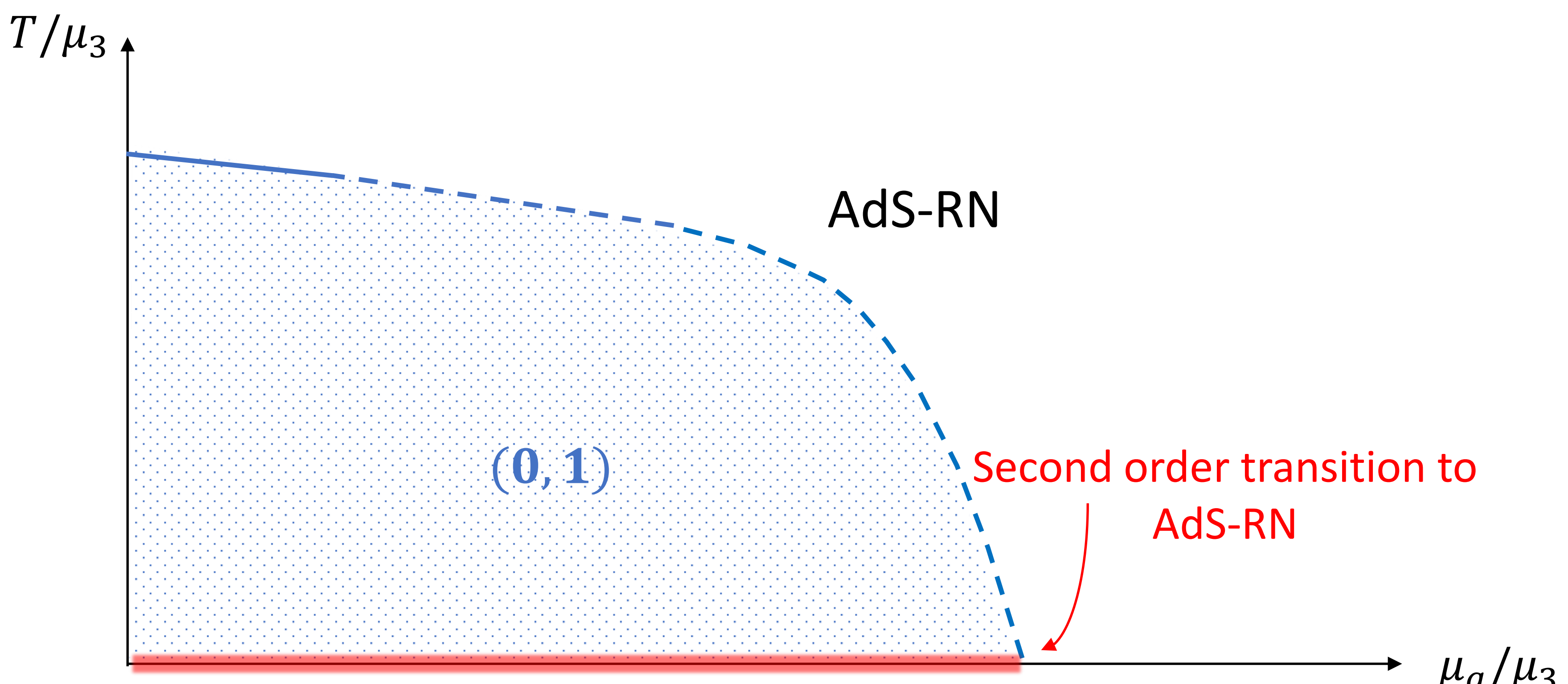


Solitonic solution

- Horizonless
- AdS^5 IR geometry

~ [Horowitz & Roberts, '09]

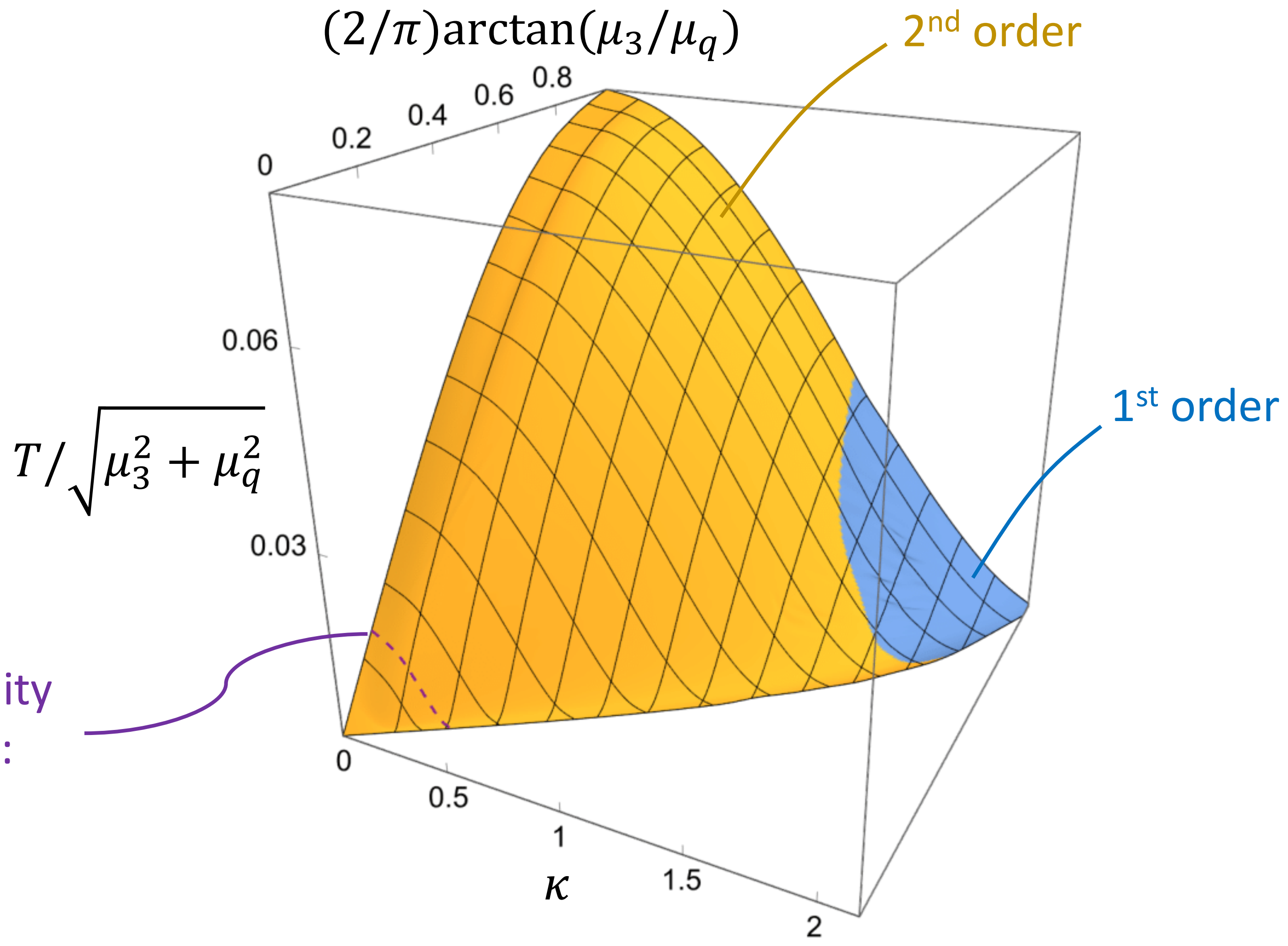
A plane at fixed $\kappa < \kappa_c$ with $\mu_q \neq 0$



Extremal hairy BH

- AdS^2 IR geometry
- Q_q carried by the horizon
- Q_3 carried by the hair

The full 3d phase diagram



From charge neutrality and β-equilibrium:

$\boldsymbol{\mu_3 < \mu_q/6}$

Summary and outlook

First step towards the description of holographic **neutrino transport** : toy model of **strongly-coupled quark matter**

- **Hydrodynamic** behavior
- **Opacity suppressed** compared with the weak coupling result
- More work is needed to **corroborate** these results

Several directions of improvement :

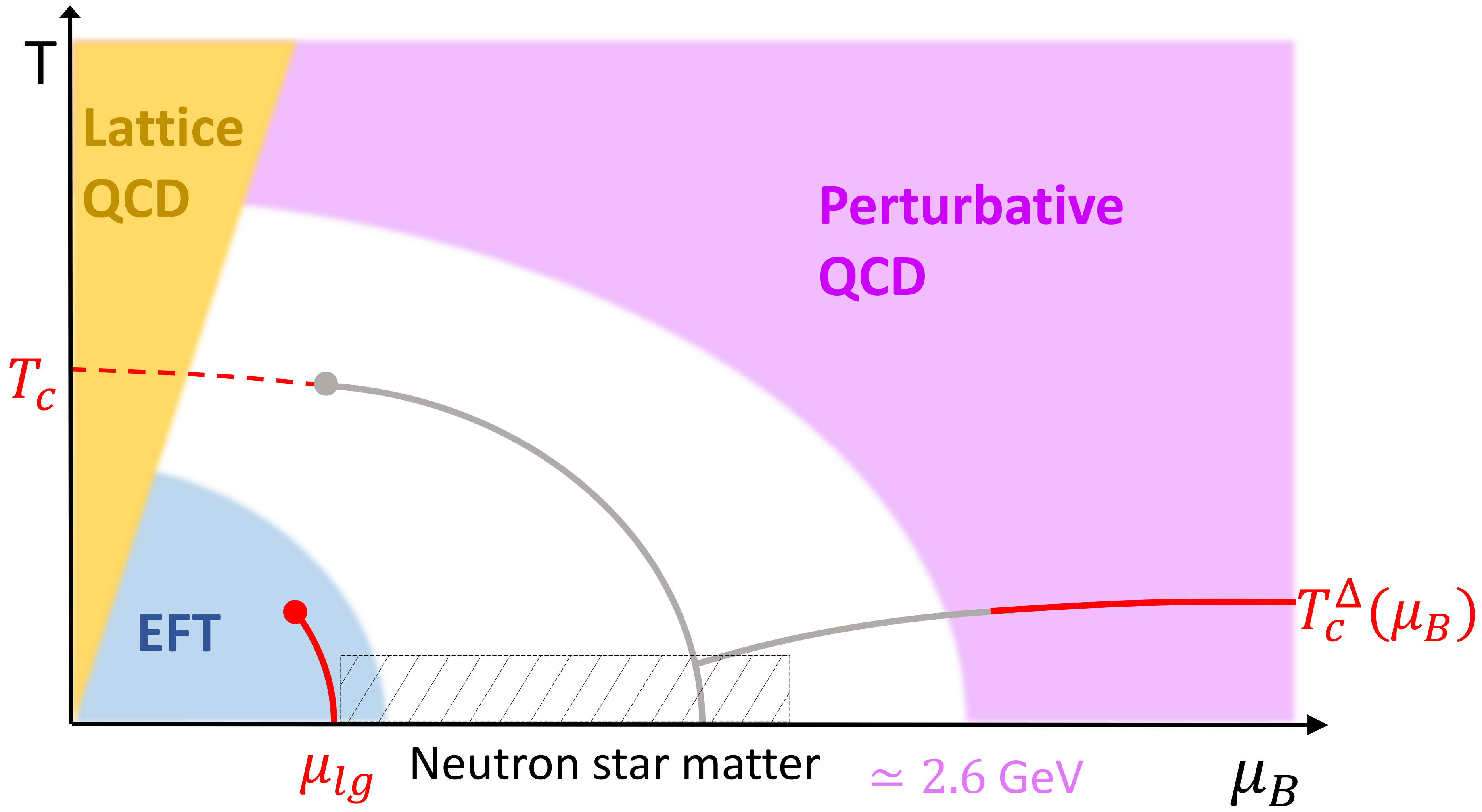
- Transport in an **isospin asymmetric** medium
- **Neutrino rates** from **neutral current** interactions
- **More realistic model** of holographic QCD

Thank you !

arXiv:2306.00192 [astro-ph.HE]
arXiv:2409.04630 [hep-th]

Appendix

Phases of QCD



H. Details about the perturbations of AdS-RN

Perturbations of AdS-RN

[Son & Starinets '02]
[Skenderis & van Rees '08]

$\langle J_\lambda J_\sigma \rangle^R$ is obtained by considering **perturbations** of the fields on top of **AdS-RN**

$$A^M_{L/R} \to \bar{A}^M_{L/R} + \delta A^M_{L/R}\,, \qquad g_{MN} \to \bar{g}_{MN} + \delta g_{MN}\,,$$

$$\forall \boldsymbol{\varphi},\ \delta\varphi = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{ik.x} C_k(z)\delta\varphi_0(k)\,, \qquad \text{At } z \sim z_H : \mathrm{C_k}(z) \sim (z_H - z)^{-\frac{ik^0 z_H}{4}}$$

Infalling boundary condition

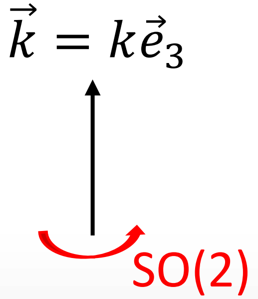
- Only $\boldsymbol{\delta T_{MN}} \propto \delta A_B$ **couples to $\boldsymbol{\delta g}$**
- The **charged current** gauge fields **decouple** from δg

Perturbations : Symmetries

The boundary plasma has an **SO(3) rotational invariance**

$$\langle J_\lambda J_\sigma \rangle^R(\omega, \vec{k}) = P^\perp(\omega, \vec{k})_{\lambda\sigma} i\boldsymbol{\Pi}^\perp(\boldsymbol{\omega}, \mathbf{k}) + P^\parallel(\omega, \vec{k})_{\lambda\sigma} i\boldsymbol{\Pi}^\parallel(\boldsymbol{\omega}, \mathbf{k})$$

For a given **mode $(\boldsymbol{\omega}, \vec{\boldsymbol{k}})$**, it reduces to an **SO(2) subgroup**



The perturbations are divided into **helicity sectors** that decouple

Helicity	Gauge field	Metric
$h = 0$	$\delta A_0\,, \delta A_3$	$\delta g_0^0\,, \delta g_0^3\,, \delta g_3^3\,, \delta g_1^1 + \delta g_2^2$
$h = 1$	$\delta A_{1,2}$	$\delta g_0^{1,2}\,, \delta g_3^{1,2}$
$h = 2$	$-$	$\delta g_2^1\,, \delta g_1^1 - \delta g_2^2$

Sector decoupled from the metric

Consider δA_μ that decouples from $\delta g_{\mu\nu}$

The modes are organized in terms of the **gauge-invariants** under

$$\mathrm{U}(1) : \delta A \to \delta A + \mathrm{d}\delta\lambda$$

$\boldsymbol{h = 1}$	$\boldsymbol{h = 0}$
$\delta A_1 \,, \delta A_2$	$E^\parallel \equiv \omega \delta A_3 + k \delta A_0$

The linearized **Maxwell equations** in each helicity sector can be written in terms of the gauge-invariants

The Π's are extracted from the **solutions near the boundary** $(z \to 0)$

$$\Pi^\perp \propto -\frac{\ell}{z} \frac{\partial_z \delta A_1}{\delta A_1}\bigg|_{z\to 0} \,, \qquad \Pi^\parallel \propto -\frac{\ell}{z} \frac{\partial_z \delta E^\parallel}{\delta E^\parallel}\bigg|_{z\to 0} \,.$$

Sector coupled to the metric

$\delta T_{MN} \propto \delta A_B$ couples to $\delta g_{\mu\nu}$

Again, organize the modes in terms of the **gauge-invariants** under :

- $U(1) : \delta X \to \delta X + \mathrm{d}\delta\lambda$
- **Diffeomorphisms** :

$$\delta X_M \to \delta X_M + \delta\xi^N \partial_N \bar{X}_M + \bar{X}_N \partial_M \delta\xi^N$$

$$\delta g_{MN} \to \delta g_{MN} + \nabla_M \delta\xi_N + \nabla_N \delta\xi_M$$

$\boldsymbol{h = 1}$	$\boldsymbol{h = 0}$
$\delta X_{1,2}$	$\delta S_1 \equiv \omega\delta X_3 + k\delta X_0 + a(z)\mu\, k(\delta g_1^1 + \delta g_2^2)$
$\delta Y^{1,2} \equiv k\delta g_0^{1,2} + \omega\delta g_3^{1,2}$	δS_2 $\equiv 2\omega k\delta g_0^3 + \omega^2\delta g_z^z - f(z)k^2\delta g_0^0 + b(z,\omega/k)k^2(\delta g_1^1 + \delta g_2^2)$

Sector coupled to the metric

The linearized **Einstein-Maxwell equations** in each helicity sector can be written in terms of the **gauge-invariants** :

- $\boldsymbol{h=1}$ **:** 2 coupled 2nd order ODE's for $\delta X_{1,2}$ and $\delta Y^{1,2}$
- $\boldsymbol{h=0}$ **:** 2 coupled 2nd order ODE's for δS_1 and δS_2

The Π's are extracted from the **solutions near the boundary** $(z \to 0)$

$$\boldsymbol{h=1}: \qquad \delta X_1 = \delta\hat{X}_1 + z^2 \delta\Pi_{X_1} + \cdots , \qquad \delta\Pi_{X_1} \equiv \boldsymbol{\Pi^{\perp}_{XX}} \delta\hat{X}_1 + \Pi^{\perp}_{XY} \delta\hat{Y}^1 ,$$

Compute **2 solutions** and invert the linear relation

$$\left(\boldsymbol{\Pi^{\perp}_{XX}} \; \Pi^{\perp}_{XY} \right) = \left(\delta\Pi^{(1)}_{X_1} \; \delta\Pi^{(2)}_{X_1} \right) \begin{pmatrix} \delta\hat{X}^{(1)}_1 & \delta\hat{X}^{(2)}_1 \\ \delta\hat{Y}^1_{(1)} & \delta\hat{Y}^1_{(2)} \end{pmatrix}^{-1}$$