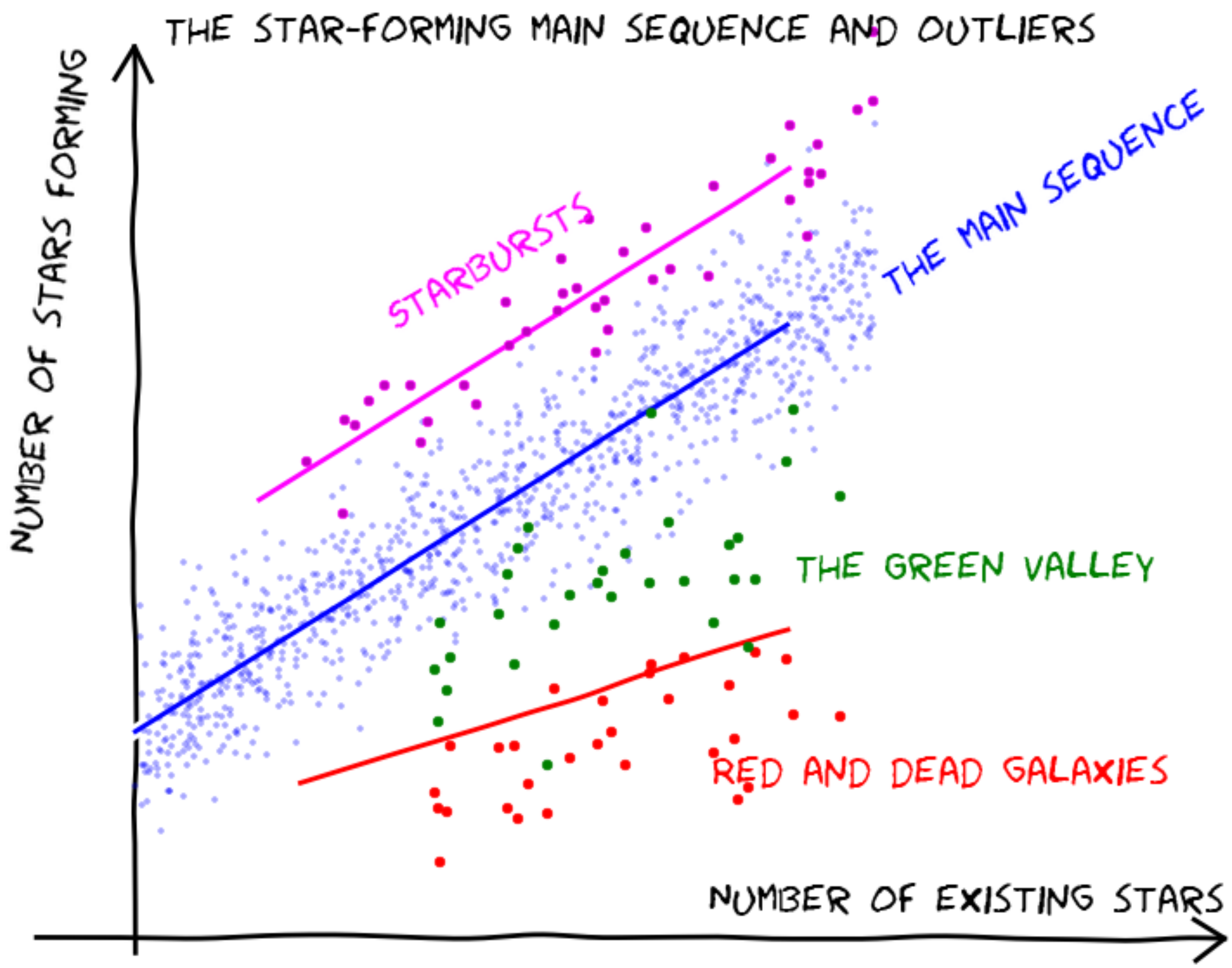
From Pixels to Parameters

- Part 1: Galaxy Surveys
  - Galaxies beyond the point-particle picture
- Part 2: From Pixels to Power Spectra
  - Systematics, systematics, systematics (& estimators)
- Part 3: From Power Spectra to Parameters
  - Covariances, inference & error bars you can trust
- Part 4: Weak Lensing
  - Galaxies beyond the spin-2 field picture

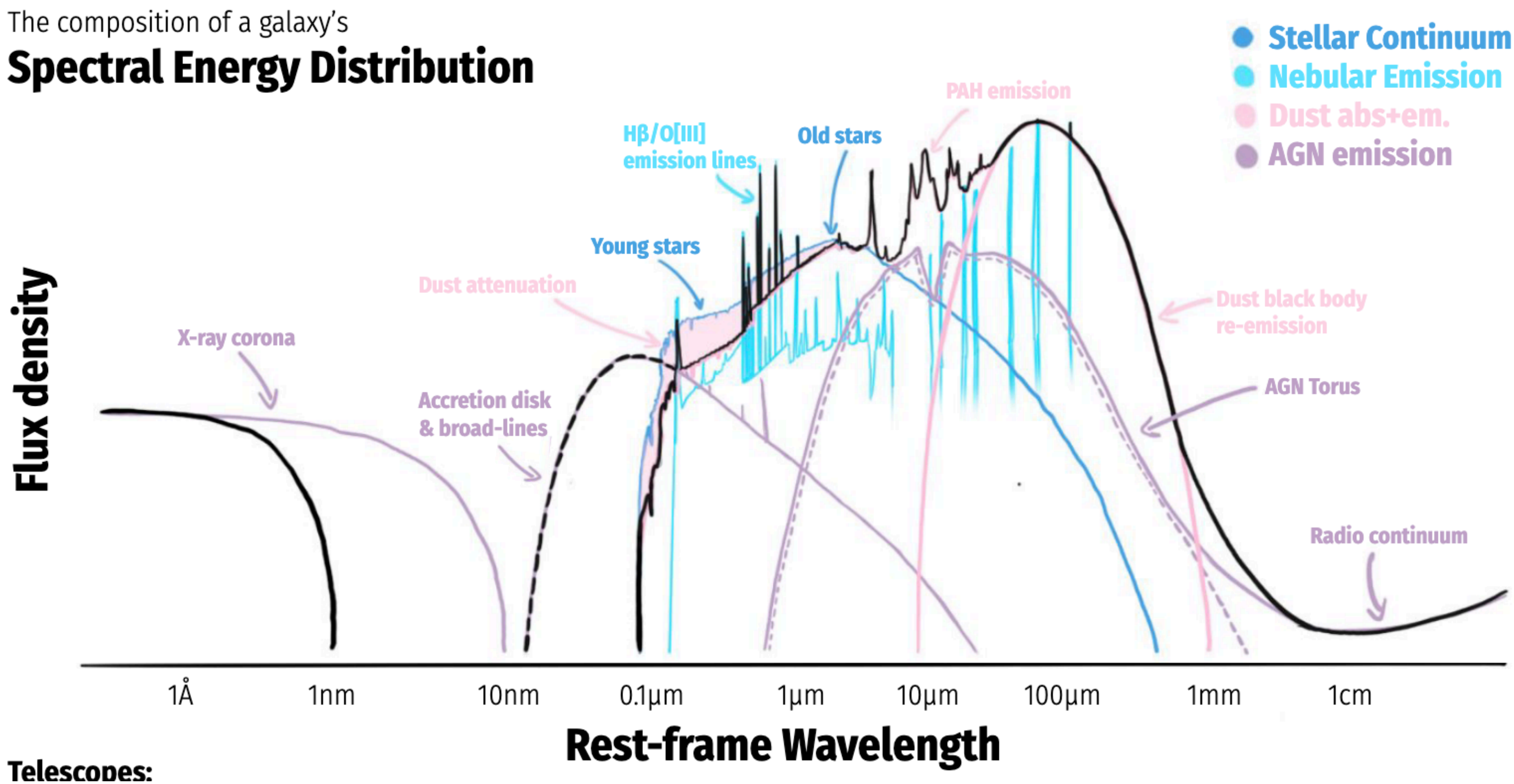
Powerful surveys → excellent statistical power → systematic effects (astrophysics, observational) more significant. Correct cosmological interpretation of observations will require more and more astrophysics.

Galaxies 101 (astronomer's version)



credit: Candels collaboration

How to Bake a Galaxy Spectrum



Iyer+ 2025

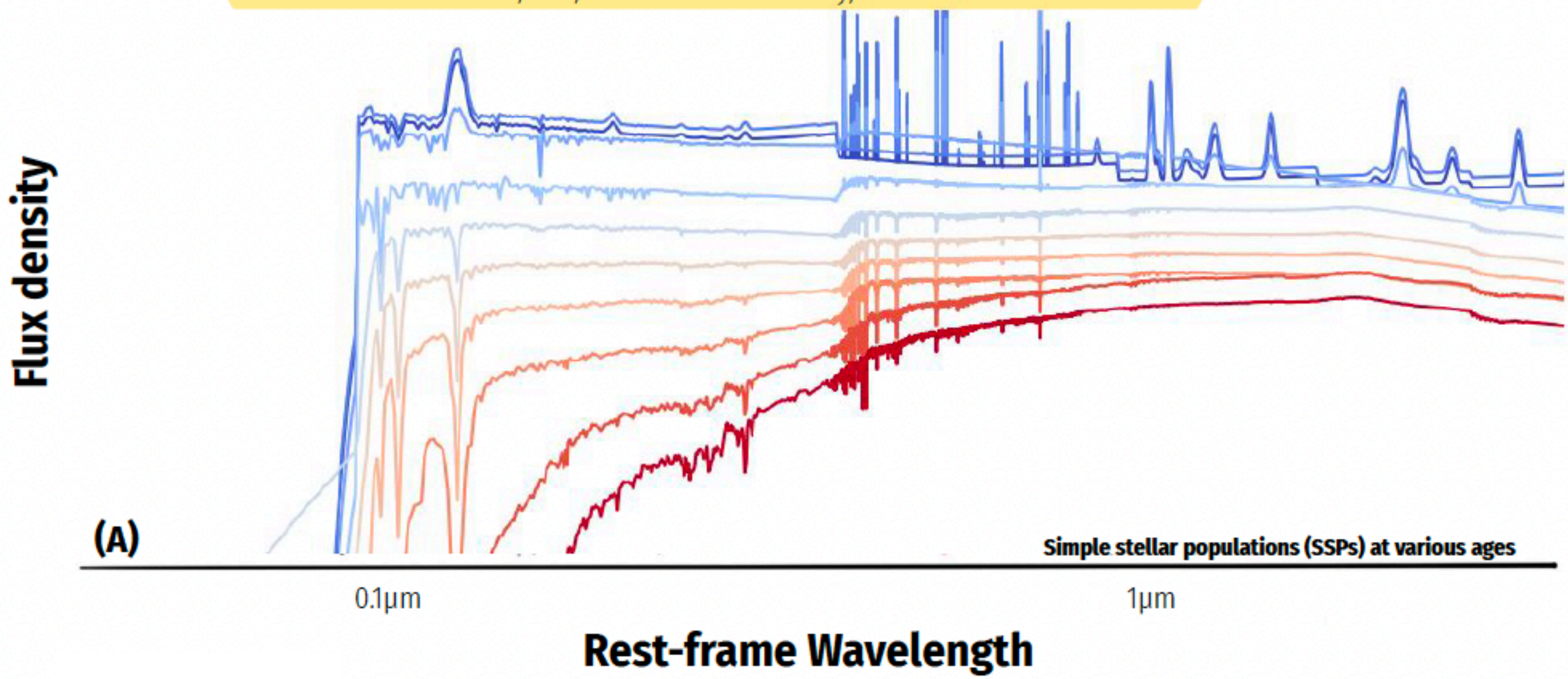
The composition of a galaxy's

Stellar continuum

The light of a galaxy's stars, shaped by their age and metallicity

> UV-to-NIR stellar continuum based on the galaxy's stellar populations

> constrains stellar mass, SFR, star formation history, stellar chemical enrichment



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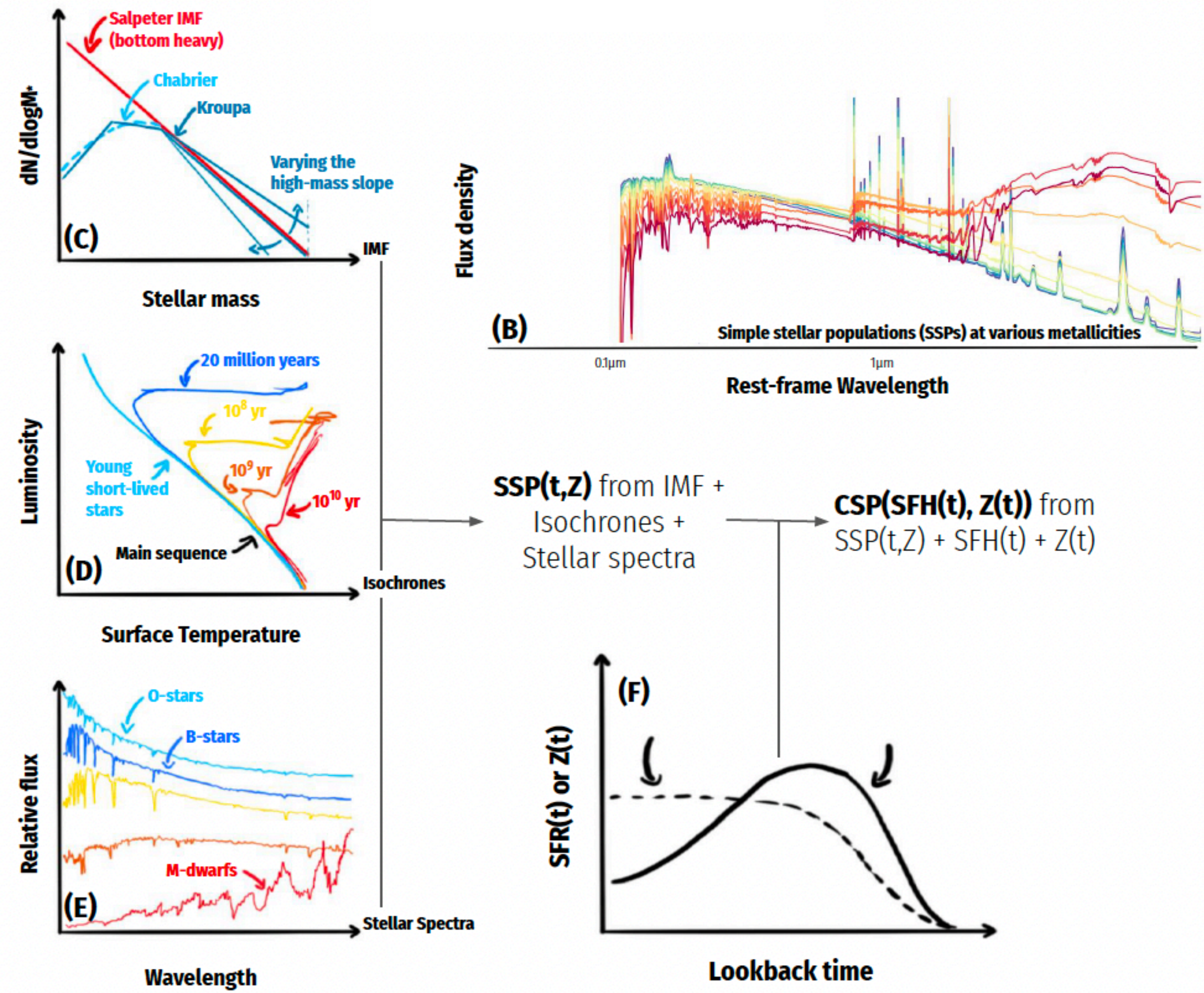

Salpeter IMF
(bottom heavy)
Chabrier
Kroupa
Varying the
high-mass slope
dN/dlogM*
(C)
IMF
Stellar mass
Flux density
(B)
Simple stellar populations (SSPs) at various metallicities
0.1μm
1μm
Rest-frame Wavelength
20 million years
10^8 yr
10^9 yr
10^{10} yr
Young
short-lived
stars
Main sequence
Luminosity
(D)
Isochrones
Surface Temperature
SSP(t,Z) from IMF +
Isochrones +
Stellar spectra
CSP(SFH(t), Z(t)) from
SSP(t,Z) + SFH(t) + Z(t)
O-stars
B-stars
M-dwarfs
Relative flux
(E)
Stellar Spectra
Wavelength
(F)
SFR(t) or Z(t)
Lookback time

Iyer+ 2025

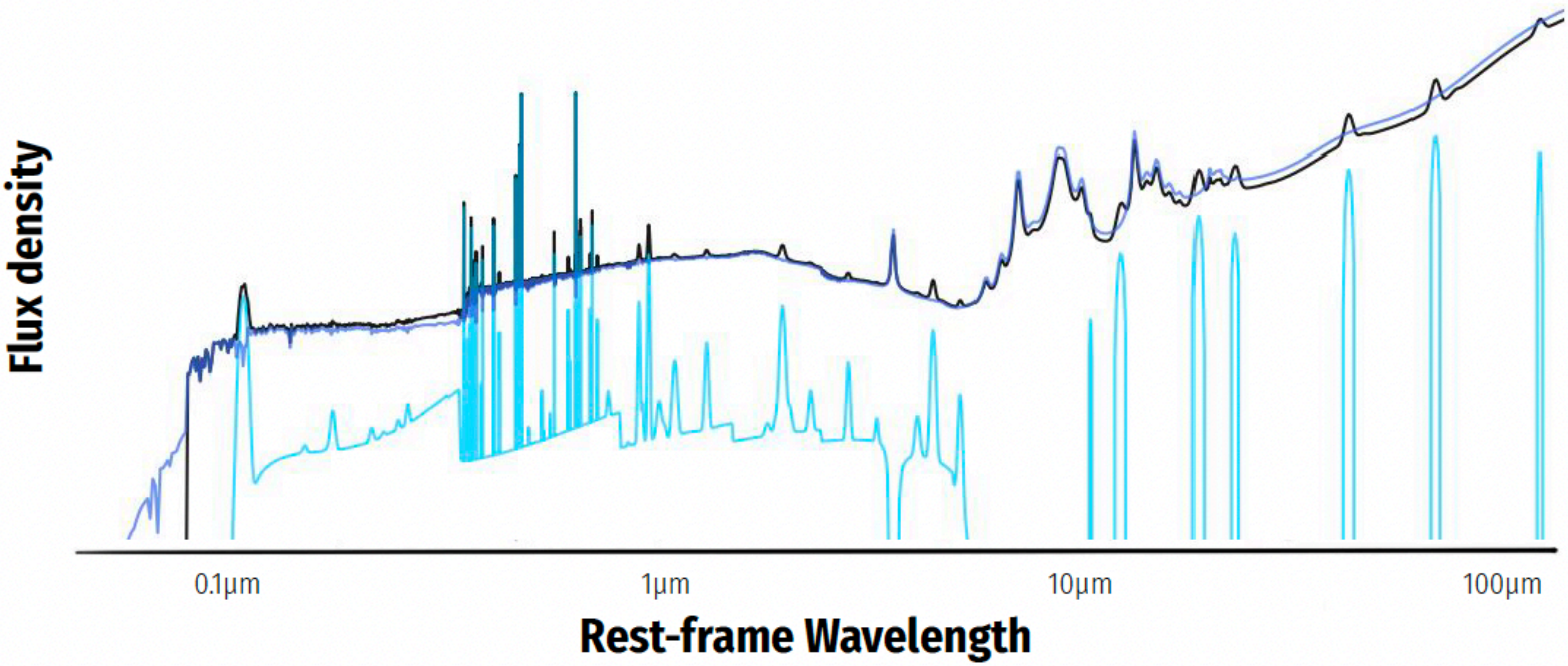
The composition of a galaxy's

Nebular emission

Light from collisionally excited gas and plasma in the interstellar medium (ISM)

> UV-to-NIR nebular continuum, UV-to-radio emission lines based on ISM gas

> probes local ISM conditions, chemical enrichment, winds, shocks and outflows

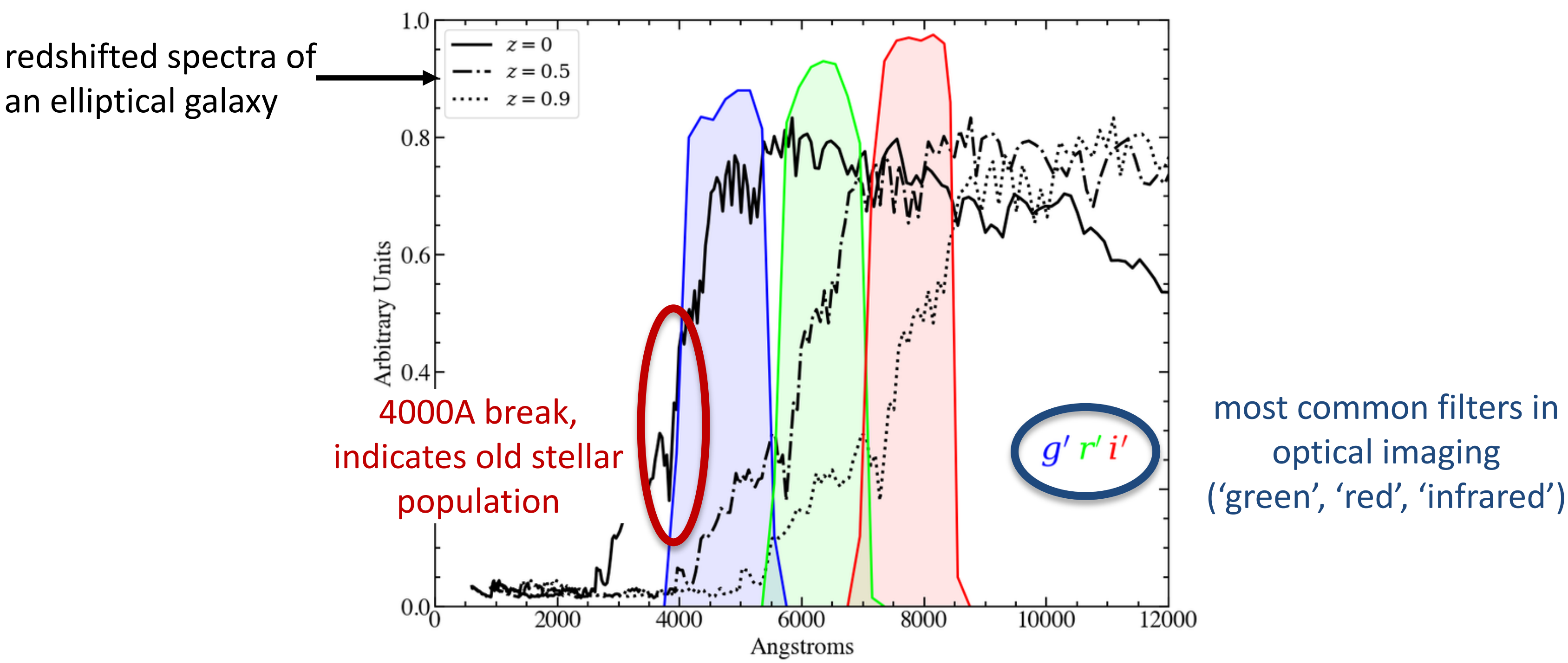


Iyer+ 2025

Galaxy Surveys

Galaxy Surveys: Imaging

- Images → angular coordinates of objects
  - In practice, astrometric calibration + object detection are major research areas!
- Need to identify galaxies, and estimate their redshifts
  - Stars are point sources, galaxies extended; occupy specific regions in color space



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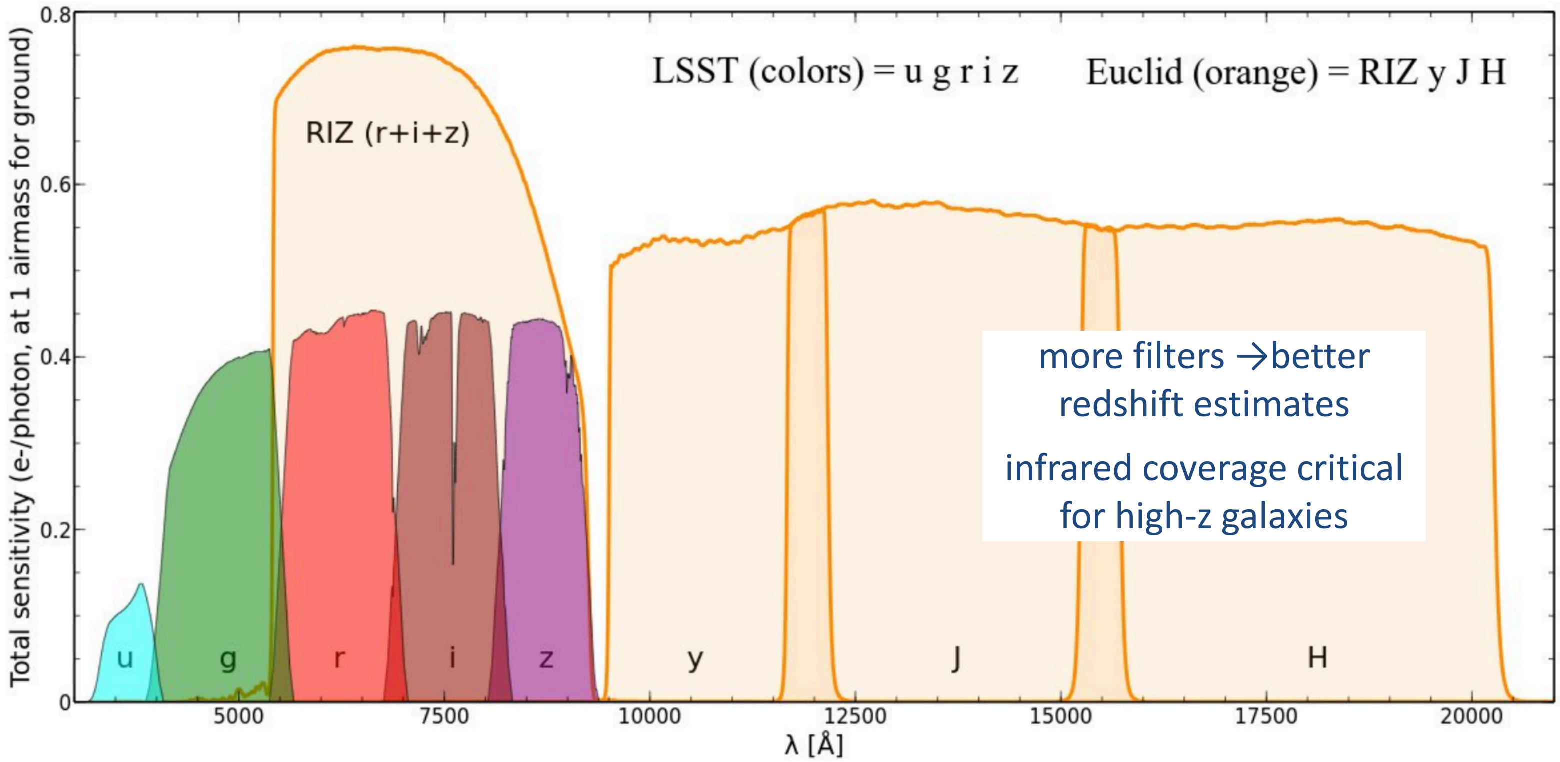
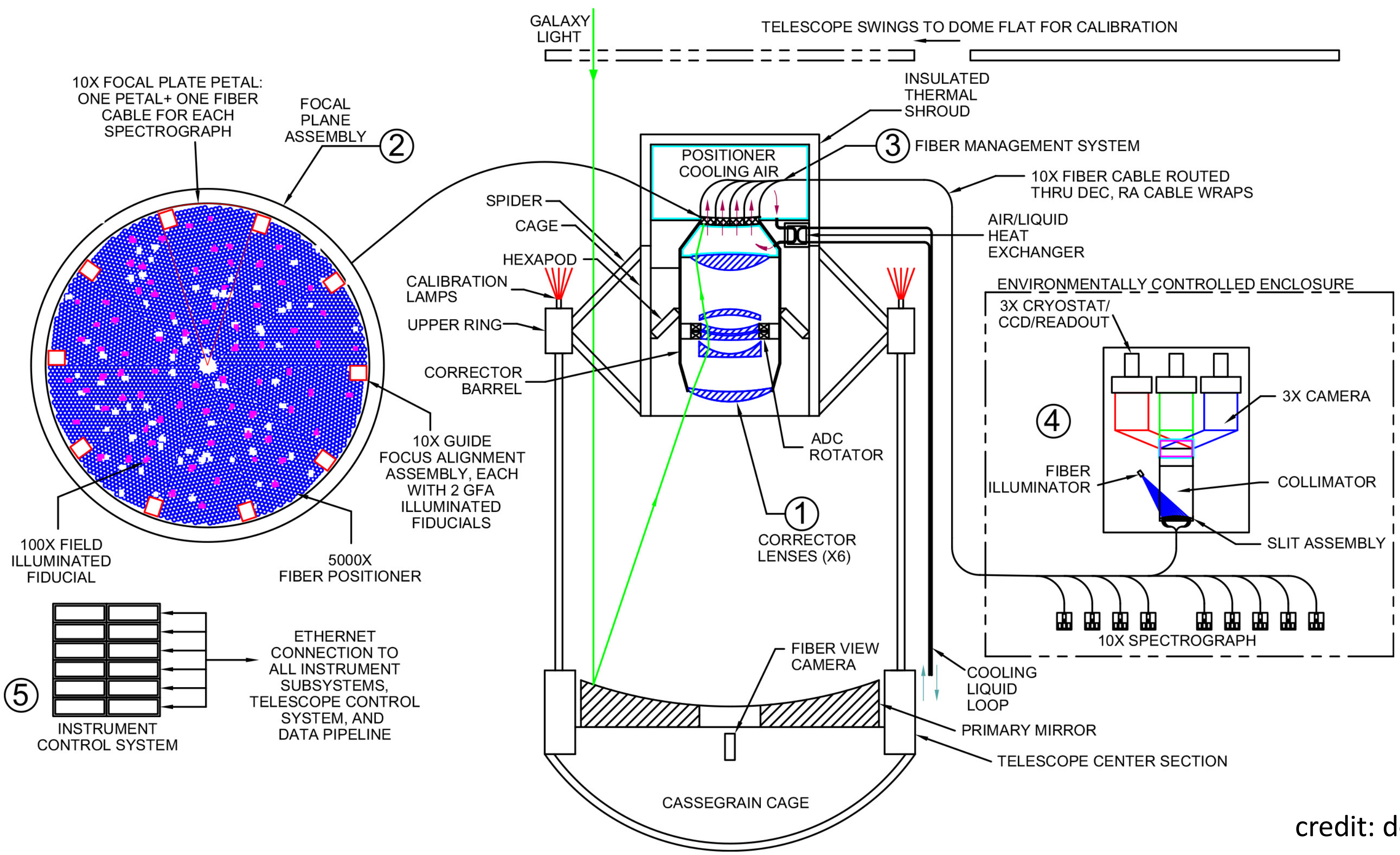


Galaxy Surveys: Spectroscopy

- Classic: point spectrograph to individual galaxies
  - Wide-field surveys use multi-object spectrographs, e.g., BOSS (1000 fibers), DESI (5000 fibers)



credit: desi.lbl.gov

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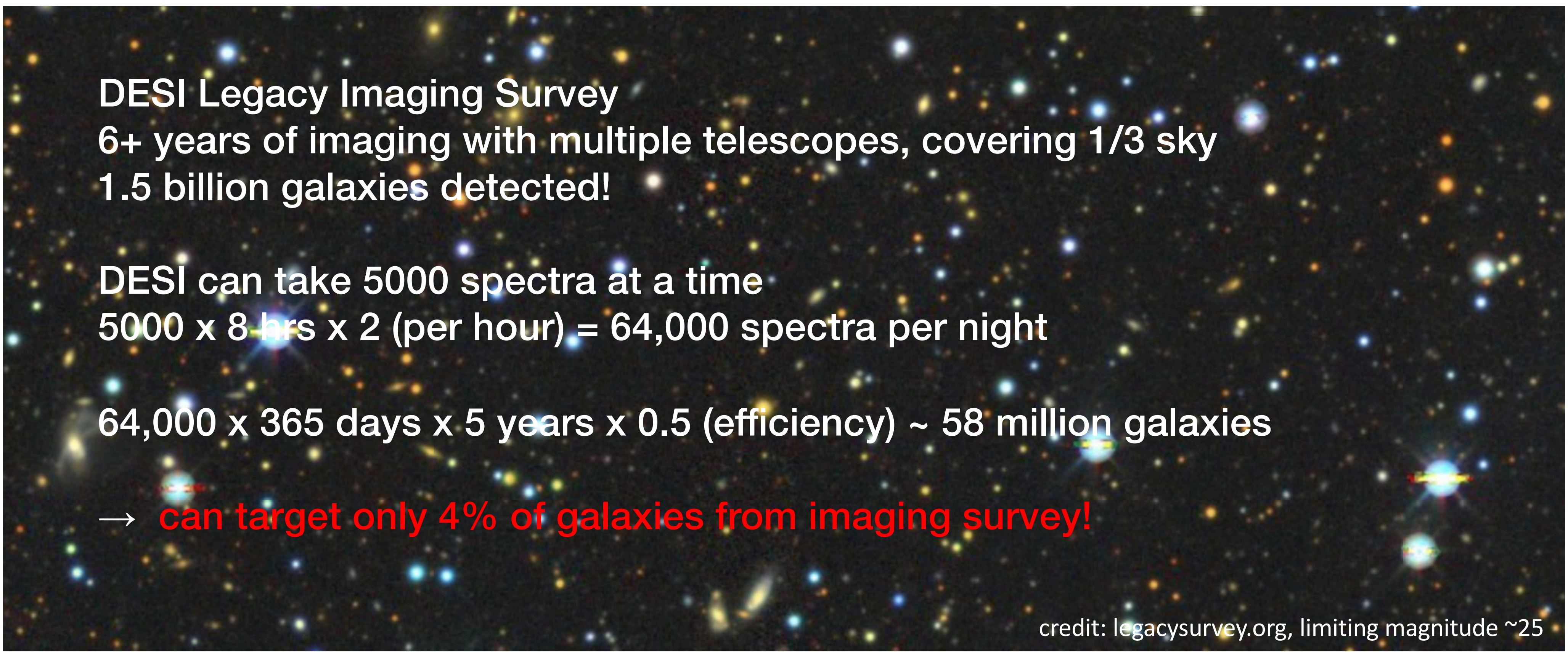

DESI Legacy Imaging Survey
6+ years of imaging with multiple telescopes, covering 1/3 sky
1.5 billion galaxies detected!

DESI can take 5000 spectra at a time
5000 x 8 hrs x 2 (per hour) = 64,000 spectra per night

64,000 x 365 days x 5 years x 0.5 (efficiency) ~ 58 million galaxies

→ can target only 4% of galaxies from imaging survey!

credit: legacysurvey.org, limiting magnitude ~25

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    - Goal 1: shot noise subdominant at BAO scales

      → **n >** $1/P(k{\sim}0.1) \sim$ **$10^{-3..-4}$ (h/Mpc)^3**
    - Goal 2: maximize chances for getting **reliable redshift during exposure time**

      → **magnitude cut**

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    - Goal 2: maximize chances for getting **reliable redshift during exposure time**

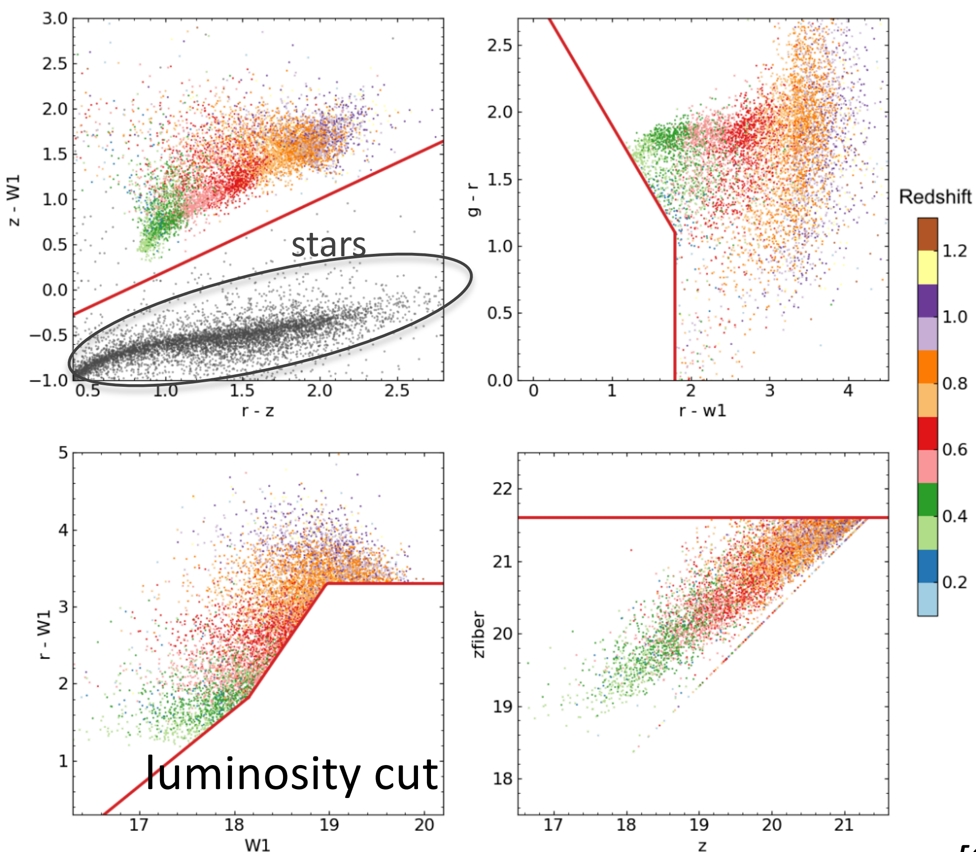
      → **magnitude cut**
    - Goal 3: select galaxy populations with **well-understood selection function**

      → different galaxy types (LRG, ELG, QSO) at different z, identified by color cuts

DESI Target Selection

Large Red Galaxies (LRG)

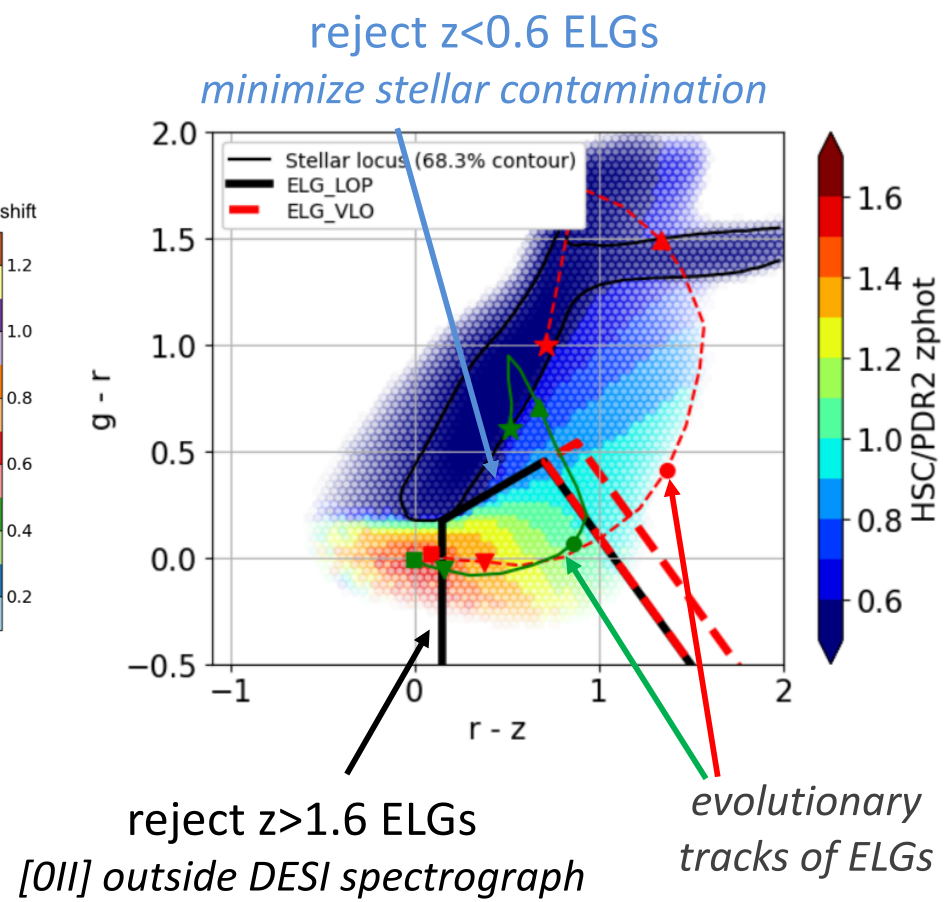
0.4 < z <~ 1.0



Zhou+ 2023

Emission Line Galaxies (ELG)

0.6 < z < 1.6



Raichoor+ 2023

Galaxy Surveys: Spectroscopy

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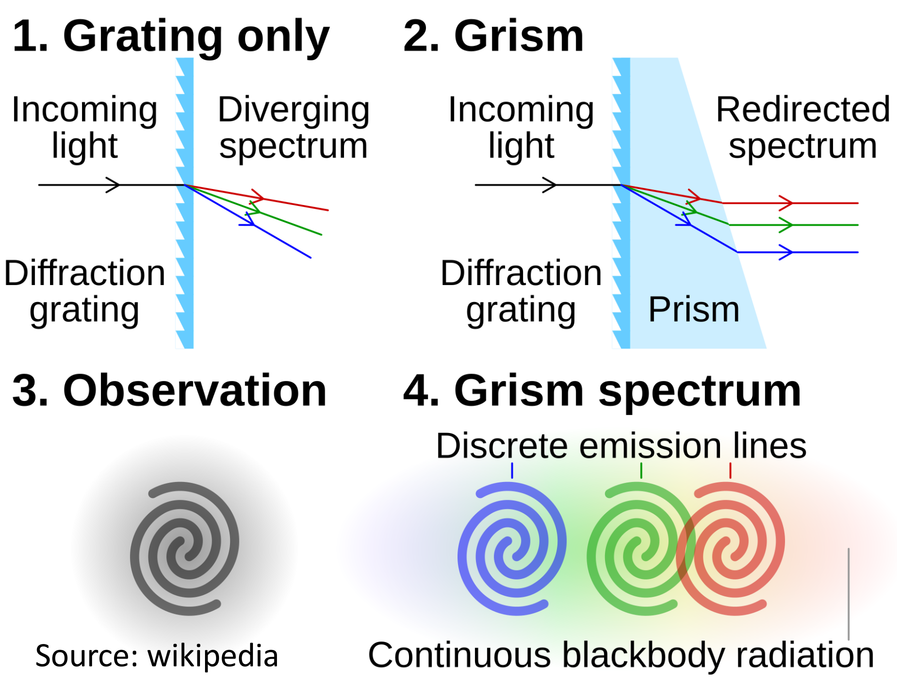
- Slit-less/grism spectroscopy: disperse all (sufficiently bright) galaxies within field of view



Source: wikipedia

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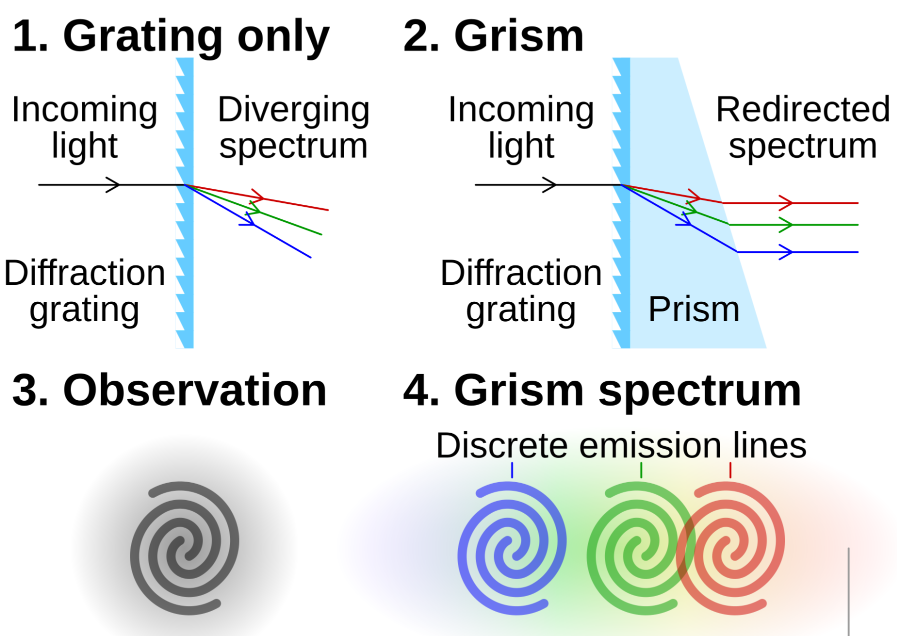
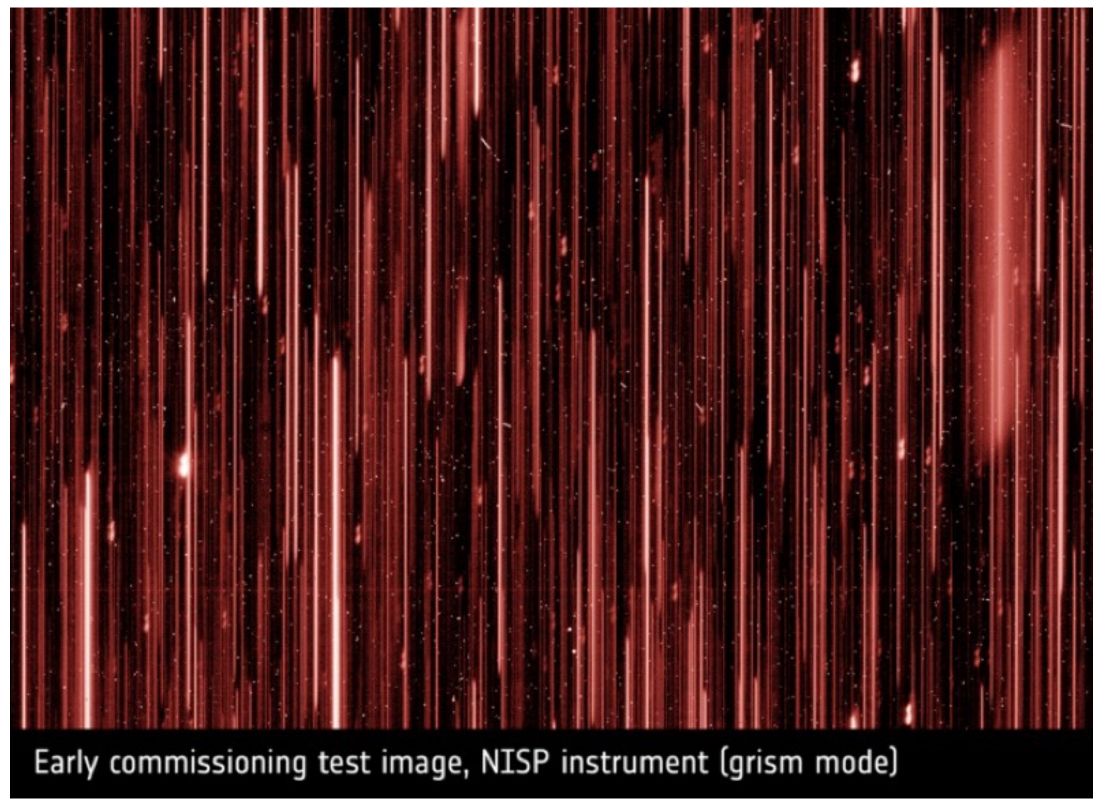


Source: wikipedia



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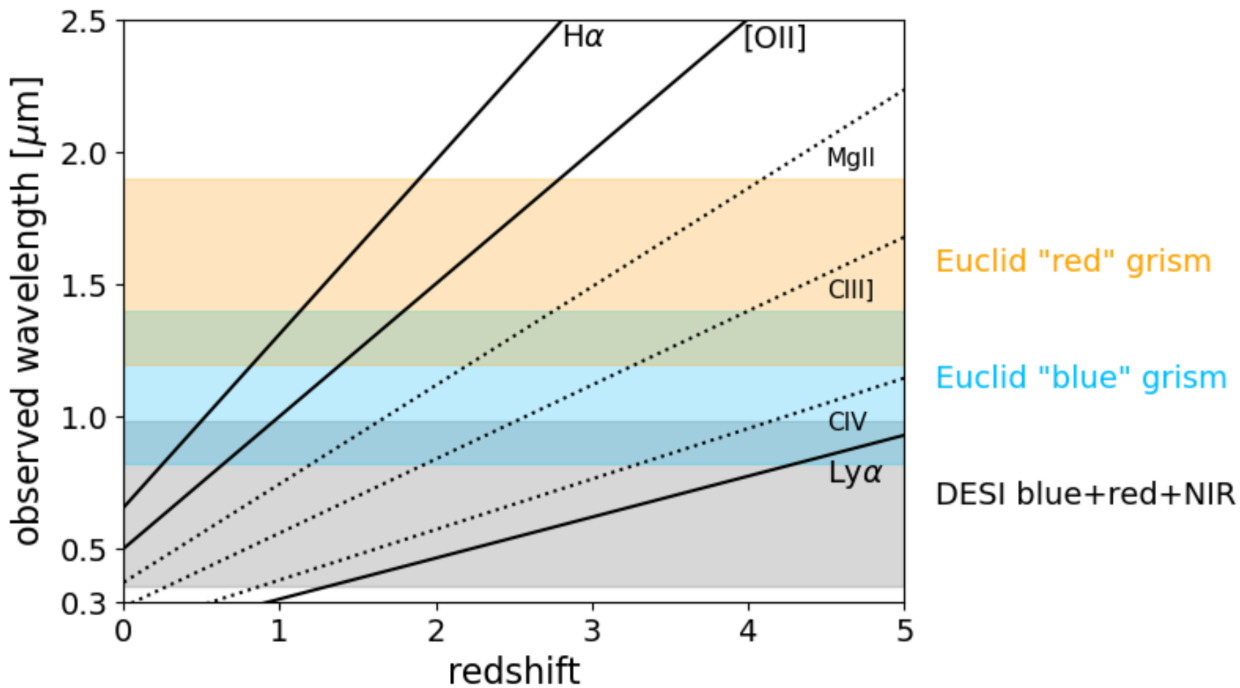


Modeling Galaxy Formation

With Misha, you calculate the evolution of small density perturbations with (linear + higher-order) perturbation theory.

Today: oversimplified, but insightful, (semi-)analytic models for the highly non-linear regime

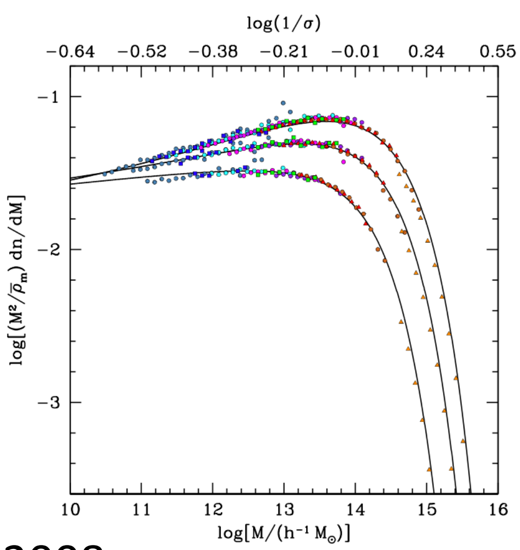
- Halo formation through **gravitational collapse**.
- Halo biasing in the **peak-background split**.
- Assuming that all matter is distributed in halos, approximate the non-linear matter power spectrum with the **halo model**.
- Include "gastrophysics" to describe **galaxy formation**.

The Halo Mass Function

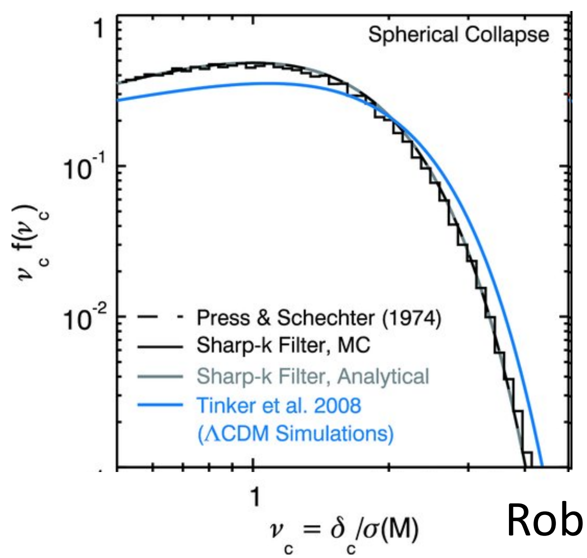
The Press-Schechter model builds intuition for the halo mass function and its cosmology + redshift dependence:

$$\frac{dn(M,z)}{d\ln M} = \frac{\bar{\rho}}{M}\sqrt{\frac{2}{\pi}}\nu e^{-\nu^2/2}\left|\frac{d\ln\sigma(M,z)}{d\ln M}\right|$$

- **High mass**: exponential cut off above M^* where $\sigma(M^*) = \delta_c$
  $M^*(z=0) \sim 10^{13} h^{-1} M_\odot$
- For higher precision, measure f(v) in simulations:

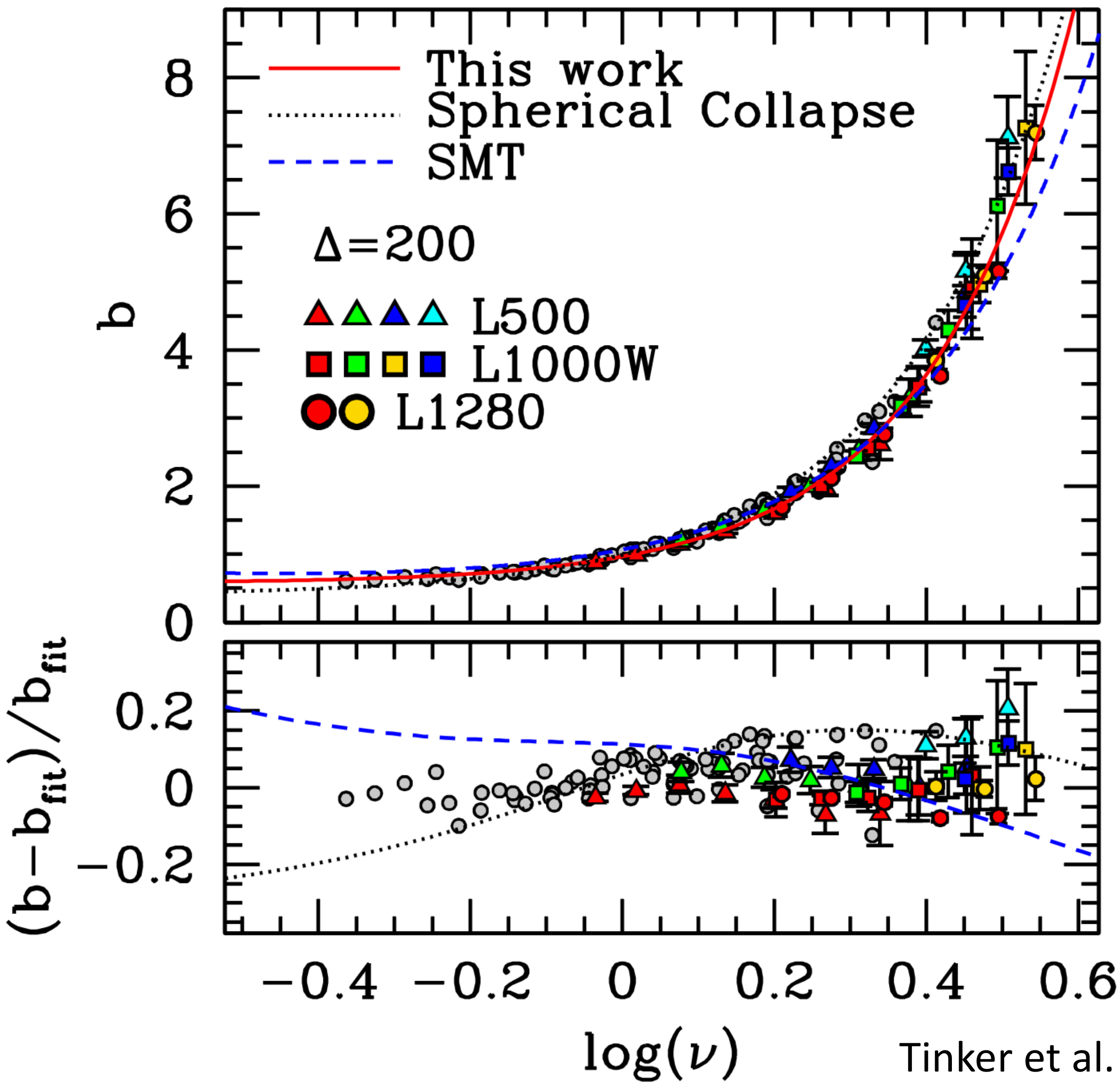


Tinker et al. 2008



Robertson et al. 2009

(Linear) Halo Bias



Tinker et al. 2010

Hierarchical Merging

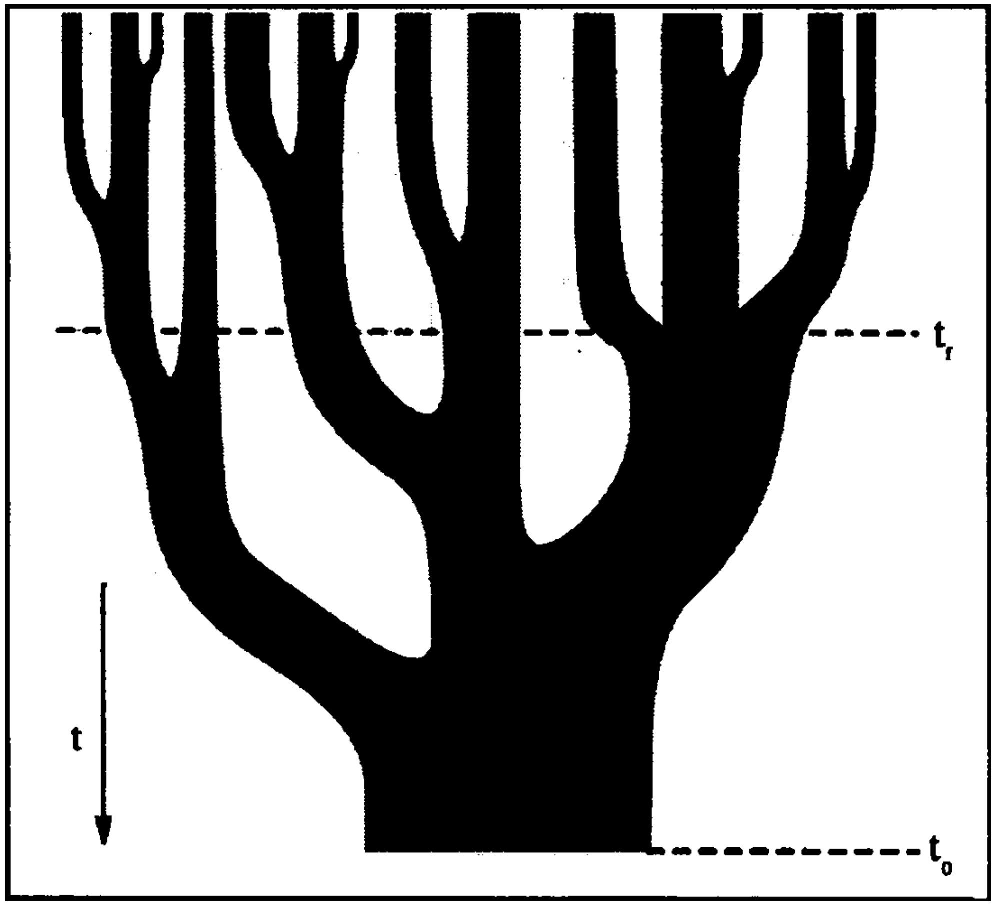
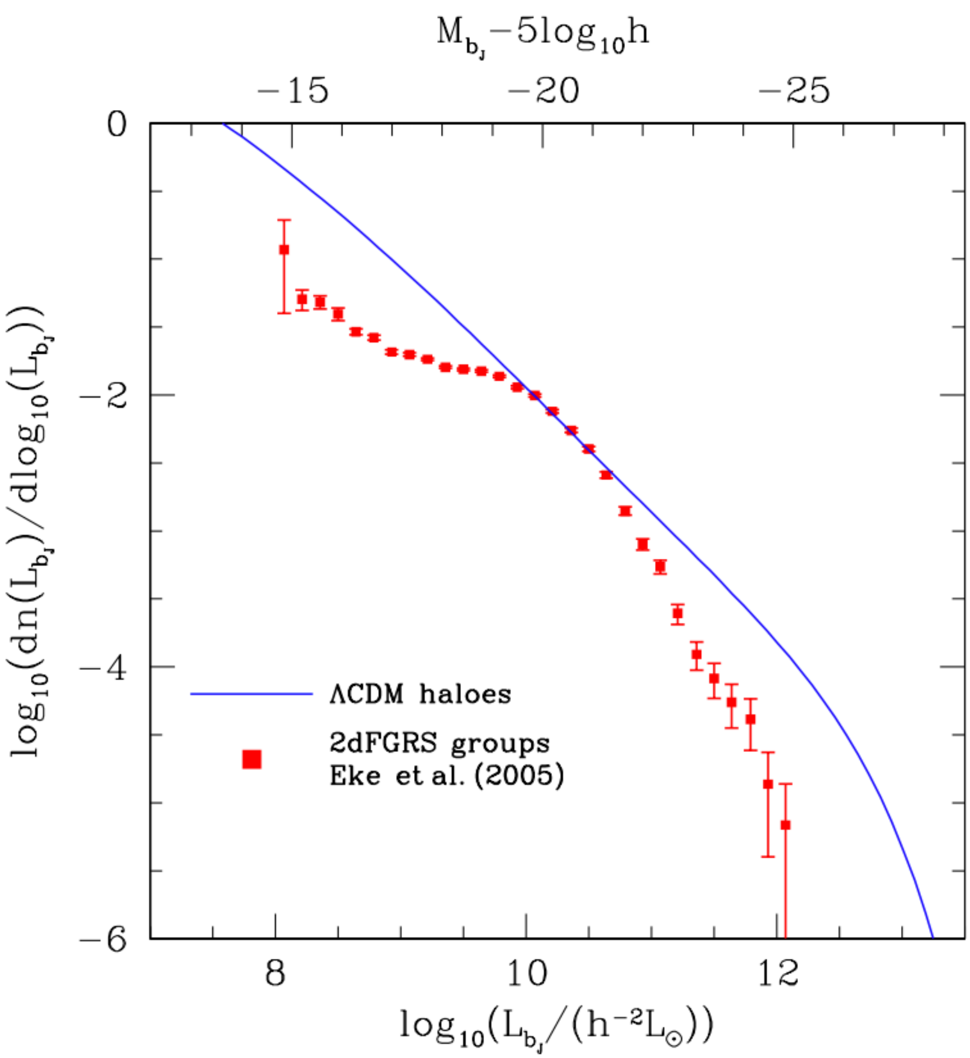


Illustration of a merger tree, depicting the growth of dark matter halos as the results of a series of mergers. A horizontal slice through the tree gives the mass distribution of progenitor halos at a given time.
(Lacey & Cole 1993)

Galaxy Formation: a Classic Recipe

1. Structure formation is driven by gravitational instability.
2. Dark-matter proto haloes get a spin due to tidal torques. Hoyle 1949
3. Galaxies form inside dark-matter haloes, via a two-stage collapse:
   a. Dissipationless collapse of the dark-matter haloes,
   b. dissipative collapse of gas: baryons collapse into the halo potential well and get shock heated. Rees & Ostriker 1977, White & Rees 1978
4. Gas cools mainly by radiative transitions: typical mass of a galaxy is set by cooling arguments . Hoyle 1953, Silk 1977, Binney 1977, Rees & Ostriker 1977
5. The formation of disk galaxies can be understood by the cooling of gas to the DM-halo centres via conservation of the angular momentum of the DM halo. Fall & Efstathiou 1980
6. (Elliptical galaxies form via the merger of disk galaxies)

From Halos to Galaxies

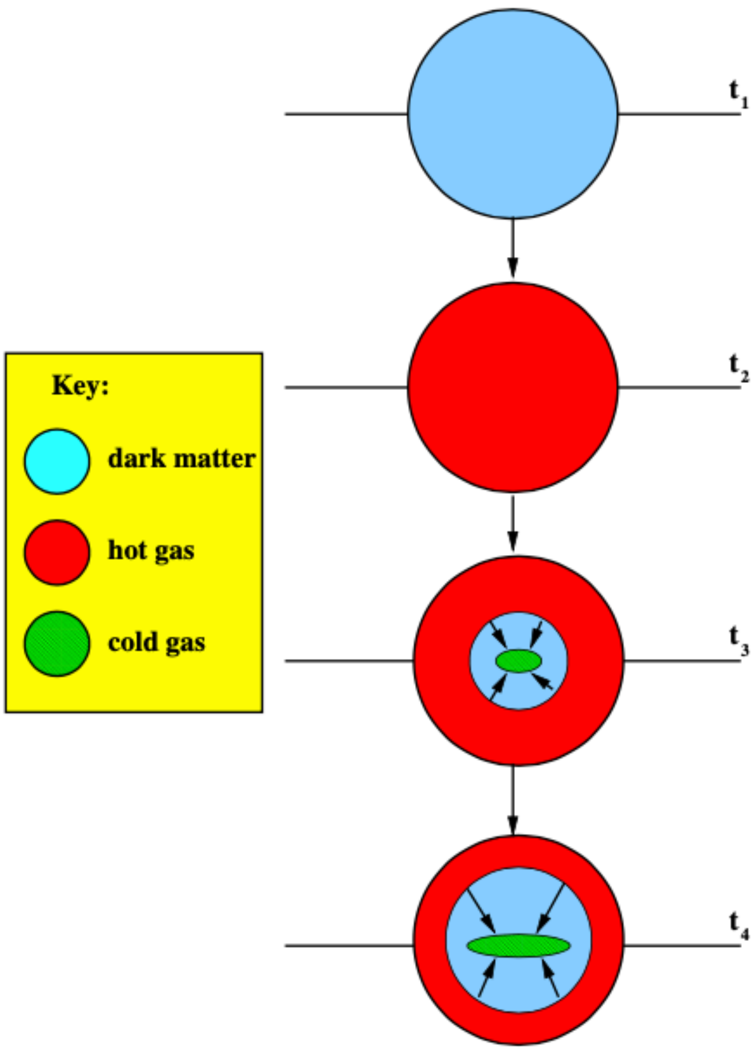


Simple prediction for the LF of galactic compared with the group luminosity function measured from the 2dFGRS (Eke+06).

Obvious discrepancy between the shape of the observed luminosity function and the one expected if all halos had the same M/L ratio.

Halos of different mass must have different efficiency with which baryons are converted into stars.

From Halos to Galaxies



t1: Baryons fall into the potential well of a dark matter halo

t2: Baryons are shock-heated to the halo temperature.

t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.

t4: The radius within which gas had sufficient time to cool increases with time.

(From Baugh 2007)

From Halos to Galaxies

t2: Baryons are shock-heated to the halo temperature.

Consider a gas cloud of mass M_{gas} falling into a halo of mass M_h with v_{in}.

If the gas falls in from large distances (where Φ(r) ~0), and has negligible initial velocity, then **$v_{in}^2 \sim 2|\Phi(r_{halo})|$.**

Assuming hydrostatic equilibrium (+ideal gas, spherical halo) and virial theorem, and ignoring external pressure, magnetic fields, etc., one can define the **virial temperature**

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2k_{\rm B}} V_{\rm c}^2 \simeq 3.6 \times 10^5\,{\rm K} \left(\frac{V_{\rm c}}{100\,{\rm km\,s^{-1}}}\right)^2$$

$$T_{\rm vir} \simeq 4 \times 10^4 \left(\frac{\mu}{1.2}\right) \left(\frac{M_{\rm h}}{10^8 h^{-1} {\rm M}_\odot}\right)^{\frac{2}{3}} \left(\frac{1+z}{10}\right) {\rm K}\,,$$

From Halos to Galaxies

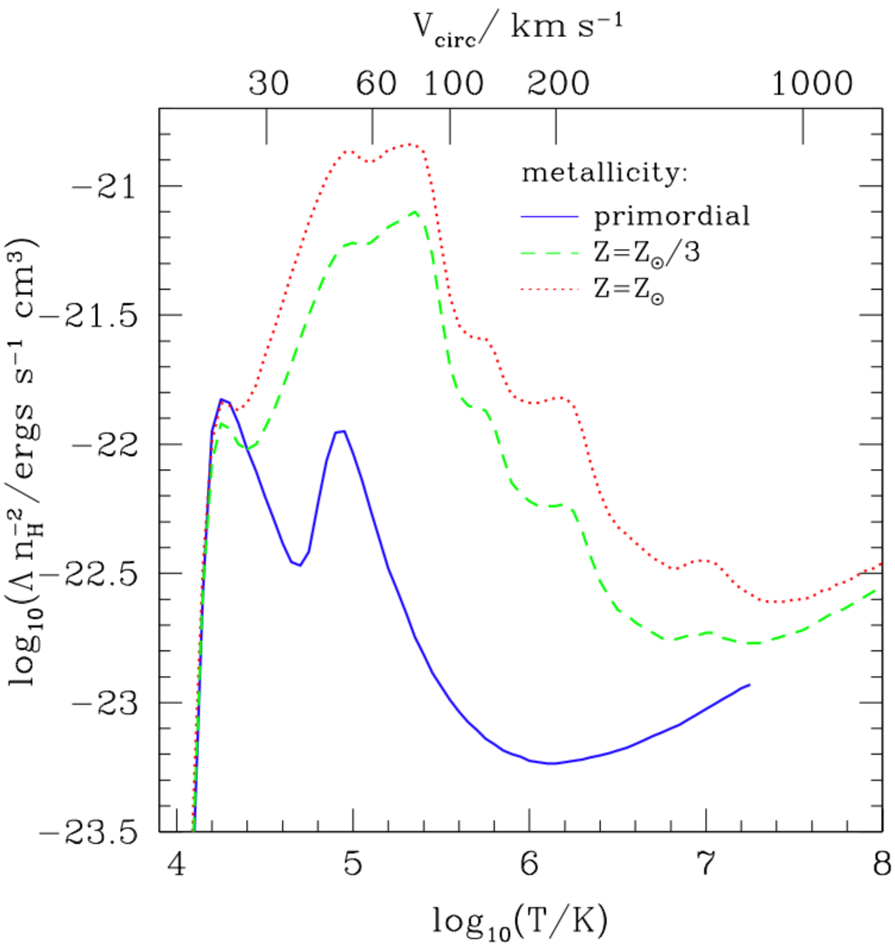
t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.

$$t_{\rm cool}(r) = \left(\frac{3}{2}\frac{\rho_{\rm gas}(r) k_{\rm B} T_{\rm vir}}{\mu m_{\rm H}}\right) / \left(\rho^2_{\rm gas}(r) \Lambda(T_{\rm vir}, Z_{\rm gas})\right)$$

- **Molecular cooling:** Collisions excite vibrational and rotational energy levels in H2. The subsequent decay of the excited states removes energy from the gas. Most important in halos with virial temperature below T ~ 10^4 K.
- **Atomic cooling:** Partly ionized atoms cool through the excitation and radiative decay of higher energy levels. Contributes while gas is partly, but not fully ionized (10^4 K < T < 10^6 K for primordial gas). Metal enrichment enhances the cooling efficiency at higher temperatures where gas of primordial composition is almost completely ionized.
- **Bremsstrahlung:** As electrons are accelerated in an ionized plasma they loose energy due to the emission of Bremsstrahlung. Dominant cooling process in cluster halos with T > 10^7 K.

From Halos to Galaxies

t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.



Cooling rate as a function of halo temperature and gas metallicity. Bremsstrahlung becomes the dominant process at T > 10^7 K. Atomic line cooling is most efficient at lower temperatures and causes the peaks at 15000 K (H) and 10^5 K (He+).

Sutherland & Dopita 1993

From Halos to Galaxies

t4: The radius within which gas had sufficient time to cool increases with time.

$$t_{\rm cool}(r) = \left(\frac{3}{2}\frac{\rho_{\rm gas}(r)k_{\rm B}T_{\rm vir}}{\mu m_{\rm H}}\right) / \left(\rho^2_{\rm gas}(r)\Lambda(T_{\rm vir}, Z_{\rm gas})\right)$$

We can implicitly define the as the radius at which the cooling time is equal to the dynamical time (or free-fall time) of the halo

$$t_{\rm cool}(r_{\rm cool}) = t_{\rm dyn}$$

If $t_{cool} > t_H$: Cooling is not important. Gas is in hydrostatic equilibrium, unless it was recently disturbed.

If $t_{dyn} < t_{cool} < t_H$: System is in quasi-hydrostatic equilibrium. It evolves on cooling time scale. Gas contracts slowly as it cools, but system has sufficient time to continue to re-establish hydrostatic equilibrium.

If $t_{cool} < t_{dyn}$: Gas cannot respond fast enough to loss of pressure. Since cooling time decreases with increasing density, cooling proceeds faster and faster. Gas falls to center of dynamic system on free-fall time.

From Halos to Galaxies

t4: The radius within which gas had sufficient time to cool increases with time.

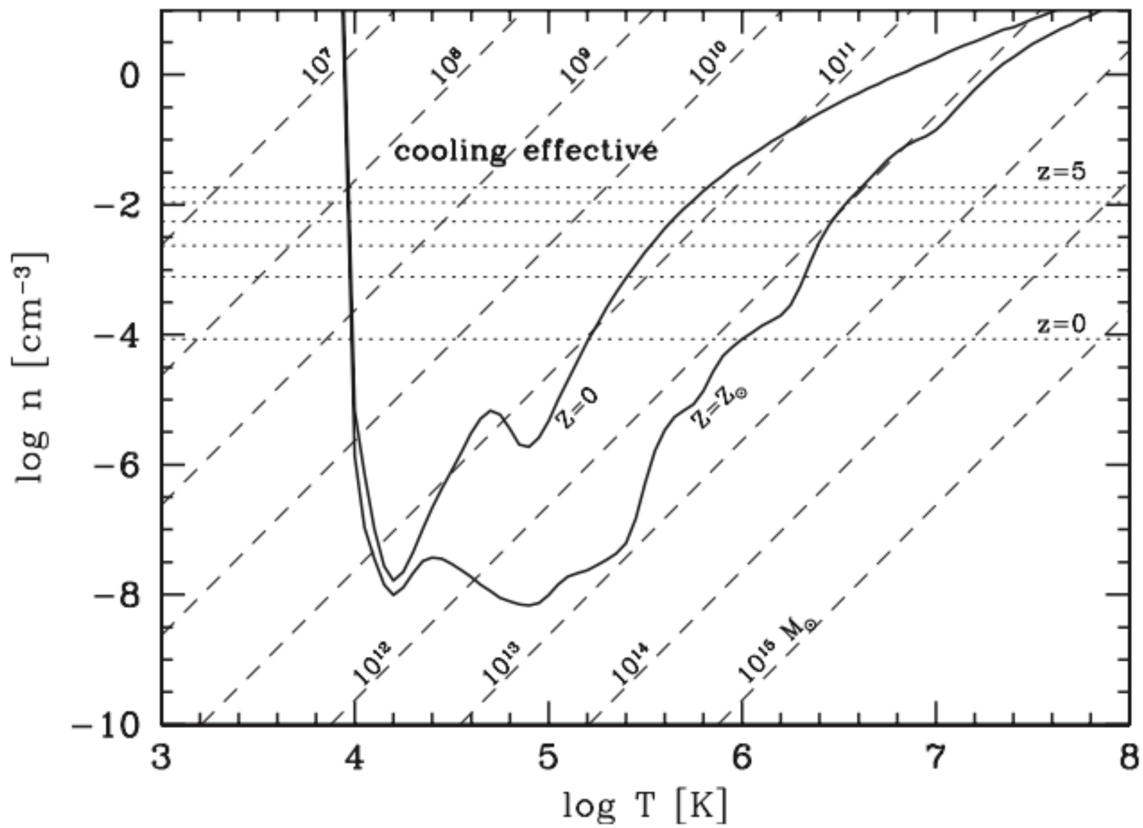


Fig. 8.6. Cooling diagram showing the locus of $t_{cool} = t_{ff}$ in the n–T plane. The upper and lower curves correspond to gas with zero and solar metallicity, respectively. The tilted dashed lines are lines of constant gas mass (in $M_\odot$), while the horizontal dotted lines show the gas densities expected for virialized halos ($\delta = 200$) at different redshifts. All calculations assume $f_{gas} = 0.15$ $\Omega_{m,0} = 0.3$, and $h = 0.7$. Cooling is effective for clouds with n and T above the locus.

From Halos to Galaxies

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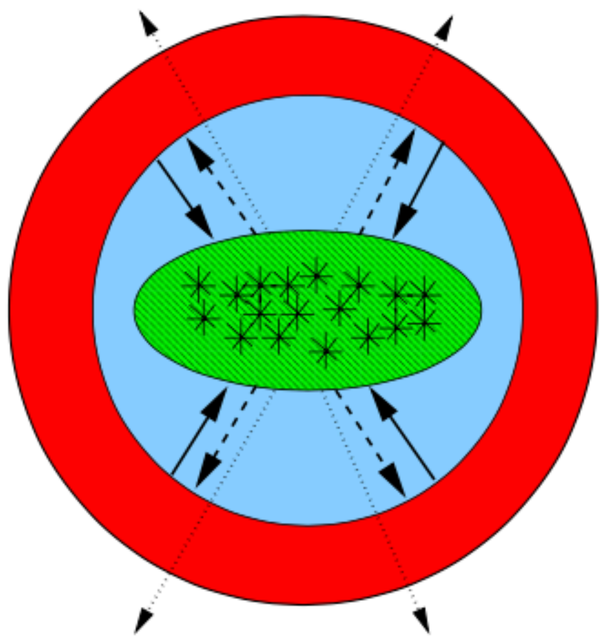
At late times and in massive systems, the cooling radius typically is smaller than the virial radius. The gas outside r_{cool} forms a quasi-static hot halo, while gas from the central regions ($r < r_{cool}$) cools and is accreted onto the center. The accretion rate can be estimated from a continuity equation

$$\dot{m}_{\rm cool} = 4\pi\rho_{\rm gas}(r_{\rm cool})r^2_{\rm cool}\dot{r}_{\rm cool} = \frac{m_{\rm hot}\dot{r}_{\rm cool}}{R_{\rm vir}} \sim \frac{m_{\rm hot}r_{\rm cool}v_{\rm vir}}{R^2_{\rm vir}} \propto f_{\rm b}r_{\rm cool}v^2_{\rm vir}H(z)G^{-1}$$

At $z = 0$ the transition between halos that are rapidly cooling and those forming a static hot halo occurs at $M_{vir} \sim 2 - 3 \times 10^{11} M_\odot$.

Star Formation & Feedback



$$\Delta m_{\rm reheat} = \frac{4}{3}\epsilon \frac{\eta_{\rm SN}\ E_{\rm SN}}{v_{\rm vir}^2}\Delta m_*$$

Feedback from supernova-driven winds. Hot halo gas cools and builds up a reservoir of cold gas in the galactic disk (solid arrows). The cold gas is turned into stars. Supernova explosions reheat a fraction of the cold gas and returns it to the hot phase (dashed arrows) or eject material from the halo (dotted arrows).

(Illustration from Baugh 2007)

Galaxy Transformations

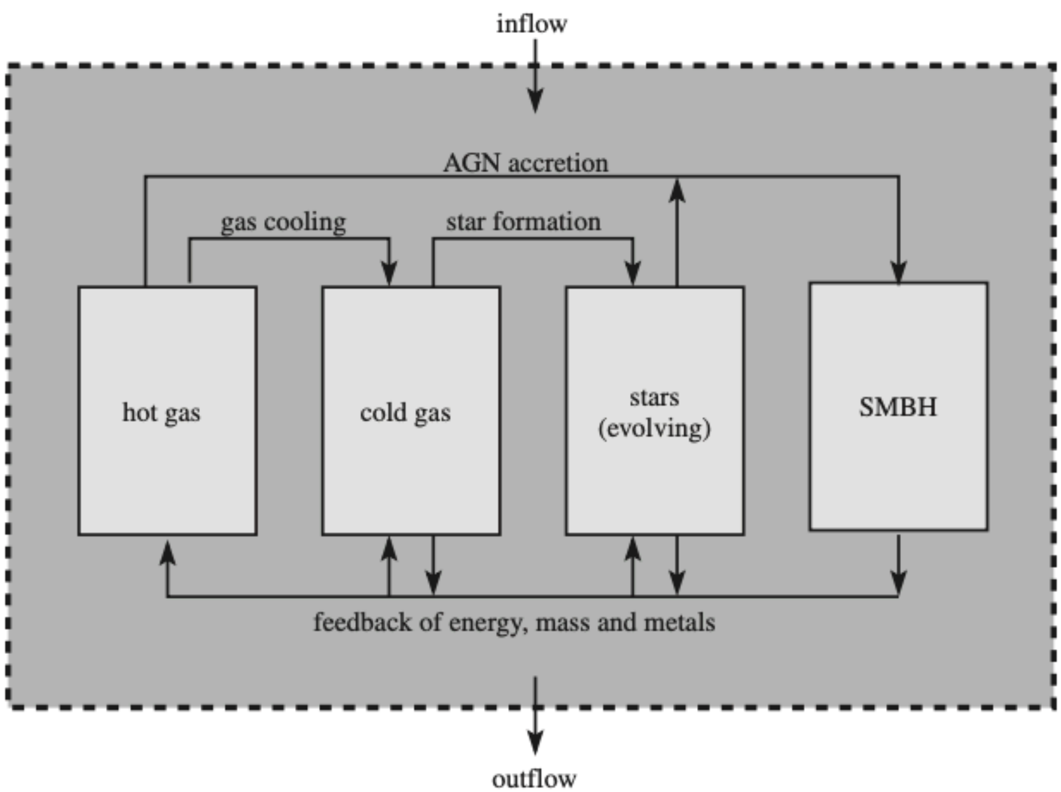


Fig. 1.2. A flow chart of the evolution of an individual galaxy. The galaxy is represented by the dashed box which contains hot gas, cold gas, stars and a supermassive black hole (SMBH). Gas cooling converts hot gas into cold gas, star formation converts cold gas into stars, and dying stars inject energy, metals and gas into the gas components. In addition, the SMBH can accrete gas (both hot and cold) as well as stars, producing AGN activity which can release vast amounts of energy which affect primarily the gaseous components of the galaxy. Note that in general the box will not be closed: gas can be added to the system through accretion from the intergalactic medium and can escape the galaxy through outflows driven by feedback from the stars and/or the SMBH. Finally, a galaxy may merge or interact with another galaxy, causing a significant boost or suppression of all these processes.

Simulating Galaxy Evolution

- Gravity & hydrodynamics
- Atomic processes (radiative cooling of the gas):
  - Heavy elements (metals = beyond He)
  - Molecules (H_2, CO, …)
- Star formation
  - Stellar evolution
  - Metal production and enrichment
  - Feedback: from supernovae, stellar winds, …
- (SuperMassive) Black Holes:
  - Formation
  - Growth: merging and gas accretion
  - Feedback: radiative, thermal, momentum
- Radiation (RHD):
  From stars, BHs, diffuse gas, reionization
- Magnetic Fields (MHD)
- Relativistic particle populations (cosmic rays)
- Dust (i.e. very large molecules)
- Plasma Physics (thermal conduction, …)

Essentially, all astrophysical phenomena are unresolved

(occur below the physical resolution of the sims)

They require some level of "subgrid" modelling:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \boldsymbol{F} = \cancel{0}$$
$$= S$$

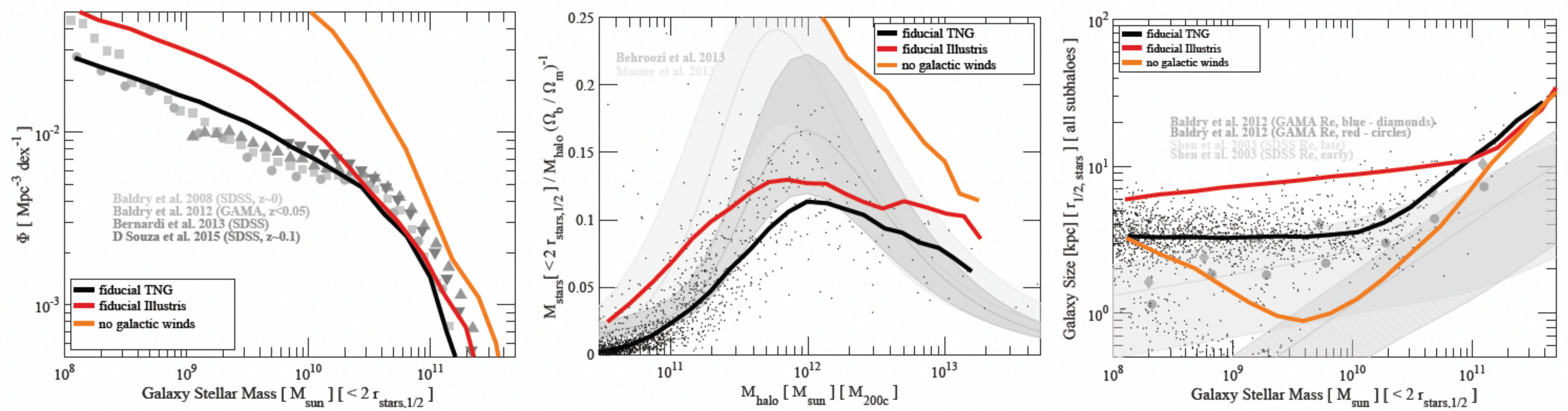
Messy astrophysics adds complex, poorly understood source terms

"Laws" suggested by observations and tailored theoretical models are invoked and implemented

Via look-up "tables"

into surrounding gas

Simulating Star Formation
Different Simulation – Different Recipe



Pillepich+2018

Without ‘a’ form of feedback acting at all masses, i.e., a mechanism regulating conversion of gas into star, halos would host way too massive and compact galaxies.

AGN Feedback

Supermassive black holes, residing in all galactic spheroids, play an important role in the formation of massive galaxies. Black holes deplete a galaxy's gas supply by accretion of gas, and injecting heat into the ISM. At times where a galaxy is not undergoing a major merger, the black hole quiescently accretes gas which is assumed to come from the hot halo.

A simple phenomenological model for the accretion rate is

$$\dot{m}_{\rm BH} = \frac{\kappa_{\rm AGN}}{10G} m_{\rm BH}\ f_{\rm hot}\ v_{\rm vir}^3\ ,$$

This accretion rate can be motivated by noting that $f_{hot}v_{vir}^3/H(z)/(10G)$ is the total mass of hot gas, so that in case of unit accretion efficiency the black hole will accrete all hot gas within a Hubble time. We then assume that a fixed fraction **η** of the accreted energy is released radiatively

$$L_{\rm BH} = \eta\ \dot{m}_{\rm BH}\ c^2$$

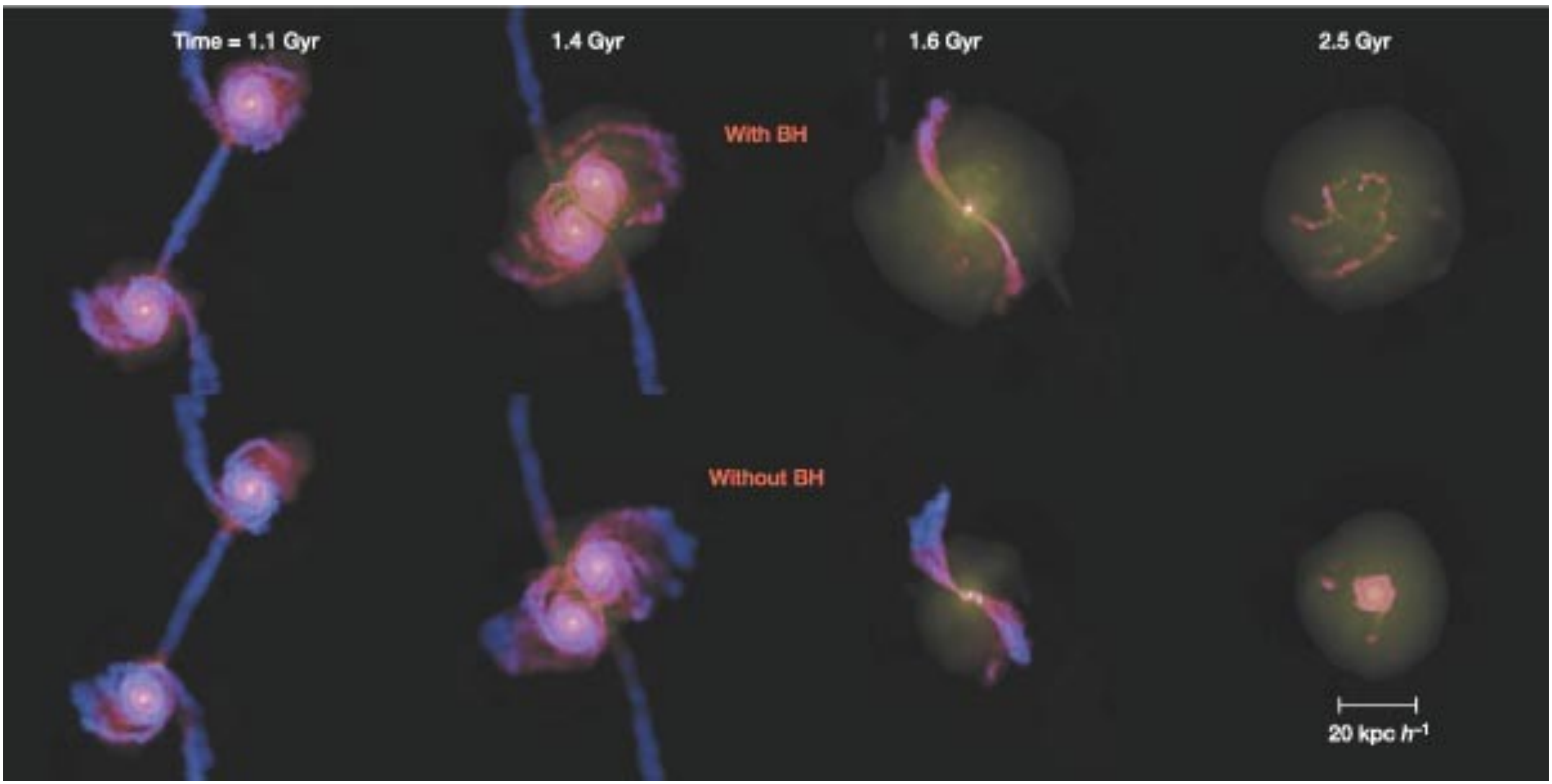
and injected into the ISM where it compensates in part for cooling, giving rise to a modified infall rate

$$\dot{m}\prime_{\rm cool} = \dot{m}_{\rm cool} - \frac{L_{\rm BH}}{\frac{1}{2}\ v_{\rm vir}^2} = \dot{m}_{\rm cool} - \frac{\eta\ \kappa_{\rm AGN}}{5G}\ m_{\rm BH}\ c^2\ f_{\rm hot}\ v_{\rm vir}\ .$$

AGN Feedback in Simulations

Di Matteo, Springel & Hernquist 2005:

Simulations of galaxy mergers with and without AGN feedback



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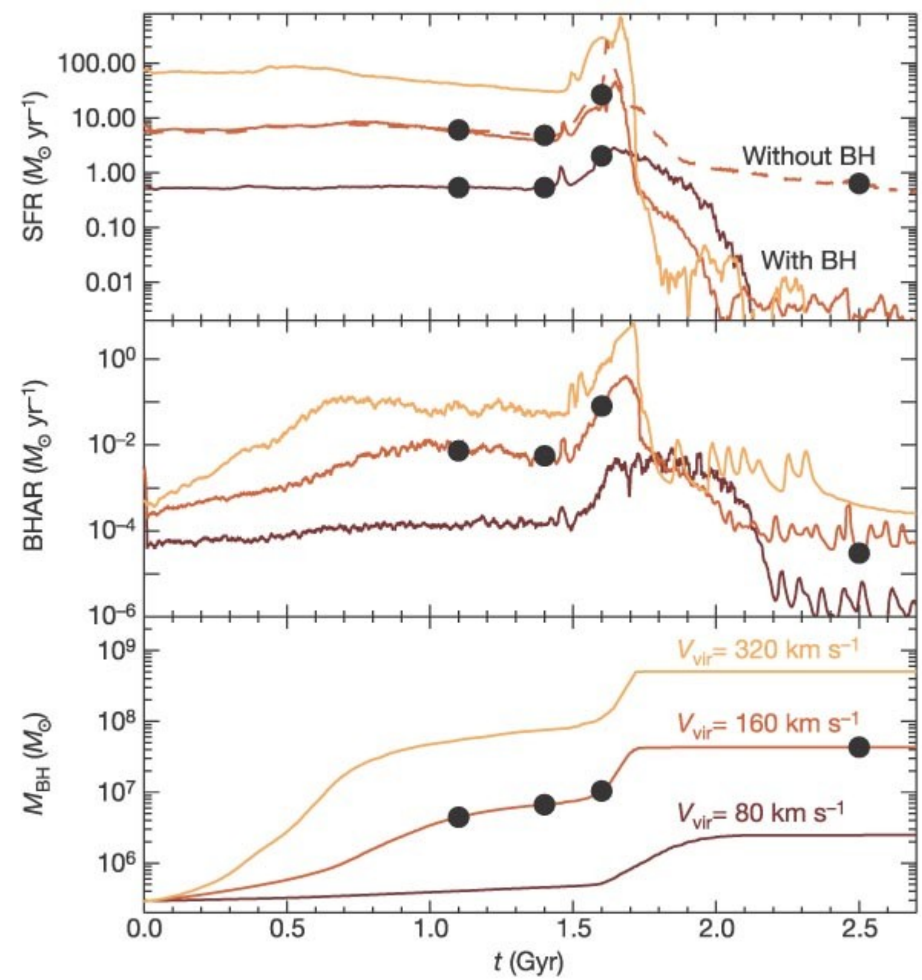
Simulations of galaxy mergers with and without AGN feedback

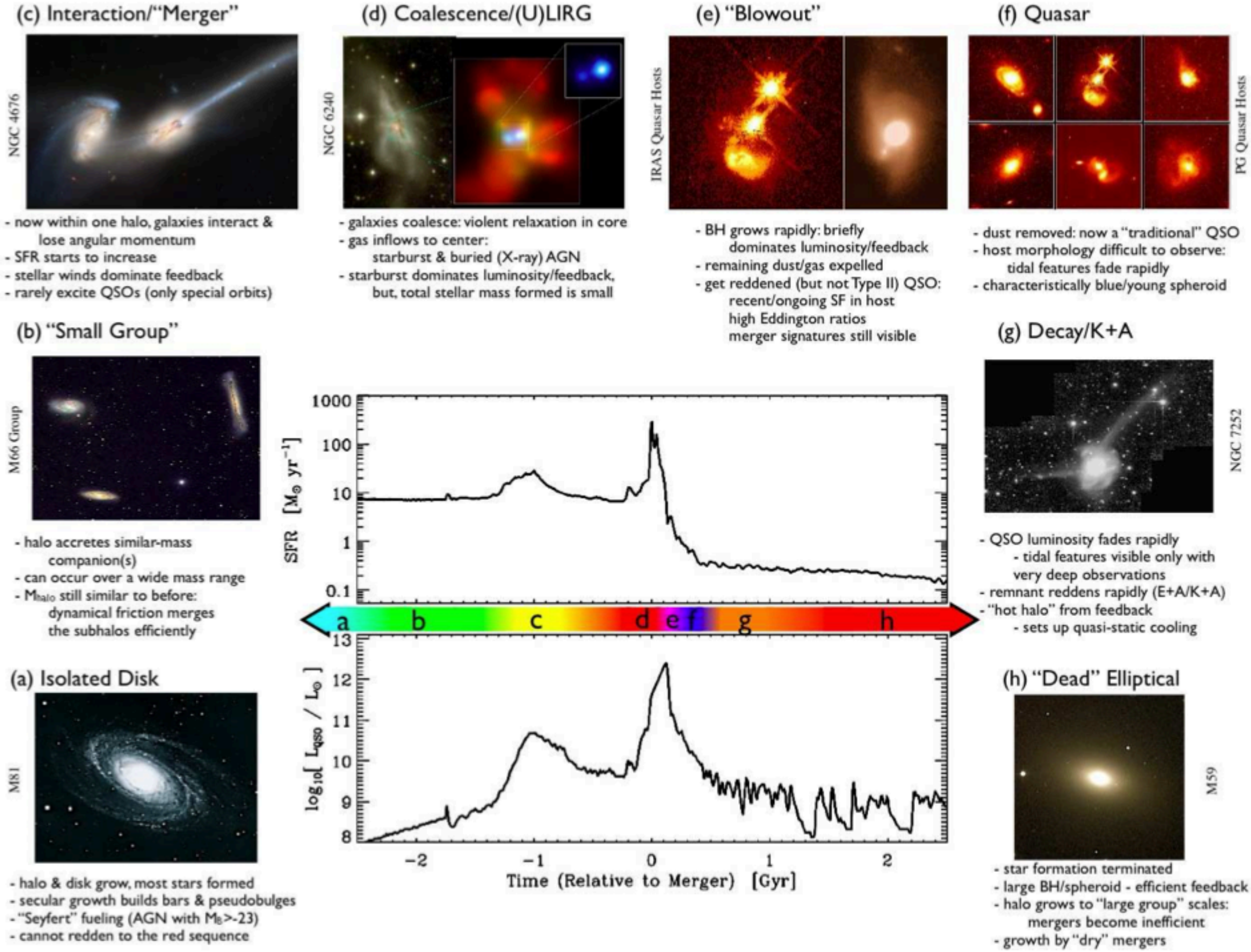
Merger leads to inflow of gas, burst of star formation.

Merger also leads to strong inflows that feed gas to the supermassive black hole and thereby power the quasar.

The energy released by the quasar expels enough gas to quench both star formation and further black hole growth.

Simulated merger remnants with AGN feedback follow observed early-type galaxy scaling relations!





Hopkins et al. 2008

Simulating AGN Feedback

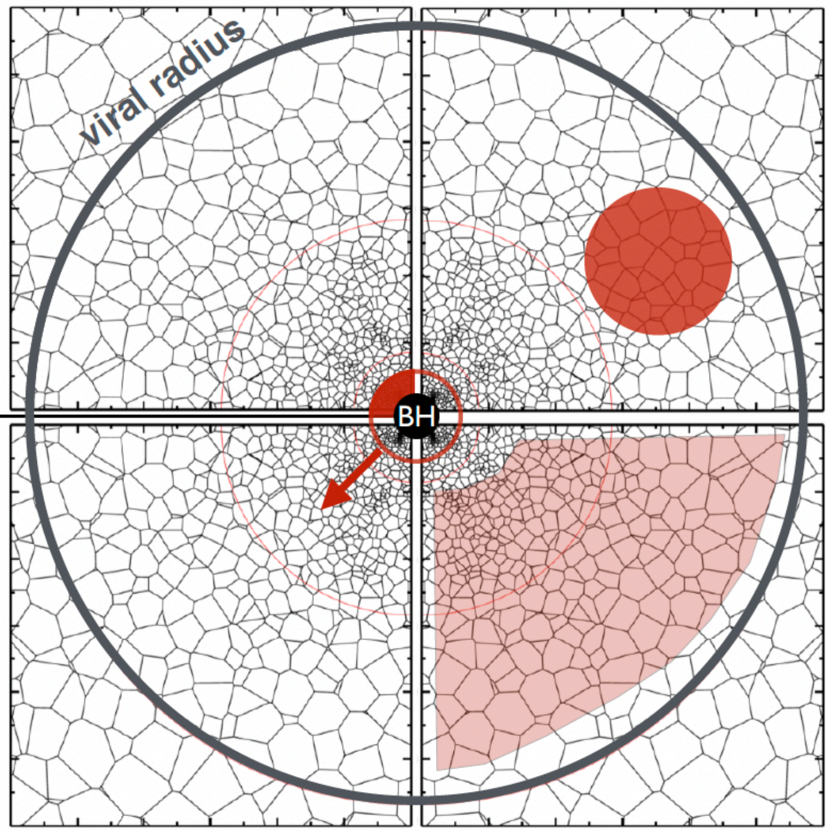
Different Simulation – Different Recipe

Thermal Dump (near the BH)

Continuous?
- yes e.g. Illustris, HorizonAGN
- no e.g. Eagle

Only at high accretion rates?
- yes e.g. Illustris
- no e.g. Eagle (all the time)

aka QUASAR MODE



aka RADIO MODE

Thermal Dump (bubbles)

Very sporadic, energetic bubbles: Illustris

More frequent, "smaller bubbles": Auriga

Only at low accretion rates?
- yes e.g. Illustris
- no e.g. Auriga (all the time)

Isotropic?
- no e.g. TNG, each time in different dirs
- no: bipolar e.g. HorizonAGN

Continuous?
- ~ e.g. TNG, each time in different dirs

Only at low accretion rates?
- yes e.g. TNG, HorizonAGN

Kinetic Kick or BH-driven winds

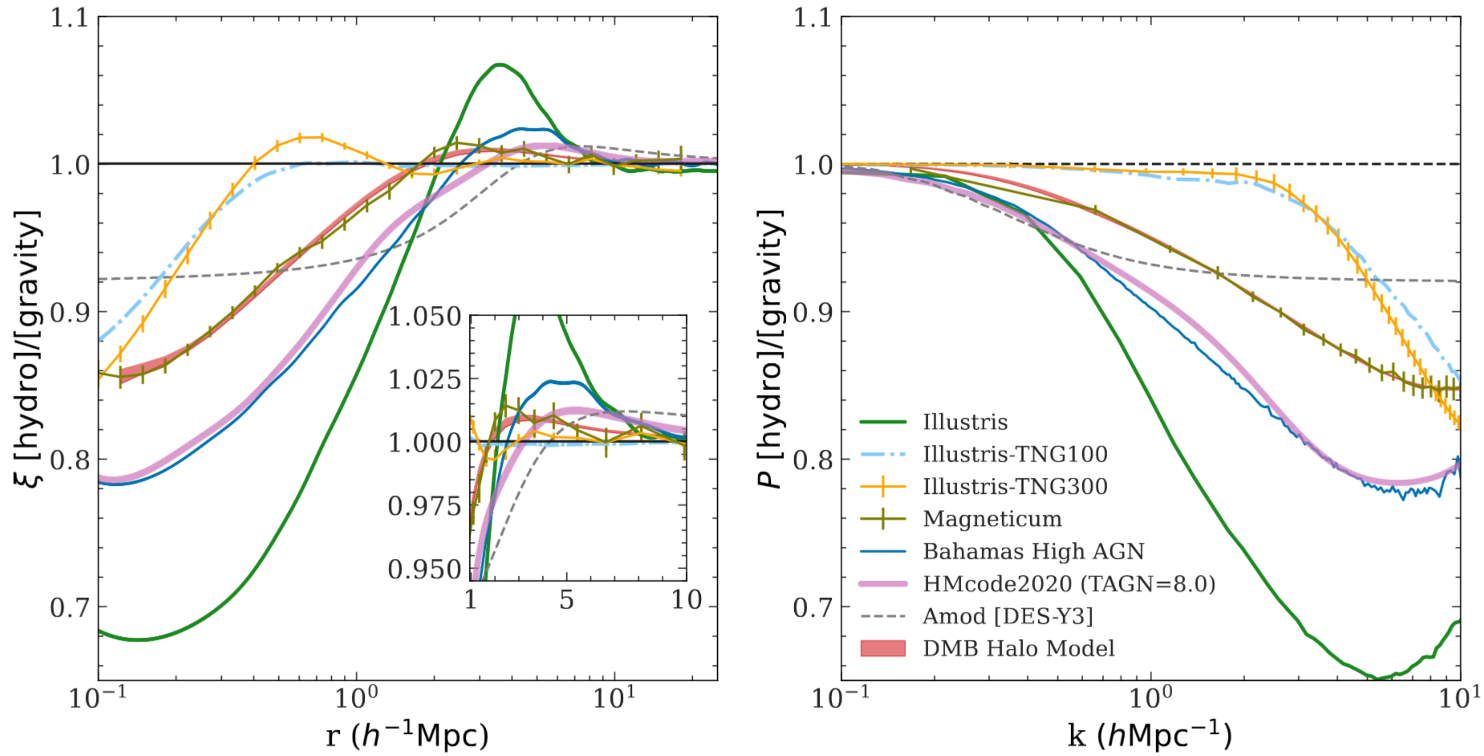
Affecting only non-self shielded gas e.g. Mufasa, NIHAO variations

"By Hand" heating of the gaseous halo

See also Choi et al. 2012, 2014, 2015; Dubois et al. 2010, 2012; Weinberger et al 2017

All simulations also require prescription for forming initial SMBHs…

Impact of Feedback on the Matter Distribution



The Halo Model

The concept of halos is a useful way of modeling clustering in the highly non-linear regime.

Assume that all mass is in dark matter halos.

If we know:

1. How halos cluster.
2. The mass distribution around each halo.

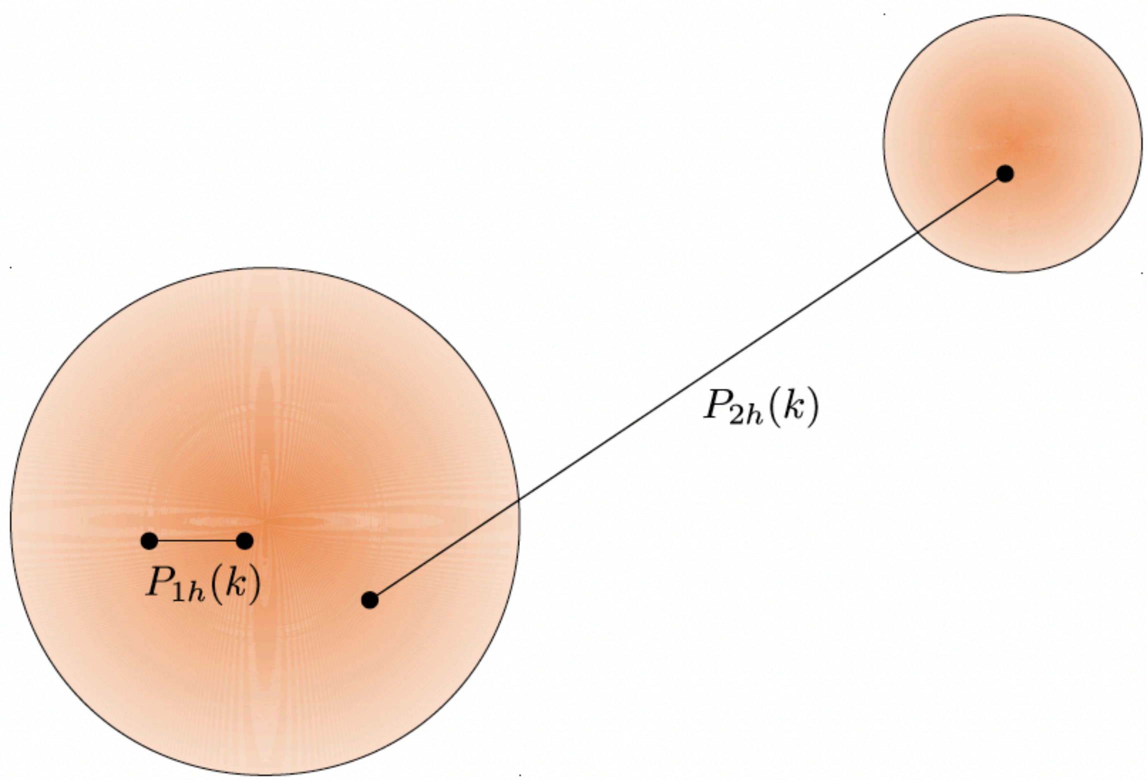
Then we can calculate how *all* matter is clustered.

This formalism can be extended to model the clustering of galaxies (and other tracers).

The Halo Model

Computation of matter clustering splits into

- 1-halo term: clustering of matter within one halo
- 2-halo term: large-scale clustering of halos



Dodelson/Schmidt: Modern Cosmoloy

The Halo Model: 1h Term

Write mass profile as $\rho(r, M) = MU(r, M)$

$U(r|M)$ is normalized to unity over halo volume (e.g., R_{200}).

On small scales, the two particles will always be in the same halo → single mass integral

$$P_{\rm mm}^{\rm 1h}(k) = \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 \hat{U}_{\rm m}^2(M, k) n(M) \, {\rm d}M$$

Note that mass function must obey $\int_0^\infty M n(M) \, {\rm d}M = \bar{\rho}$

(all mass is contained in halos)

The Halo Model: 2h Term

For two halo term, halos may have different masses

$$P_{\rm hh}(M_1, M_2, k) = b(M_1)b(M_2)P_{\rm mm}^{\rm lin}(k)[1 + \beta^{\rm nl}(M_1, M_2, k)]$$

non-linear corrections,
→ Asgari+(2023) review

→ double mass integral

$$P_{\rm mm}^{\rm 2h}(k) = P_{\rm mm}^{\rm lin}(k) \times \left[\frac{1}{\bar{\rho}} \int_0^\infty M \hat{U}_{\rm m}(M, k) b(M) n(M) \, {\rm d}M\right]^2$$

Note that bias function must obey $\int_0^\infty M b(M) n(M) \, {\rm d}M = \bar{\rho}$

(mass is on average unbiased)

Halo Profile

Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

Free parameter: halo concentration **c(M)** requires fitting function

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Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

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Reference	Definition	Notes
Navarro et al. (1997)	200c	Depends on a cosmology-dependent halo-collapse redshift that is calculated semi-analytically.
Bullock et al. (2001)	virial	Two relations presented in paper: a simple model where c is a power-law in M (although scaled by a cosmology-dependent non-linear mass) and a more complicated model where c is related to a cosmology-dependent halo formation redshift, which is calculated semi-analytically.
Eke et al. (2001)	virial	Depends on a cosmology-dependent halo-collapse redshift that is calculated semi-analytically.
Neto et al. (2007)	200c	Only considered the Millennium Springel et al. (2005) cosmology at $z = 0$.
Macciò et al. (2008)	virial	Modified version of the Bullock et al. (2001) algorithm.
Duffy et al. (2008)	200, 200c, virial	Simple $c(M)$ power-law relations are presented that are fitted to simulations of *WMAP* 5 cosmology. Explicit z dependence. Separate relations for 'relaxed' and 'full' samples of haloes.
Prada et al. (2012)	200c	'Cosmology dependent' relation presented as a function of $\sigma(M, a)$. Upturn in halo concentration for high-mass haloes.
Kwan et al. (2013)	200c	Emulated relation for a variety of wCDM cosmologies.
Ludlow et al. (2014)	200c	Relates halo concentration to mass-accretion history.
Klypin et al. (2014)	200c	Parametrised in terms of ν.
Diemer & Kravtsov (2015)	200c	Present a semi-analytical, cosmology-dependent model parametrised in terms of ν and $n_{\rm eff}$ – the effective slope of the power spectrum on collapse scales. Demonstrates that concentration–mass relation is 'most universal' when masses are defined via 200c.
Correa et al. (2015)	200c	Relates halo concentration to mass-accretion history. Only applies to relaxed haloes.
Okoli & Afshordi (2016)	200c	Focusses on relaxed low-mass haloes using analytical arguments. Cosmology dependence incorporated via ν dependence.
Ludlow et al. (2016)	200c	Applies for WDM as well as for CDM cosmologies. Depends on a collapse redshift that is calculated semi-analytically.
Child et al. (2018)	200c	Power-law relation but scaled via the cosmology-dependent non-linear mass. Also consider Einasto profiles. Individual and stacked halo profiles considered separately.
Diemer & Joyce (2019)	200c	Improved version of Diemer & Kravtsov (2015) with additional dependence on the logarithmic linear growth rate to capture non-standard expansion histories.
Ishiyama et al. (2021)	200c, virial	Uses the same functional form as Diemer & Joyce (2019) but fitted to a larger simulation resulting in up to 5% errors for a wide range of masses and redshifts.

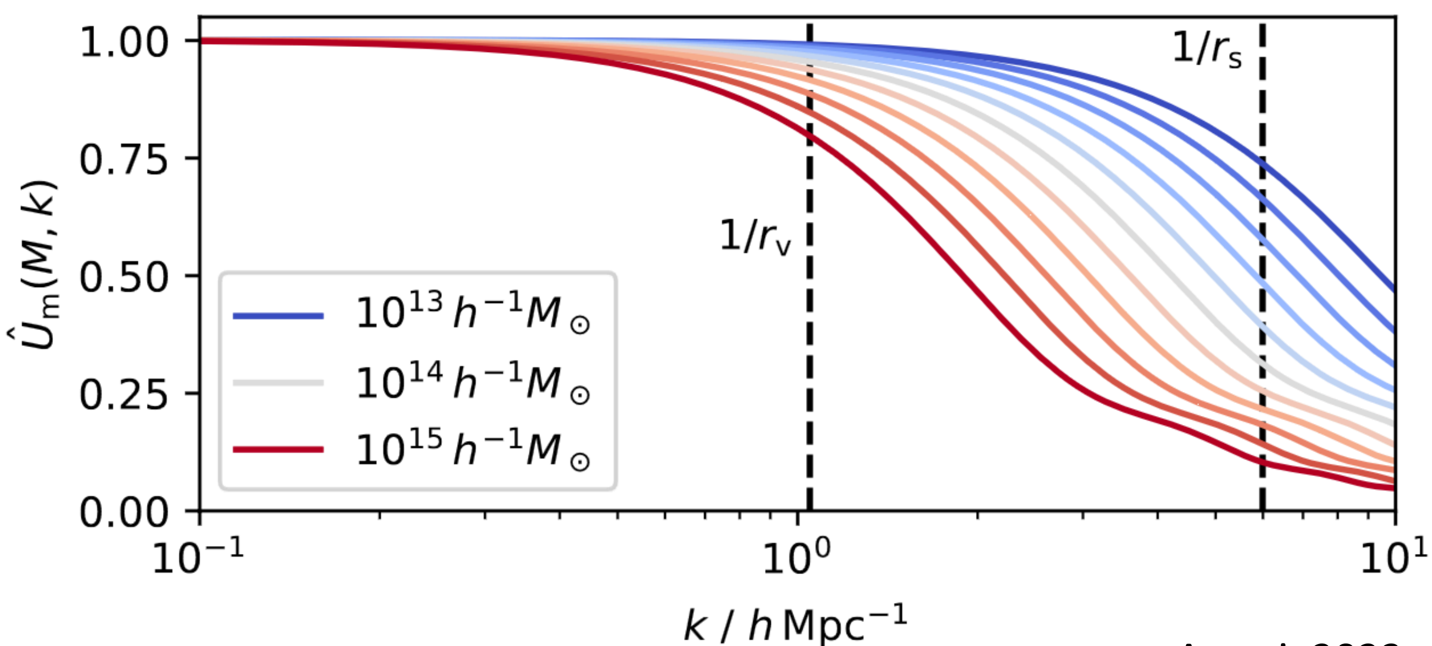
Non-exhaustive list of c(M) fitting functions. Note different definitions!

Asgari+2023

Halo Profile

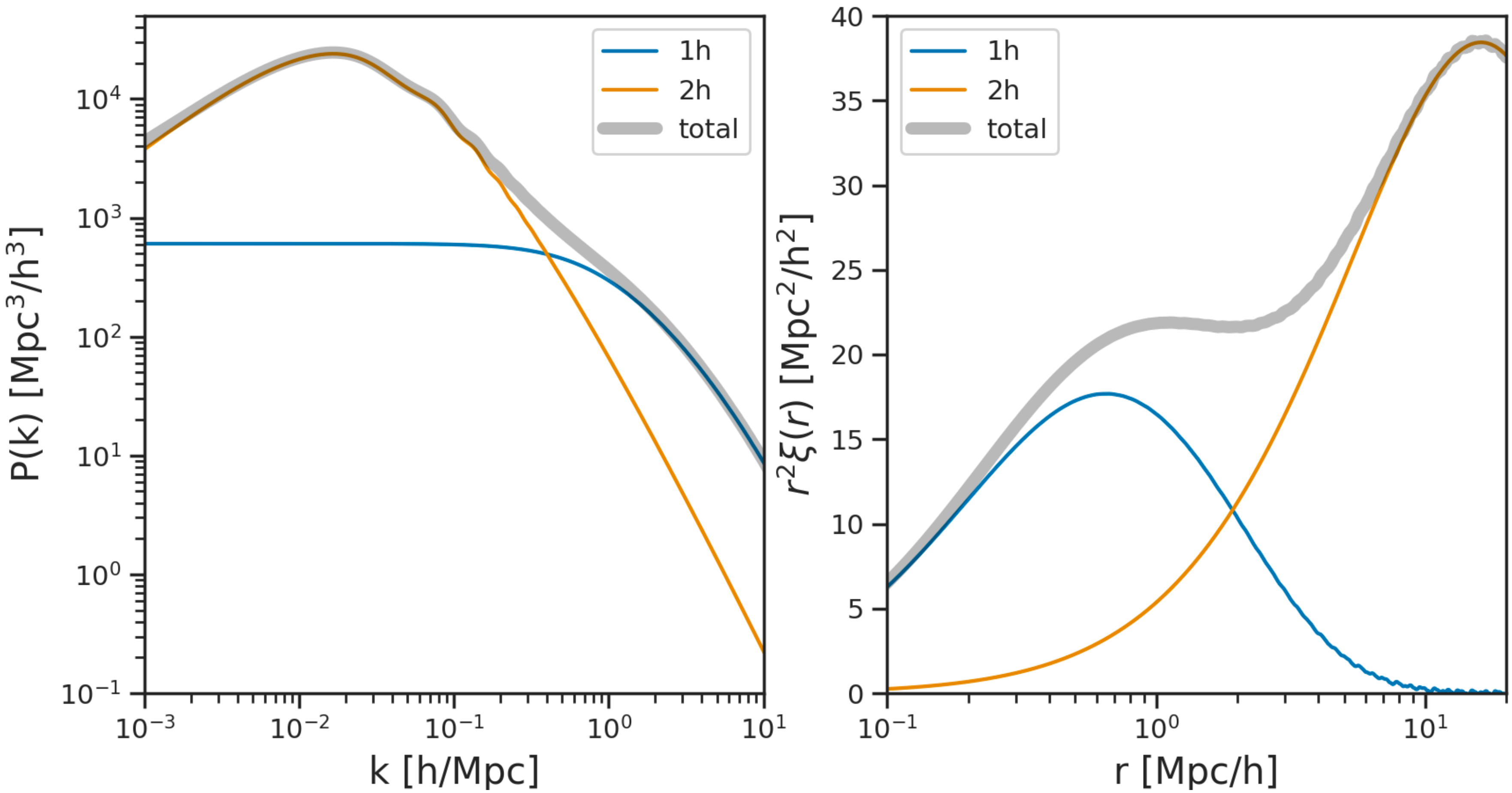
Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

Free parameter: halo concentration **c(M)** requires fitting function



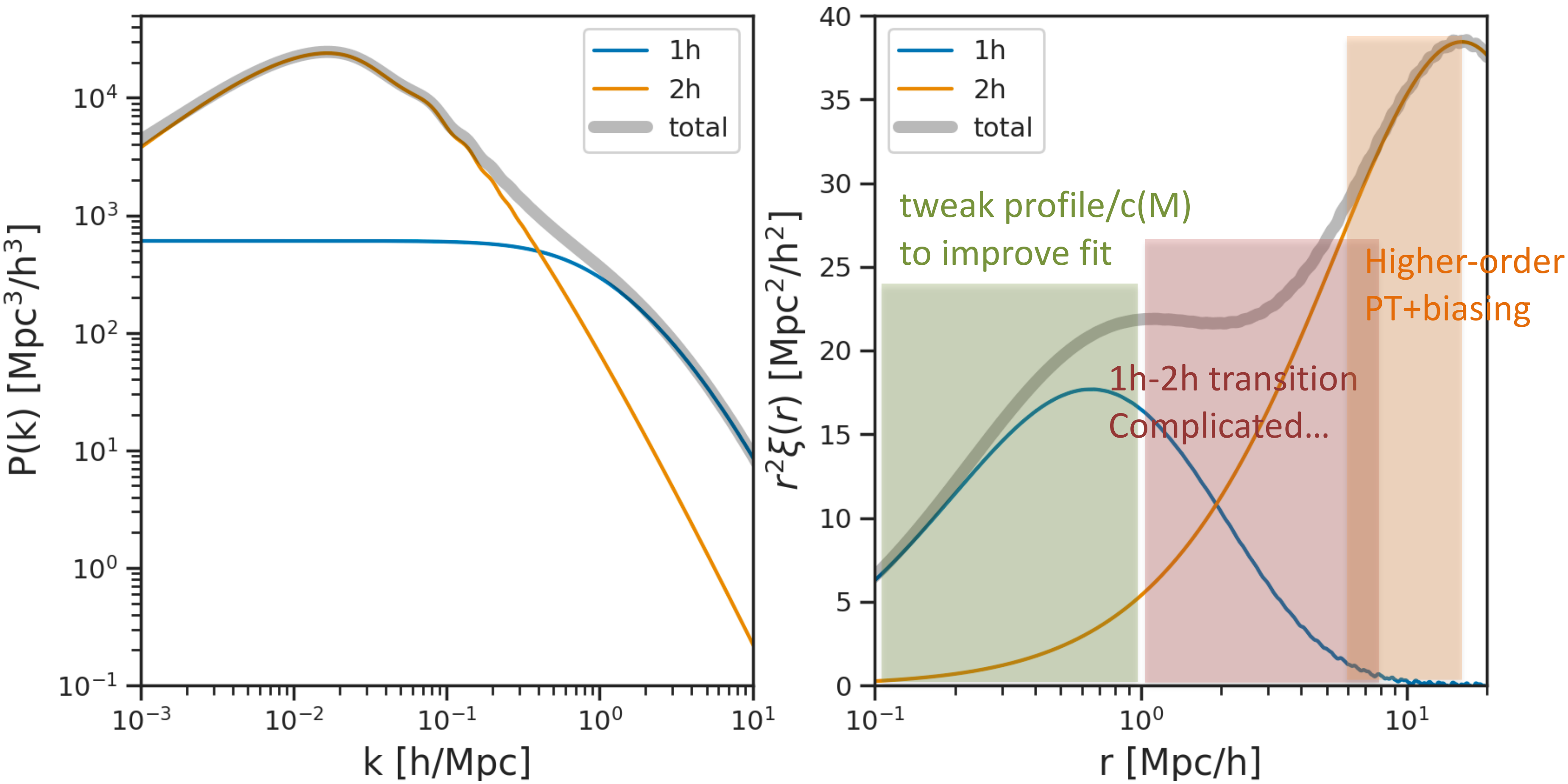
Asgari+2023

The Halo Model



Typically agrees with simulations at 10s of % level.

The Halo Model

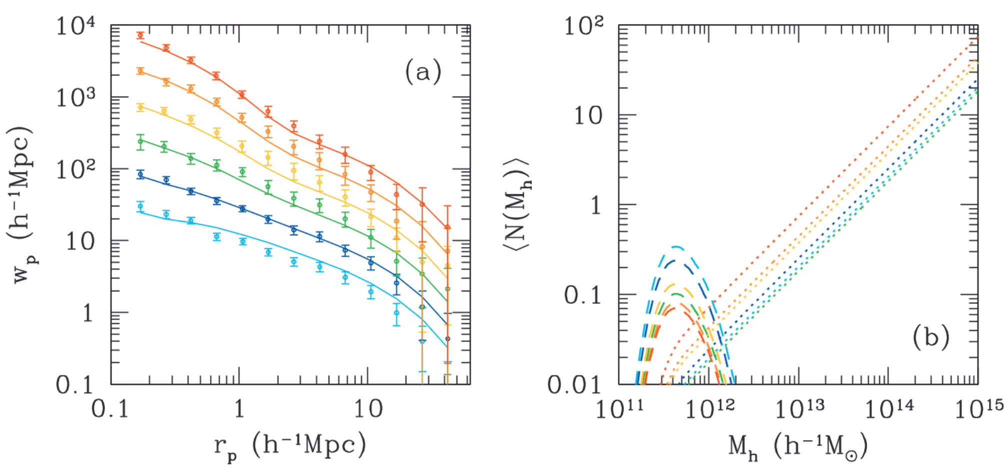


Typically agrees with simulations at 10s of % level.

Halo Occupation Distribution

Despite the rich astrophysics of feedback, the empirical halo occupation distribution (HOD) has been successful to model 2pt clustering of many different galaxy samples (so far…):

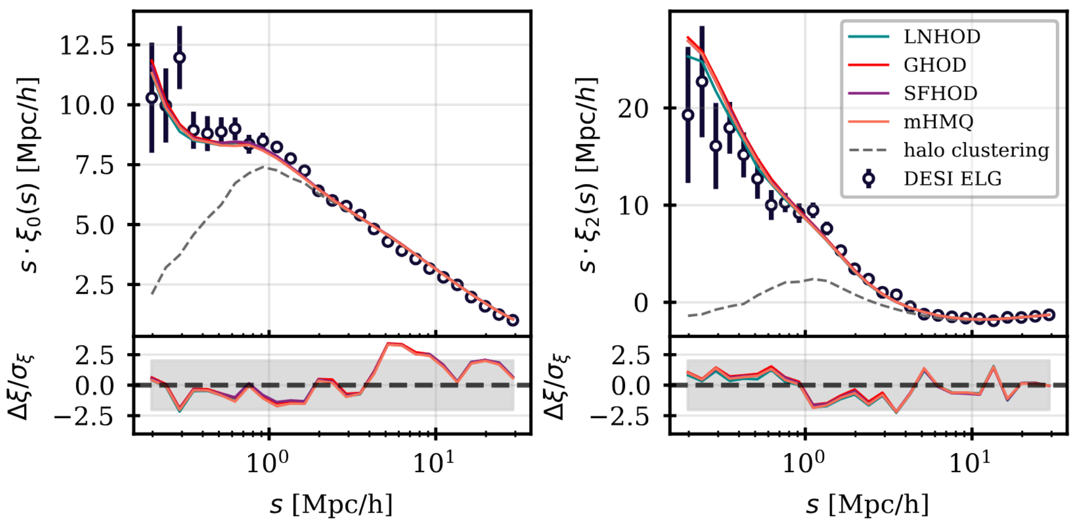
- Split galaxies into central galaxies (c) and satellites (s)
- Introduce empirical forms for halo occupation $N_{c/s}(M)$ and radial galaxy distribution.
- Compute galaxy power spectrum $P_{\rm gg}(k) = P_{\rm cc}(k) + 2P_{\rm cs}(k) + P_{\rm ss}(k)$

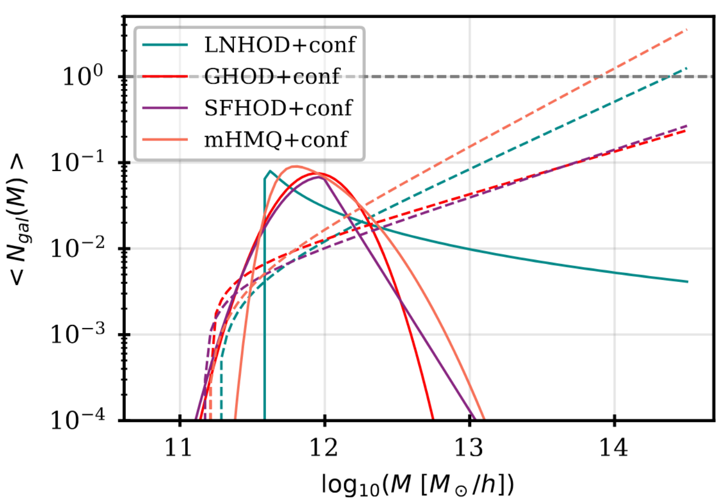


Zehavi+ 2010

Halo + HOD model fits to luminosity bin galaxy samples in SDSS (w_p measurements offset for clarity).
To match precision of recent data, many extensions needed already…

HODs for (DESI) ELGs





Rocher+ 2024

Fitting DESI ELG 2pt clustering already requires

- Flexible parameterization of $N_s(M)$
- Parameterized satellite conformity ($N_c(M)$ depends on whether central galaxy is ELG)
- Radial satellite distributions extending beyond R_{vir}

Beyond-2pt statistics will likely require (many) morel parameters.