Structure Formation with Warm White Noise

Mustafa A. Amin

RICE

Structure formation with Warm Wave Dark Matter

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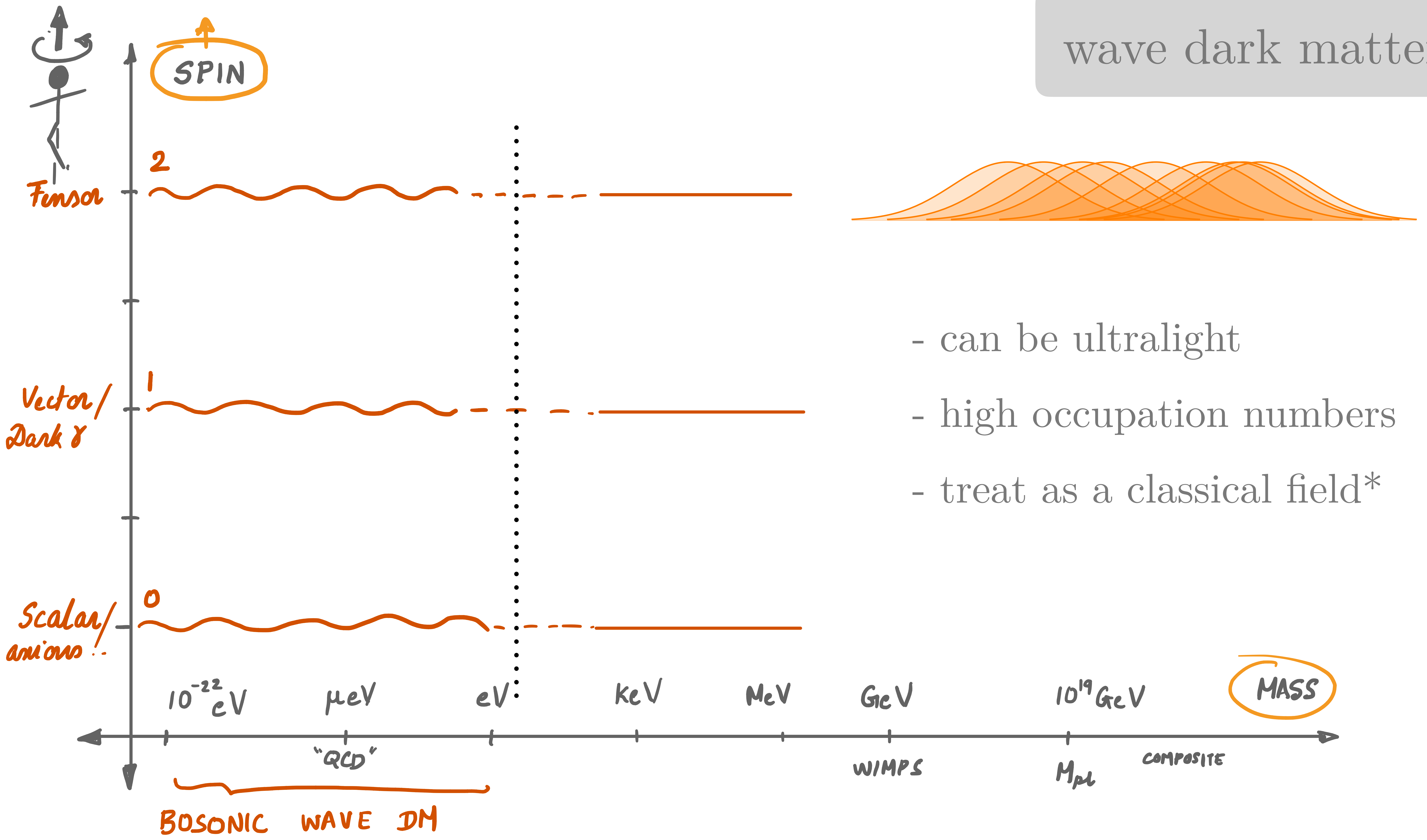
wave dark matter

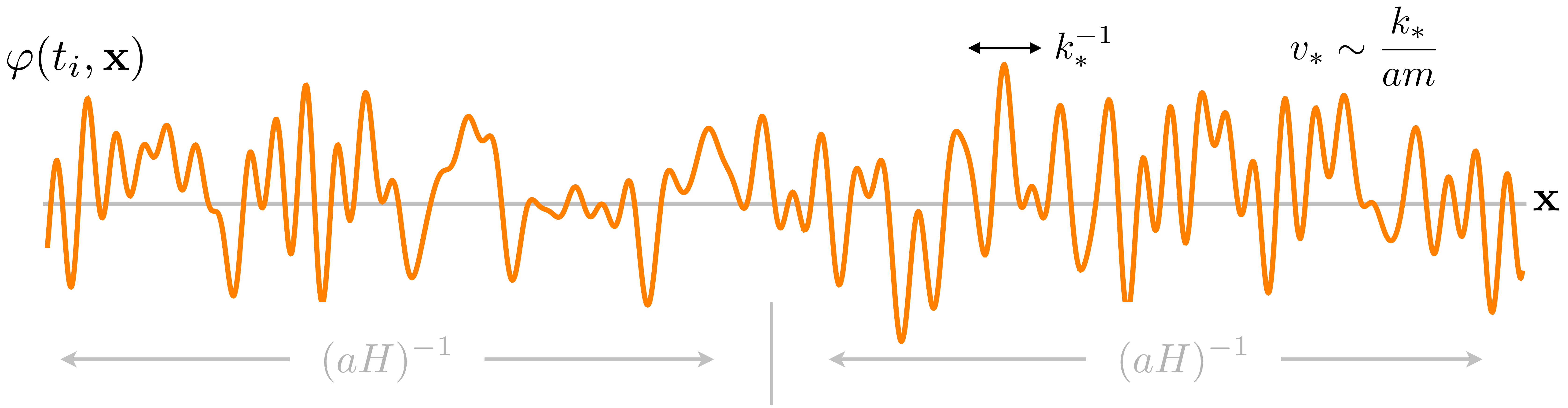


- can be ultralight
- high occupation numbers
- treat as a classical field*

axions with ~~PQ~~ before inflation

Cold vs. Warm Wave Dark Matter

axions with ~~PQ~~ before inflation

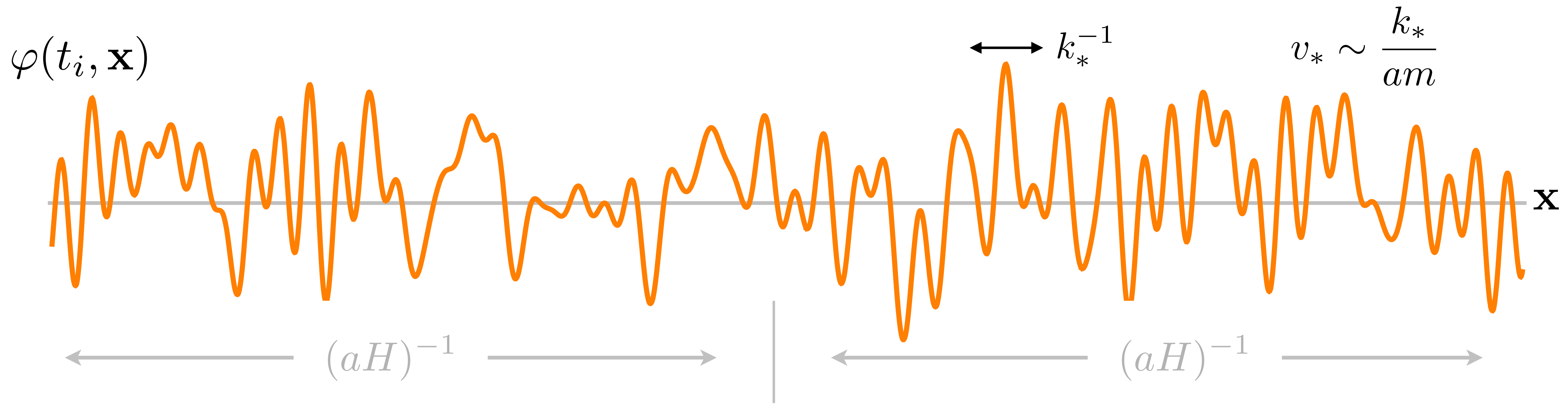


axions with ~~PQ~~ after inflation

*quantitatively, warmth will depend on parameters

Cold vs. Warm Wave Dark Matter

axions with ~~PQ~~ before inflation

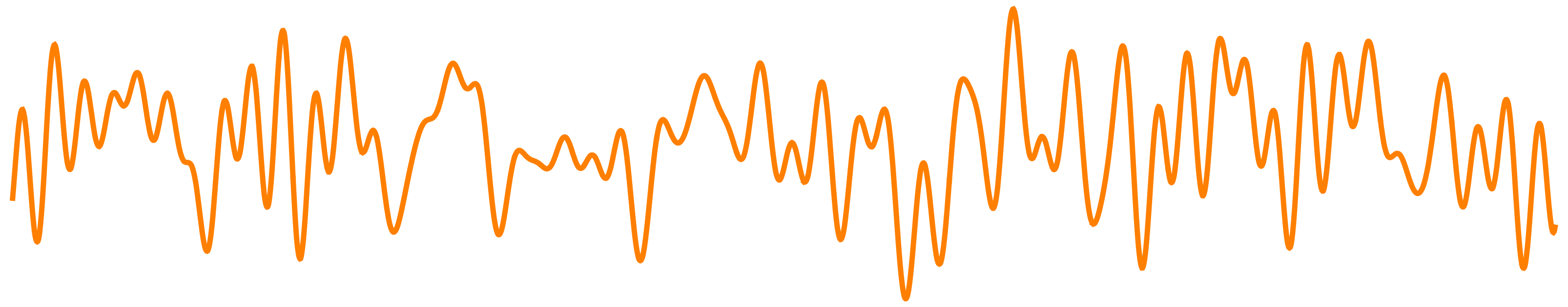


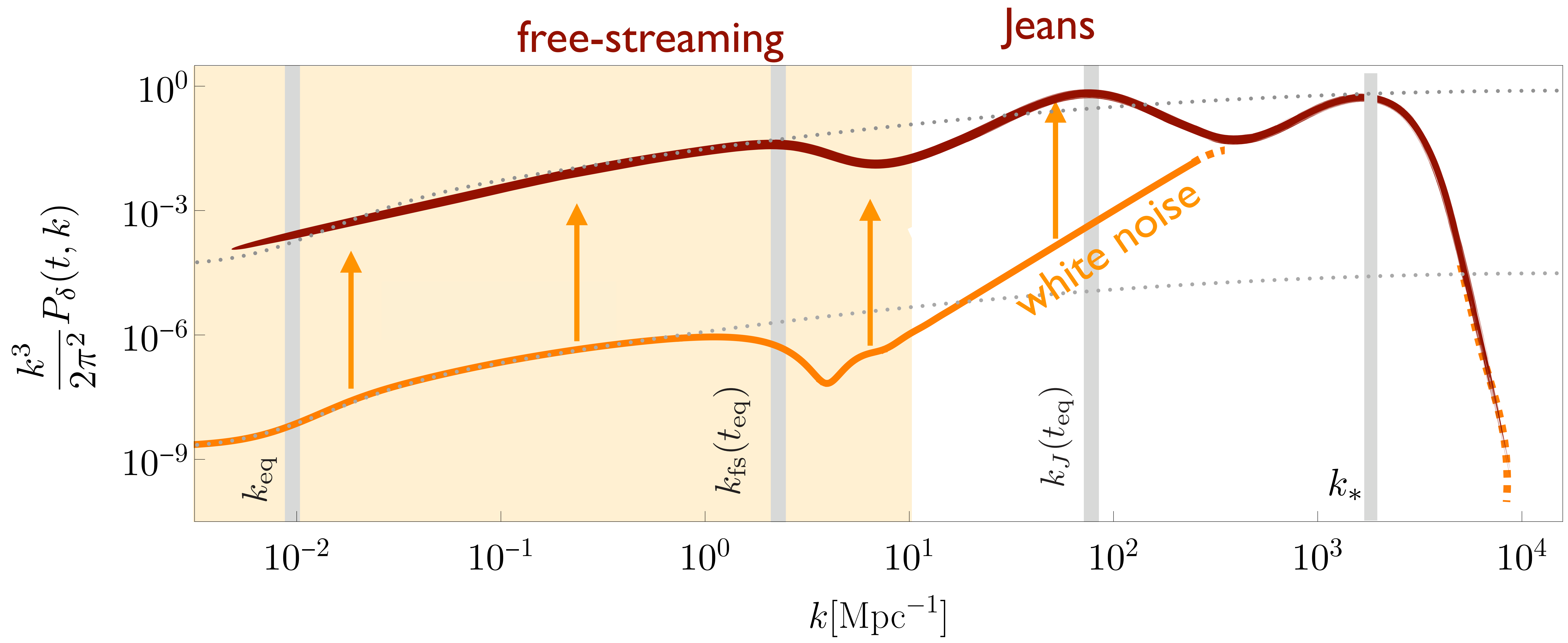
axions with ~~PQ~~ after inflation

*quantitatively, warmth will depend on parameters

main takeaways

If dark matter density is dominated by sub-Hubble field modes





Observationally accessible,

scale-dependent features

in the matter power spectrum

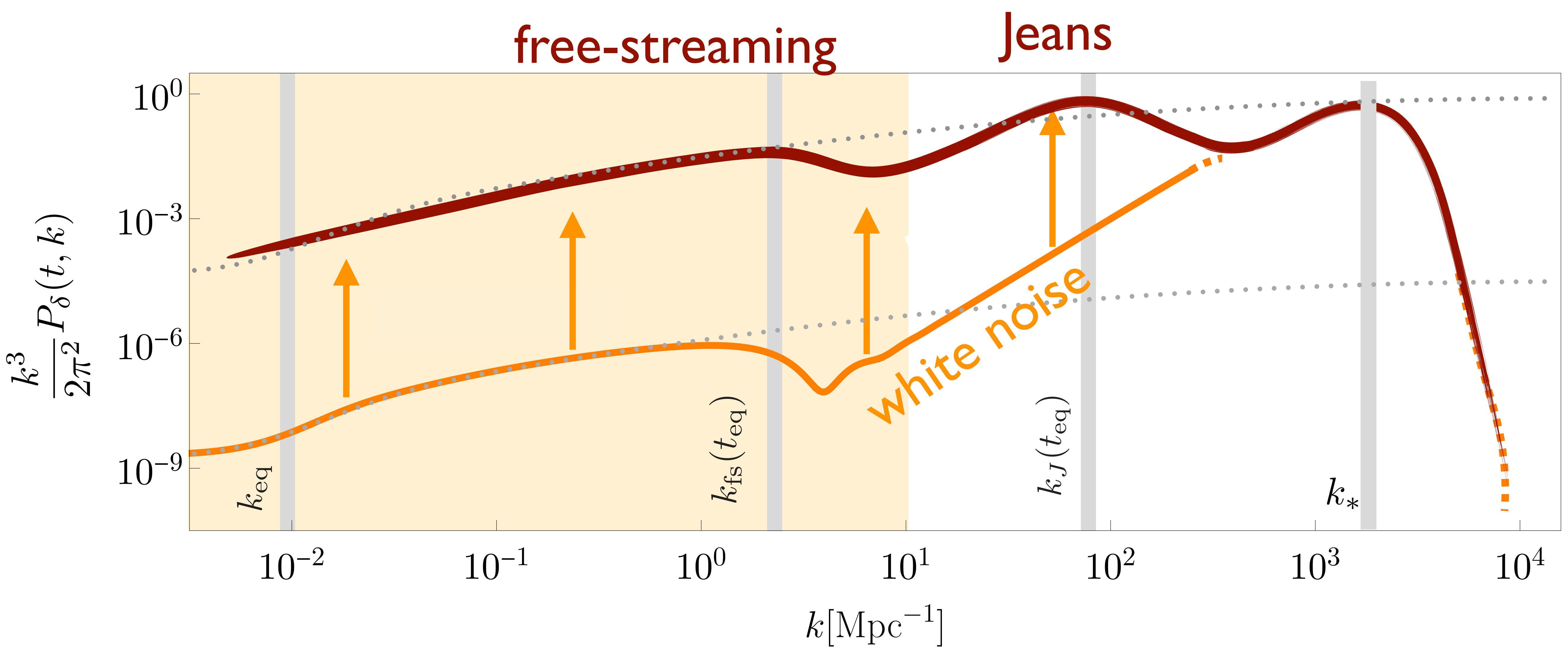
$k_{\rm fs}$ free-streaming

$k_{\rm J}$ Jeans

k_* white noise

main takeaways

If dark matter density is dominated by sub-Hubble field modes



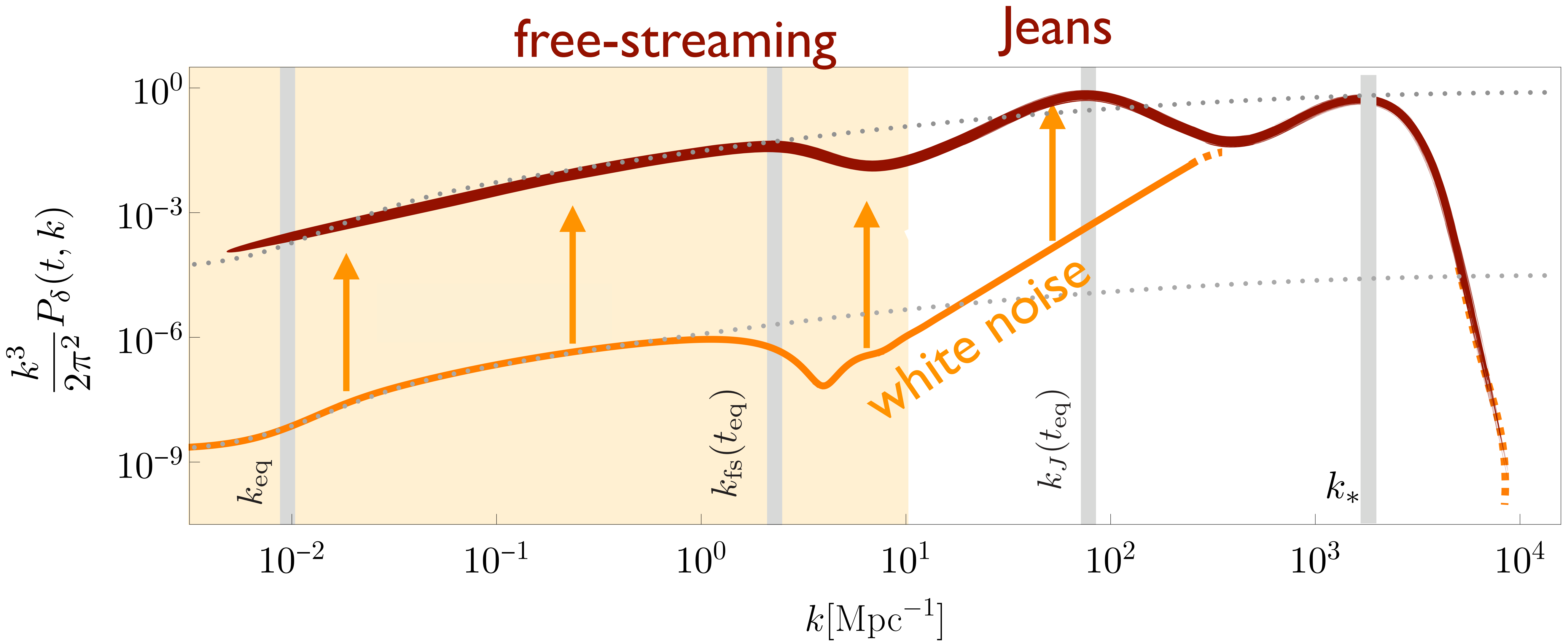
Observationally accessible,
scale-dependent features
in the matter power spectrum

Non-detection — $m \gtrsim 10^{-19}\,\mathrm{eV}$

Lower bound on dark matter mass

main takeaways

If dark matter density is dominated by sub-Hubble field modes

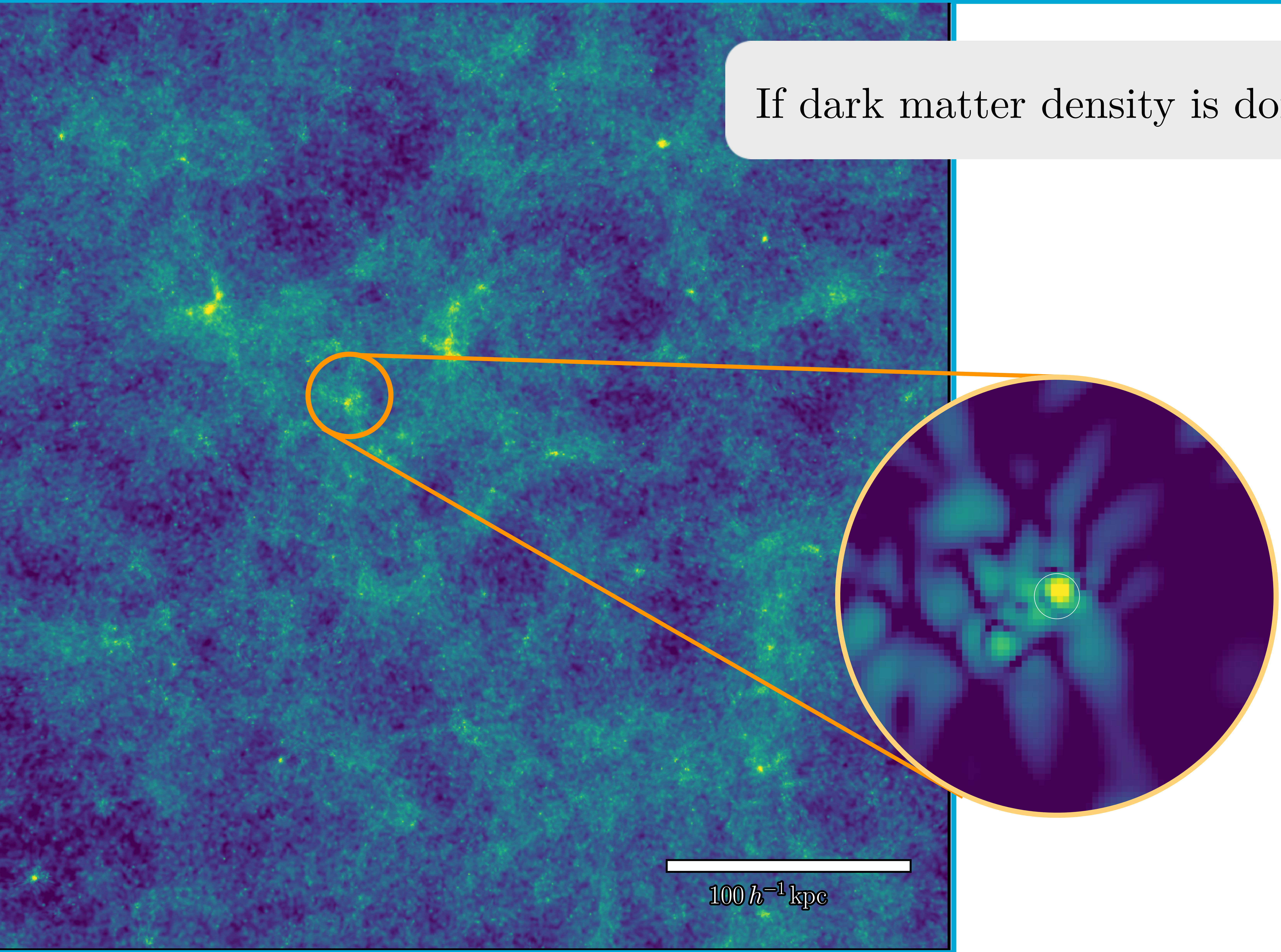


Observationally accessible, scale-dependent features in the matter power spectrum

Non-detection — $m \gtrsim 10^{-19}\,\mathrm{eV}$
$k_* \gtrsim 10^3\,\mathrm{Mpc}^{-1}$

Probe of particle identity formation mechanism

main takeaways

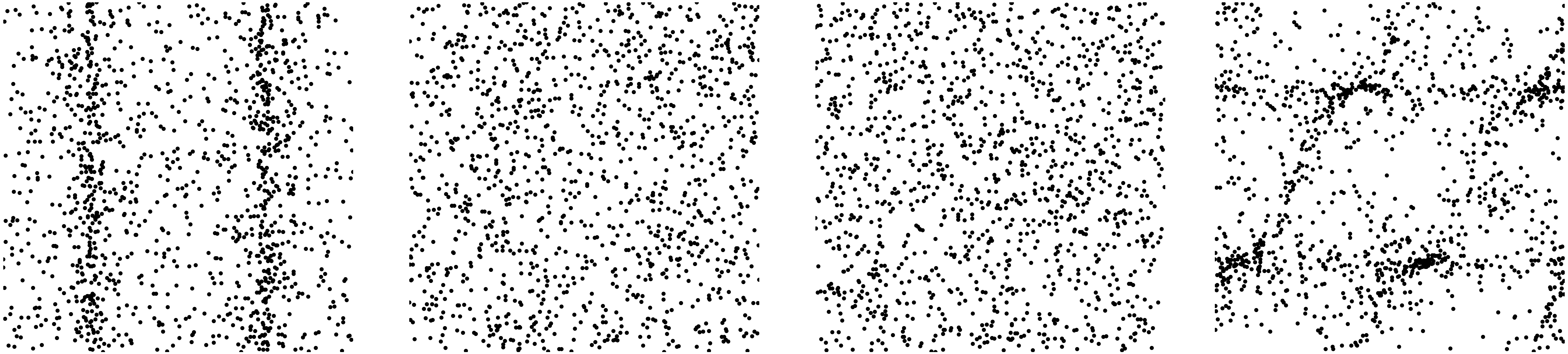


If dark matter density is dominated by sub-Hubble field modes

Observationally accessible,
scale-dependent features
in the matter power spectrum
(+ miniclusters, solitons etc.)

structure formation with finite number density and warmth

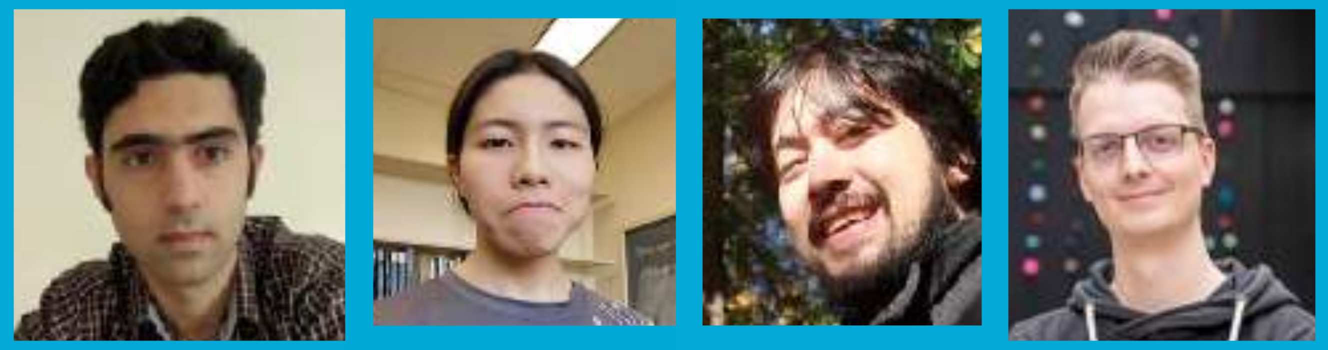
radiation domination   matter domination



(a) —free streaming→ (b) —free streaming→ (c) — grav. clustering → (d)

*dots represent de Broglie interference granules/black holes/ etc.

what we have/can provide so far



Mirbabayi Ling Delos May

1. relevant scales of features in power spectrum, and rough constraints from observations

MA & Mirbabayi (2022)

2. analytic framework for calculating the scale dependent power spectrum & its evolution
3. cosmological simulations of particle & wave dark matter (confirms analytics)
4. predictions of halo mass functions, soliton formation & comparison with simulations

Ling & MA (2024), MA, Delos & Mirbabayi (2025) MA, May & Mirbabayi (2025)

some relevant *recent* work: Liu, Hu and Xiao (2024,25), Venegas & Long (2025) models. physics. analytics. simulations. applications

models with DM density dominated by sub-Hubble fields

non-gravitational production *after* inflation ("causal")

phase transitions

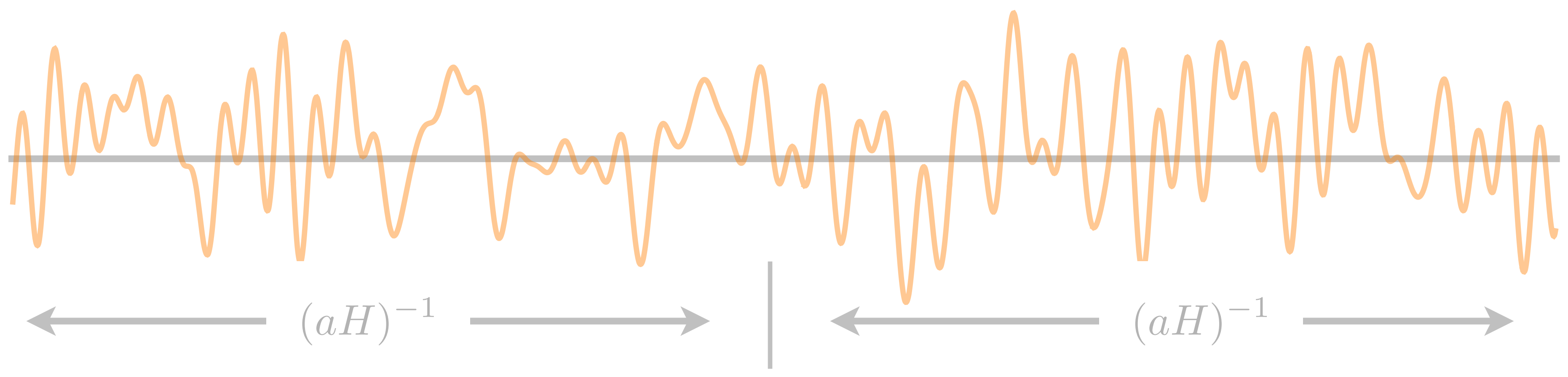
- *axion-like fields (including QCD)*

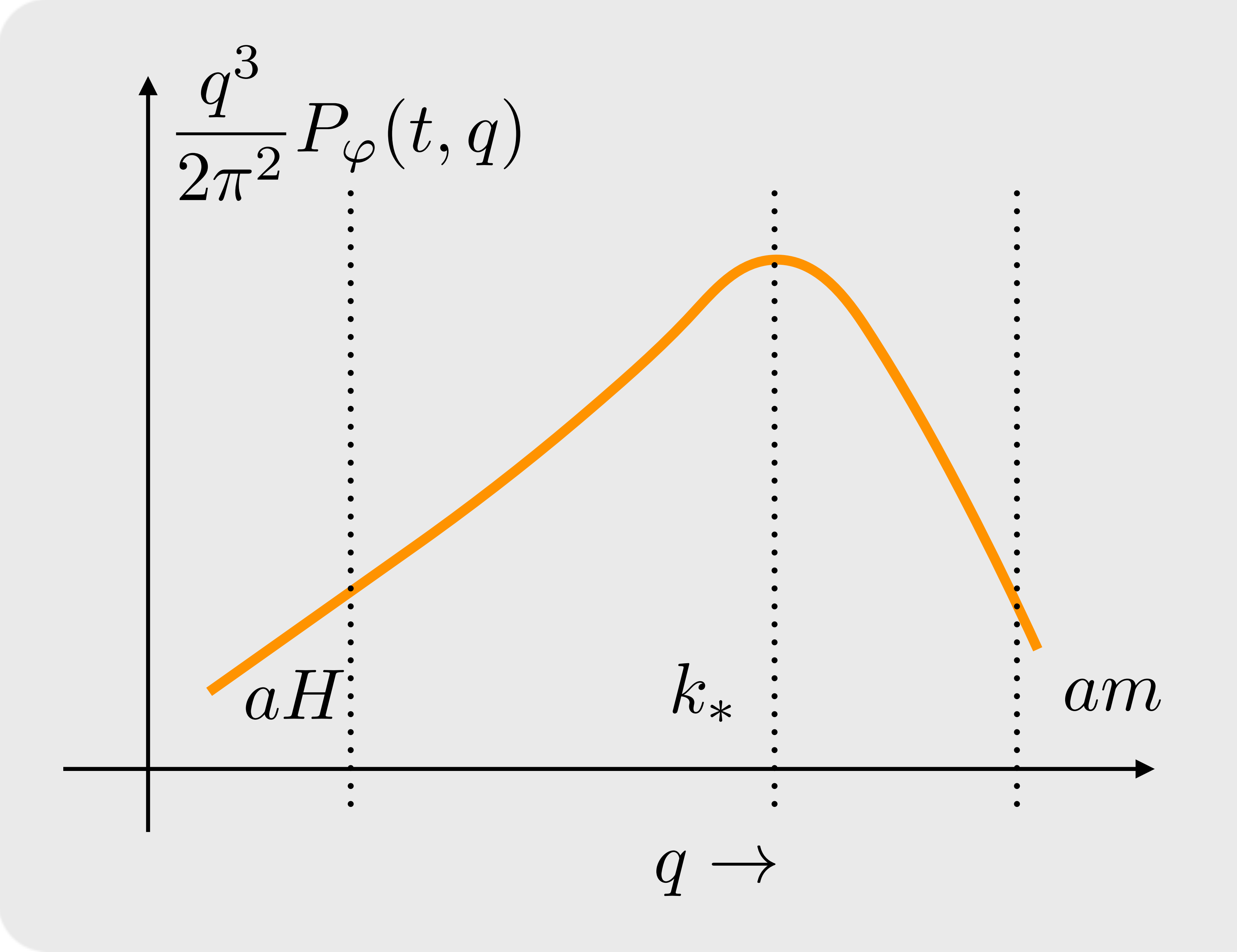
resonant/tachyonic energy transfer from fields, defects eg. strings

also works for thermal particle production, but nothing new there

inflationary gravitational particle production

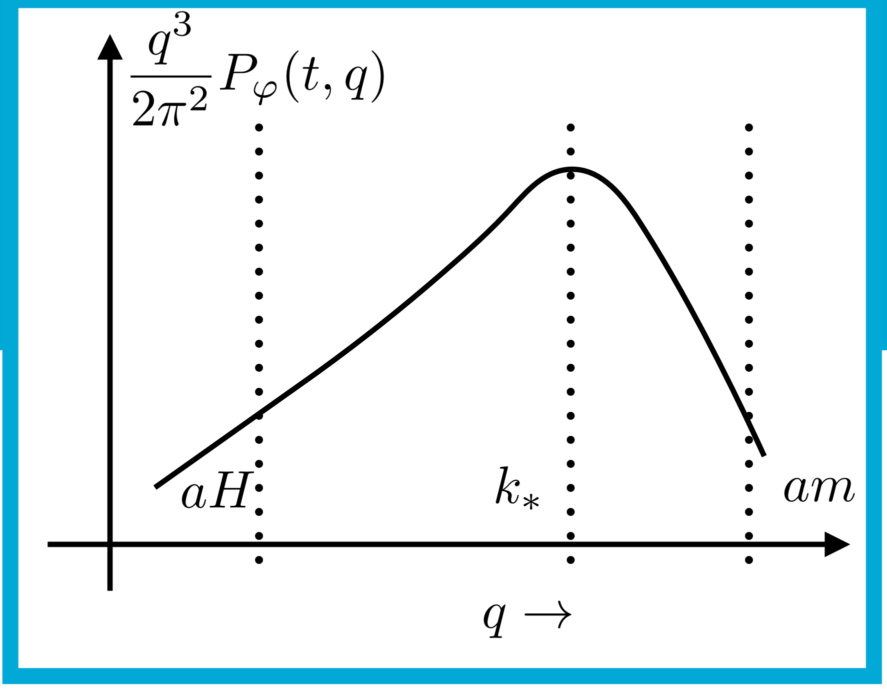
- eg. dark photon dark matter





relevant physics: initial conditions + evolution

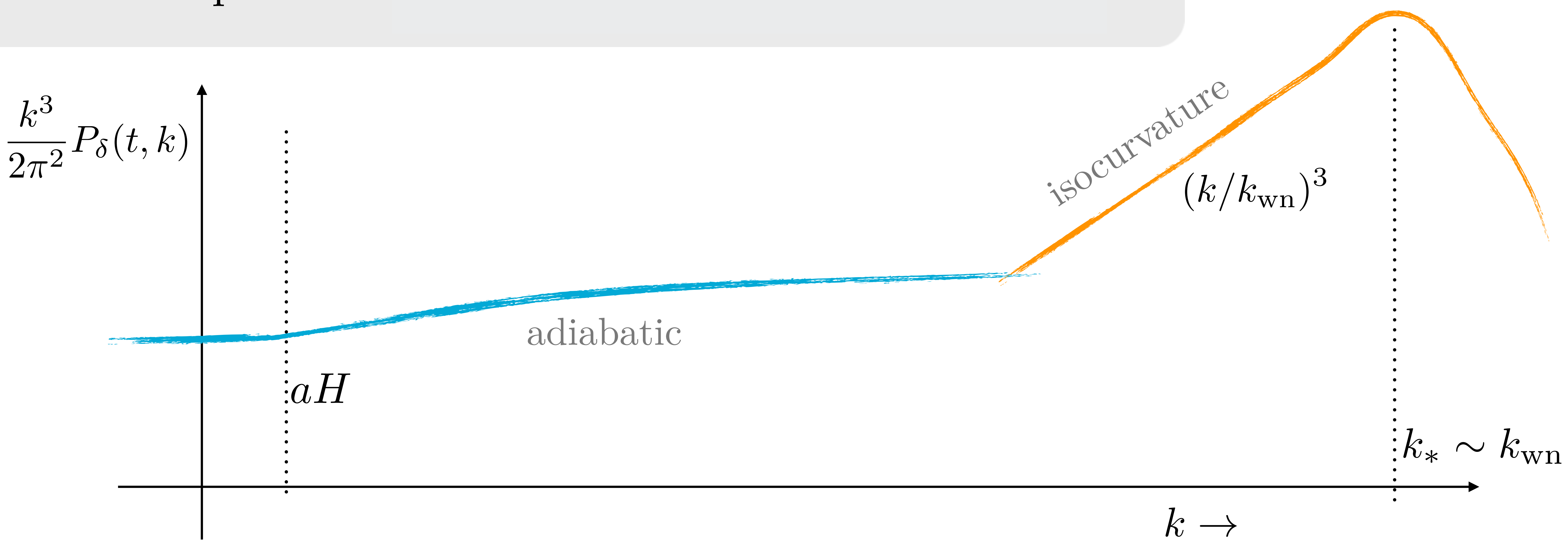
initial conditions



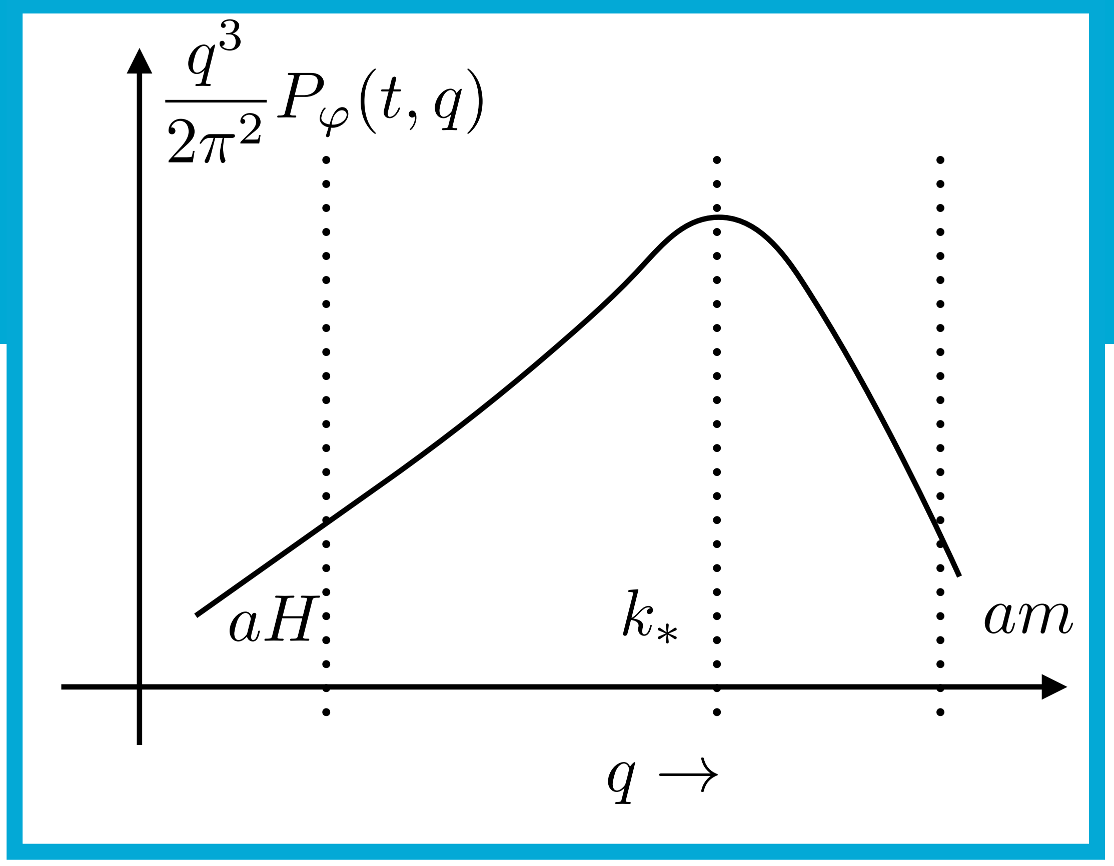
Dark matter density dominated by sub-Hubble field modes

1. small length scale “isocurvature” component

2. larger scale adiabatic component

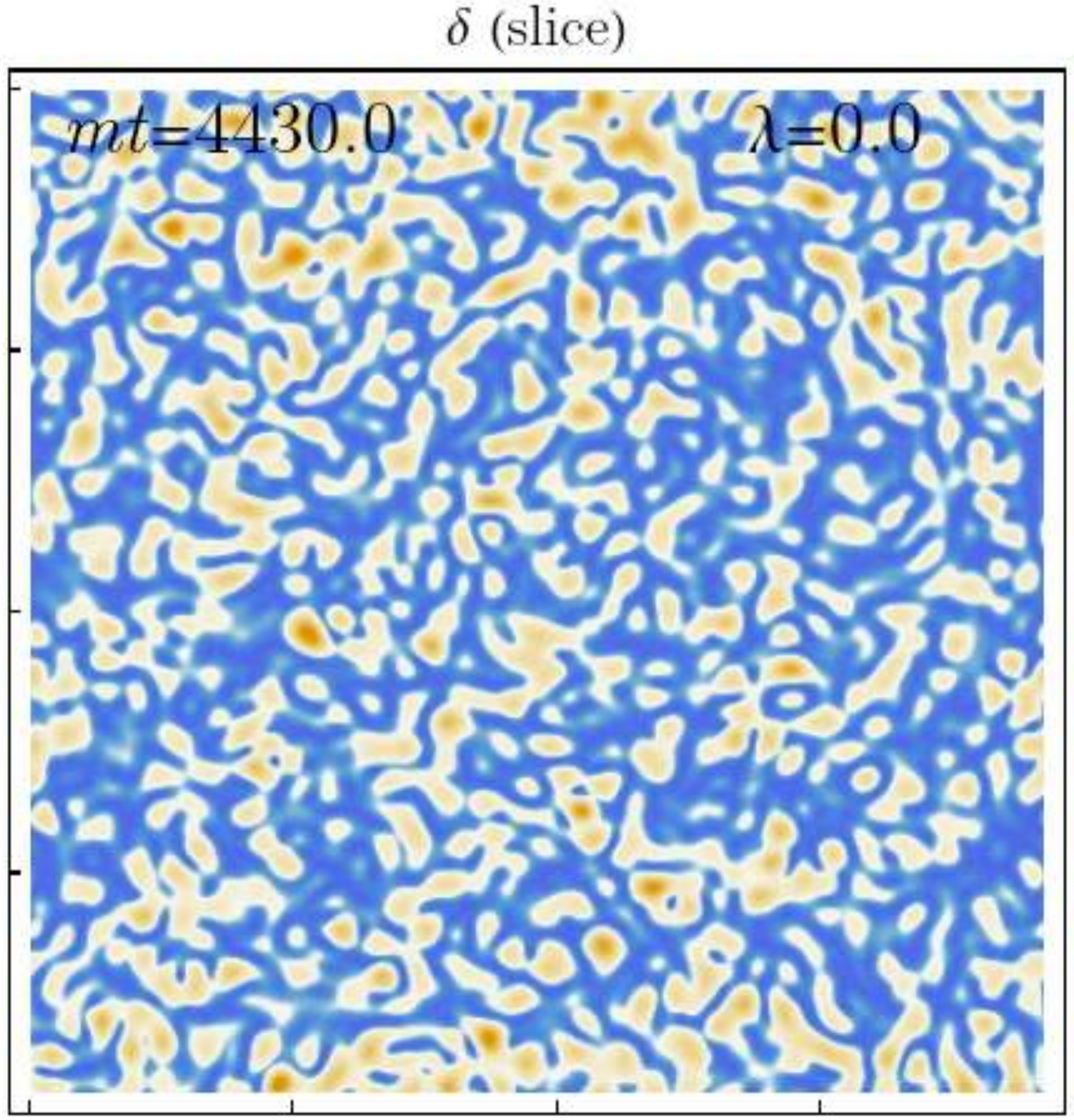


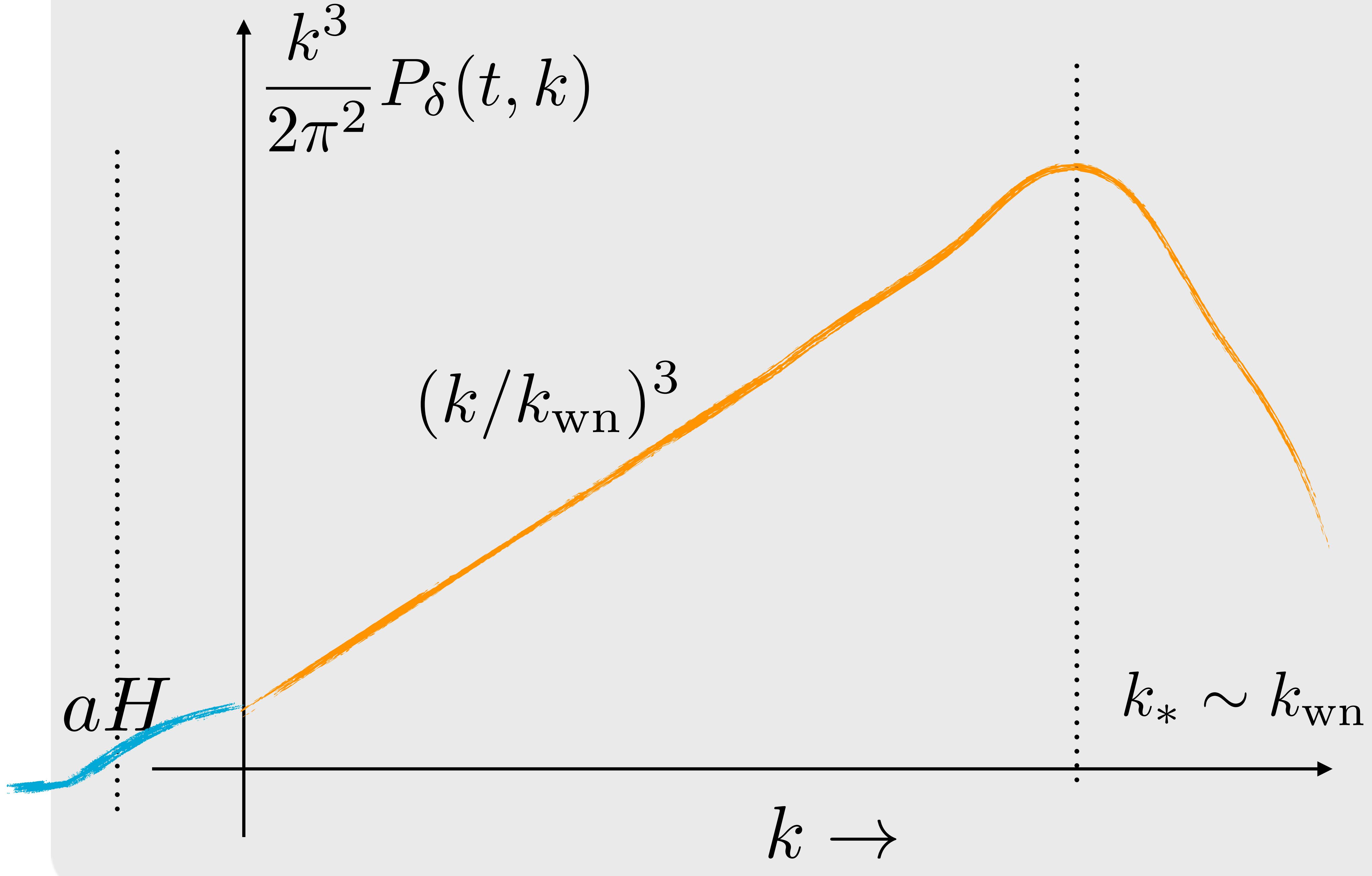
causality & `isocurvature' density enhancement



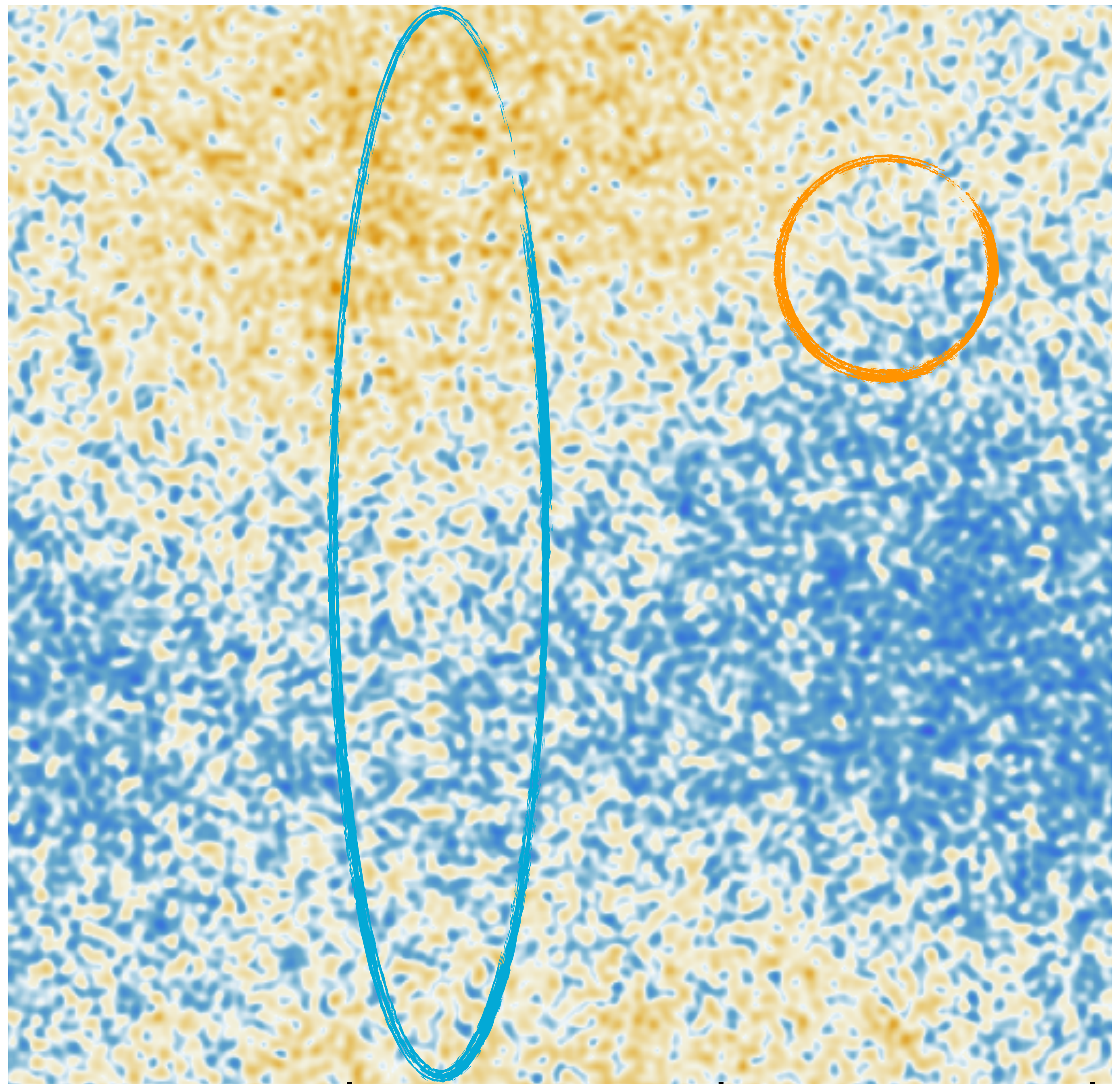
independendent of k for $k \ll k_*$

$$P_\delta(t,k) \approx \frac{m^4}{\bar{\rho}^2(t)} \int d\ln q \, \frac{q^3}{2\pi^2} \left[P_\varphi(q,t)\right]^2 \equiv \frac{2\pi^2}{k_{\rm wn}^3}$$

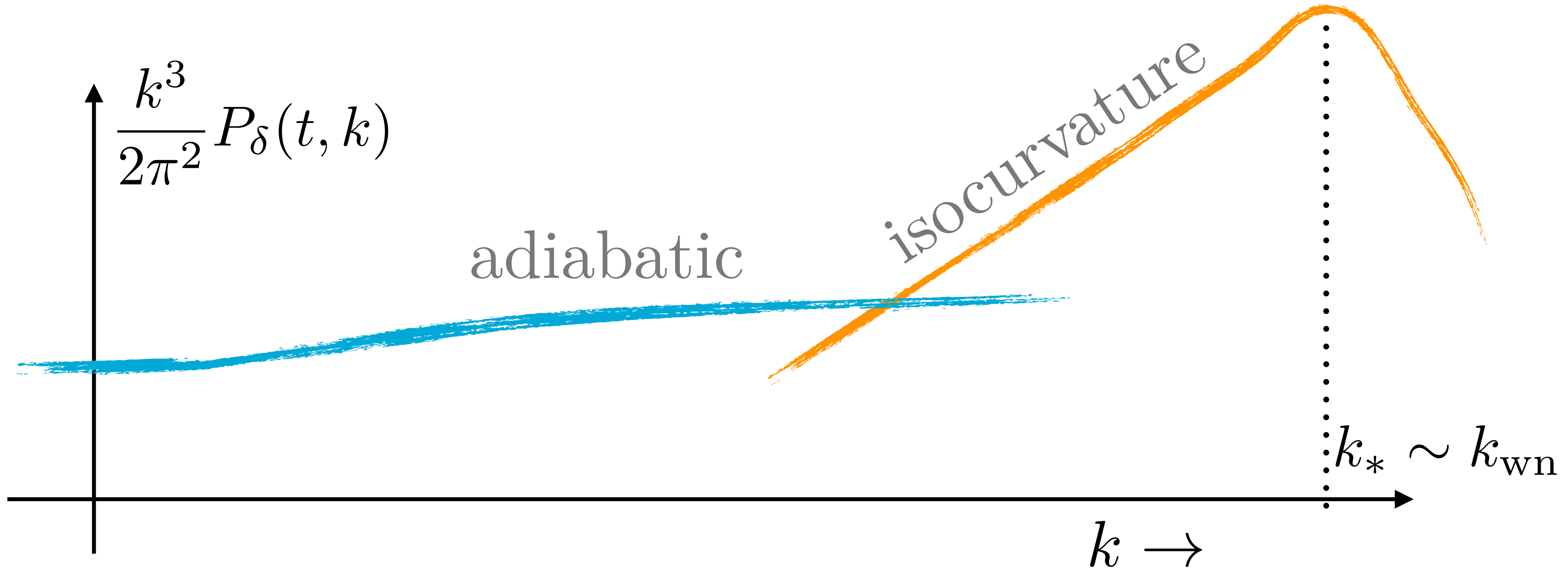


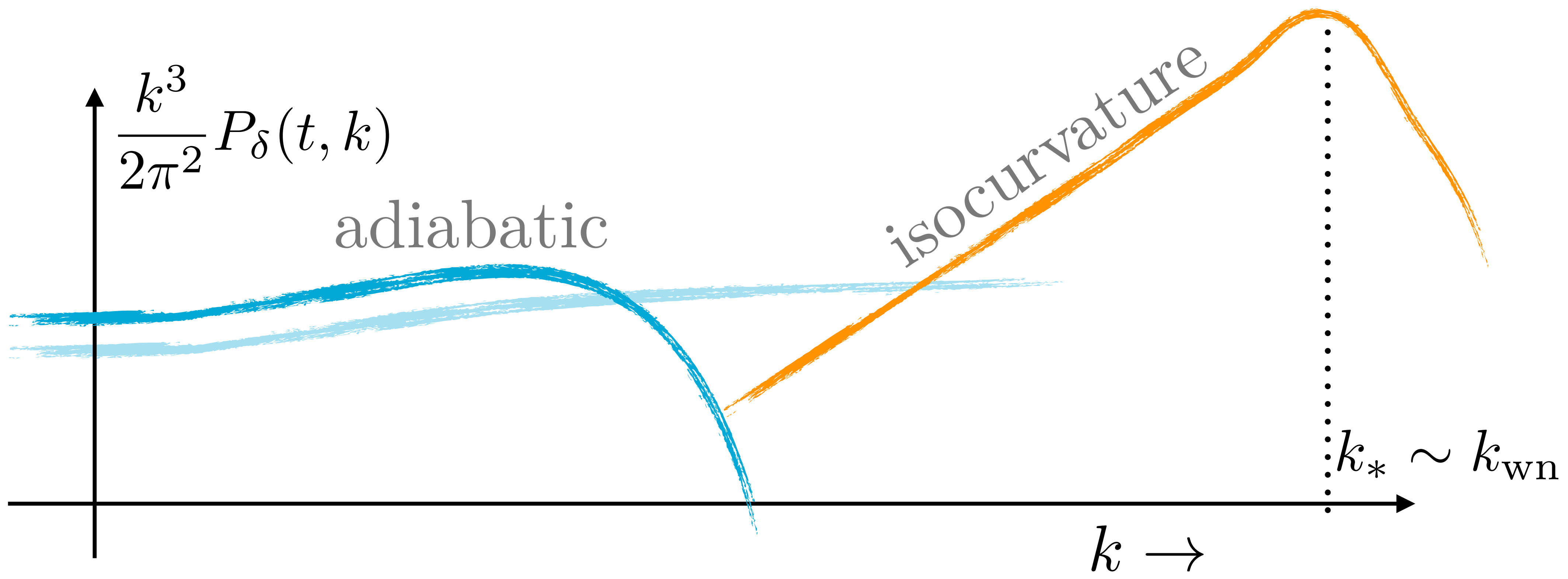


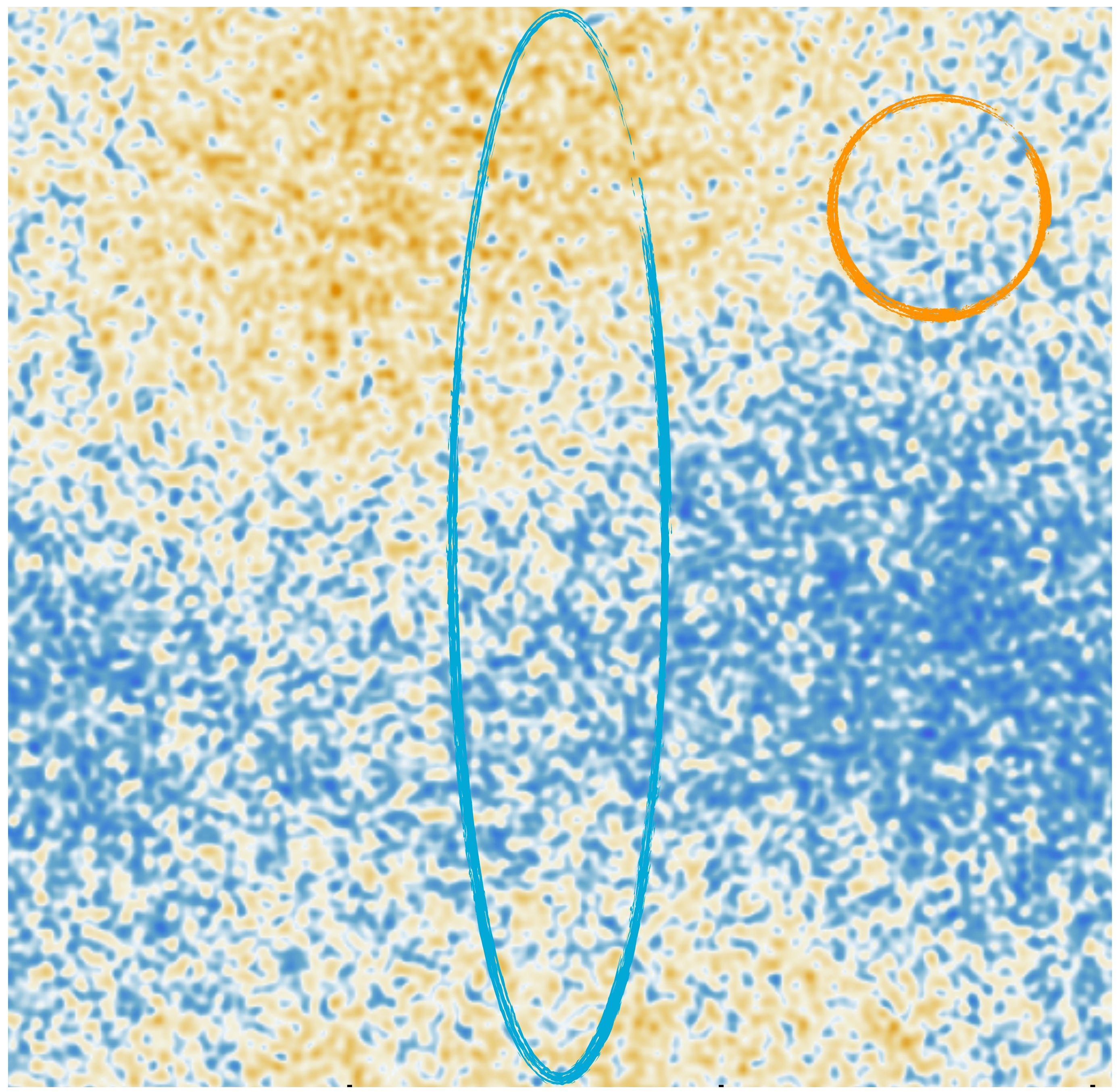
density evolution during radiation domination

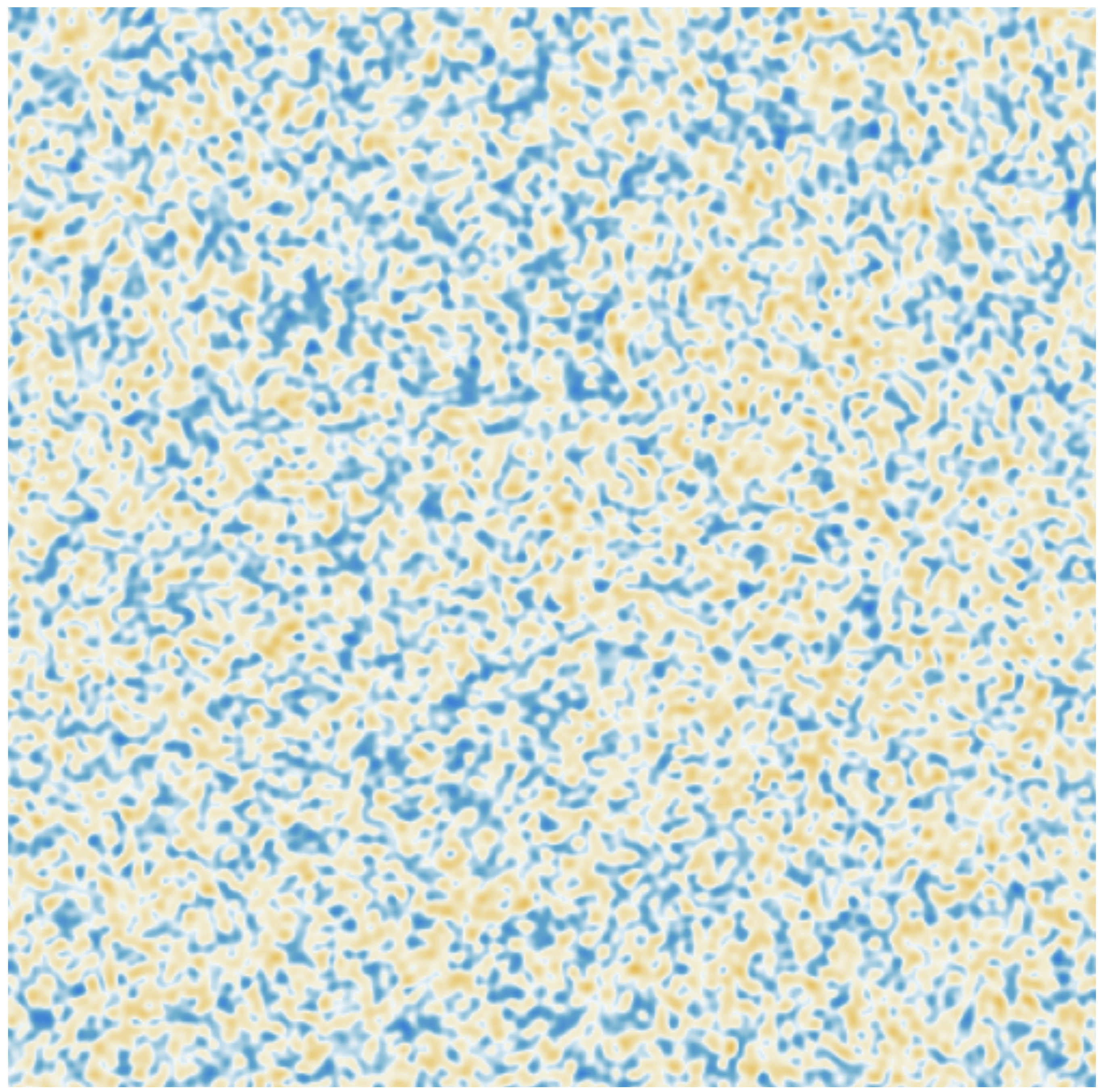


density evolution during radiation domination

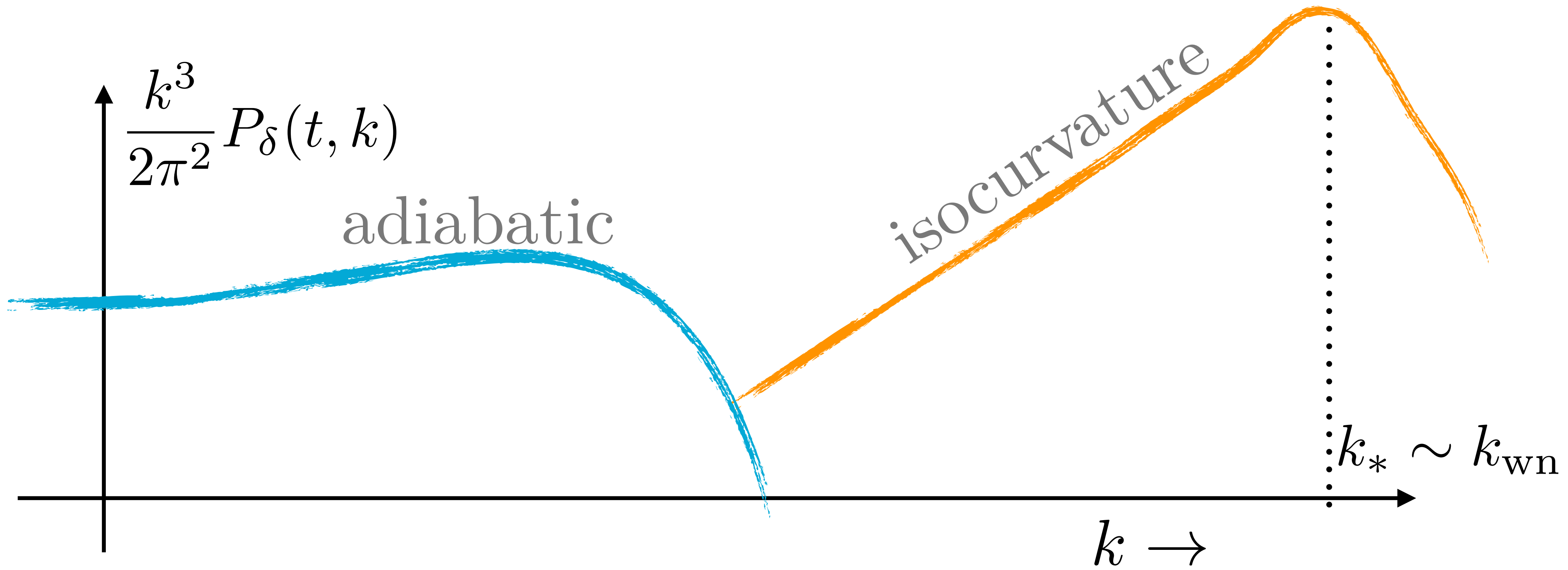


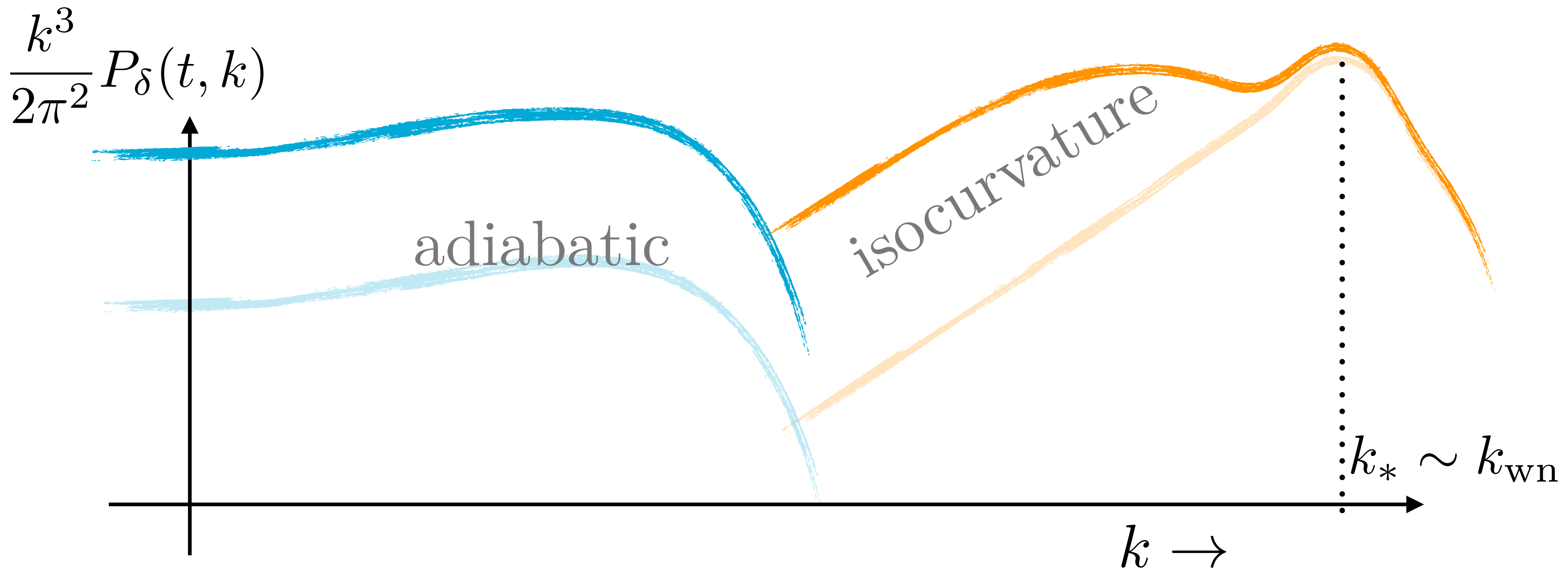


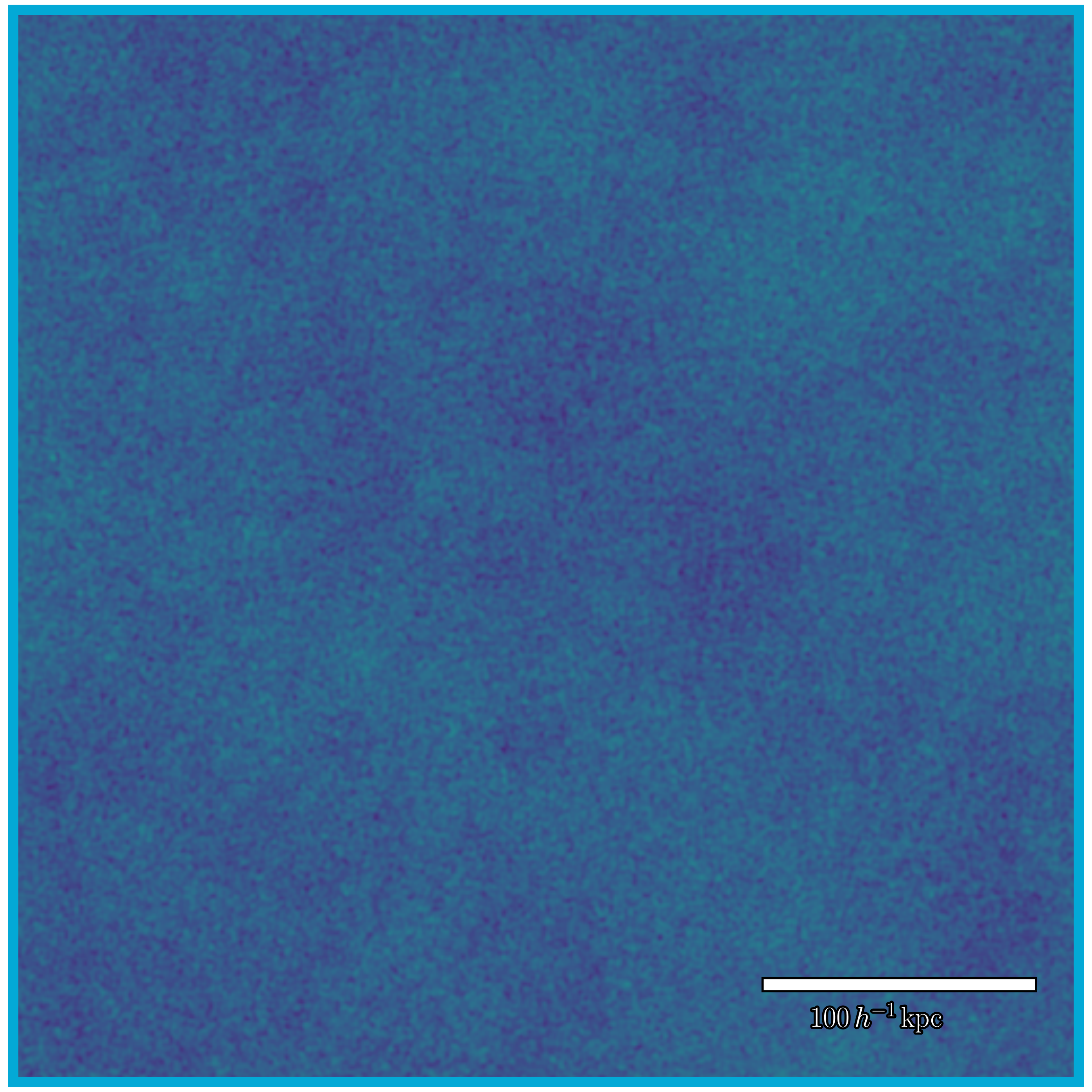


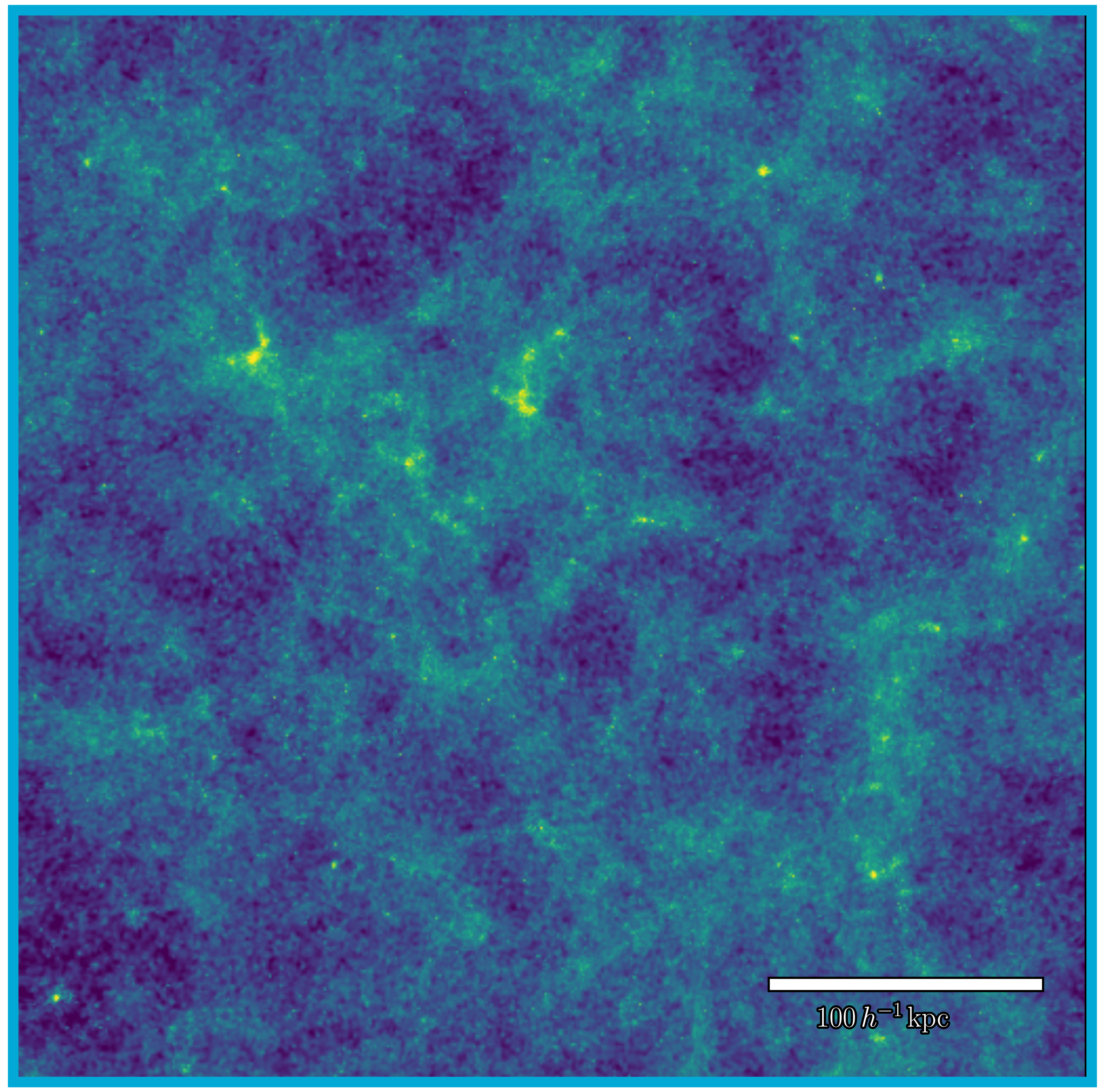


density evolution during matter domination

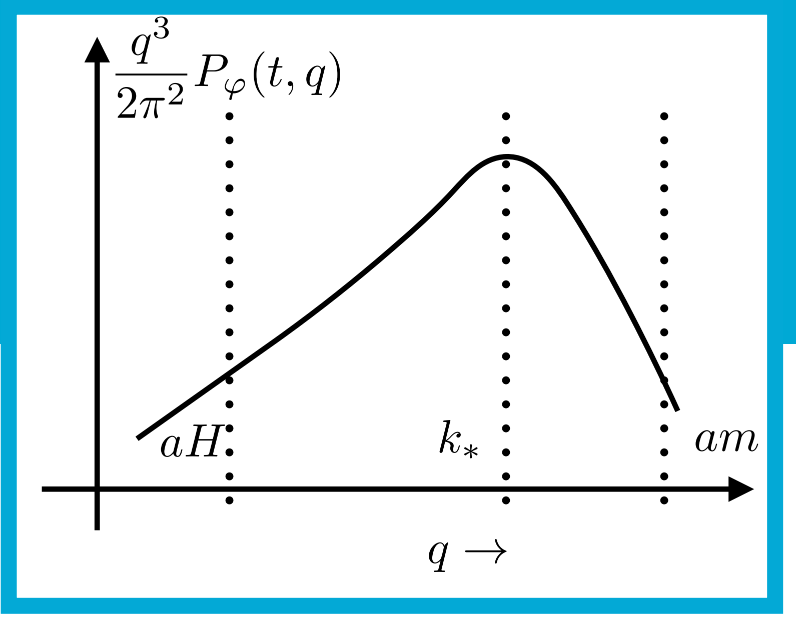


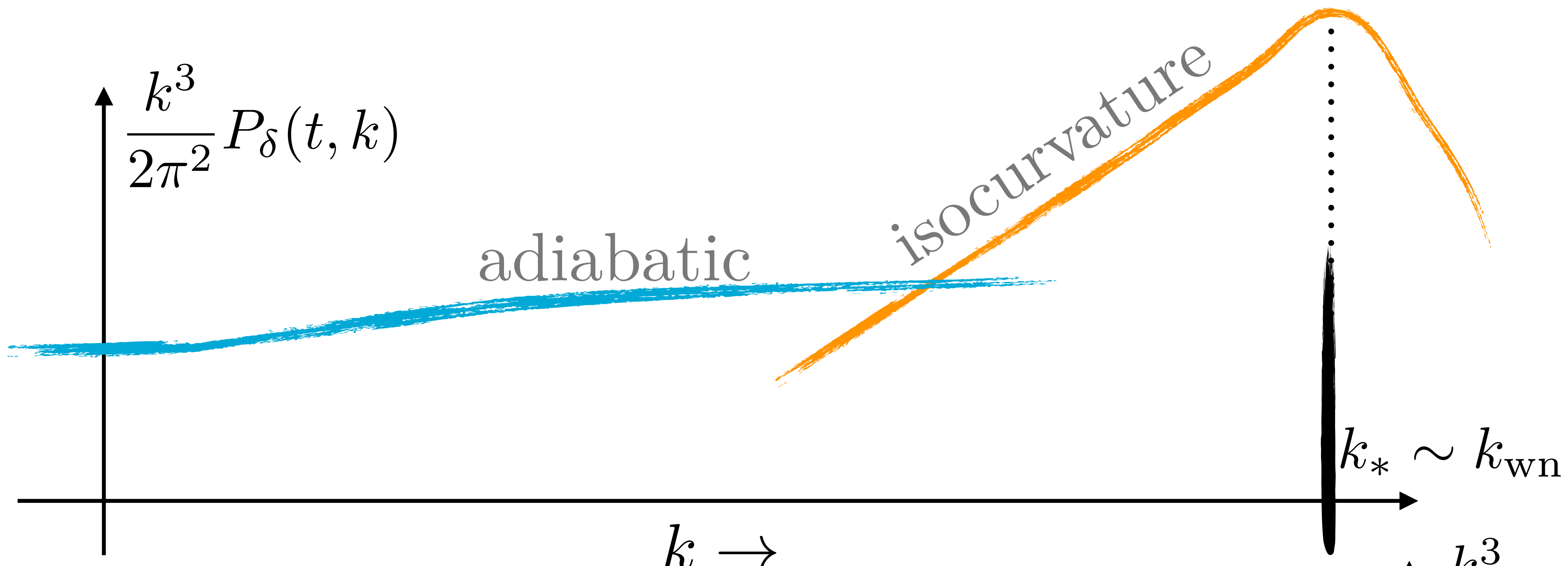




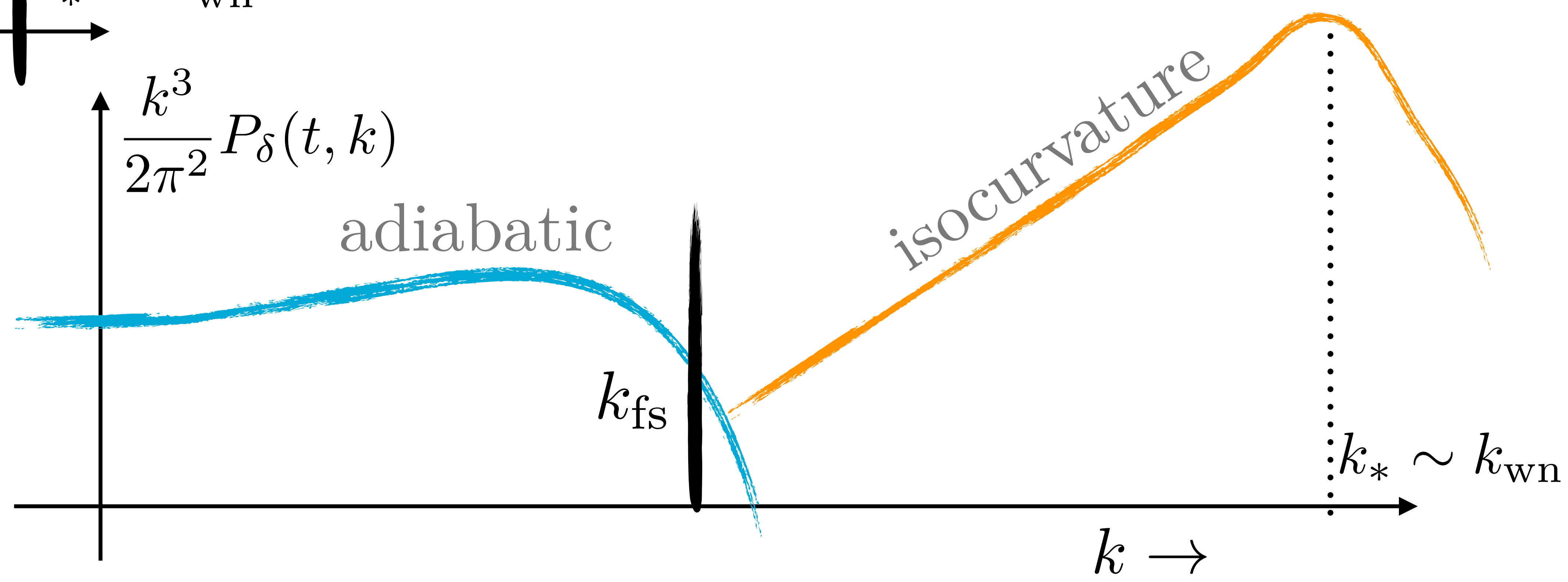


evolution of the matter power spectrum

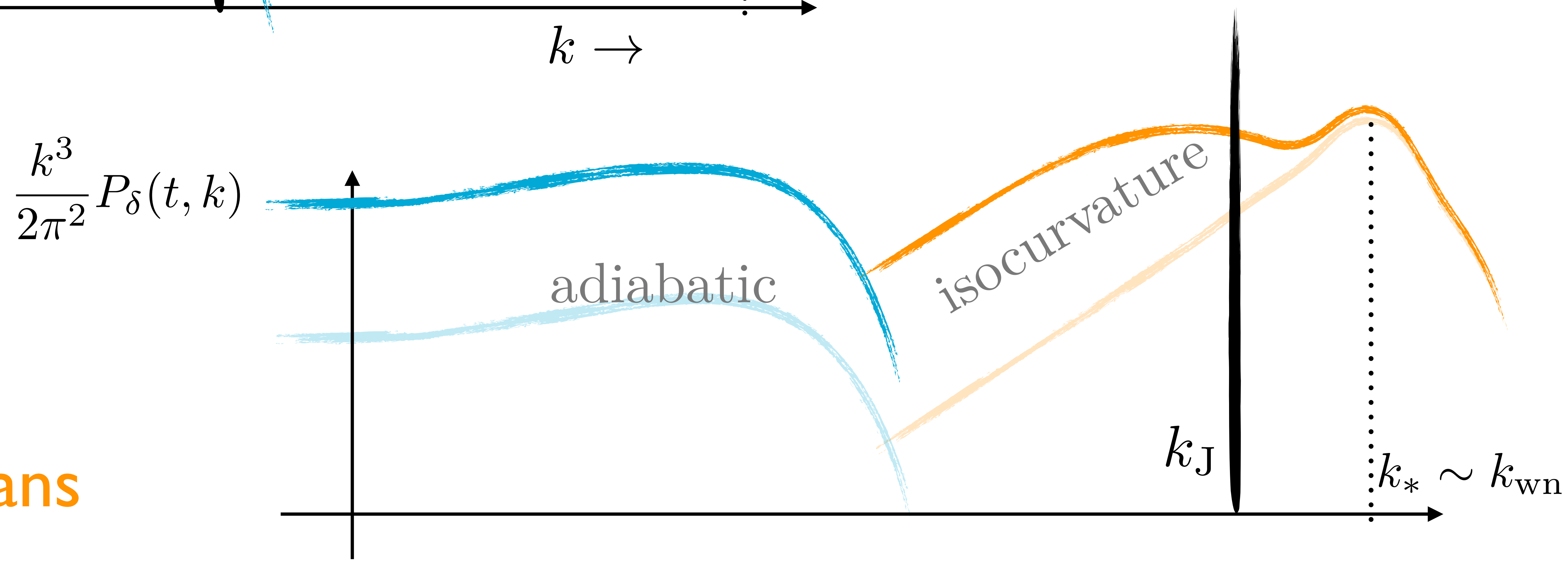




white noise

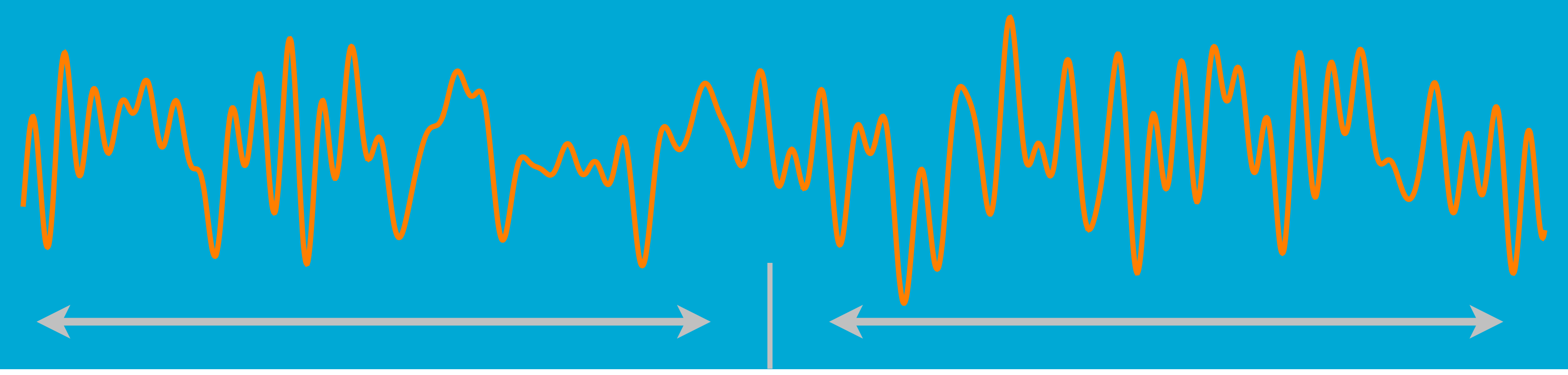


free-streaming



Jeans

key physical effects in the density



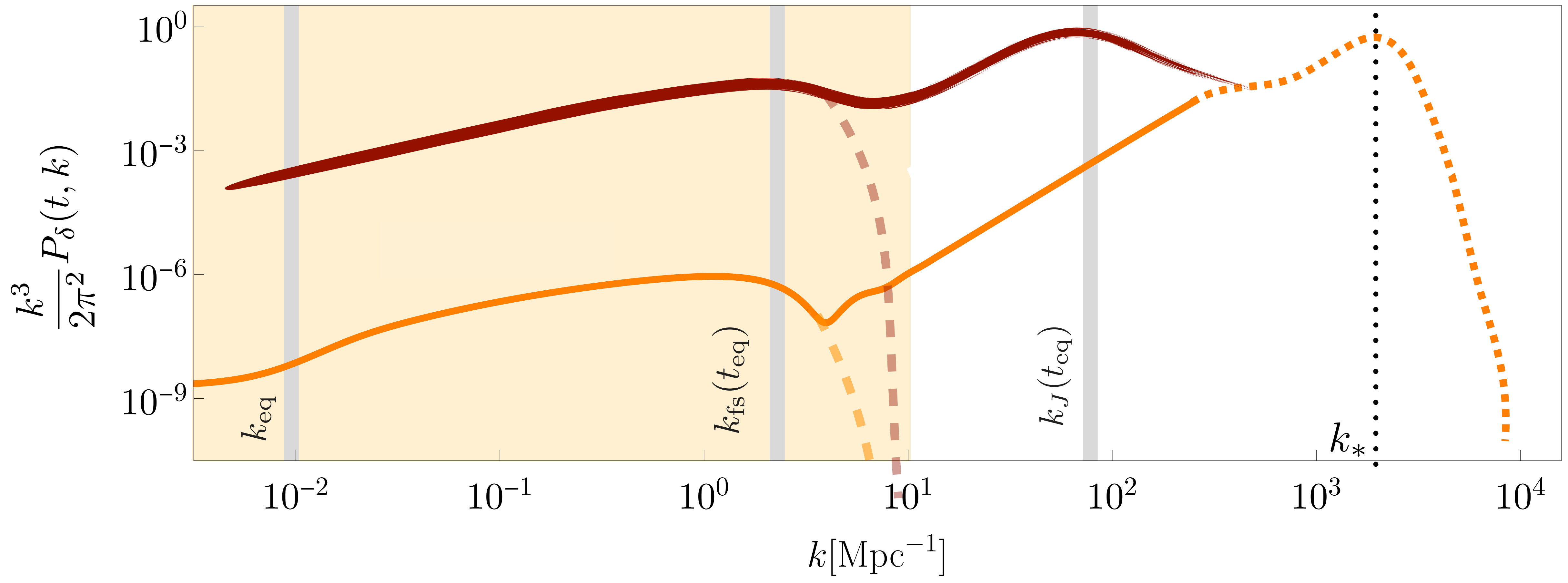
Dark matter density dominated by sub-Hubble field modes

1. Scale-dependent density enhancement in an “isocurvature” component

   causality + Jeans

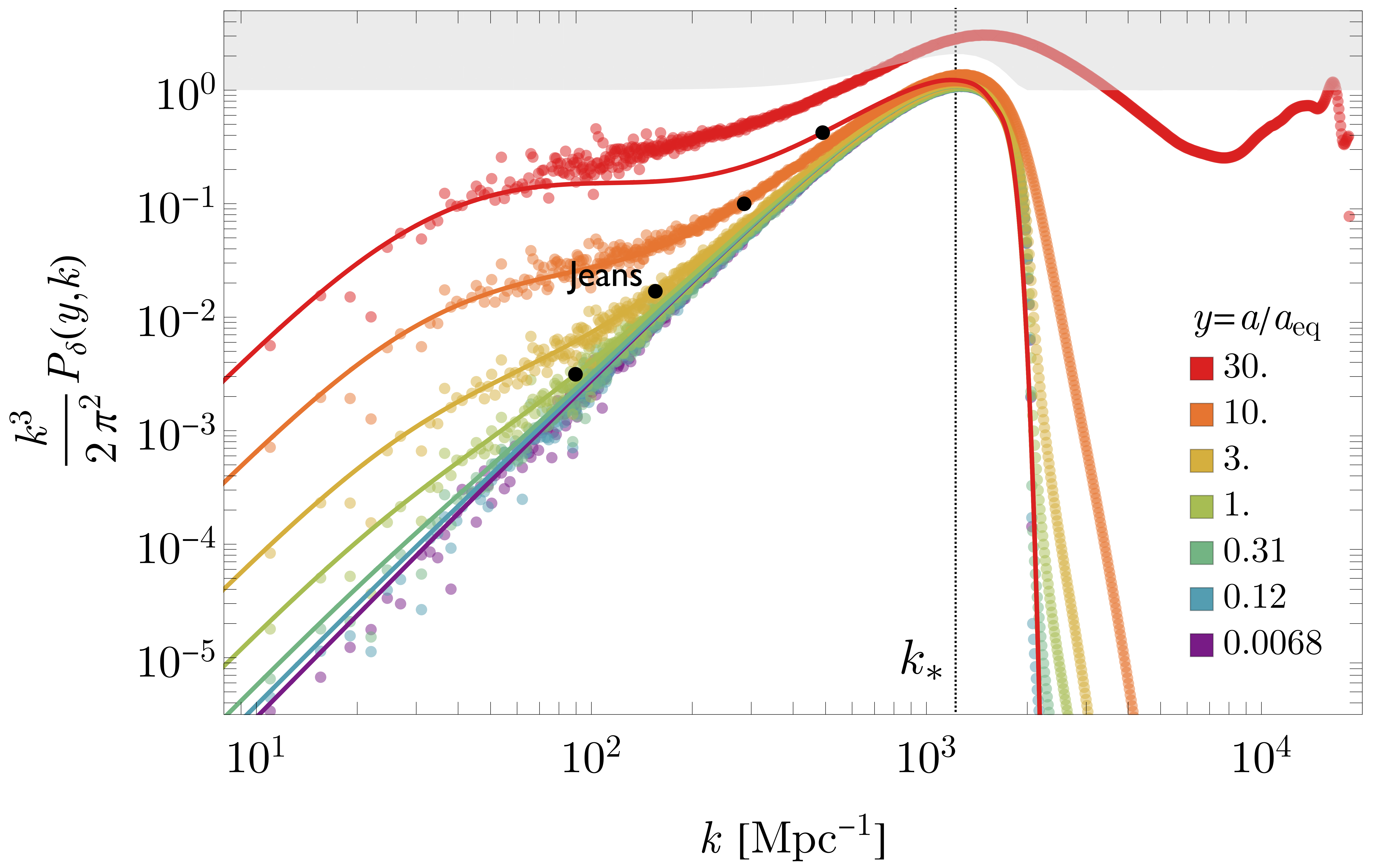
2. Scale-dependent suppression of the adiabatic perturbations

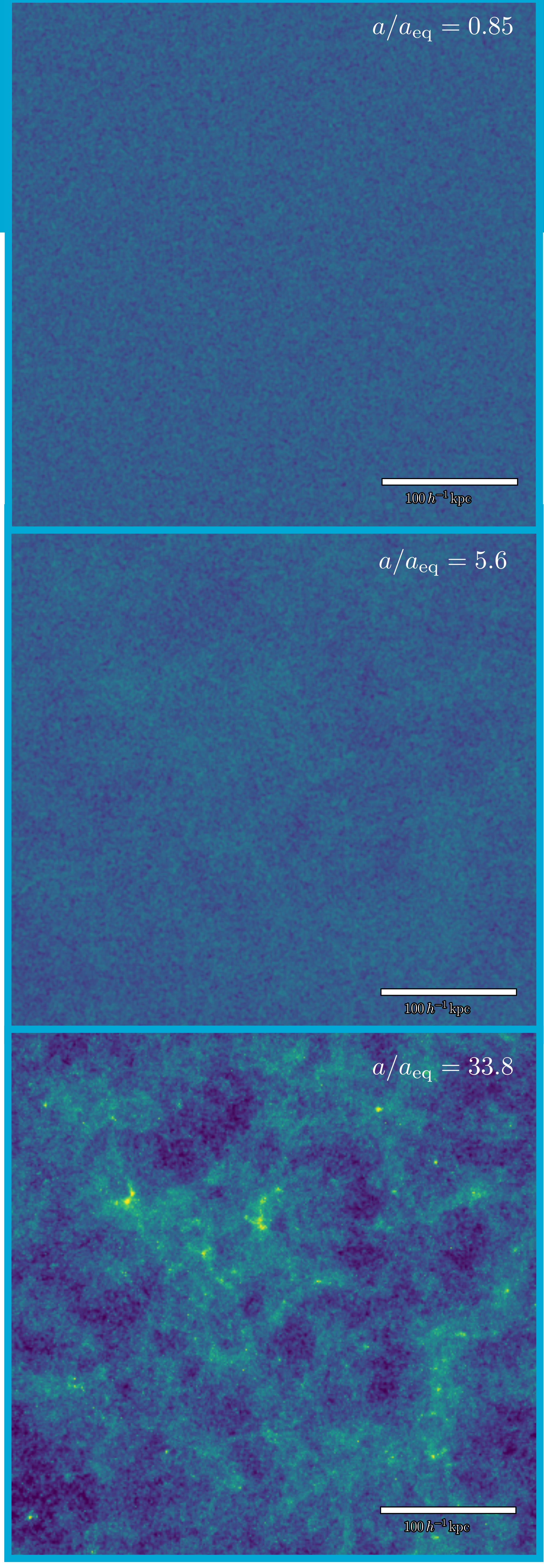
   free streaming



“isocurvature” density evolution — Jeans

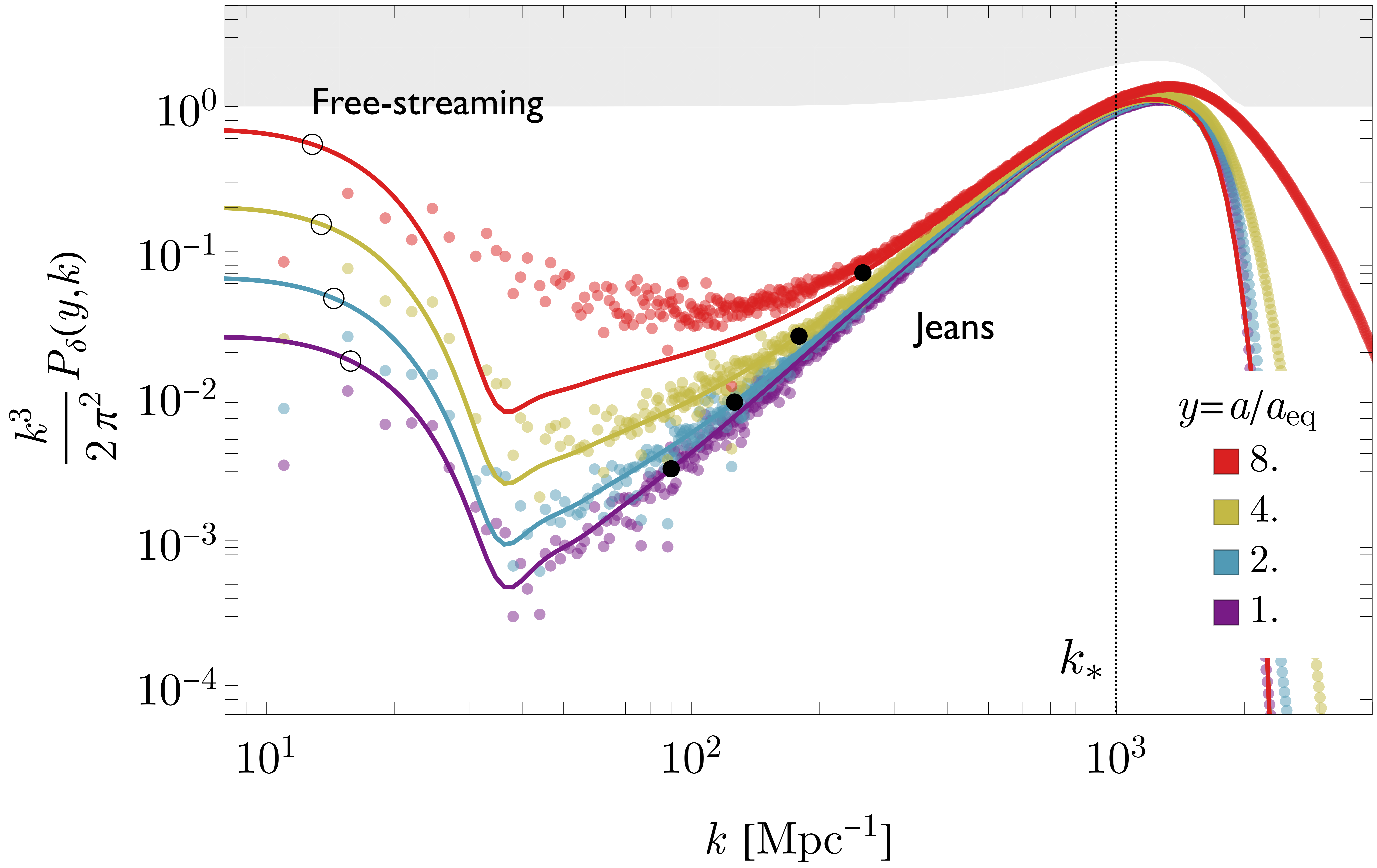
*solid curves are ‘analytic’ results

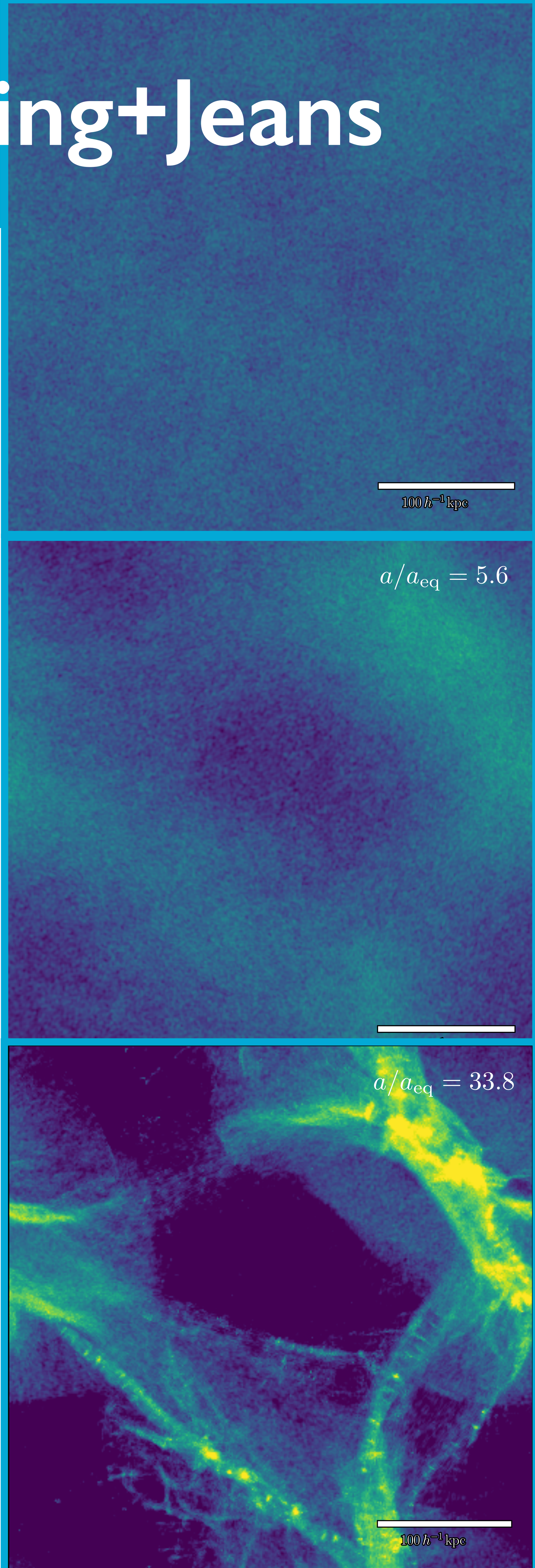




(enahanced) adiabatic + isocurvature — free-streaming+Jeans

MA, May & Mirbabayi (2025)





the analytic framework

MA, Delos & Mirbabayi (2025), MA, May & Mirbabayi (2025)

$$P_\delta(t,k)$$

$$\Uparrow$$

$$\underbrace{f^{(2)}(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{p}_1,\boldsymbol{p}_2)}_{\text{2-particle phase space distribution function}}$$

$$\Uparrow$$

$$\underbrace{\mathcal{L}_t^{(1)} f^{(1)}(t,\boldsymbol{p}_1) = S[f^{(2)}], \quad \mathcal{L}_t^{(2)} f^{(2)}(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{p}_1,\boldsymbol{p}_2) \approx S[f^{(1)}]}_{\text{(truncated) BBGKY hierarchy}}$$

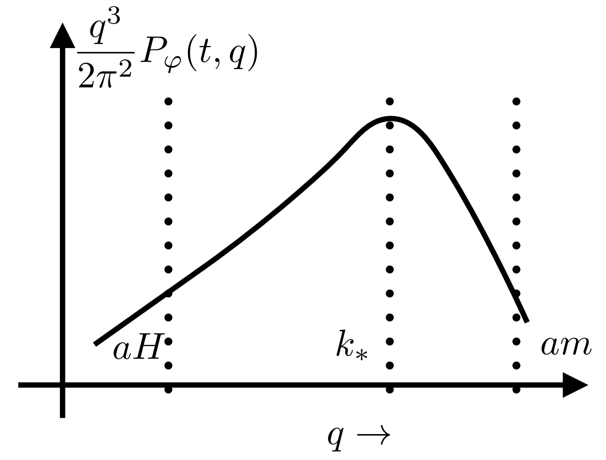
$$\uparrow$$

$$i\,(\partial_t + 3H/2)\,\Psi = -(2ma^2)^{-1}\nabla^2\Psi + m\Phi\Psi, \qquad \nabla^2\Phi = 4\pi G a^2\,(\rho - \bar{\rho})$$

works for classical particles and waves

the analytic framework - solve one Volterra eq.

$$f_0(p) = (m/a^3\bar{\rho})P_\psi(p)$$



the analytic framework - solve one Volterra eq.

$$\mathcal{F}(y,y') \;=\; \ln\left[(y/y')(1+\sqrt{1+y'})^2/(1+\sqrt{1+y})^2\right]$$

$$T^{(a)}_{\rm fs}(y,y',k) = \cos[\gamma\alpha_k^2\mathcal{F}(y,y')]\int_{\boldsymbol{q}} f_0(q)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]$$

$$T^{(b)}_{\rm fs}(y,y',k) = \frac{1}{\gamma\alpha_k^2}\sin[\gamma\alpha_k^2\mathcal{F}(y,y')]\int_{\boldsymbol{q}} f_0(q)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]$$

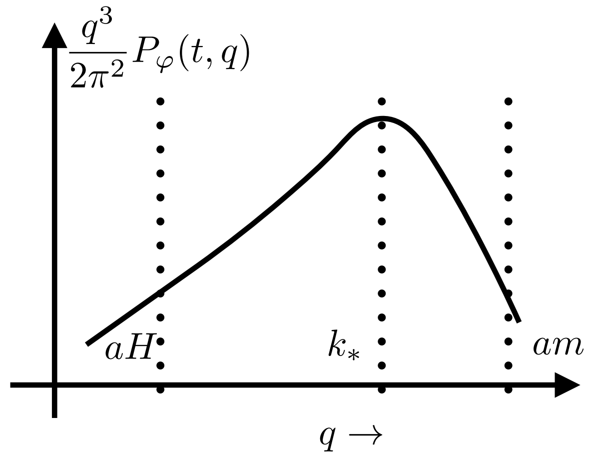
$$T^{(c)}_{\rm fs}(y,y',k) = \frac{\int_{\boldsymbol{q}} f_0(|\boldsymbol{q}+\boldsymbol{k}/2|)f_0(|\boldsymbol{q}-\boldsymbol{k}/2|)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]}{\int_{\boldsymbol{q}} f_0(|\boldsymbol{q}+\boldsymbol{k}/2|)f_0(|\boldsymbol{q}-\boldsymbol{k}/2|)}$$

$$\alpha_k \equiv \sqrt{2}\frac{k}{k_{\rm eq}}\frac{k_*}{a_{\rm eq}m},$$

$$\gamma \equiv \frac{1}{2\sqrt{2}}\frac{a_{\rm eq}m}{k_*}\frac{k_{\rm eq}}{k_*}$$

$$y \;=\; a/a_{\rm eq}$$

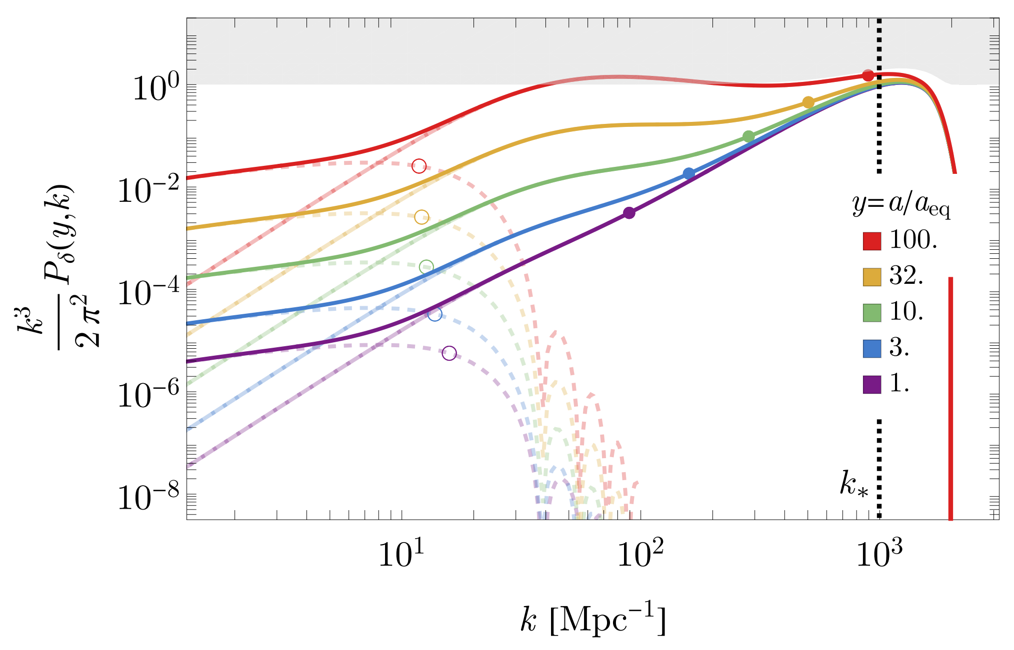
$$f_0(p) = (m/a^3\bar{\rho})P_\psi(p)$$



the analytic framework - solve one Volterra eq.

$$\Delta^2_\delta(y,k) = \Delta^{(\mathrm{ad})}_\delta(y_0,k)\left[\mathcal{T}^{(\mathrm{ad})}_k(y,y_0)\right]^2 + \Delta^{(\mathrm{iso})}_\delta(y_0,k)\left[\mathcal{T}^{(\mathrm{iso})}_k(y,y_0)\right]^2$$

$$y \;=\; a/a_{\mathrm{eq}}$$



$$\mathcal{T}^{(a,b,c)}_k(y,y') = T^{(a,b,c)}_{\mathrm{fs}}(y,y',k) + \frac{3}{2}\int_{y'}^{y}\frac{dy''}{\sqrt{1+y''}}\mathcal{T}^{(b)}_k(y,y'')T^{(a,b,c)}_{\mathrm{fs}}(y'',y',k)$$

< 1 sec

$$\mathcal{F}(y,y') \;=\; \ln\left[(y/y')(1+\sqrt{1+y'})^2/(1+\sqrt{1+y})^2\right]$$

$$T^{(a)}_{\mathrm{fs}}(y,y',k) = \cos[\gamma\alpha_k^2\mathcal{F}(y,y')]\int_{\boldsymbol{q}} f_0(q)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]$$

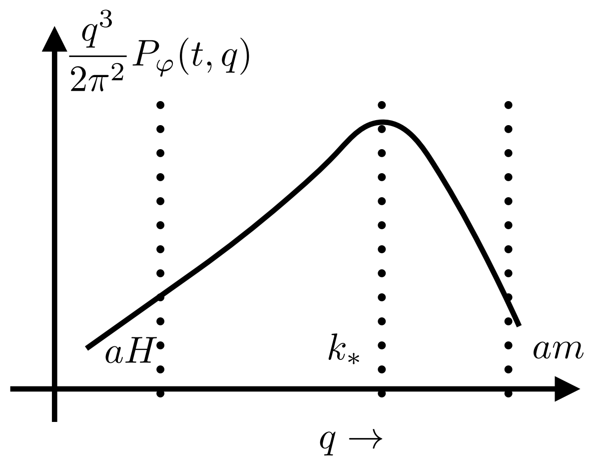
$$T^{(b)}_{\mathrm{fs}}(y,y',k) = \frac{1}{\gamma\alpha_k^2}\sin[\gamma\alpha_k^2\mathcal{F}(y,y')]\int_{\boldsymbol{q}} f_0(q)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]$$

$$T^{(c)}_{\mathrm{fs}}(y,y',k) = \frac{\int_{\boldsymbol{q}} f_0(|\boldsymbol{q}+\boldsymbol{k}/2|)f_0(|\boldsymbol{q}-\boldsymbol{k}/2|)\exp\left[-i\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\frac{q}{k_*}\alpha_k\mathcal{F}(y,y')\right]}{\int_{\boldsymbol{q}} f_0(|\boldsymbol{q}+\boldsymbol{k}/2|)f_0(|\boldsymbol{q}-\boldsymbol{k}/2|)}$$

$$\alpha_k \equiv \sqrt{2}\frac{k}{k_{\mathrm{eq}}}\frac{k_*}{a_{\mathrm{eq}}m},$$

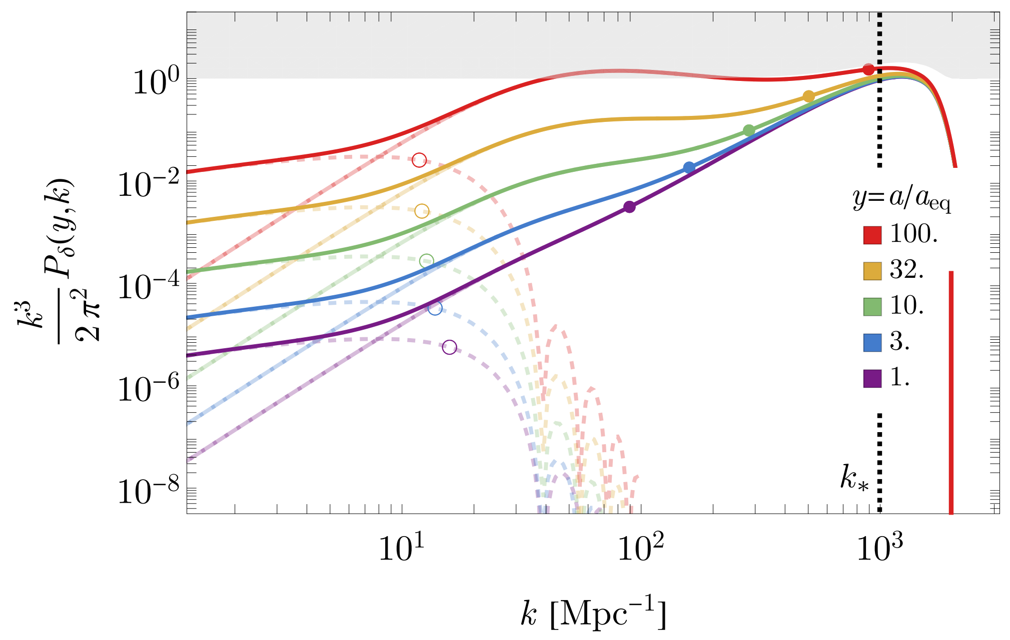
$$\gamma \equiv \frac{1}{2\sqrt{2}}\frac{a_{\mathrm{eq}}m}{k_*}\frac{k_{\mathrm{eq}}}{k_*}$$

$$f_0(p) = (m/a^3\bar{\rho})P_\psi(p)$$



the isocurvature transfer function is non-trivial

$$\Delta_\delta^2(y,k) = \Delta_\delta^{(\mathrm{ad})}(y_0,k)\left[\mathcal{T}_k^{(\mathrm{ad})}(y,y_0)\right]^2 + \Delta_\delta^{(\mathrm{iso})}(y_0,k)\left[\mathcal{T}_k^{(\mathrm{iso})}(y,y_0)\right]^2$$

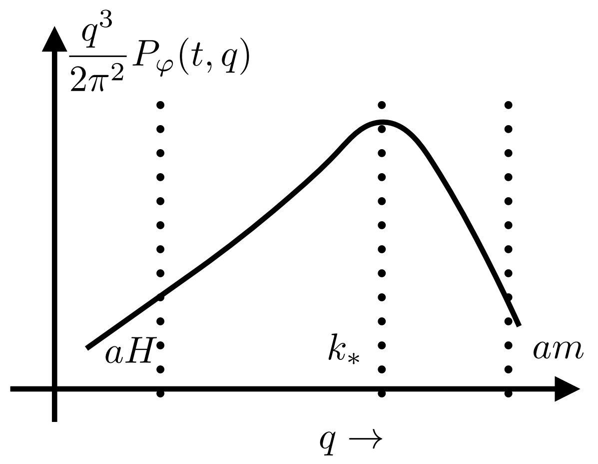


$$\mathcal{T}_k^{(\mathrm{iso})}(y,y_0) = \left[1 + 3\int_{y_0}^{y}\frac{\mathrm{d}y'}{\sqrt{1+y'}}\mathcal{T}_k^{(\mathrm{b})}(y,y')\mathcal{T}_k^{(\mathrm{c})}(y,y')\right]^{1/2},$$

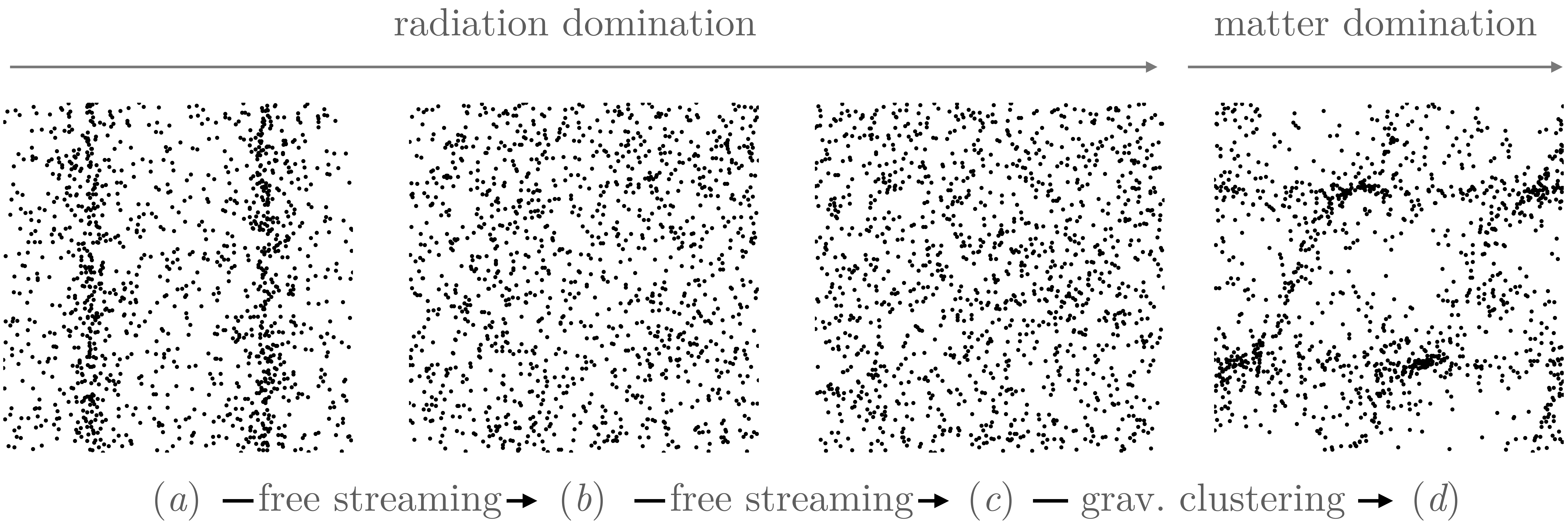
$$\mathcal{T}_k^{(\mathrm{ad})}(y,y_0) = \mathcal{T}_k^{(\mathrm{a})}(y,y_0) + \frac{1}{2}\frac{\mathrm{d}\ln(P_\delta^{(\mathrm{ad})}(y_0,k))}{\mathrm{d}\ln(y_0)}\sqrt{1+y_0}\,\mathcal{T}_k^{(\mathrm{b})}(y,y_0)$$

$$f_0(p) = (m/a^3\bar{\rho})P_\psi(p)$$

$$y = a/a_{\mathrm{eq}}$$



structure formation with small number density and warmth



*dots represent interference granules/quasi-particles/black holes etc.

applications

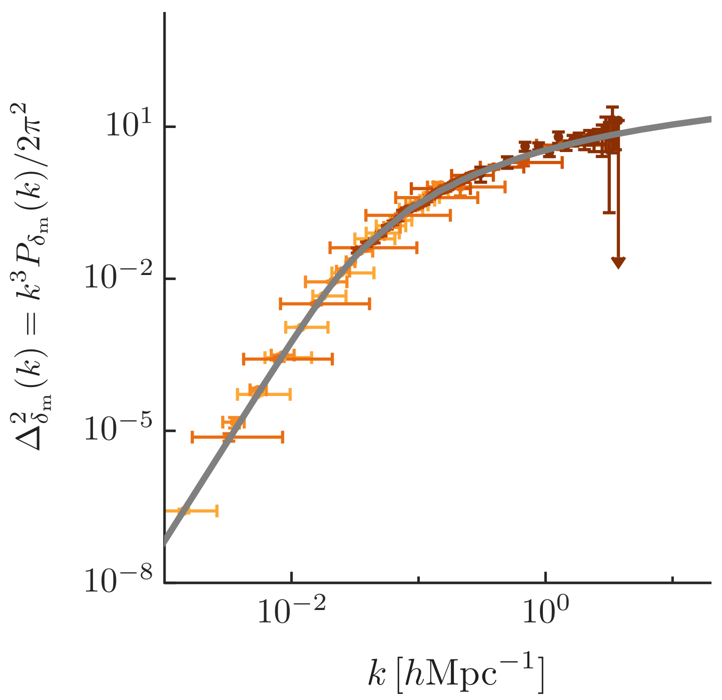
the lower bound on wave DM mass

Dark matter density dominated by sub-Hubble field modes

Then in density power spectrum:

1. white-noise excess
2. free-streaming suppression

1. and 2. not seen for $k < k_{\rm obs} \sim 10\,{\rm Mpc}^{-1}$



$$m \gtrsim 10^{-19}\,{\rm eV}$$

MA & Mirbabayi 2022

comparison with literature

$m \gtrsim 2 \times 10^{-21}\,\mathrm{eV}$	Irsic et. al (2017) — Lyα
$m \gtrsim 1 \times 10^{-21}\,\mathrm{eV}$	Banik et. al (2019) — Streams
$m \gtrsim 3 \times 10^{-21}\,\mathrm{eV}$	Nadler et. al (2021) — MW satellites
$m \gtrsim 3 \times 10^{-19}\,\mathrm{eV}$	Dalal & Kravtsov (2022) — dynamical heating of stars
$m \gtrsim 4 \times 10^{-21}\,\mathrm{eV}$	Powell et. al (2023) — lensing
$m \gtrsim 10^{-19}\,\mathrm{eV}$	MA & Mirbabayi (2022/24)

* constraints come from different observations, compare with care.

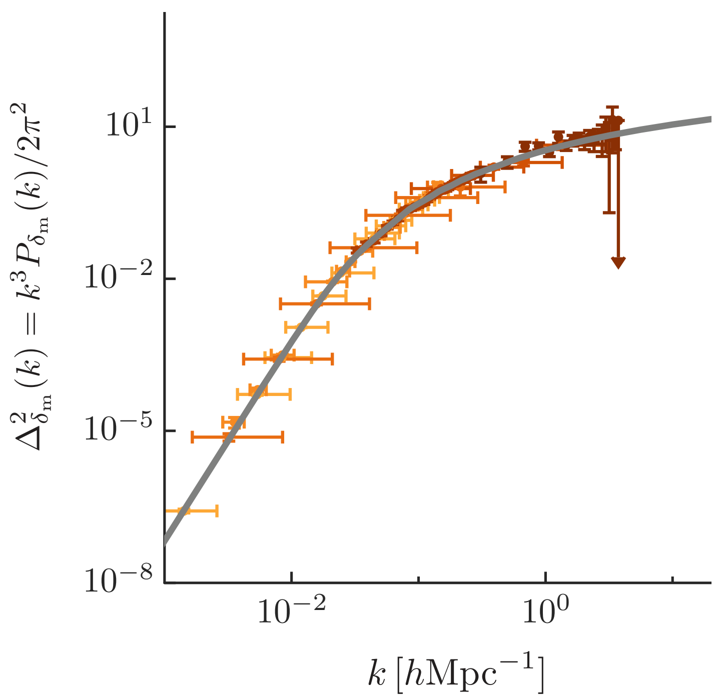
Assumptions matter: For thermal production, this becomes many **keV.**

the lower bound on “PBH” mass

Then in density power spectrum:

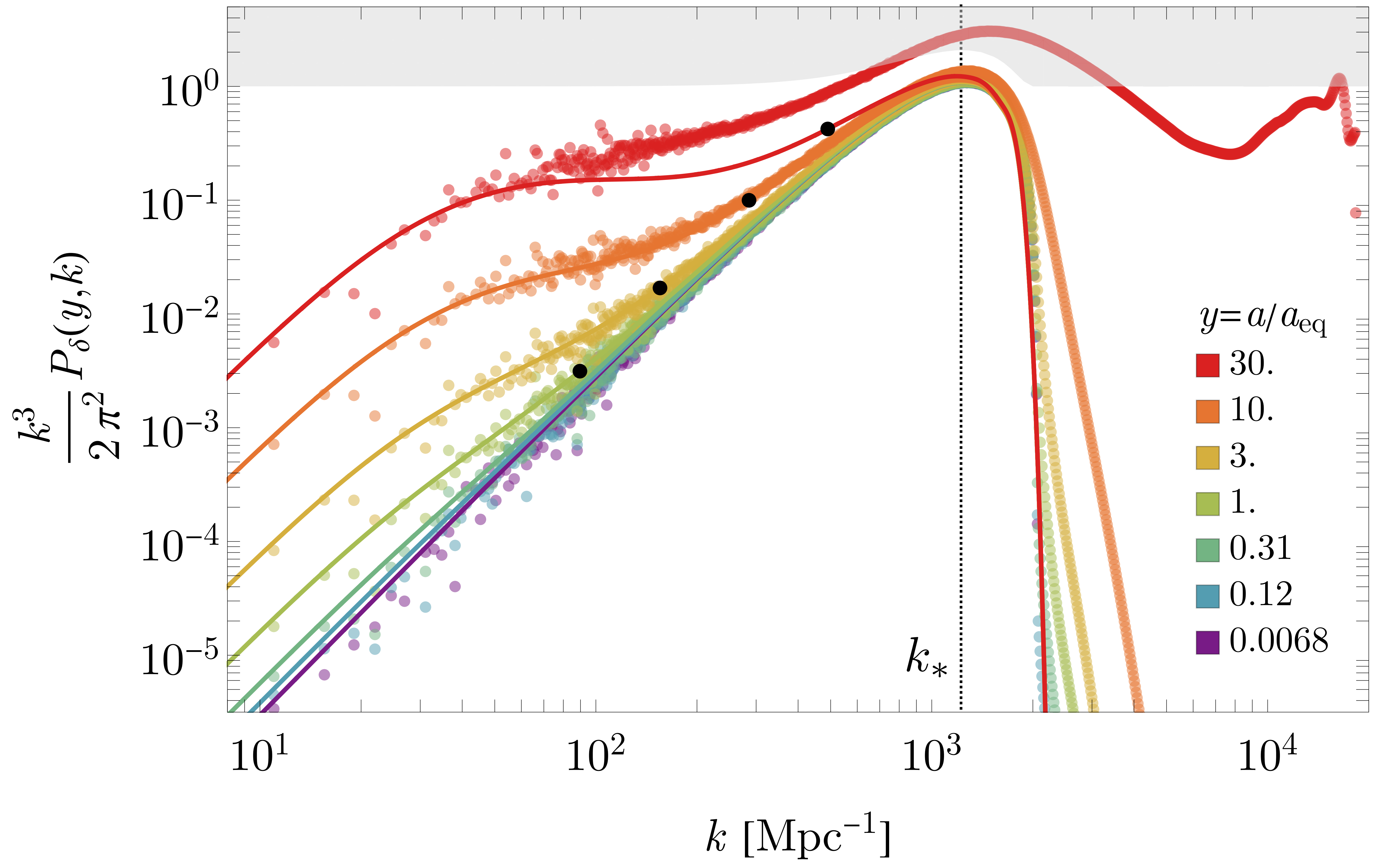
1. white-noise excess
2. free-streaming suppression

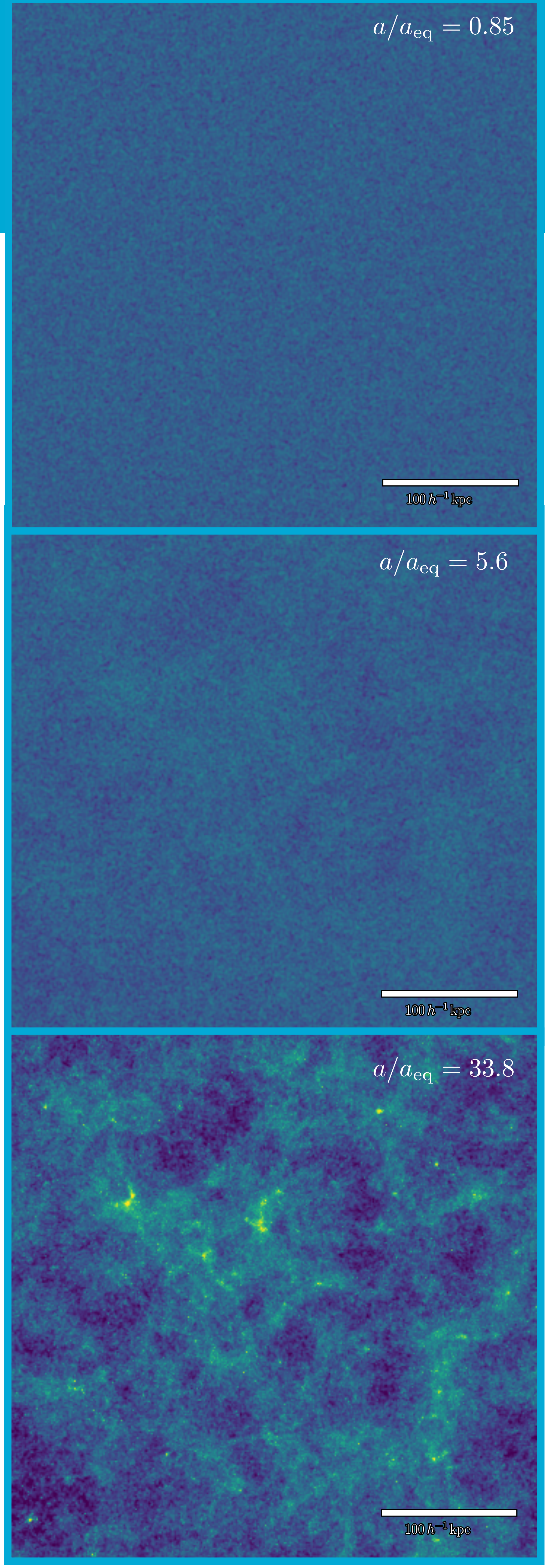
1. and 2. not seen for $k < k_{\rm obs} \sim 10\,{\rm Mpc}^{-1}$



$M_{\rm pbh} \gtrsim {\rm few} \times 10^2 M_\odot$ $\sigma_{\rm eq} \lesssim {\rm few} \times 10\,{\rm km}\,s^{-1}$

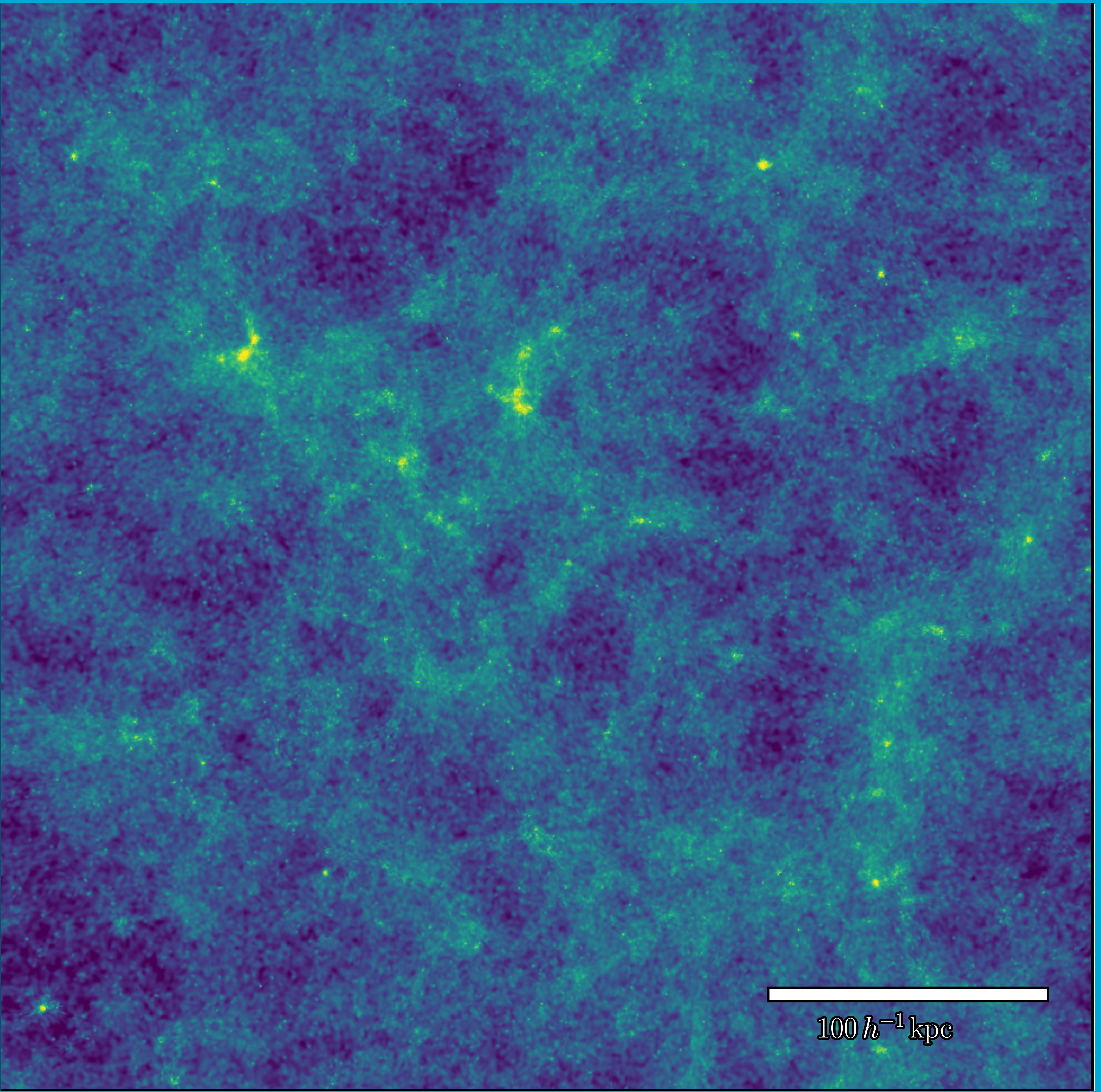
enhanced power on small scales

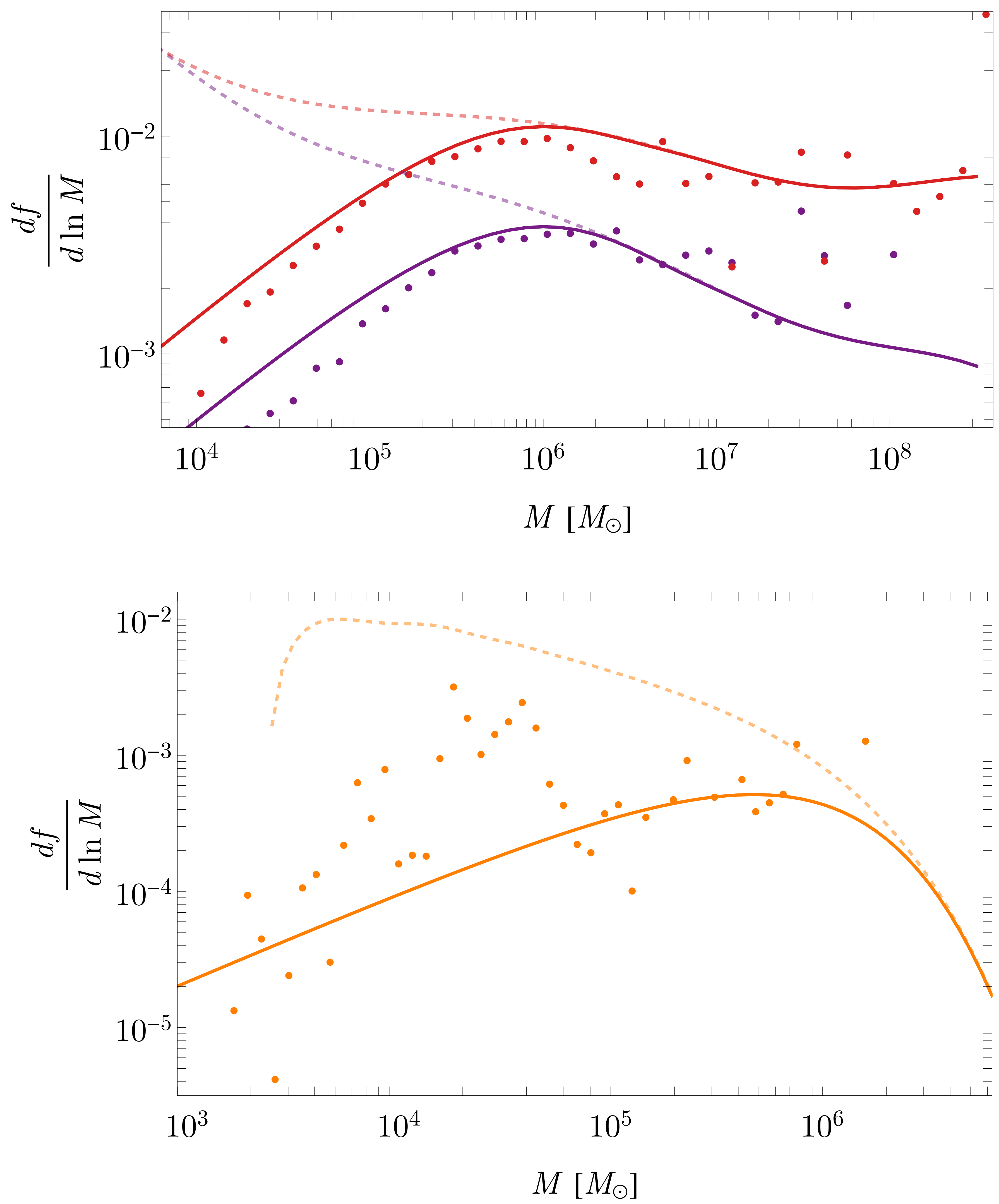




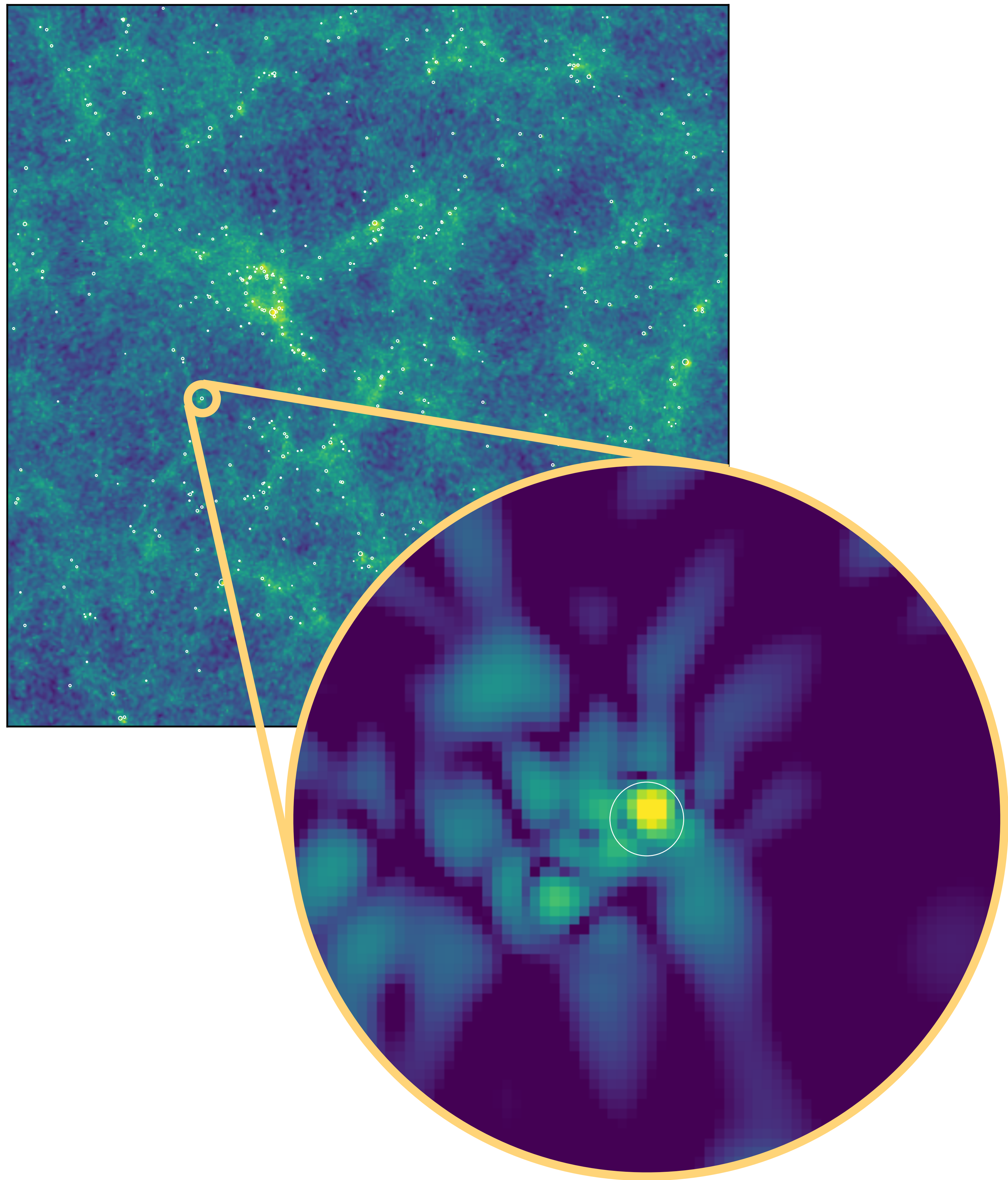
- $k_J(a) = a\frac{\sqrt{4\pi G\bar{\rho}}}{\sigma}$ $\qquad$ $\sigma \sim \frac{k_*}{am}$

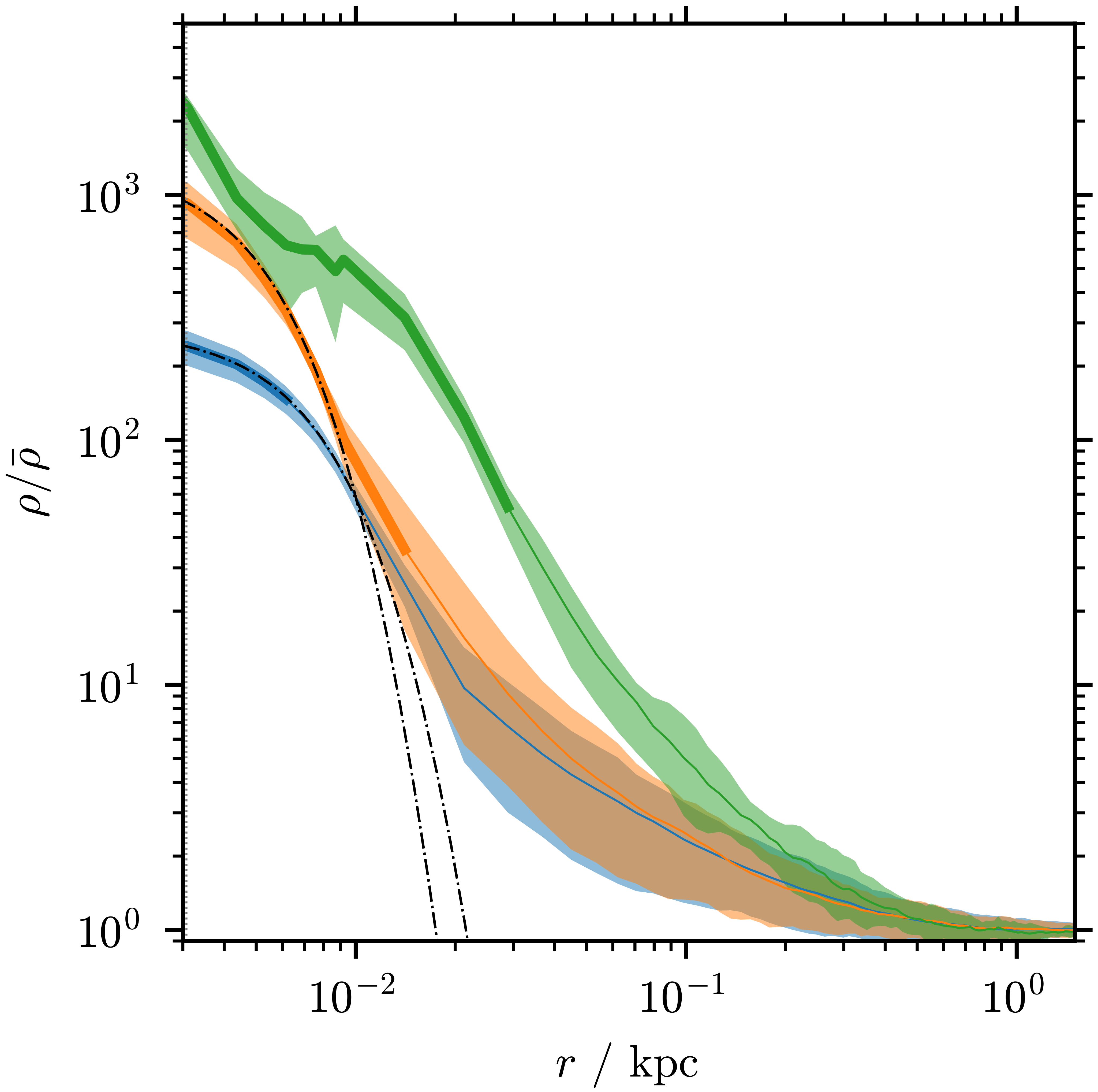
we can predict the halo mass function





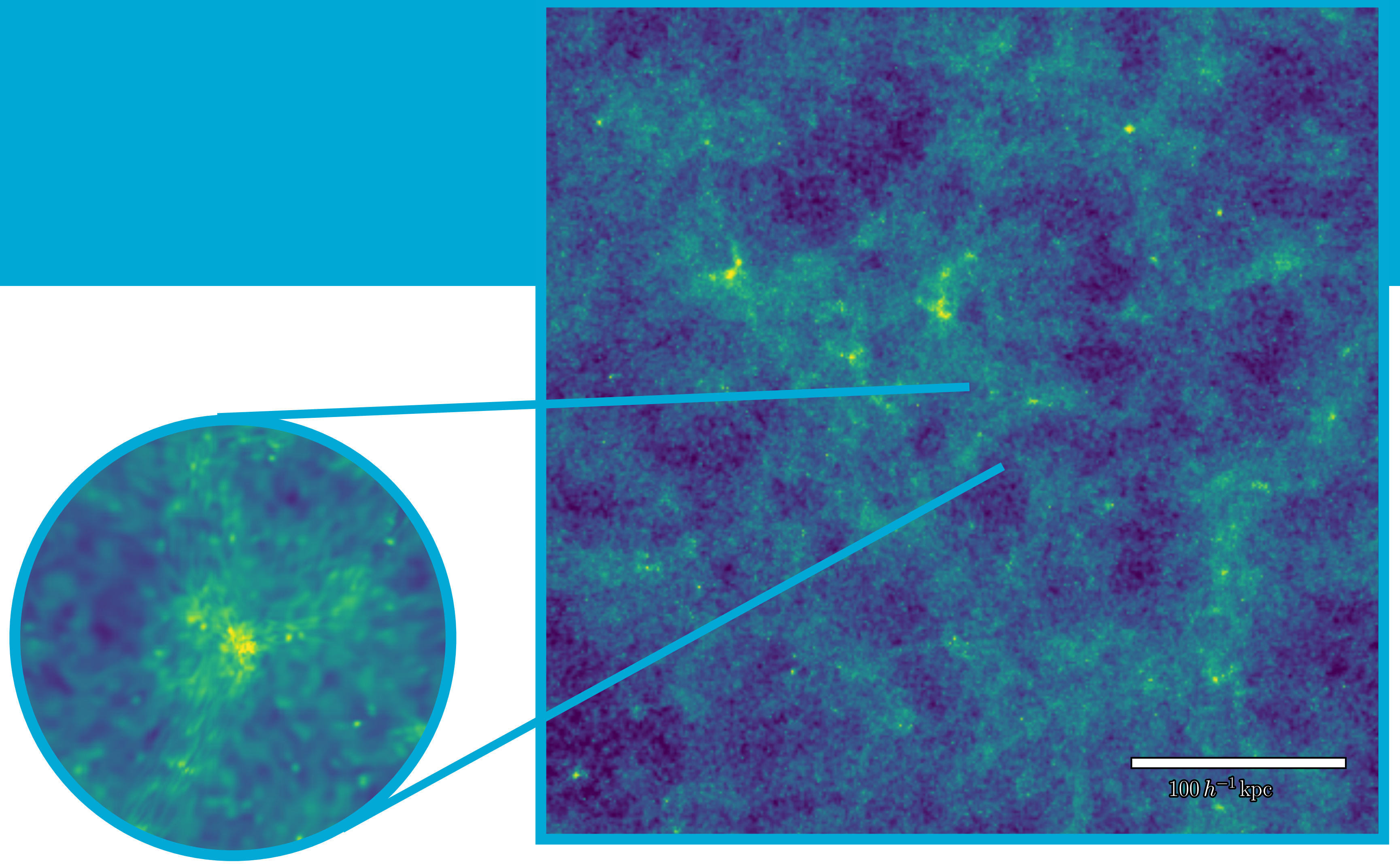
solitons — time scales, abundance …



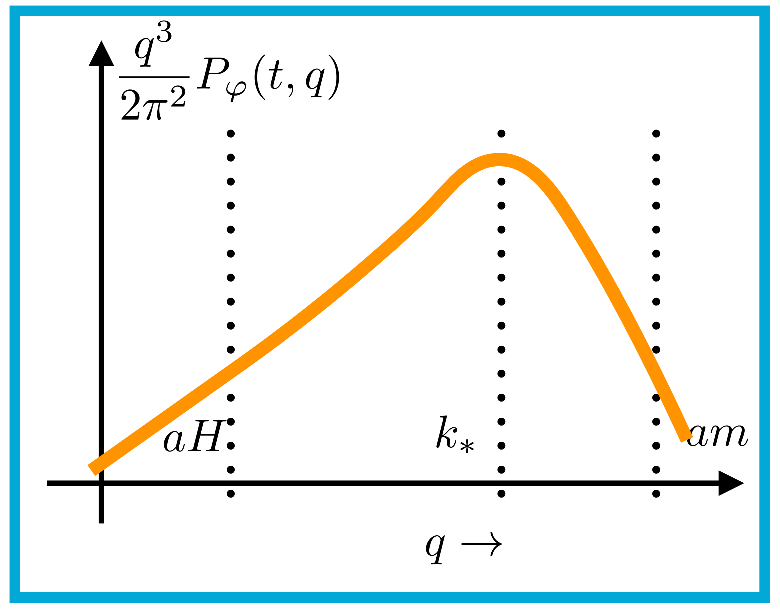


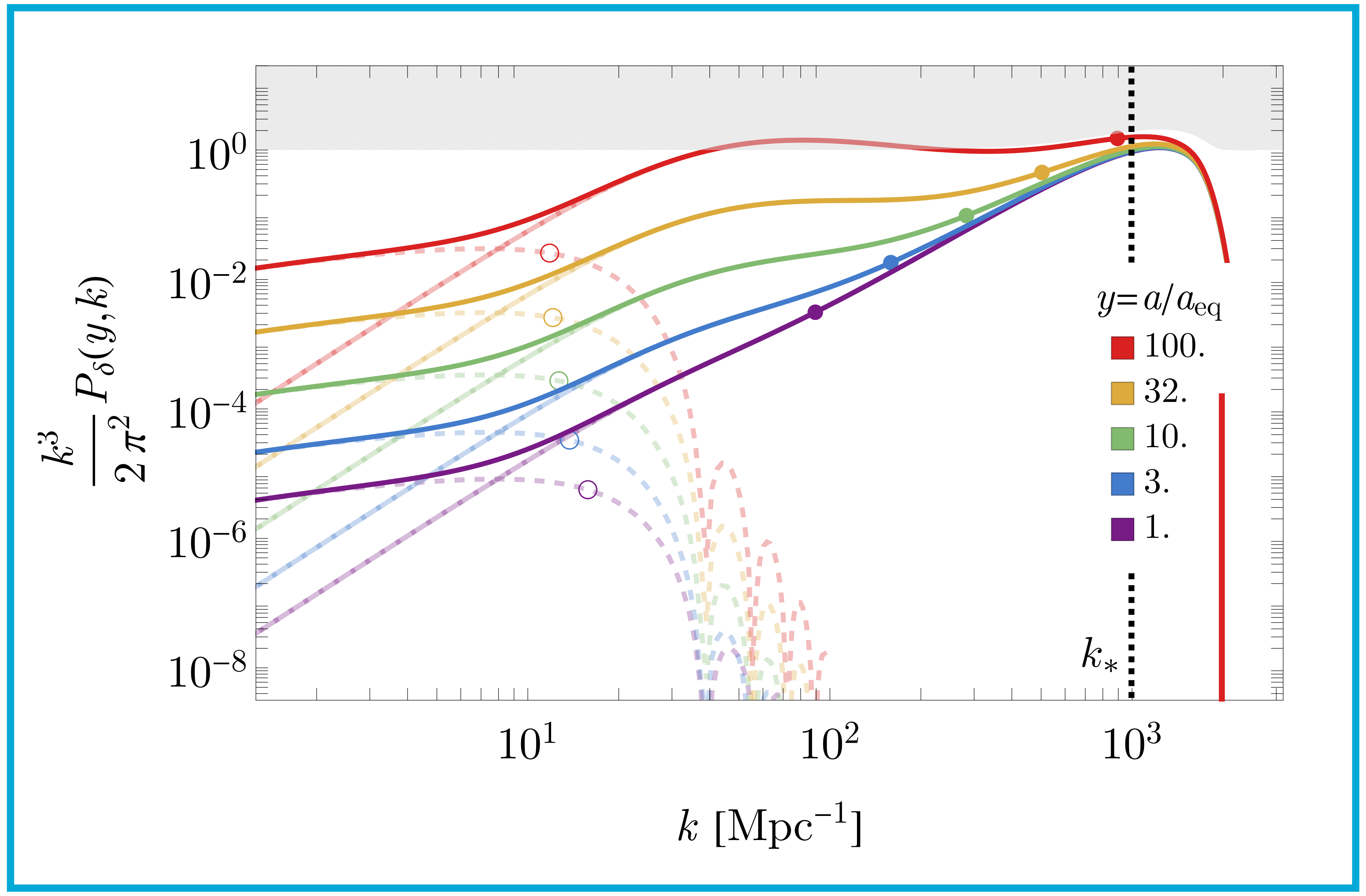
* can be order unity fraction of mass in collapsed objects, at least at early times

summary



If dark matter density is dominated by sub-Hubble field modes



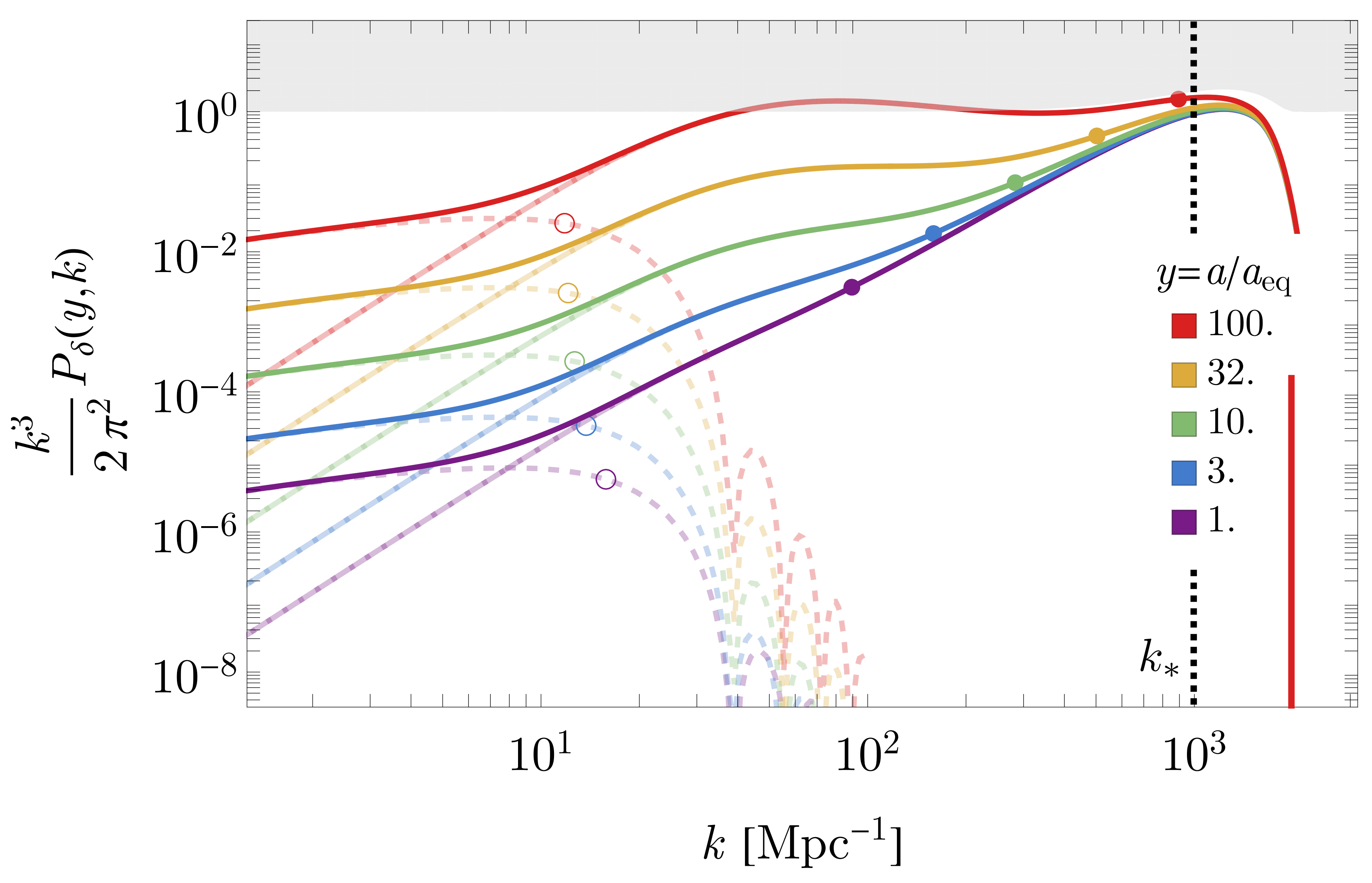


$$\Delta^2_\delta(y,k) = \underbrace{\Delta^{2(\mathrm{ad})}_\delta(y_0,k)\left[\mathcal{T}^{\mathrm{ad}}_k(y,y_0)\right]^2}_{\text{adiabatic IC + evolution}} + \underbrace{\Delta^{2(\mathrm{iso})}_\delta(y_0,k)\left[1 + 3\int_{y_0}^{y}\frac{dy'}{\sqrt{1+y'}}\mathcal{T}^{(b)}_k(y,y')\mathcal{T}^{(c)}_k(y,y')\right]}_{\text{isocurvature IC + evolution}}$$

isocurvature enhancement
+Jeans & free-streaming features
(+ miniclusters, solitons etc.)

Non-detection — $m \gtrsim 10^{-19}$ eV
Probe of formation mechanism

generality — warmth + shot noise



- also works for PBHs, solitons, miniclusters etc

- fractional amounts

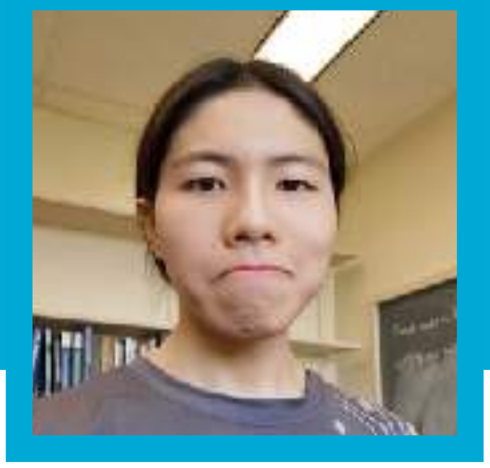
- on-grav. interactions

For mixed cases, see: Verdiani, Castorina, Salvioni (2025), Celik & Schmidt (2025)

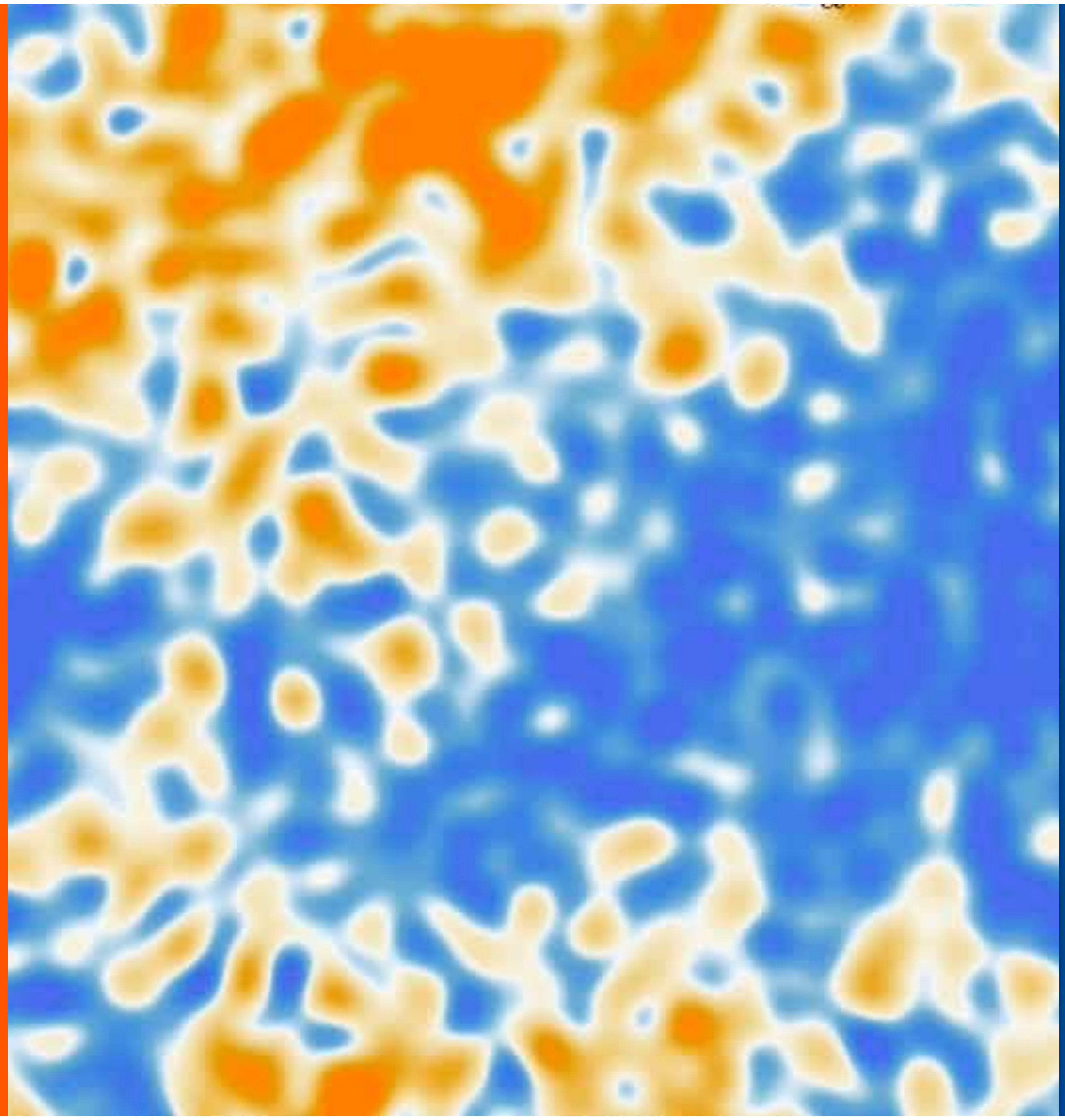
PBH bounds from Lya= Ivanov & Trifonopolis (2025)

soliton formation

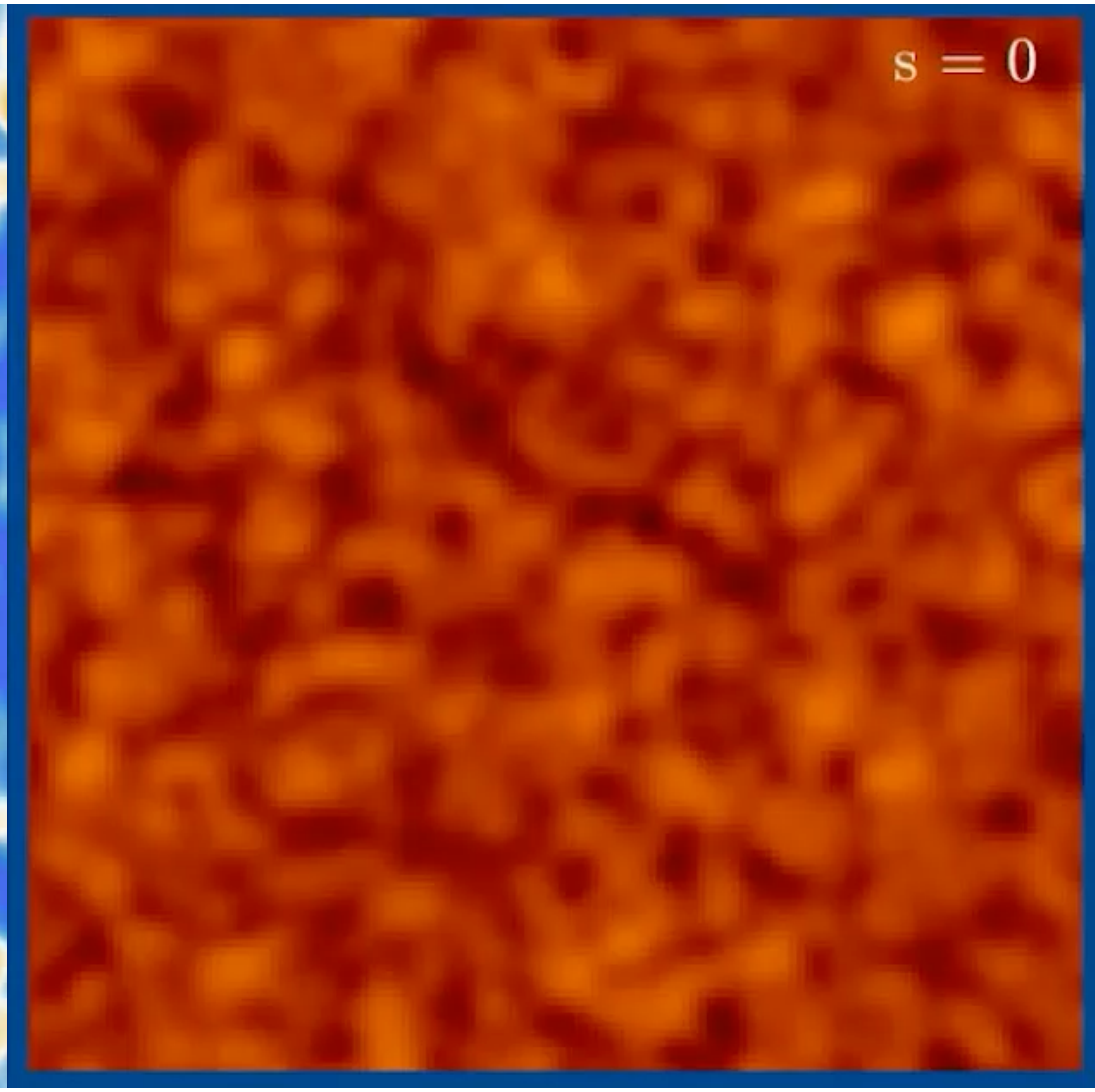


with Mocz (2018)

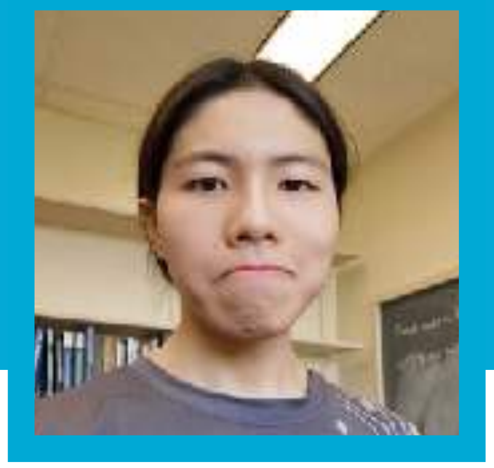


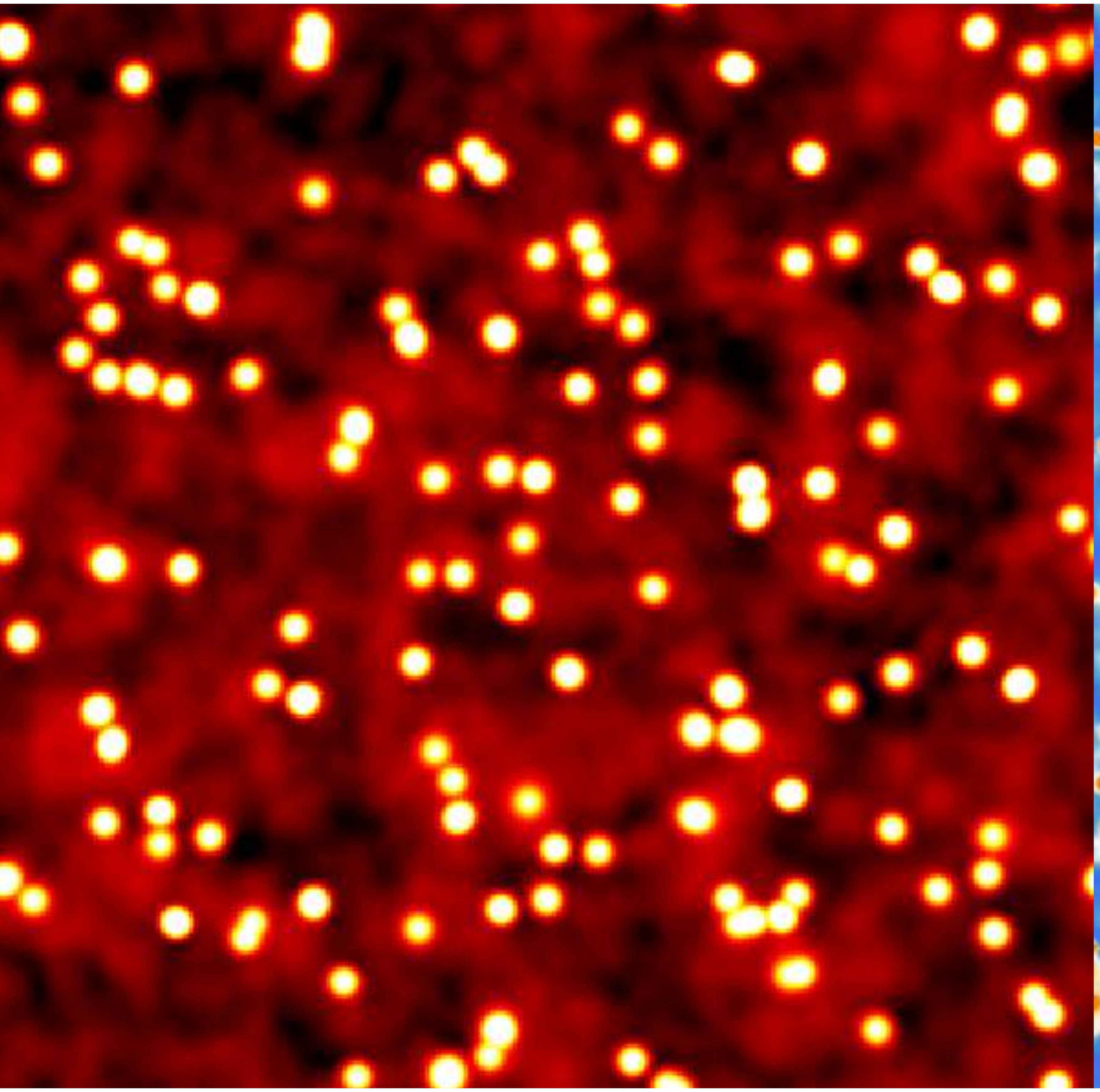
with Ling (2024)



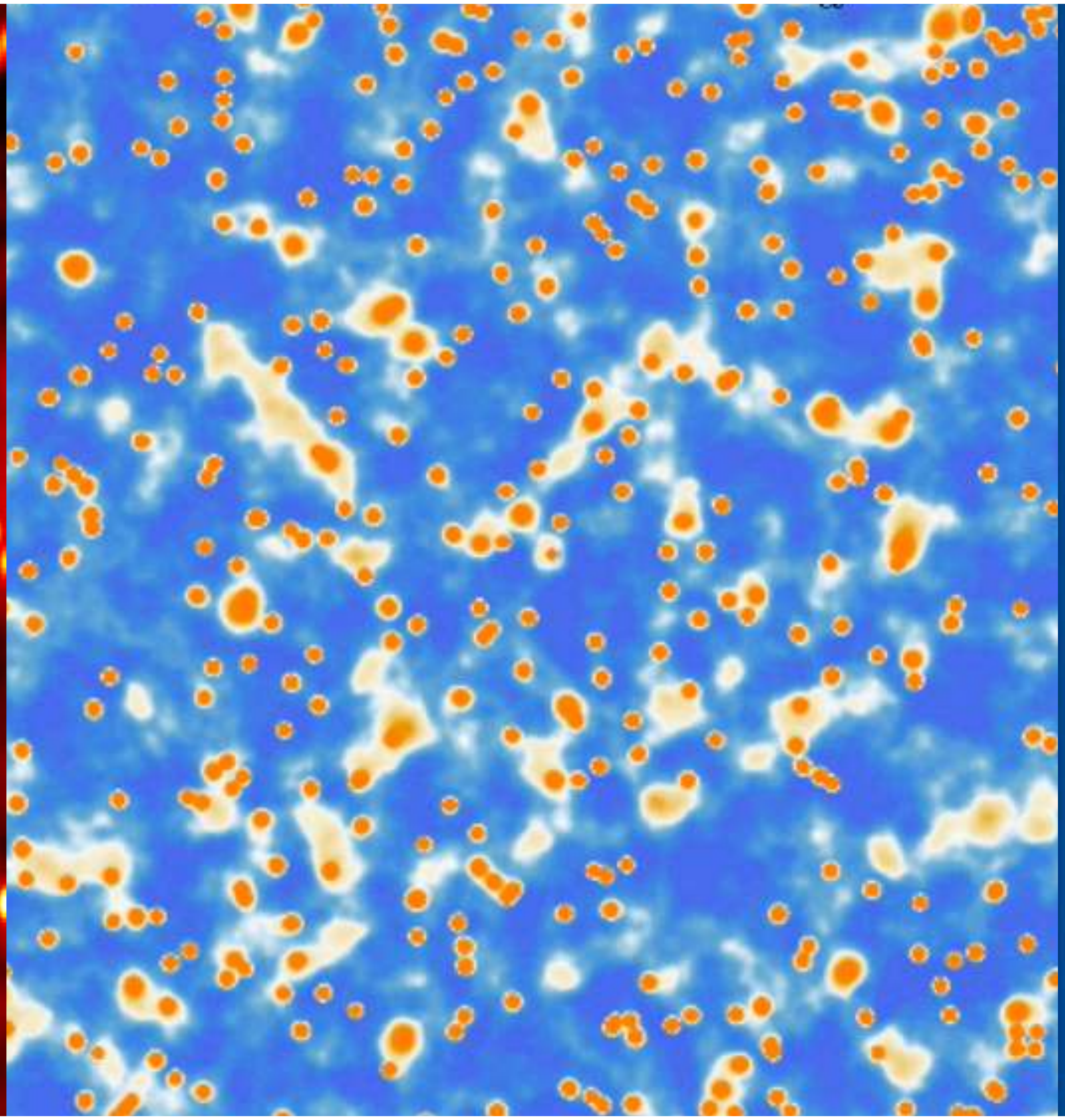
with Jain, Thomas, Wanichwecharungruang (2023)

soliton formation

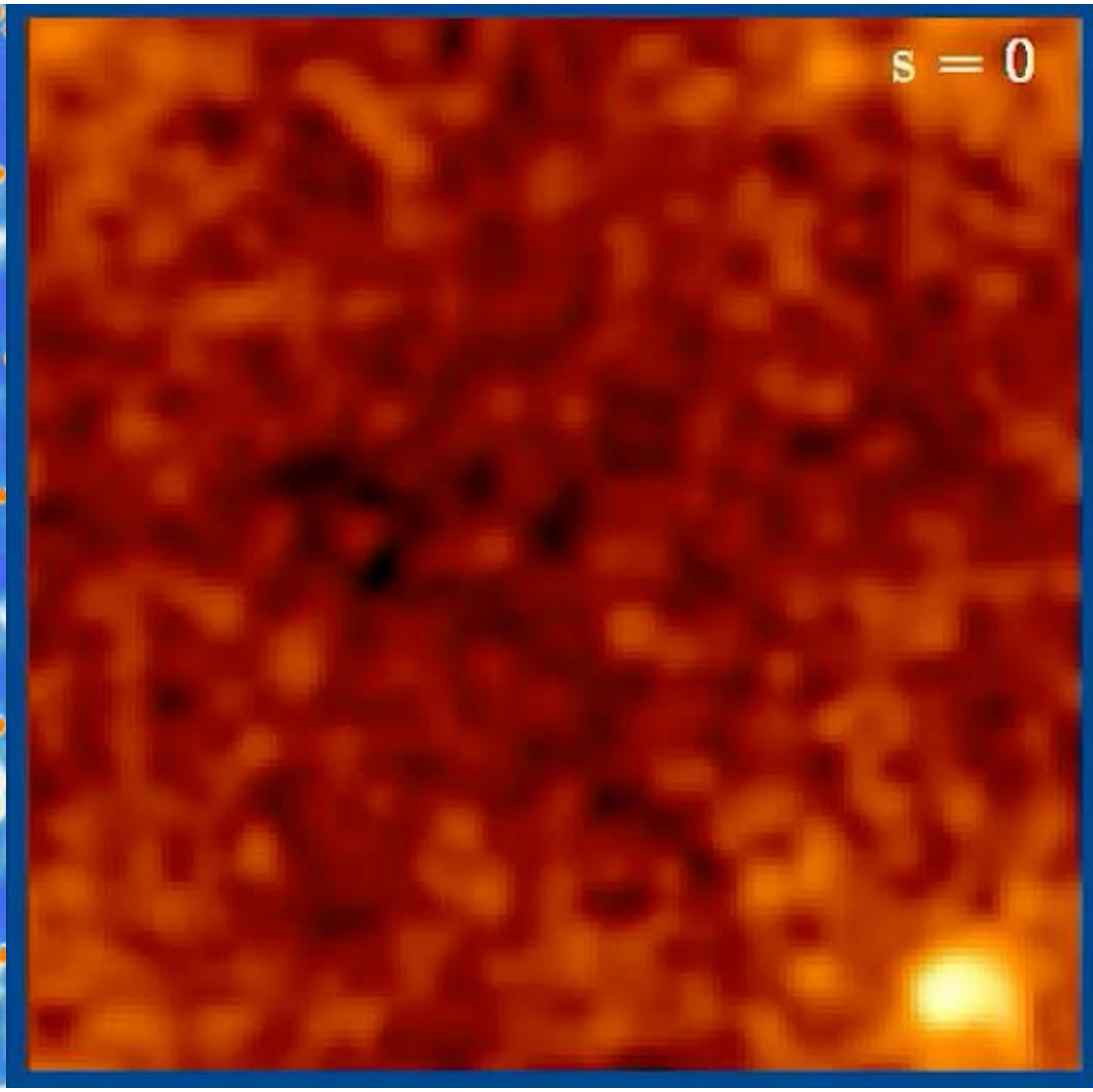




with Mocz (2018)



with Ling (2024)



with Jain, Thomas, Wanichwecharungruang (2023)