New Inference on / New Physics from Galaxy Clustering

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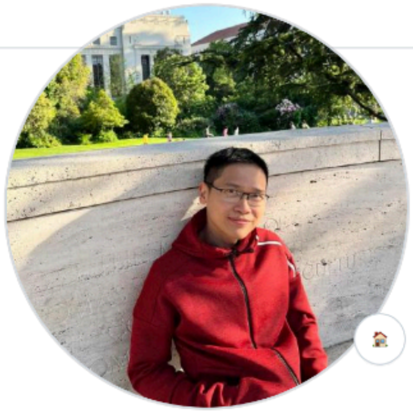
with
Ivana Babić, Sten Delos, Marco Celoria, Sam Goldstein, Andrija Kostić, Toshiki Kurita, Minh Nguyen, Ivana Nikolac, Antti Rantala, Henrique Rubira, Julia Stadler, Beatriz Tucci, Sam Young

New Physics from Galaxy Clustering, GGI, Firenze, Sep 8, 2025

Protagonists



Minh Nguyen
MinhMPA
Follow

Cosmologist, interested in cosmology from large-scale structure and the cosmic background radiation. Previously at MPA Garching, now at LCTP Michigan.

Field-level inference
Intrinsic alignments

Ivana Nikolac
- SBI
- novel summaries

Ivana Babić

- PhD student
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BAO scale inference

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SBI, BAO scale inference

Main Focus
cosmology, large-scale structure, forward modeling, likelihood-free inference, assembly bias

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Redshift-space distortions
Field-level inference

Protagonists

Sten Delos (Carnegie Observatories)

Sam Goldstein (Columbia)

Antti Rantala (MPA)

Marco Celoria (HPC Cineca)

Sam Young (Sussex)

Toshiki Kurita (MPA)

New Inference

from Galaxy Clustering

Inference in Cosmology

- Given a cosmological model with parameters θ, we can hope to predict

1. Statistics of initial conditions — Prior $P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$
2. How a given $\delta_{\rm in}(x)$ evolves into the final density field — deterministic evolution

$$\vec{\delta}_{\rm fwd}[\vec{\delta}_{\rm in}, \theta]$$

Field-level inference (FLI)

- Let's put galaxies on a grid: $\delta_g(\boldsymbol{x}) = n_g(\boldsymbol{x})/\langle n_g \rangle - 1$

  No effective loss of information provided k_{Ny} >= k_{max} of our analysis.

- The *full joint posterior of initial conditions and cosmological parameters given the data* is then given by

$$P(\theta, \boldsymbol{\delta}_{\rm in}|\delta_g) \propto P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

conditional probability of galaxy density given forward-modeled density field - contains all physics of galaxy formation

Field-level inference (FLI)

- Let's put galaxies on a grid: $\delta_g(\boldsymbol{x}) = n_g(\boldsymbol{x})/\langle n_g \rangle - 1$

  No effective loss of information provided k_{Ny} >= k_{max} of our analysis.

- The *posterior of cosmological parameters* is obtained by marginalizing over δ_{in}:

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

Extremely high dimensional integral

Field-level inference

- So let's try to tackle this challenge: Infer cosmology *without* data compression

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- In other words, we want to solve the extremely high dimensional integral via Monte Carlo sampling

Field-level inference

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Scheme:
  - Discretize field on grid/lattice
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Evaluate likelihood on data and repeat

Field-level inference

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Scheme:
  - Discretize field on grid/lattice
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Evaluate likelihood on data and repeat
- Results in samples from the *joint posterior of initial conditions and cosmological parameters*

Pioneered by Jasche, Kitaura, Ensslin; Mo et al

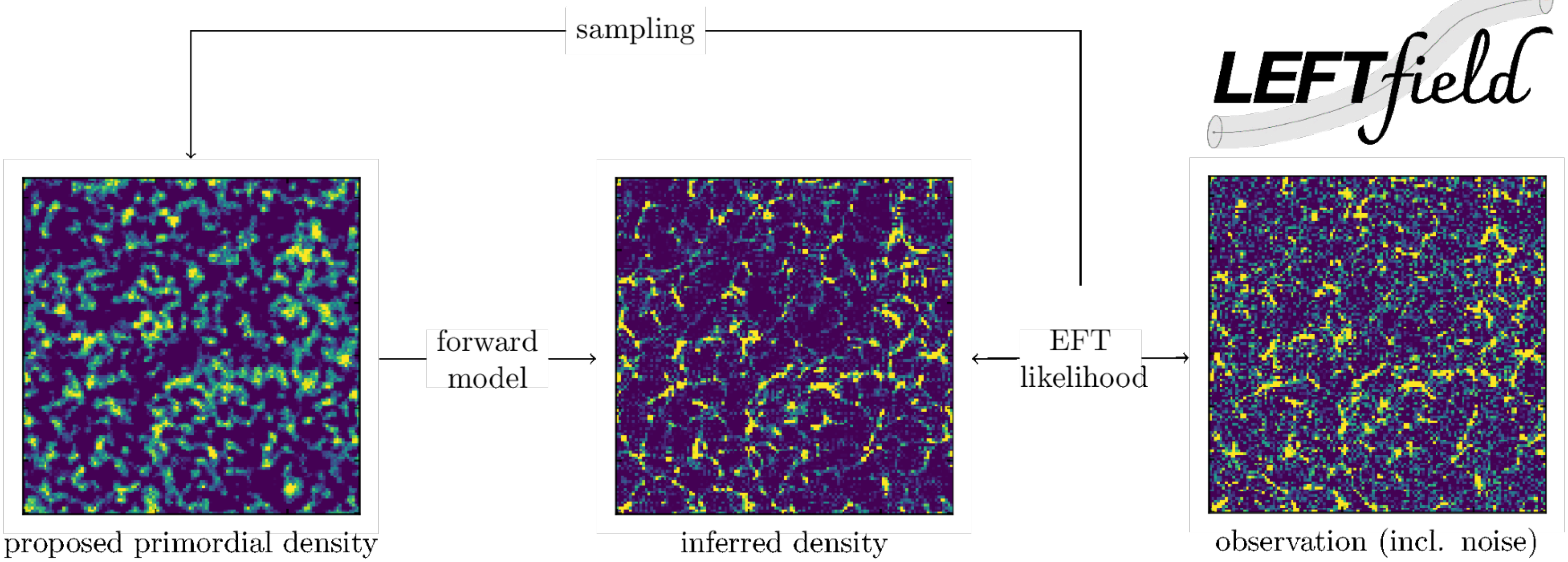
Field-level inference

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Scheme:
  - Discretize field on grid/lattice (Nyquist frequency = cutoff Λ)
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Evaluate likelihood on data and repeat
- Challenge: even with fairly coarse resolution, have to sample million(s) of parameters
  - Key: Hamiltonian Monte Carlo

Visualization: results from field-level inference on mock data

- Slices through linear density, evolved biased (mean) field, and mock data



LEFTfield

https://www.mpa-garching.mpg.de/1095991/hl202405

Field-level requires a *galaxy likelihood*

- We need an expression for the *field-level galaxy likelihood:*
    - conditional probability of galaxy density given matter density

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Will see later that field-level inference puts *very stringent requirements* on forward model! The price of extracting full information.

Field-level requires a *galaxy likelihood*

- We need an expression for the *field-level galaxy likelihood:*
  - conditional probability of galaxy density given matter density

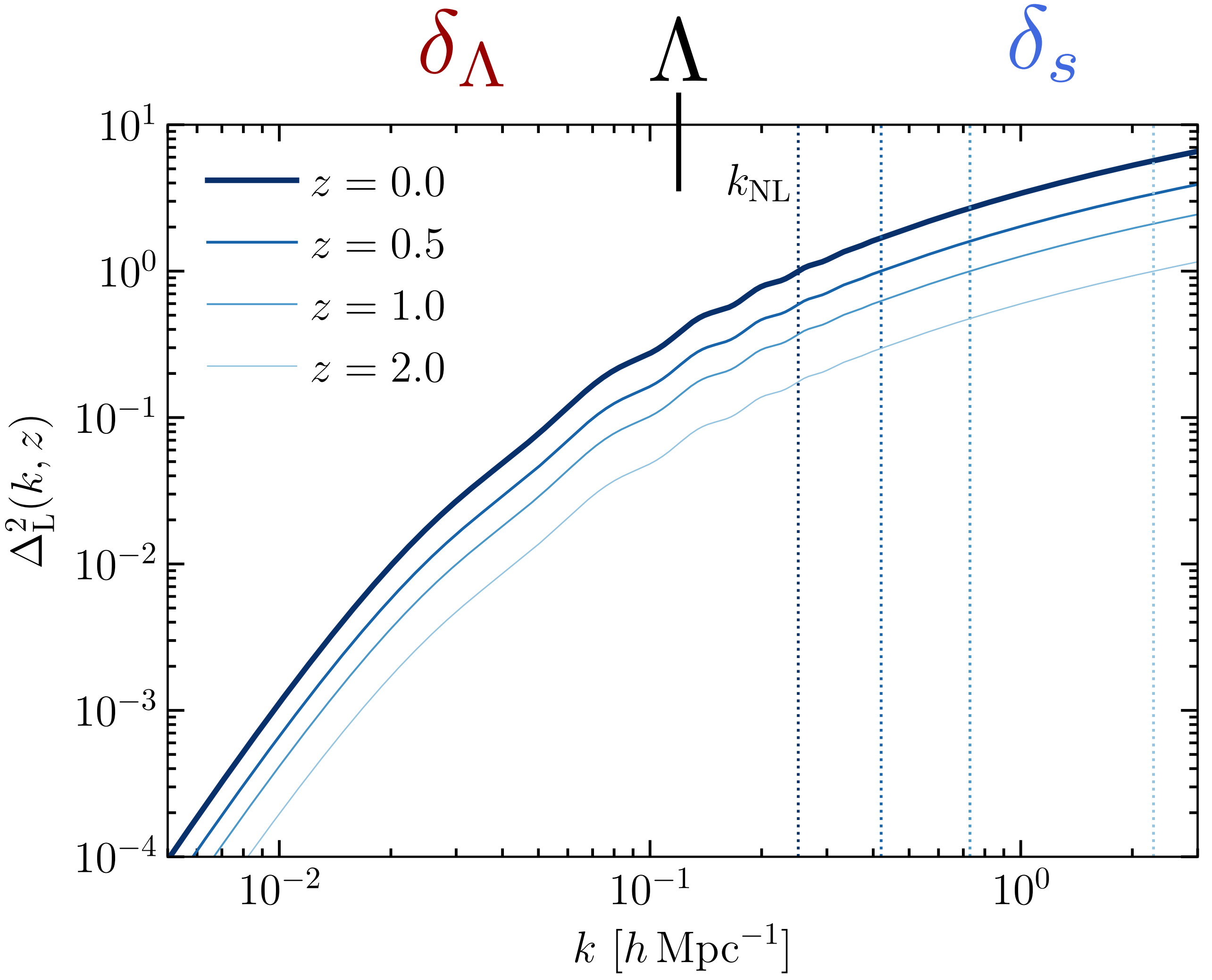
$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Our approach: integrate out modes *above a scale Λ analytically* (-> EFT), and *sample modes below Λ explicitly.* Recall that we evolve density on a grid, hence Λ has to be finite.

EFT approach

- Idea: trust our theory for *k* < *Λ*, where fractional density perturbations are << 1
- Split *initial* perturbations into large scale (< Λ) and small scale (>= Λ):

$$\delta(\boldsymbol{x}, \tau) \equiv \frac{\rho_m(\boldsymbol{x}, \tau)}{\bar{\rho}\,(\tau)} - 1 = \delta_\Lambda + \delta_s$$

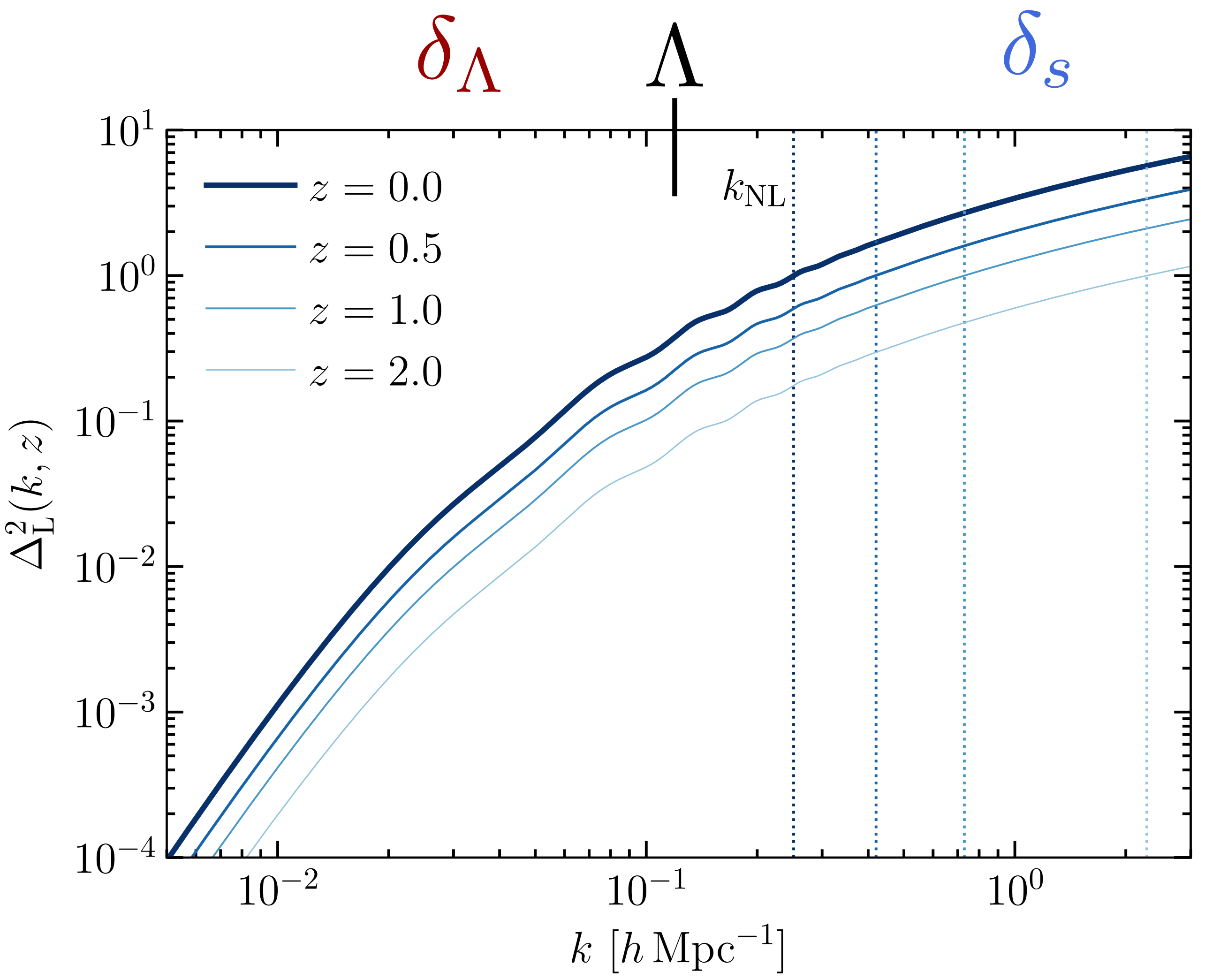


EFT approach

- Idea: trust our theory for *k* < *Λ*, where fractional density perturbations are << 1
- Split *initial* perturbations into large scale (< Λ) and small scale (>= Λ):

$$\delta(\boldsymbol{x},\tau) \equiv \frac{\rho_m(\boldsymbol{x},\tau)}{\bar{\rho}\,(\tau)} - 1 = \delta_\Lambda + \delta_s$$

- Then, we integrate out (marginalize over) perturbations with k > Λ

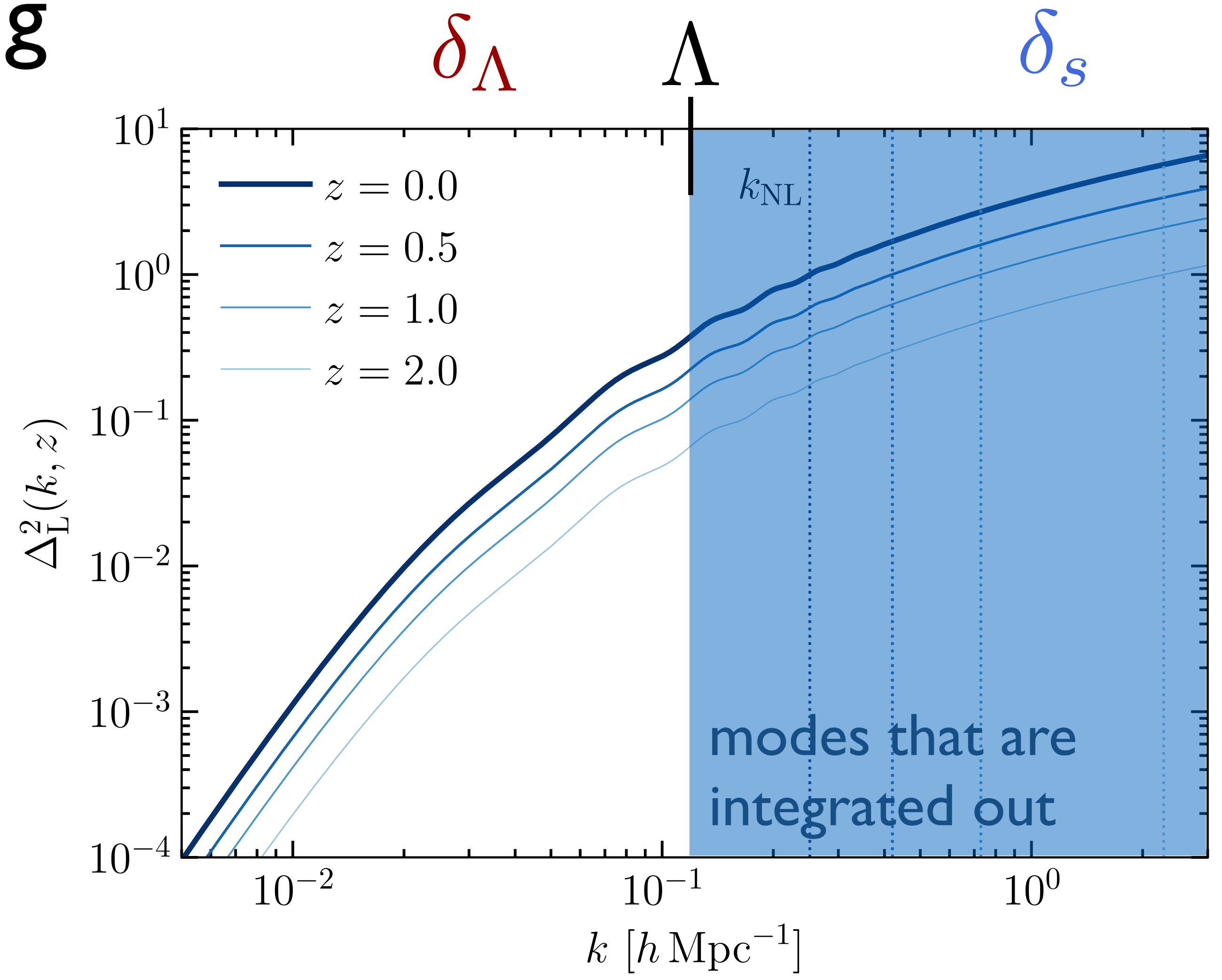


(A) Bias

- Incorporate effect of large-scale perturbations explicitly using bias expansion, with free coefficients b_O

$$\delta_g(\boldsymbol{x}) = \sum_O b_O(\Lambda) O[\delta_\Lambda^{\text{in}}](\boldsymbol{x})$$

  - Fields O are constructed from $\delta_\Lambda^{\text{in}}$



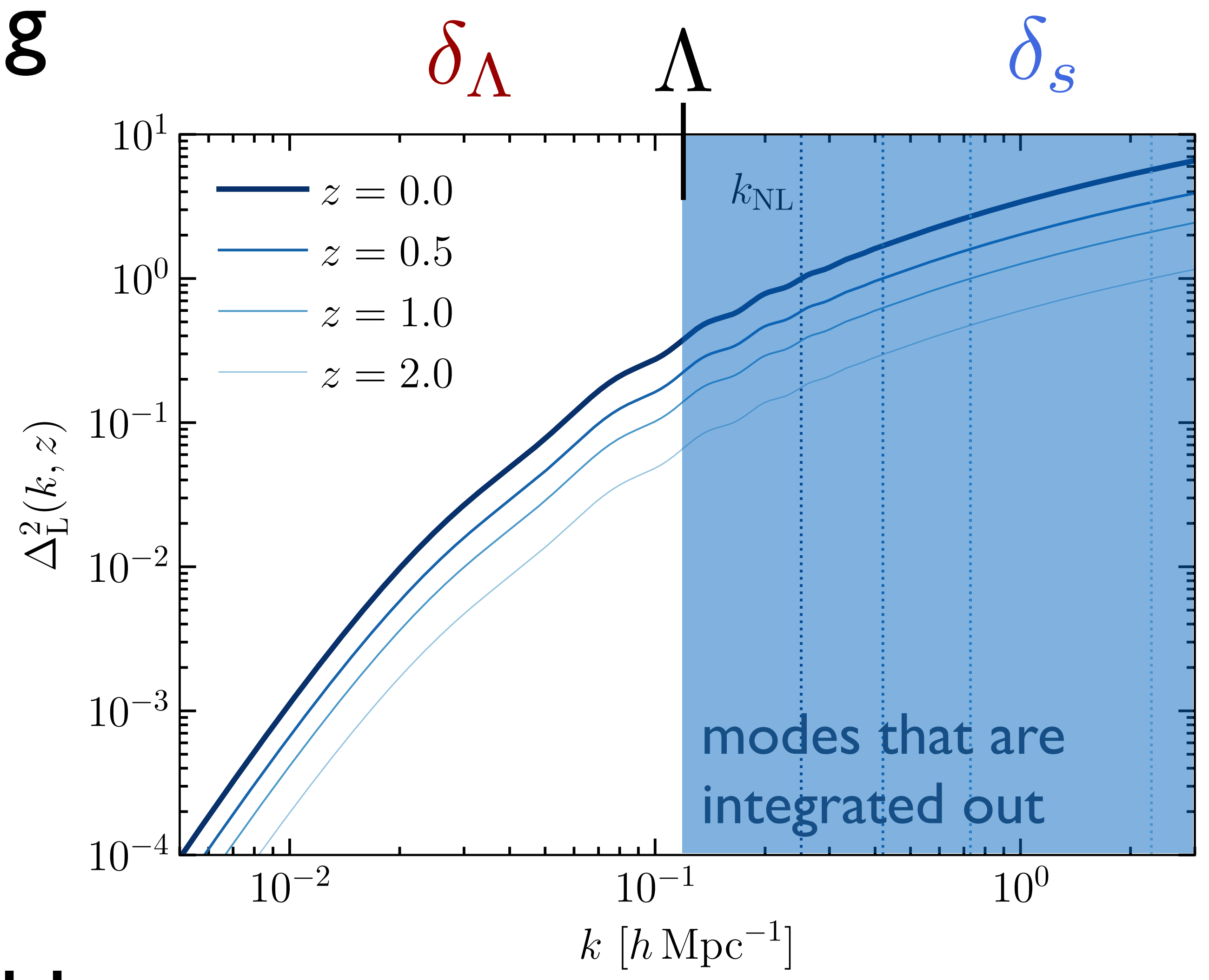
(B) Stochasticity

- Incorporate effect of large-scale perturbations explicitly using bias expansion, with free coefficients b_O

$$\delta_g(\boldsymbol{x}) = \sum_O b_O O(\boldsymbol{x}) + \varepsilon(\boldsymbol{x})$$

  - Fields O are constructed from δ_Λ

- Small-scale perturbations add noise ε



(B) Stochasticity

- ε arises from local (in real space) superposition of many small-scale perturbations

- Central limit theorem: ε(k) is approximately Gaussian distributed (the lower k, the more Gaussian it is)
    - Local in real space: power spectrum is white noise at low k, with corrections* $\sim k^2$:

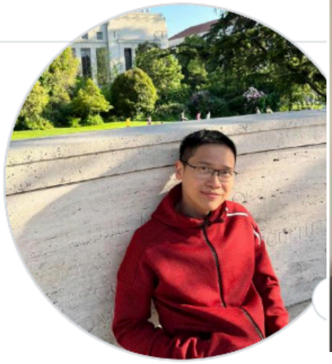
* Also, density dependence: coupling of ε and δ

Desjacques, Jeong, FS; Phys. Rept. (2018)

Cosmology results I: Inferring σ_8 from rest-frame tracers

Cosmology results I: Inferring σ_8 from rest-frame tracers

- Can we recover unbiased $\mathcal{A}_s$ (σ_8) from a tracer (halo, HOD, …) catalog, treating bias parameters as unknown ?
- Perfect degeneracy between b_1 and σ_8 at linear order; nonlinear information essential



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Idealized test: Inferring σ_8 from rest-frame tracers

- Results on field-level σ_8 inference from dark matter halos in real space
  - Marginalizing over bias and stochastic terms
- Idea: compare field-level result with power spectrum + bispectrum using the same forward model and modes of the data
  - Via simulation-based inference (SBI) using the same forward model as in the field-level analysis

Nguyen, FS, Tucci, Reinecke, Kostić PRL 2024, arXiv:2403.03220

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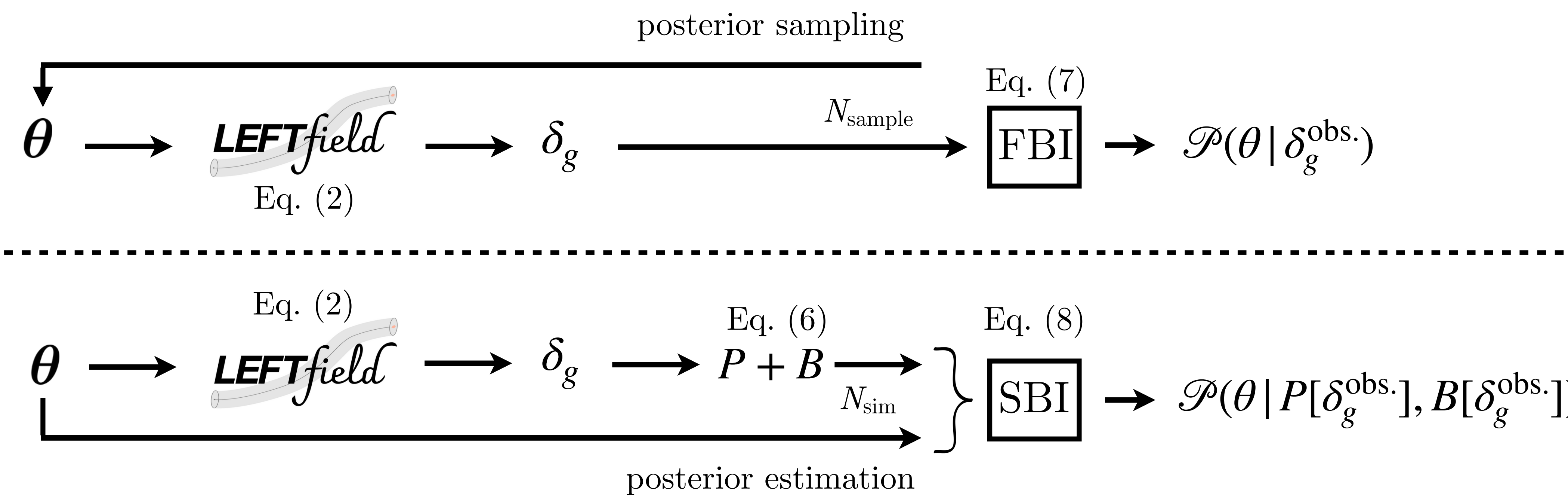
Idealized test: Inferring σ₈ from rest-frame tracers

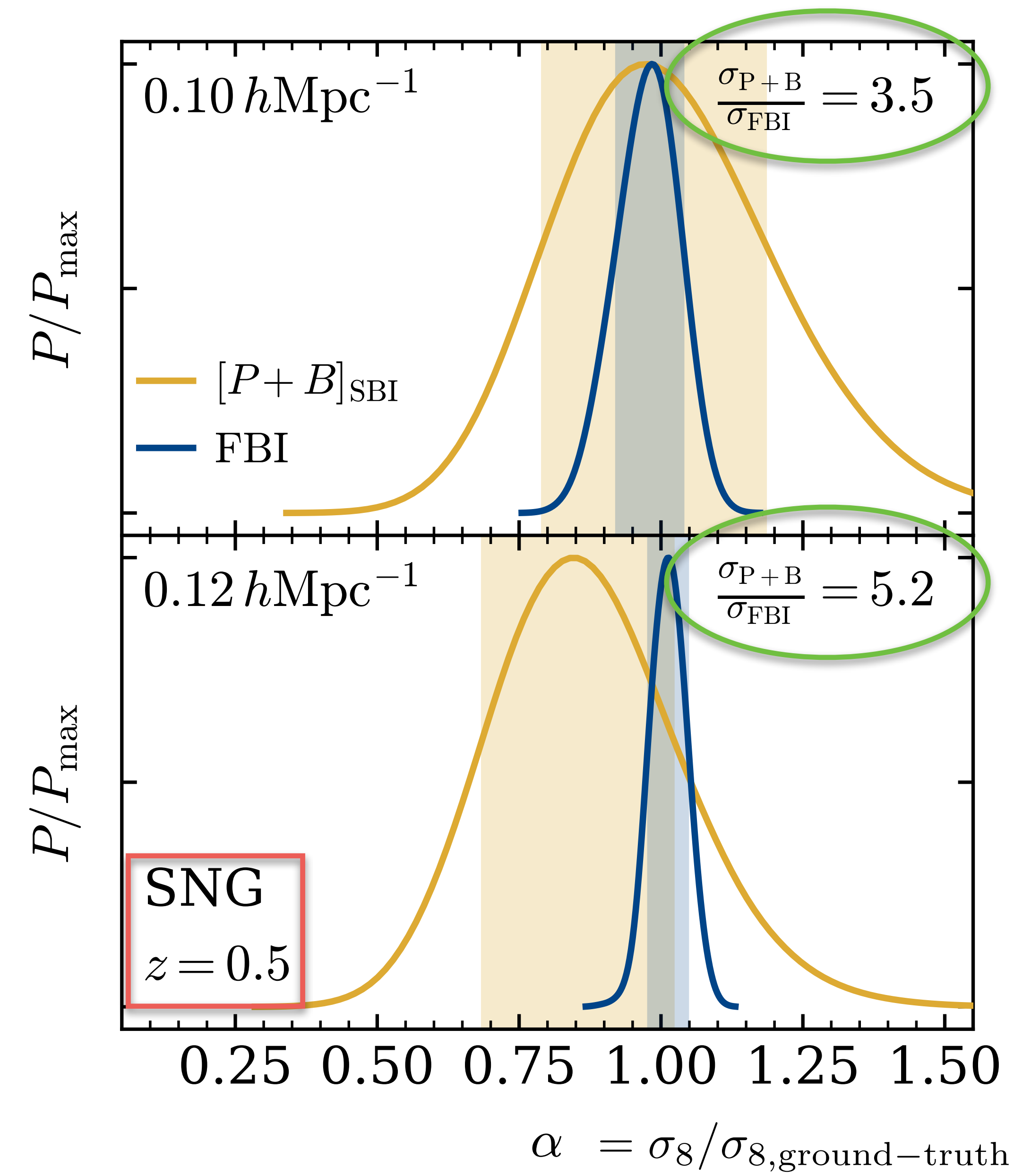


- Idea: compare field-level result with power spectrum + bispectrum using the same forward model and modes of the data
  - Via simulation-based inference (SBI) using the same forward model as in the field-level analysis

Nguyen, FS, Tucci, Reinecke, Kostić PRL 2024, arXiv:2403.03220

Idealized test: Inferring σ8 from rest-frame tracers

- First results on field-level σ8 inference from dark matter halos in real space
  - Marginalizing over bias and stochastic terms
- Field-level inference vs power spectrum + bispectrum using the same forward model and modes of the data



FBI = FLI

Nguyen, FS, Tucci, Reinecke, Kostić PRL 2024, arXiv:2403.03220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Idealized test: Inferring σ8 from rest-frame tracers

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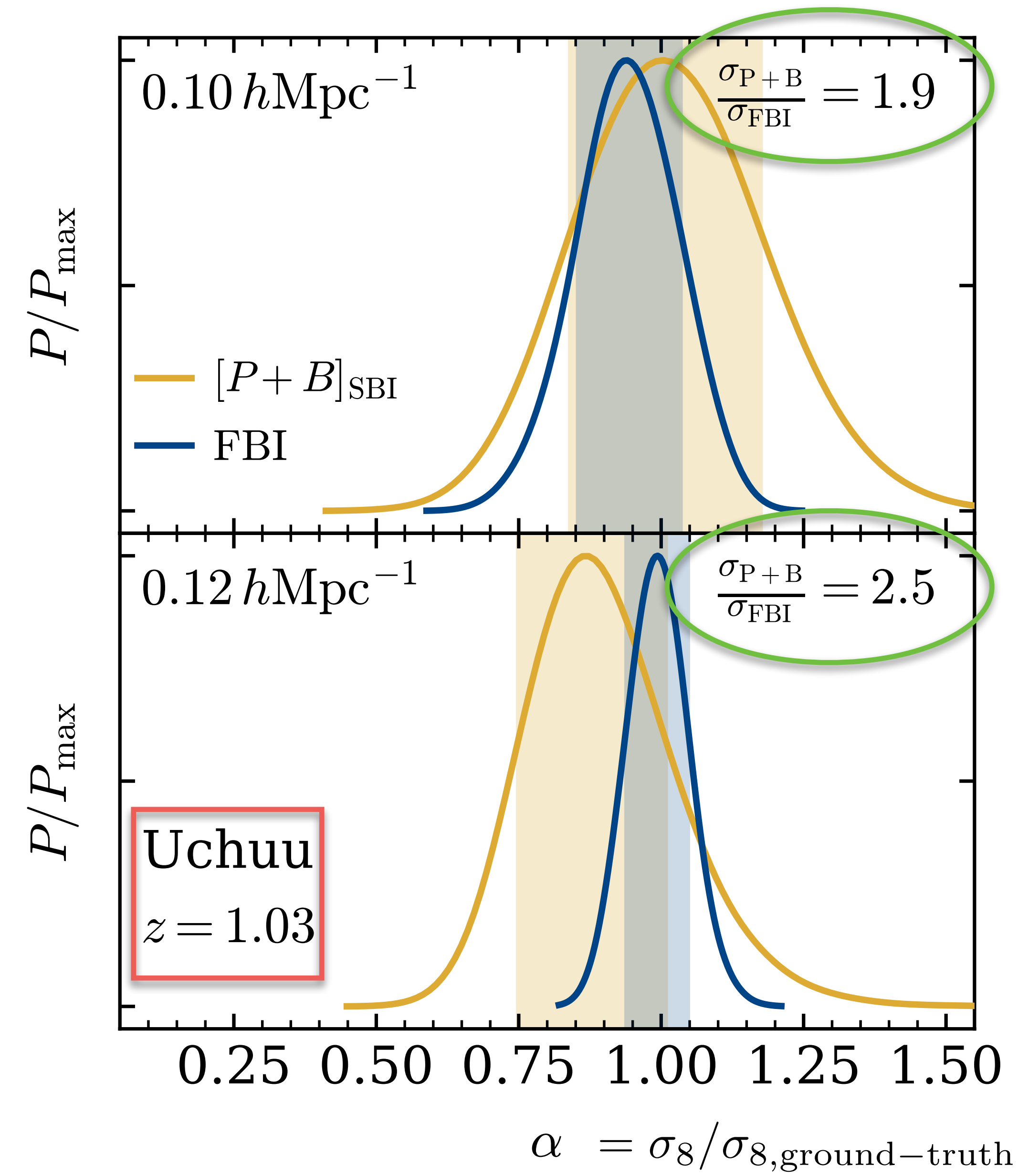
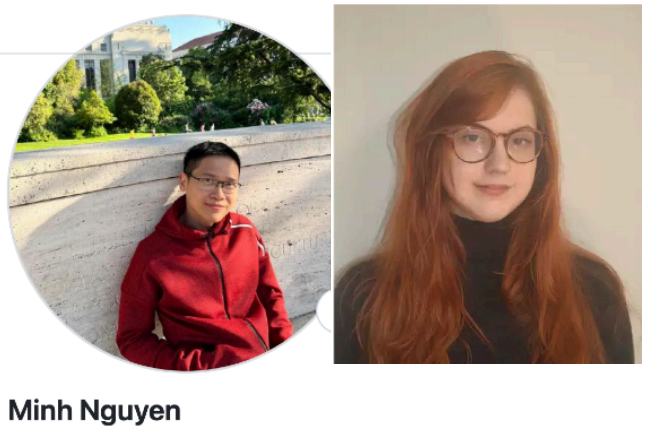


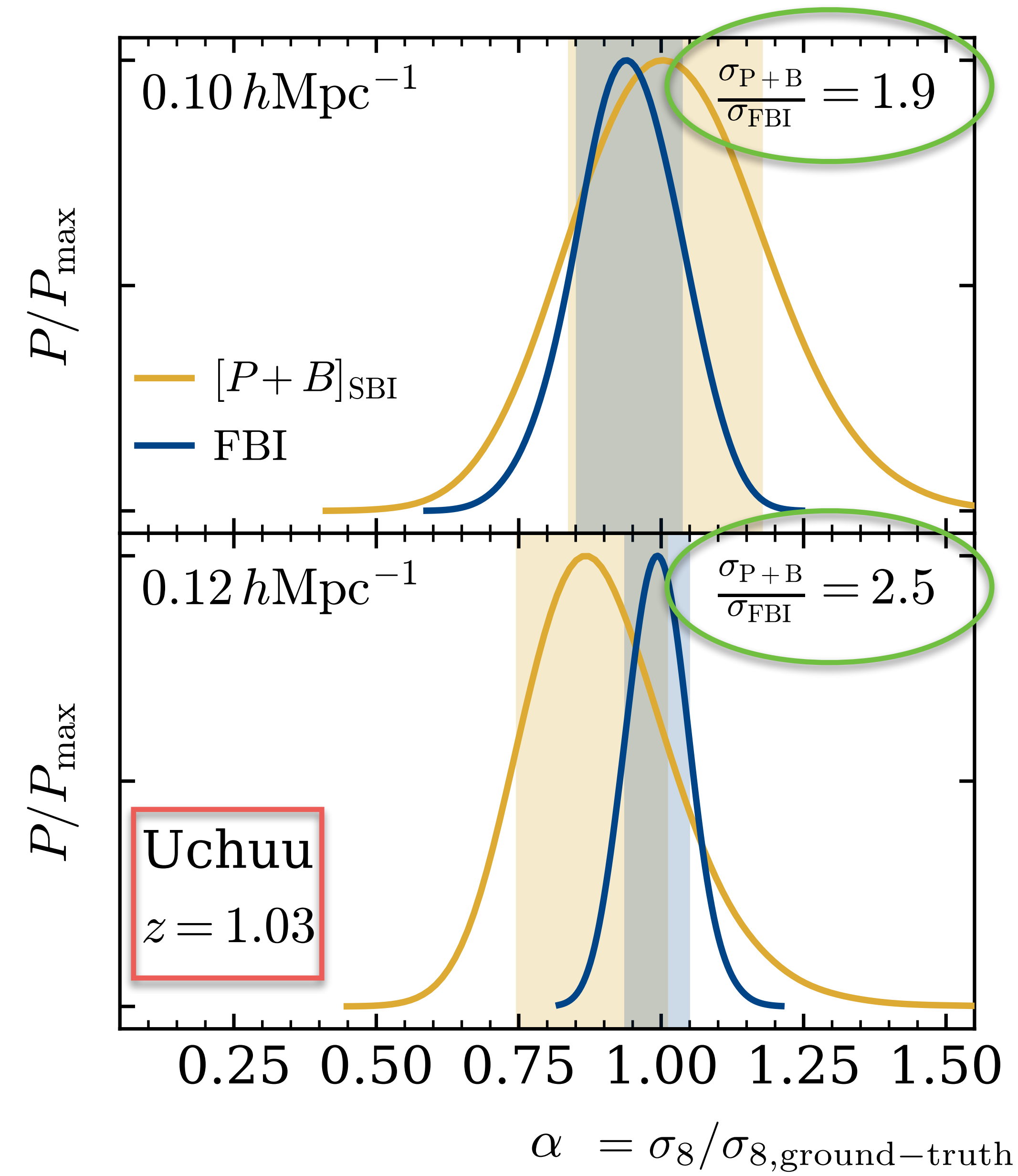
FBI = FLI

Nguyen, FS, Tucci, Reinecke, Kostić PRL 2024, arXiv:2403.03220



Idealized test: Inferring σ_8 from rest-frame tracers

- Caveats:
  - third-order bias (bispectrum constrains only second order at LO)
  - no non-Gaussian noise included in either analysis



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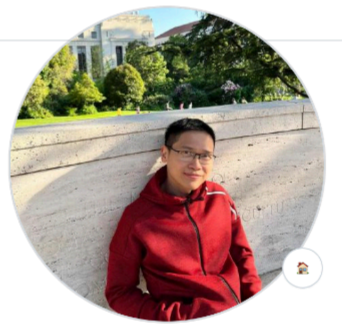

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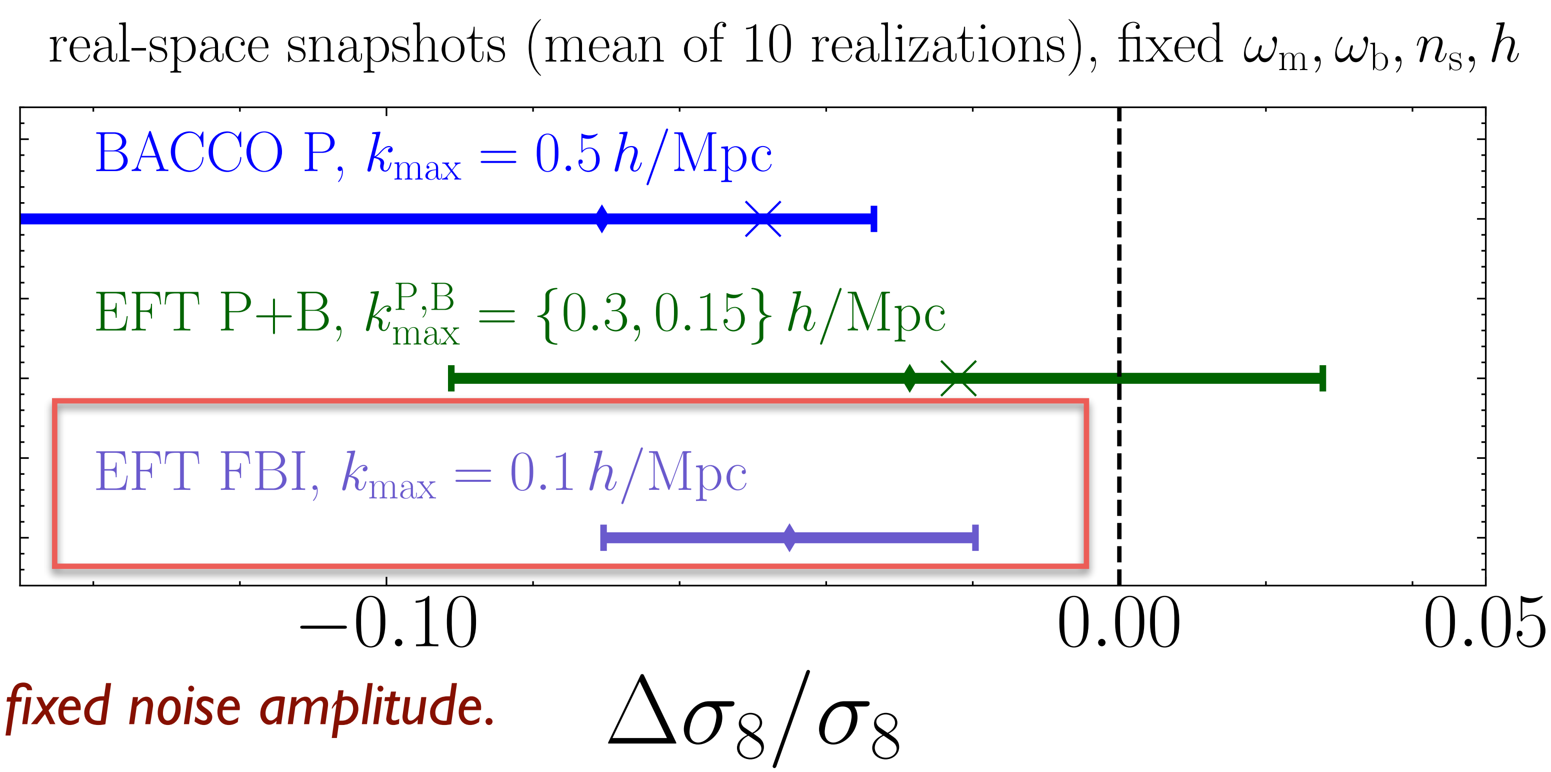
Results from blind challenge

A Parameter-Masked Mock Data Challenge for Beyond-Two-Point Galaxy Clustering Statistics*

The Beyond-2pt Collaboration

Elisabeth Krause,[1] Yosuke Kobayashi,[1,2] Andrés N. Salcedo,[1] Mikhail M. Ivanov,[3] Tom Abel,[4,5,6] Kazuyuki Akitsu,[7] Raul E. Angulo,[8,9] Giovanni Cabass,[10] Sofia Contarini,[11,12,13] Carolina Cuesta-Lazaro,[14,15,16] ChangHoon Hahn,[17] Nico Hamaus,[18,19] Donghui Jeong,[20,21] Chirag Modi,[22,23] Nhat-Minh Nguyen,[24,25] Takahiro Nishimichi,[2,26,27] Enrique Paillas,[28,29] Marcos Pellejero Ibañez,[30] Oliver H. E. Philcox,[31,32] Alice Pisani,[33,22,34,17] Fabian Schmidt,[35] Satoshi Tanaka,[26] Giovanni Verza,[36,22] Sihan Yuan,[4,6] Matteo Zennaro,[37]

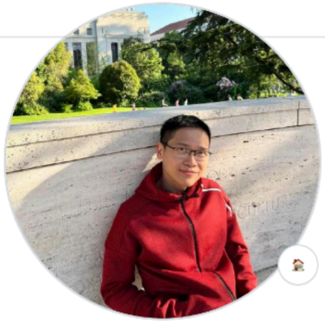
EFT-based field-level inference on blind catalogs:



Second-order bias

Results for fixed noise amplitude.

Thanks to Y. Kobayashi, A. Salcedo, E. Krause, and M. Ivanov, M. Pellejero !



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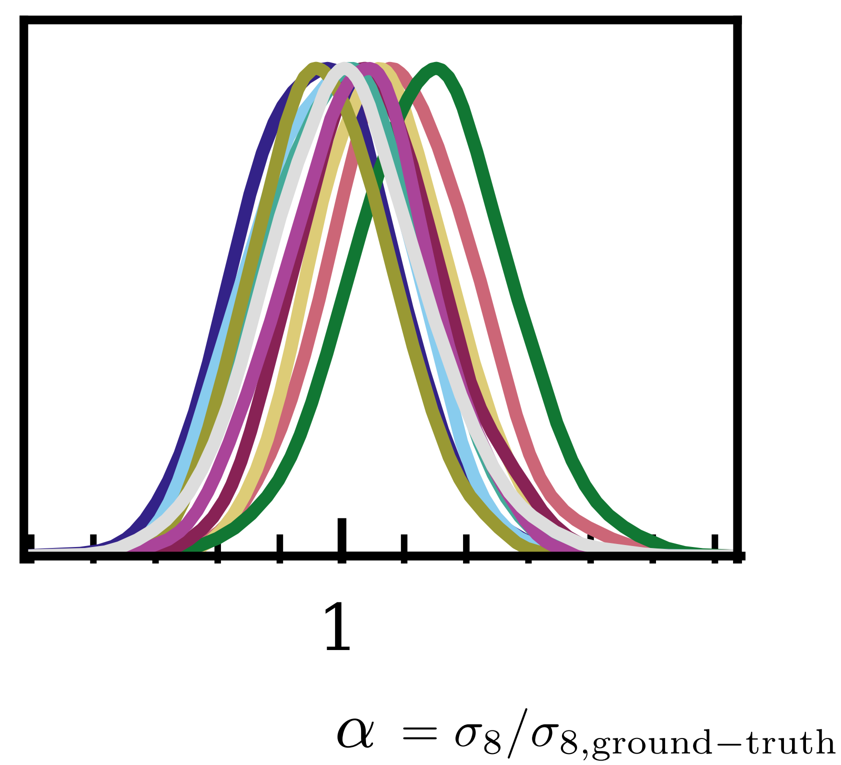
Posterior for 10 independent mock catalog realizations:

No sign of underestimated posterior variance.



Thanks to Y. Kobayashi, A. Salcedo, E. Krause, and M. Ivanov, M. Pellejero !

LEFTfield

Where does the field-level information come from?

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Let's consider the zero-noise limit of the field-level posterior, such that likelihood becomes Dirac delta
- We can then formally perform integration over initial conditions $\delta_{\rm in}$ analytically to obtain marginalized posterior:

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}] \middle| \theta\right) \mathcal{J}[\delta_g, \{b_O\}]$$

Jacobian $|D\delta_{\rm fwd}/D\delta_{\rm in}|^{-1}$

$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

Cabass, Simonović, Zaldarriaga (2023); FS (2025)

Where does the field-level information come from?

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}]\middle|\theta\right) \mathcal{J}[\delta_g, \{b_O\}]$$
$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

- Involves inverse of forward model, evaluated on the data
- In case of linear forward model, $\delta_{\rm fwd} = b_1\delta_{\rm in}$, marginalized field-level posterior is function of the power spectrum of the data - *$P_g(k)$ is sufficient statistic*

Cabass, Simonović, Zaldarriaga (2023); FS (2025)

Where does the field-level information come from?

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta^{-1}_{\rm fwd}[\delta_g, \{b_O\}]\middle|\theta\right) \mathcal{J}[\delta_g, \{b_O\}]$$
$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta^{-1}_{\rm fwd}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

- If forward model is nonlinear, $\delta_{\rm fwd}^{-1}$ is a *nonlinear functional of the data* δ_g: effectively, we add higher *n*-point functions to the posterior
- Each term in the forward model adds a new, specific statistic to the posterior
  - Complete forward model at 2nd order: power spectrum + bispectrum
  - Complete forward model at 3d order: power spectrum + bispectrum + trispectrum …

FS (2025), arXiv:2504.15351

Where does the field-level information come from?

- Specifically, have shown this at the level of the maximum-a-posteriori value of bias coefficents and σ_8:

$$\sum_{O'^{(3)}} \left\langle b_{O'}^{\mathrm{MAP}} \right\rangle A_{O'O} = Y_O$$

where A_{OO}, Y_O are functionals of the data:

- N-point functions of the data enter the MAP expressions in quite nontrivial way beyond leading order

FS (2025), arXiv:2504.15351

Where does the field-level information come from?

- Ensemble-mean of MAP expression for third-order bias

$$\sum_{O'^{(3)}} \left\langle b_{O'}^{\mathrm{MAP}} \right\rangle A_{O'O} = Y_O$$

where A_{OO}, Y_O are functionals of the data:

$$\begin{aligned}
Y_O = &\frac{1}{b_\delta^5} \sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k)} \left\langle \delta_g(-\boldsymbol{k}) O[\delta_g, \delta_g, \delta_g](\boldsymbol{k}) \right\rangle_c \\
&+ \frac{1}{b_\delta^3} \sum_{\boldsymbol{k},\boldsymbol{p}}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k)} \left[S_O(\boldsymbol{k}, \boldsymbol{p}, -\boldsymbol{p}) + S_O(\boldsymbol{p}, \boldsymbol{k}, -\boldsymbol{k}) \right] P_{\mathrm{L}}(k) P_{g,\Lambda}^{1-\mathrm{loop}}(p) \\
&- \frac{1}{b_\delta^7} \sum_{O'^{(2)}} b_{O'} \sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k)} \left\langle O[\delta_g, \delta_g, \delta_g](-\boldsymbol{k}) O'[\delta_g, \delta_g](\boldsymbol{k}) \right\rangle \\
&- \frac{6}{b_\delta^6} \sum_{O'^{(2)}} b_{O'} \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k_1) P_{\mathrm{L}}(k_2)} \left\langle \underbrace{\delta^{(1)}(-\boldsymbol{k}_2) O[\delta^{(1)}}, \delta_g, \delta_g](\boldsymbol{k}_1) \underbrace{\delta^{(1)}(-\boldsymbol{k}_1) O'[\delta^{(1)}}, \delta_g](\boldsymbol{k}_2) \right\rangle \\
&- \frac{12}{b_\delta^8} \sum_{O_1^{(2)}, O_2^{(2)}} b_{O_1} b_{O_2} \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k_1) P_{\mathrm{L}}(k_2) P_{\mathrm{L}}(k_3)} \\
&\qquad \times \left\langle \tilde{O}_1 \underbrace{[\delta^{(1)}, \delta_g, \delta_g](\boldsymbol{k}_3) \delta^{(1)}}(-\boldsymbol{k}_1) \tilde{O}_2 \underbrace{[\delta^{(1)}, \delta_g](\boldsymbol{k}_1) \delta^{(1)}}(-\boldsymbol{k}_2) \tilde{O}_3 \underbrace{[\delta^{(1)}, \delta_g](\boldsymbol{k}_2) \delta^{(1)}}(-\boldsymbol{k}_3) \right\rangle .
\end{aligned} \tag{3.24}$$

$$\begin{aligned}
A_{OO'} = &\frac{1}{b_\delta^8} \sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k)} \left\langle O[\delta_g, \delta_g, \delta_g](-\boldsymbol{k}) O'[\delta_g, \delta_g, \delta_g](\boldsymbol{k}) \right\rangle \\
&+ \frac{9}{b_\delta^6} \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}^{\Lambda} \frac{1}{P_{\mathrm{L}}(k_1) P_{\mathrm{L}}(k_2)} \left\langle \underbrace{\delta^{(1)}(-\boldsymbol{k}_2) O[\delta^{(1)}}, \delta_g, \delta_g](\boldsymbol{k}_1) \underbrace{\delta^{(1)}(-\boldsymbol{k}_1) O'[\delta^{(1)}}, \delta_g, \delta_g](\boldsymbol{k}_2) \right\rangle .
\end{aligned} \tag{3.25}$$

FS (2025), arXiv:2504.15351

Where does the field-level information come from?

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta^{-1}_{\rm fwd}[\delta_g, \{b_O\}]\Big|\theta\right) \mathcal{J}[\delta_g, \{b_O\}]$$
$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta^{-1}_{\rm fwd}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

- Each term in the forward model adds a new, specific statistic to the posterior
- Lagrangian, LPT-based forward model as in LEFTfield: *correctly describes displacement terms at all orders, precisely those terms responsible for the degeneracy breaking*
- Impact of missing operators in forward model is proportional to scalar product of missing $O_{\rm missing}[\delta]$ with $O[\delta]$ of interest

FS (2025), arXiv:2504.15351

A toy scenario

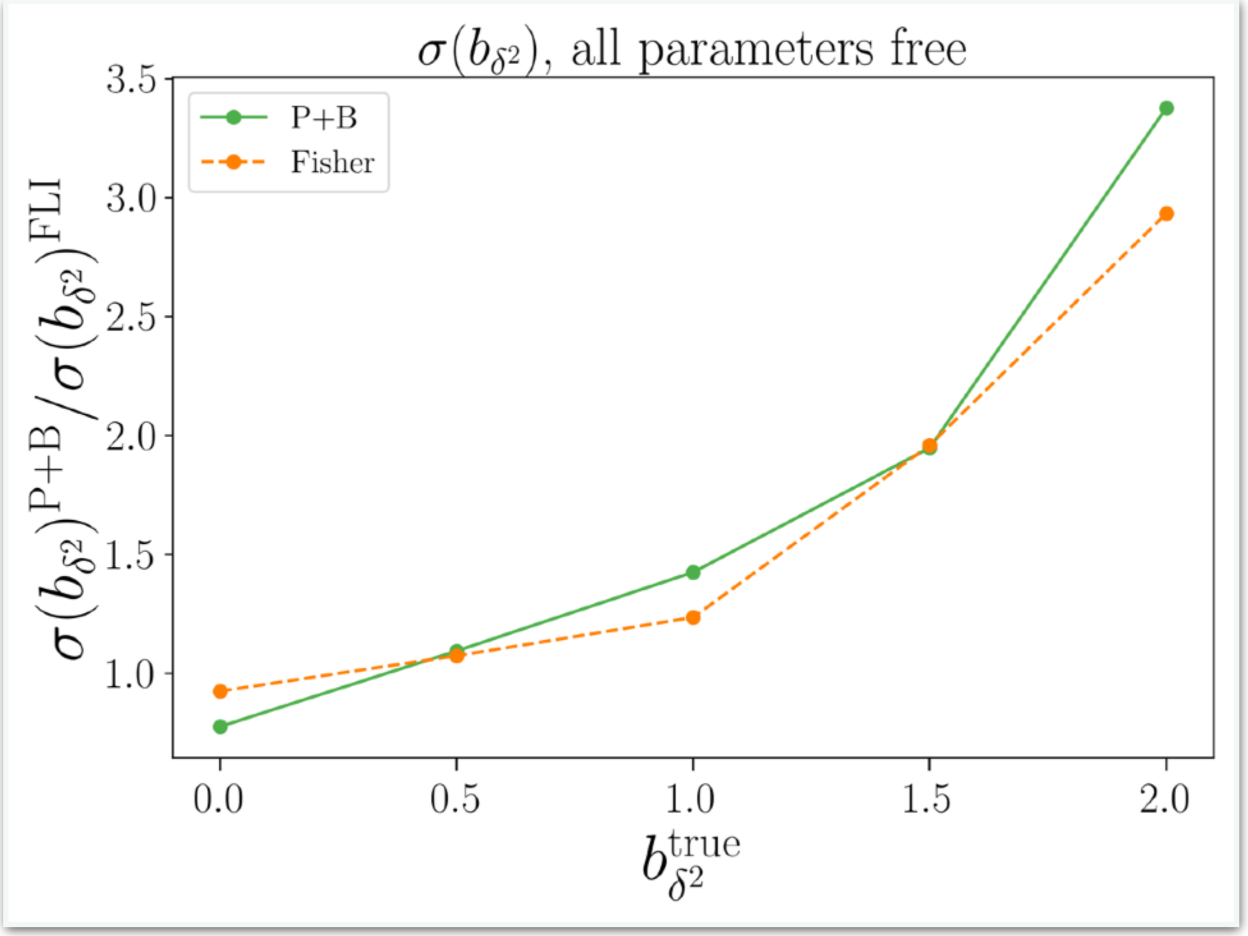
- We can look at a much simpler case:

$$\delta_g(\boldsymbol{x}) = b_\delta \delta^{(1)}(\boldsymbol{x}) + b_{\delta^2}[\delta^{(1)}(\boldsymbol{x})]^2 + \epsilon(\boldsymbol{x})$$

- Compare FLI with P+B as a function of the ground-truth value of b_{δ^2}
- As expected, for $b_{\delta^2,\text{true}}=0$, FLI recovers same constraint as P+B
- For nonzero $b_{\delta^2,\text{true}}$, FLI yields more information effectively extracted from higher n-point functions

Marginalized constraint on b_{δ^2}

$\Lambda = k_{\max} = 0.14h\ \text{Mpc}^{-1}$



SBI and Fisher calculation using sample covariance

Nikolac, FS, Tucci, in prep.

A toy scenario

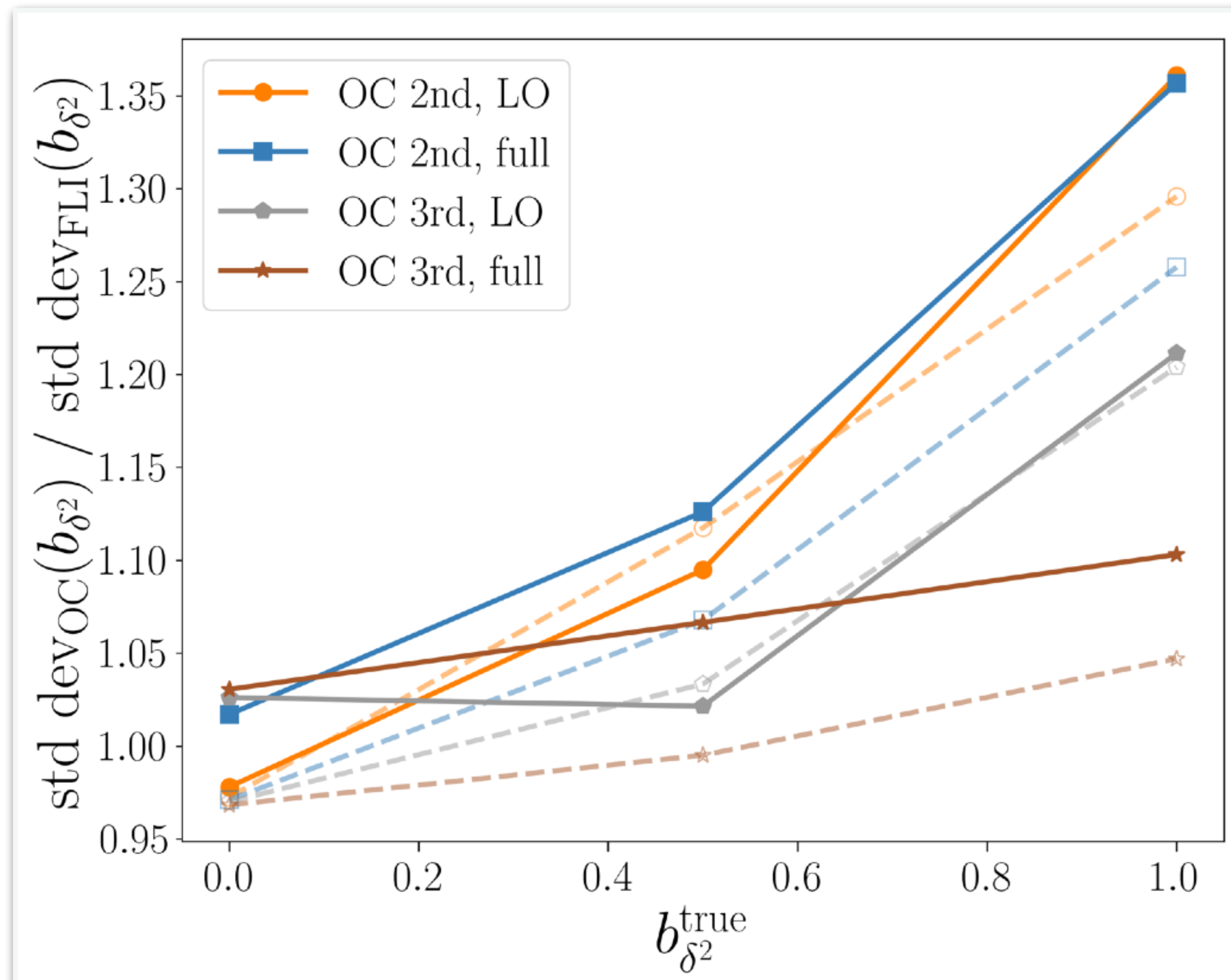
- For nonzero $b_{\delta^2}^{\text{true}}$, FLI yields more information effectively extracted from higher n-point functions
- For this simple forward model, can access the information via compressed statistics: correlations of local powers of the data:

$$\langle [\delta_g(\boldsymbol{x})]^n [\delta_g(\boldsymbol{x})^m \rangle_c$$

- Indeed, higher-order statistics recover the information gain in FLI

Marginalized constraint on b_{δ^2}

$\Lambda = k_{\max} = 0.14h\ \mathrm{Mpc}^{-1}$



SBI (solid) & Fisher (dashed) results

Nikolac, FS, Tucci, in prep.

Cosmology results II: Field-level inference of BAO scale

Cosmology results II: Field-level inference of BAO scale

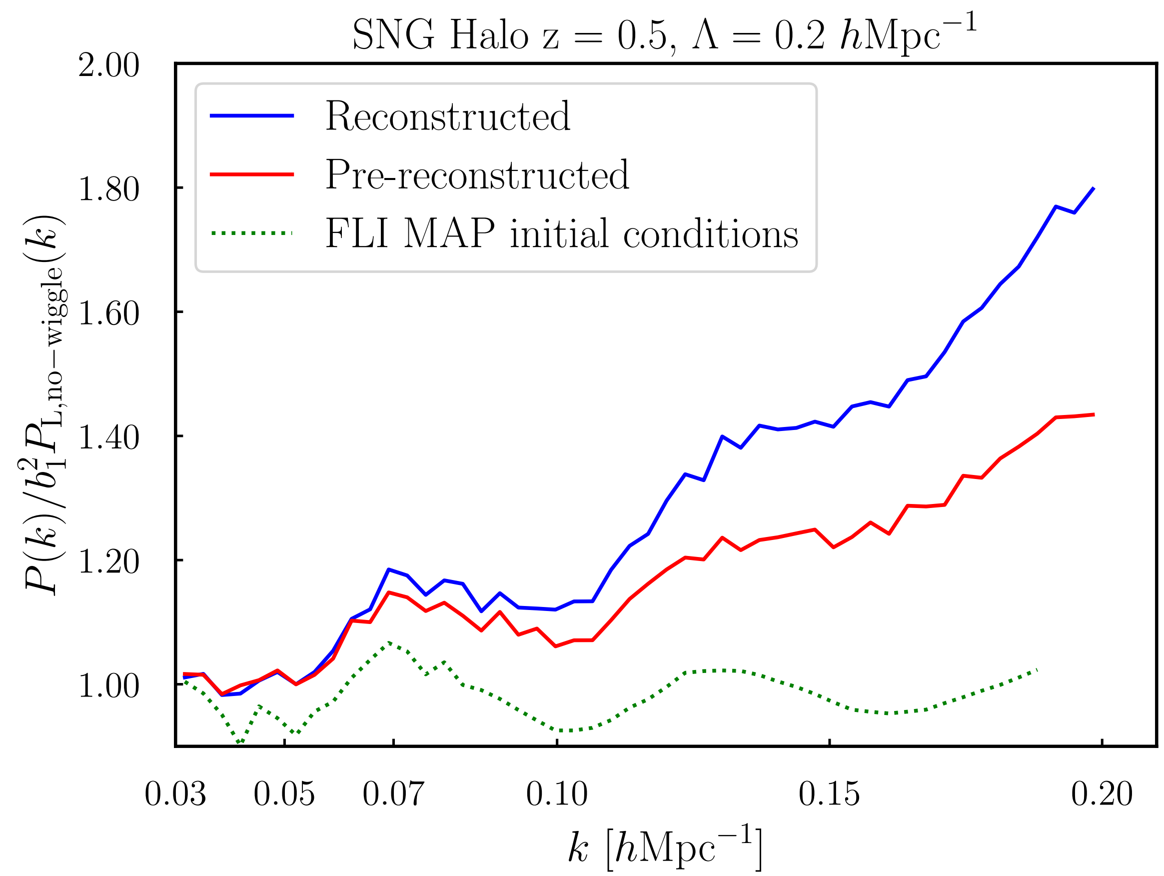
- Constraints on expansion history (dark energy) from galaxy clustering are based on the BAO standard ruler (cf. DESI results)
- These are commonly inferred by performing reconstruction procedure on galaxies, and then using the *post-reconstruction galaxy power spectrum*

Cosmology results II: Field-level inference of BAO scale

- Reconstruction idea: estimate large-scale displacements from galaxy density field, then move galaxies *back* to inferred initial positions

- Improves error bar on BAO scale by up to 50%

- Can we also do this in a *forward* approach by performing joint field-level inference of initial density field and BAO scale?



Babić, FS, Tucci (2025), arXiv:2505.13588

Cosmology results II: Field-level inference of BAO scale

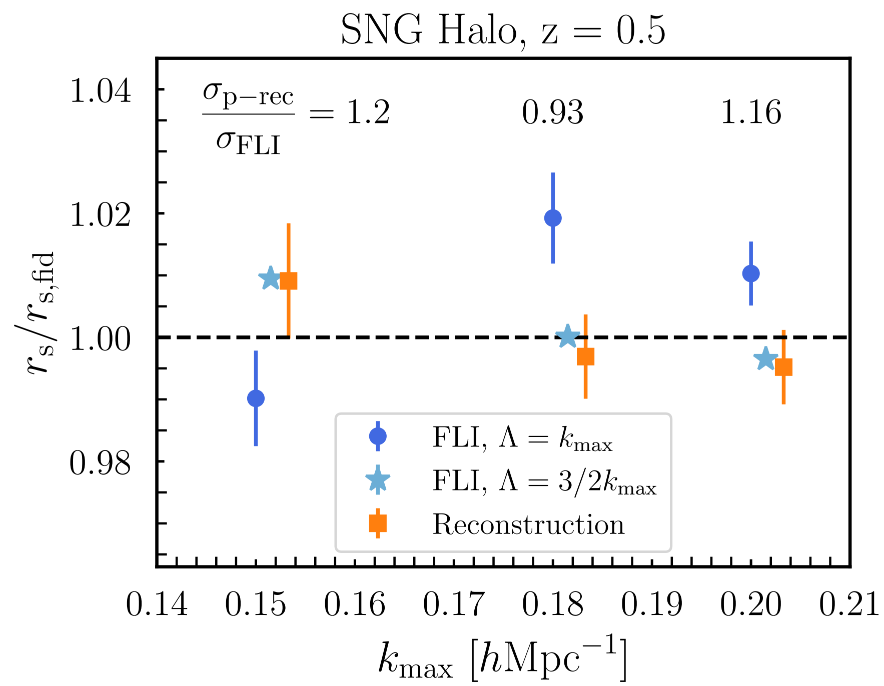
- Field-level inference of BAO scale using a trick: moving BAO feature in linear (initial) density field:

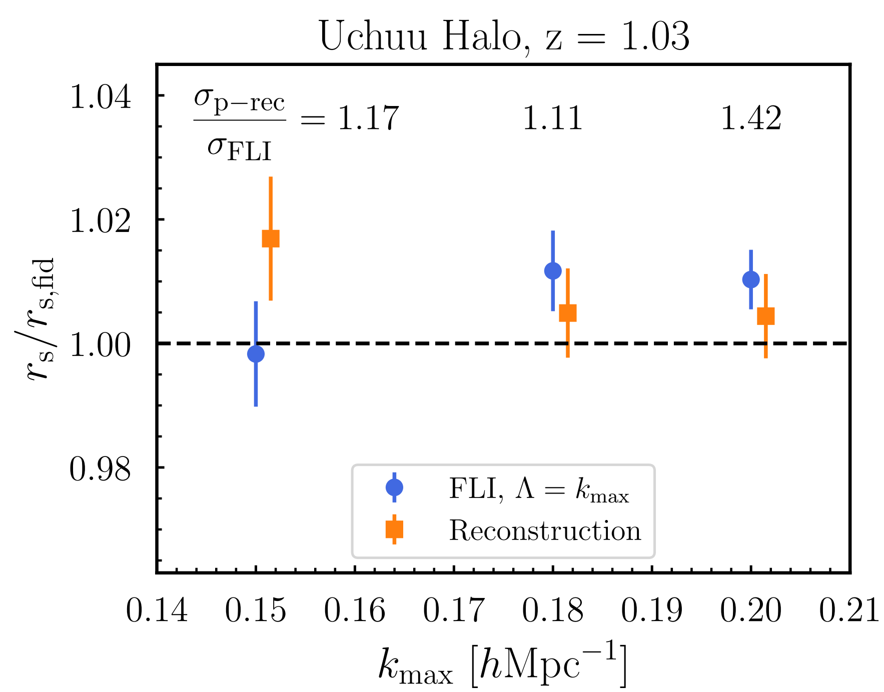
$$f(k, r_s) = \frac{T^2_{\rm BAO}(k|r_s)}{T^2_{\rm BAO}(k|r_{s,\rm fid})}\,,$$

$$T^2_{\rm BAO}(k|r_s) = 1 + A\sin(k\,r_s + \phi)\exp(-k/k_{\rm D})$$

- Compare with reconstruction analysis applied to the same scales of the data

- Note: reconstruction uses fixed linear bias, field-level inference infers all bias coefficients jointly with BAO scale





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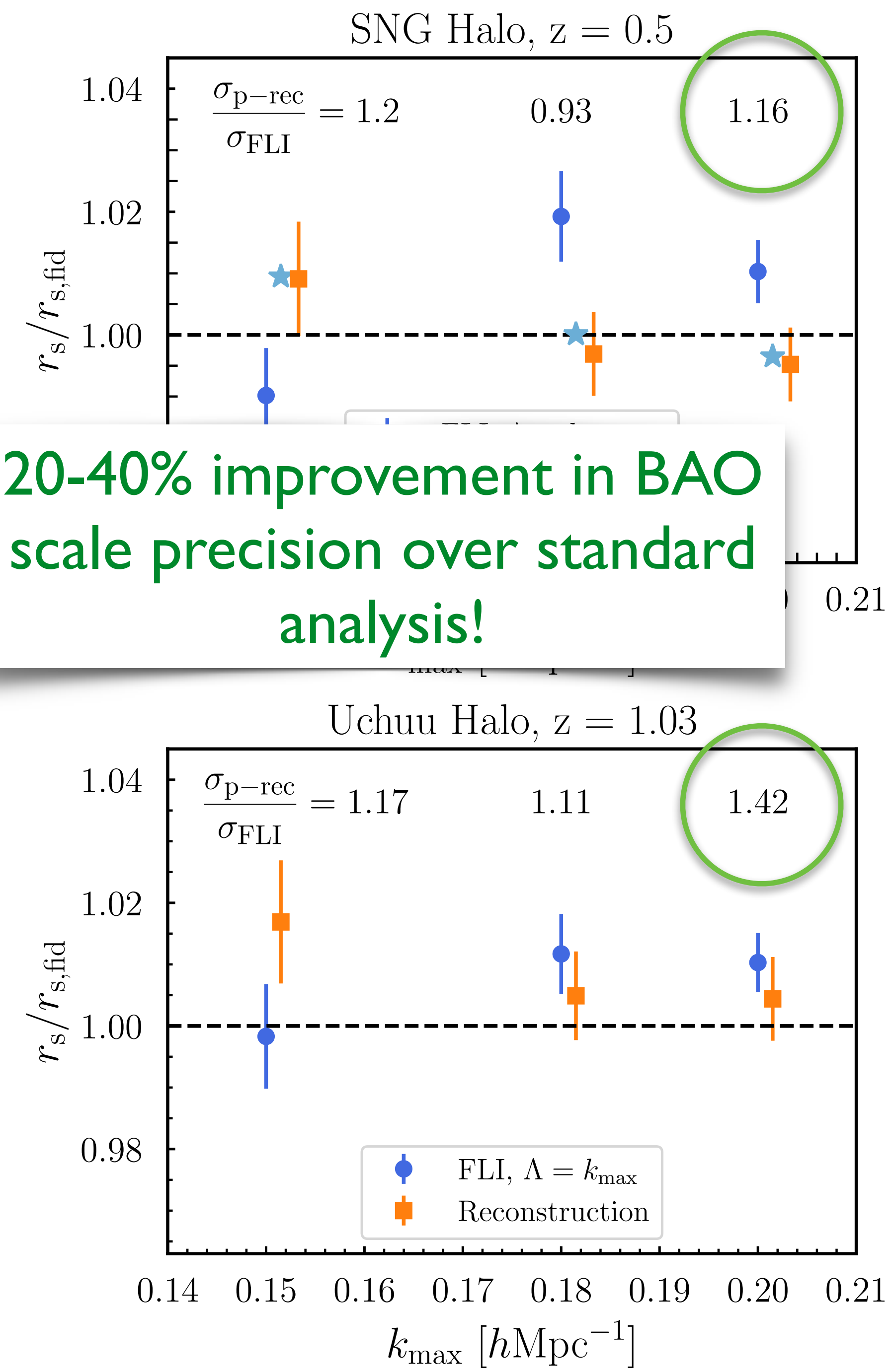


20-40% improvement in BAO scale precision over standard analysis!

Babić, FS, Tucci (2025), arXiv:2505.13588

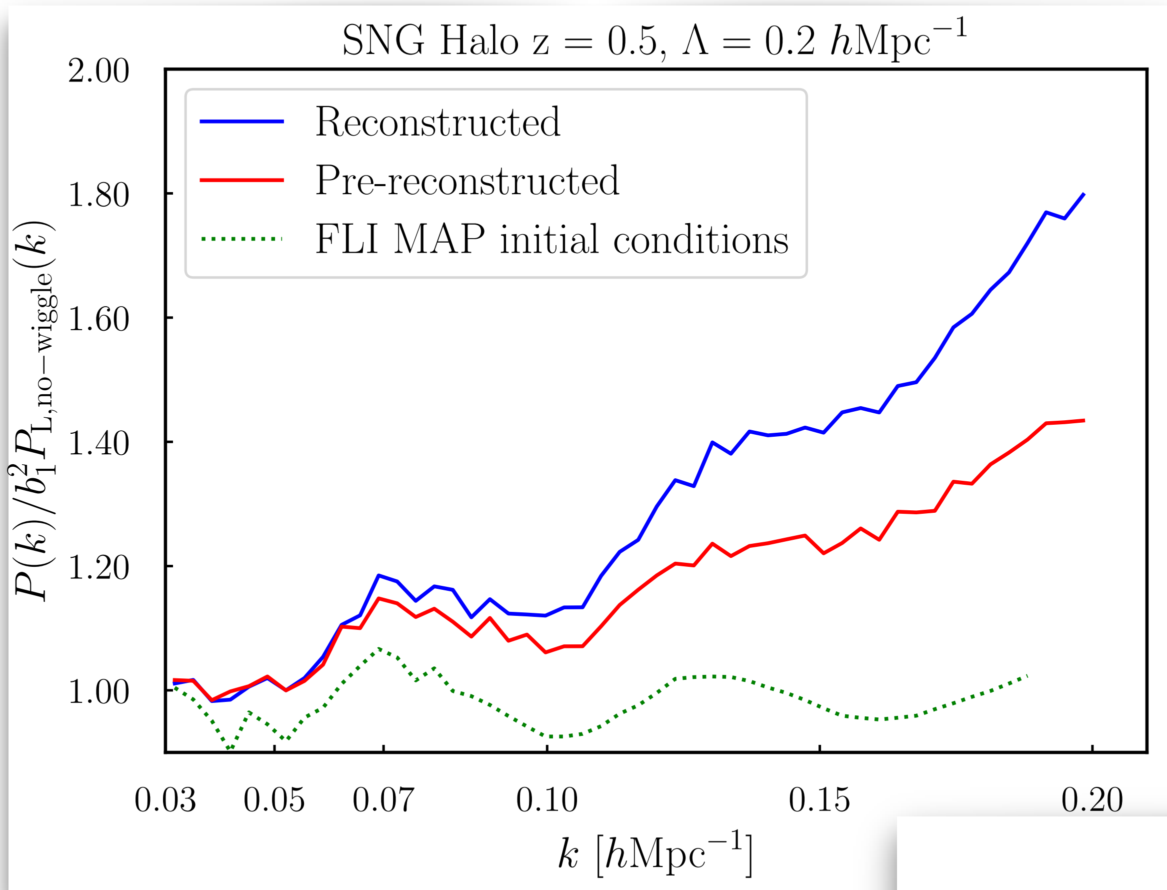
Where does the field-level *BAO* information come from?

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}]\Big|\theta\right) \mathcal{J}[\delta_g, \{b_O\}]$$
$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

- In case of *perfect* forward model, $\delta_{\rm fwd}^{-1}$ is a sample from prior (Gaussian linear density field) - in fact, information obtained is precisely that contained in linear density field: *optimal inference*
  - Field-level inference "undoes" nonlinear evolution as well as nonlinear bias
- On the other hand, standard BAO reconstruction leaves substantial broad-band contribution to $\delta_g^{\rm post\text{-}rec}$; this explains information gain found at field level
- Cannot easily be recuperated using higher-order n-pt functions

Babić, FS, Tucci (2025), arXiv:2505.13588

Where does the field-level *BAO* information come from?



$$\ldots{}^{-1}_{\mathrm{vd}}[\delta_g,\{b_O\}]\Big|\theta\Big)\,\mathcal{J}[\delta_g,\{b_O\}]$$

$$\int_{\boldsymbol{k}} \frac{|\delta^{-1}_{\mathrm{fwd}}[\delta_g,\{b_O\}](\boldsymbol{k})|^2}{P_{\mathrm{L}}(k|\theta)}\Big]\,\mathcal{J}[\delta_g,\{b_O\}]$$

$\delta_{\mathrm{fwd}}{}^{-1}$ is a sample from prior (Gaussian

rmation obtained is precisely that contained

rence

nonlinear evolution as well as nonlinear

$$F^{\mathrm{FLI}}_{r_s r_s} = -\left\langle \frac{\partial^2}{\partial r_s^2} \ln \mathcal{P}_{\mathrm{FLI}}[\{b_O\}, r_s|\delta_g] \right\rangle = \frac{1}{2}\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{[P_{\mathrm{L}}(k|r_{s,\mathrm{fid}})]^2} \left(\frac{\partial P_{\mathrm{L}}(k|r_{s,\mathrm{fid}})}{\partial r_{s,\mathrm{fid}}}\right)^2$$

$$F^{\mathrm{rec-P(k)}}_{r_s r_s} = -\left\langle \frac{\partial^2}{\partial r_s^2} \ln \mathcal{P}_{\mathrm{rec-P(k)}}[r_s|\delta_g] \right\rangle = \sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{\mathrm{Var}[P_{\text{p-rec}}(k|r_{s,\mathrm{fid}})]} \left(\frac{\partial P_{\text{p-rec}}(k|r_s)}{\partial r_s}\right)^2$$
$$= \frac{1}{2}\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{[P_{\text{p-rec}}(k|r_{s,\mathrm{fid}})]^2} \left(\frac{\partial P_{\text{p-rec}}(k|r_s)}{\partial r_s}\right)^2 .$$

- On the other hand, stan band contribution to δ_g level
- Cannot easily be recupe

Babić, FS, Tucci (2025), arXiv:2505.13588

Summary (New Inference)

- Field-level inference (FLI) uses all information up to given k_{max}
  - guaranteed to be optimal (for correct model)
- LEFTfield is a consistent EFT-based field-level forward model, ready for idealized data at this point
  - >~ 100% gain in σ_8 from rest-frame tracers (unbiased inference highly nontrivial as well)
  - Self-consistent BAO reconstruction with gain in BAO scale precision ~20-40% over standard reconstruction
- Both of these probes could yield very interesting insights on dark energy going forward!

Summary (New Inference)

- Analytical results in zero-noise limit yield useful insights into where the information is coming from
- If perturbation theory valid up to k_{max} considered, FLI corresponds to combined inference from finite (but not necessarily small) set of n-point functions

FS (2025), arXiv:2504.15351

Summary (New Inference)

- FLI *beyond perturbative regime:* forward model needs to correctly describe n-point functions of arbitrary order — not easy when attempting to describe real galaxies.
  - Typically, empirical models struggle to describe bispectrum up to same k as power spectrum…

FS (2025), arXiv:2504.15351

New Physics

from Galaxy Clustering

New Physics

from Galaxy Clustering and other things

1. Dark Energy can cross phantom divide
2. Galaxy shapes can probe parity violation
3. Fun with PBH: a UV-complete dark matter scenario

FS (2017)

1. Dark Energy *can* cross phantom divide

- If observations are consistent with w=-1, have we proven that DE=Λ?

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + p(\phi, X) + \mathcal{L}_m\right]$$

$$X \equiv -\frac{1}{2}(\partial_\mu\phi)^2$$

  - Canonical scalar field: yes

$$p(\phi) = X + V(\phi) \quad \Rightarrow \quad w = \frac{\dot\phi^2/2 - V(\phi)}{\dot\phi/2 + V(\phi)}$$

- Not true in general: could have equation of state that *varies around* w=-1

- *Monodromic k-essence:*

$$p(\phi, X) = \tilde V(\phi)\left[-X/M^4 + (X/M^4)^2\right]$$

$$\tilde V(\phi) = C\left(\frac{\phi}{\phi_0}\right)^{-\alpha}\left[1 - A\sin(\nu H_0\phi + \delta)\right].$$

1. Dark Energy *can* cross phantom divide

- If observations are consistent with w=-1, have we proven that DE=Λ?
  - Canonical scalar f

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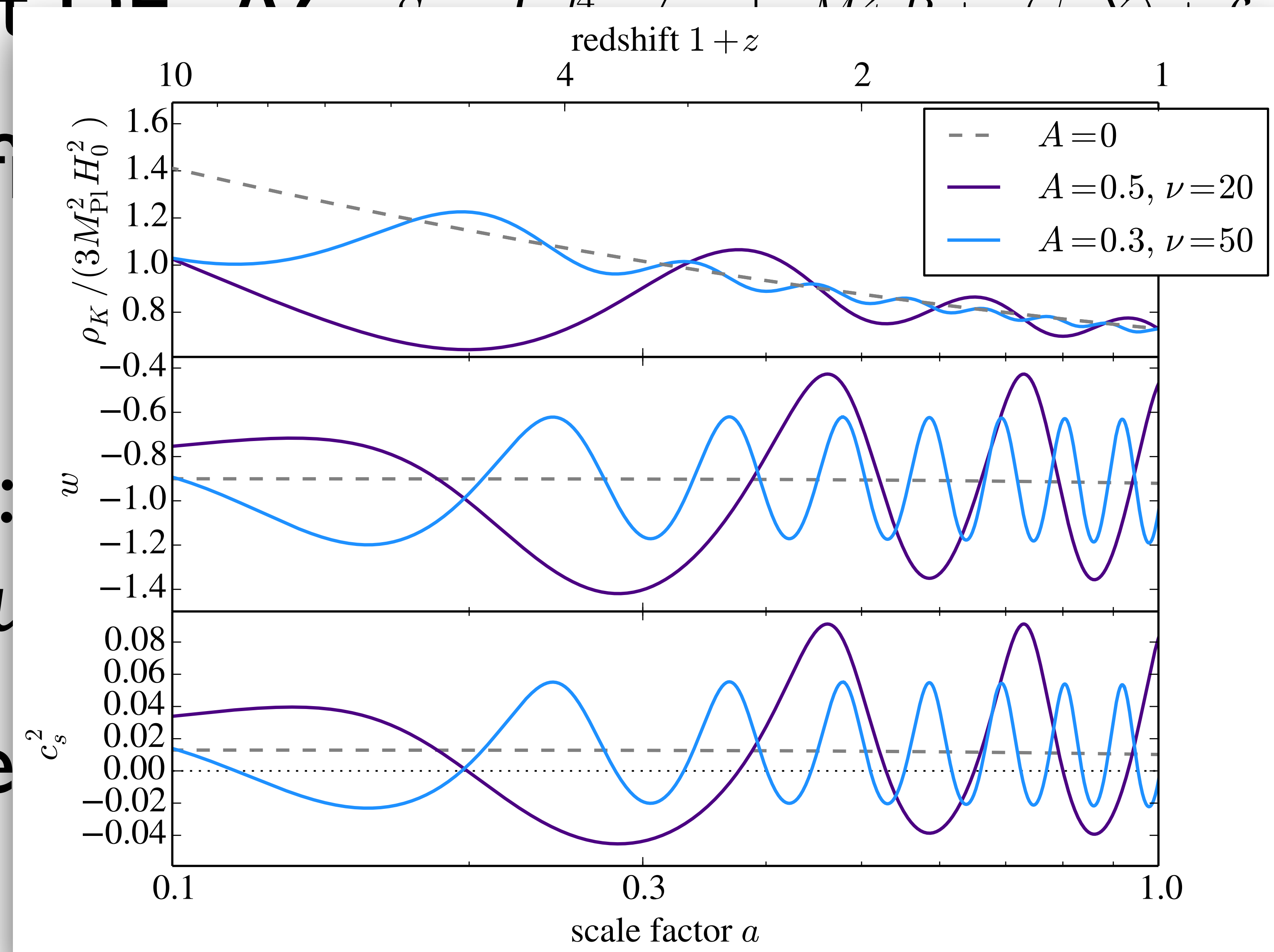


Dark Energy *can* cross phantom divide

- Fine at the background level, but DE perturbations suffer tachyonic instabilities if $c_s^2 < 0$
- k-essence case naturally has $c_s^2 \ll 1$; in fact, $c_s^2 \sim (1+w)$ in $1+w \to 0$ limit, leading to tachyonic instabilities as $1+w < 0$
- These can be dealt with consistently if
  - Higher-derivative contributions are present:

$$\ddot{\delta\phi} \sim -c_s^2 k^2 \delta\phi + \frac{k^4}{\bar{M}^2}\delta\phi + \ldots$$

e.g., from

$$\Delta\mathcal{L}_{\mathrm{DE,h.deriv.}} = -\frac{\bar{M}^2}{2}\left[\Box\phi + 3H(\phi)\right]^2$$

  - c_s^2 stays infinitesimally below 0
- Lowers cutoff of the theory, but not ruled out.

Creminelli, D'Amico, Noreña, Vernizzi (2009)

Dark Energy *can* cross phantom divide

- An example viable model (due to Marco Celoria):

$$p(\phi, X) = \frac{\bar{M}^4}{2}(2X-1)^2 - F(\phi) + G(\phi)(2X+1)$$
$$F(\phi) = V_0\left[1 - \tilde{A}\sin(\tilde{\nu}H_0\phi)\right]$$
$$G(\phi) = V_0\tilde{A}\tilde{\nu}H_0\cos(\tilde{\nu}H_0\phi).$$

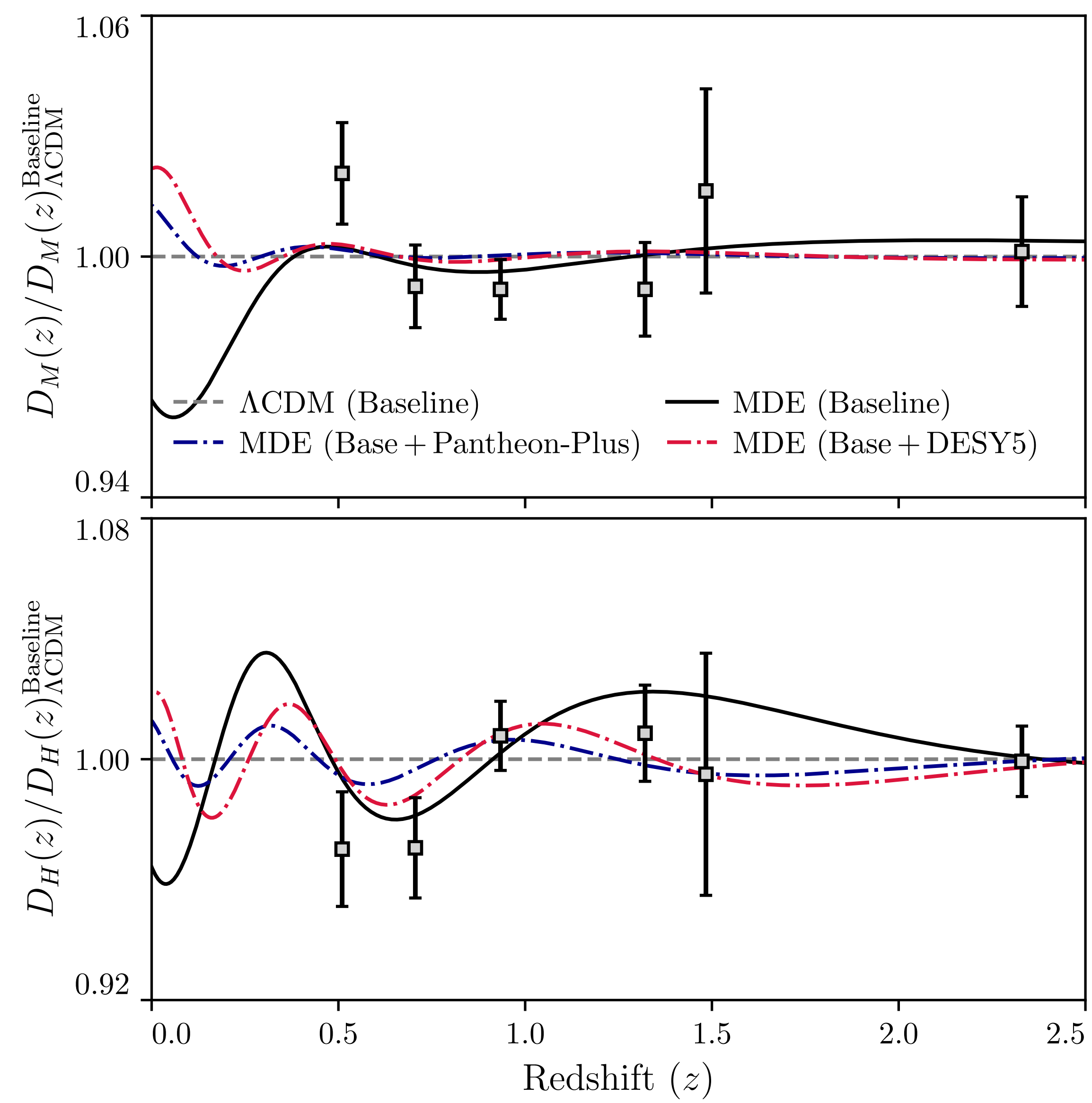
- Oscillations with amplitude Δw~0.1 around w=-1 easily possible while satisfying constraints on instabilities and having cutoff > eV scale.

Goldstein, Celoria, FS (2025)

Goldstein, Celoria, FS (2025)

Monodromic k-essence and DESI

- 3 free parameters (FS 2017 model) in addition to Ω_{de}, potential tilt α <=> mean w:
    - amplitude, frequency, phase of oscillations
- Exclude all observables sensitive to perturbations here



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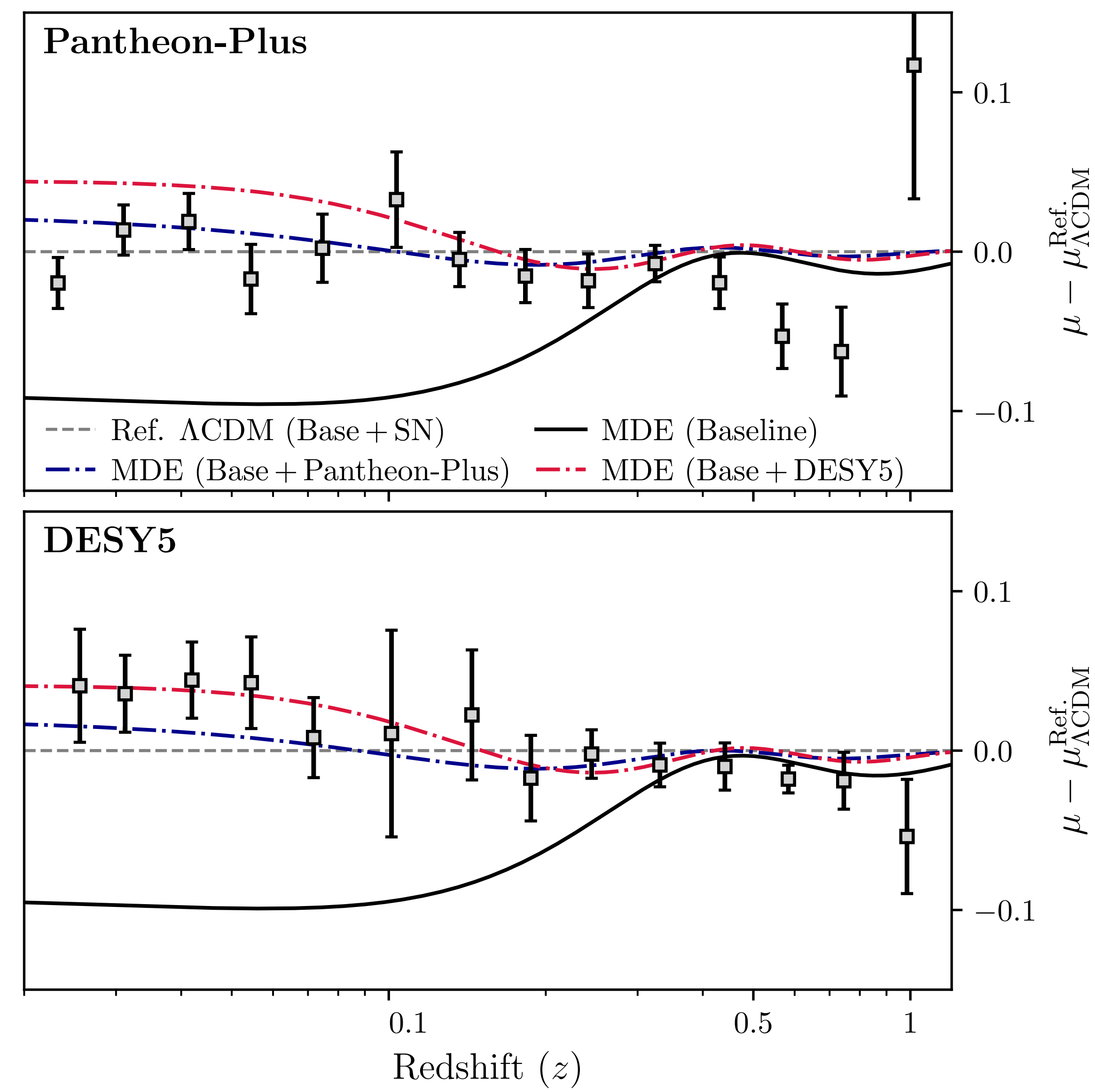
- 3 free parameters (FS 2017 model) in addition to Ω_{de}, potential tilt α <=> mean w:
    - amplitude, frequency, phase of oscillations
- Exclude all observables sensitive to perturbations here
- Similar fit quality to DESI BAO + SN as w_0, w_a
- Mean w consistent with -1 (motivated by theory as well); then, only 1 more free parameter than w_0, w_a !



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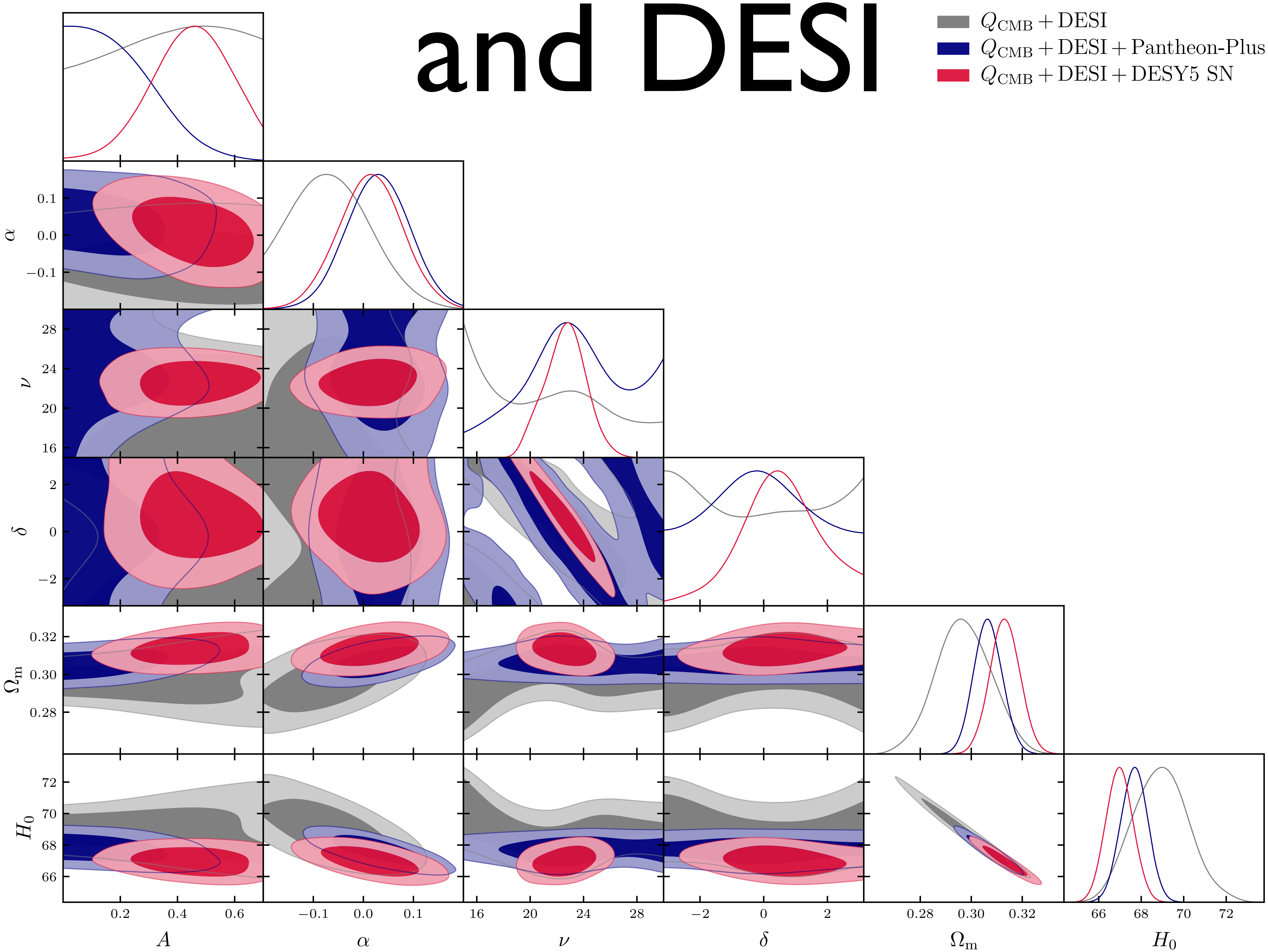

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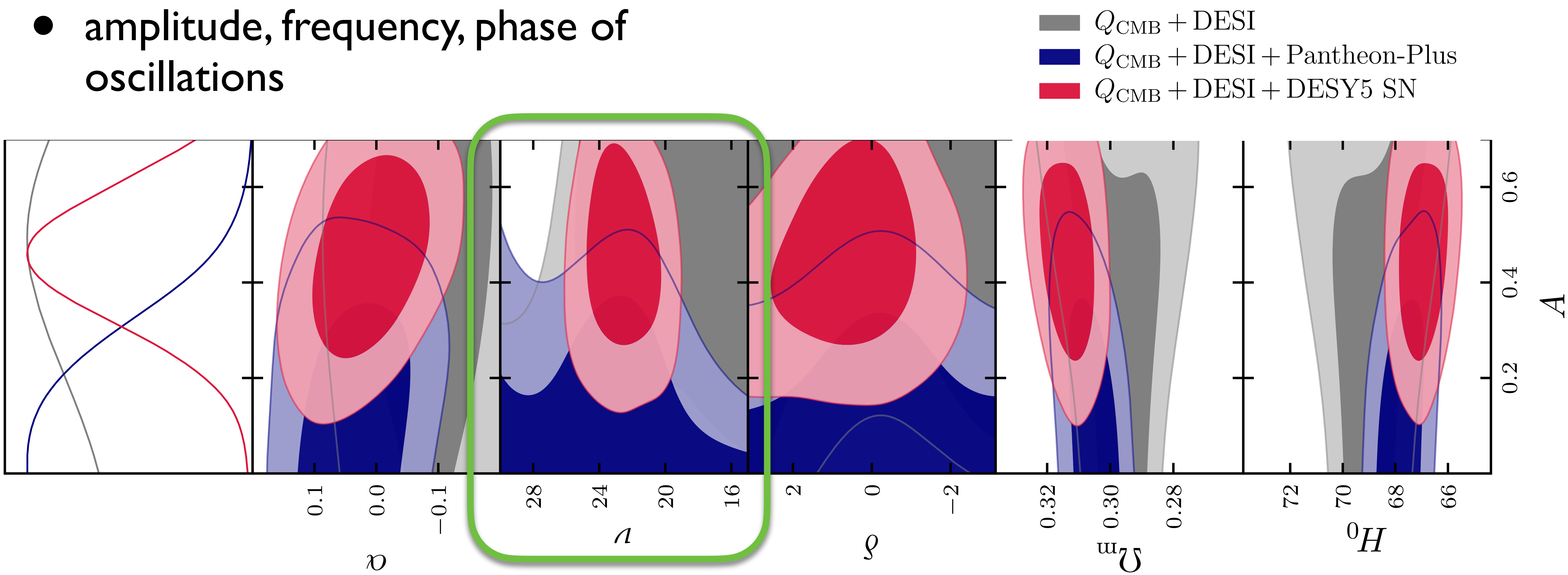
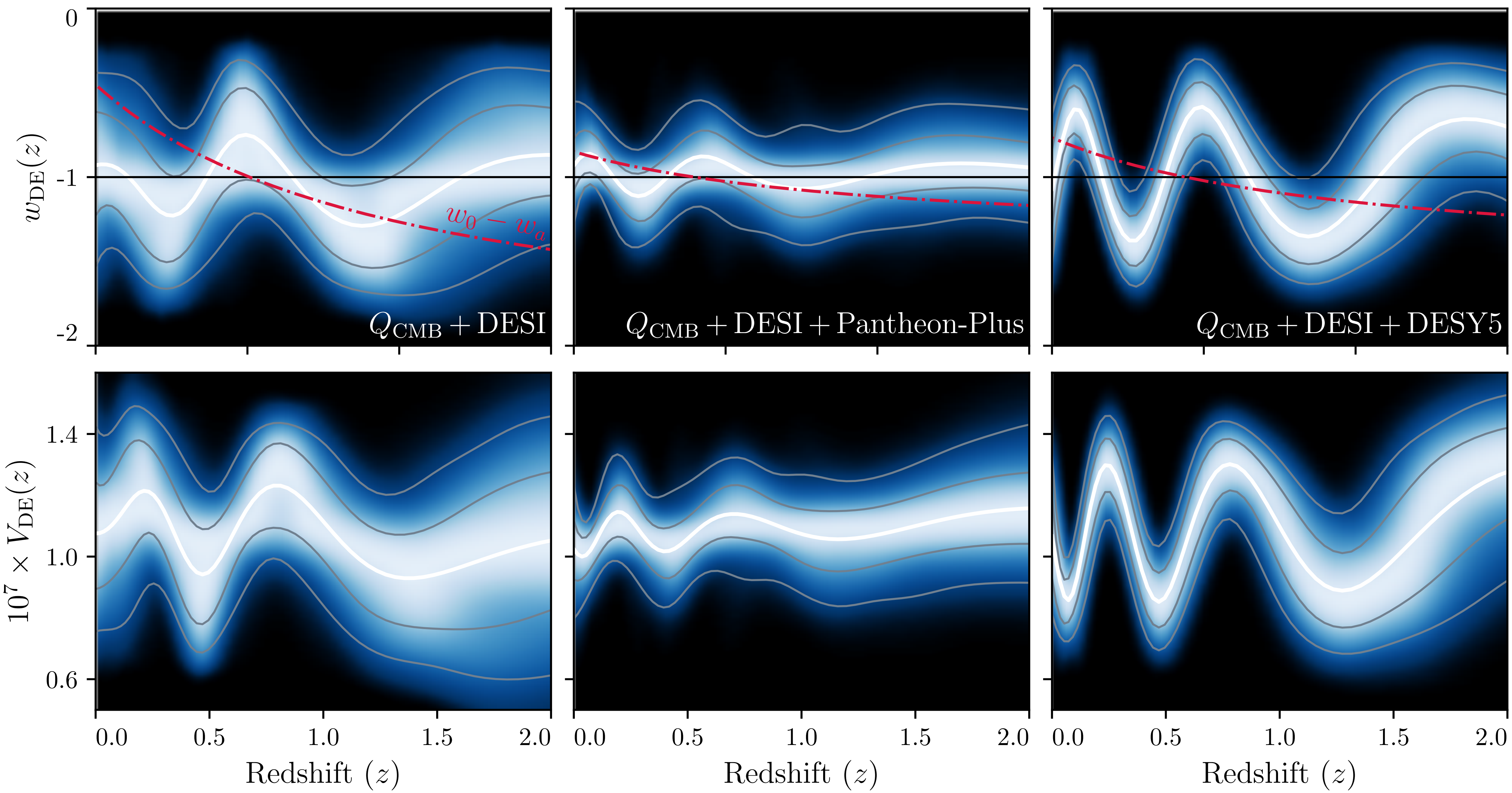


Goldstein, Celoria, FS (2025)

Monodromic k-essence and DESI

- Reconstruction of w(z) and k-essence "potential"



2. Galaxy shapes as probes of parity

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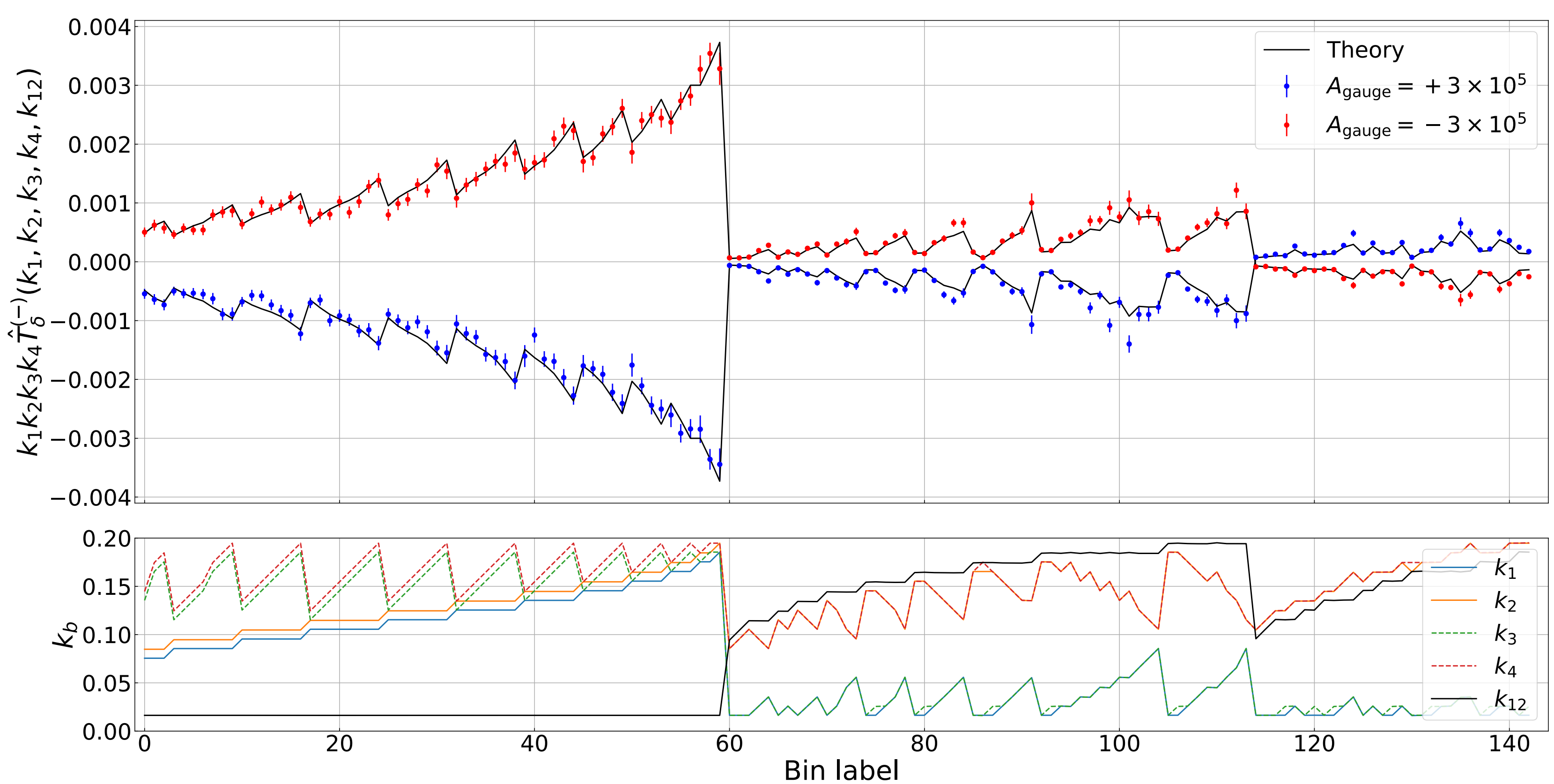
- Enhanced large-scale parity-odd correlation induced in case of enhanced collapsed limit of primordial trispectrum
- A new probe of parity violation in primordial perturbations



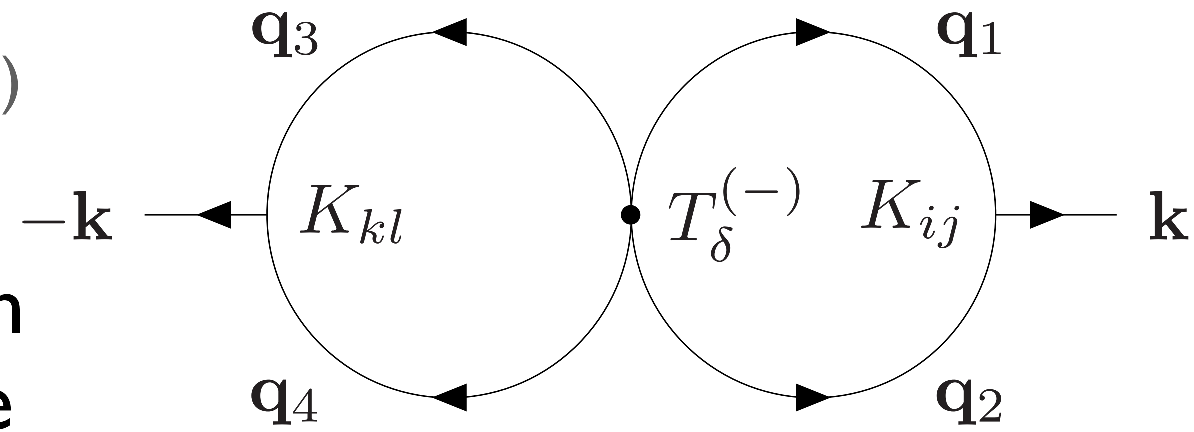
Kurita, Jamieson, Komatsu, FS (2025)

Primordial parity violation

- The leading signature of parity violation in primordial curvature perturbations is in connected 4-point function (trispectrum)
- Parity-odd primordial trispectrum can always be written as:
$$T_\Phi^{(-)} = i\,[\mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)]\,\tau_-(k_1, k_2, k_3, k_4, k_{12}, k_{14})$$
- Interesting case is when τ_- is enhanced in collapsed limit: k_{12}->0 or k_{14}->0
- Physical scenario: primordial chiral U(1) field that couples to inflaton, $\mathcal{L} \supset 1/4 f(\varphi)(-F^2 + \gamma F\tilde{F})$

Galaxy shapes as probes of parity

- Compute 1-loop parity-odd shape statistics in the "EFT of shapes" - EFTofLSS applied to a 3D 2-tensor observable Vlah, Chisari, FS (2020)
- New divergence appears for primordial trispectrum - absorbed in counterterm: scale-dependent shape bias
- Enhanced large-scale signal in 1-to-1 correspondence with collapsed limit of primordial trispectrum
- Smoking gun of parity violation



Kurita, Jamieson, Komatsu, FS (2025)

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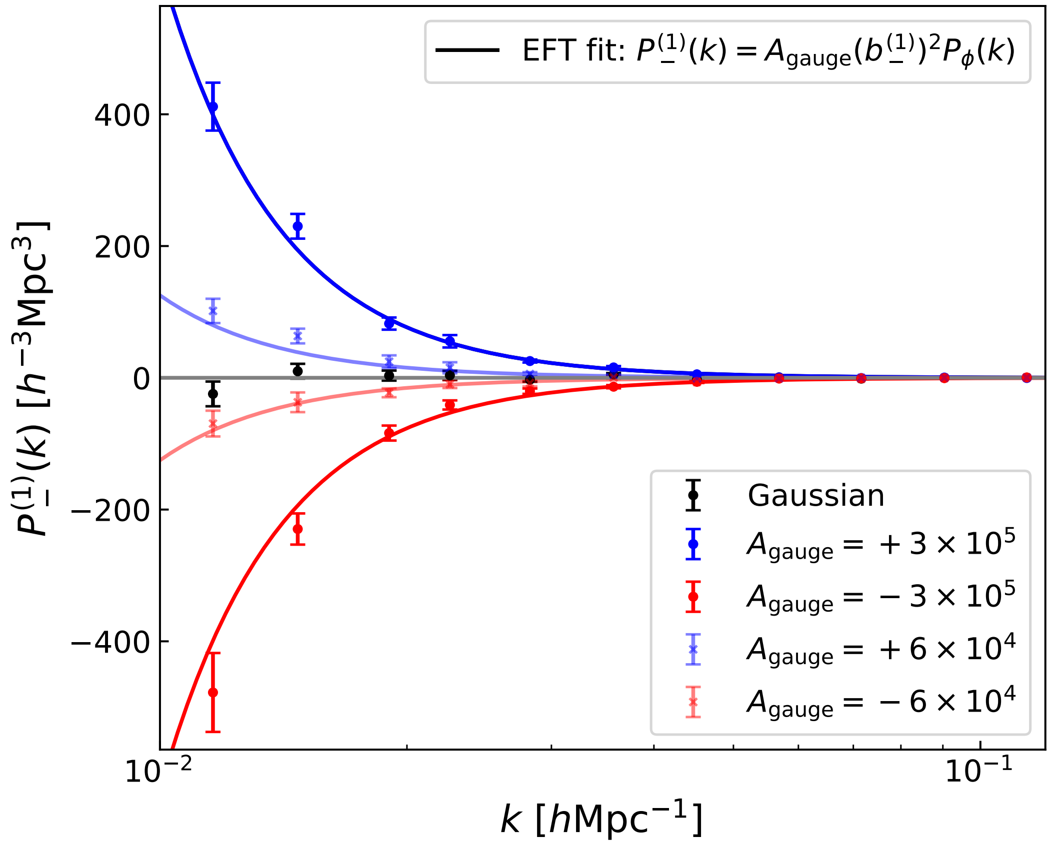

Validation against halo shape statistics in N-body simulations with primordial trispectrum

Kurita, Jamieson, Komatsu, FS (2025)

Galaxy shapes as probes of parity

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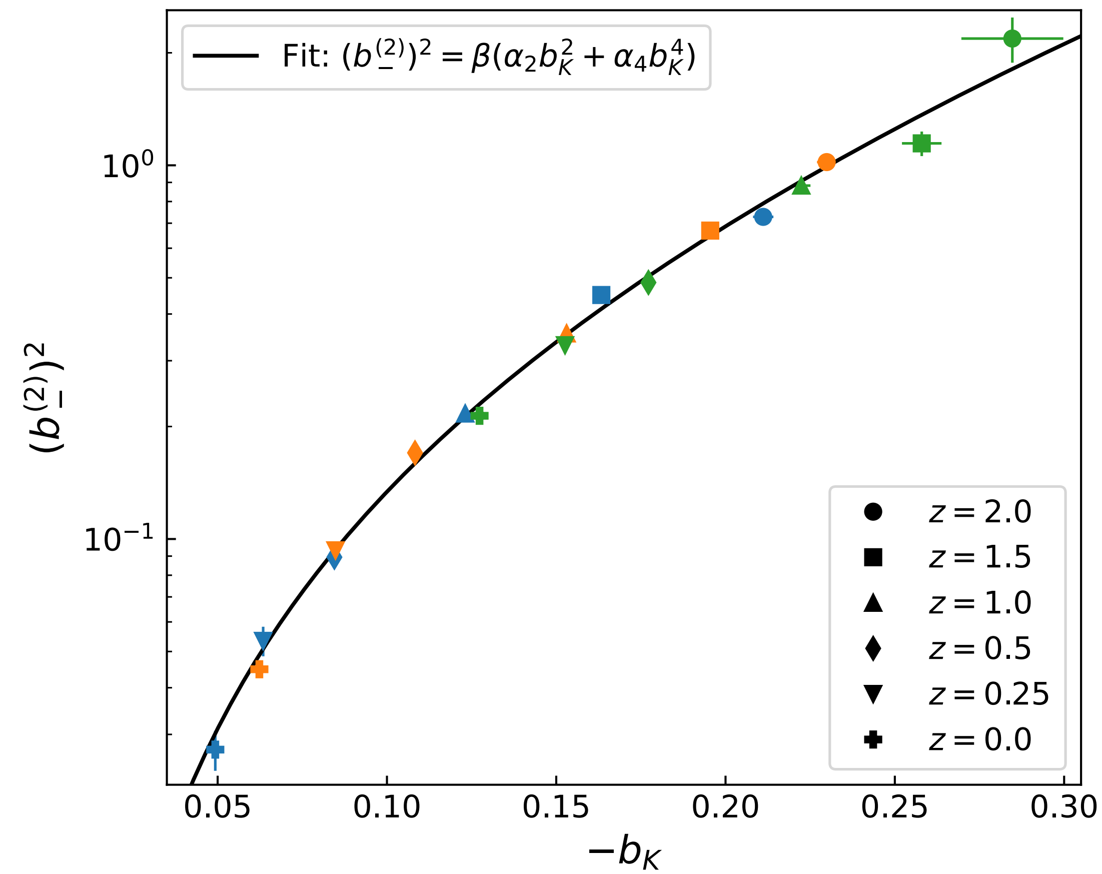
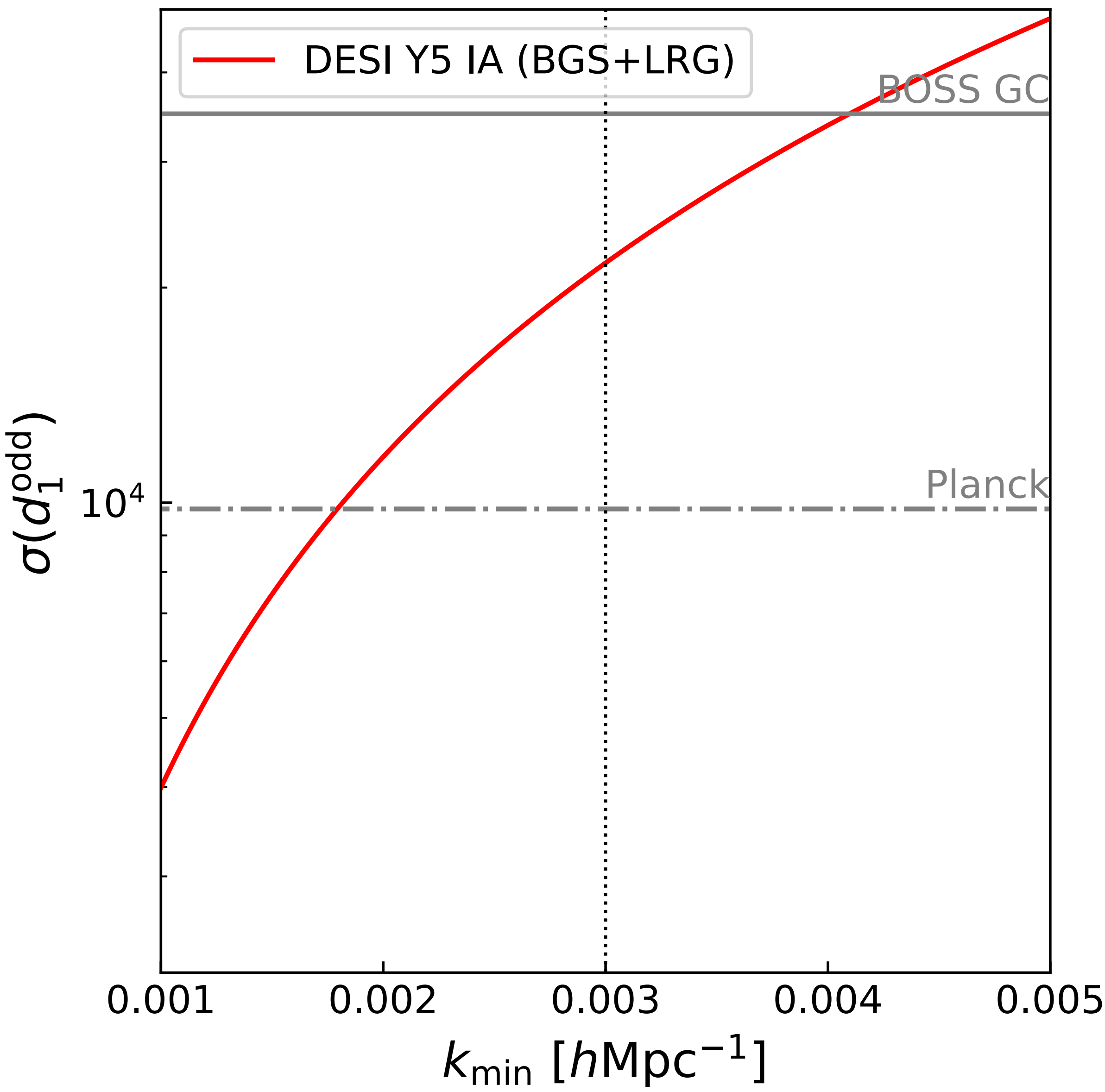


Measured shape bias (helicity-2) vs linear shape bias

Kurita, Jamieson, Komatsu, FS (2025)

Galaxy shapes as probes of parity

- U(1) gauge field scenario: on large scales, galaxy shapes directly trace the "fossil" helical U(1) field
- Forecast for DESI 3D shape statistic contraints on this scenario



Kurita, Jamieson, Komatsu, FS (2025)

3. Primordial black holes: an UV-complete dark matter scenario

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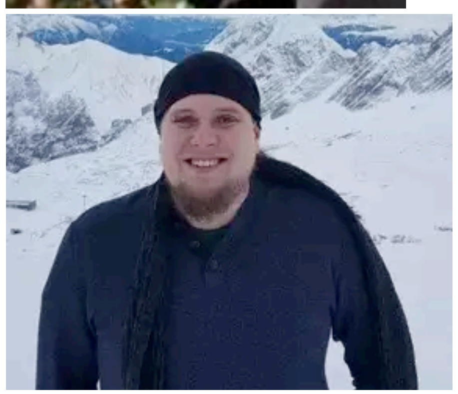


- Idea: simulate the full nonlinear evolution of an overdense region in a universe with PBH dark matter
- Key tool: BIFROST code for hierarchical N-body integration including multi-body dynamics and relativistic corrections
  - Black hole mergers included using recipe calibrated on full GR simulations
- In other words: fully calculate the UV theory of structure formation with PBH dark matter
  - Except for baryons…

Bifrost

[20] A. Rantala, T. Naab, F.P. Rizzuto, M. Mannerkoski, C. Partmann and K. Lautenschütz, *BIFROST: simulating compact subsystems in star clusters using a hierarchical fourth-order forward symplectic integrator code*, Monthly Notices of the Royal Astronomical Society **522** (2023) 5180 [2210.02472].

- Direct-summation N-body code written by Antti Rantala
- 4th-order symplectic integrator
- regularization for close encounters and hard bound systems (LogH)
- BH spin is followed
- including post-Newtonian corrections (in regularized regime) up to order 3.5 (v^7), including GW radiation reaction
- Out-state of binary BH mergers described by fitting formulae derived from numerical-relativity simulations

[20] A. Rantala, T. Naab, F.P. Rizzuto, M. Mannerkoski, C. Partmann and K. Lautenschütz, *BIFROST: simulating compact subsystems in star clusters using a hierarchical fourth-order forward symplectic integrator code*, Monthly Notices of the Royal Astronomical Society **522** (2023) 5180 [2210.02472].

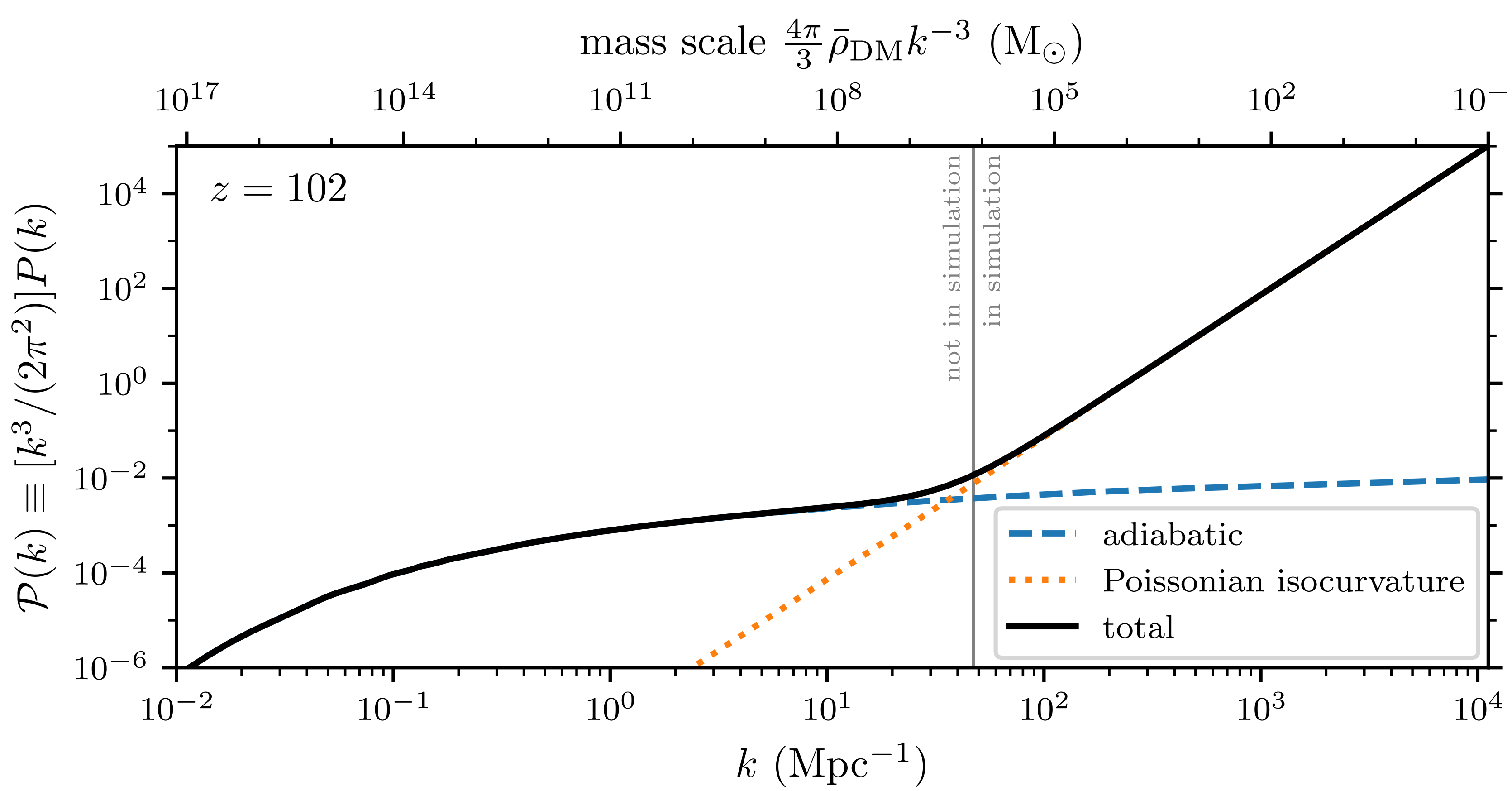
Bifrost

- But:
  - Bifrost uses physical coordinates
  - Bifrost assumes vacuum boundary conditions
- Hence, choose isolated overdense region for our simulation

arXiv:2410.01876

Initial conditions

- Consider slightly overdense spherical region within volume of ~100 kpc (comoving)
- This region contains ~10^6 PBH drawn from a lognormal mass function (<M> = 16 M_{sun})
- Initialize at *a=3*10*$^{-12}$, *actual formation time*
- Evolution through radiation domination with high-precision-tuned Gadget4 (no softening)



arXiv:2410.01876

Three simulations

- **Particle dark matter simulation**
- Standard CDM ICs from same adiabatic realization

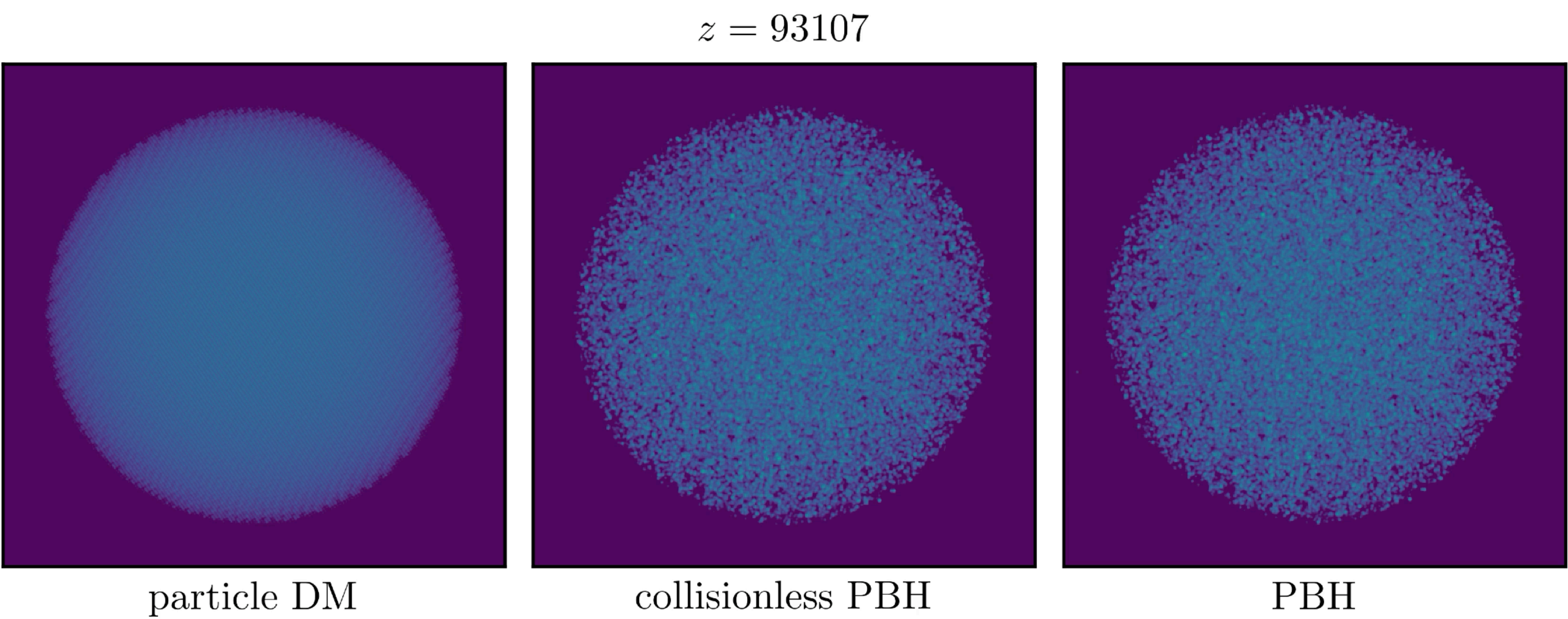
- **Collisionless PBH simulation**
- Gadget4 (softened) from PBH ICs

- **Collisional PBH simulation**
- Bifrost code

arXiv:2410.01876

Three simulations

- **Particle dark matter simulation**
- **Collisionless PBH simulation**
- **Collisional PBH simulation**



arXiv:2410.01876

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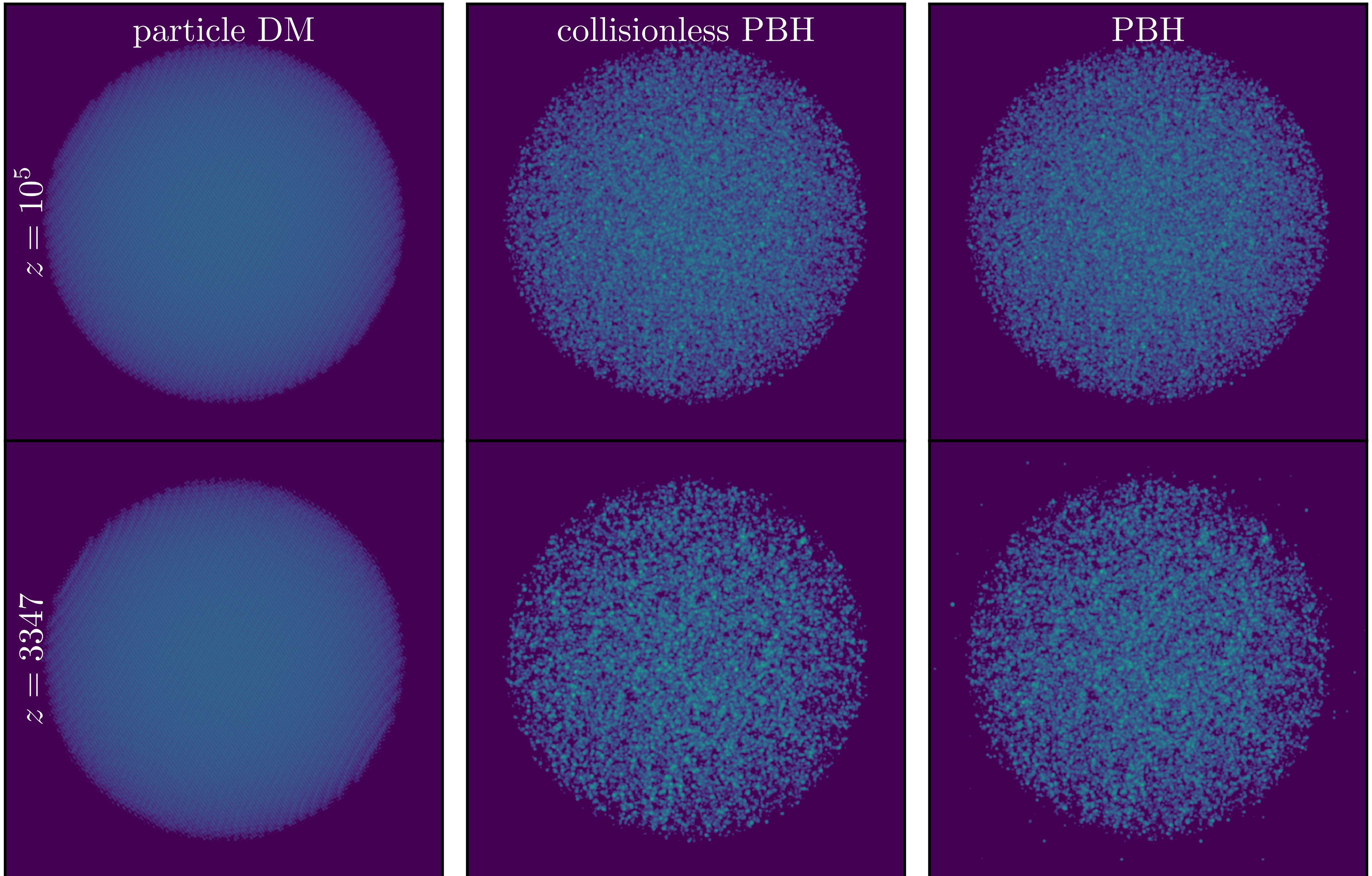

arXiv:2410.01876

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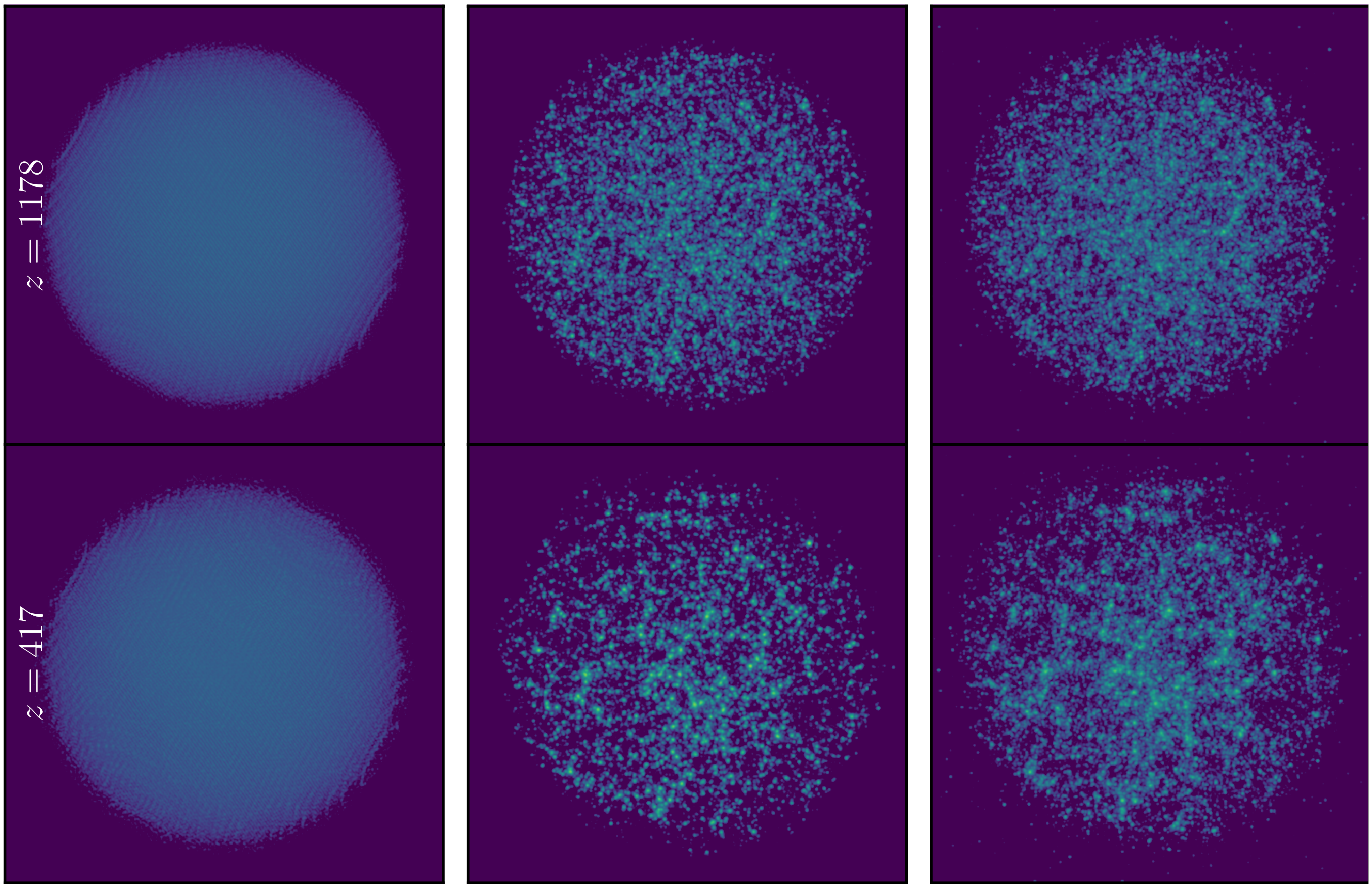
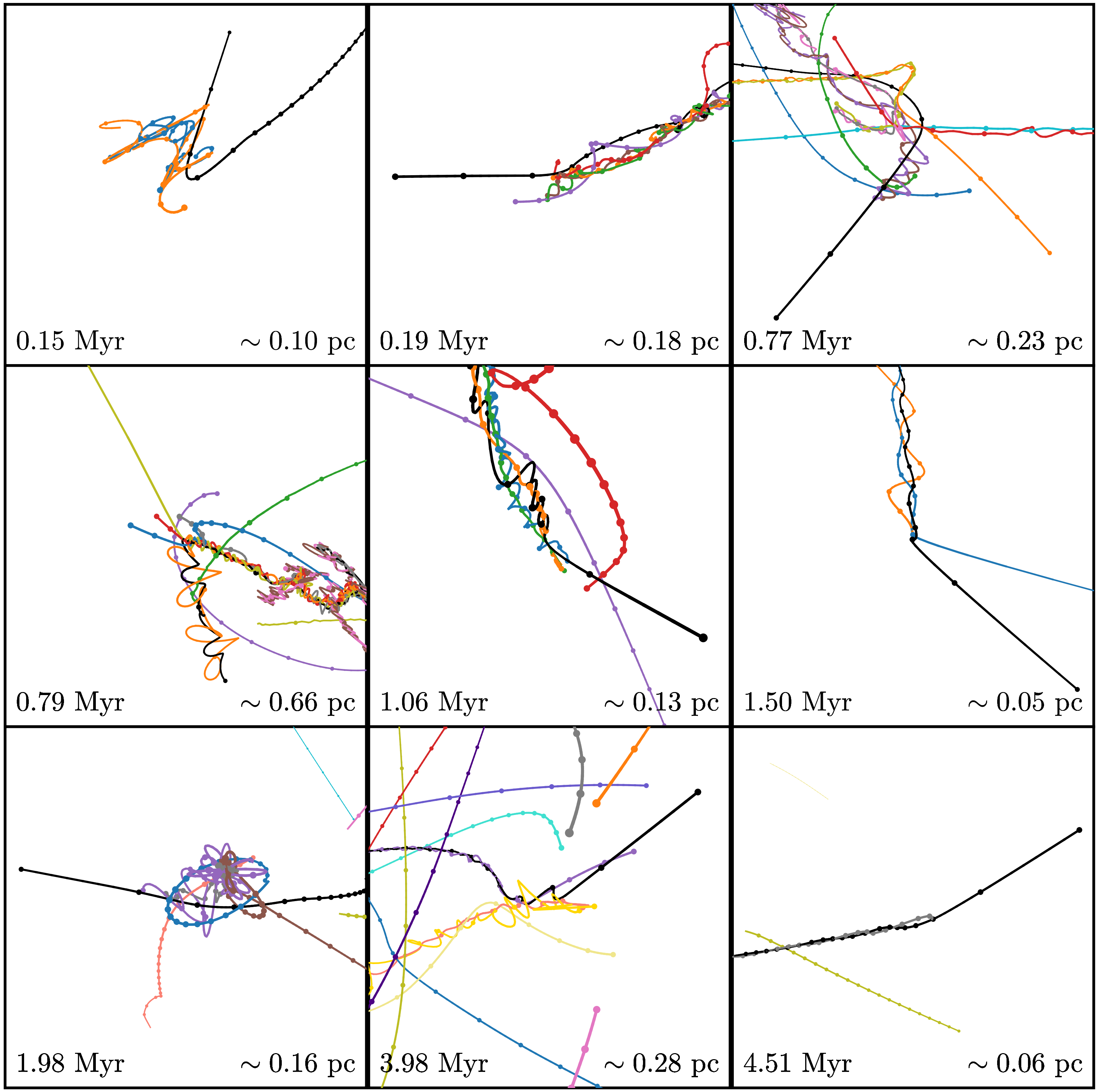


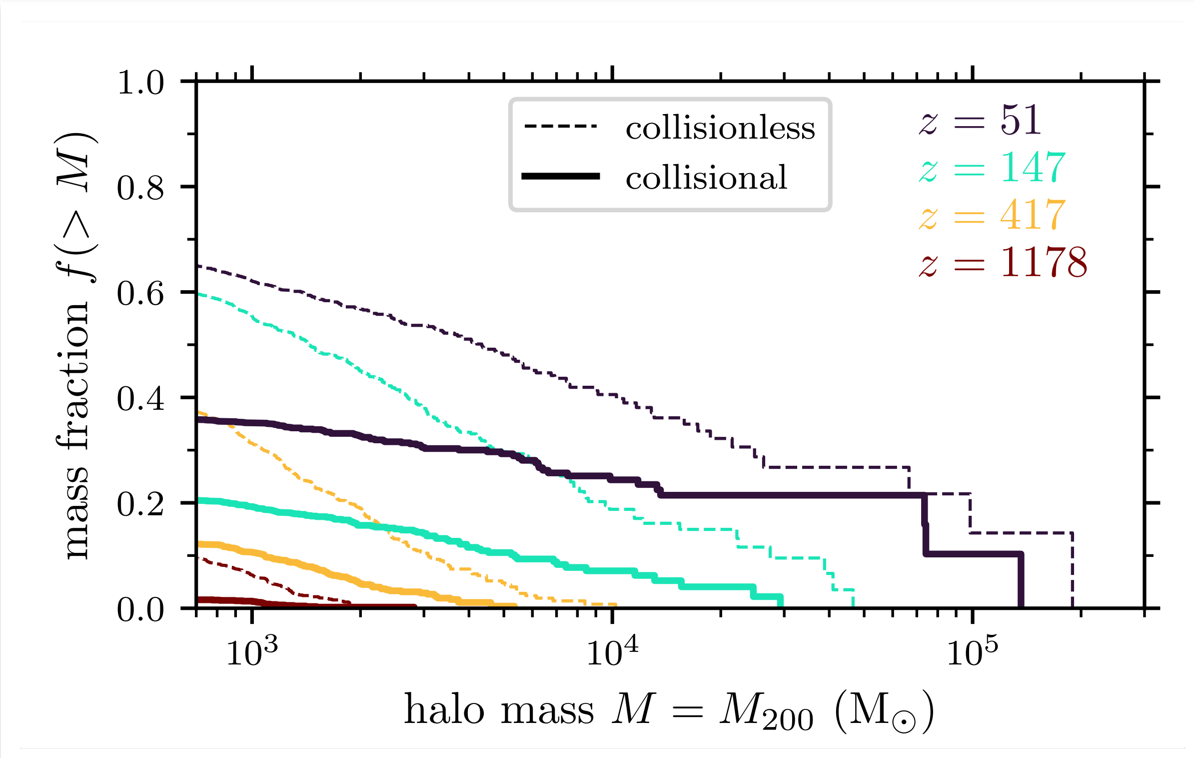
A plethora of multi-body interactions

A plethora of multi-body interactions

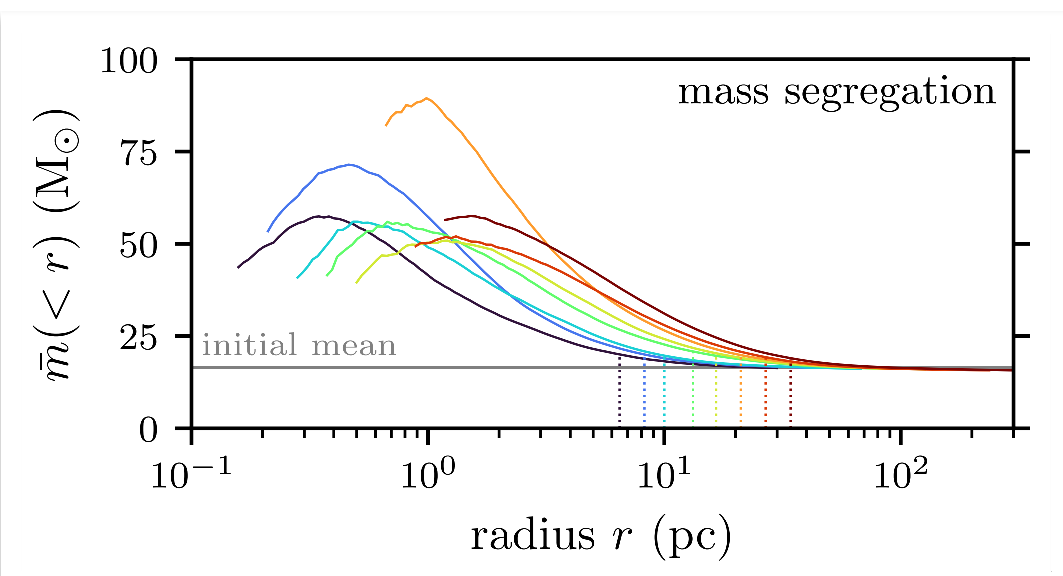


Dynamical effects on halo formation

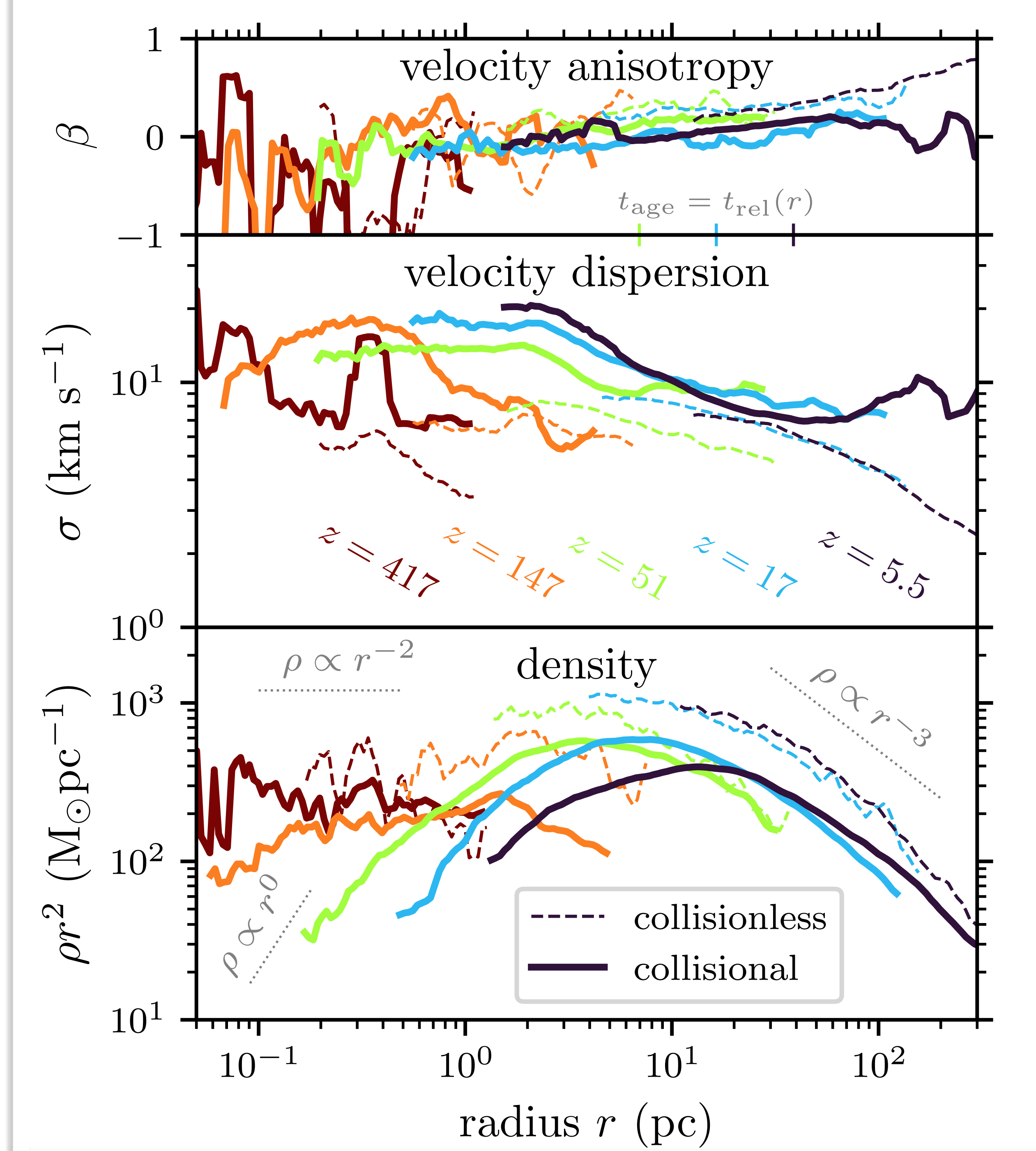
Suppression of substructure



Mass segregation among PBH



Formation of core, erasure of velocity anisotropy

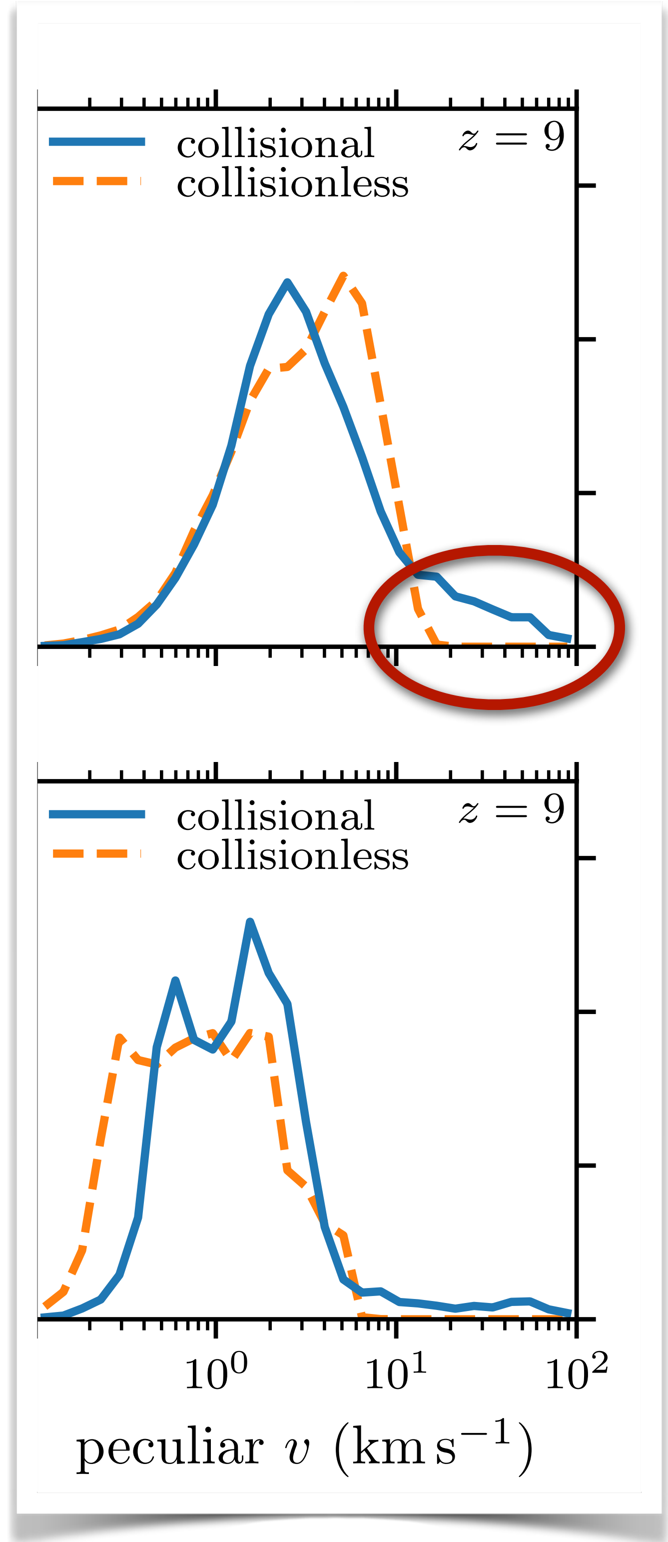


Dynamical effects on halo formation

High-velocity tail from 3-body interactions & kicks

-> W/HDM component is generated dynamically!

Quite interesting, as it violates the standard EFTof LSS treatments.



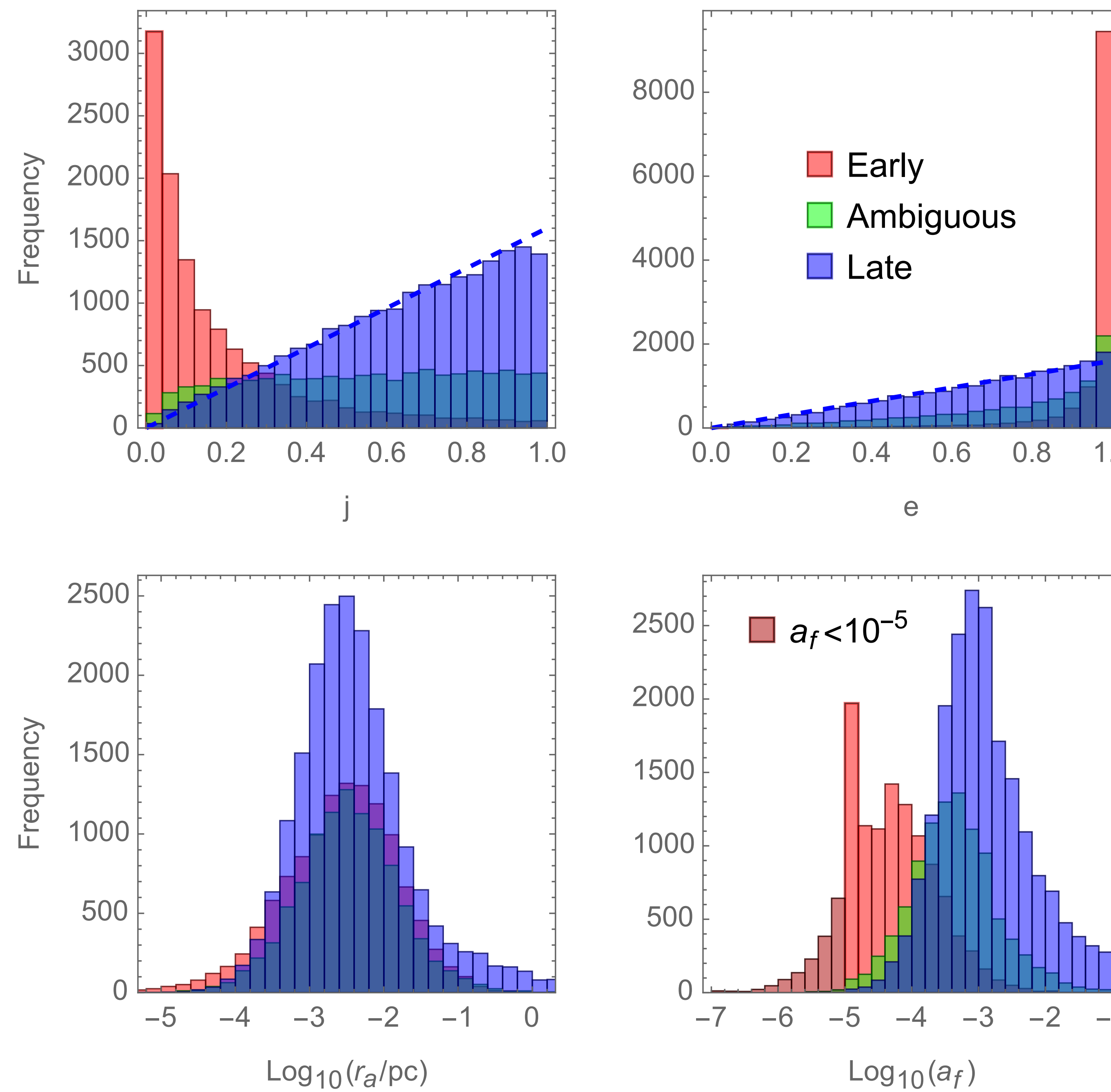
Lower panel:

Attempt to remove binary velocities

Properties of binary PBH population

- Distinguish between binaries formed during radiation domination (early) and during nonlinear structure formation (late)

Angular momentum	Eccentricity
Semi-major axis	Formation time



Total in simulation:
~ 4000 early binaries
~ 5000 late binaries

Generation of gravitational background

- Current LIGO/Virgo/Kagra limit: $\Omega_{\rm GW} \lesssim 10^{-8}$ per e-fold
- Expect to provide very tight constraint on PBH mass fraction, but keep in mind that our simulations assume 100% PBH fraction

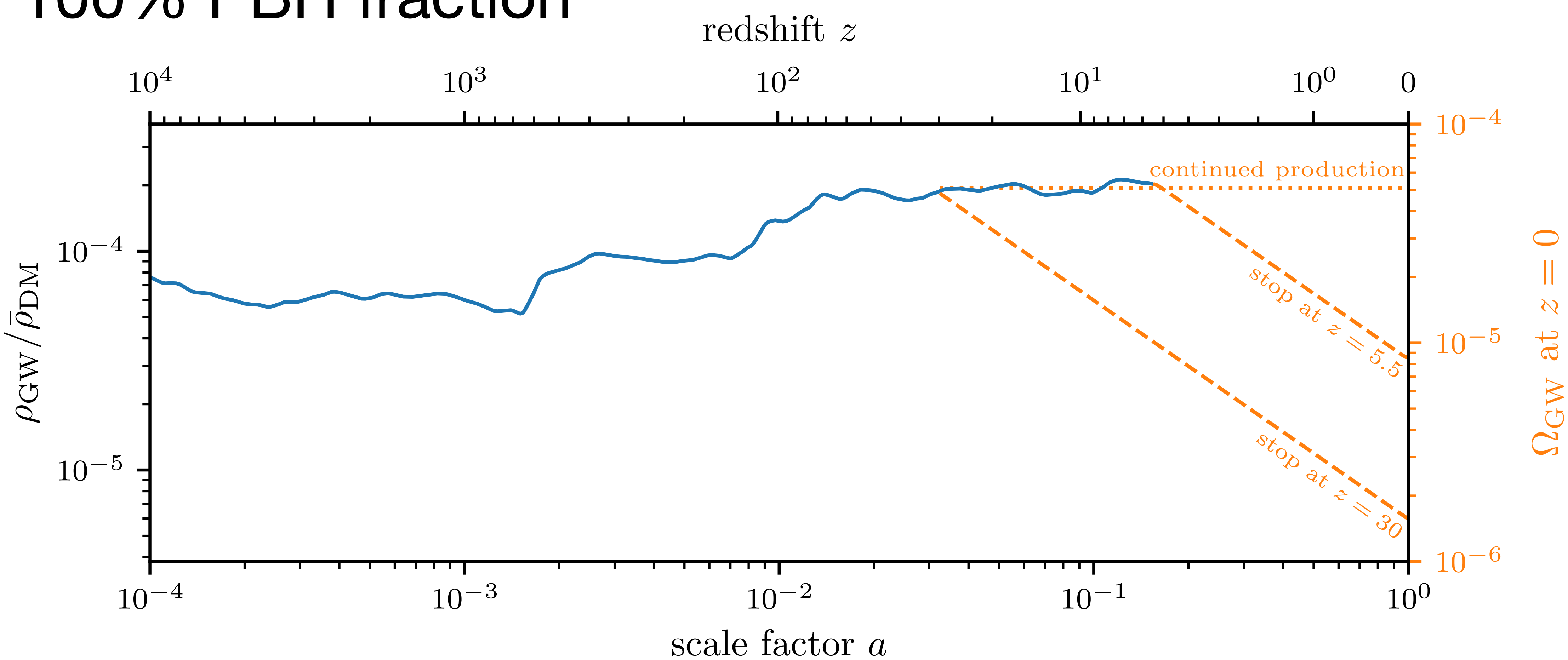


Figure 21: Energy density $\rho_{\rm GW}$ in gravitational radiation compared to the mass density $\bar{\rho}_{\rm DM}$ of the PBHs. At late times, the ratio is steady at about $\rho_{\rm GW}/\bar{\rho}_{\rm DM} \simeq 2 \times 10^{-4}$.

Generation of gravitational background

- Current LIGO/Virgo/Kagra limit: $\Omega_{\rm GW} \lesssim 10^{-8}$ per e-fold
- Expect to provide very tight constraint on PBH mass fraction, but keep in mind that our simulations assume 100% PBH fraction
- Roughly constant value of 10^{-4} suggests universality: scale-free problem
- Not quite true however: formation time dictates length of evolutionary period during radiation domination, which influences properties of primordial binaries

Summary (New Physics)

- LSS offers still quite a bit of discovery space - many corners we haven't looked at yet
- **Dark energy:** w(a) is not necessarily slowly-varying, and not monotonic - worth looking beyond w_0-w_a
  - Both as theorists/phenomenologists and observers
- **Inflation:** Galaxy shapes are parity- and spin-sensitive probes of primordial perturbations
- **Dark matter:** PBH is a phenomenologically rich scenario - motivates investigations of multi-component dark matter
  - GW as clean and powerful probe - but over limited frequency range
  - Guaranteed relative perturbations: new modes that need to be included in LSS modeling Verdiani+; Çelik & FS
  - Additionally, poorly constrained (primordial) isocurvature modes