



UNIVERSITY OF
CAMBRIDGE



Parity Violation on Cosmological Scales

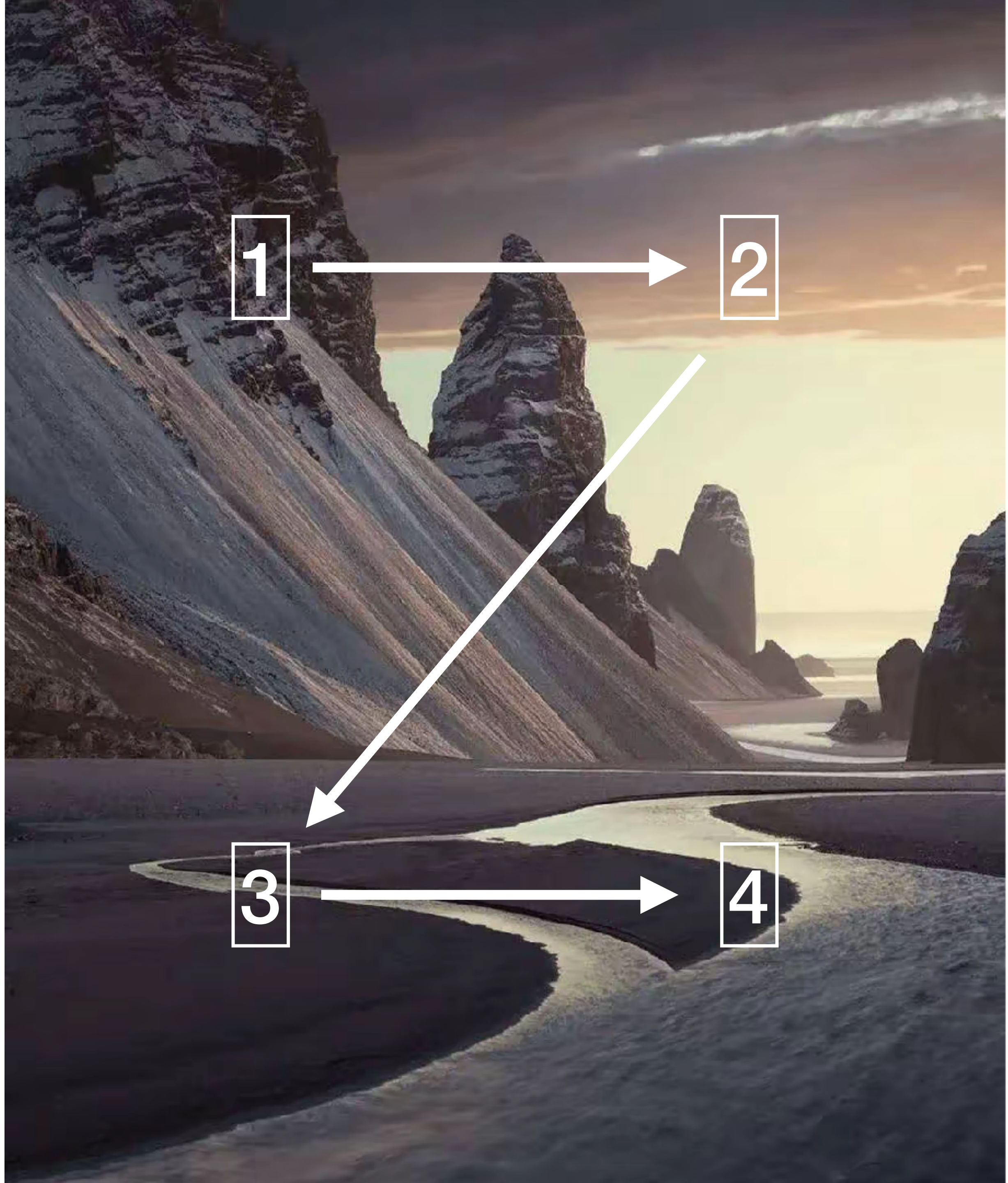
New Physics from Galaxy Clustering, GGI

9 September, 2025

Jiamin Hou







Parity Transformation

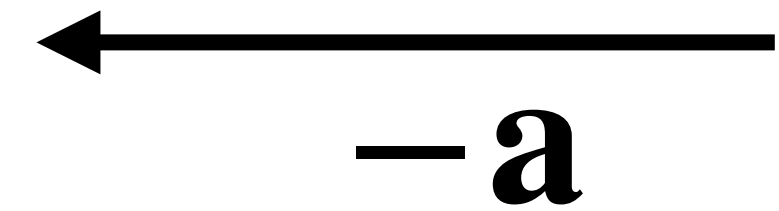
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{\hat{\mathbb{P}}} \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

How to extract parity information
from scalar field?

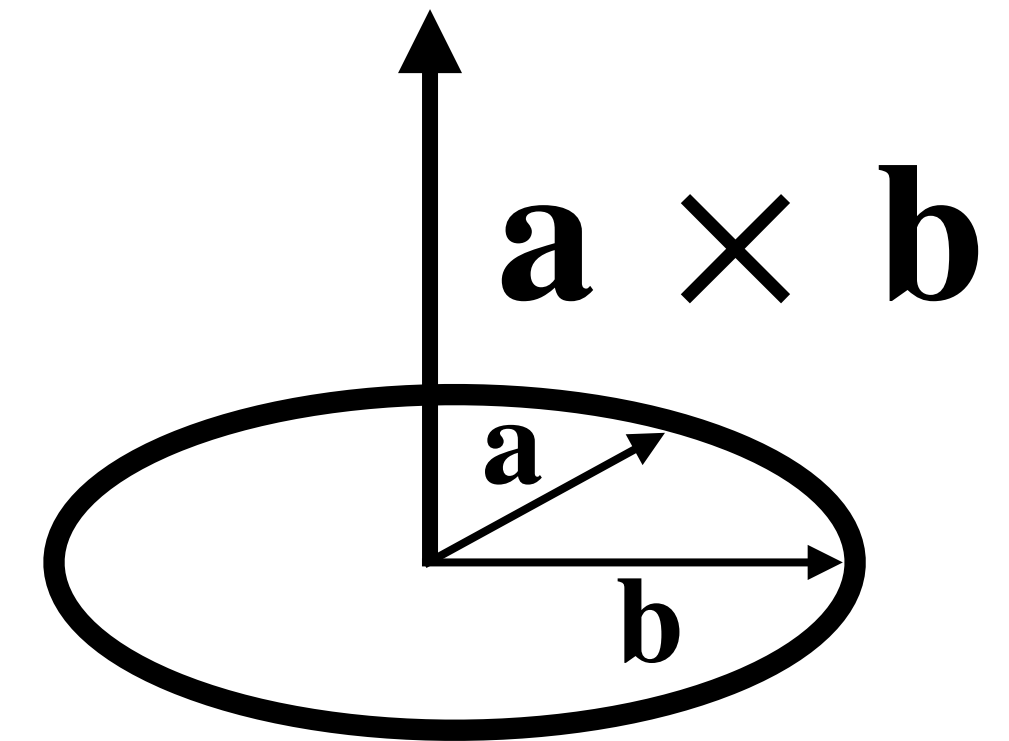
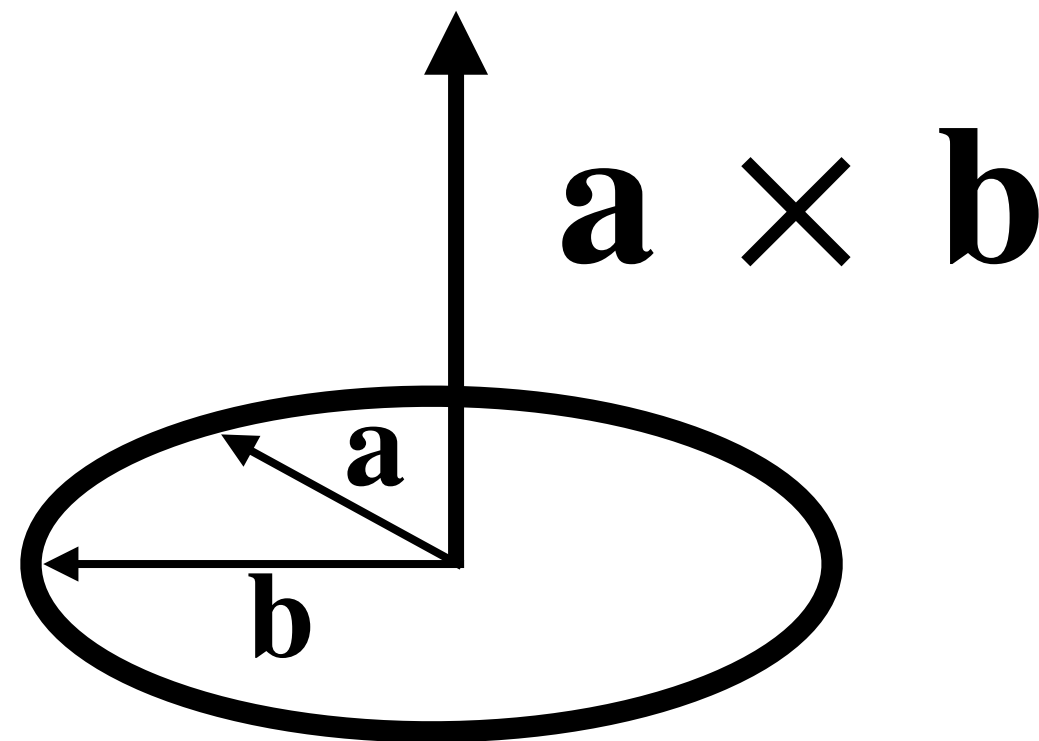
$$\Phi(\mathbf{x}) \longrightarrow \Phi(-\mathbf{x})$$

Parity Transformation for Vectors

Polar Vector



Axial Vector
(Pseudovector)



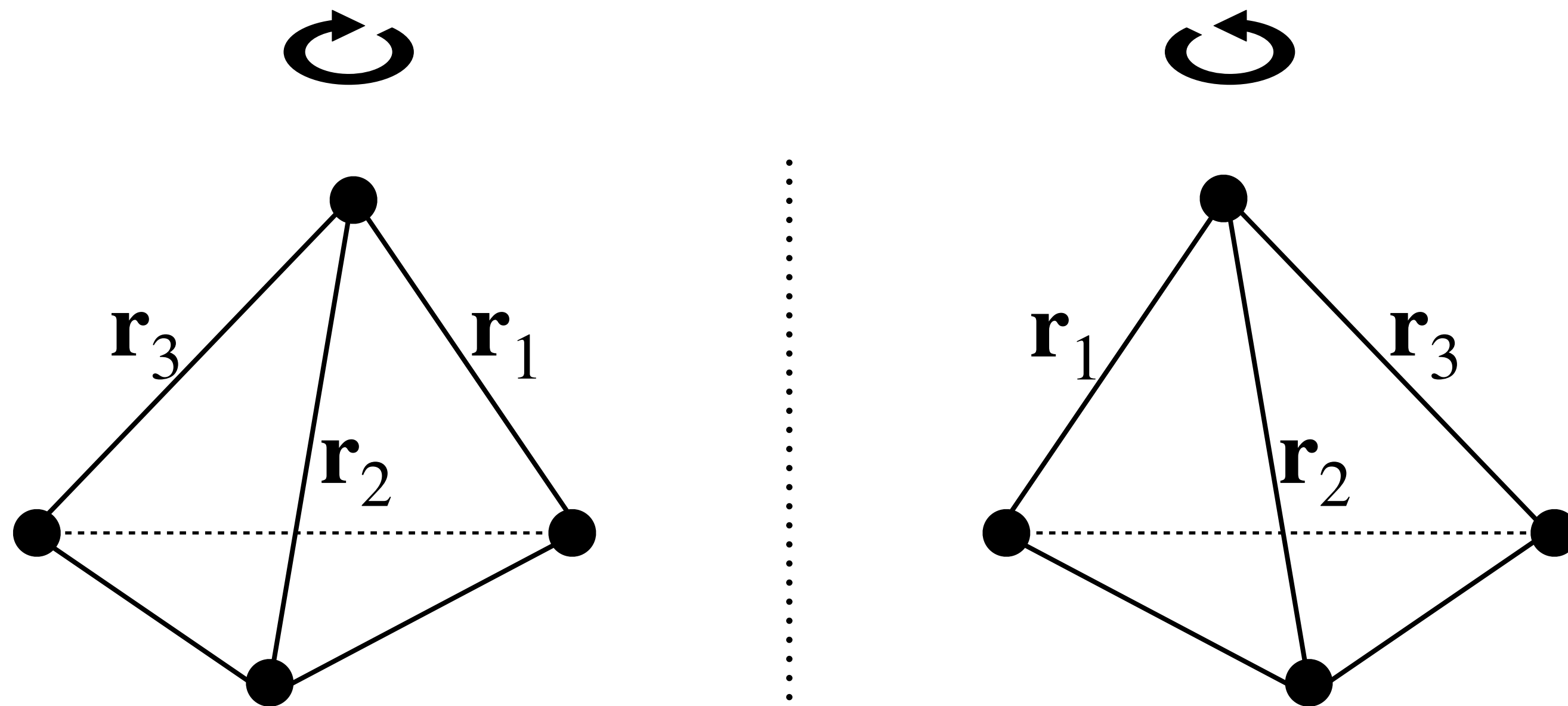
An “imagined” mirror

Polar Vector $\times$ Axial Vector
(Pseudovector) = Pseudoscalar

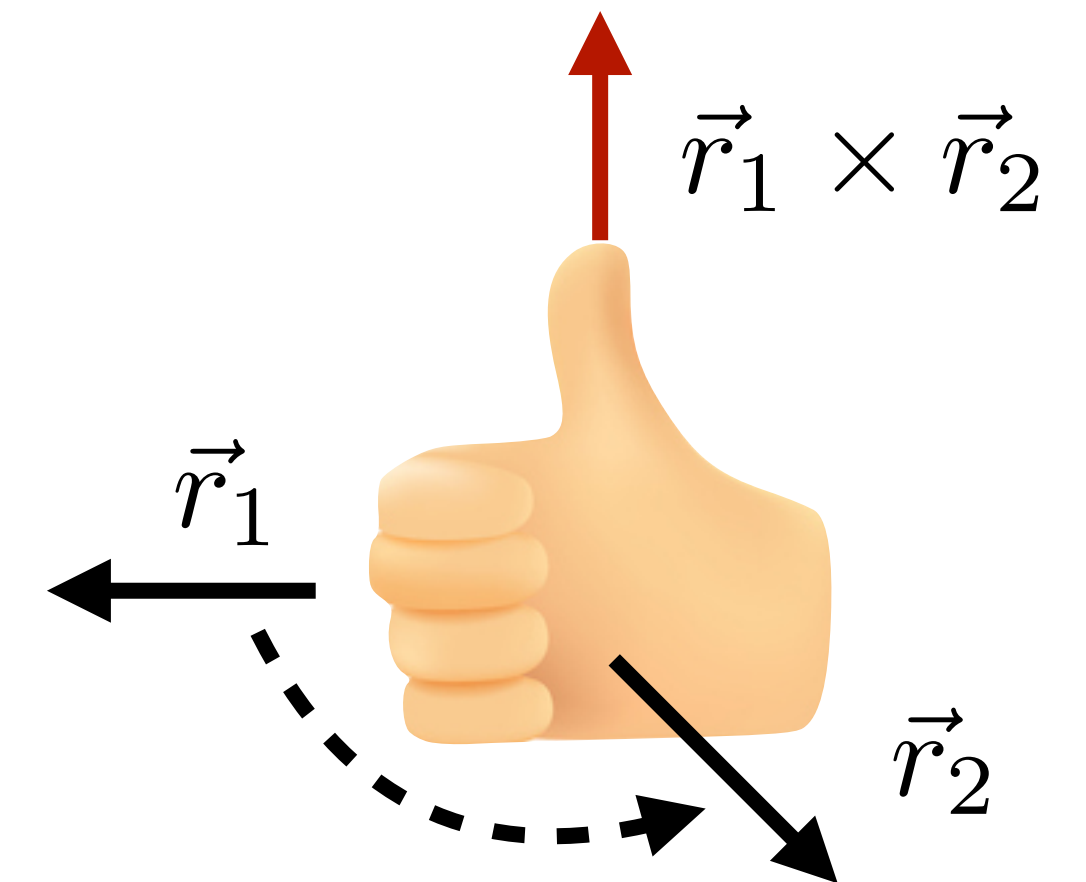
$$\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3) \longrightarrow -\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3)$$

Test of Parity Symmetry with Tetrahedra

$$\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3)$$



An "imagined" mirror

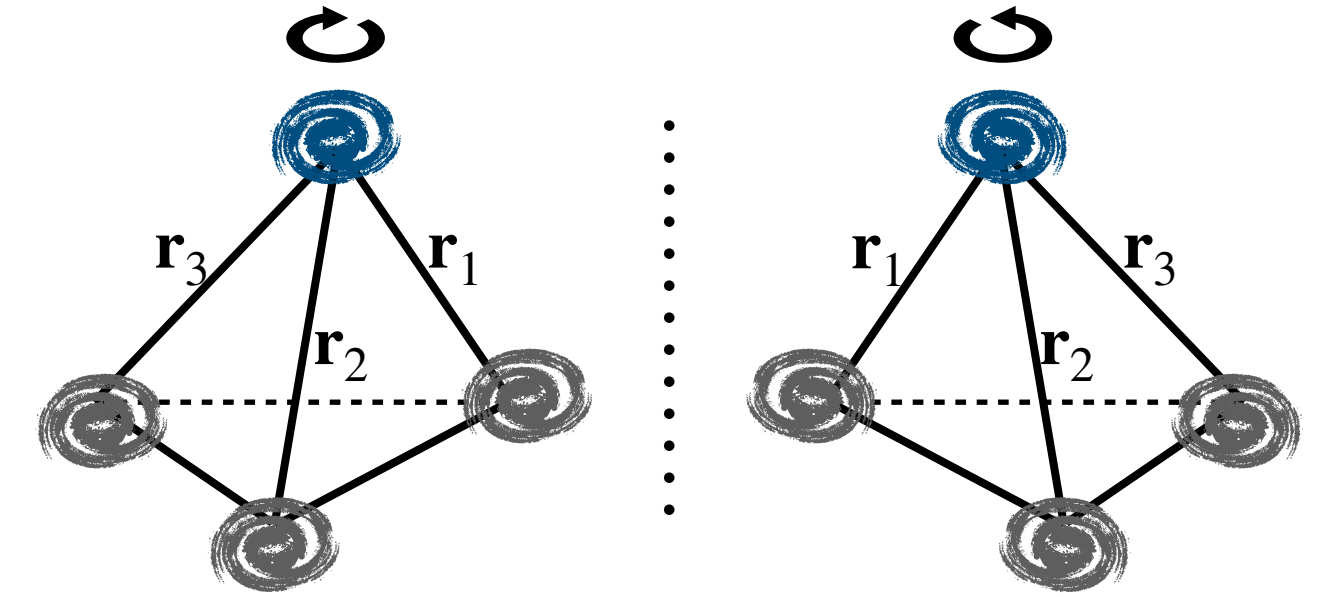


Need at least
4 points
to probe parity

Parity-Sensitive Cosmological Probes

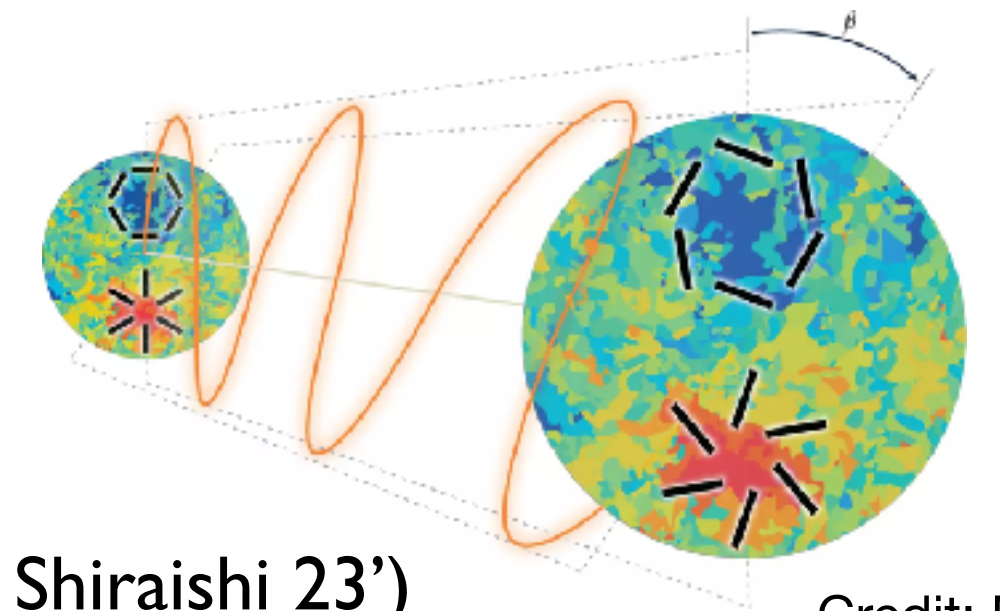
Scalar

- Galaxy 3D distribution (Hou+ 23', Philcox 22')
- CMB Temperature (Shiraishi 16', Philcox 23'), CMB lensing (Grecco++JH++, 25')



Vector/ Tensor

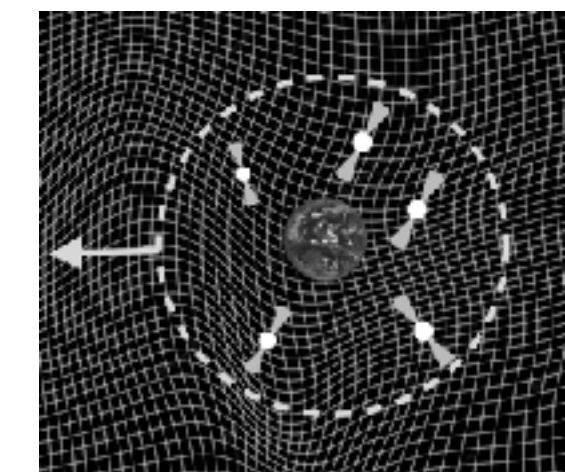
- Galaxy spin (Yu+ 20', Shim+ 24'), Intrinsic alignment (Kurita+, in prep), Shear & Shape (Philcox+, 23')
- CMB birefringence (Minami & Komatsu 20'; Eskilt & Komatsu 20'; Diego-Palazuelos+22')
- CMB polarisation (Lue+ 99', Saito+ 07', Kamionkowski & Souradeep 10', Masui + 17', Philcox & Shiraishi 23')



Credit: Minami

Tensor

- GW birefringence (Yagi+ 12', Berti+ 18', Qiao+ 19', Zhao+ 19', Alexander & Yunes 18', Yamada & Tanaka 20', Jenks+ 23', Ng+ 23'), Stochastic fore/background (Liang+ 24' Keisuke+ 24', Sato-Polito & Kamionkowski 24')



Credit: astrobites

Parity-violating Mechanisms

- Early-time: inflation
- Late-time: dark energy / dark matter

Parity-violating Mechanism and No-go theorem

Liu ++ 20'; Cabass ++ 22';

- **Bunch Davis vacuum** (Cabass, Ivanov, Philcox 23')
- **Scale invariance** (Creque-Sarbinowski ++ 23'; Niu ++ 23'; Reinhard++ JH ++ 24')
- **Unitarity & Locality** (Stefanyszyn, Tong, Zhu 22')
- **Tree-level** (Lee, McCulloch, Pajer 23')

4PCF can be decomposed into a
parity-even and parity-odd component

$$\zeta = \zeta_+ + i\zeta_-$$

Even: gravity-induced non-Gaussianity

Odd: beyond standard model physics

$$\zeta_- = \mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3) \tau_-(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

Extract Parity Violation from Scalar field and Choice of Basis

Rotational invariant — statistical isotropy

Sensitive to parity transformation

Allow to construct fast algorithms

Spherical Harmonics as a Convenient Choice

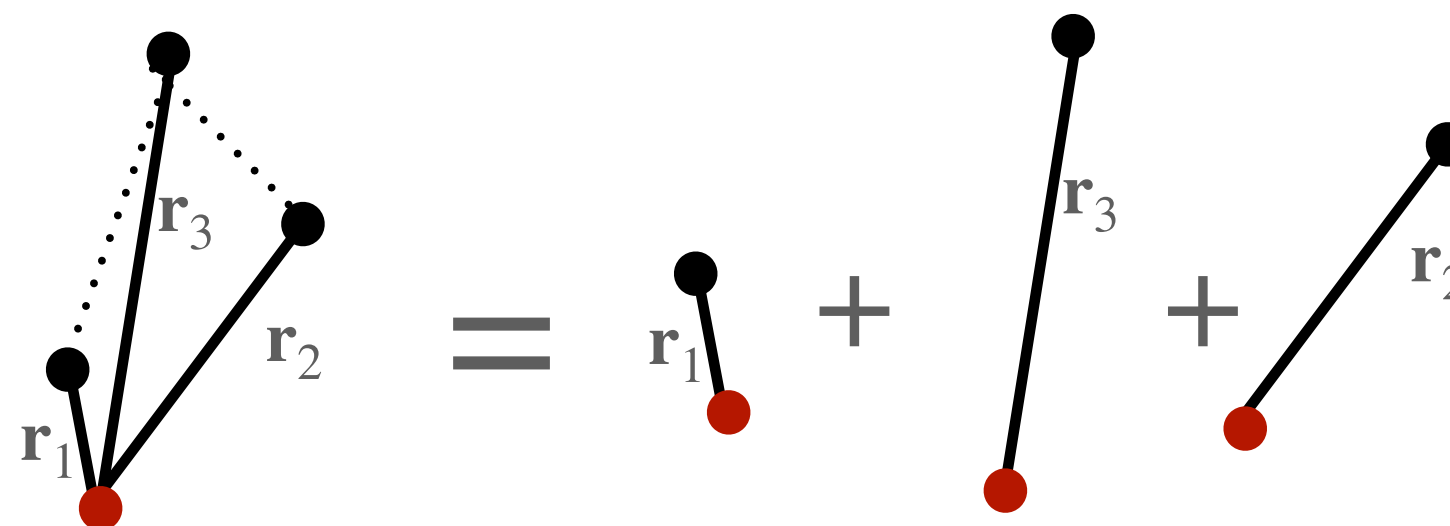
Rotational invariant

$$V^{(\ell_1)} \otimes V^{(\ell_2)} = \bigoplus_{L=|\ell_1-\ell_2|}^{\ell_1+\ell_2} V^{(L)}$$

Sensitive to parity transformation

$$\mathbb{P}[Y_{\ell m}(\hat{n})] \rightarrow (-1)^\ell Y_{\ell m}(\hat{n})$$

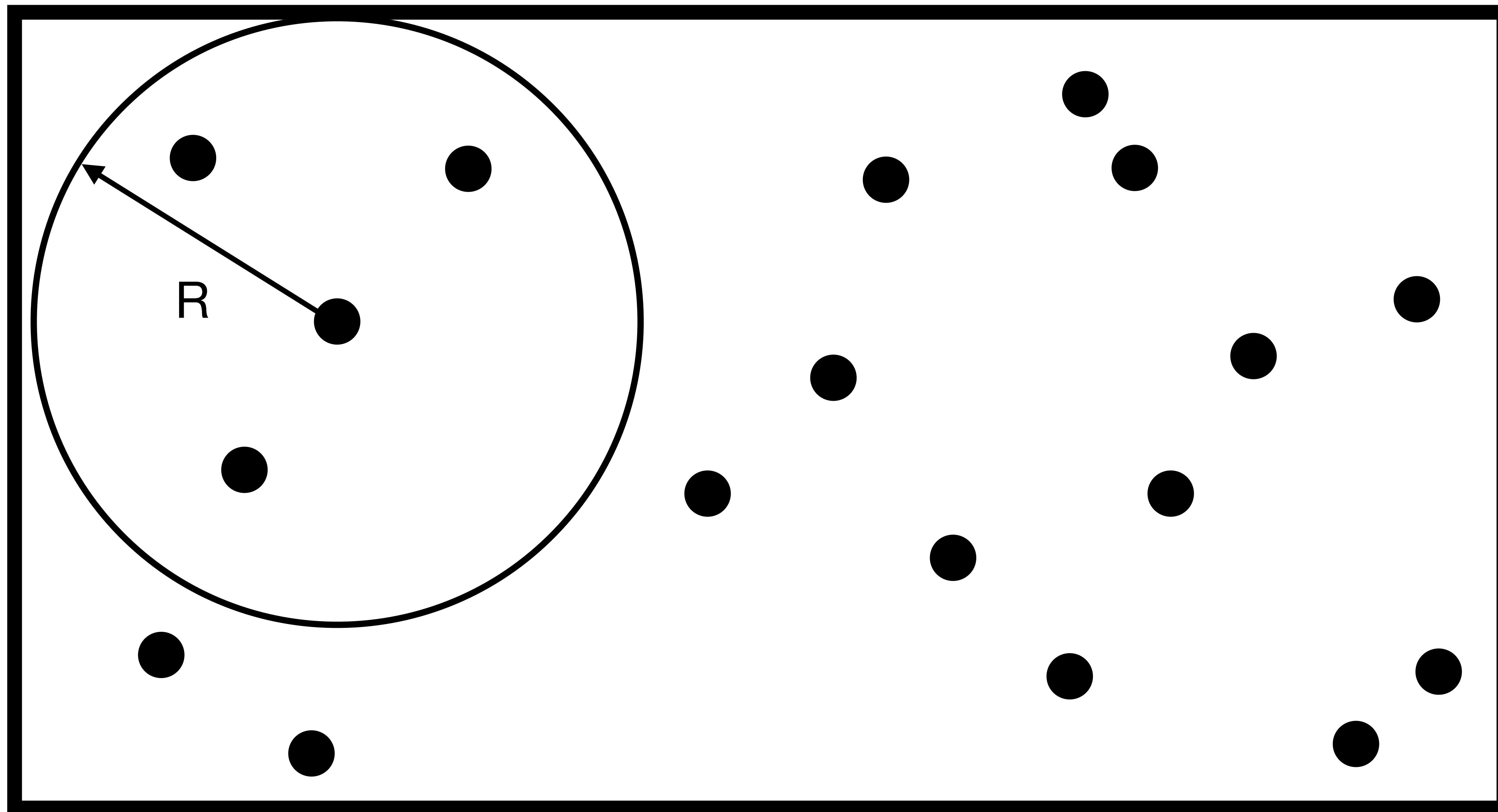
Speed and
Factorizability

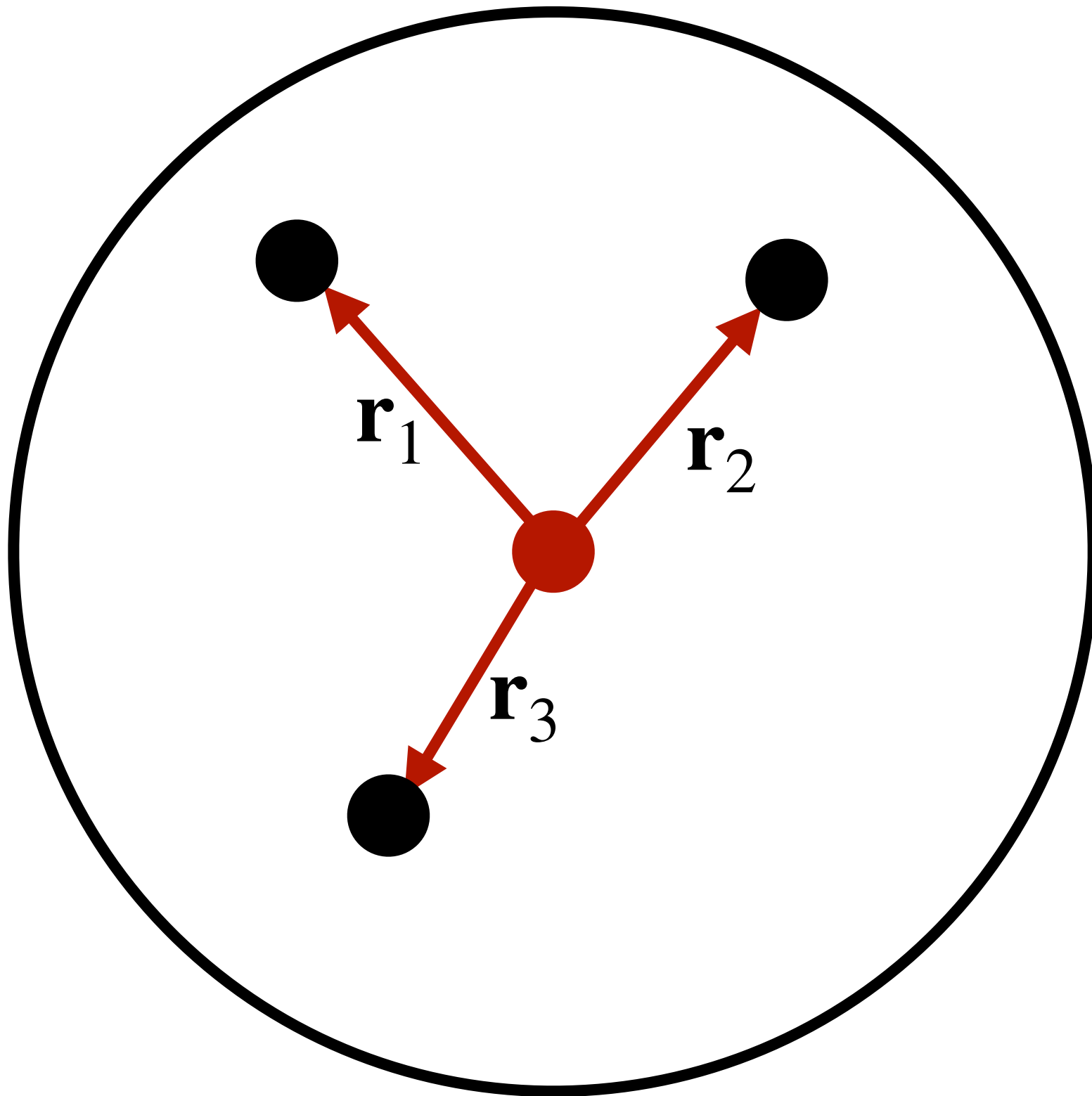


Harmonic Basis for 4-Point Function

$$\begin{aligned} & \mathcal{P}_{\ell_1 \ell_2 \ell_3}(\hat{r}_1, \hat{r}_2, \hat{r}_3) \\ & \equiv \sum_{m_1, m_2, m_3} (-1)^{\ell_1 + \ell_2 + \ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y_{\ell_1 m_1}(\hat{r}_1) Y_{\ell_2 m_2}(\hat{r}_2) Y_{\ell_3 m_3}(\hat{r}_3) \end{aligned}$$

How are spherical harmonics related to galaxies?





Single density field

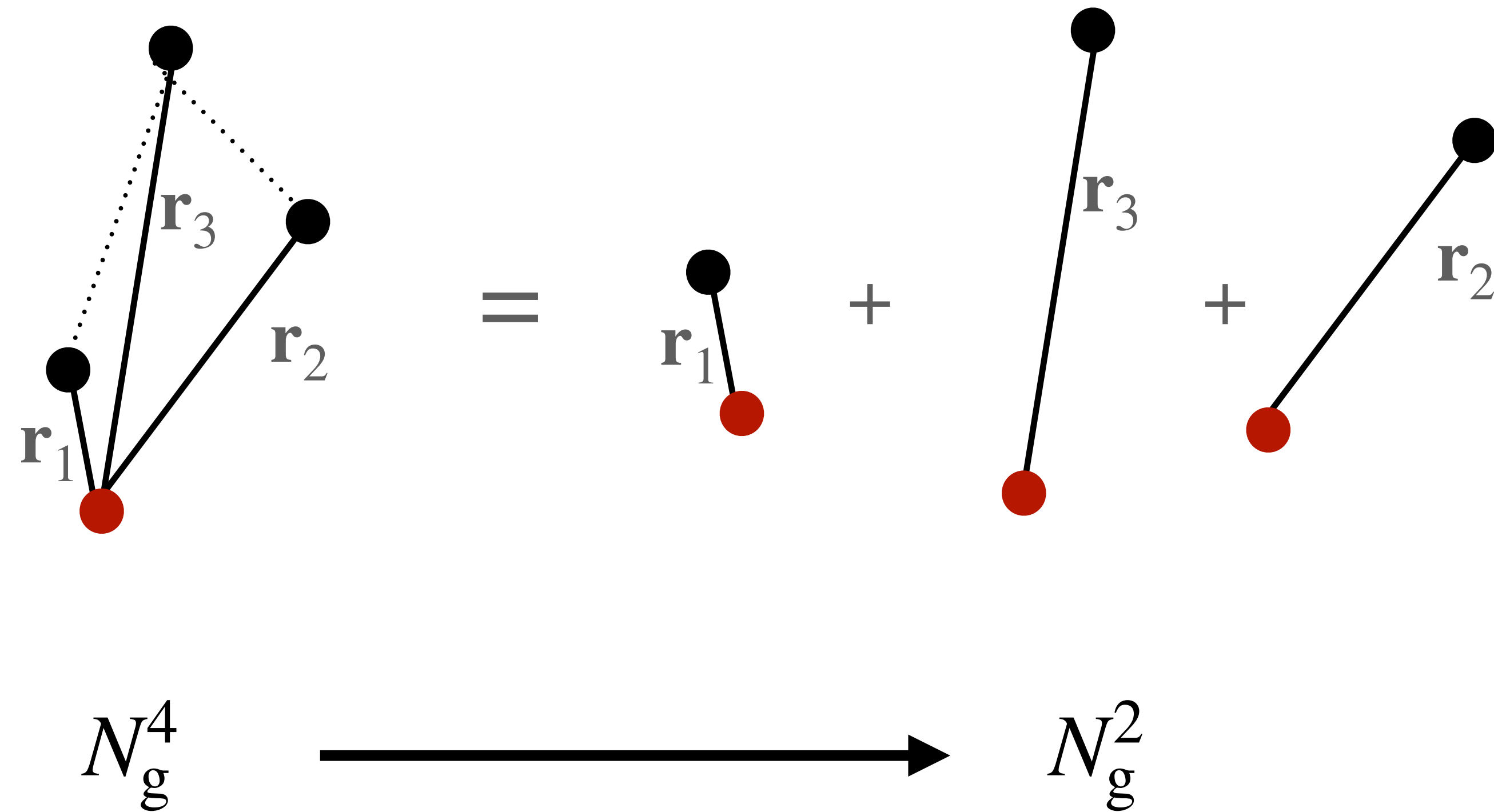
$$\delta(\mathbf{x} + \mathbf{r}_i) = \sum_{\ell_i m_i} a_{\ell_i m_i}(r_i | \mathbf{x}) Y_{\ell_i m_i}(\hat{r}_i)$$



N -density fields

$$\begin{aligned} & \langle \delta(\mathbf{x}) \prod_{i=1}^{N-1} \delta(\mathbf{x} + \mathbf{r}_i) \rangle \\ &= \sum_{\ell_1, \dots, \ell_{N-1}} \zeta_{\Lambda}(r_1, \dots, r_{N-1}) \mathcal{P}_{\Lambda}(\hat{r}_1, \dots, \hat{r}_{N-1}) \end{aligned}$$

Angular Basis is Factorizable



Reduce the scaling from
“quartet” to “pair”

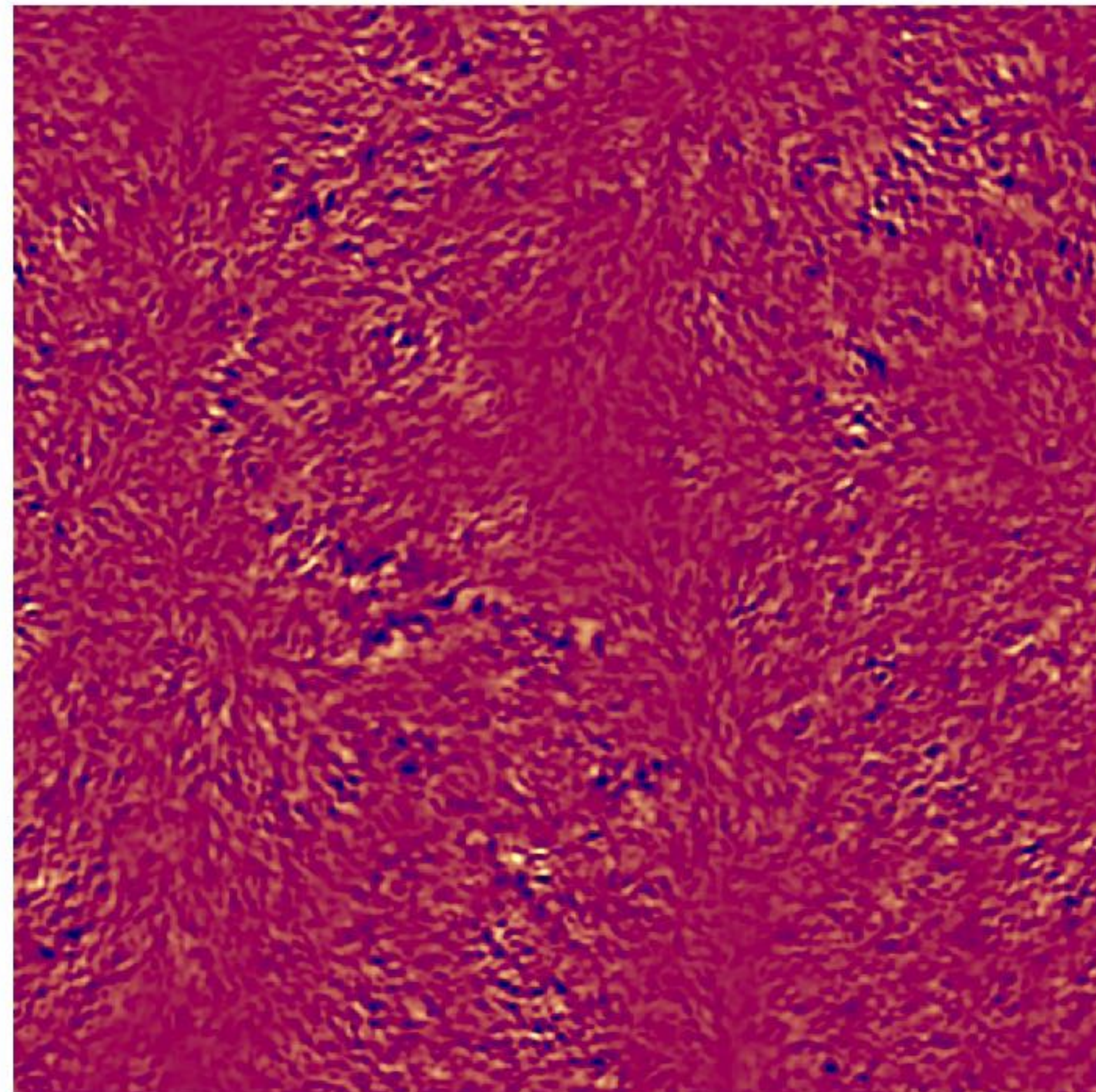
Measurement of 4-point function for $\mathcal{O}(10^6)$ Galaxies

$\sim 10^{24}$ galaxy quartets

Centuries of calculation $\rightarrow$ **One minute!***

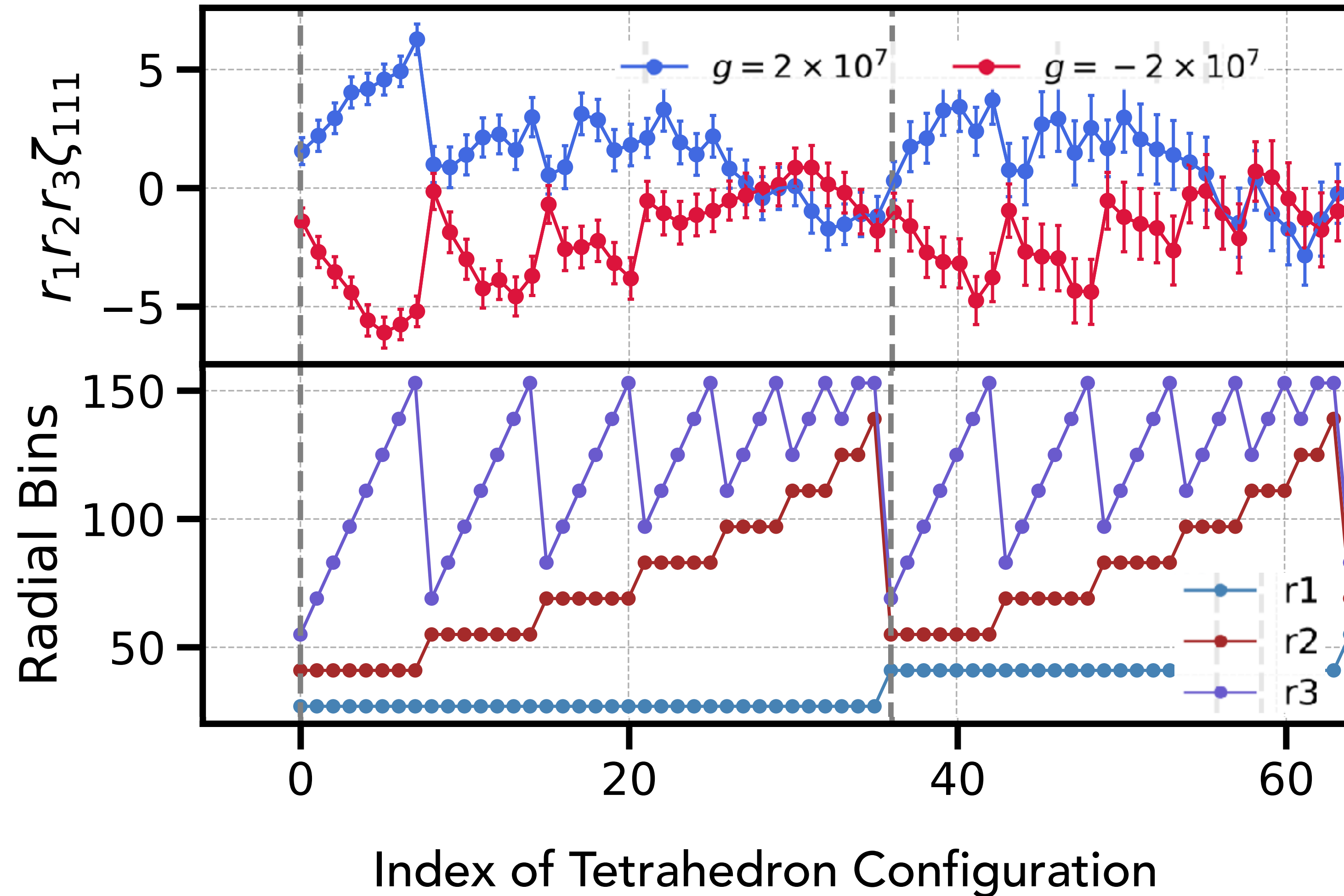
Parity Violation in Density Field

$$\Phi_{\text{NG}} = \nabla \Phi_G^{[\alpha]} \cdot \left[\nabla \Phi_G^{[\beta]} \times \nabla \Phi_G^{[\gamma]} \right]$$



JH, Slepian, Jamieson (2024)
Jamieson, Caravano, **JH** ++ (2024)
Also see Coulton ++ (2023)

Measured Odd 4-Point Function from Simulation

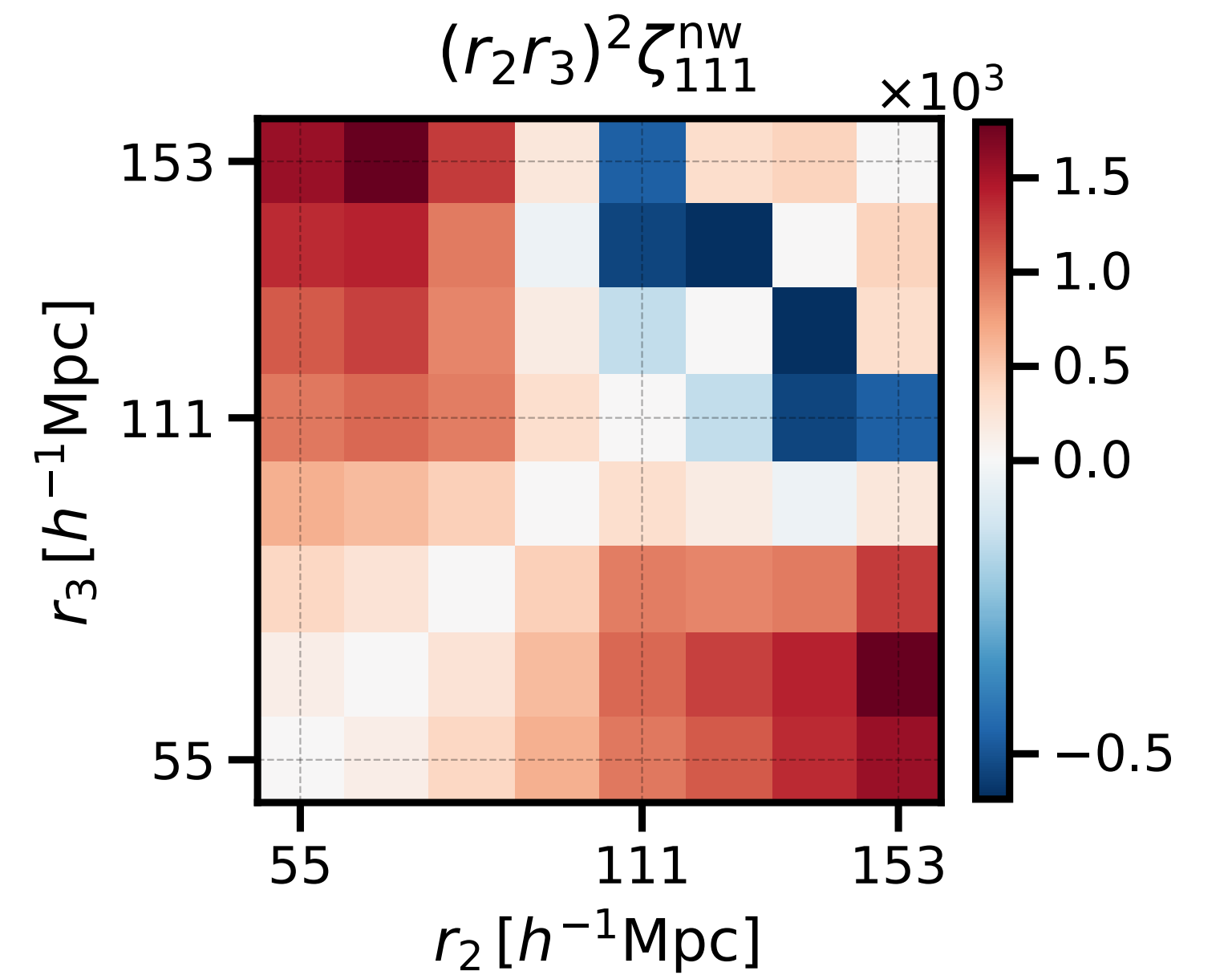
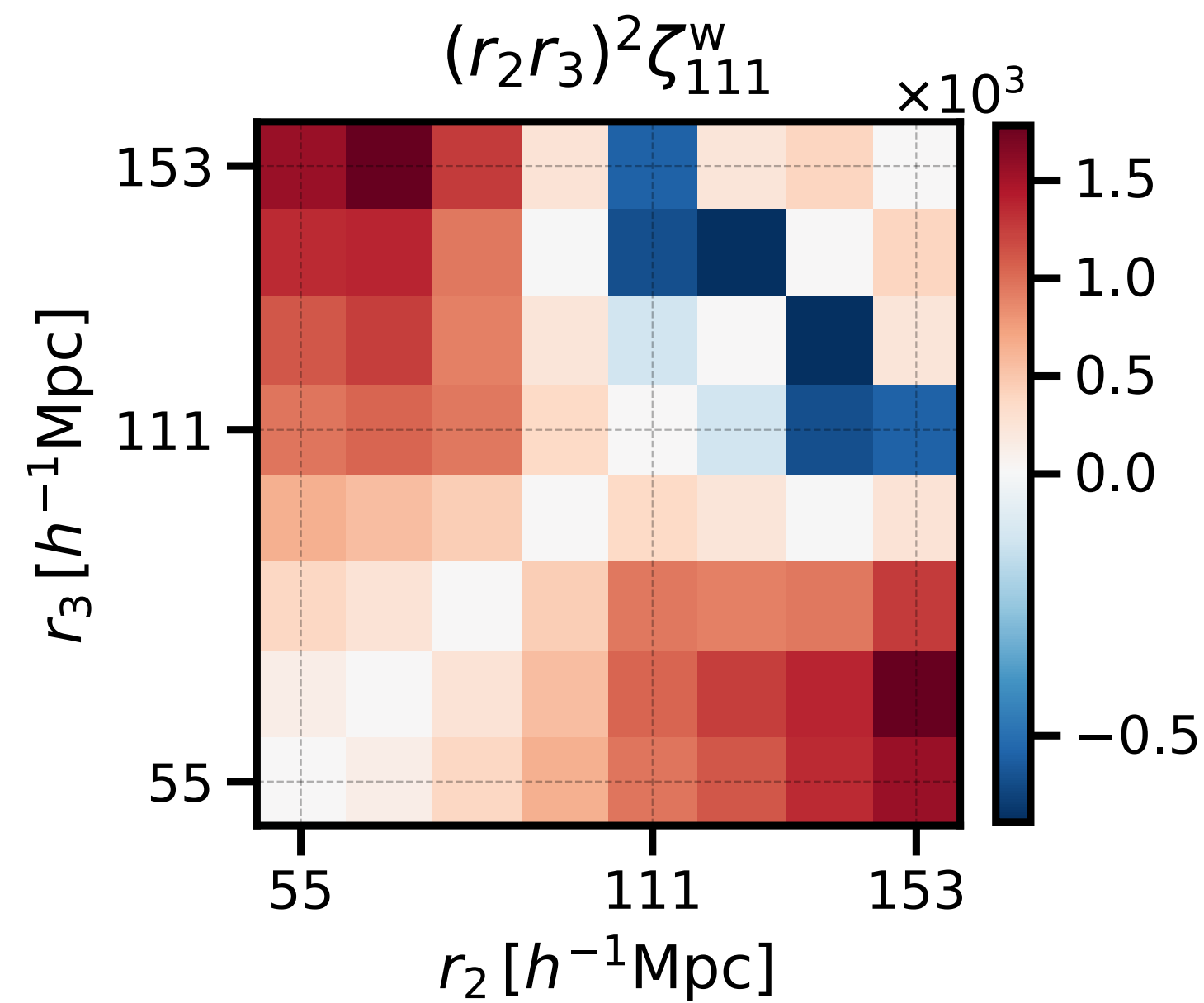
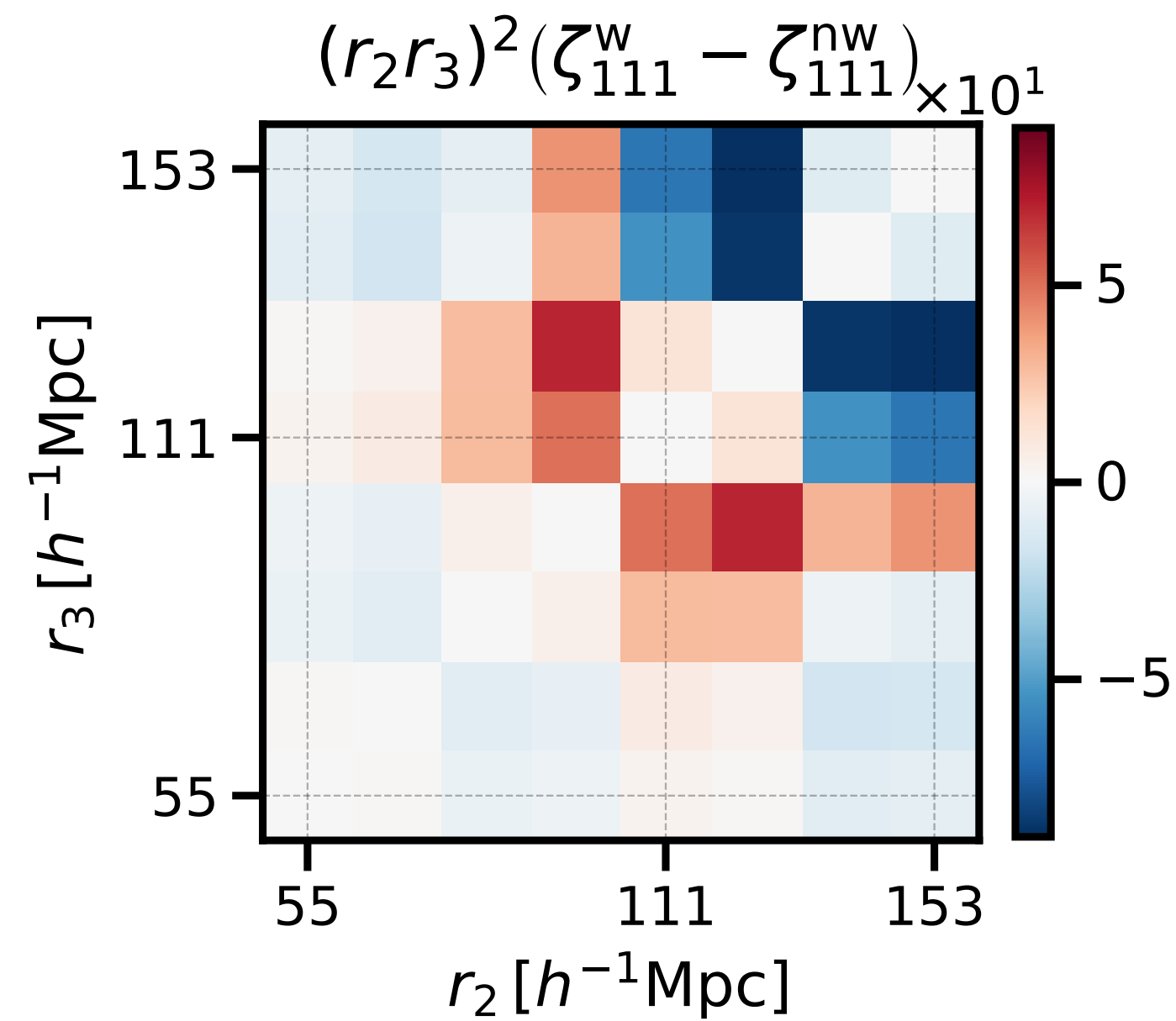


$$\zeta_{\Lambda}(R) = \int d\hat{R} \, \zeta(\mathbf{R}) \, \mathcal{P}_{\Lambda}^*(\hat{R})$$

The background of the slide is a Cosmic Microwave Background (CMB) fluctuation map. It shows a complex pattern of temperature variations across the sky, with warmer regions (yellow and orange) and cooler regions (blue and purple). The pattern is characterized by a series of concentric, irregular rings and patches of varying intensity, representing the acoustic oscillations in the early universe. The overall color palette is dominated by cool blues and purples, with occasional bright yellow and orange spots.

Can we find baryon acoustic oscillations
in the odd sector?

Baryon Acoustic Oscillation in the Odd 4-Point Correlation Function



For scalar (spin-0) field, what are the necessary ingredients for probing the parity transformation?

- 4 fields: $\Phi_1, \Phi_2, \Phi_3, \Phi_4$
- Scalar triple product: $\hat{\mathbf{k}}_1 \cdot (\hat{\mathbf{k}}_2 \times \hat{\mathbf{k}}_3)$

Group and Weight the Fields

$$\langle \Phi \Phi \Phi \Phi \rangle$$

Trispectrum / 4PCF

4 Fields:

$$\left. \begin{array}{l} \langle \Phi \Phi, \Phi \Phi \rangle \\ \langle \Phi, \Phi \Phi \Phi \rangle \end{array} \right\}$$

Compressed
4-point statistics

A Different Way for Extracting Parity: Parity-odd Power Spectra (*POP*)

$$\left\langle \tilde{\mathbf{V}}(\mathbf{k}) \cdot \tilde{\mathbf{A}}(\mathbf{k}') \right\rangle \equiv (2\pi)^3 \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') P_V(k)$$

↓
Vector
(Odd)

↓ $\tilde{\mathbf{A}}(\mathbf{k}) \propto i\mathbf{k} \times \tilde{\mathbf{V}}(\mathbf{k})$
Pseudovector
(Even)

Jamieson, Caravano, **JH** ++ (24')

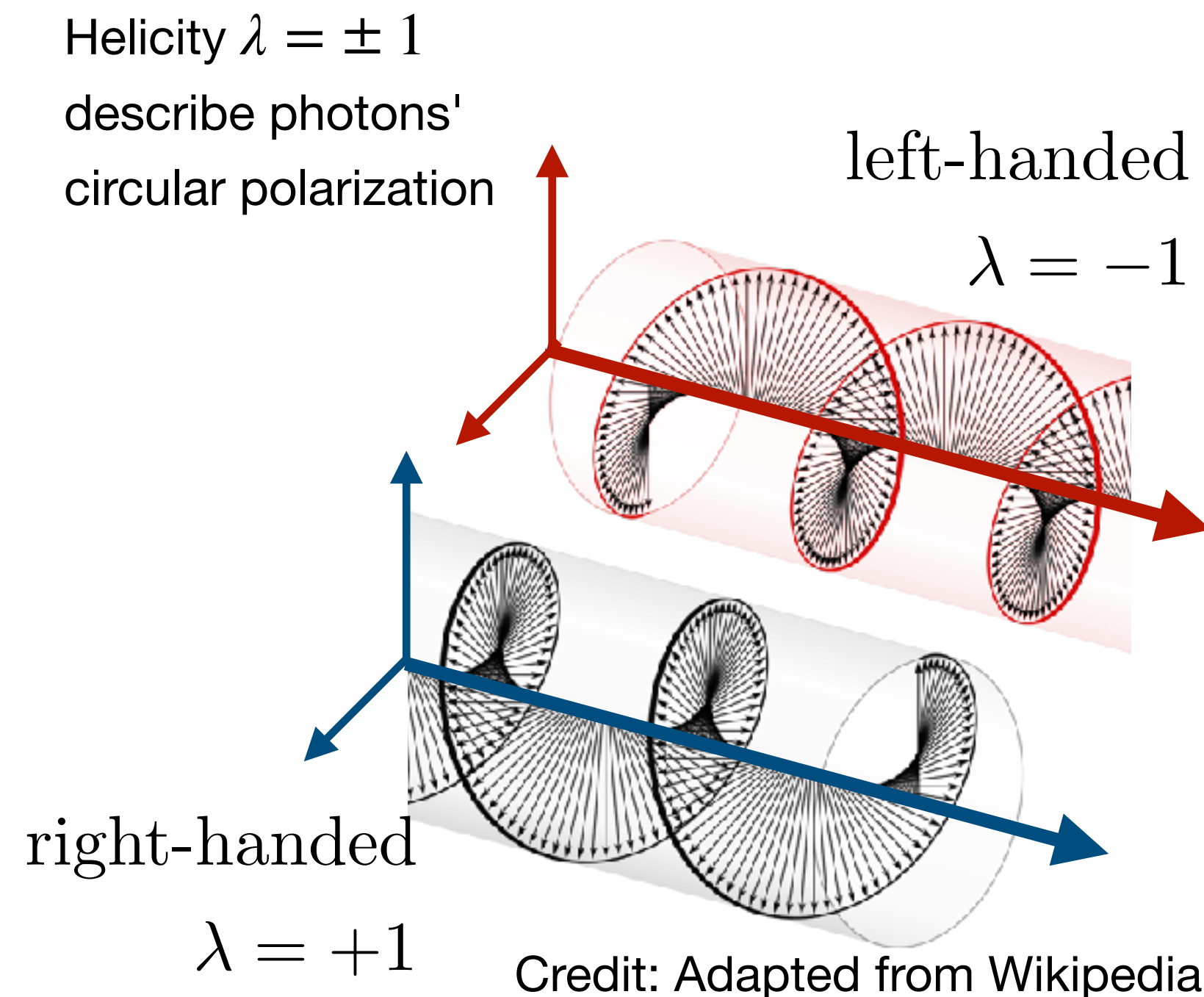
Also see:

Jeong & Kamionkowski (12'), Schmittfull ++ (14'), Munshi (21'), Gao ++ (in prep.)

POP and Polarization Basis

$$\begin{aligned}\mathbf{V}(\mathbf{x}) &= -\nabla\psi(\mathbf{x}) + \nabla\times\mathbf{A}(\mathbf{x}) \\ &= v_{\parallel}\hat{e}_{\parallel}(\mathbf{x}) + \underbrace{v_{+}\hat{e}_{+}(\mathbf{x}) + v_{-}\hat{e}_{-}(\mathbf{x})}\end{aligned}$$

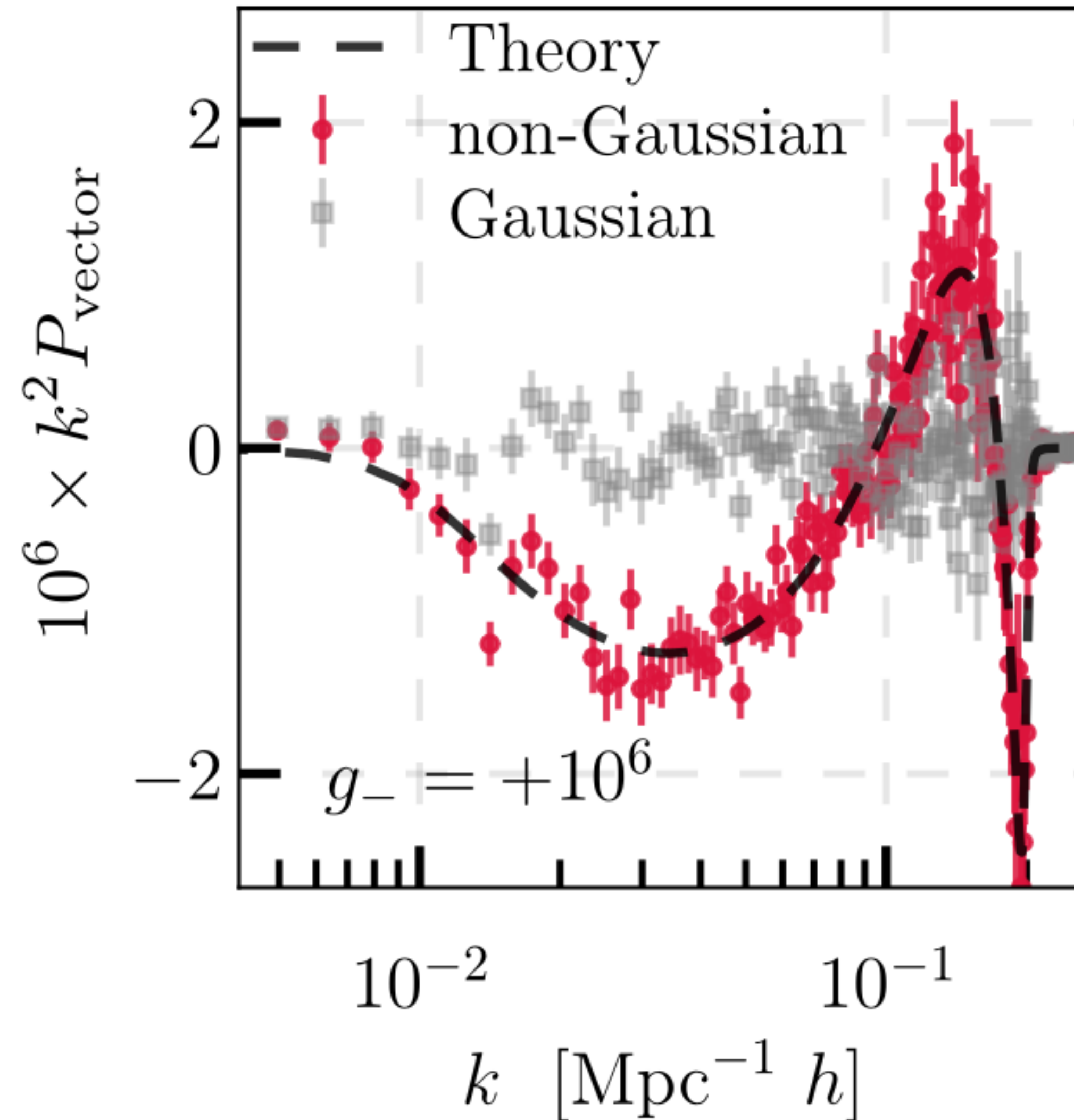
$\hat{e}_{\{+,-\}}$: Polarization vectors



$$\left\langle \tilde{\mathbf{V}}(k) \cdot \tilde{\mathbf{A}}(k') \right\rangle = ik^{-1} \left\langle v_{+}(k)v_{-}(k') \right\rangle + \text{cyc.}$$

$$\rightarrow P_V(k) \propto P_{+-}(k) + \text{cyc.}$$

Measured P_{OP} from Parity-Violating Simulations



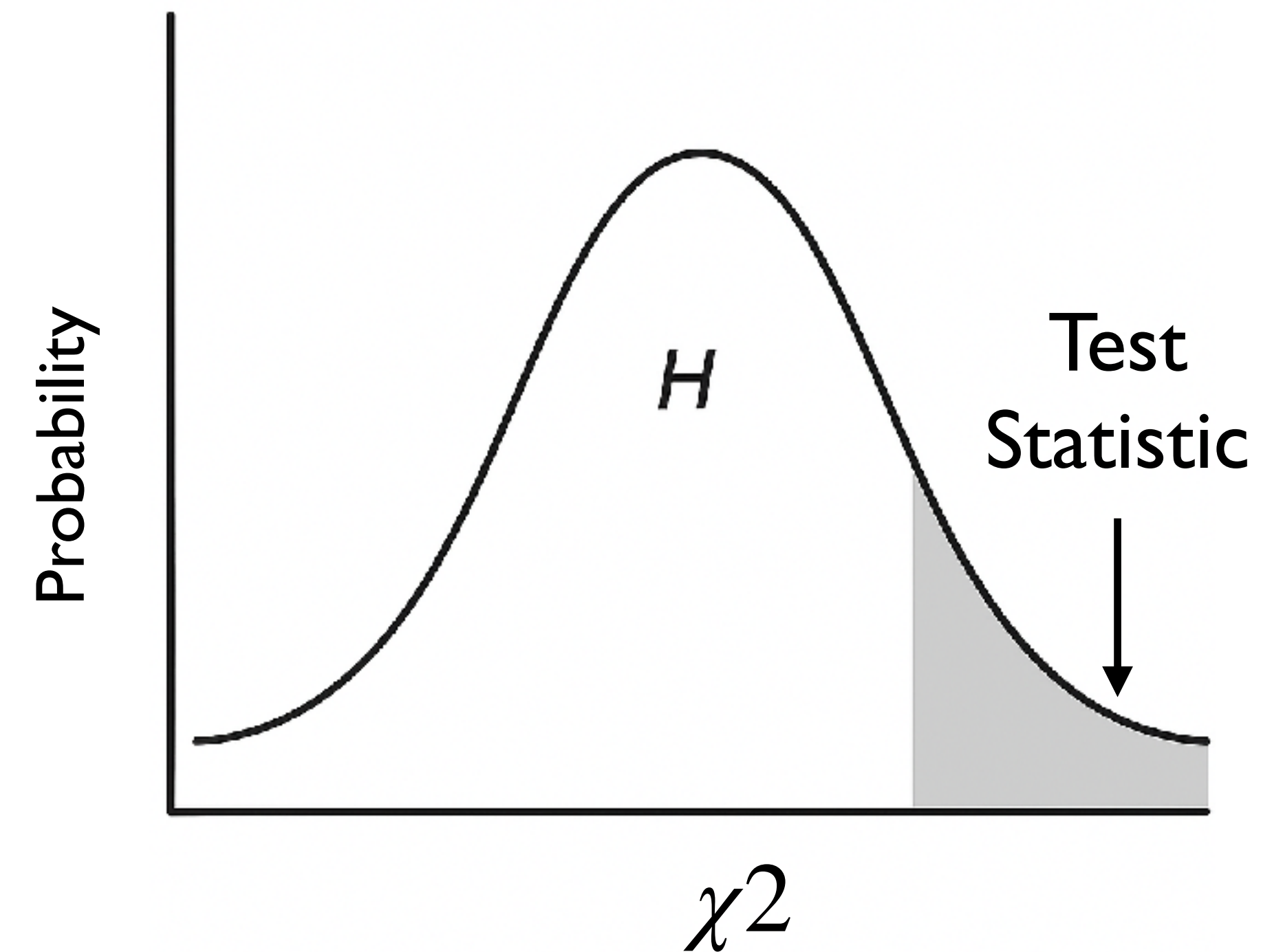
Measurements on the Real Galaxy Data

Signal, Uncertainties, and Significance

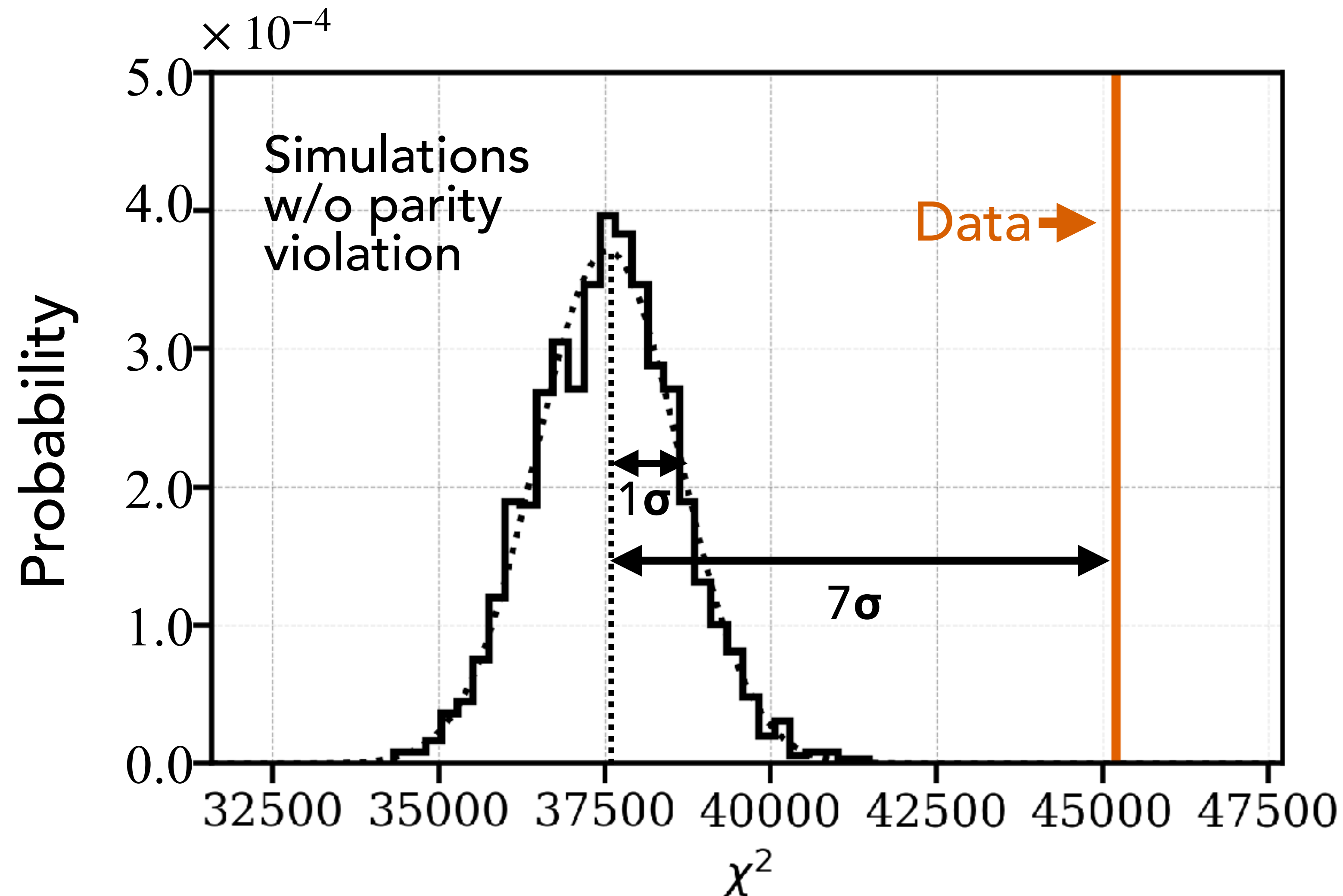
Observed data compose several components:

$$\hat{\zeta} = \zeta + \hat{\epsilon} \text{ — signal, cosmic variance}$$

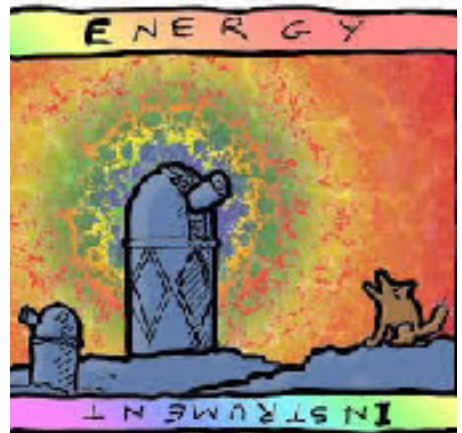
Often, use $\chi^2 = \hat{\zeta}^T \hat{\mathbb{C}}^{-1} \hat{\zeta}$ to quantify significance with null hypothesis



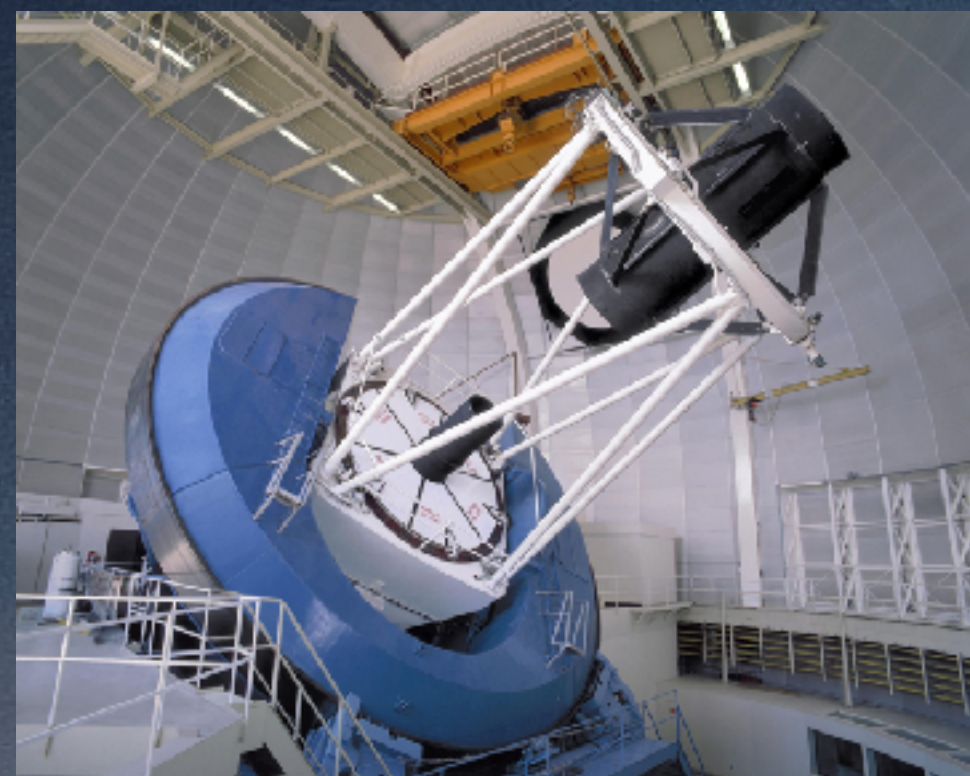
Intriguing Measurement of Parity Violation in Real Galaxy Data from SDSS



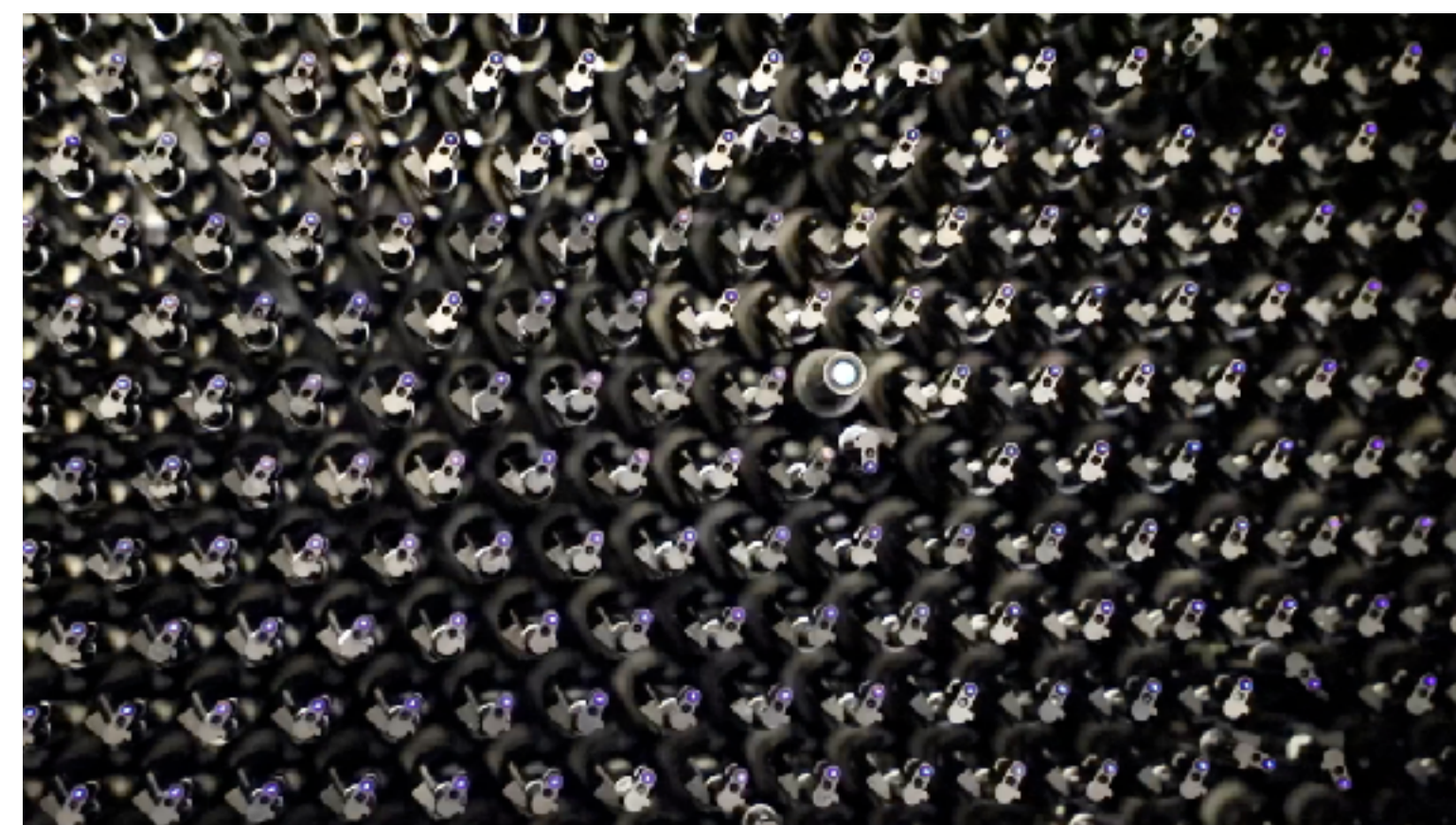
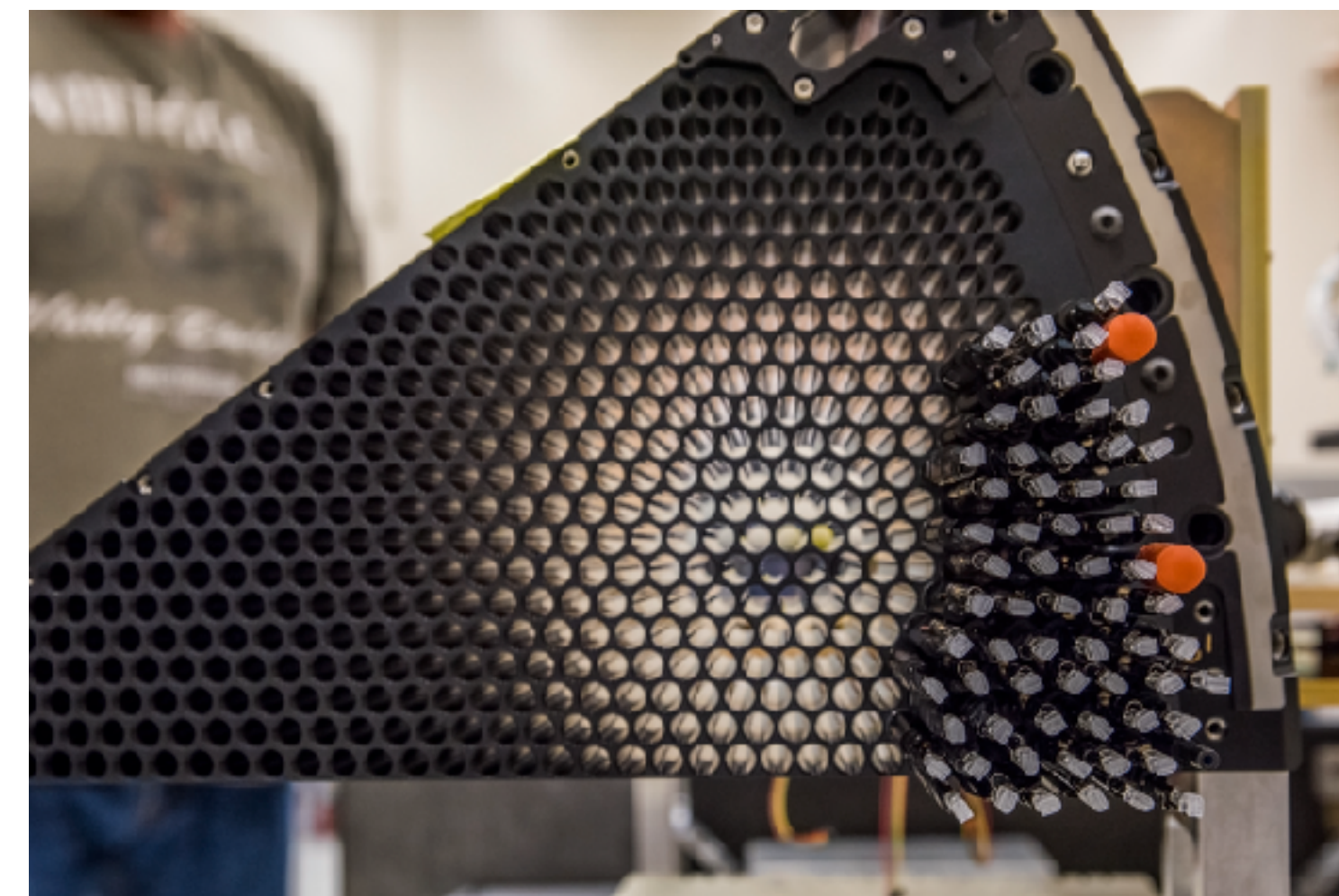
SDSS BOSS: Baryon Oscillation
Spectroscopic Survey



Dark Energy Spectroscopic Instrument (DESI)

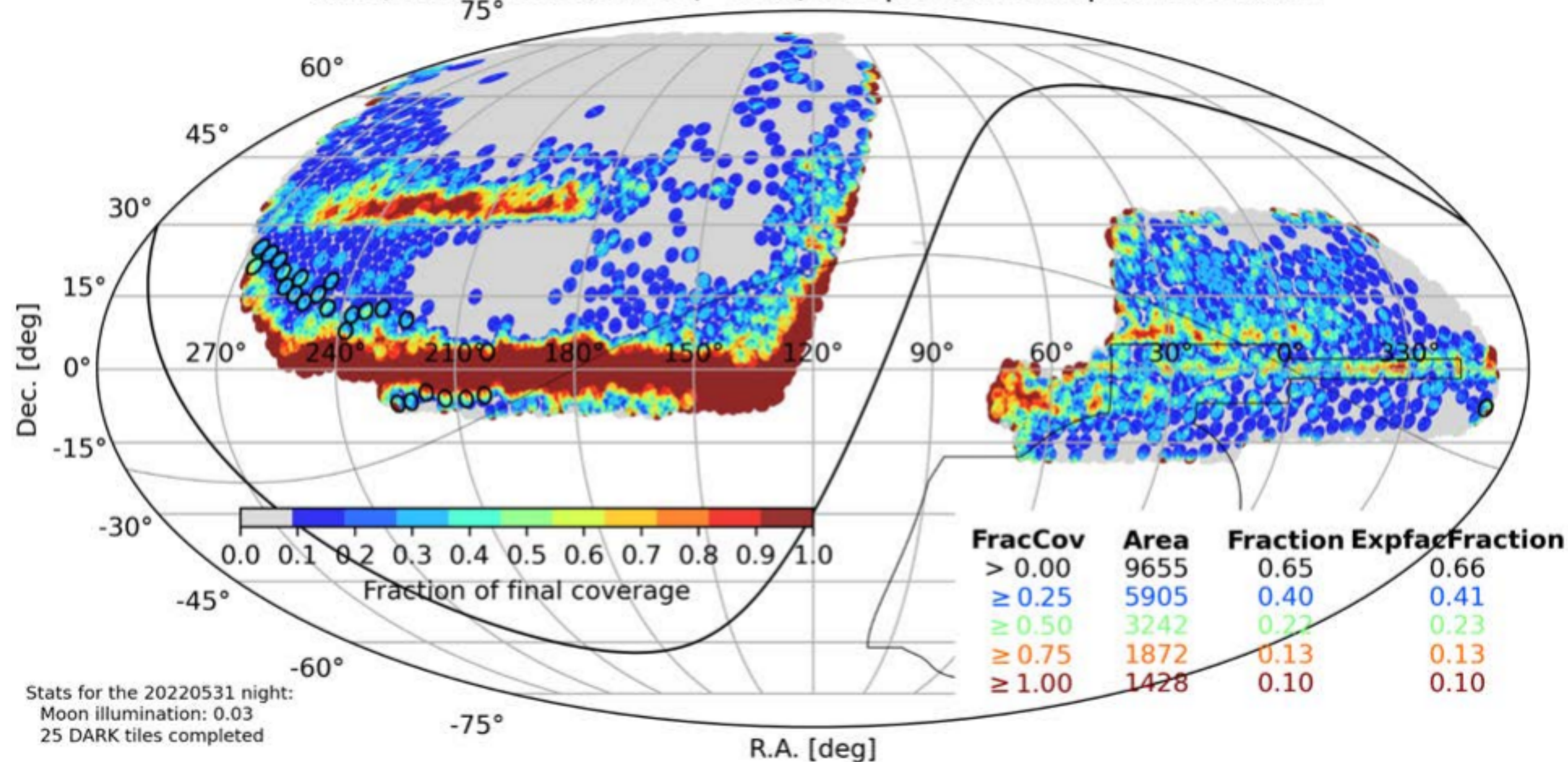
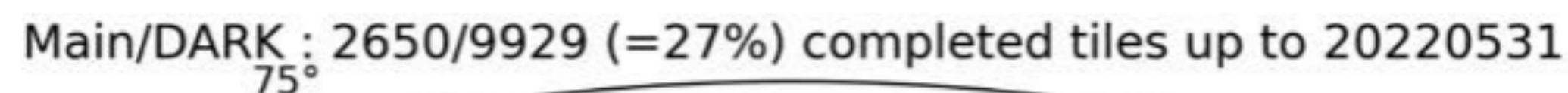


- 3.8 meter telescope @Kitt Peak
- 5,000 fibers
- Controlled by robotic positioners





U.S. Department of Energy Office of Science

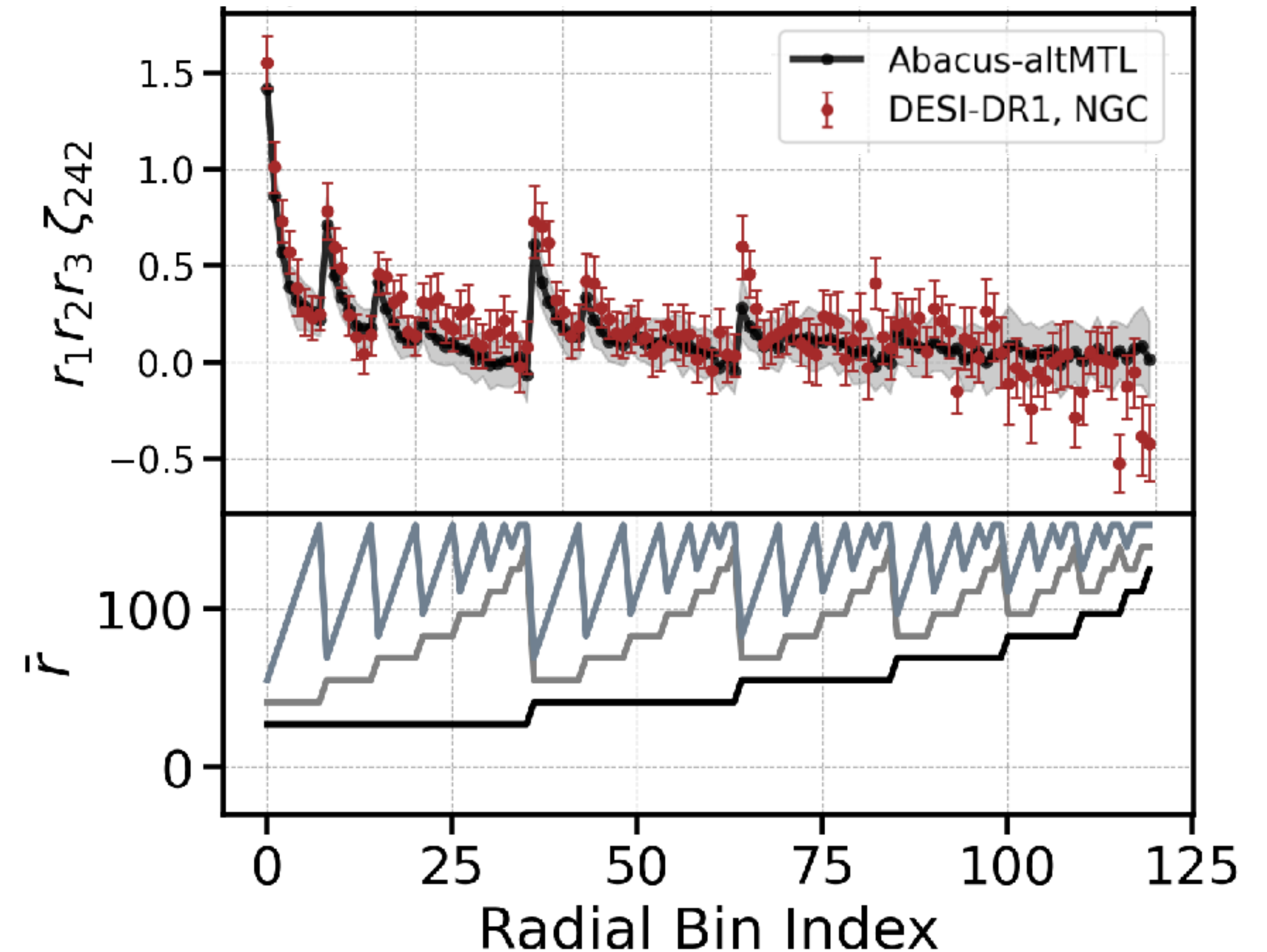
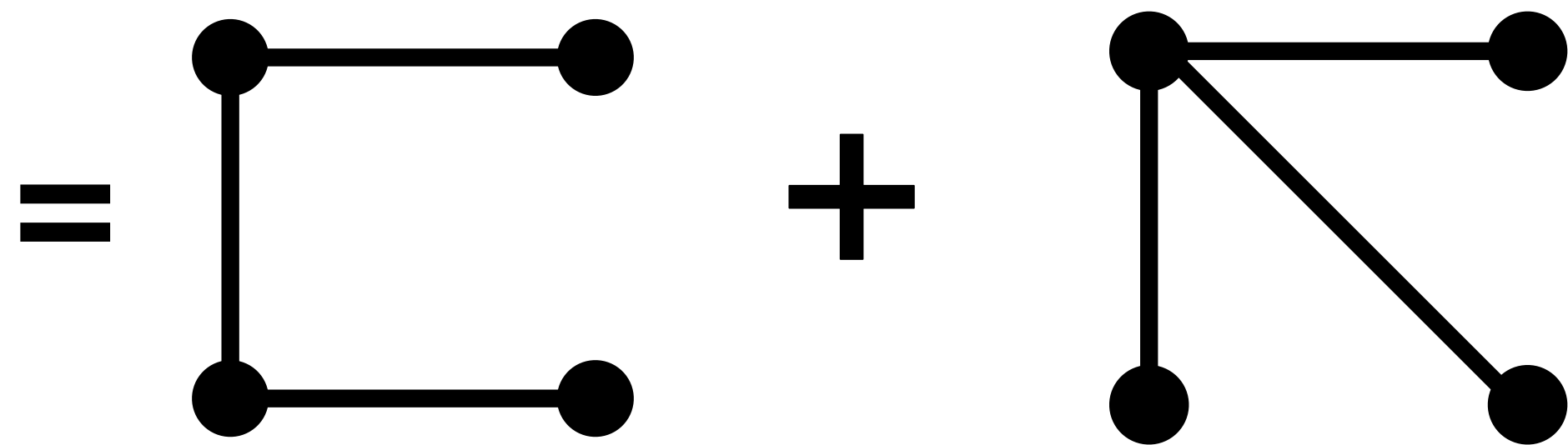


~ 50 % completeness

Gravity-induced Connected 4PCF from DESI

$$\zeta_+ = \zeta_+^{\text{dc}} + \zeta_+^{\text{c}}$$

Gravity-induced Connected
4-Point Function ζ_+^{c}



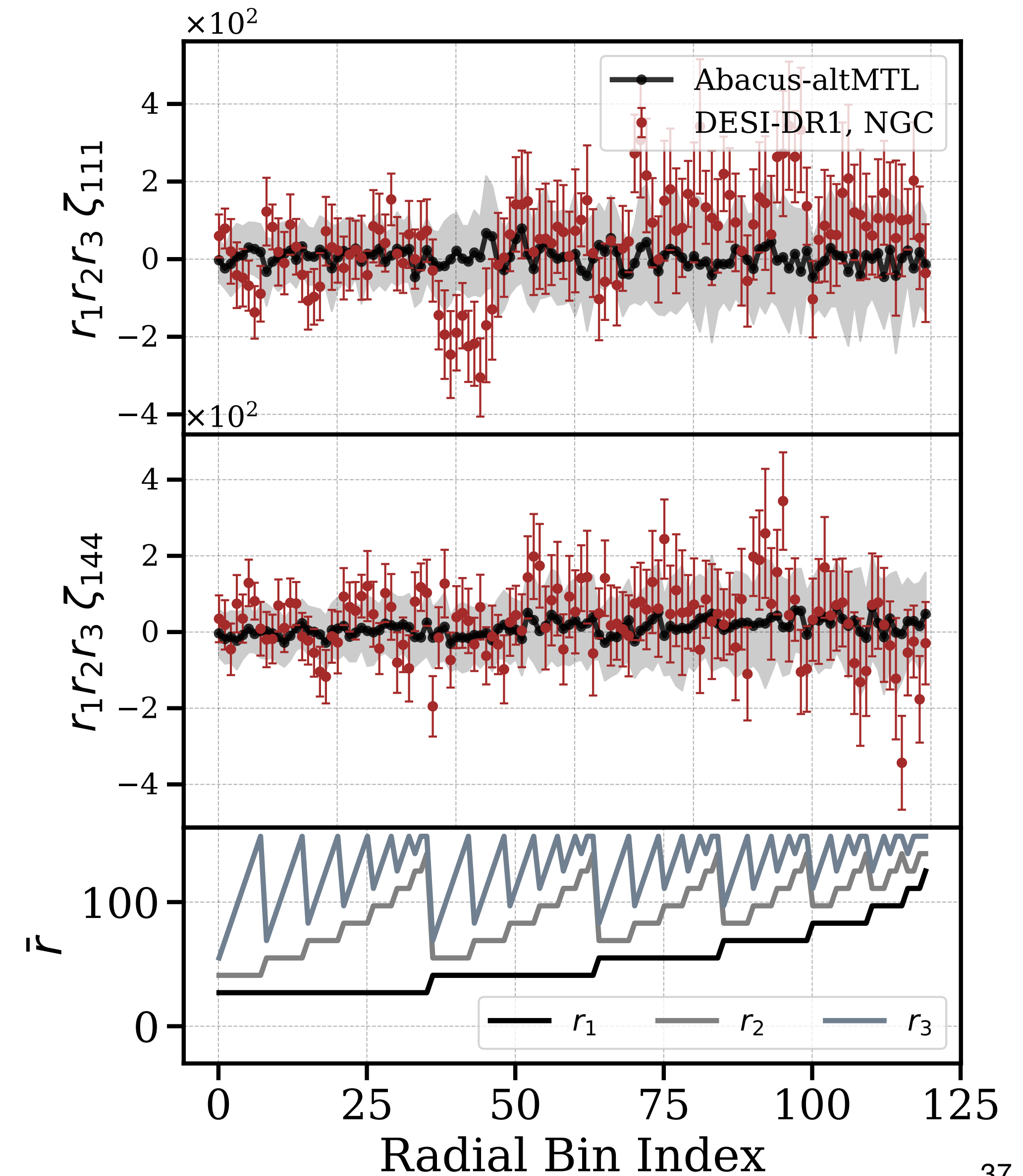
**Larger than 14σ in Gravity-induced
even 4PCF in DESI Y1**

Parity-odd Measurements of DESI DRI

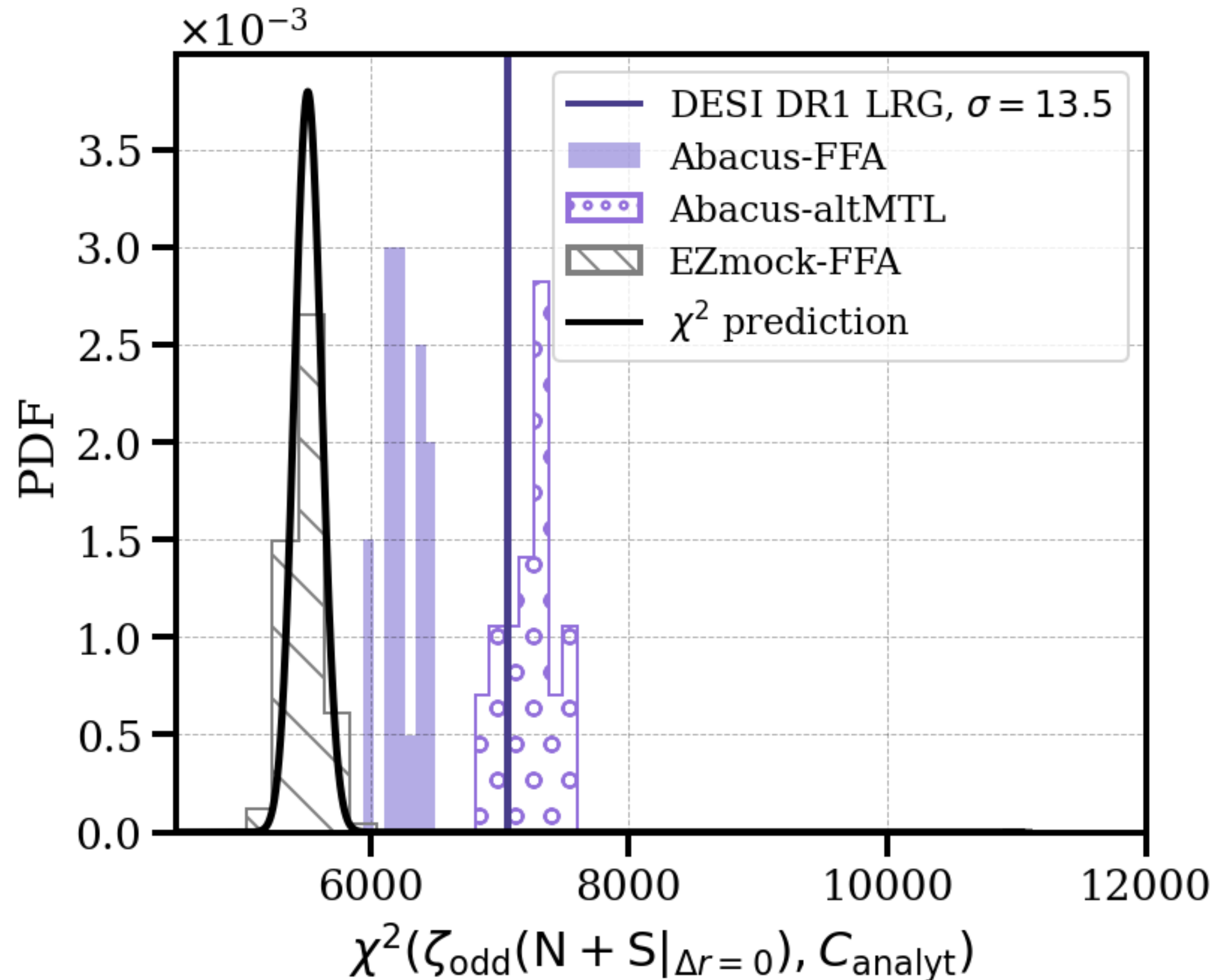
Compared to the even parity, odd sector is noise dominated

Observed data:

$$\hat{\zeta} = \zeta + \hat{\zeta}_s + \hat{\epsilon} \text{ — signal, systematics, cosmic variance}$$



Significance of Parity-Odd 4PCF before Calibration



Two effects on the covariance :

Sample incompleteness due to limited fiber

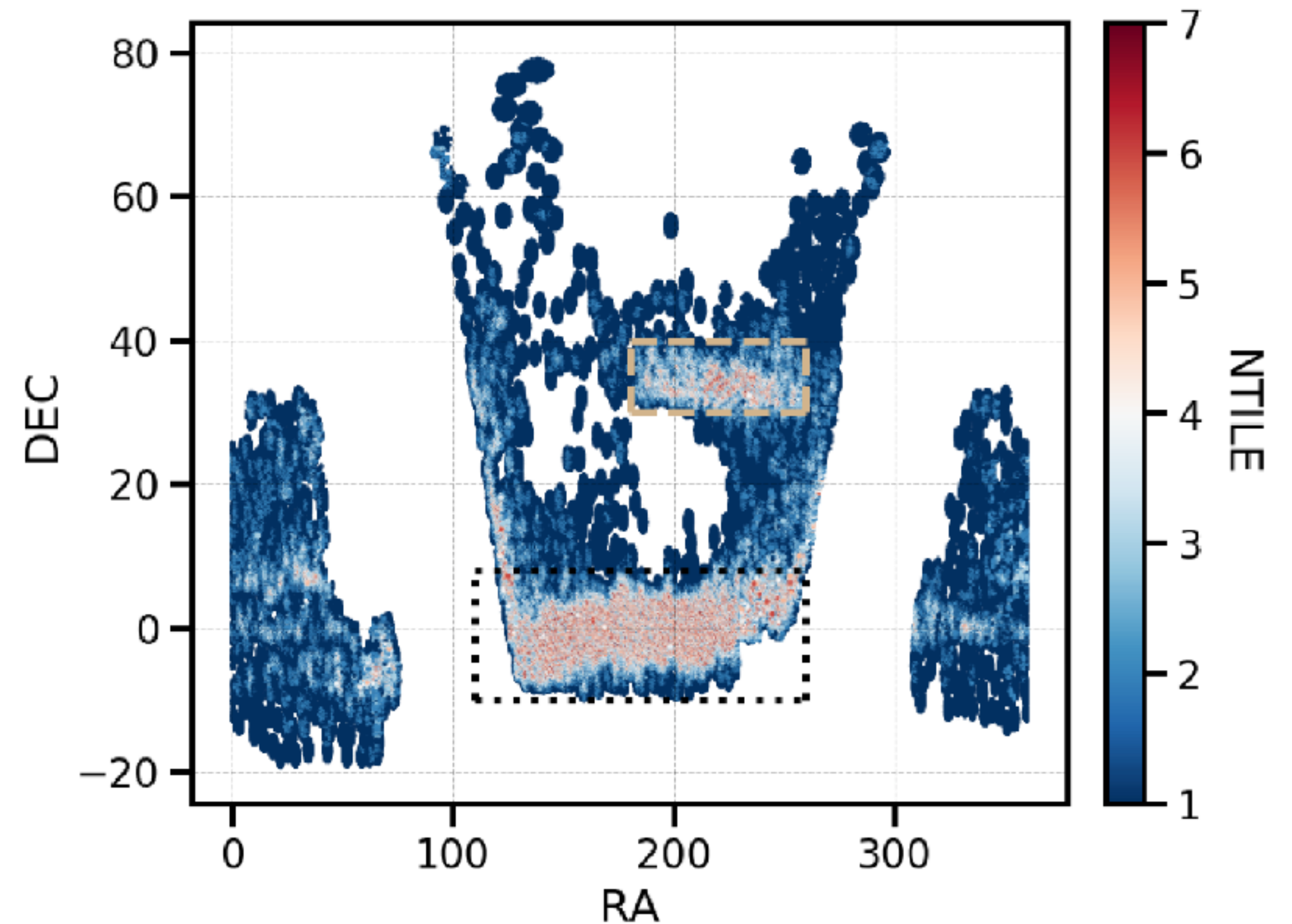
Volume replication due to box size of N-body simulation

Cross-correlation as Consistency Test

Cross-correlate different sky patches

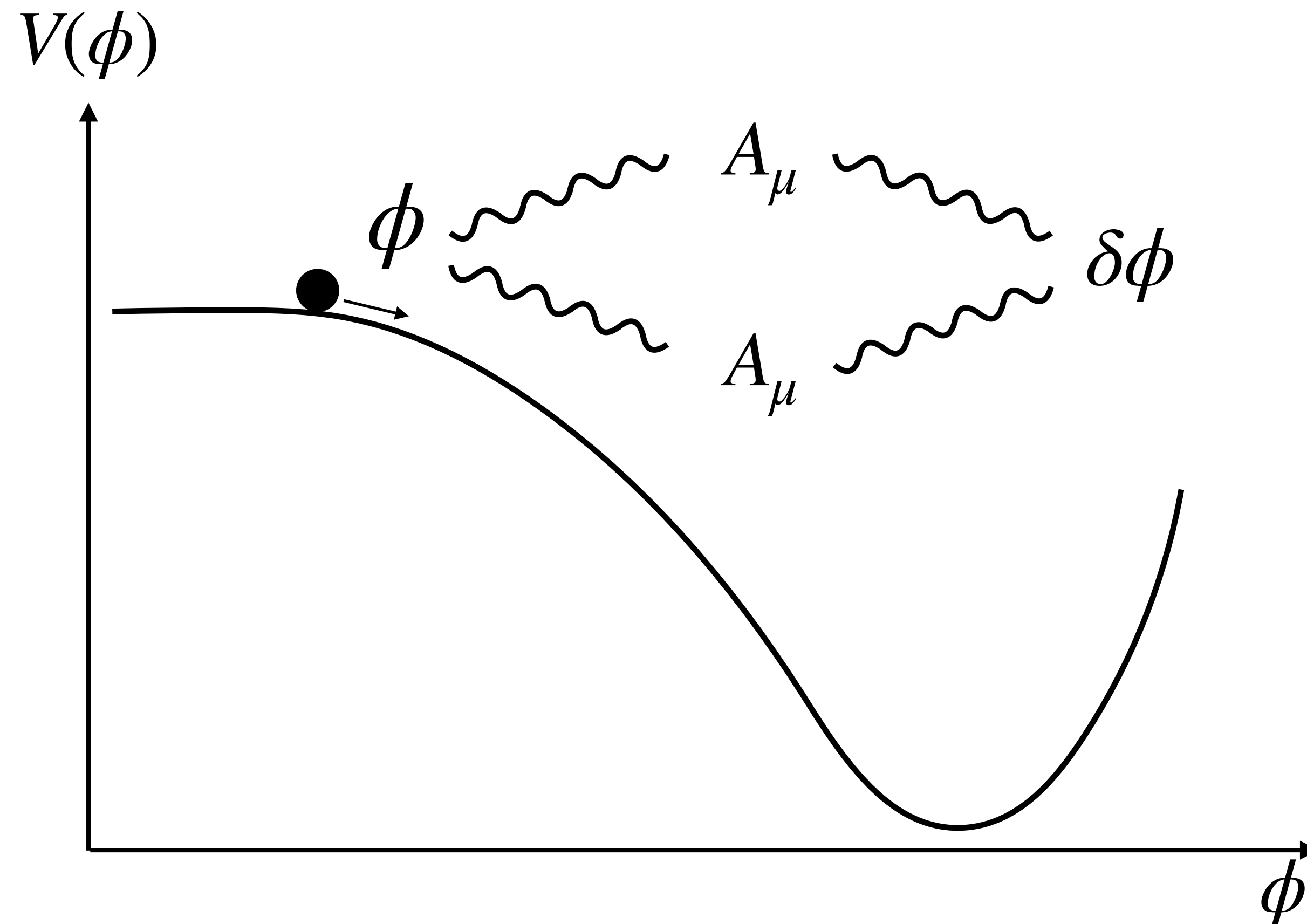
$$\chi^2_{\times} \propto \hat{\xi}^{\mu,i} (\hat{\mathbb{C}}^{-1})_{ij} \hat{\xi}^{\nu,j}$$

→ deterministic contributions



Beyond Single-field Inflation

Axion-U(1) Inflation $\mathcal{L} \supset \boxed{-\frac{1}{4\Lambda}\phi\tilde{F}^{\mu\nu}F_{\mu\nu}}$



Equations of Motion

Gauge field

$$A_{\pm}''(\tau, k) + k^2 A_{\pm}(\tau, k) \mp \frac{2k\xi}{\tau} A_{\pm}(\tau, k) = 0$$

Inflaton
perturbation

$$\delta\phi'' + 2\mathcal{H}(\tau)\delta\phi' - \nabla^2\delta\phi + a(\tau)^2 \frac{d^2V}{d\bar{\phi}^2} \delta\phi = \frac{a(\tau)^2}{\Lambda} (J - \langle J \rangle)$$

Source

$$J \equiv \mathbf{E} \cdot \mathbf{B}$$

Primordial Curvature Trispectrum

$$\left\langle \prod_{i=1}^4 \zeta(\tau, \mathbf{k}_i) \right\rangle \supset \prod_{i=1}^4 \int d\tau_i G(\tau, \tau_i, k_i) \int d^3 \mathbf{q}_i q_i [\hat{\mathbf{e}}(\mathbf{q}_i) \cdot \hat{\mathbf{e}}(\mathbf{k}_i - \mathbf{q}_i)] A'_+(\tau_i, |\mathbf{k}_i - \mathbf{q}_i|) A_+(\tau_i, q_i) \\ \times \delta_D(\mathbf{q}_2 + \mathbf{k}_4 - \mathbf{q}_4) \delta_D(\mathbf{q}_1 + \mathbf{k}_3 - \mathbf{q}_3) \delta_D(\mathbf{k}_1 + \mathbf{q}_1 - \mathbf{q}_4) \delta_D(\mathbf{k}_2 + \mathbf{q}_2 - \mathbf{q}_3)$$

16-D integral

Parity-Odd and Even Trispectrum from Axion Inflation

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^bSchool of Physics & Astronomy, University of Southampton, Southampton SO17 1BJ, UK

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Abstract. The four-point correlation function of primordial scalar perturbations has parity-even and parity-odd contributions and the parity-odd signal in cosmological observations is opening a novel window to look for new physics in the inflationary epoch. We study the distinct parity-odd and even prediction from the axion inflation model, in which the inflaton couples to a vector field via a Chern-Simons interaction, and the vector field is considered to be either approximately massless ($m_A \ll \text{Hubble scale } H$) or very massive ($m_A \sim H$). The parity-odd

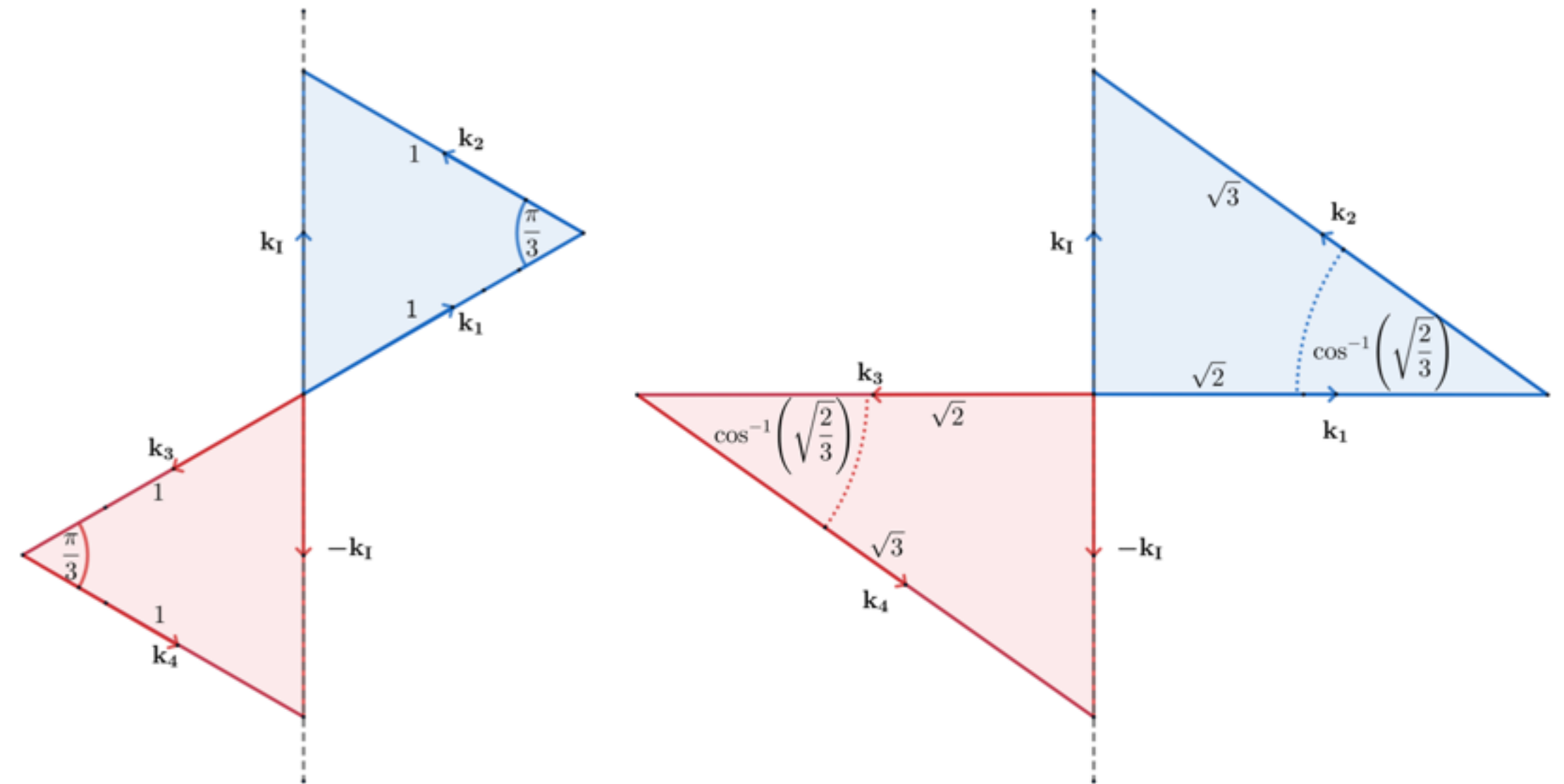
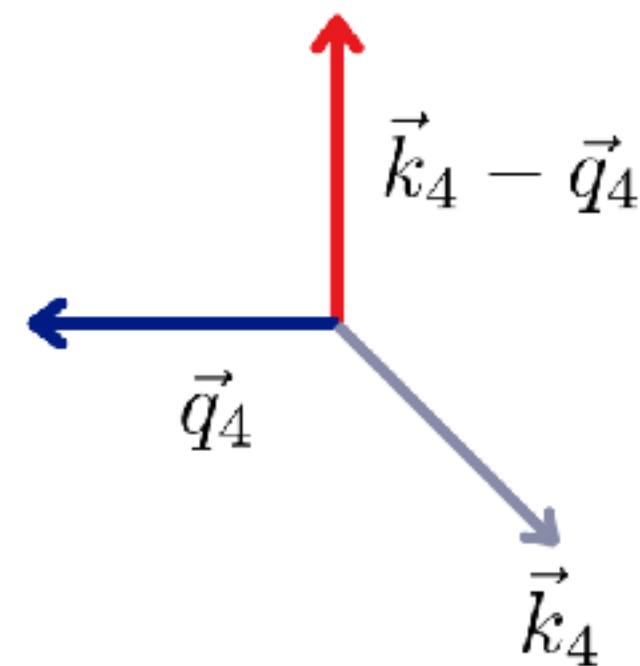
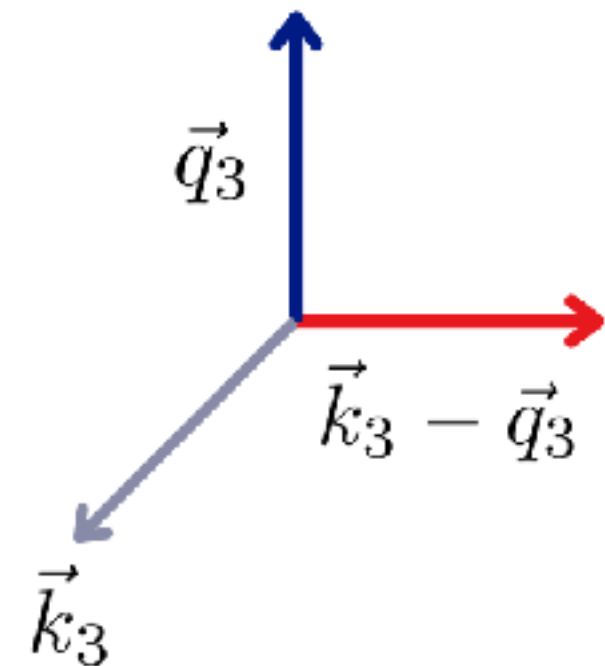
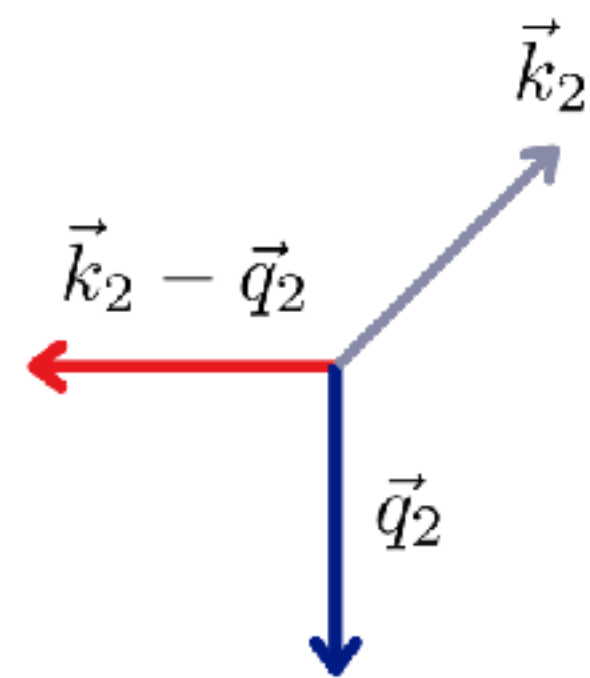
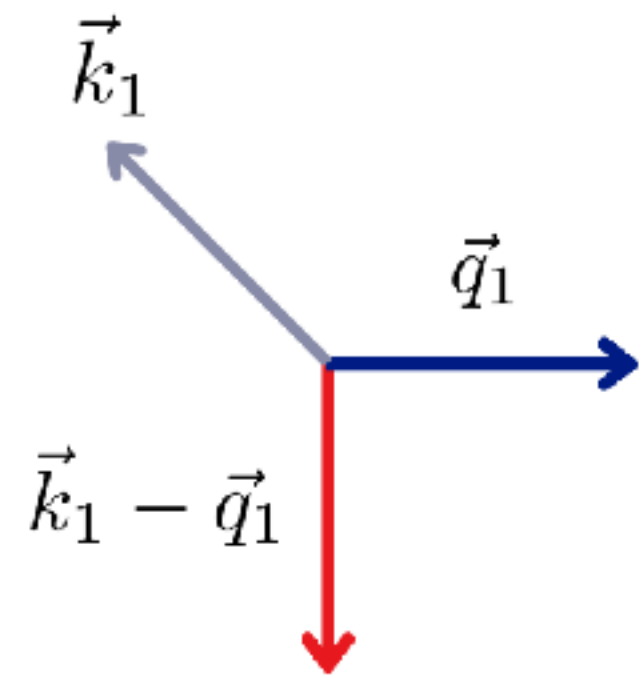


Figure 5. Momentum configurations I (left) and II (right). The red and the blue triangles are in different planes.

Can we simplify the calculation?

Angular Structure of the Integral



At each vertex:

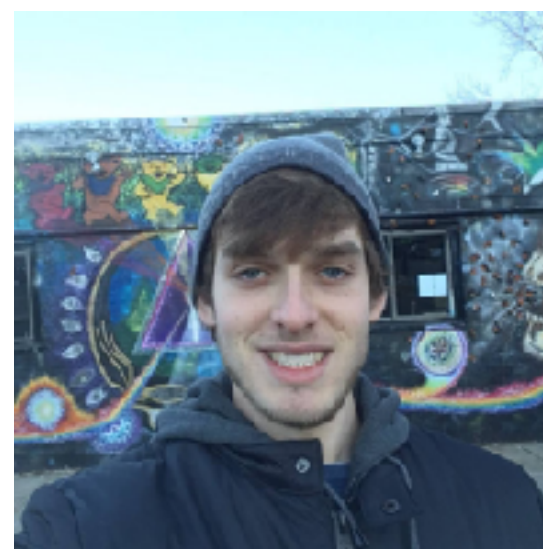
- Momentum conservation — Convolutional structure
- Polarization state of gauge field

Across the vertices:

- Wick theorem: 3-dimensional Dirac delta distribution

Reduced Trispectrum Calculation

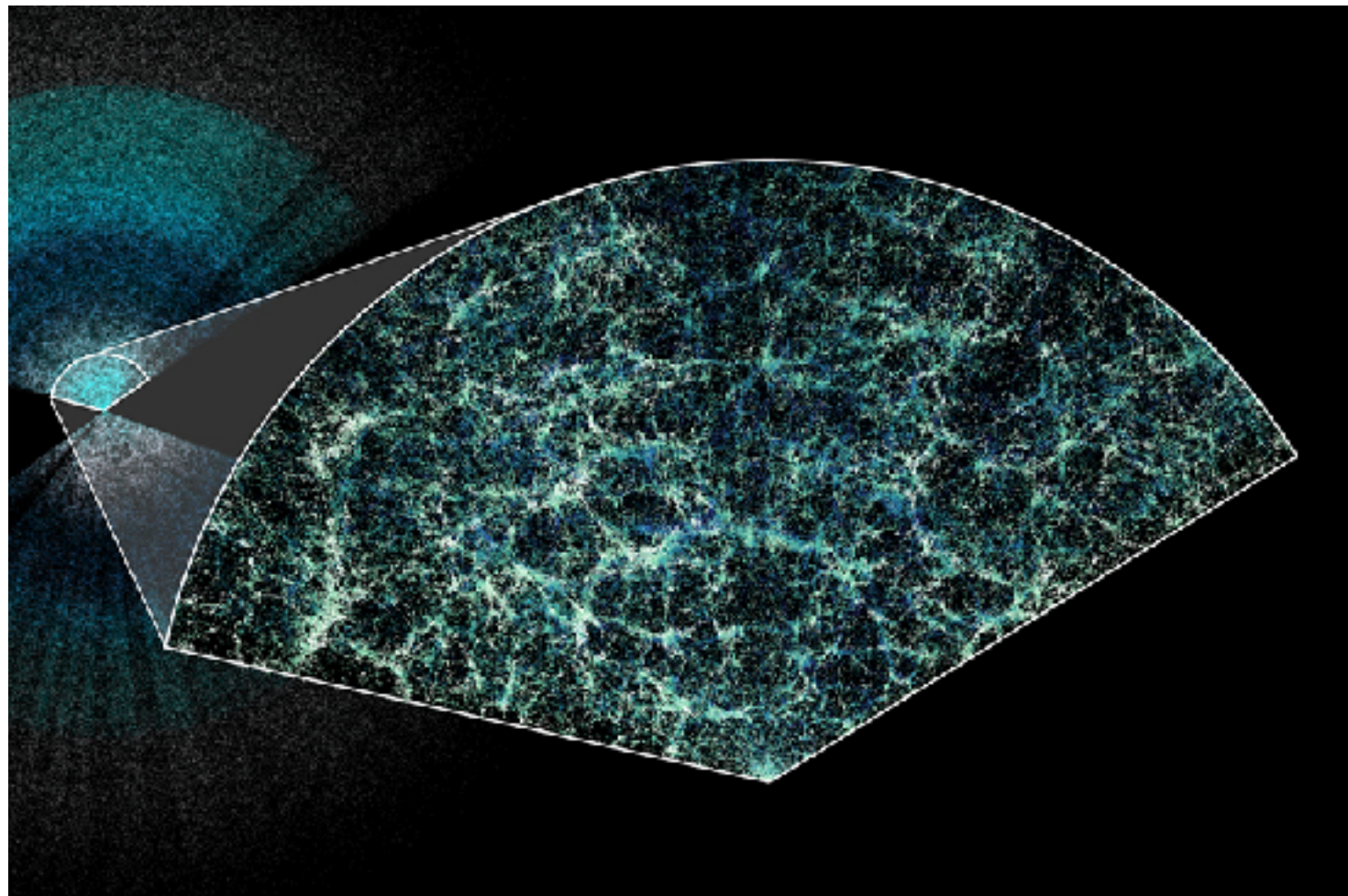
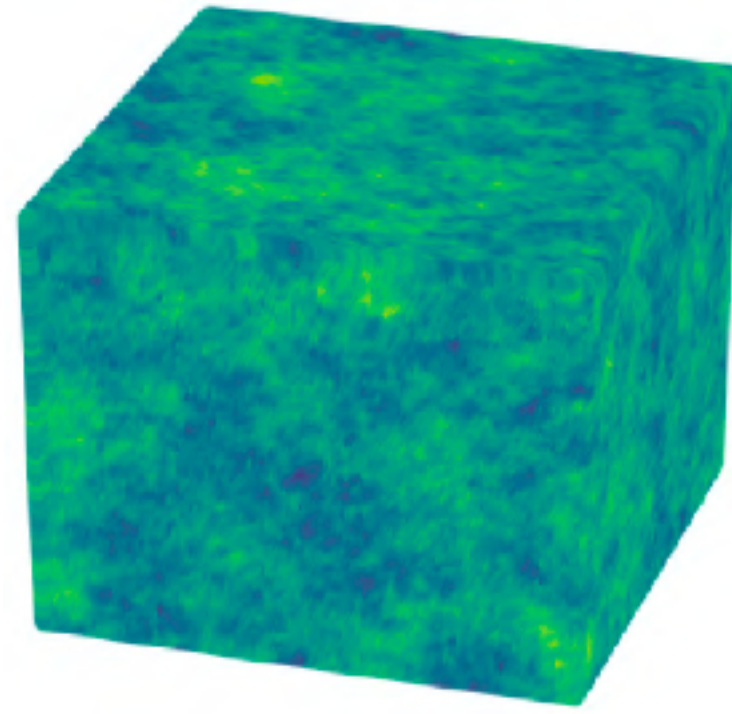
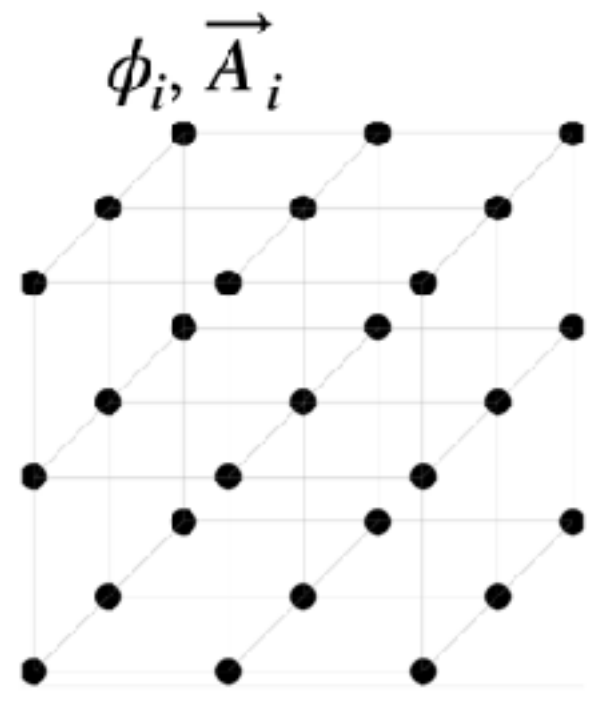
$$\left\langle \prod_{i=1}^4 \zeta(\tau, \mathbf{k}_i) \right\rangle \supset \prod_{i=1}^4 \int d\tau_i G(\tau, \tau_i, k_i) \int d^3 \mathbf{q}_i q_i [\hat{\mathbf{e}}(\mathbf{q}_i) \cdot \hat{\mathbf{e}}(\mathbf{k}_i - \mathbf{q}_i)] A'_+(\tau_i, |\mathbf{k}_i - \mathbf{q}_i|) A_+(\tau_i, q_i) \\ \times \delta_D(\mathbf{q}_2 + \mathbf{k}_4 - \mathbf{q}_4) \delta_D(\mathbf{q}_1 + \mathbf{k}_3 - \mathbf{q}_3) \delta_D(\mathbf{k}_1 + \mathbf{q}_1 - \mathbf{q}_4) \delta_D(\mathbf{k}_2 + \mathbf{q}_2 - \mathbf{q}_3)$$



Matthew Reinhard
PhD@UF

After the angular reduction

$$\supset \sum_{\ell, \mathbf{m}} \sum_{\lambda, \mu} \sum_{\lambda', \mu'} (4\pi)^{12} \left[\prod_{j=1}^4 i^{\ell_j + \lambda_j + \lambda'_j} (-1)^{\lambda'_j + \mu_j + \mu'_j} Y_{\ell_j}^{m_j*}(\hat{\mathbf{k}}_j) \sum_{\Lambda_j M_j} (-1)^{M_j} \mathcal{G}_{\lambda'_j \lambda_j \Lambda_j}^{-\mu'_j - \mu_j - M_j} \right] \\ \times \mathcal{G}_{\lambda_2 \ell_4 \lambda'_4}^{\mu_2 m_4 \mu'_4} \mathcal{G}_{\lambda_1 \ell_3 \lambda'_3}^{\mu_1 m_3 \mu'_3} \mathcal{G}_{\ell_1 \lambda'_1 \lambda_4}^{m_1 \mu'_1 \mu_4} \mathcal{G}_{\ell_2 \lambda'_2 \lambda_3}^{m_2 \mu'_2 \mu_3} \mathcal{T}_{\ell \Lambda \lambda}^{(1)M}(k_1, k_2, k_3, k_4).$$



Compare the analytic calculation of primordial trispectra to lattice simulations (Caravano+ 23', Jamieson+25')

- Trispectrum shape from axion inflation
- Limits in the analytic calculation
- Renormalizability (?) — slow-roll regime

Parity-violating simulations for galaxies

- Impact of gravity, observational effects on initial conditions

Parity Violation from Home 2025

18–21 November

Early Registration is now open

<https://parity.cosmodiscussion.com/>



&



Please contact for Postdoc and PhD Student Positions

