



renormalization and efficient evaluation of the dark-matter two-loop power spectrum

Matthew Lewandowski
CERN TH

New Physics from Galaxy Clustering - GGI
10/09/25

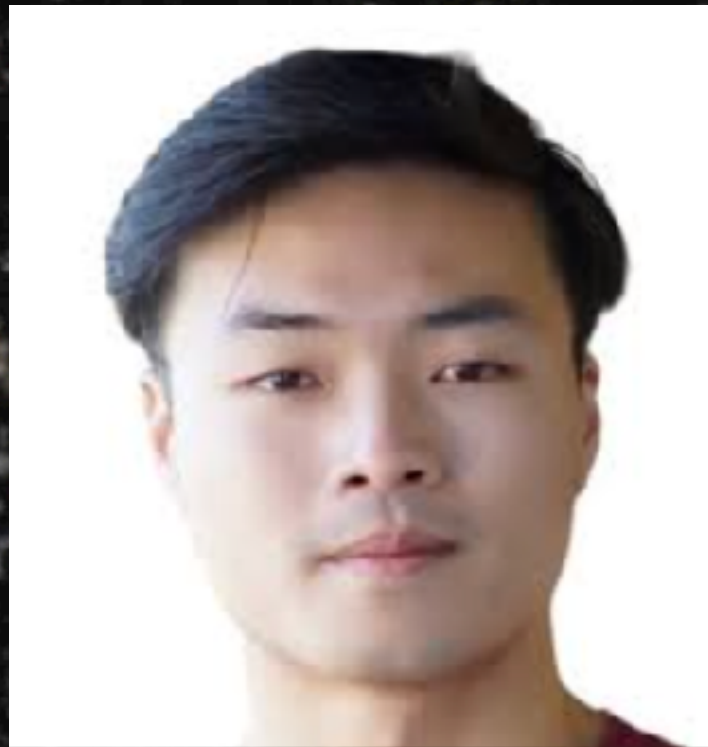


renormalization and efficient evaluation of the dark-matter two-loop power spectrum

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New Physics from Galaxy Clustering - GGI
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in collaboration with
Anastasiou, Favorito,
Senatore, Zheng

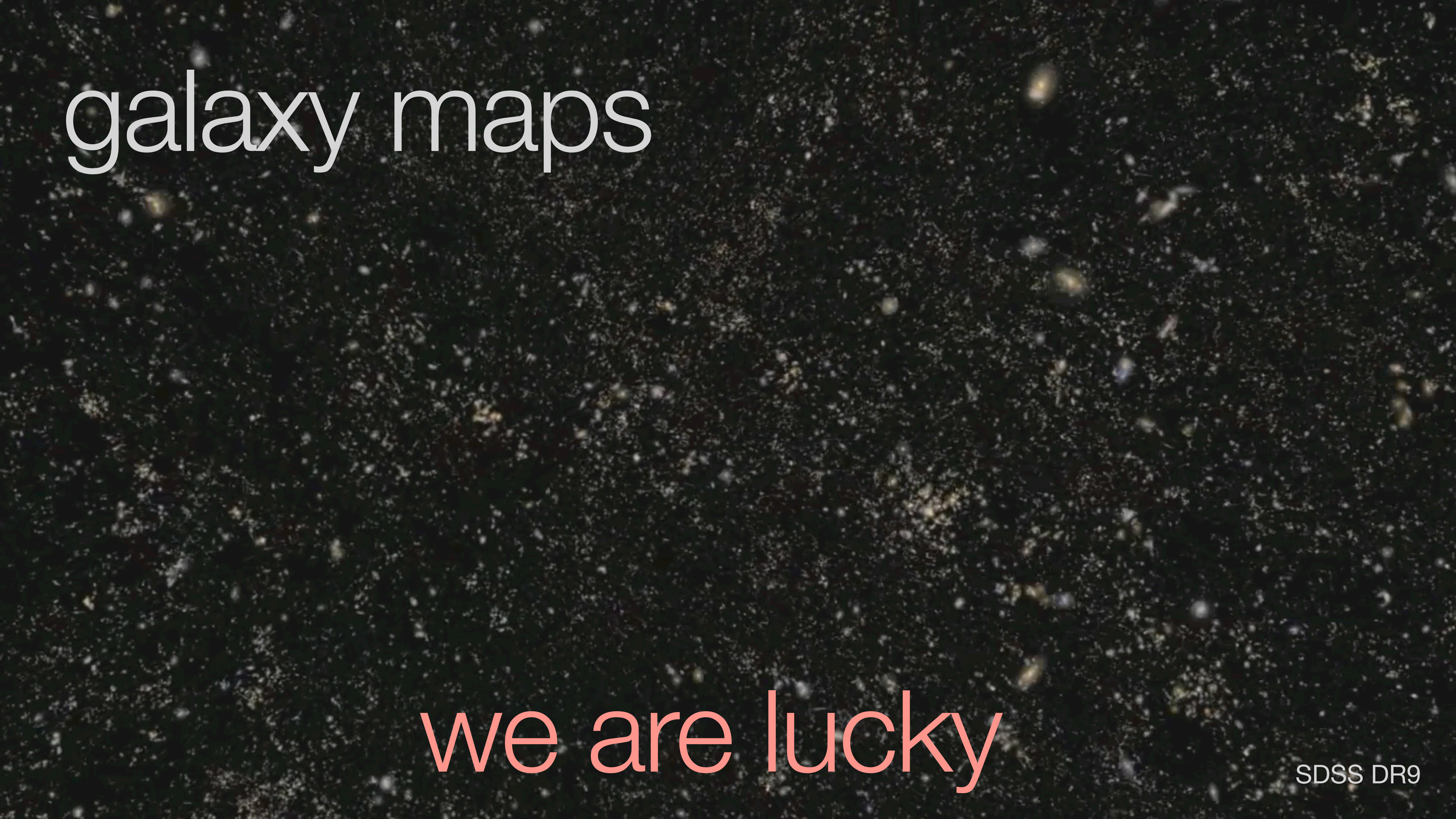


Henry Zheng, Stanford



Andrea Favorito, ETH

ETH zürich



galaxy maps

we are lucky

SDSS DR9

the promise

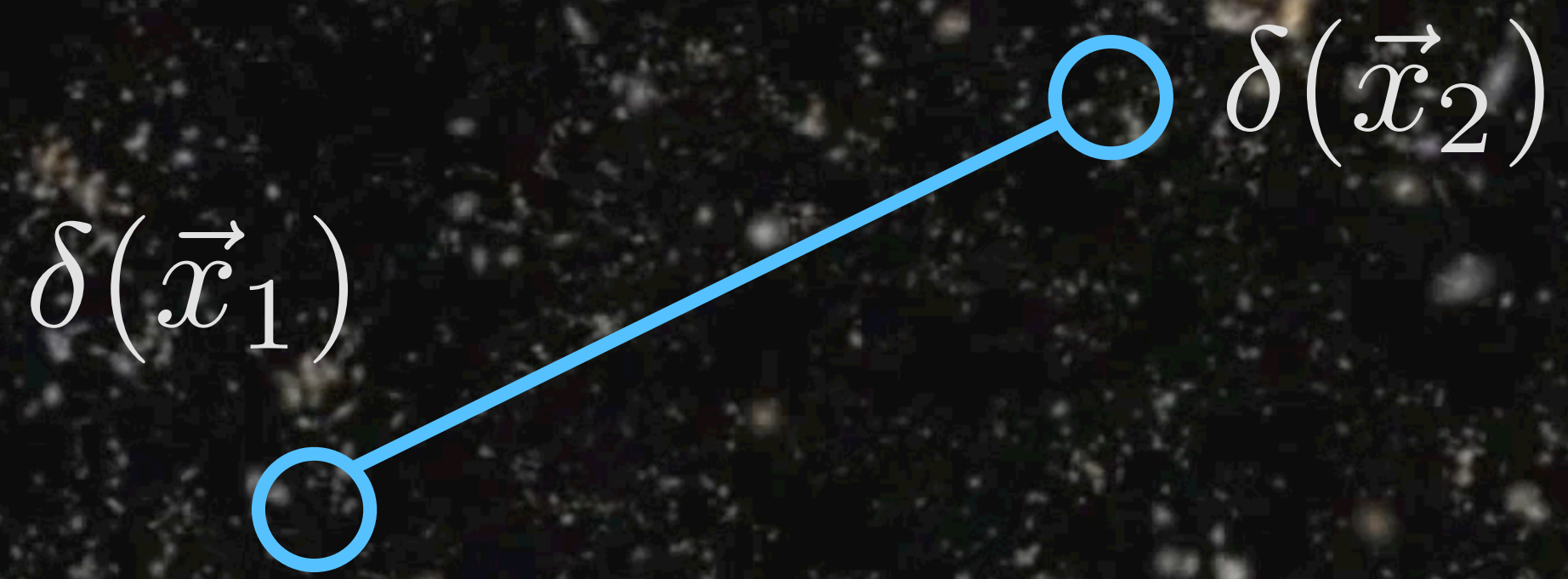
- this map is sensitive to the Universe's contents, initial conditions, and interactions
- standard evolution - Λ CDM parameters like Ω_m , H_0 , and σ_8
- but also many other effects we've discussed

$$\mathcal{O} = \mathcal{O}_{\text{DM, gravity}} + \epsilon \mathcal{O}_{\text{BSM}}$$

the method

summary statistics

two point/power spectrum



$$\langle \delta(\vec{x}_1) \delta(\vec{x}_2) \rangle$$

$$P(k)$$

overdensity: $\delta(\vec{x})$

three point/bispectrum



$$\langle \delta(\vec{x}_1) \delta(\vec{x}_2) \delta(\vec{x}_3) \rangle$$

$$B(k_1, k_2, k_3)$$

the method

summary statistics

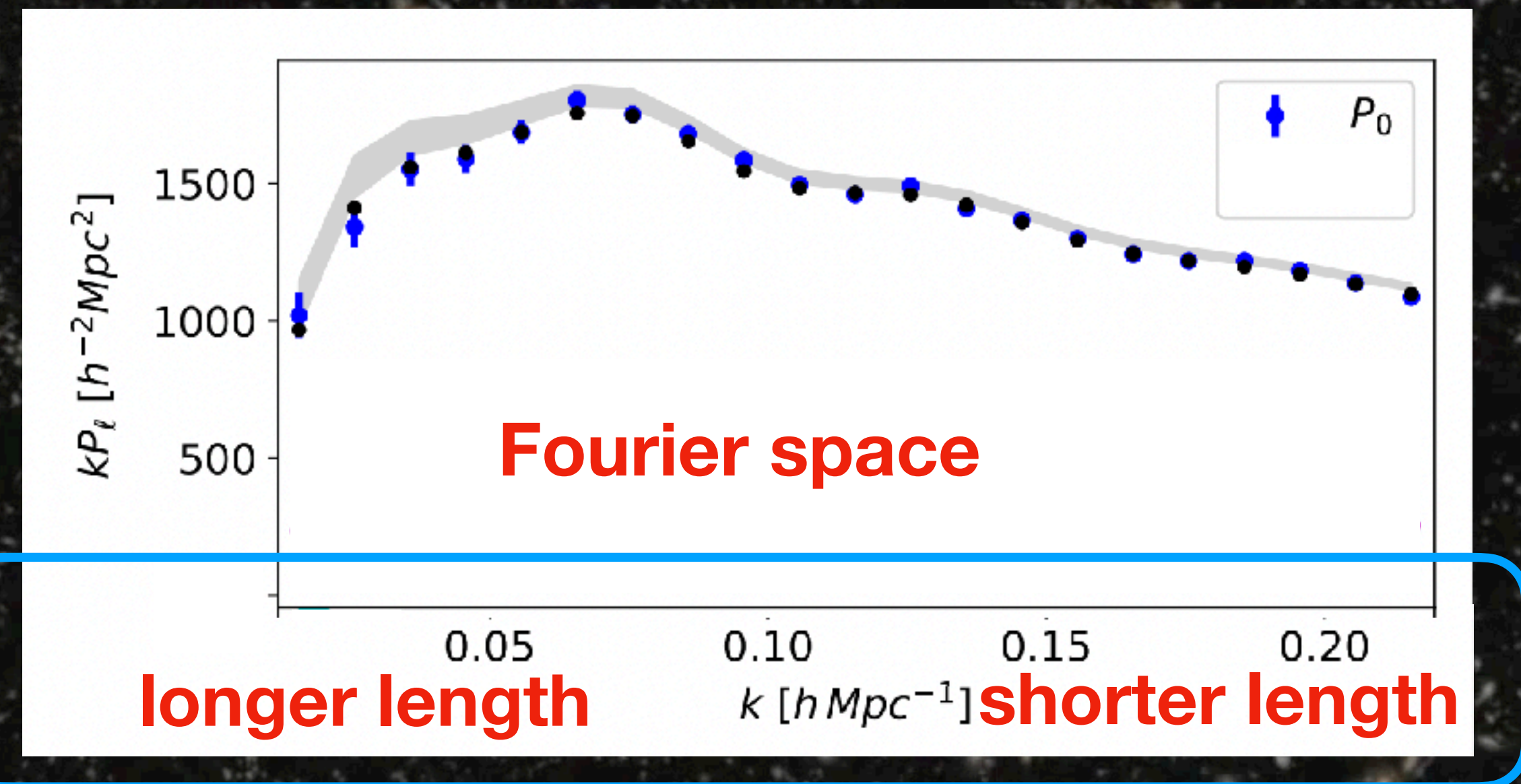
two point/power spectrum



$$\langle \delta(\vec{x}_1) \delta(\vec{x}_2) \rangle$$

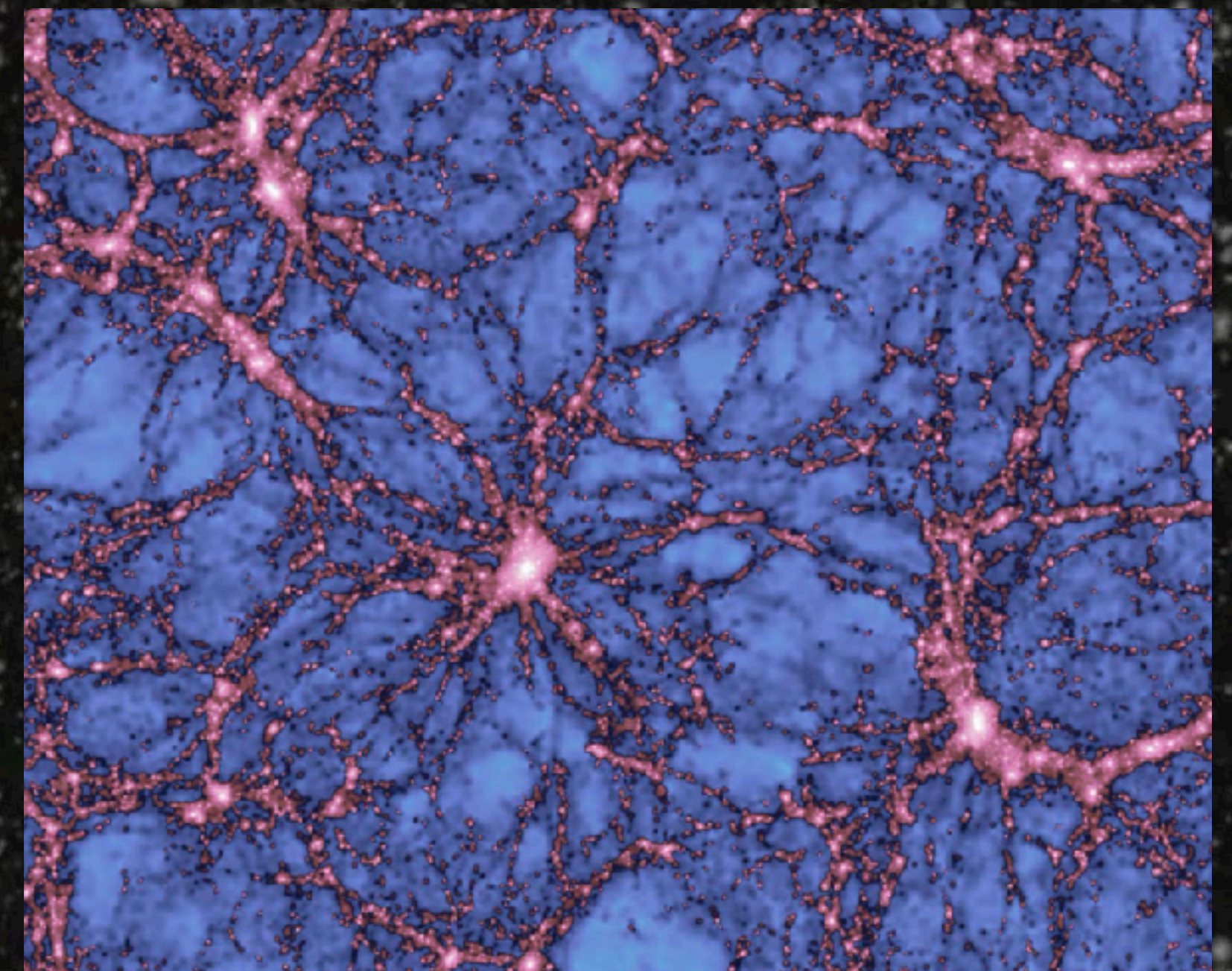
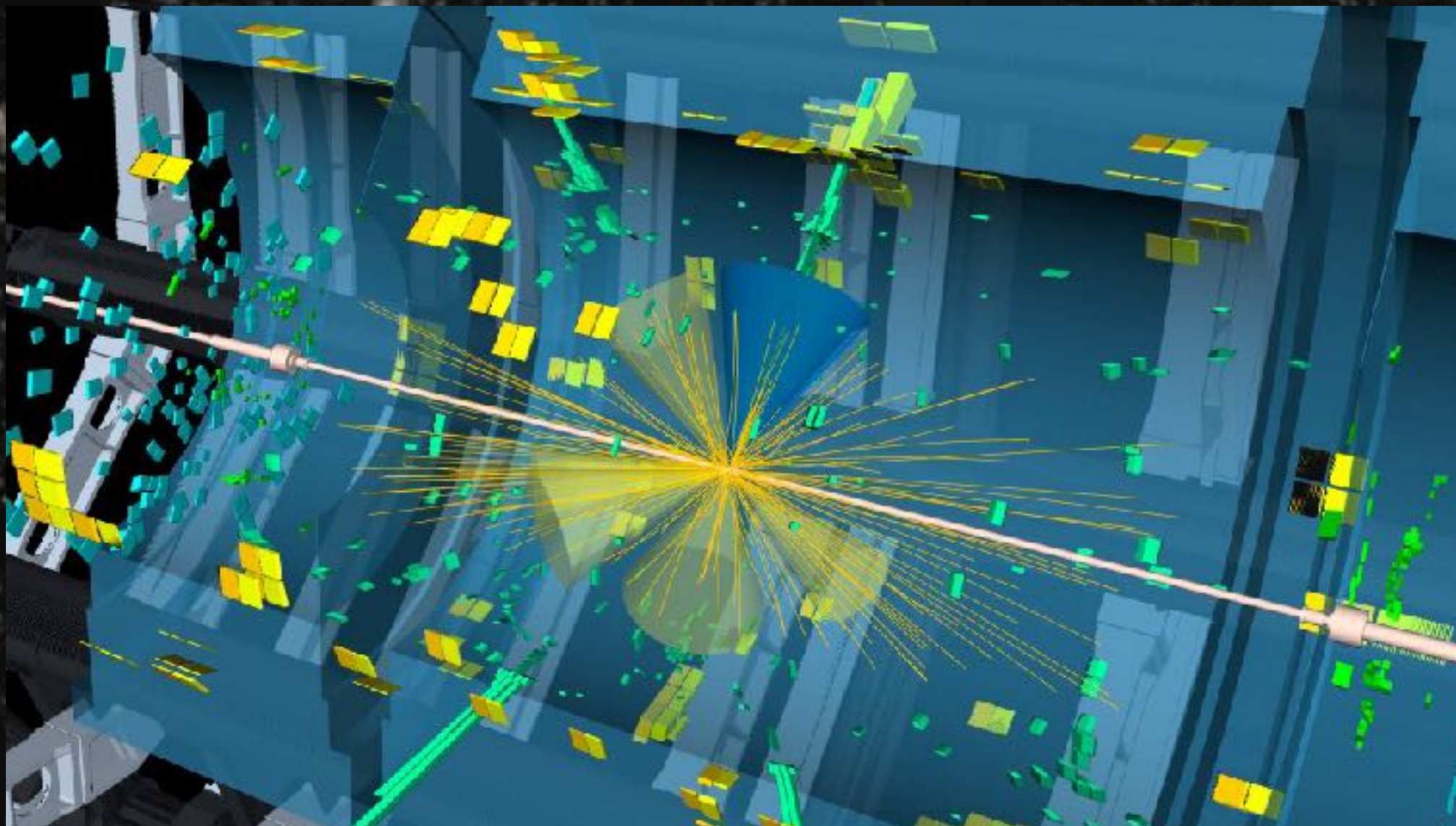
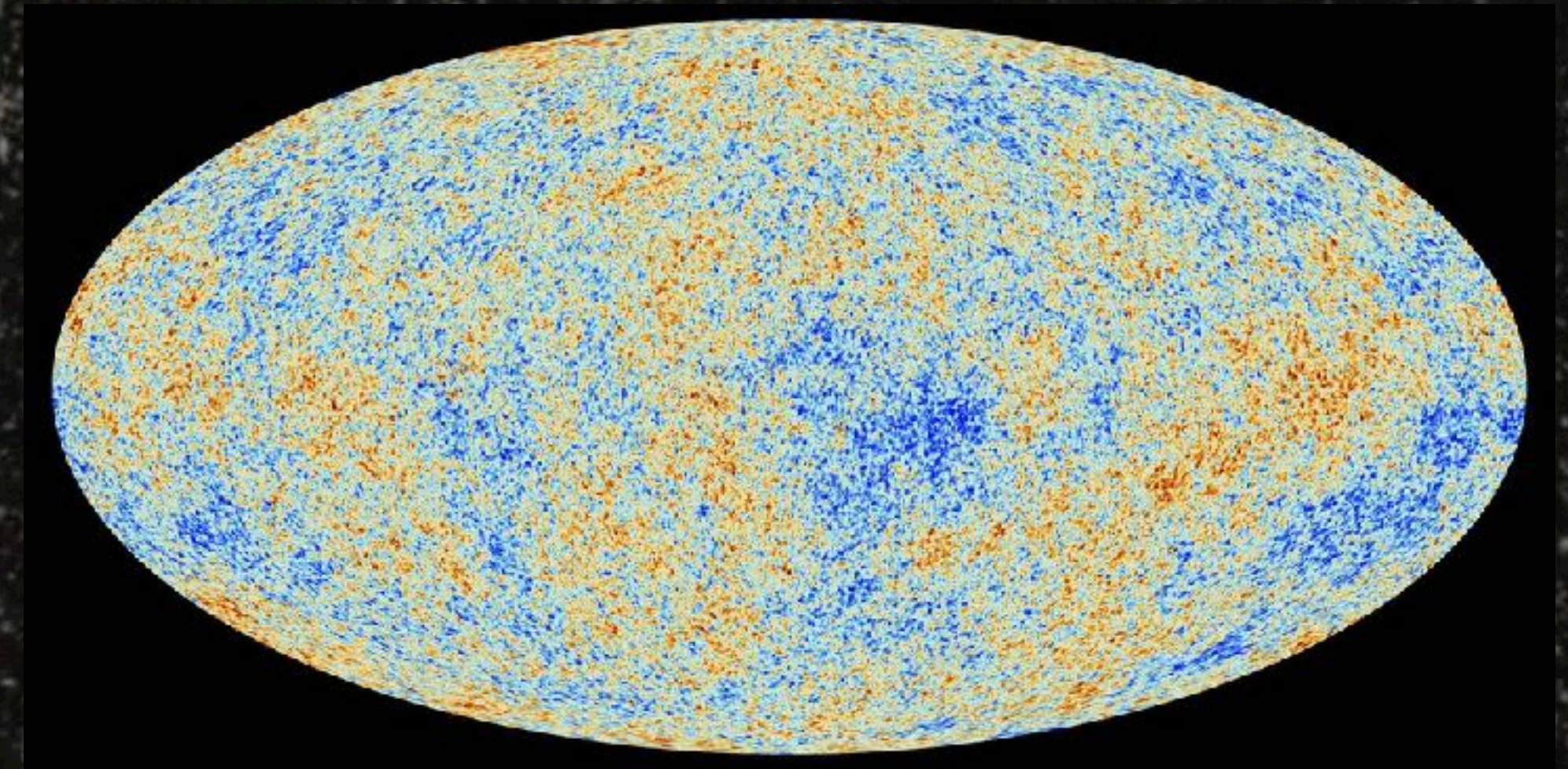
$$P(k)$$

overdensity: $\delta(\vec{x})$



the challenge

- CMB is linear - lucky again
- think how lucky we are that standard model is weakly coupled
- LSS is highly nonlinear on small scales



the payoff

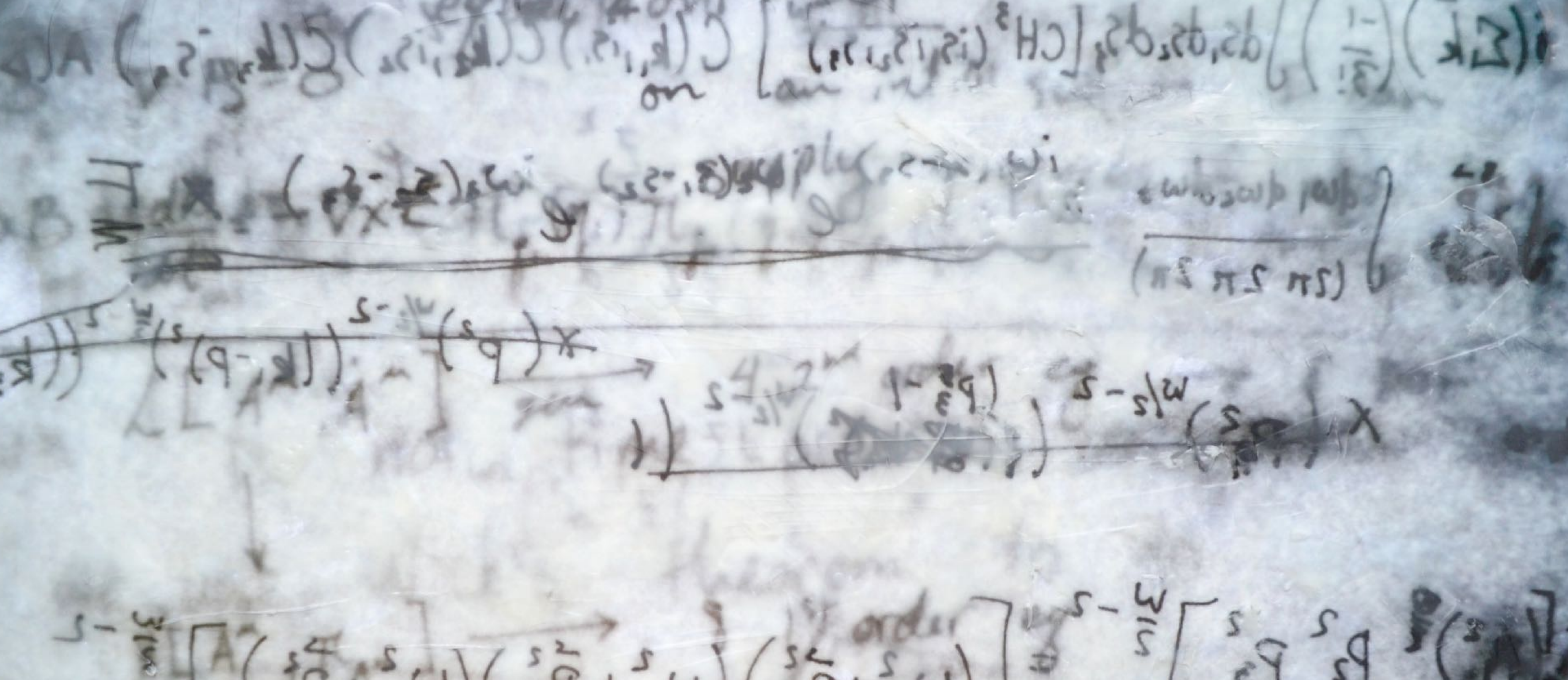
- but LSS is three-dimensional
- many more available modes

$$\text{error bars} \sim \frac{1}{\sqrt{N_{\text{modes}}}} \sim \frac{1}{\sqrt{V_{\text{survey}} k_{\text{max}}^3}}$$

largest k , smallest
length at which we
trust the theory

**next-generation surveys
(DESI, Euclid, MegaMapper,
PUMA, etc.) delivering data
in the next 2-25 years**

- interesting point from McDonald and Roy 2009:
 - we pay about \$1,000 per mode for the $\sim 10^6$ modes we have
 - so increasing k_{max} by a factor of 1.3 is essentially worth a billion dollars



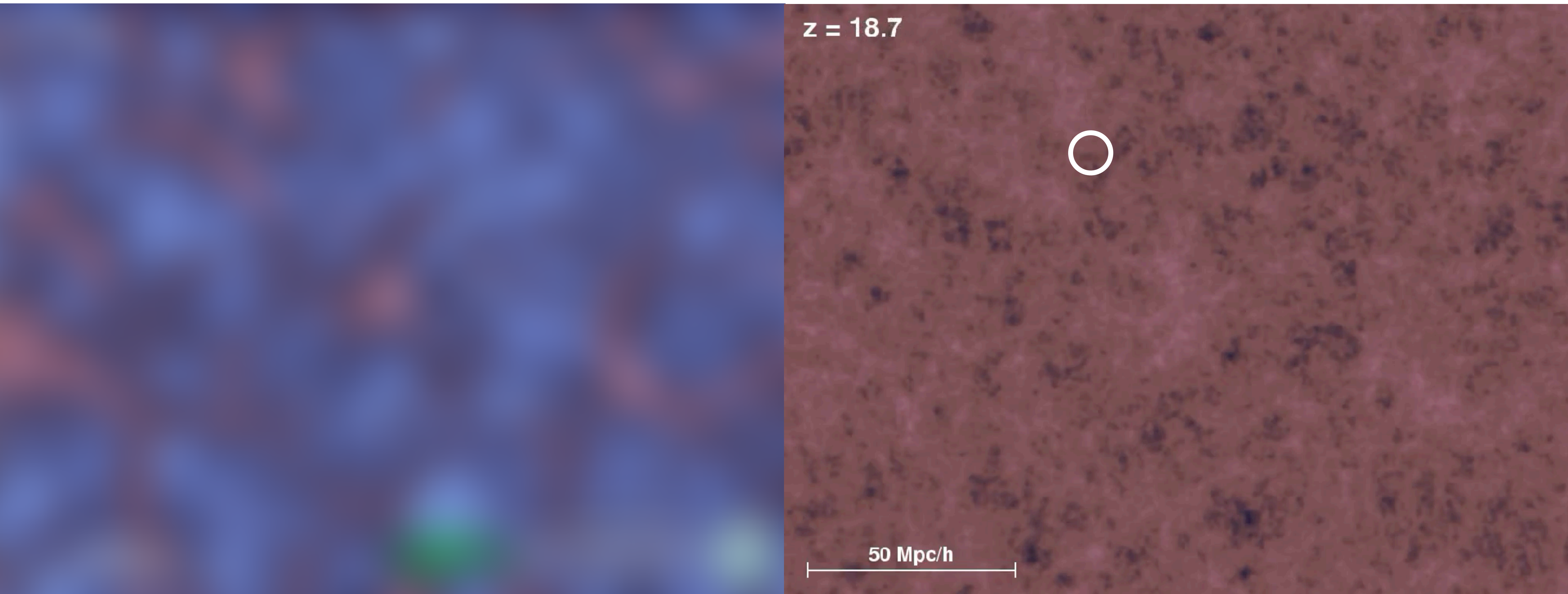
EFT of LSS

dark-matter clustering

Zel'dovich
Peebles
Blumenthal
Faber
Primack
Rees
Feldman
Kaiser
Peacock
...

dark-matter clustering

Max-Planck-Institute for Astrophysics,
Volker Springel



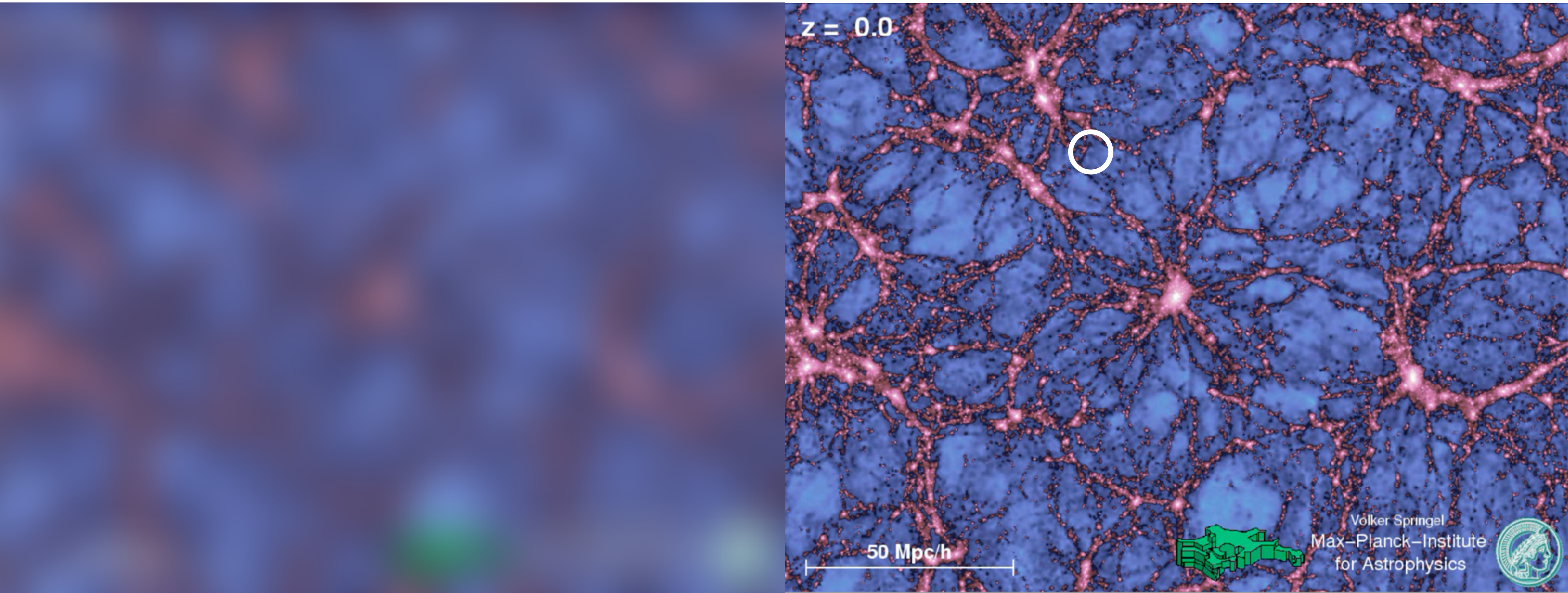
effective field theory of LSS

- controlled, analytic, perturbative expansion
- how can small scales affect large scales?

Baumann, Nicolis,
Senatore, Zaldarriaga 12

Carrasco, Hertzberg,
Senatore 12

Max-Planck-Institute for Astrophysics,
Volker Springel



effective field theory of LSS

Baumann, Nicolis,
Senatore, Zaldarriaga 12
Carrasco, Hertzberg,
Senatore 12

expanding universe: $a(t)$

time scale: $H = \dot{a}/a$

DM fluid variables: δ, v^i

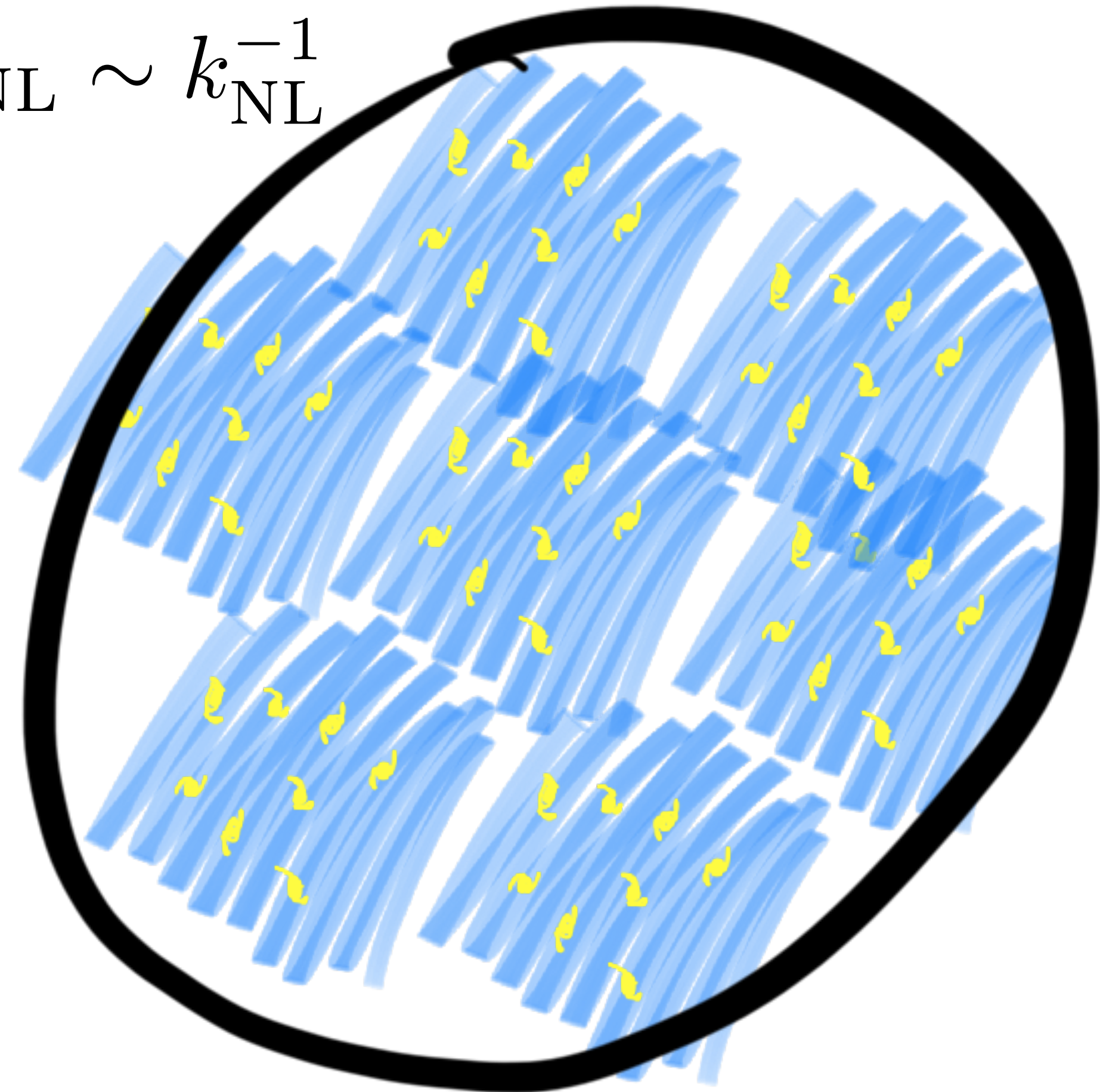
nonlinear scale

$$k_{\text{NL}}^{-1} \sim \frac{v}{aH}$$

EFT expansion param.

$$k/k_{\text{NL}} \lesssim 1$$

$$L_{\text{NL}} \sim k_{\text{NL}}^{-1}$$



$$L/L_{\text{NL}} \gtrsim 1$$

effective field theory of LSS

equations of motion for dark matter

Baumann, Nicolis,
Senatore, Zaldarriaga 12

Carrasco, Hertzberg,
Senatore 12

what do we actually solve?

continuity $[\delta, v]$ $= 0$

Euler $[\delta, v]$ $= \frac{\partial_i \partial_j}{k_{\text{NL}}^2} \tau_{\text{EFT}}^{ij}$

perfect fluid

EFT counterterms

effective field theory of LSS

equations of motion for dark matter

Baumann, Nicolis,
Senatore, Zaldarriaga 12

Carrasco, Hertzberg,
Senatore 12

what do we actually solve?

EOM local for π_S and π_V^i

$$\partial^2 \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2 \delta$$

$$a \mathcal{H} \delta' + \frac{\pi_S}{\bar{\rho}} = 0,$$

$$\mathcal{H} \pi'_S + \frac{4\mathcal{H}}{a} \pi_S + \frac{3}{2a} \bar{\rho} \Omega_m \mathcal{H}^2 \delta = -\frac{\partial_i \partial_j}{a} \left(\frac{2}{3} \frac{\bar{\rho}}{\Omega_m \mathcal{H}^2} \left(\partial_i \Phi \partial_j \Phi - \frac{1}{2} \delta_{ij} (\partial \Phi)^2 \right) + \frac{\pi^i \pi^j}{\rho} + \tau^{ij} \right),$$

$$\mathcal{H} \pi'_V^i + 4 \frac{\mathcal{H}}{a} \pi_V^i = -\frac{\epsilon^{ijk}}{a} \partial_j \partial_l \left(\frac{2}{3} \frac{\bar{\rho}}{\Omega_m \mathcal{H}^2} \partial_k \Phi \partial_l \Phi + \frac{\pi^k \pi^l}{\rho} + \tau^{kl} \right)$$

over and momentum density

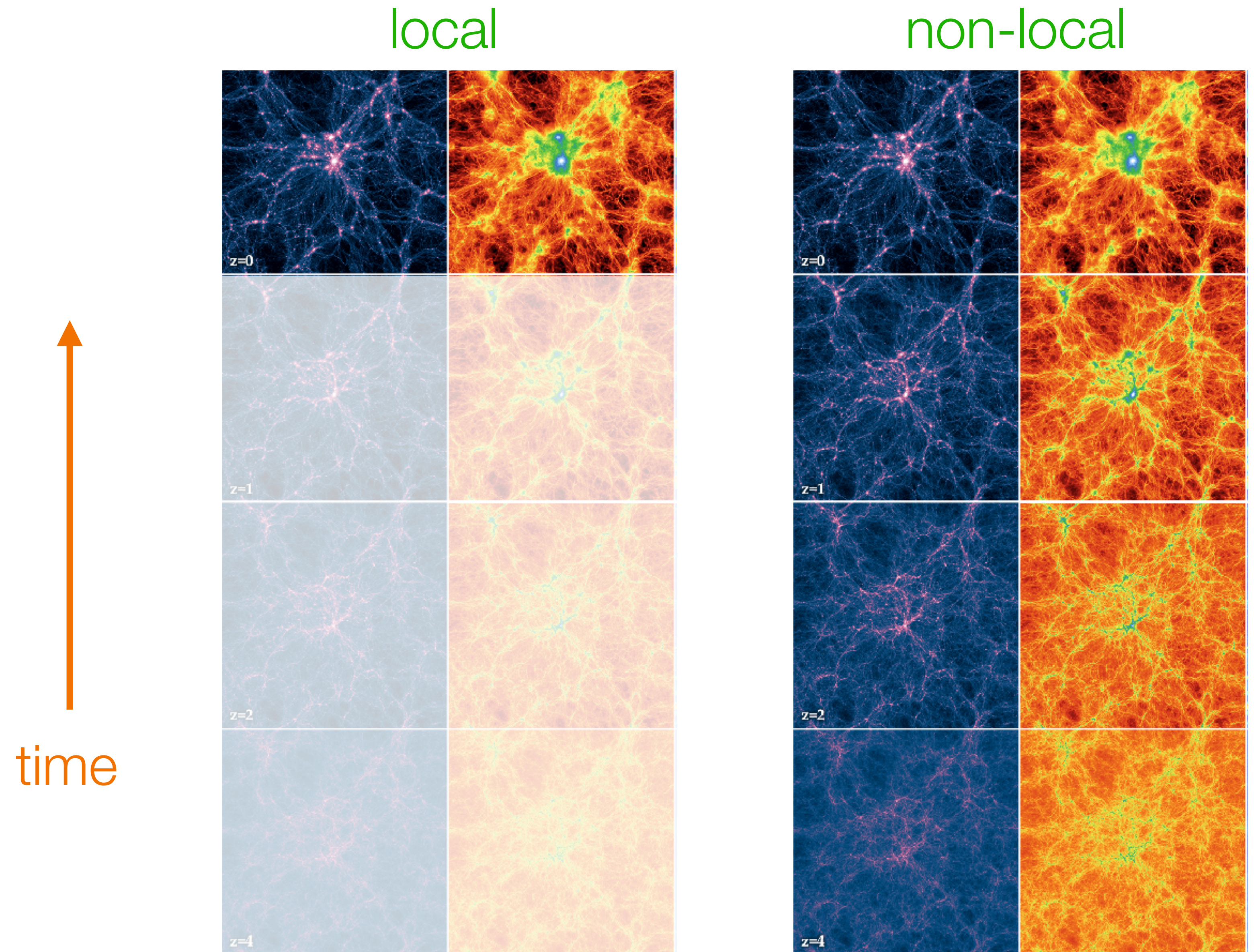
$$\begin{aligned} \delta(\vec{x}, a) &\equiv (\rho(\vec{x}, a) - \bar{\rho}(a)) / \bar{\rho}(a) & \pi_S &\equiv \partial_i \pi^i \\ \pi_V^i &\equiv \epsilon^{ijk} \partial_j \pi^k, & \pi_V^i &\equiv \epsilon^{ijk} \partial_j \pi^k, \\ \pi^i(\vec{x}, a) &\equiv \rho(\vec{x}, a) v^i(\vec{x}, a) & \pi^i &= \frac{\partial_i}{\partial^2} \pi_S - \epsilon^{ijk} \frac{\partial^j}{\partial^2} \pi_V^k \end{aligned}$$

non-local-in-time structure formation

the question

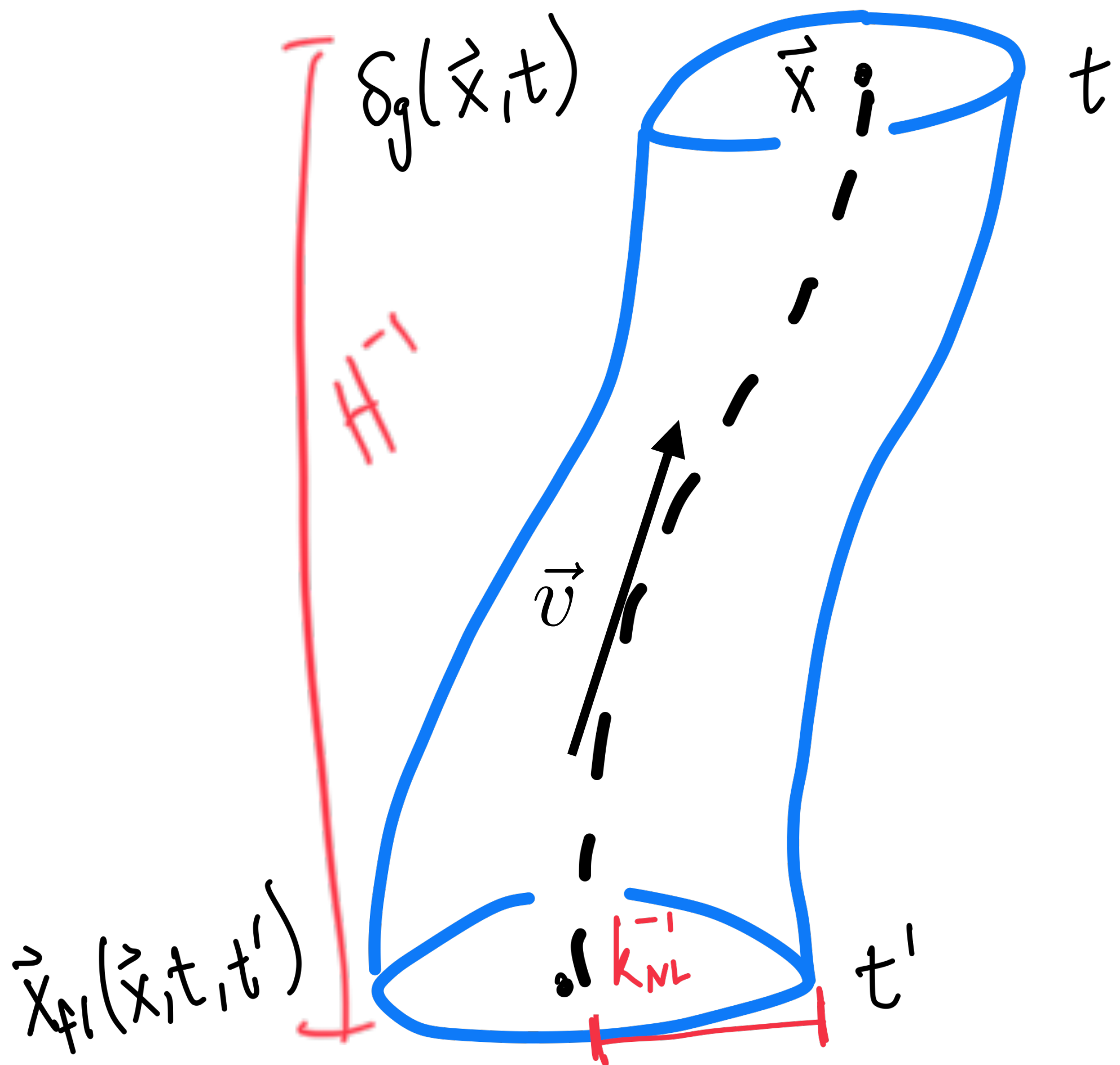
given two identical localized dark-matter configurations at a given time, will the same galaxies always form, or do we need to know the whole history of that configuration?

Illustris simulations



non-local-in-time structure formation

$$\tau_{ij}(\vec{x},t) = f_{\text{very complicated}} \left(\left\{ H, \Omega_m, \dots, m_{\text{dm}}, g_{\text{ew}}, \dots, \rho_{\text{dm}}, \partial_i v^j_{\text{dm}}, \dots \right\}_{\text{past tube } (\vec{x},t)} \right)_{ij}$$



Carrasco, Foreman,
Green, Senatore 13

Senatore 14

Desjacques, Jeong,
Schmidt 16

Donath, ML, Senatore, 23

Anastasiou, Favorito, ML,
Senatore, Zheng 25

non-local-in-time structure formation

$$\tau_{ij}(\vec{x}, t) = f_{\text{very complicated}} \left(\left\{ H, \Omega_m, \dots, m_{\text{dm}}, g_{\text{ew}}, \dots, \rho_{\text{dm}}, \partial_i v_{\text{dm}}^j, \dots \right\}_{\text{past tube}(\vec{x}, t)} \right)_{ij}$$

$$\tau_{ij}(\vec{x}, t) = \sum_{n=0}^{\infty} \int_{\text{past tube}(\vec{x}, t)} d^4 x_1 \dots \int_{\text{past tube}(\vec{x}, t)} d^4 x_n \sum_{\text{all subsets } \{\mathcal{O}_1, \dots, \mathcal{O}_n\} \subset \{\mathcal{O}\}} \left\{ \frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta \mathcal{O}_1(x_1) \dots \delta \mathcal{O}_n(x_n)} \Big|_0 \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \right\}_{ij},$$

$$\frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta \mathcal{O}_1(x_1) \dots \delta \mathcal{O}_n(x_n)} \Big|_0 \simeq \left\langle \frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta \mathcal{O}_1(x_1) \dots \delta \mathcal{O}_n(x_n)} \Big|_0 \right\rangle + \epsilon_{1, \dots, n}(x_1, \dots, x_n).$$

Carrasco, Foreman,
Green, Senatore 13

Senatore 14

Donath, ML, Senatore, 23

Anastasiou, Favorito, ML,
Senatore, Zheng 25

non-local-in-time structure formation

Carrasco, Foreman,
Green, Senatore 13

Senatore 14

Donath, ML, Senatore, 23

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Senatore, Zheng 25

$$\vec{x}_{\text{fl}}(\vec{x}, t, t') = \vec{x} + \int_t^{t'} \frac{dt_1}{a(t_1)} \vec{v}(\vec{x}_{\text{fl}}(\vec{x}, t, t_1), t_1)$$

$$\begin{aligned} \tau_{ij}(\vec{x}, t) &\simeq \sum_{n=0}^{\infty} \int dt_1 \dots \int dt_n \sum_{\text{all subsets } \{\mathcal{O}_1, \dots, \mathcal{O}_n\} \subset \{\mathcal{O}\}} \\ &\times \left\{ (K_{1, \dots, n}(t, t_1, \dots, t_n) + \epsilon_{1, \dots, n}(t, (\vec{x}_{\text{fl}}(\vec{x}, t, t_1), t_1), \dots, (\vec{x}_{\text{fl}}(\vec{x}, t, t_n), t_n))) \right. \\ &\quad \left. + \frac{\partial}{k_{\text{NL}}} \epsilon_{1, \dots, n, \partial}(t, (\vec{x}_{\text{fl}}(\vec{x}, t, t_1), t_1), \dots, (\vec{x}_{\text{fl}}(\vec{x}, t, t_n), t_n)) + \dots \right) \\ &\times \mathcal{O}_1(\vec{x}_{\text{fl}}(\vec{x}, t, t_1), t_1) \dots \mathcal{O}_n(\vec{x}_{\text{fl}}(\vec{x}, t, t_n), t_n) \}_{ij} \end{aligned}$$

non-local-in-time structure formation

Carrasco, Foreman,
Green, Senatore 13

Senatore 14

Donath, ML, Senatore, 23

Anastasiou, Favorito, ML,
Senatore, Zheng 25

$$\begin{aligned} \tau_{ij}(\vec{x}, t) = & \frac{\Omega_m \mathcal{H}^2 \bar{\rho}}{k_{\text{NL}}^2} \sum_l \int^t dt' H(t') \left(c_{1, \mathcal{O}_l}(t, t') \mathcal{O}_{1,l}(\vec{x}_{\text{fl}}(\vec{x}, t, t'), t')_{ij} + \right. \\ & + \int^t dt'' H(t'') \left(c_{2, \mathcal{O}_l}(t, t', t'') \mathcal{O}_{2,l}((\vec{x}_{\text{fl}}(\vec{x}, t, t'), t'), (\vec{x}_{\text{fl}}(\vec{x}, t, t''), t''))_{ij} \right. \\ & \left. \left. + \int^t dt''' H(t''') c_{3, \mathcal{O}_l}(t, t', t'', t''') \mathcal{O}_{3,l}((\vec{x}_{\text{fl}}(\vec{x}, t, t'), t'), (\vec{x}_{\text{fl}}(\vec{x}, t, t''), t''), (\vec{x}_{\text{fl}}(\vec{x}, t, t'''), t'''))_{ij} \right) \right) , \end{aligned}$$

$$\mathcal{O}_{1,l} \in \left\{ r_{ij}, p_{ij}, \delta_{ij}^K \delta, \delta_{ij}^K \theta, \delta_{ij}^K \frac{\partial^2}{k_{\text{NL}}^2} \delta \right\}$$

$$r_{ij} = \partial_i \partial_j \Phi, \quad p_{ij} = \partial_i v^j$$

$$\begin{aligned} \mathcal{O}_{2,l} \in \left\{ \delta_{ij}^K \delta^2, \delta_{ij}^K \theta^2, \delta_{ij}^K \delta \theta, r_{ij} \delta, p_{ij} \delta, r_{ij} \theta, p_{ij} \theta, r_{ik} r_{kj}, p_{ik} p_{kj}, \right. \\ \left. \frac{1}{2} (r_{ik} p_{kj} + r_{jk} p_{ki}), \text{Tr}(r^2) \delta_{ij}^K, \text{Tr}(p^2) \delta_{ij}^K, \text{Tr}(rp) \delta_{ij}^K \right\} \end{aligned}$$

$$\mathcal{O}_{3,l} \in \left\{ r_{ik} r_{km} r_{mj}, \text{Tr}(r^2) r_{ij}, \text{Tr}(r^3) \delta_{ij}^K, r_{ik} r_{kj} \delta, \text{Tr}(r^2) \delta \delta_{ij}^K, r_{ij} \delta^2, \delta^3 \delta_{ij}^K \right\}$$

non-local-in-time structure formation

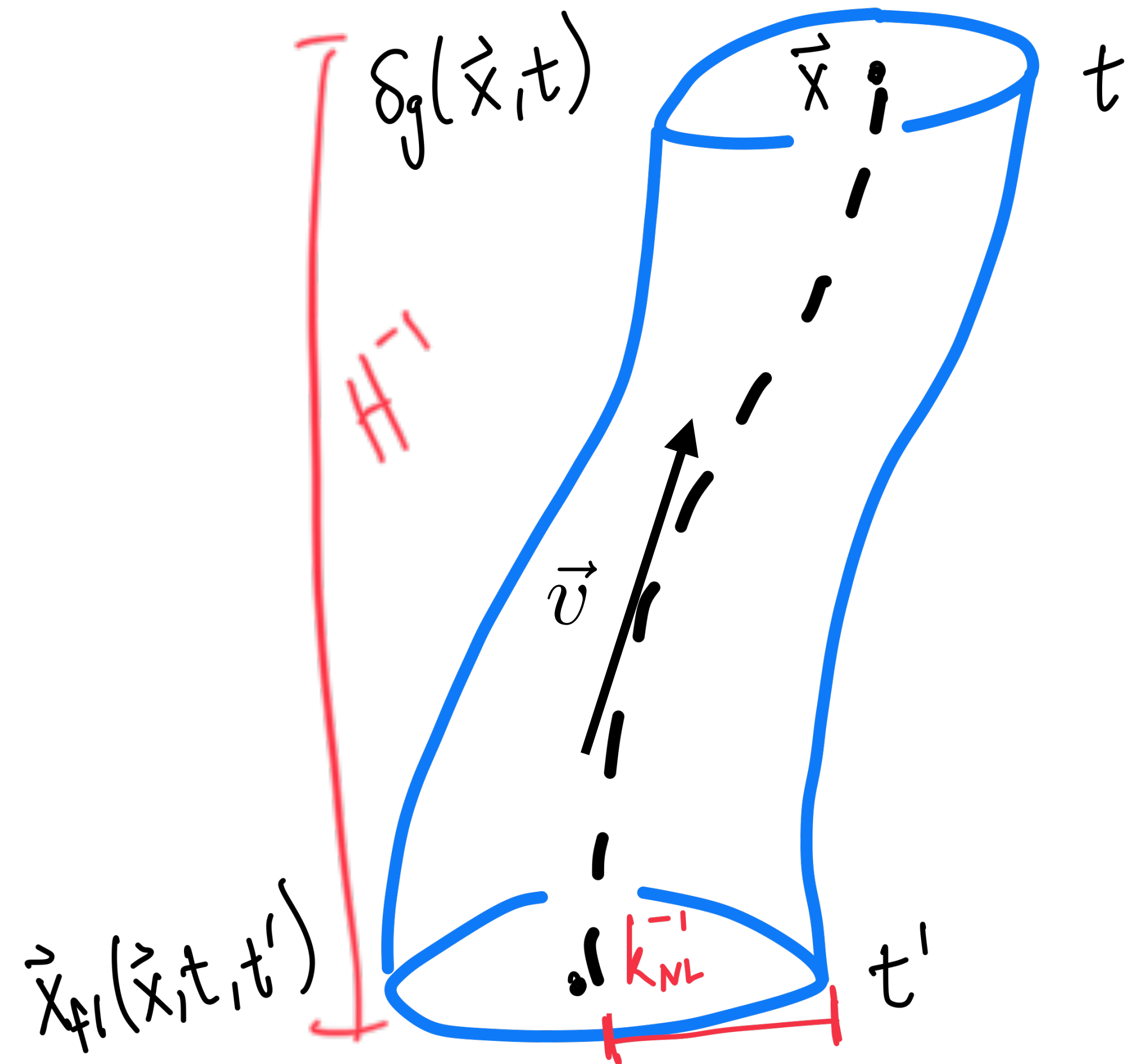
$$\tau_{(n)}^{ij}(\vec{x}, t) = \sum_{\Gamma \in \Gamma_n} c_{\Gamma}(t) [\mathbb{C}_{\Gamma}^{ij}]^{(n)}(\vec{x}, t)$$

**18 total at 3rd order;
17 for local-in-time**

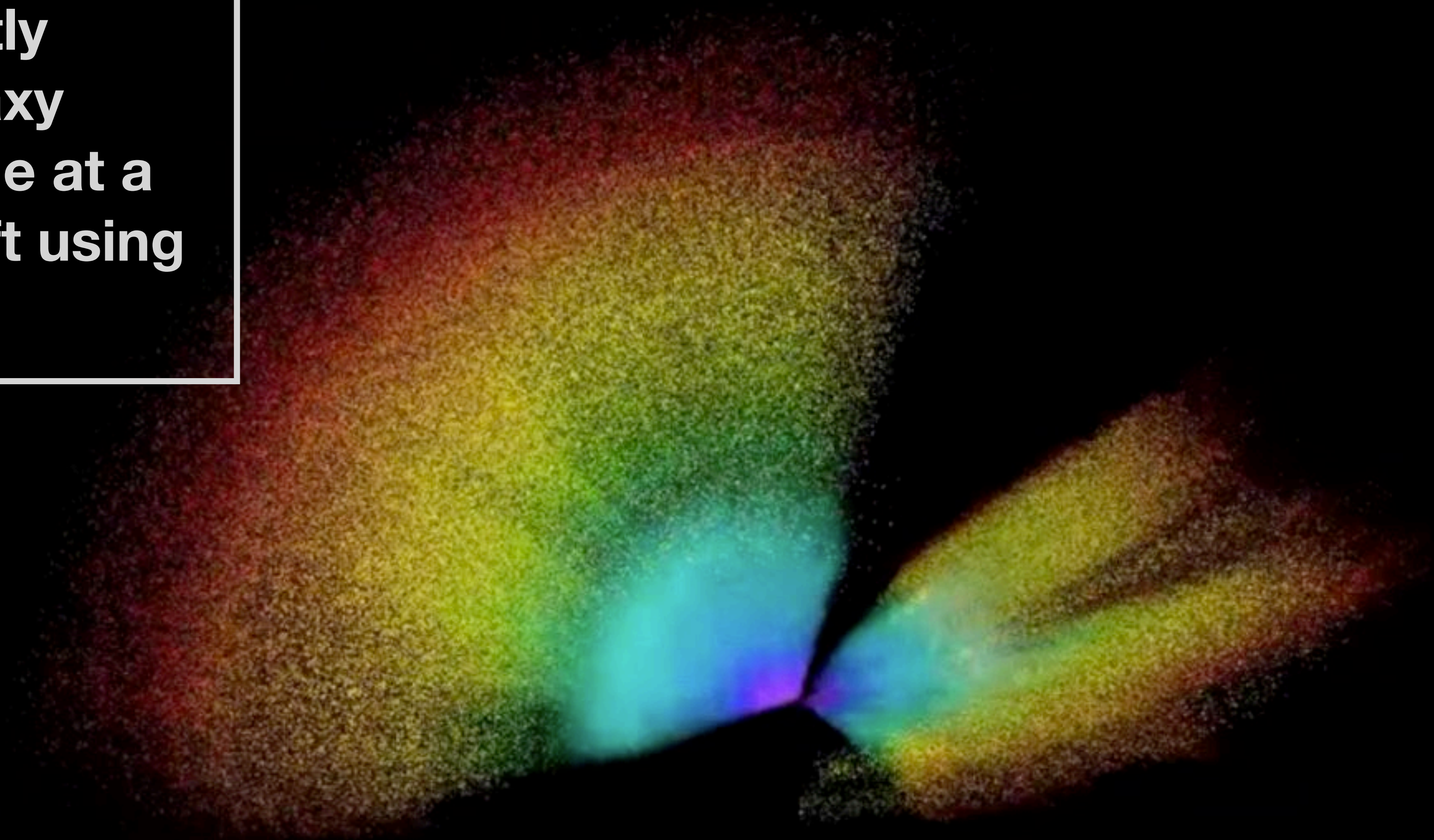
galaxy bias
@ 5th order
non-local: 29
local: 26

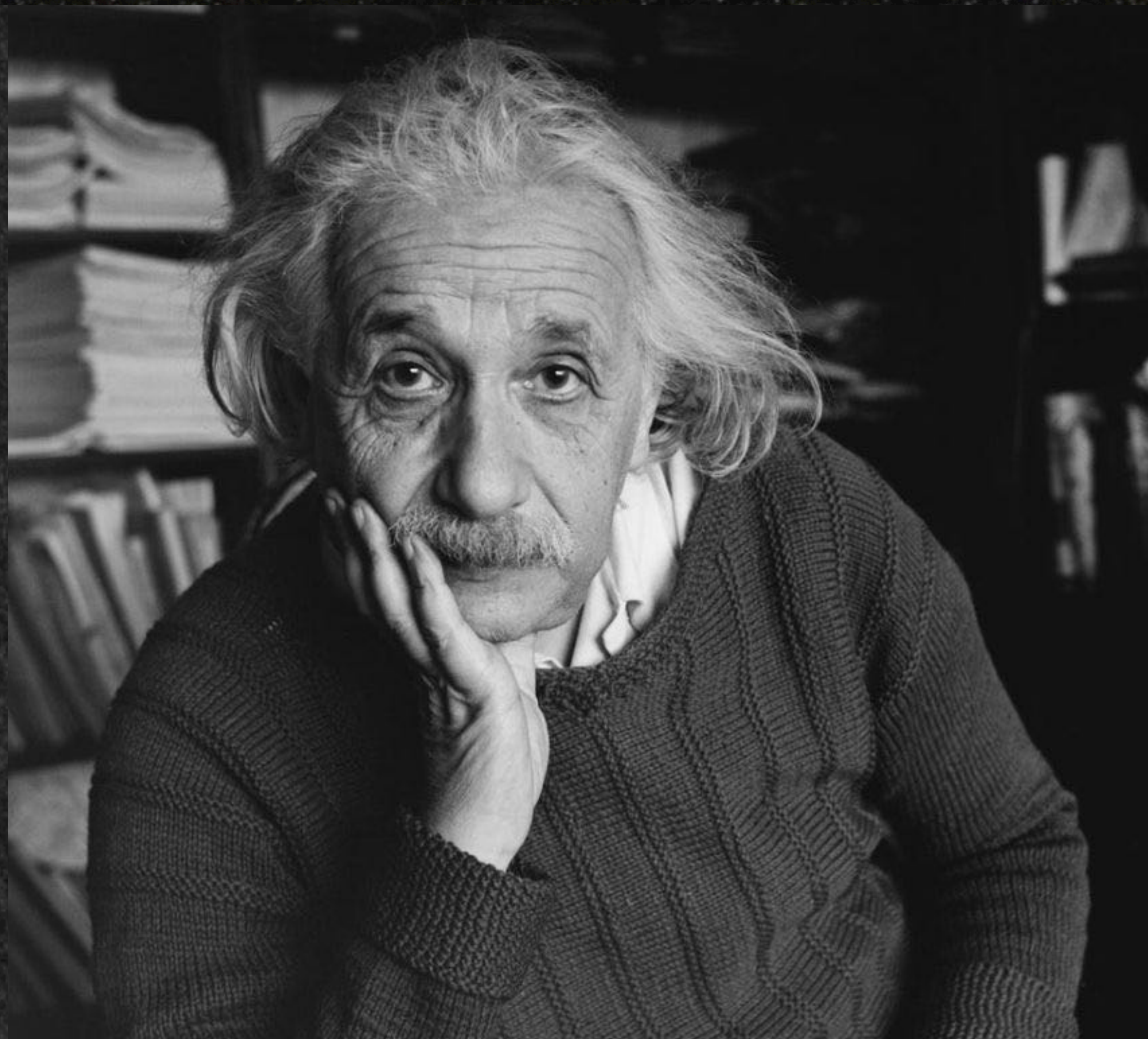
the galaxy-clustering signal at fifth order
is sensitive to whether or not galaxies
form on time scales of order Hubble

Donath, ML, Senatore, 23
Anastasiou, Favorito, ML,
Senatore, Zheng 25
Ansari, Banerjee, Jain,
Padhyegurjar 24
Vlah, Chisari, Schmidt 19



**we can directly
measure galaxy
formation time at a
single redshift using
field theory.**





solution of DM EOM

Anastasiou, Favorito, ML,
Senatore, Zheng 25

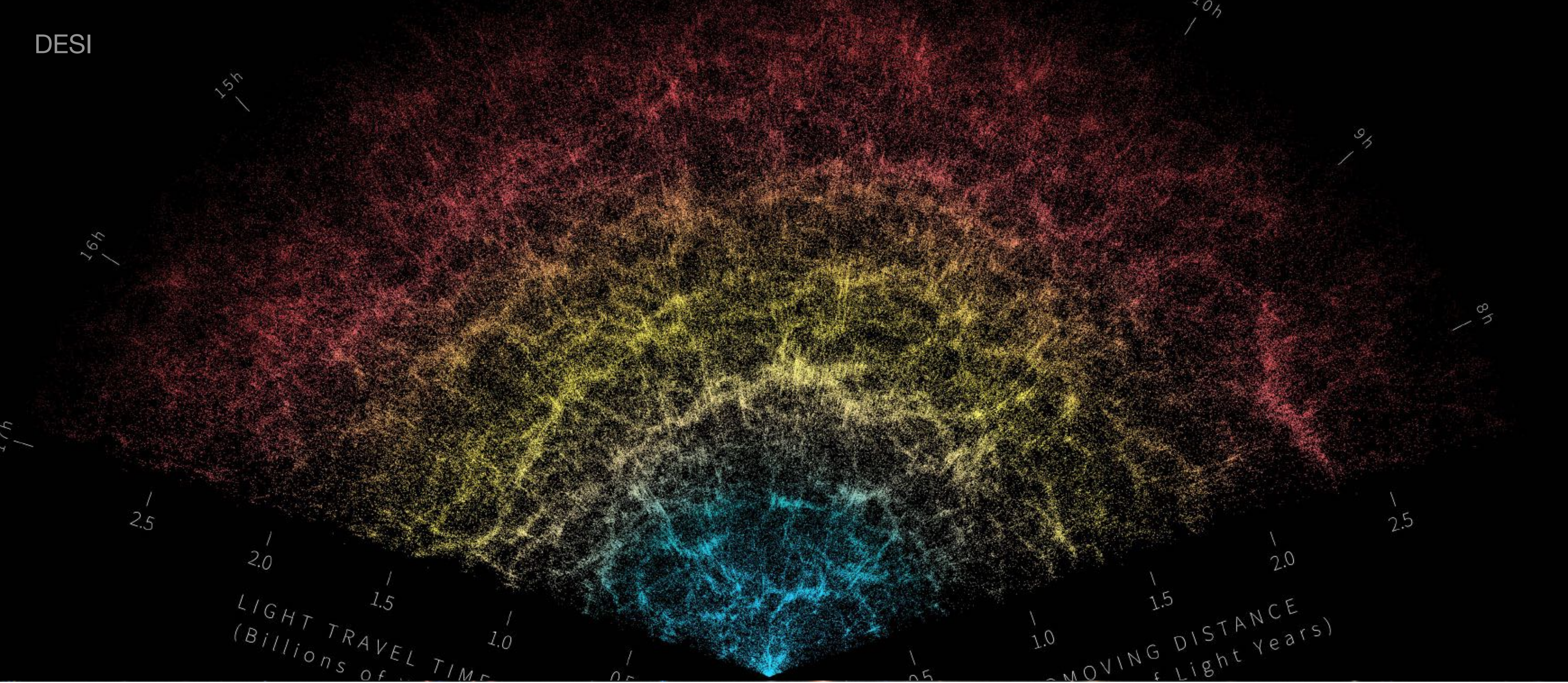
with τ^{ij} , solve EOM for δ

$$\delta_{\text{ct}}^{(3)}(a) = \frac{D(a)^5 \partial_i \partial_j}{26} \left[3 \left(\frac{\partial_i \tilde{\delta}_{\text{ct}}^{(2)}}{\partial^2} \frac{\partial_j \tilde{\delta}^{(1)}}{\partial^2} + \frac{\partial_i \tilde{\delta}_{\text{ct}}^{(1)}}{\partial^2} \frac{\partial_j \tilde{\delta}^{(2)}}{\partial^2} \right) \right. \\ \left. - \frac{3}{2} \delta_{ij} \left(\frac{\partial_k \tilde{\delta}_{\text{ct}}^{(2)}}{\partial^2} \frac{\partial_k \tilde{\delta}^{(1)}}{\partial^2} + \frac{\partial_k \tilde{\delta}_{\text{ct}}^{(1)}}{\partial^2} \frac{\partial_k \tilde{\delta}^{(2)}}{\partial^2} \right) + 2 \left(\tilde{\pi}_{\text{ct},(2)}^i \tilde{\pi}_{(1)}^j + \tilde{\pi}_{\text{ct},(1)}^i \tilde{\pi}_{(2)}^j \right) \right. \\ \left. - 2 \tilde{\pi}_{\text{ct},(1)}^i \tilde{\pi}_{(1)}^j \tilde{\delta}^{(1)} - \tilde{\pi}_{(1)}^i \tilde{\pi}_{(1)}^j \tilde{\delta}_{\text{ct}}^{(1)} + \frac{1}{k_{\text{NL}}^2} \tilde{\tau}_{\text{ct},(3)}^{ij} \right], \quad \text{vorticity generated, contributes to DM at 3rd order}$$

lower-order counterterms
must appear consistently

$$\omega_{(2)}^i \sim \epsilon^{ijk} \partial_j \partial_l \tau_{(2)}^{lk}$$

DESI



dark matter at two loops

two-loop DM power spectrum

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P(k)$$

$$\delta = \delta_{\text{no-ct}} + \delta_{\text{ct}}$$

$$\delta_{\text{no-ct}} = \delta^{(1)} + \dots + \delta^{(5)}$$

$$\delta_{\text{ct}} = \delta_{\text{ct}}^{(1)} + \delta_{\text{ct}}^{(2)} + \delta_{\text{ct}}^{(3)}$$

$$P(k) = P_{\text{no-ct}}(k) + P_{\text{ct}}(k)$$

Anastasiou, Favorito, ML,
Senatore, Zheng 25

Bakx, Rubira, Chisari,
Vlah 25

Garny, Taule 23

Garny, Taule 22

Fasiello, Fujita,
Vlah 22

Konstandin, Porto,
Rubira 19

Foreman, Perrier,
Senatore 15

Carrasco, Foreman,
Green, Senatore 13

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

$$\mathcal{P}_{\text{no-ct}}(k) = \mathcal{P}_{\text{lin.}}(k) + \mathcal{P}_{1\text{-loop}}(k) + \mathcal{P}_{2\text{-loop}}(k)$$

linear power spectrum/propagator

$$\mathcal{P}_{\text{lin.}}(k) \equiv \begin{array}{c} \mathcal{P}_{\text{lin}}(k) \\ \longleftarrow \bullet \longrightarrow \end{array}$$

$$p_{22}(\vec{k}, \vec{q}) = 2 \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(|\vec{k} - \vec{q}|) \left[F_2(\vec{q}, \vec{k} - \vec{q}) \right]^2,$$

one-loop power spectrum

$$p_{13}(\vec{k}, \vec{q}) = 6 \mathcal{P}_{\text{lin.}}(k) \mathcal{P}_{\text{lin.}}(q) F_3(\vec{q}, -\vec{q}, \vec{k}).$$

$$\mathcal{P}_{13}(k) \equiv \begin{array}{c} \mathcal{P}_{\text{lin}}(q) \\ \bullet \\ \text{---} \bullet \text{---} \\ \text{---} \square \text{---} \\ \text{---} \bullet \text{---} \\ \mathcal{P}_{\text{lin}}(k) \end{array}$$

$k \quad F_3^{(s)} \quad k$

$$\mathcal{P}_{22}(k) \equiv \begin{array}{c} \mathcal{P}_{\text{lin}}(q) \\ \bullet \\ \text{---} \square \text{---} \\ \text{---} \square \text{---} \\ \text{---} \bullet \text{---} \\ \mathcal{P}_{\text{lin}}(|\vec{k} - \vec{q}|) \end{array}$$

$k \quad F_2^{(s)} \quad F_2^{(s)} \quad k$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

two-loop power spectrum

$$\mathcal{P}_{15}(k) \equiv \text{diagram},$$

$$\mathcal{P}_{42}(k) \equiv \text{diagram},$$

$$\mathcal{P}_{33}^{(\text{I})}(k) \equiv \text{diagram},$$

$$\mathcal{P}_{33}^{(\text{II})}(k) \equiv \text{diagram}.$$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

two-loop power spectrum

$$p_{15}(\vec{k}, \vec{p}, \vec{q}) = 30 \mathcal{P}_{\text{lin.}}(k) \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(p) F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) ,$$

$$p_{42}(\vec{k}, \vec{p}, \vec{q}) = 24 \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(p) \mathcal{P}_{\text{lin.}}(|\vec{k} - \vec{q}|) \\ \times F_2(\vec{q}, \vec{k} - \vec{q}) F_4(-\vec{q}, \vec{q} - \vec{k}, \vec{p}, -\vec{p}) ,$$

$$p_{33}^{(\text{I})}(\vec{k}, \vec{p}, \vec{q}) = 9 \mathcal{P}_{\text{lin.}}(k) \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(p) F_3(\vec{k}, \vec{q}, -\vec{q}) F_3(-\vec{k}, \vec{p}, -\vec{p}) ,$$

$$p_{33}^{(\text{II})}(\vec{k}, \vec{p}, \vec{q}) = 6 \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(p) \mathcal{P}_{\text{lin.}}(|\vec{k} - \vec{q} - \vec{p}|) \\ \times F_3(\vec{q}, \vec{p}, \vec{k} - \vec{q} - \vec{p}) F_3(-\vec{q}, -\vec{p}, -\vec{k} + \vec{q} + \vec{p}) .$$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

counterterm diagrams

$$\mathcal{P}_{13}^{\text{ct}}(k, a) = 2D(a)^4 \langle \tilde{\delta}_{\text{ct}}^{(1)}(\vec{k}) \tilde{\delta}^{(1)}(\vec{k}') \rangle' ,$$

$$\mathcal{P}_{15}^{\text{ct}}(k, a) = 2D(a)^6 \left(\langle \tilde{\delta}_{\text{ct}}^{(3)}(\vec{k}) \tilde{\delta}^{(1)}(\vec{k}') \rangle' + \langle \tilde{\delta}_{\partial^2 \text{ct}}^{(1)}(\vec{k}) \tilde{\delta}^{(1)}(\vec{k}') \rangle' \right) ,$$

$$\mathcal{P}_{33}^{\text{ct}}(k, a) = D(a)^6 \left(2 \langle \tilde{\delta}_{\text{ct}}^{(1)}(\vec{k}) \tilde{\delta}^{(3)}(\vec{k}') \rangle' + \langle \tilde{\delta}_{\text{ct}}^{(1)}(\vec{k}) \tilde{\delta}_{\text{ct}}^{(1)}(\vec{k}') \rangle' \right) ,$$

$$\mathcal{P}_{42}^{\text{ct}}(k, a) = 2D(a)^6 \langle \tilde{\delta}_{\text{ct}}^{(2)}(\vec{k}) \tilde{\delta}^{(2)}(\vec{k}') \rangle' ,$$

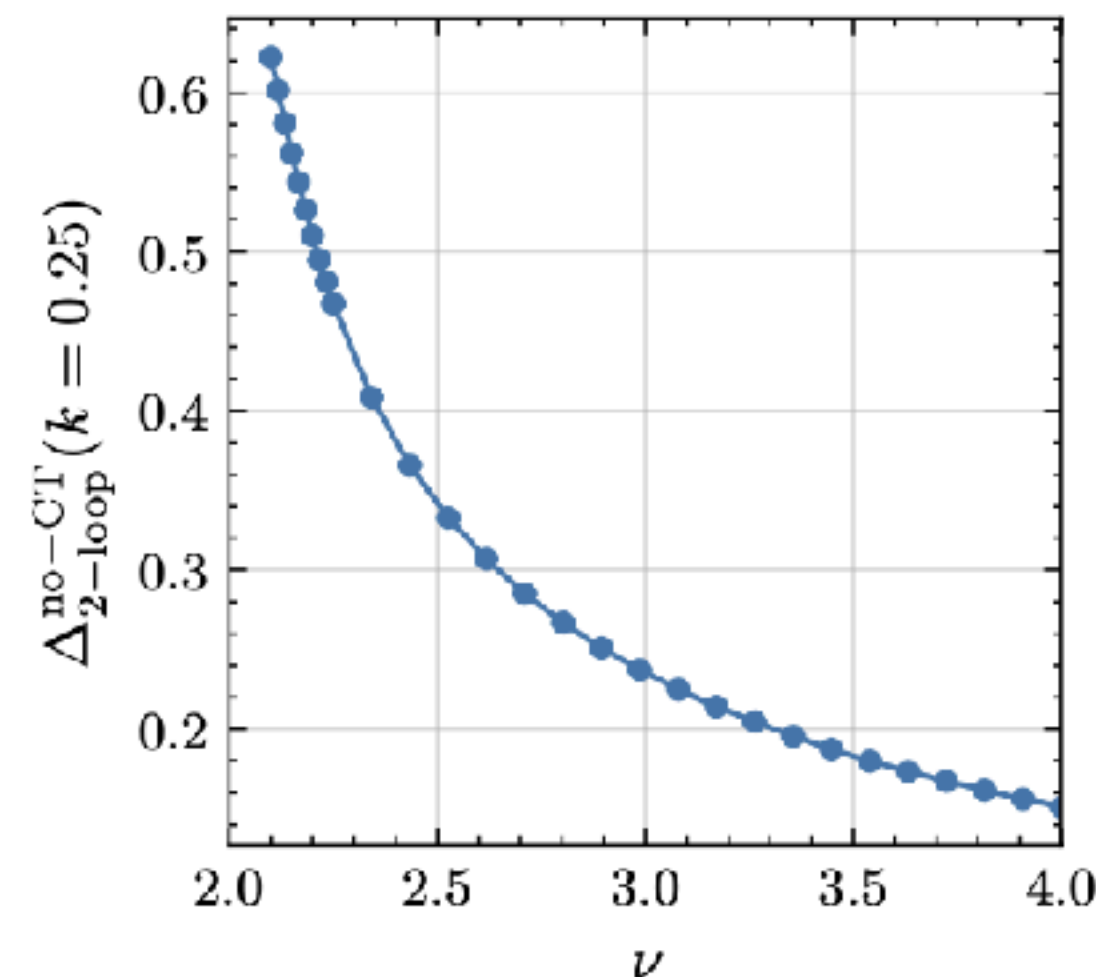
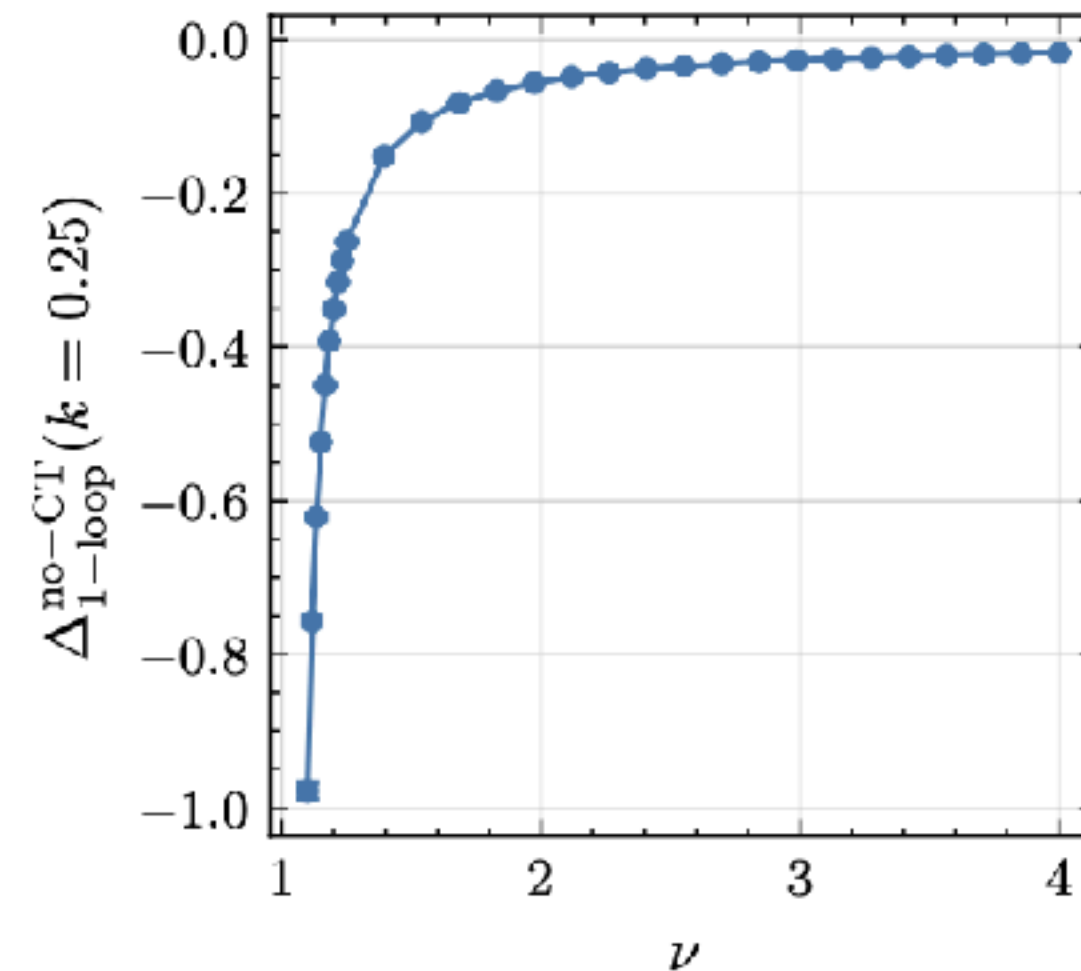
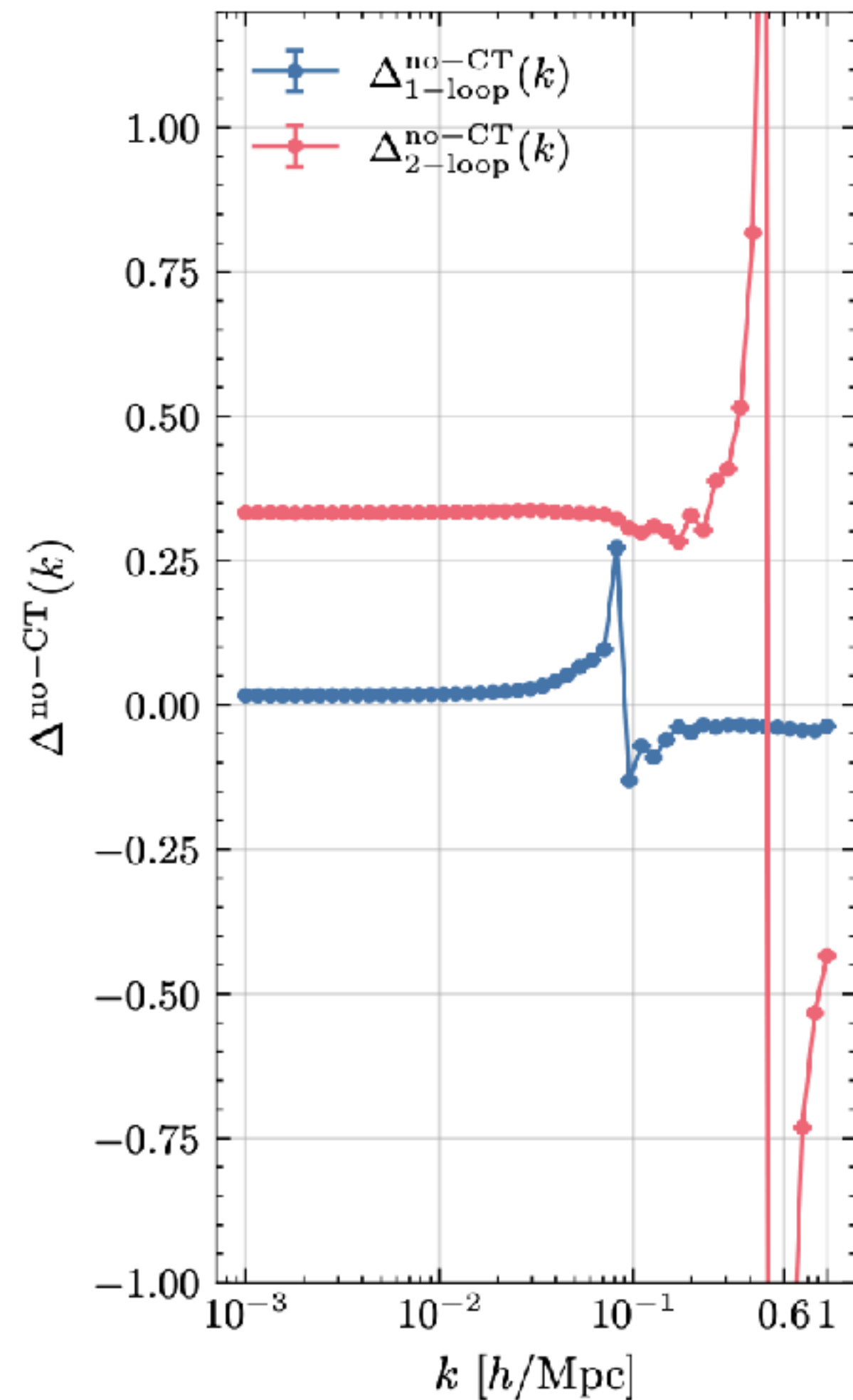
**in terms of 18
EFT coefficients
in τ^{ij} we found
before**

after contractions, 8 independent EFT coefficients

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

UV regulation and renormalization



measure contribution
from UV

$$\Delta_{n\text{-loop}} = \frac{\int_{1.3}^{\infty} p_{n\text{-loop}}^{\text{no-ct}}}{\int_0^{\infty} p_{n\text{-loop}}^{\text{no-ct}}}$$

not UV divergent, but
slightly UV dependent

$$\mathcal{P}_{\text{lin.}}(k; \nu) = \mathcal{P}_{\text{lin.}}(k) \theta(q_{UV} - k) + \mathcal{P}_{\text{lin.}}(q_{UV}) \left(\frac{q_{UV}}{k} \right)^{\nu} \theta(k - q_{UV}).$$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
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UV regulation and renormalization one loop

$$\begin{array}{c}
 \text{Diagram 1: } \mathcal{P}_{\text{lin}}(q) \text{ loop with external momentum } k \text{ and } F_3^{(s)} \text{ vertex} \\
 = \\
 \text{Diagram 2: } \mathcal{P}_{\text{lin}}(q) \text{ loop with external momentum } k \text{ and } F_3^{(s), \text{UV-reg.}} \text{ vertex} \\
 + \\
 \text{Diagram 3: } \mathcal{P}_{\text{lin}}(q) \text{ loop with external momentum } k \text{ and } F_3^{(s), \text{UV}} \text{ vertex}
 \end{array}$$

UV regulated
UV limit

$$\begin{array}{c}
 \text{Diagram 3: } \mathcal{P}_{\text{lin}}(q) \text{ loop with external momentum } k \text{ and } F_3^{(s), \text{UV}} \text{ vertex} \\
 = C_{13}(M) k^2 \text{ } \mathcal{P}_{\text{lin}}(k) \text{ } ,
 \end{array}$$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

UV regulation and renormalization

two loops

$$\mathcal{P}_{2\text{-loop}} = \mathcal{P}_{2\text{-loop}}^{\text{UV-reg.}} + \mathcal{P}_{2\text{-loop}}^{\text{UV-(1)}} + \mathcal{P}_{2\text{-loop}}^{\text{UV-(2)}}$$

single and double
hard limits

regulated, locally
UV finite

$$\mathcal{P}_{2\text{-loop}}^{\text{UV-reg.}} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4}.$$

UV limits

$$\mathcal{P}_{2\text{-loop}}^{\text{UV-(1)}} = 2 \times \text{diagram 1} + \text{diagram 2} + 2 \times \text{diagram 3} + \mathcal{P}_{2\text{-loop}}^{\text{UV-(2)}} = \text{diagram 4} + \text{diagram 5}.$$

Anastasiou, Haindl, Sterman,
Yang, Zeng 20
Anastasiou, Sterman 22

Herzog, Ruijl 17
Anastasiou, Sterman 18

Anastasiou, Karlen,
Sterman, Venkata 24

two-loop DM power spectrum

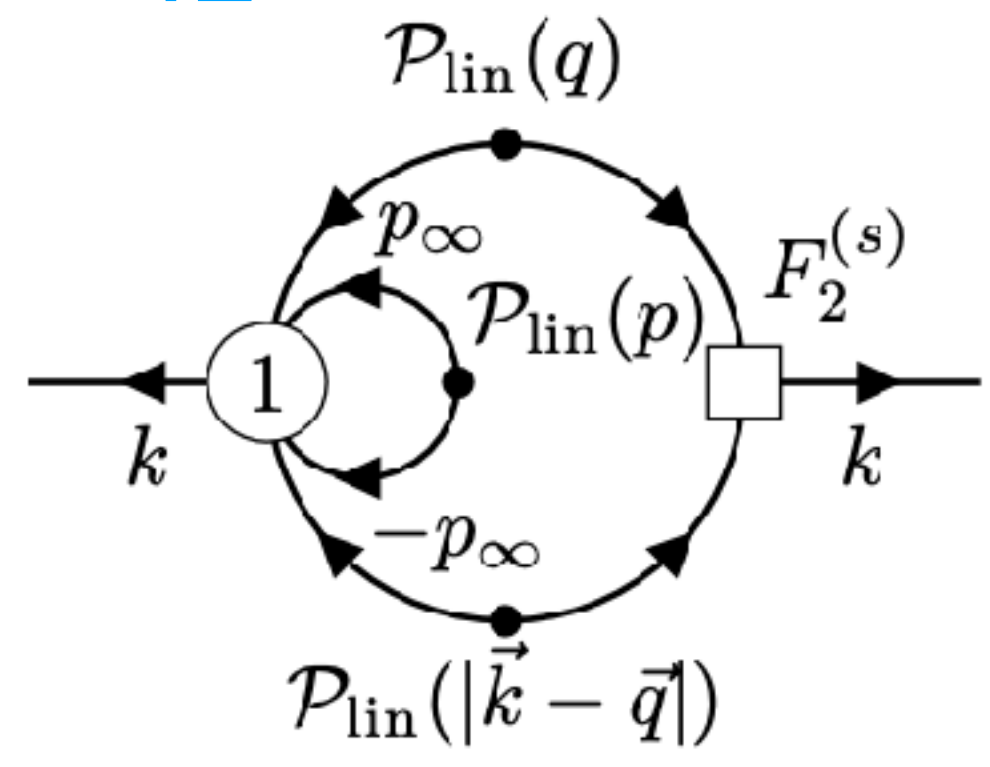
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UV regulation and renormalization

local UV subtractions in the integrand

$$p_{2\text{-loop}}^{\text{UV-reg}} = p_{2\text{-loop}} - \mathcal{R}_{1\text{-UV}}[p_{2\text{-loop}}] - \mathcal{R}_{2\text{-UV}}[p_{2\text{-loop}} - \mathcal{R}_{1\text{-UV}}[p_{2\text{-loop}}]]$$

P_{42} eg.



$$= \left(\int_{\vec{p}} \frac{\mathcal{P}_{\text{lin.}}(p)}{p^2} f_{\text{screen}}(p^2) \right) \int_{\vec{q}} \frac{\mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(|\vec{k} - \vec{q}|) F_2(\vec{q}, \vec{k} - \vec{q})}{169785 q^2 |\vec{k} - \vec{q}|^2} \times$$

$$\times (48096 (\vec{k} \cdot \vec{q})^3 + (16892 k^2 - 48096 q^2) (\vec{k} \cdot \vec{q})^2 +$$

$$+ (-32879 (k^2)^2 + 35744 q^2 k^2) (\vec{k} \cdot \vec{q}) - 25933 q^2 (k^2)^2 + 6176 (q^2)^2 k^2)$$

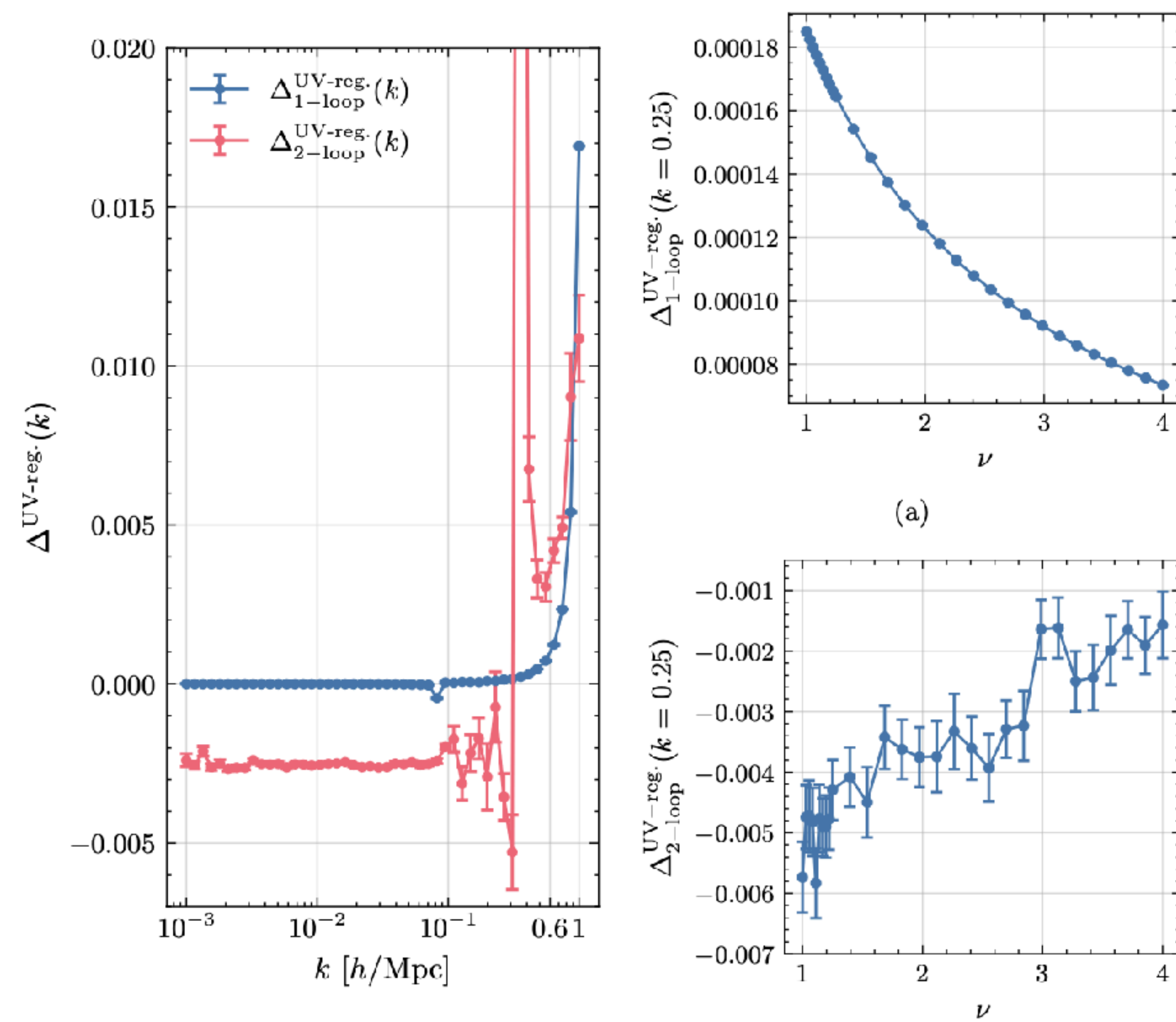
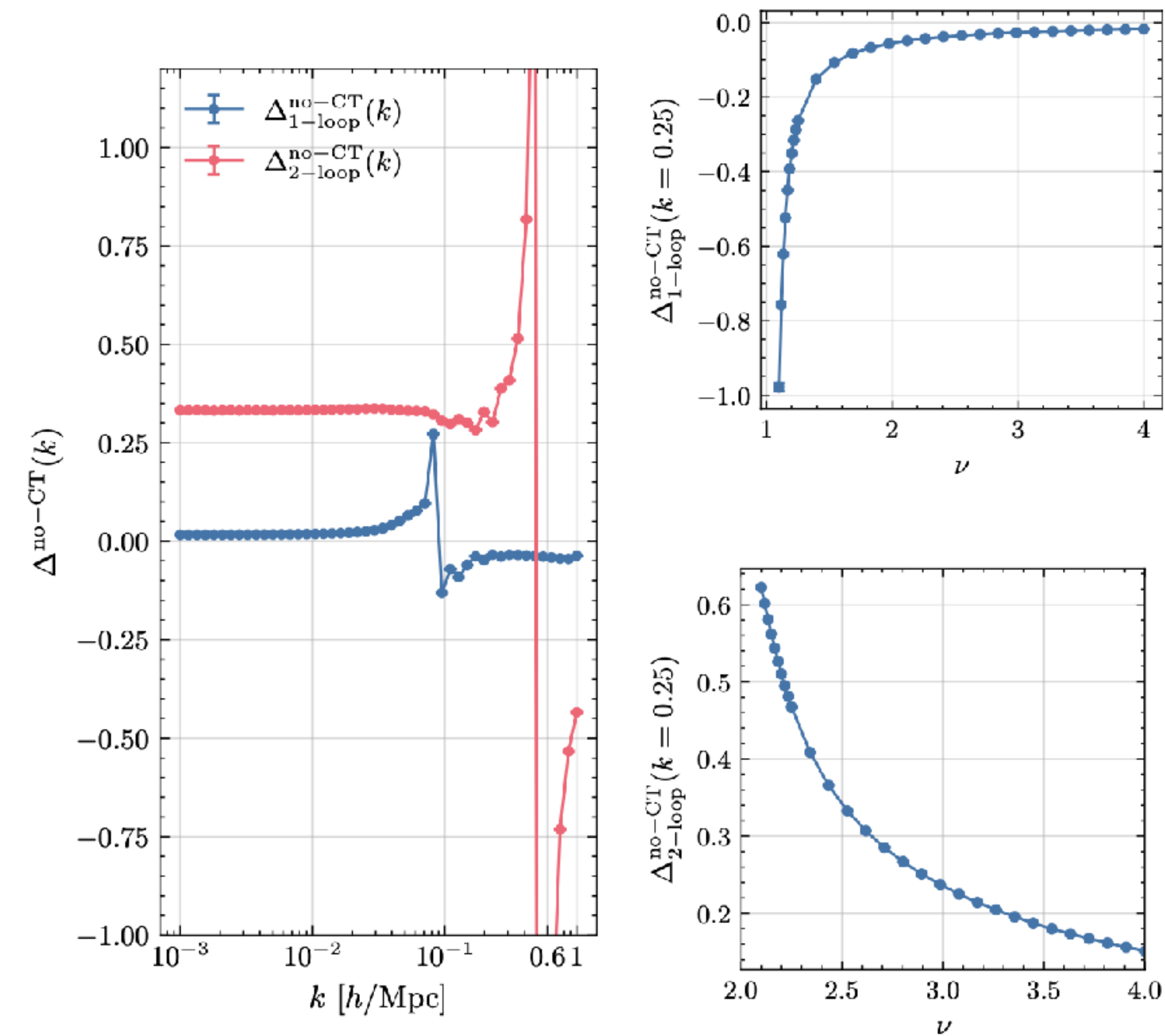
$$f_{\text{screen}}(q^2) = \theta(q^2 - (0.6 h/\text{Mpc})^2)$$

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UV sensitive

UV regulated



two-loop DM power spectrum

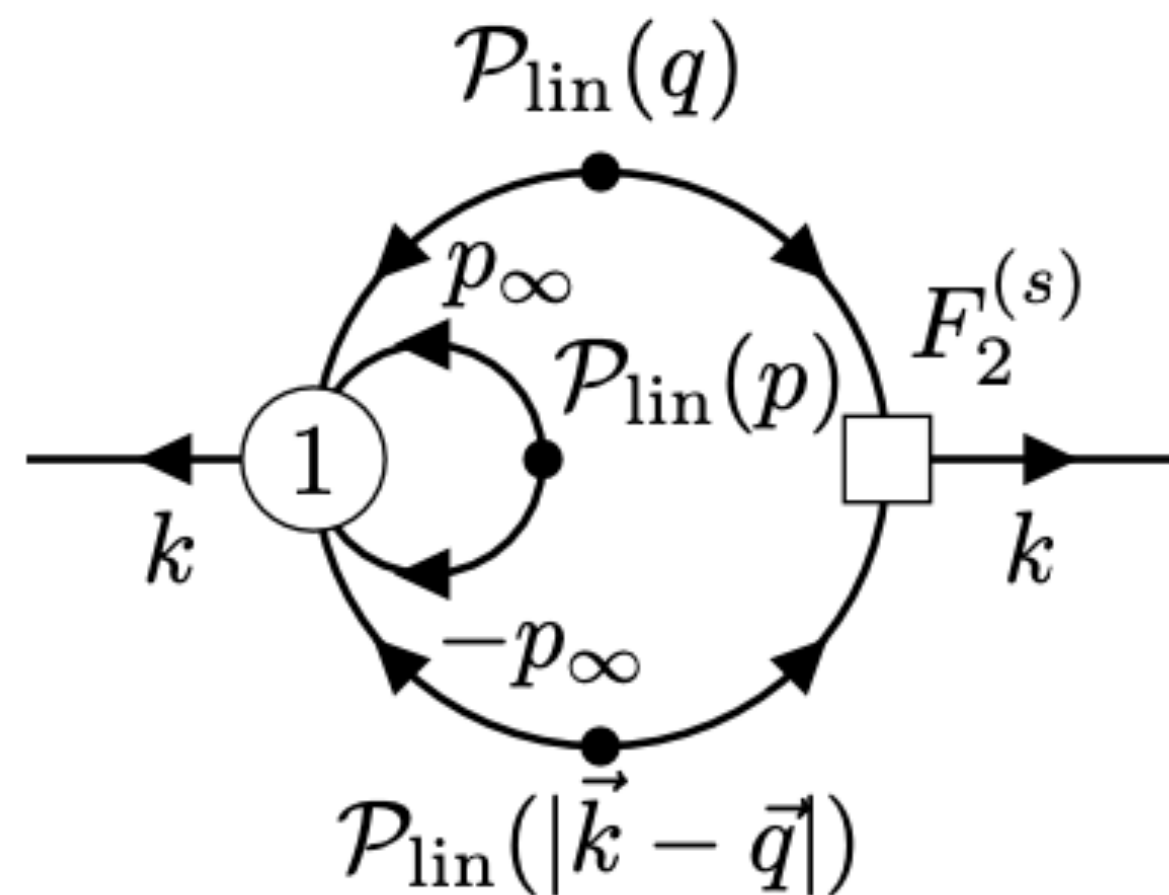
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Senatore, Zheng 25

UV matching

every UV divergence should be
cancelled by an EFT counterterm

eg. UV limit of P_{42}

determines 4 EFT
coefficients in $\delta_{ct}^{(2)}$



=

$$2\langle\tilde{\delta}_{ct}^{(2)}(\vec{k})\tilde{\delta}^{(2)}(\vec{k}')\rangle$$

two-loop DM power spectrum

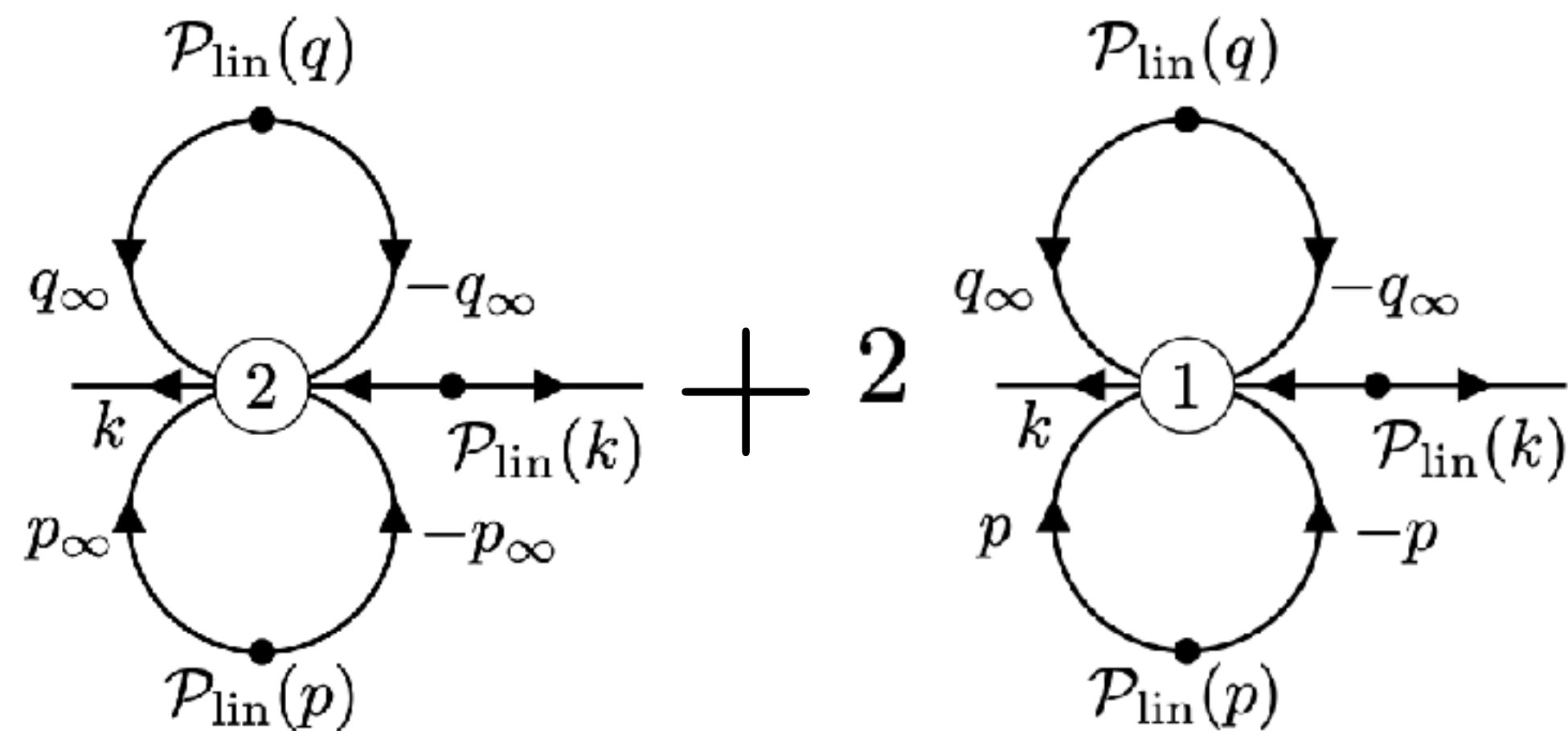
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UV matching

every UV divergence should be
cancelled by an EFT counterterm

similarly for P_{15}

determines 4 EFT
coefficients



$$= 2 \left(\langle \tilde{\delta}_{\text{ct}}^{(3)}(\vec{k}) \tilde{\delta}^{(1)}(\vec{k}') \rangle + \langle \tilde{\delta}_{\partial^2 \text{ct}}^{(1)}(\vec{k}) \tilde{\delta}^{(1)}(\vec{k}') \rangle \right)$$

all 8 EFT coefficients
are necessary and
sufficient to cancel UV

two-loop DM power spectrum

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importance of EFT counterterms

how many of the 8 are needed for data?

$$\theta^i = \{c_{\delta,1}, c_{\delta,2}, c_{\delta^2,1}, c_{r\delta,1}, c_{\delta,3}, c_{r\delta,2}, c_{r\delta^2,1}, c_{\partial^2\delta,1}\}$$

$$F_{ij} = \frac{1}{2} \frac{\partial^2}{\partial \theta^i \partial \theta^j} \sum_{n,m} \mathcal{P}(k_n) C^{-1}(k_n, k_m) \mathcal{P}(k_m) \Big|_{\theta=0}$$

**Fisher matrix,
DESI-size covariance**

diagonalizing

$$D^{-1/2} \approx \{0.03, 0.08, 0.25, 14, 180, 590, 14000, 27000\}$$

**so a DESI-size survey would
determine 3 parameters well**

**previous works have
used only 3/4 EFT
coefficients, tested
UV sensitivity
numerically (but
integrals are actually
convergent)**

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EFT counterterms

P_{42} diagram

~ 20x more terms

than counterterms

[illegible]

DES

15h

16h

17h

18h

19h

20h

21h

22h

23h

24h

25h

26h

27h

28h

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421h

integration

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

IR limits

**IR divergences cancel between diagrams
because of the equiv. principle and Galilean
invariance**

**but this can be a problem when numerically
computing diagrams separately**

**particularly difficult when IR divergences are
located at different points in integration
domain**

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IR limits

eg. P_{22}

divergence for $\vec{q} \rightarrow 0$ and $\vec{k} - \vec{q} \rightarrow 0$

can use partition of unity

$$\begin{aligned}\mathcal{P}_{22}(k) &= \int_{\vec{q}} p_{22}(\vec{q}, \vec{k}) = \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k}) (|\vec{k} - \vec{q}|^2)^2}{(q^2)^2 + (|\vec{k} - \vec{q}|^2)^2} + \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k}) (q^2)^2}{(q^2)^2 + (|\vec{k} - \vec{q}|^2)^2} \\ &= 2 \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k}) (|\vec{k} - \vec{q}|^2)^2}{(q^2)^2 + (|\vec{k} - \vec{q}|^2)^2}\end{aligned}$$

now only divergence for $\vec{q} \rightarrow 0$, which is actually numerically finite because of the integration measure

now we have IR
and UV finite
integrals at the
level of the
integrands

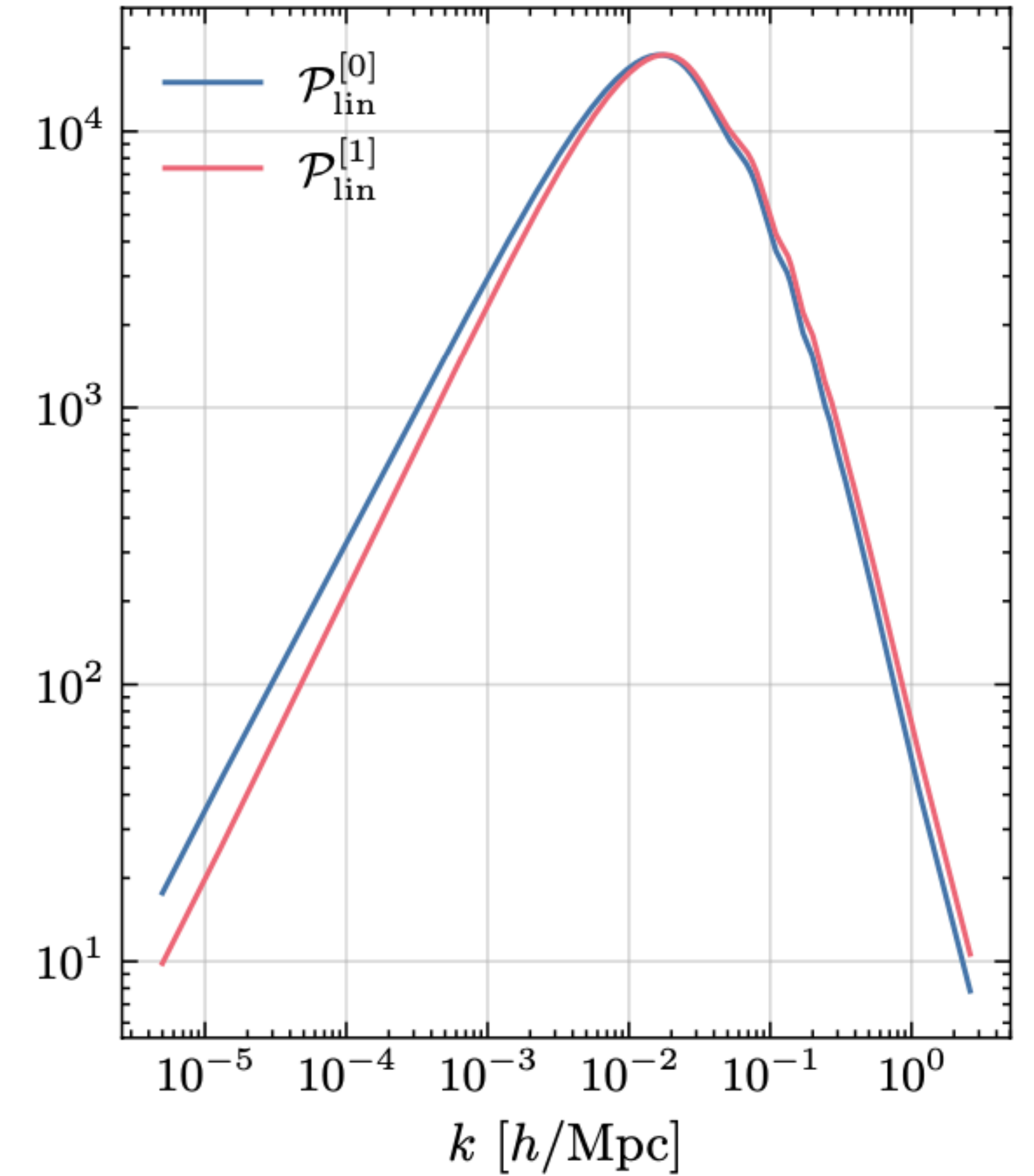
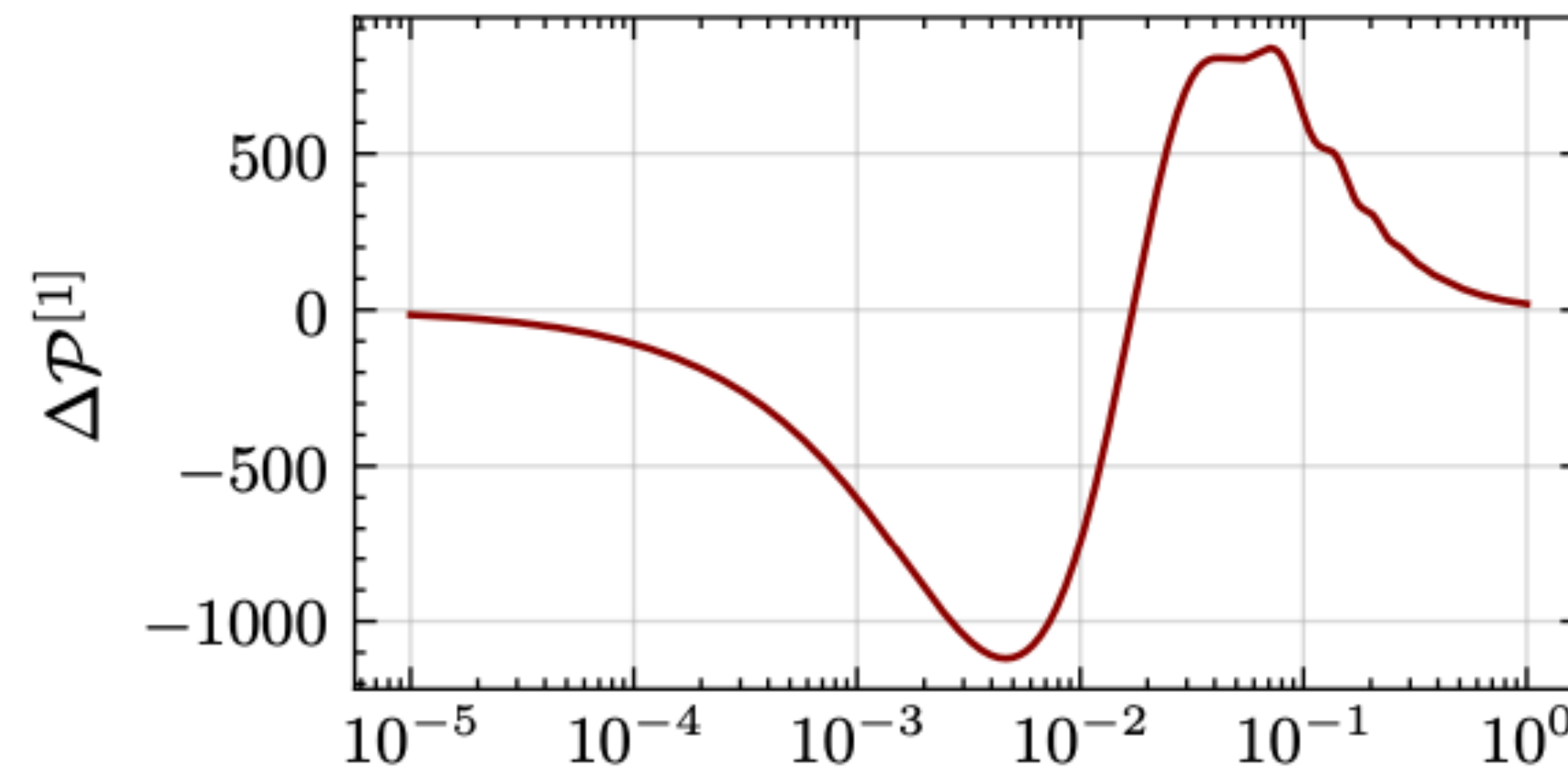
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integration strategy

- use Monte Carlo integration for a base cosmology and do loops numerically
- for a general cosmology, expand the difference in a convenient basis

$$\Delta\mathcal{P}^{[j]}(k) = \mathcal{P}_{\text{lin.}}^{[j]}(k) - \mathcal{N}^{[j]} \mathcal{P}_{\text{lin.}}^{[0]}(k)$$



two-loop DM power spectrum

integration strategy

decompose each linear power spectrum difference

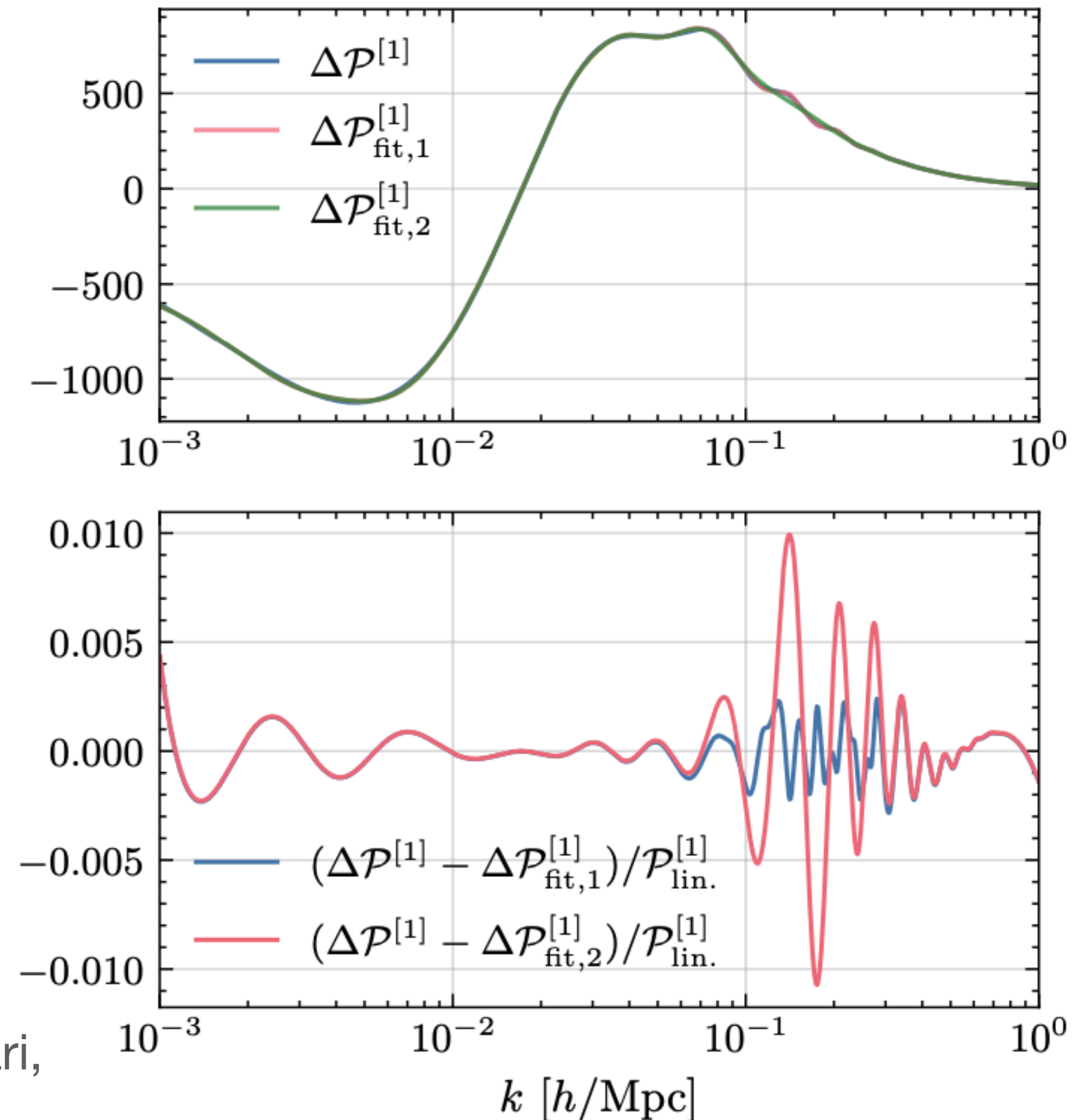
$$\Delta \mathcal{P}^{[j]}(k) = \sum_i c_i^{[j]} \left(\frac{(k/k_0)^{\alpha_i}}{\left(1 + \frac{k^2 - k_{peak}^2}{k_{UV}^2}\right)^{\beta_i}} \right)$$

cosmology
dependent

fixed basis,
~20 functions

COBRA uses singular
value decomposition

Bakx, Rubira, Chisari,
Vlah 25



two-loop DM power spectrum

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Senatore, Zheng 25

integration strategy

now plug into integrals, eg. P_{15}

$$\begin{aligned}\mathcal{P}_{15}(k) &\propto \mathcal{P}_{\text{lin.}}(k) \int_{\vec{p}, \vec{q}} F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) \mathcal{P}_{\text{lin.}}(q) \mathcal{P}_{\text{lin.}}(p) \\ &= \mathcal{P}_{\text{lin.}}(k) \int_{\vec{p}, \vec{q}} F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) (\mathcal{P}_{\text{lin.}}^{\text{Planck}}(q) + \Delta\mathcal{P}(q)) (\mathcal{P}_{\text{lin.}}^{\text{Planck}}(p) + \Delta\mathcal{P}(p)) \\ &\approx \mathcal{P}^{\text{Planck}}(k) + 2\mathcal{P}_{\text{lin.}}(k) \int_{\vec{p}, \vec{q}} F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) \Delta\mathcal{P}(q) \mathcal{P}_{\text{lin.}}^{\text{Planck}}(p) + \dots\end{aligned}$$

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integration strategy

now plug into integrals, eg. P_{15}

$$\mathcal{P}_{\text{lin.}}(k) \int_{\vec{p}, \vec{q}} F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) \Delta \mathcal{P}(q) \mathcal{P}_{\text{lin.}}^{\text{Planck}}(p) =$$

$$\sum_i c_i \mathcal{P}_{\text{lin.}}(k) \int_{\vec{p}, \vec{q}} F_5(\vec{k}, \vec{q}, -\vec{q}, \vec{p}, -\vec{p}) f_i(q) \mathcal{P}_{\text{lin.}}^{\text{Planck}}(p)$$

cosmology
dependent

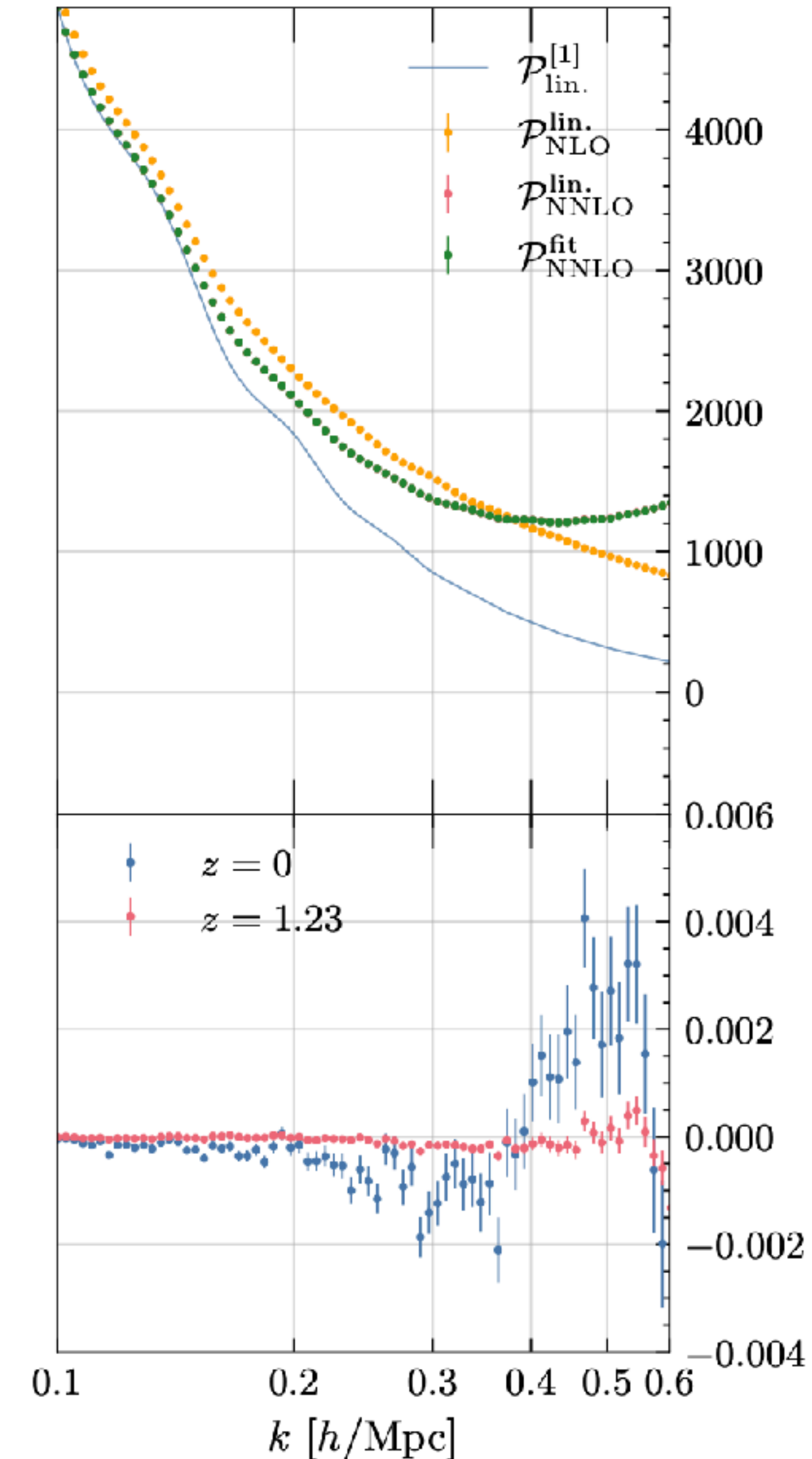
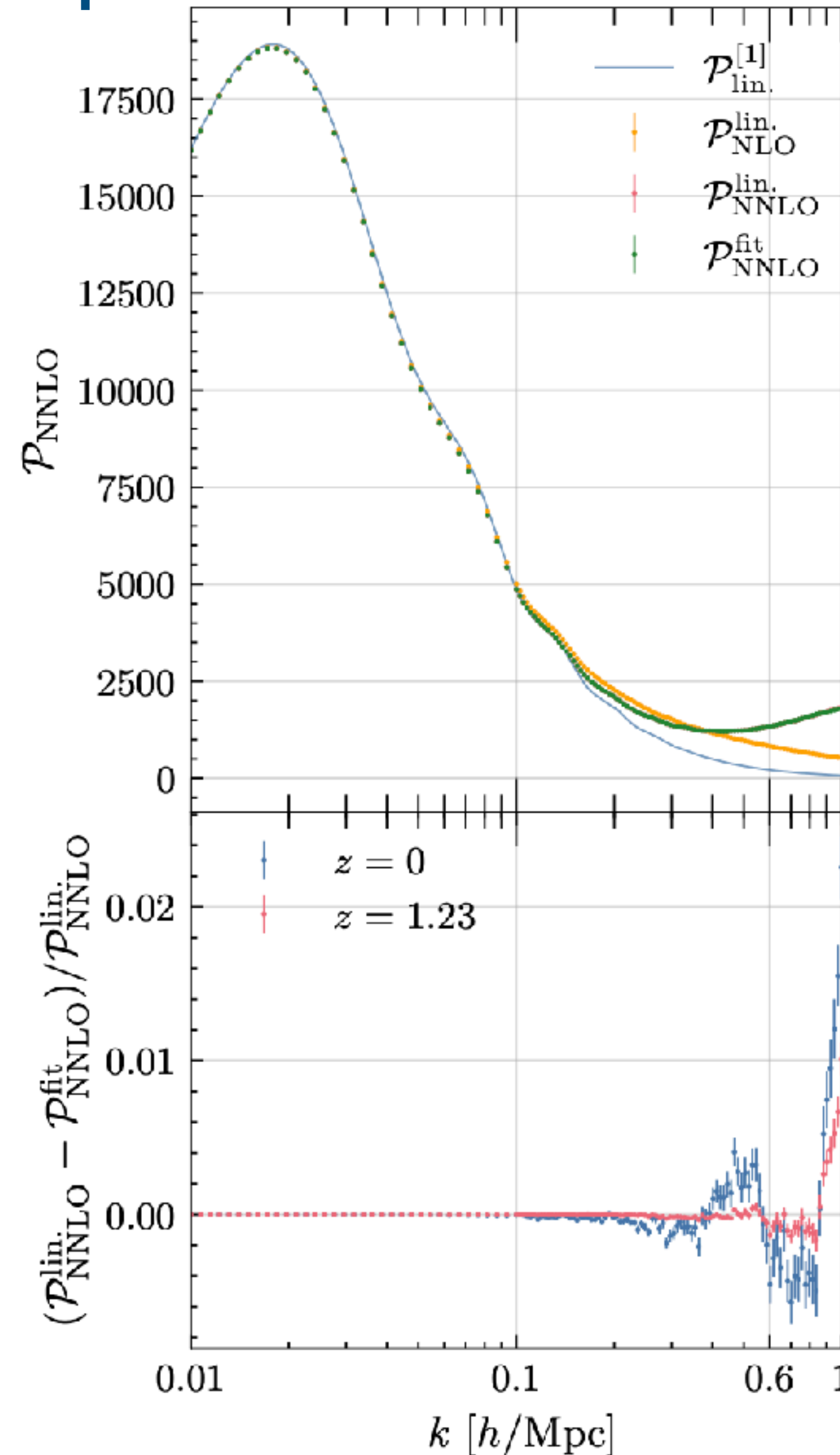
~ 3500 precomputed integrals total

two-loop DM power spectrum

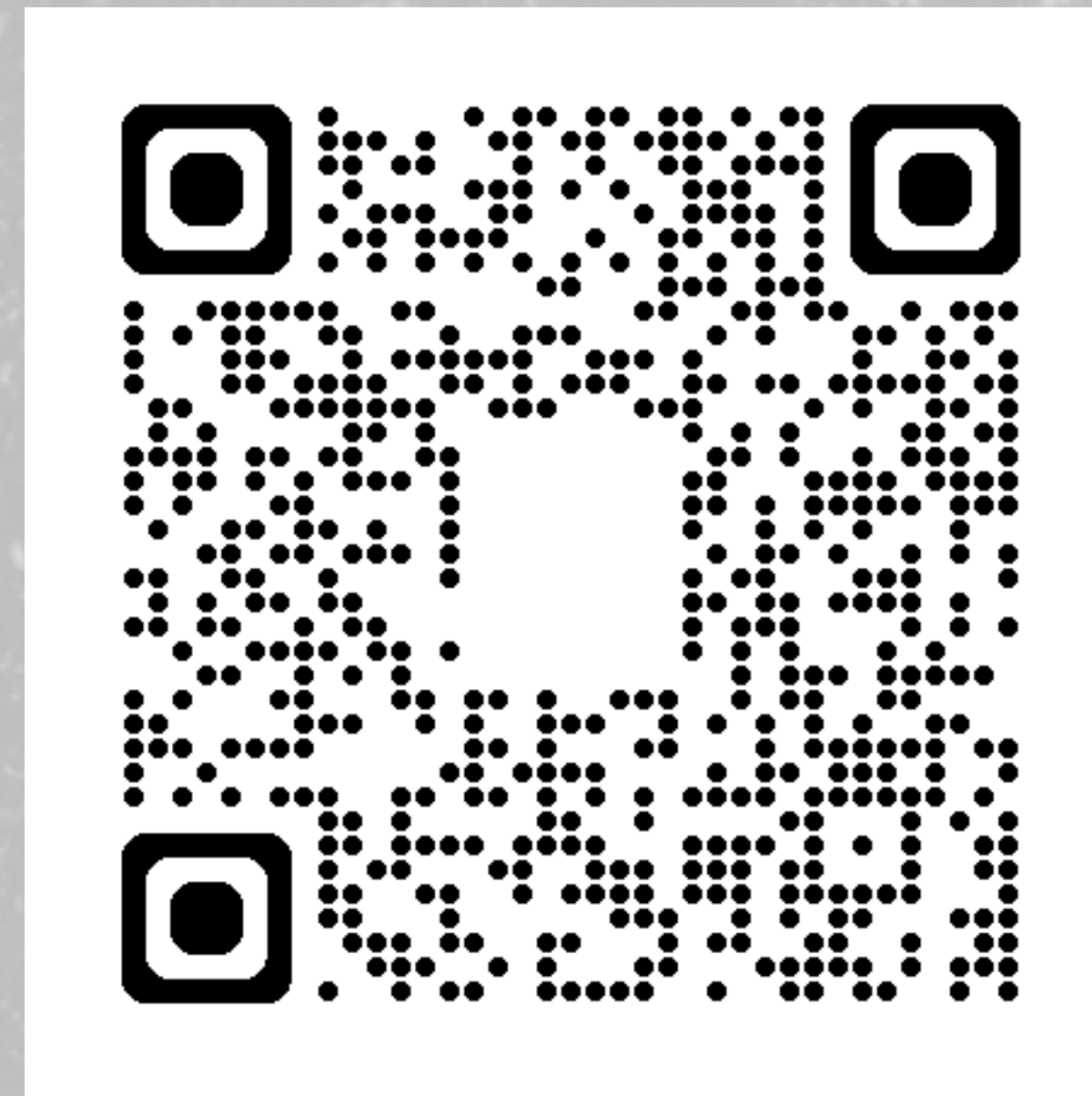
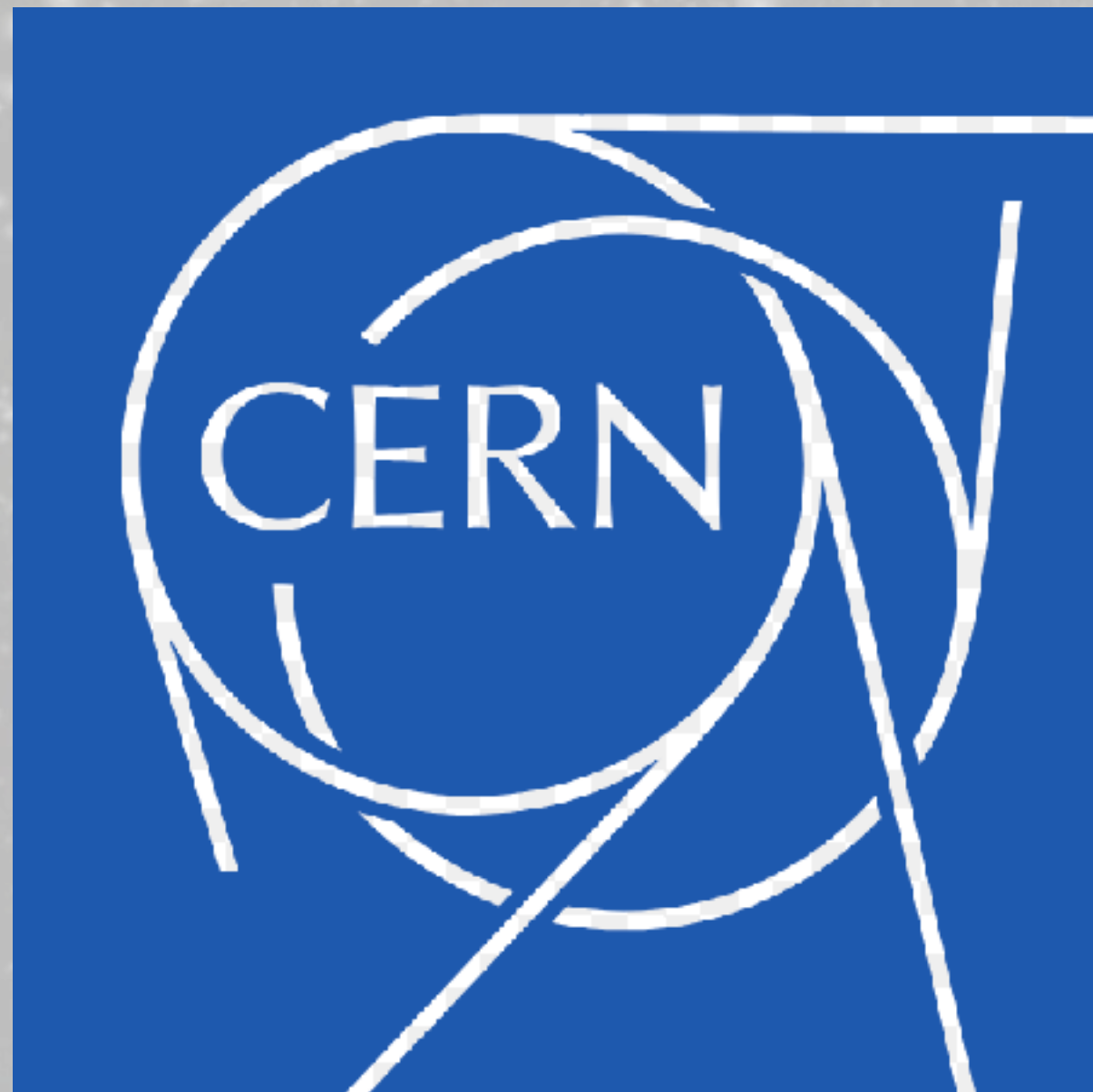
results

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Mathematica code to
compute any
cosmology and any EFT
parameters in <1s



exciting time for LSS!



<https://theory.cern/jobs>