renormalization and efficient evaluation of the dark-matter two-loop power spectrum

Matthew Lewandowski
CERN TH

New Physics from Galaxy Clustering - GGI
10/09/25

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in collaboration with

Anastasiou, Favorito, Senatore, Zheng

Henry Zheng, Stanford

Andrea Favorito, ETH

ETH zürich


galaxy maps
we are lucky
SDSS DR9

the promise

- this map is sensitive to the Universe's contents, initial conditions, and interactions
- standard evolution - ΛCDM parameters like Ω_m , H_0, and σ_8
- but also many other effects we've discussed

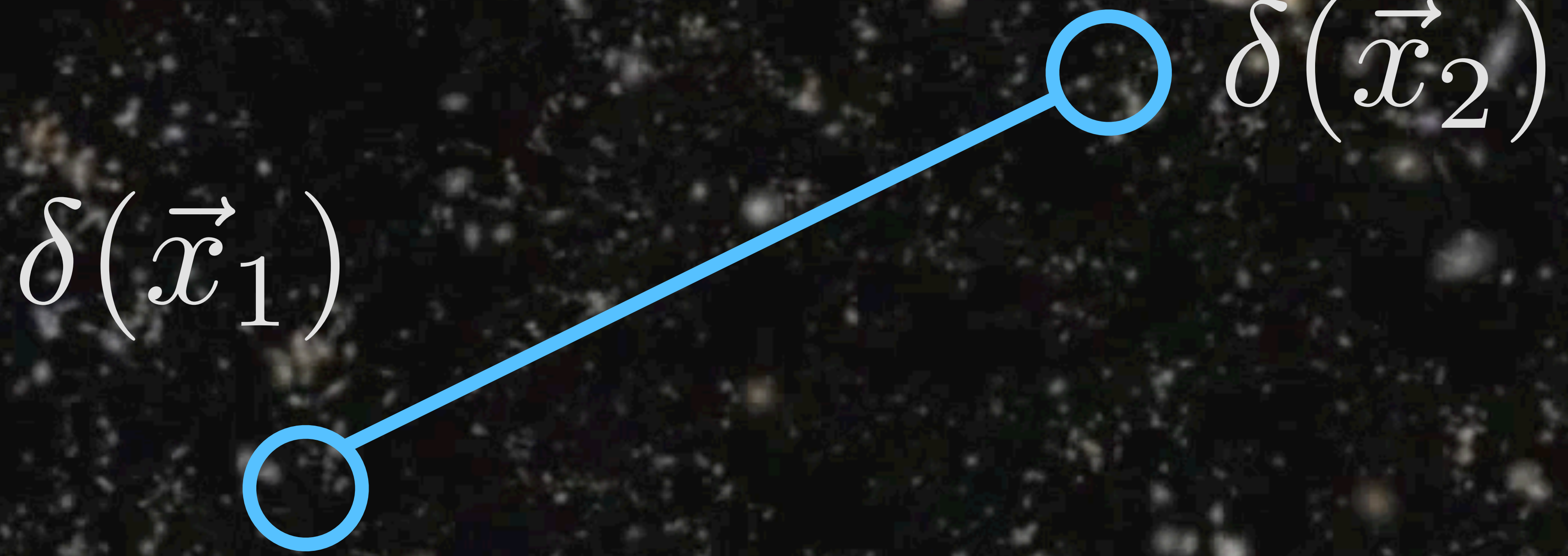
$$\mathcal{O} = \mathcal{O}_{\text{DM, gravity}} + \epsilon\mathcal{O}_{\text{BSM}}$$

the method

overdensity: $\delta(\vec{x})$

summary statistics

two point/power spectrum



$$\langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$$

$$P(k)$$

three point/bispectrum

$$\langle\delta(\vec{x}_1)\delta(\vec{x}_2)\delta(\vec{x}_3)\rangle$$

$$B(k_1, k_2, k_3)$$

the method

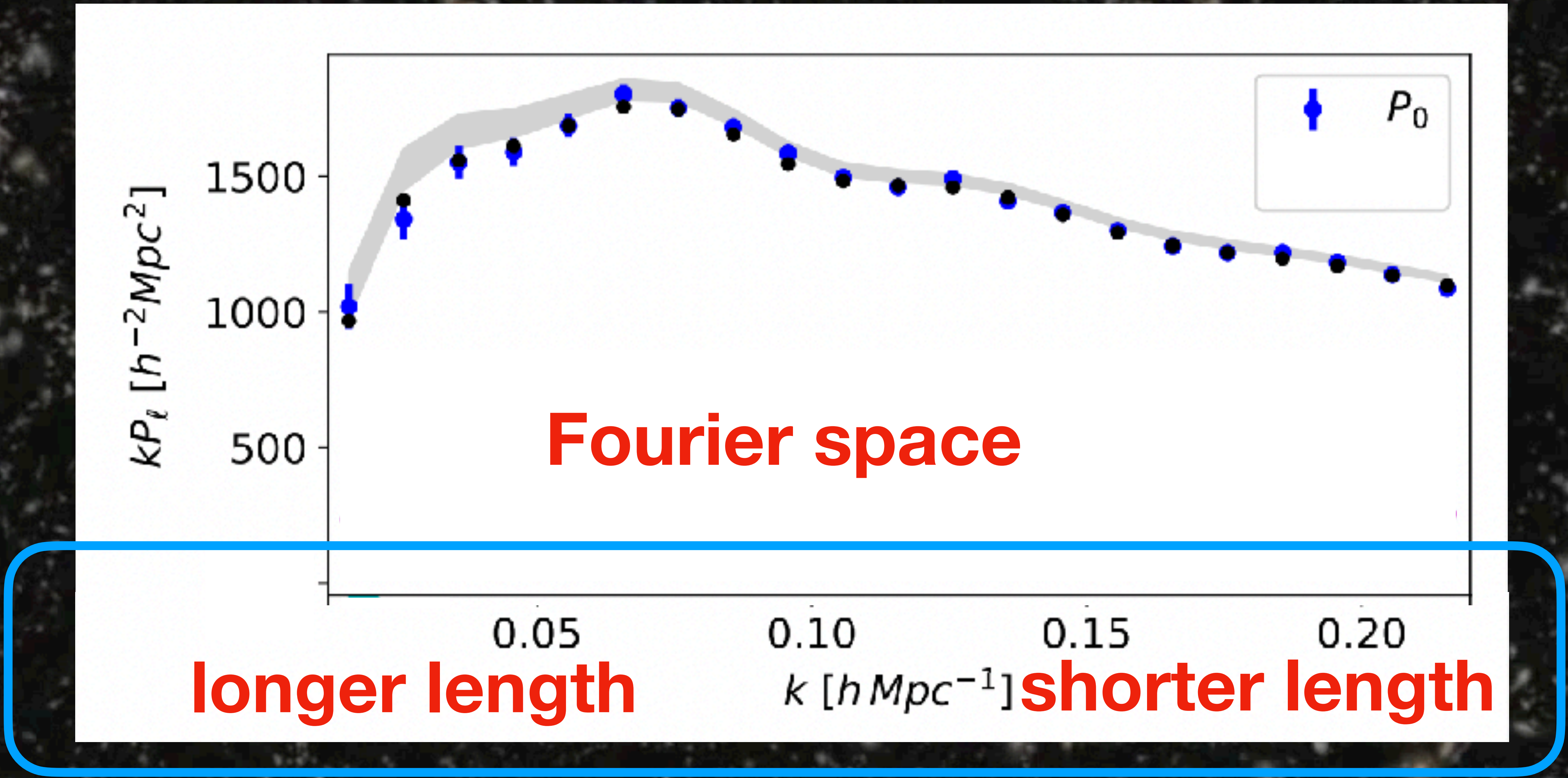
overdensity: $\delta(\vec{x})$

summary statistics

two point/power spectrum

$\delta(\vec{x}_2)$

$\delta(\vec{x}_1)$

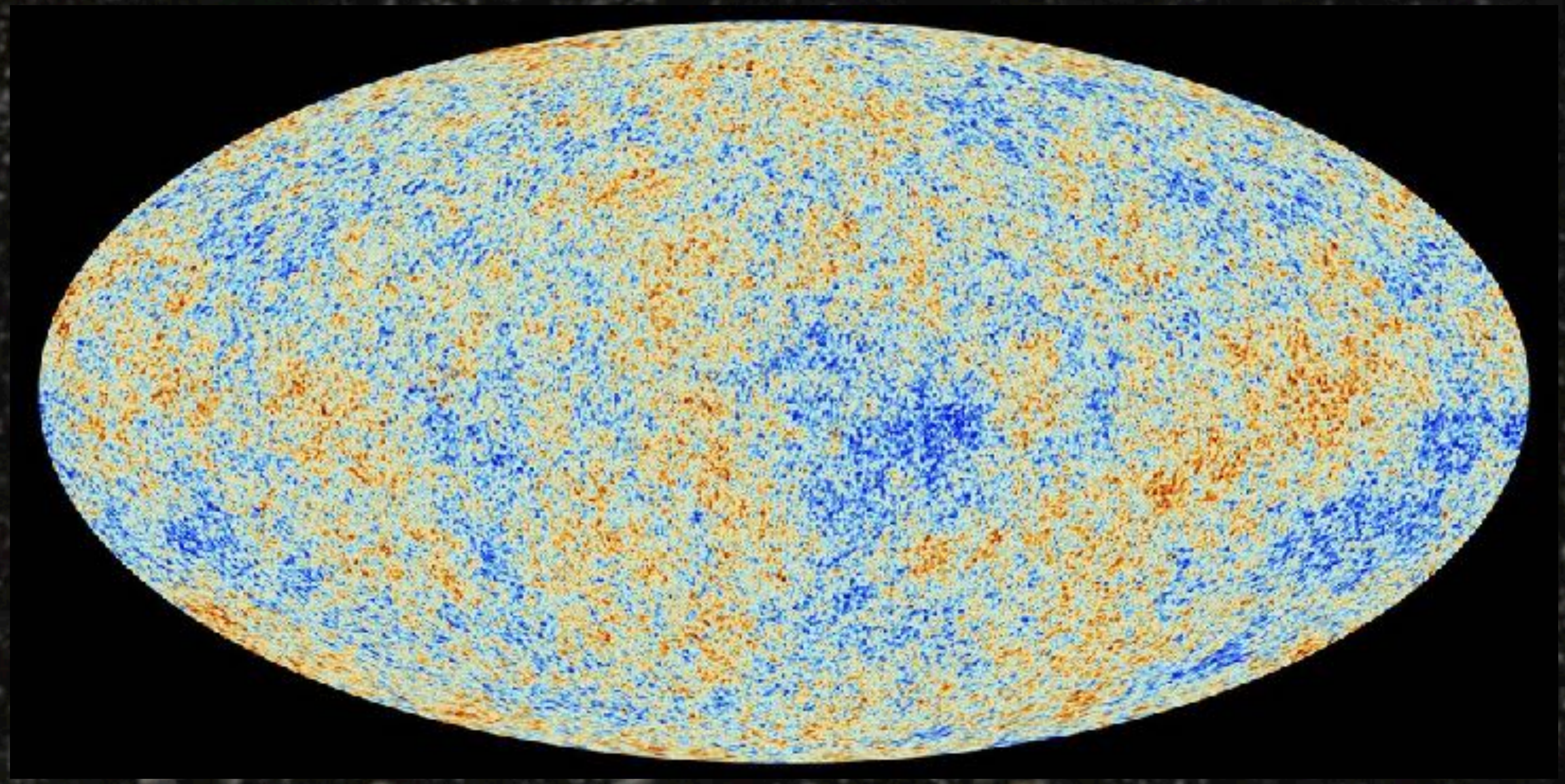


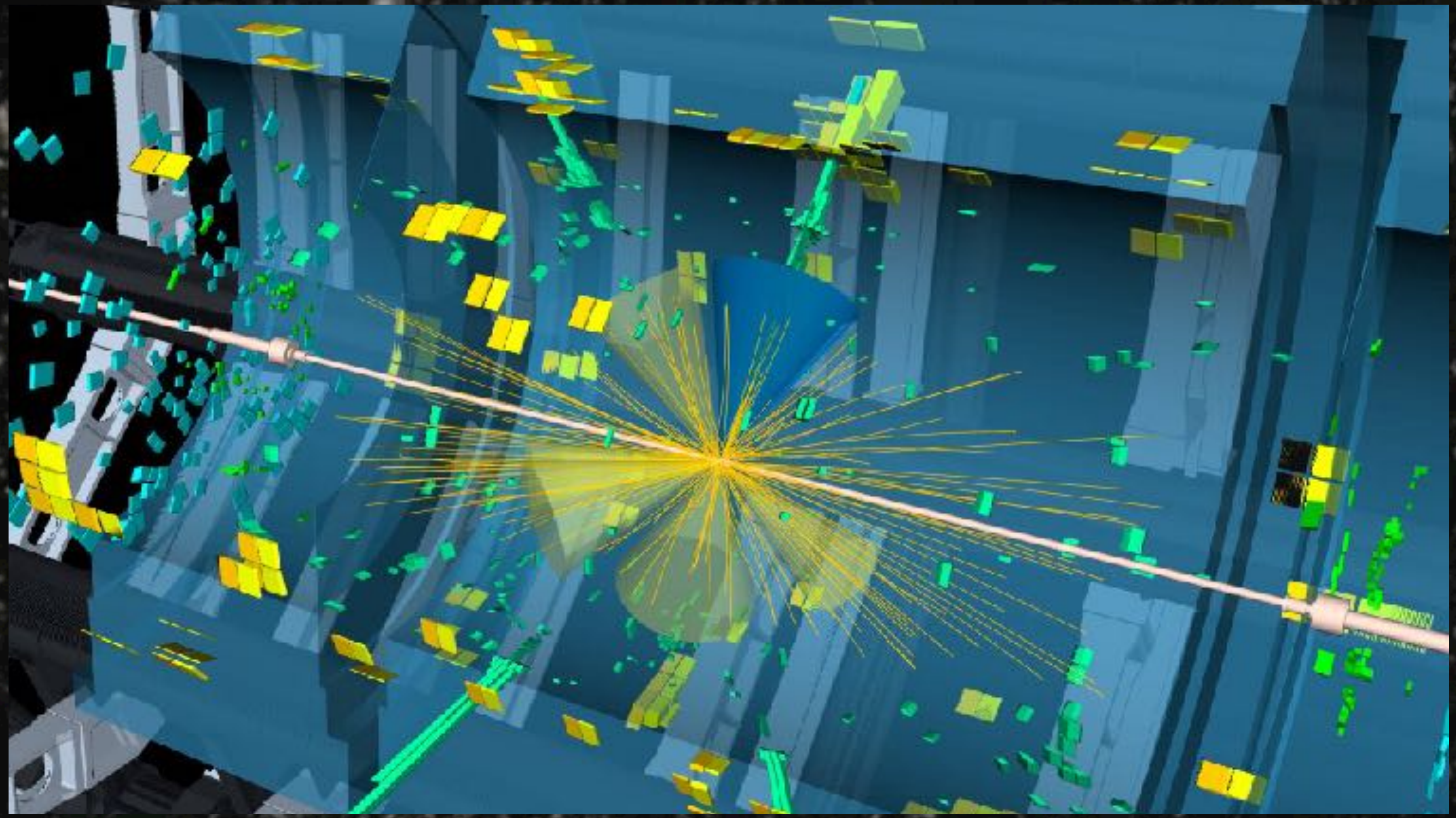
$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$$

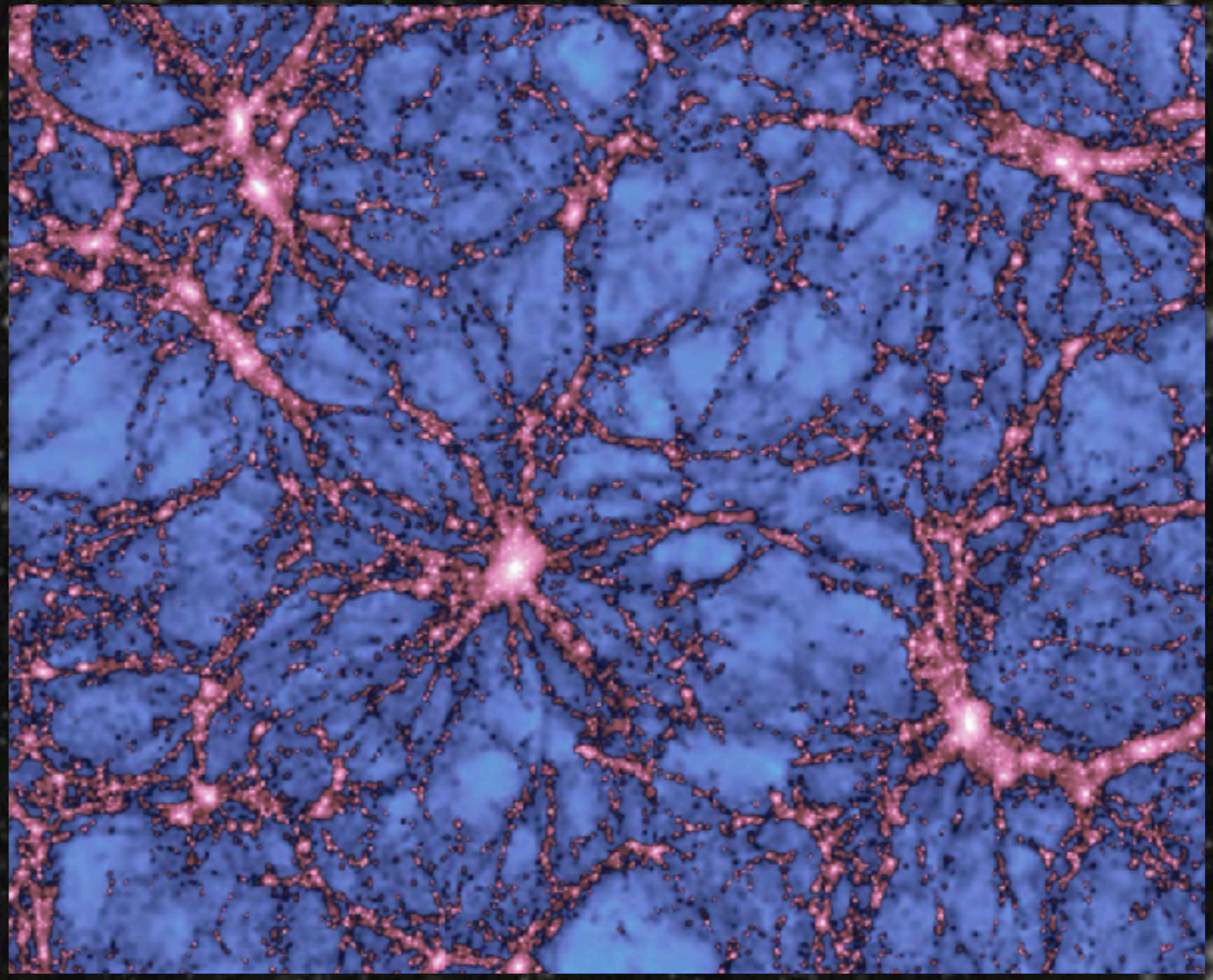
$$P(k)$$

the challenge

- CMB is linear - lucky again
- think how lucky we are that standard model is weakly coupled
- LSS is highly nonlinear on small scales







the payoff

largest k, smallest length at which we trust the theory

- but LSS is three-dimensional
- many more available modes

$$\text{error bars} \sim \frac{1}{\sqrt{N_{\text{modes}}}} \sim \frac{1}{\sqrt{V_{\text{survey}} k_{\text{max}}^3}}$$

next-generation surveys (DESI, Euclid, MegaMapper, PUMA, etc.) delivering data in the next 2-25 years

- interesting point from McDonald and Roy 2009:
  - we pay about $1,000 per mode for the ~ 10^6 modes we have
  - so increasing k_{max} by a factor of 1.3 is essentially worth a billion dollars

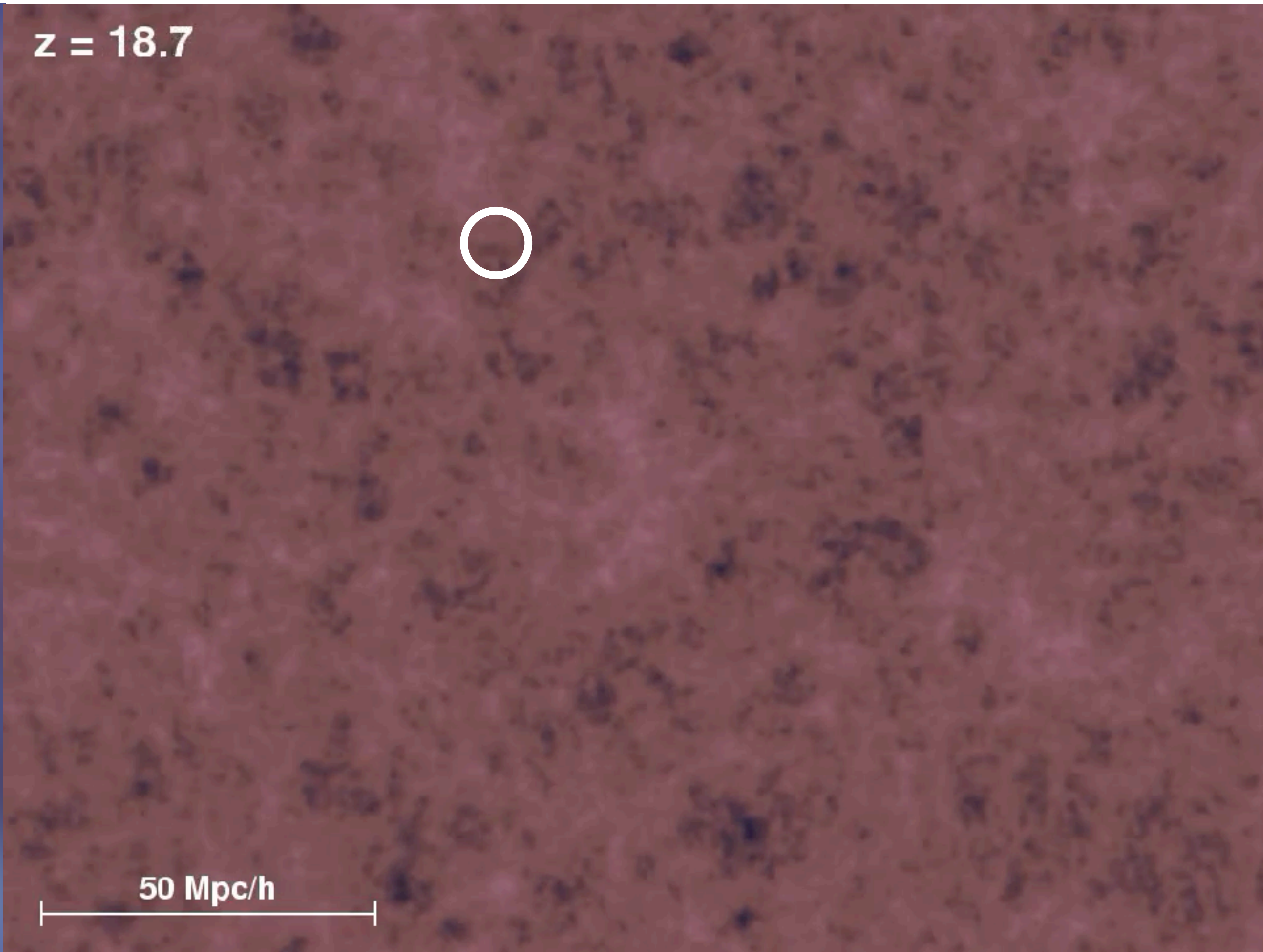
EFT of LSS

dark-matter clustering

Zel'dovich
Peebles
Blumenthal
Faber
Primack
Rees
Feldman
Kaiser
Peacock
…

dark-matter clustering

Max-Planck-Institute for Astrophysics,
Volker Springel



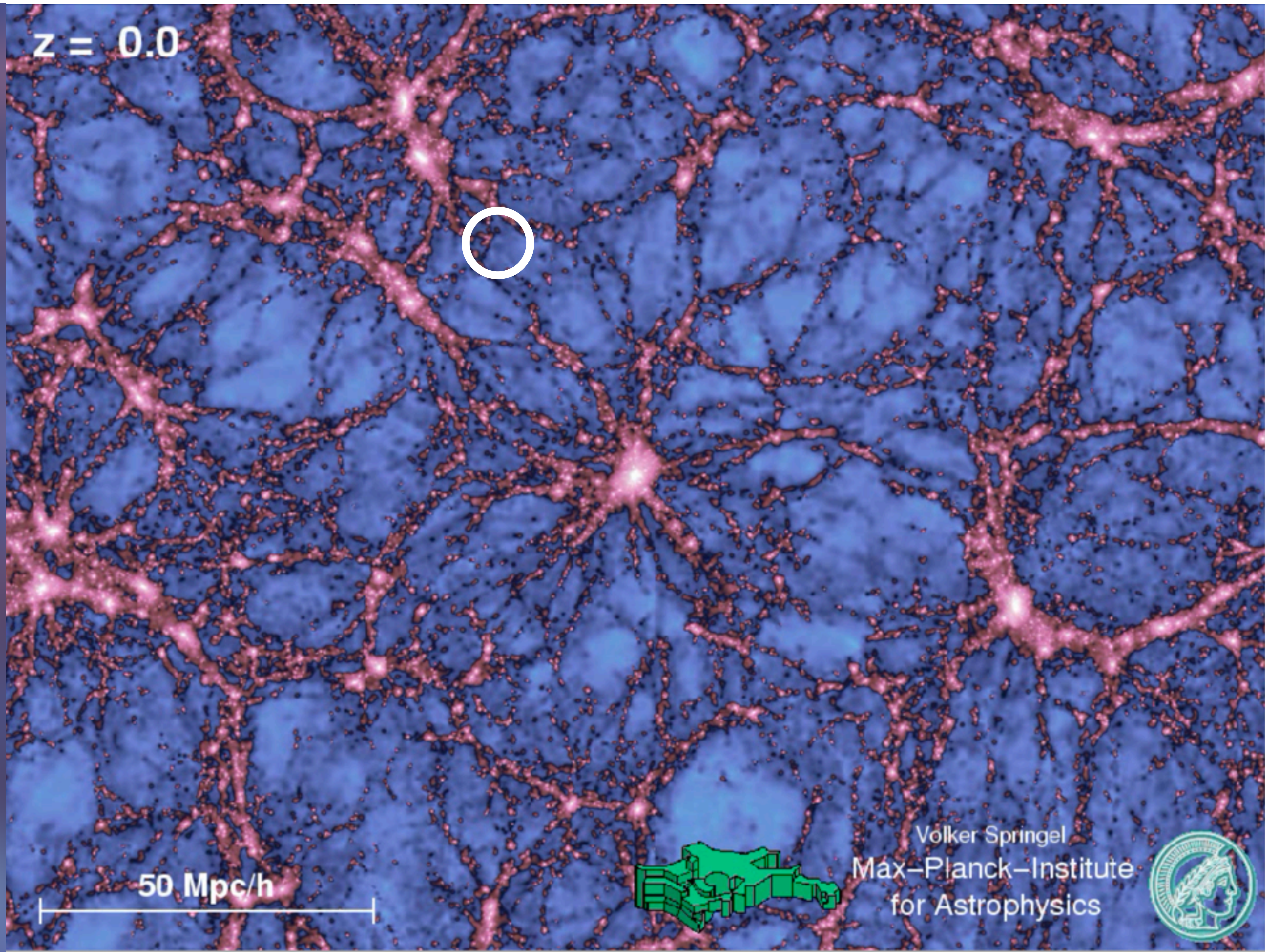
effective field theory of LSS

Baumann, Nicolis, Senatore, Zaldarriaga 12

Carrasco, Hertzberg, Senatore 12

- controlled, analytic, perturbative expansion
- how can small scales affect large scales?

Max-Planck-Institute for Astrophysics, Volker Springel



effective field theory of LSS

Baumann, Nicolis, Senatore, Zaldarriaga 12

Carrasco, Hertzberg, Senatore 12

expanding universe: $a(t)$

time scale: $H = \dot{a}/a$

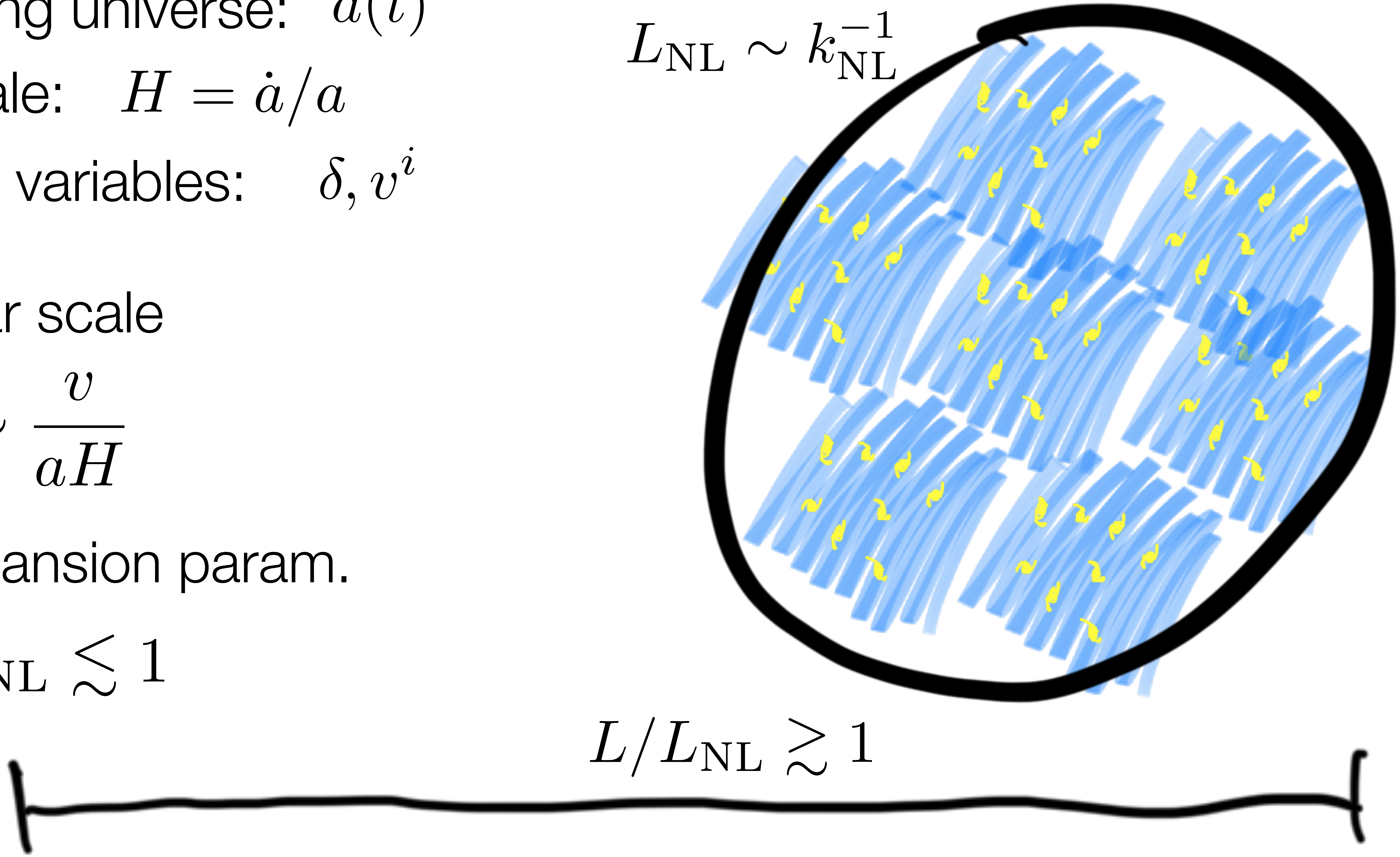
DM fluid variables: δ, v^i

nonlinear scale

$$k_{\rm NL}^{-1} \sim \frac{v}{aH}$$

EFT expansion param.

$$k/k_{\rm NL} \lesssim 1$$



effective field theory of LSS

equations of motion for dark matter

Baumann, Nicolis, Senatore, Zaldarriaga 12

Carrasco, Hertzberg, Senatore 12

what do we actually solve?

continuity [δ,v] $= 0$

Euler [δ,v] $= \dfrac{\partial_i \partial_j}{k_{\rm NL}^2} \tau^{ij}_{\rm EFT}$

perfect fluid   EFT counterterms

effective field theory of LSS

equations of motion for dark matter

Baumann, Nicolis, Senatore, Zaldarriaga 12

Carrasco, Hertzberg, Senatore 12

what do we actually solve?

EOM local for π_S and π_V^i

over and momentum density

$$\delta(\vec{x}, a) \equiv (\rho(\vec{x}, a) - \bar{\rho}(a))/\bar{\rho}(a)$$

$$\pi^i(\vec{x}, a) \equiv \rho(\vec{x}, a) v^i(\vec{x}, a)$$

$$\pi_S \equiv \partial_i \pi^i$$

$$\pi_V^i \equiv \epsilon^{ijk} \partial_j \pi^k ,$$

$$\pi^i = \frac{\partial_i}{\partial^2} \pi_S - \epsilon^{ijk} \frac{\partial^j}{\partial^2} \pi_V^k$$

$$\partial^2 \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2 \delta$$

$$a\mathcal{H}\delta' + \frac{\pi_S}{\bar{\rho}} = 0 ,$$

$$\mathcal{H}\pi_S' + \frac{4\mathcal{H}}{a}\pi_S + \frac{3}{2a}\bar{\rho}\Omega_m \mathcal{H}^2 \delta = -\frac{\partial_i \partial_j}{a} \left(\frac{2}{3} \frac{\bar{\rho}}{\Omega_m \mathcal{H}^2} \left(\partial_i \Phi \partial_j \Phi - \frac{1}{2} \delta_{ij} (\partial \Phi)^2 \right) + \frac{\pi^i \pi^j}{\rho} + \tau^{ij} \right) ,$$

$$\mathcal{H}\pi_V'^i + 4\frac{\mathcal{H}}{a}\pi_V^i = -\frac{\epsilon^{ijk}}{a} \partial_j \partial_l \left(\frac{2}{3} \frac{\bar{\rho}}{\Omega_m \mathcal{H}^2} \partial_k \Phi \partial_l \Phi + \frac{\pi^k \pi^l}{\rho} + \tau^{kl} \right)$$

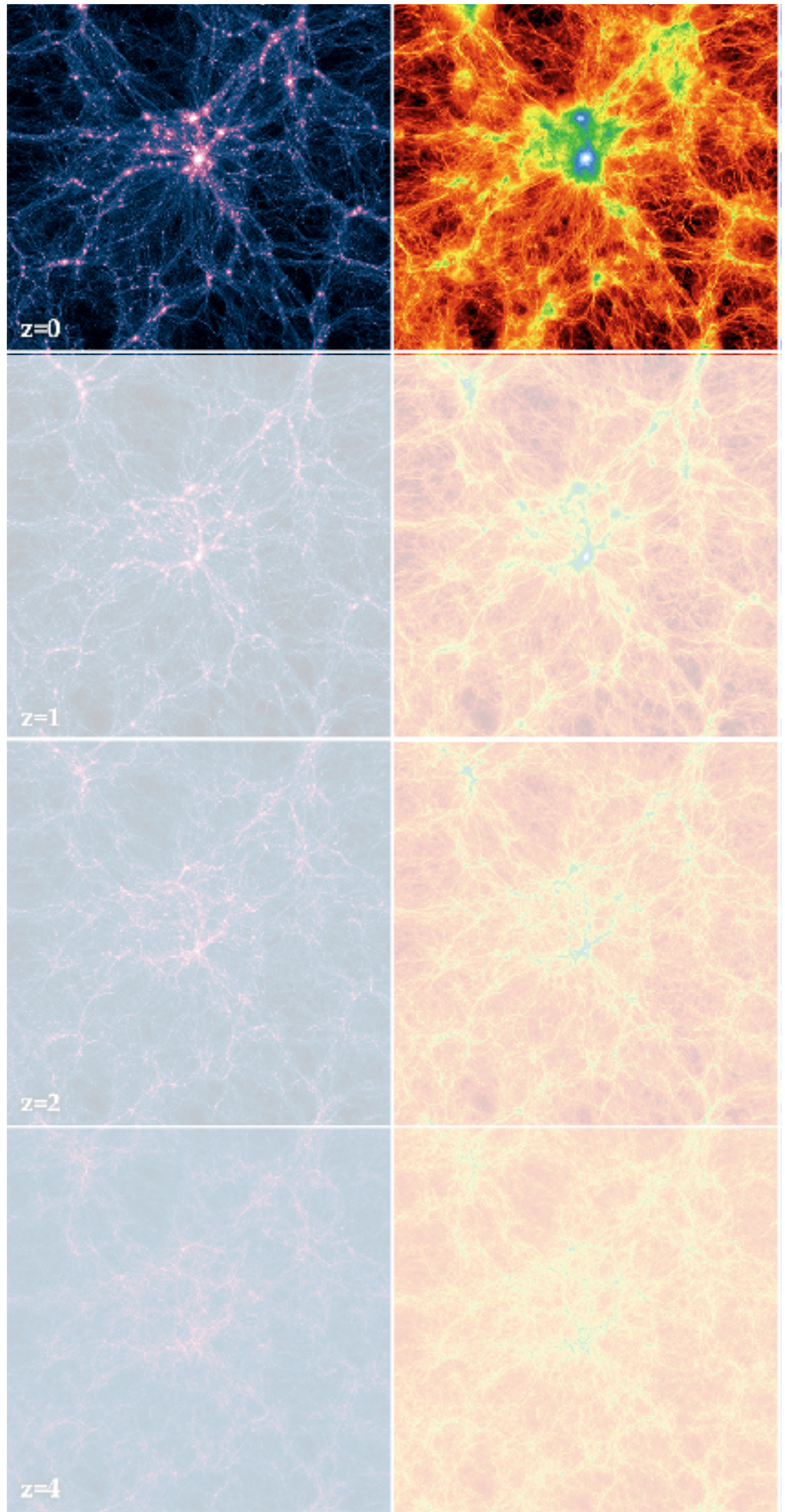
non-local-in-time structure formation

the question

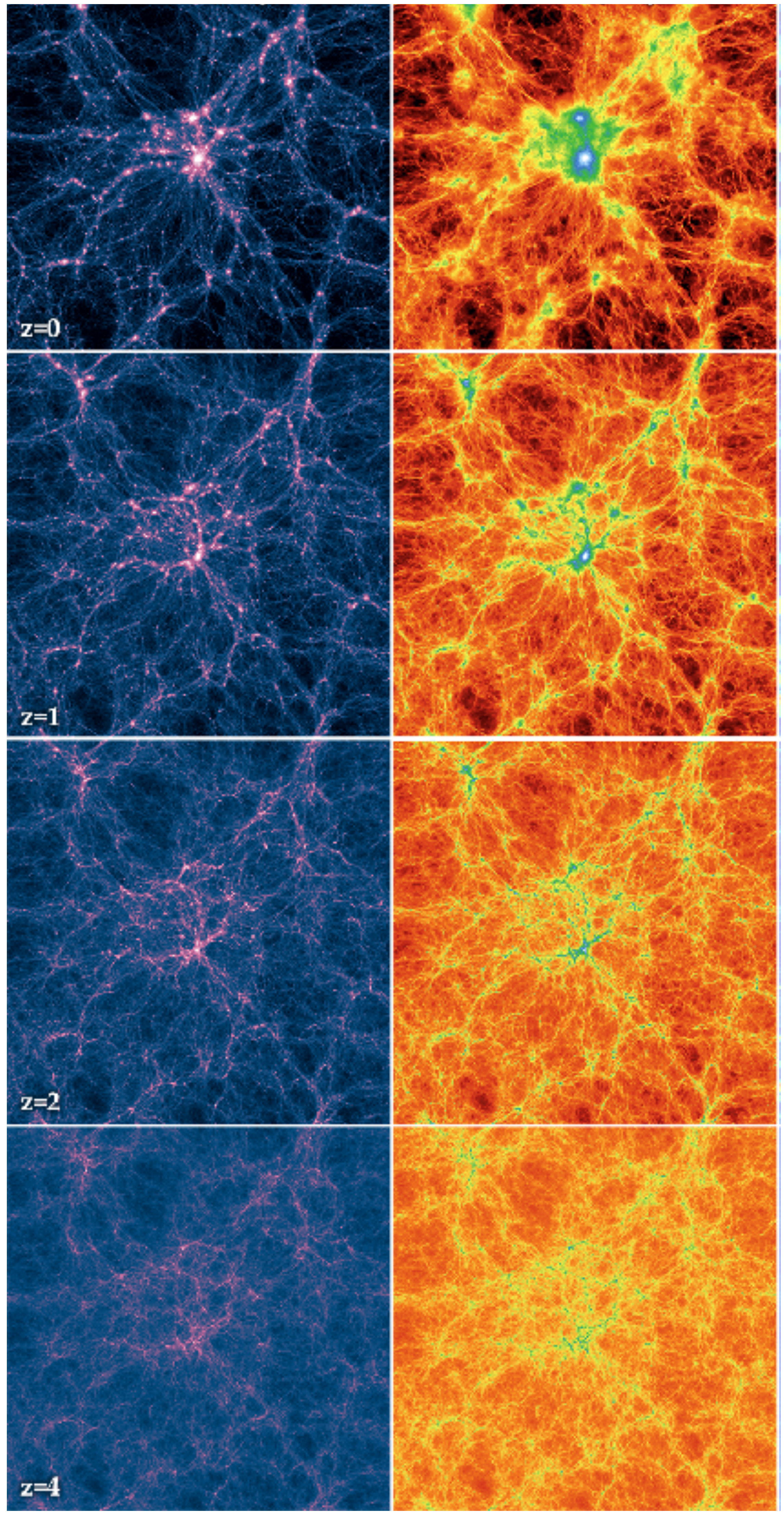
given two identical localized dark-matter configurations at a given time, will the same galaxies always form, or do we need to know the whole history of that configuration?

time

local



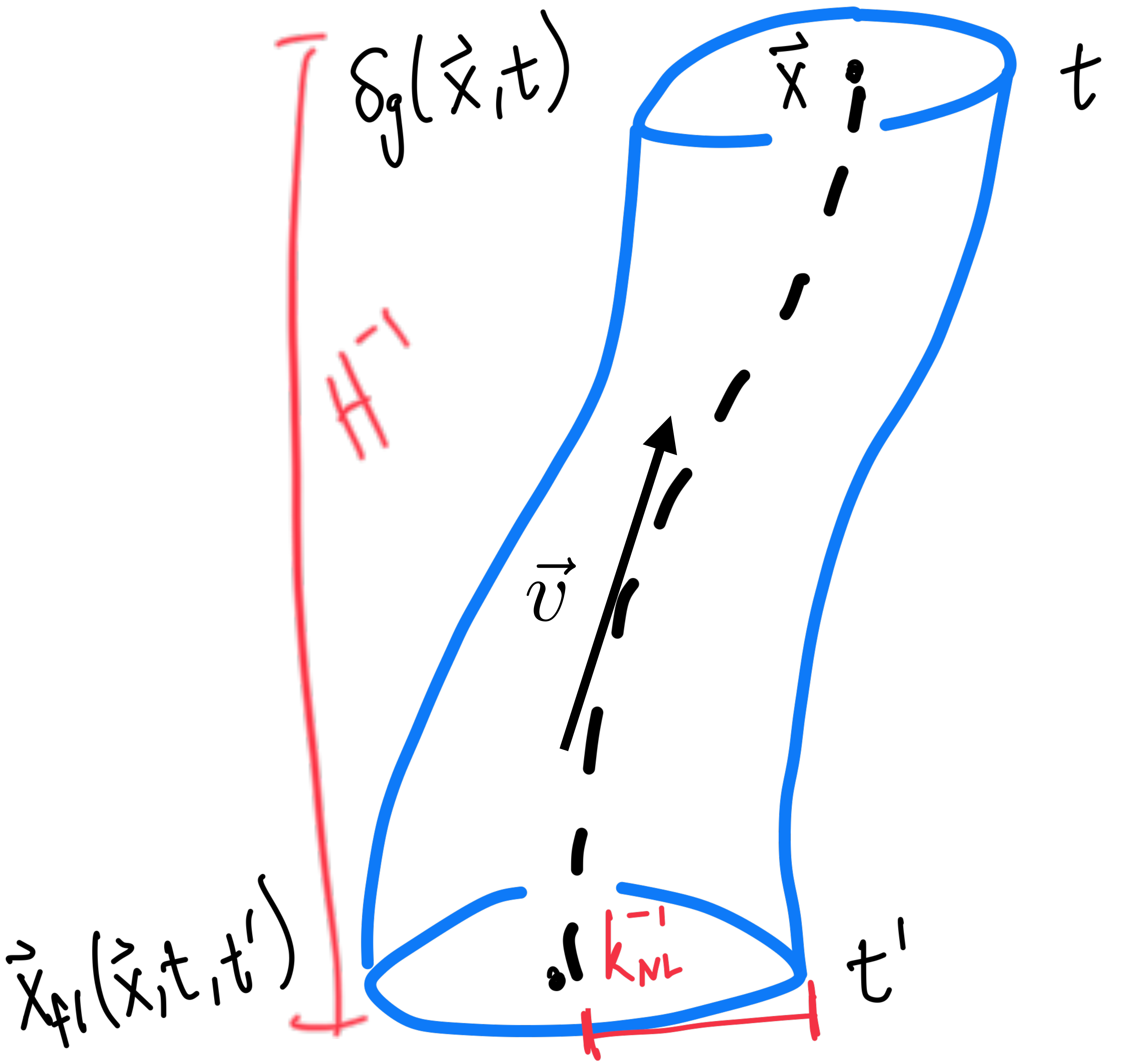
non-local



Illustris simulations

non-local-in-time structure formation

$$\tau_{ij}(\vec{x},t) = f_{\text{very complicated}}\left(\left\{H, \Omega_m, \ldots, m_{\rm dm}, g_{\rm ew}, \ldots, \rho_{\rm dm}, \partial_i v^j_{\rm dm}, \ldots\right\}_{\text{past tube}\,(\vec{x},t)}\right)_{ij}$$



Carrasco, Foreman, Green, Senatore 13

Senatore 14

Desjacques, Jeong, Schmidt 16

Donath, ML, Senatore, 23

Anastasiou, Favorito, ML, Senatore, Zheng 25

non-local-in-time structure formation

$$\tau_{ij}(\vec{x},t) = f_{\text{very complicated}}\left(\left\{H, \Omega_m, \ldots, m_{\rm dm}, g_{\rm ew}, \ldots, \rho_{\rm dm}, \partial_i v^j_{\rm dm}, \ldots\right\}_{\text{past tube}\,(\vec{x},t)}\right)_{ij}$$

$$\tau_{ij}(\vec{x},t) = \sum_{n=0}^{\infty} \int_{\text{past tube}\,(\vec{x},t)} d^4x_1 \ldots \int_{\text{past tube}\,(\vec{x},t)} d^4x_n \sum_{\text{all subsets}\,\{\mathcal{O}_1,\ldots,\mathcal{O}_{\rm n}\}\subset\{\mathcal{O}\}}$$

$$\left\{ \left.\frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta\mathcal{O}_1(x_1)\ldots\delta\mathcal{O}_n(x_n)}\right|_0 \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\right\}_{ij} ,$$

$$\left.\frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta\mathcal{O}_1(x_1)\ldots\delta\mathcal{O}_n(x_n)}\right|_0 \simeq$$

$$\langle \left.\frac{\delta^n f_{\text{very complicated}}(\{\mathcal{O}\}_{\text{past tube}})}{\delta\mathcal{O}_1(x_1)\ldots\delta\mathcal{O}_n(x_n)}\right|_0 \rangle + \epsilon_{1,\ldots,n}(x_1,\ldots,x_n) \,.$$

Carrasco, Foreman, Green, Senatore 13

Senatore 14

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non-local-in-time structure formation

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Senatore 14
Donath, ML, Senatore, 23
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$$\vec{x}_{\rm fl}(\vec{x},t,t') = \vec{x} + \int_t^{t'} \frac{dt_1}{a(t_1)} \vec{v}(\vec{x}_{\rm fl}(\vec{x},t,t_1),t_1)$$

$$\begin{aligned}\tau_{ij}(\vec{x},t) \simeq \sum_{n=0}^{\infty} \int dt_1 \ldots \int dt_n \sum_{\text{all subsets } \{\mathcal{O}_1,\ldots,\mathcal{O}_{\rm n}\}\subset\{\mathcal{O}\}} \\ \times \{(K_{1,\ldots,n}(t,t_1,\ldots,t_n) + \epsilon_{1,\ldots,n}(t,(\vec{x}_{\rm fl}(\vec{x},t,t_1),t_1),\ldots,(\vec{x}_{\rm fl}(\vec{x},t,t_n),t_n)) \\ + \frac{\partial}{k_{\rm NL}}\epsilon_{1,\ldots,n,\partial}(t,(\vec{x}_{\rm fl}(\vec{x},t,t_1),t_1),\ldots,(\vec{x}_{\rm fl}(\vec{x},t,t_n),t_n)) + \ldots\Big) \\ \times\ \mathcal{O}_1(\vec{x}_{\rm fl}(\vec{x},t,t_1),t_1)\ldots\mathcal{O}_n(\vec{x}_{\rm fl}(\vec{x},t,t_n),t_n)\}_{ij}\end{aligned}$$

non-local-in-time structure formation

Carrasco, Foreman, Green, Senatore 13

Senatore 14

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Anastasiou, Favorito, ML, Senatore, Zheng 25

$$\tau_{ij}(\vec{x},t) = \frac{\Omega_m \mathcal{H}^2 \bar{\rho}}{k_{\rm NL}^2} \sum_l \int^t dt' H(t') \Big(c_{1,\mathcal{O}_l}(t,t') \mathcal{O}_{1,l}(\vec{x}_{\rm fl}(\vec{x},t,t'),t')_{ij} +$$

$$+ \int^t dt'' H(t'') \Big(c_{2,\mathcal{O}_l}(t,t',t'') \mathcal{O}_{2,l}((\vec{x}_{\rm fl}(\vec{x},t,t'),t'),(\vec{x}_{\rm fl}(\vec{x},t,t''),t''))_{ij}$$

$$+ \int^t dt''' H(t''') \; c_{3,\mathcal{O}_l}(t,t',t'',t''') \mathcal{O}_{3,l}((\vec{x}_{\rm fl}(\vec{x},t,t'),t'),(\vec{x}_{\rm fl}(\vec{x},t,t''),t''),(\vec{x}_{\rm fl}(\vec{x},t,t'''),t'''))_{ij} \Big) \Big) \; ,$$

$$\mathcal{O}_{1,l} \in \left\{ r_{ij}, p_{ij}, \delta^K_{ij}\delta, \delta^K_{ij}\theta, \delta^K_{ij}\frac{\partial^2}{k_{\rm NL}^2}\delta \right\}$$

$$r_{ij} = \partial_i \partial_j \Phi, \quad p_{ij} = \partial_i v^j$$

$$\mathcal{O}_{2,l} \in \Big\{ \delta^K_{ij}\delta^2, \delta^K_{ij}\theta^2, \delta^K_{ij}\delta\theta, r_{ij}\delta, p_{ij}\delta, r_{ij}\theta, p_{ij}\theta, r_{ik}r_{kj}, p_{ik}p_{kj}, \frac{1}{2}(r_{ik}p_{kj} + r_{jk}p_{ki}), {\rm Tr}(r^2)\delta^K_{ij}, {\rm Tr}(p^2)\delta^K_{ij}, {\rm Tr}(rp)\delta^K_{ij} \Big\}$$

$$\mathcal{O}_{3,l} \in \left\{ r_{ik}r_{km}r_{mj}, {\rm Tr}(r^2)r_{ij}, {\rm Tr}(r^3)\delta^K_{ij}, r_{ik}r_{kj}\delta, {\rm Tr}(r^2)\delta\delta^K_{ij}, r_{ij}\delta^2, \delta^3\delta^K_{ij} \right\}$$

non-local-in-time structure formation

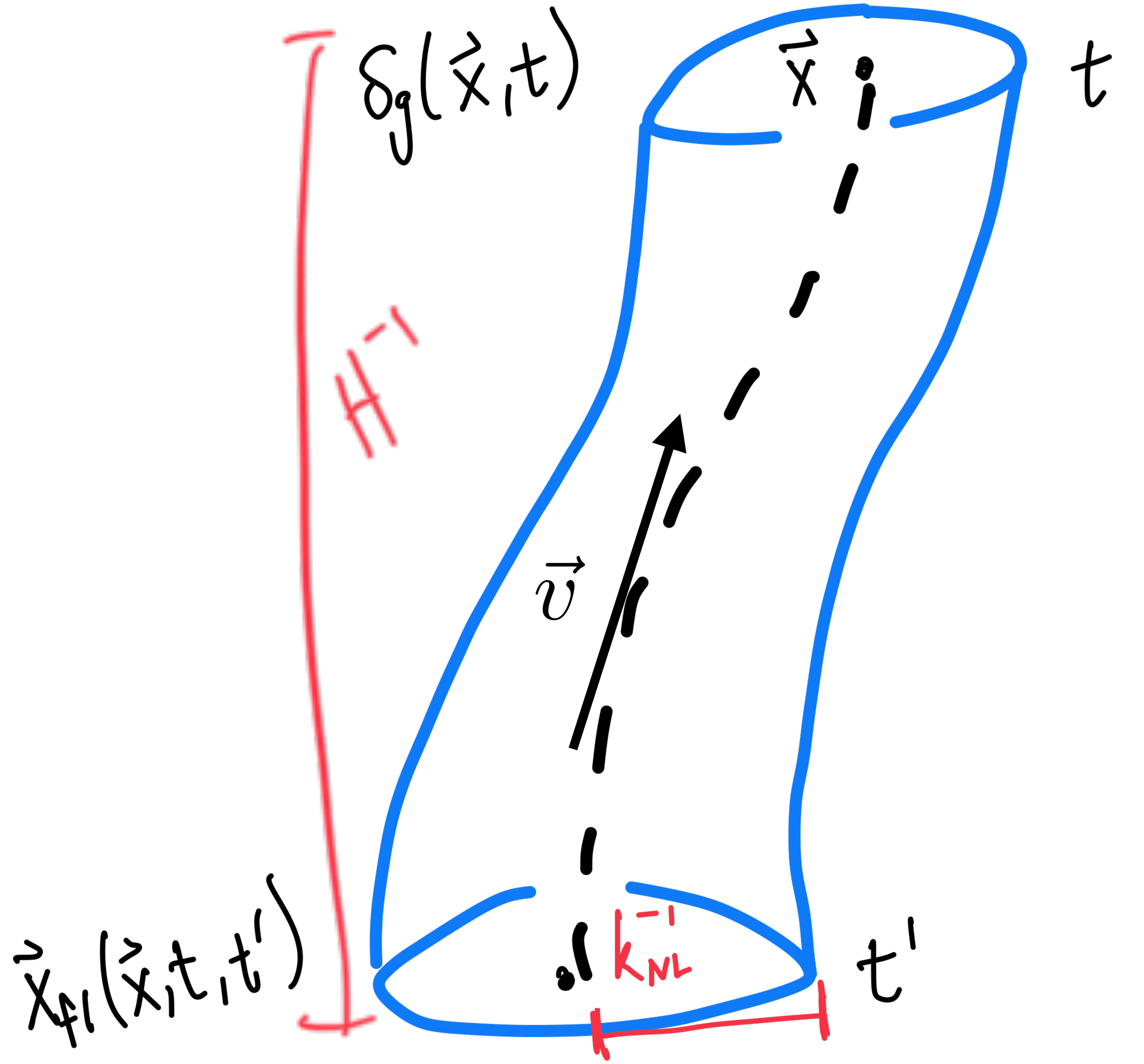
Donath, ML, Senatore, 23
Anastasiou, Favorito, ML, Senatore, Zheng 25
Ansari, Banerjee, Jain, Padhyegurjar 24
Vlah, Chisari, Schmidt 19

$$\tau^{ij}_{(n)}(\vec{x},t) = \sum_{\Gamma\in\Gamma_n} c_\Gamma(t)[\mathbb{C}^{ij}_\Gamma]^{(n)}(\vec{x},t)$$

18 total at 3rd order;
17 for local-in-time

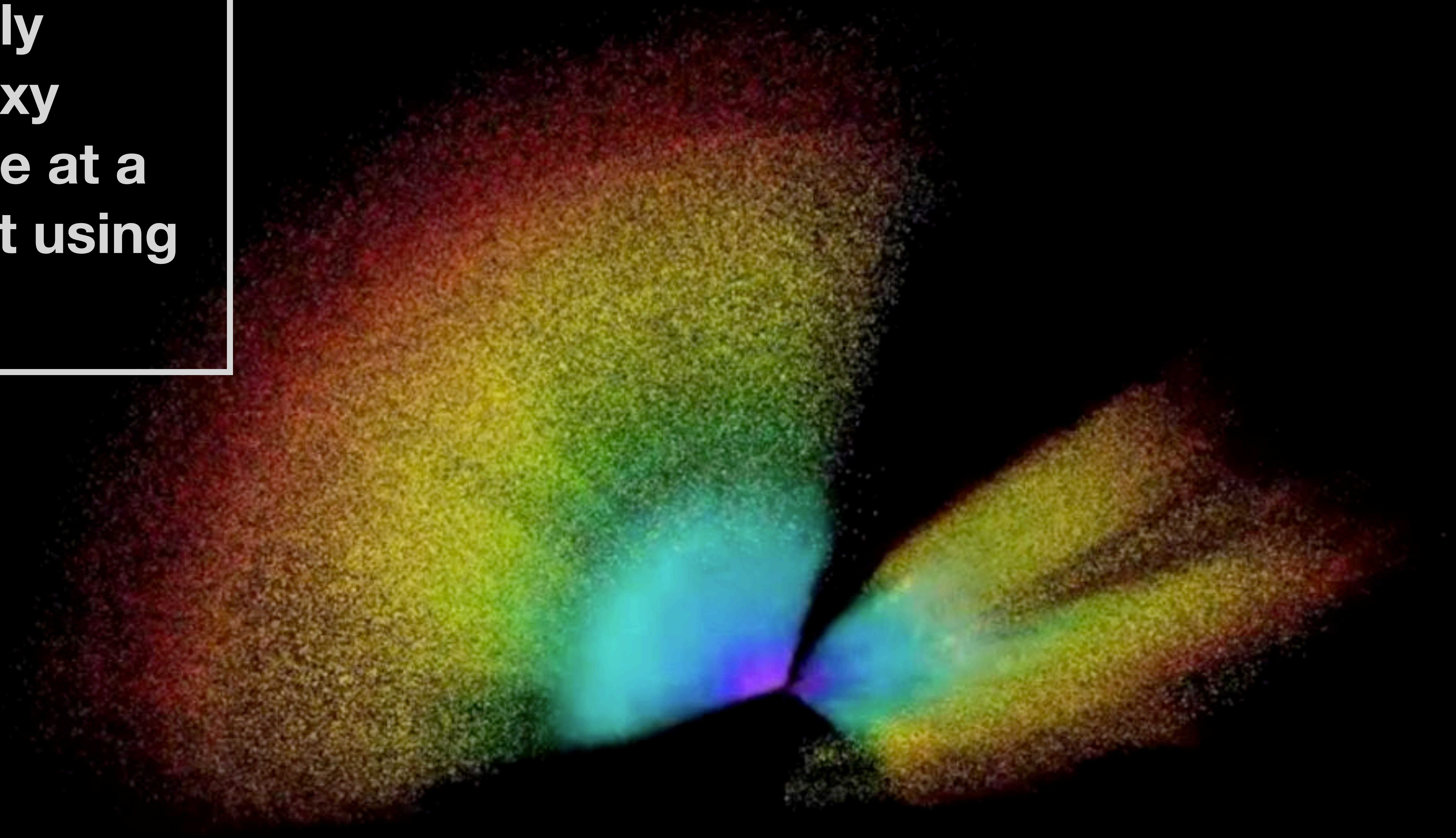
galaxy bias
@ 5th order
non-local: 29
local: 26

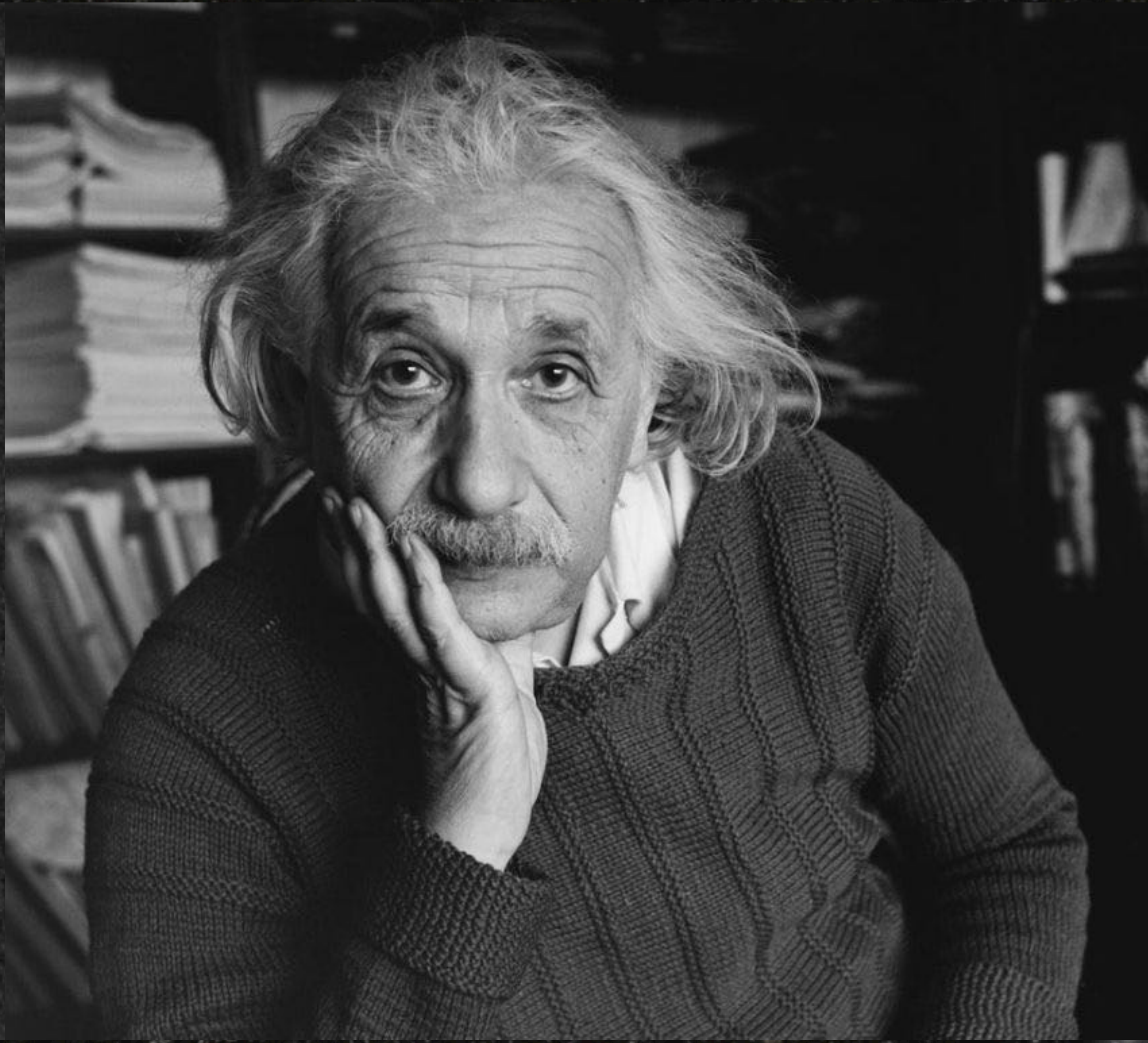
the galaxy-clustering signal at fifth order is sensitive to whether or not galaxies form on time scales of order Hubble



Donath, ML, Senatore, 23

we can directly measure galaxy formation time at a single redshift using field theory.



solution of DM EOM

Anastasiou, Favorito, ML, Senatore, Zheng 25

with τ^{ij}, solve EOM for δ

$$\delta_{\rm ct}^{(3)}(a) = \frac{D(a)^5 \partial_i \partial_j}{26} \Bigg[3 \left(\frac{\partial_i \tilde\delta_{\rm ct}^{(2)}}{\partial^2} \frac{\partial_j \tilde\delta^{(1)}}{\partial^2} + \frac{\partial_i \tilde\delta_{\rm ct}^{(1)}}{\partial^2} \frac{\partial_j \tilde\delta^{(2)}}{\partial^2} \right) - \frac{3}{2}\delta_{ij} \left(\frac{\partial_k \tilde\delta_{\rm ct}^{(2)}}{\partial^2} \frac{\partial_k \tilde\delta^{(1)}}{\partial^2} + \frac{\partial_k \tilde\delta_{\rm ct}^{(1)}}{\partial^2} \frac{\partial_k \tilde\delta^{(2)}}{\partial^2} \right) + 2 \left(\tilde\pi^i_{{\rm ct},(2)} \tilde\pi^j_{(1)} + \tilde\pi^i_{{\rm ct},(1)} \tilde\pi^j_{(2)} \right) - 2 \tilde\pi^i_{{\rm ct},(1)} \tilde\pi^j_{(1)} \tilde\delta^{(1)} - \tilde\pi^i_{(1)} \tilde\pi^j_{(1)} \tilde\delta^{(1)}_{\rm ct} + \frac{1}{k_{\rm NL}^2} \tilde\tau^{ij}_{{\rm ct},(3)} \Bigg],$$

vorticity generated, contributes to DM at 3rd order

$$\omega^i_{(2)} \sim \epsilon^{ijk} \partial_j \partial_l \tau^{lk}_{(2)}$$

lower-order counterterms must appear consistently

dark matter at two loops

two-loop DM power spectrum

Anastasiou, Favorito, ML, Senatore, Zheng 25

Bakx, Rubira, Chisari, Vlah 25

Garny, Taule 23

Garny, Taule 22

Fasiello, Fujita, Vlah 22

Konstandin, Porto, Rubira 19

Foreman, Perrier, Senatore 15

Carrasco, Foreman, Green, Senatore 13

$$\langle \delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')P(k)$$

$$\delta = \delta_{\text{no-}ct} + \delta_{ct}$$

$$\delta_{\text{no}-ct} = \delta^{(1)} + \cdots + \delta^{(5)}$$

$$\delta_{ct} = \delta_{ct}^{(1)} + \delta_{ct}^{(2)} + \delta_{ct}^{(3)}$$

$$P(k) = P_{\text{no-}ct}(k) + P_{ct}(k)$$

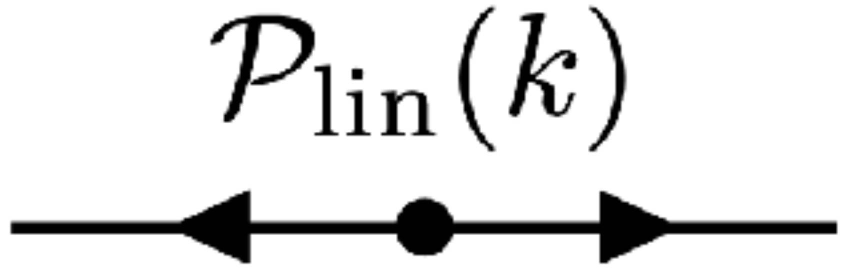
two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

$$\mathcal{P}_{\text{no-}ct}(k) = \mathcal{P}_{\text{lin.}}(k) + \mathcal{P}_{\text{1-loop}}(k) + \mathcal{P}_{\text{2-loop}}(k)$$

linear power spectrum/propagator

$\mathcal{P}_{\text{lin.}}(k) \equiv$

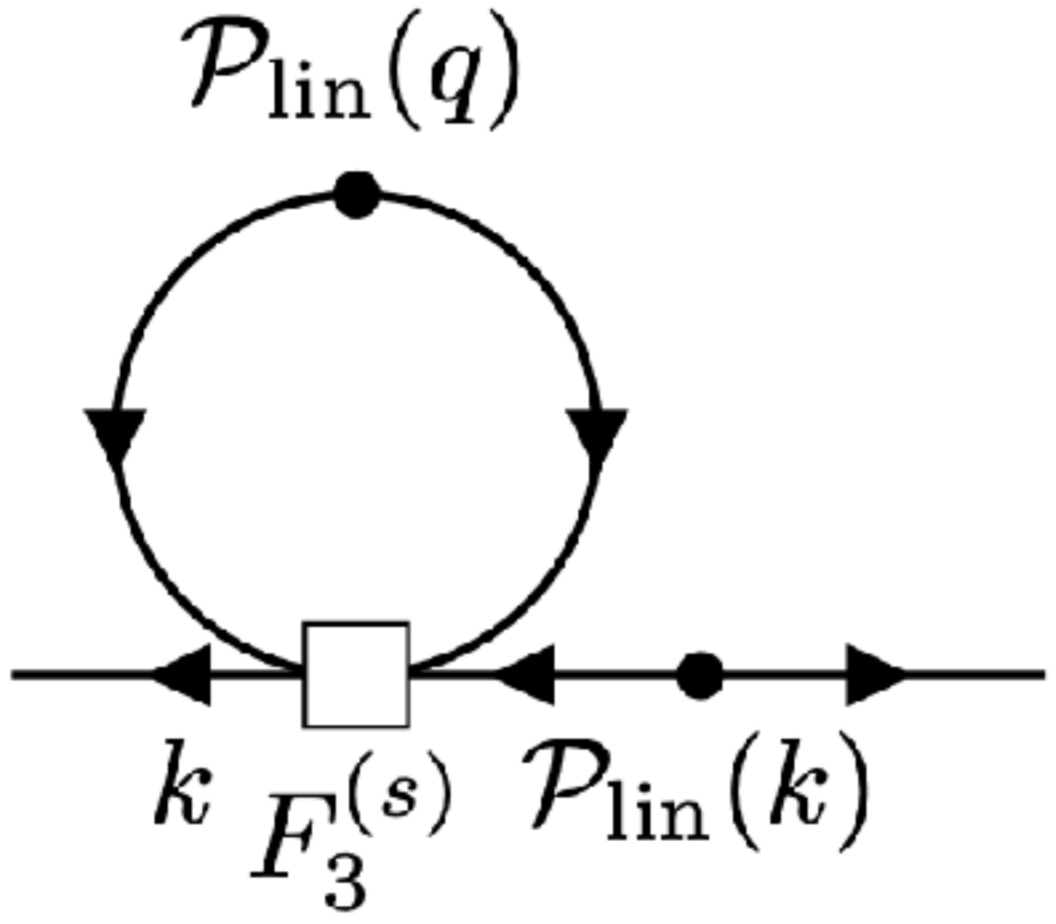


one-loop power spectrum

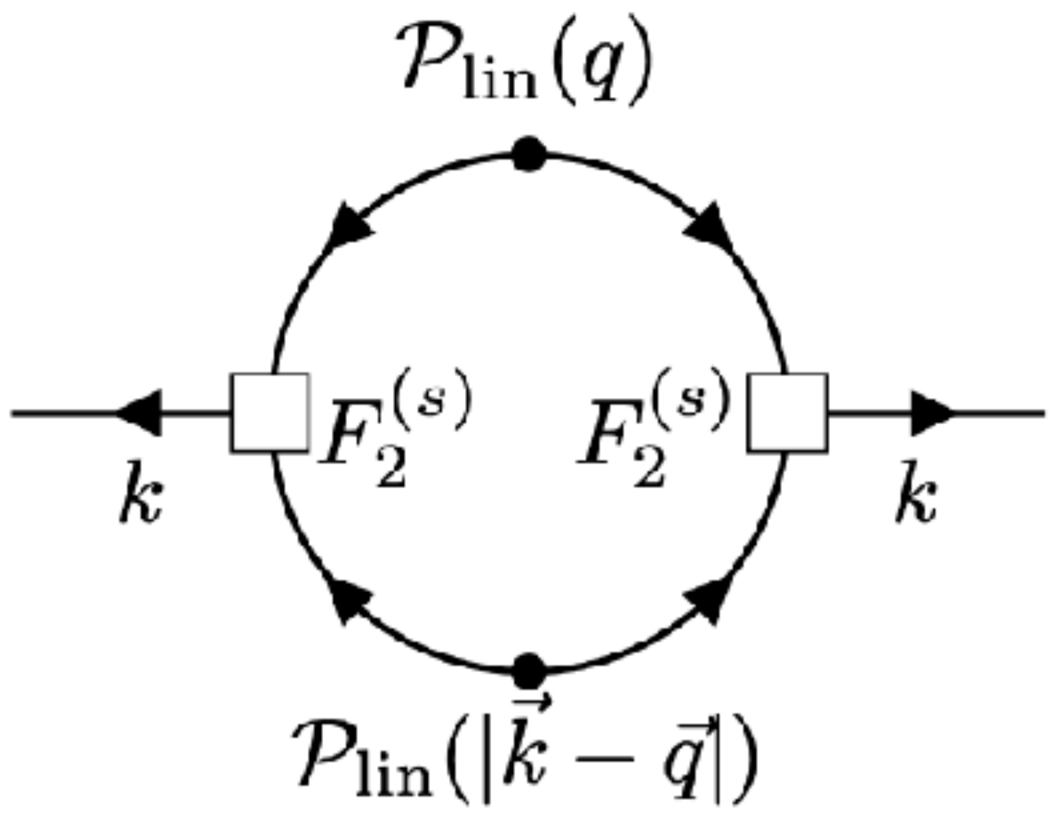
$$p_{22}(\vec{k}, \vec{q}) = 2\,\mathcal{P}_{\text{lin.}}(q)\,\mathcal{P}_{\text{lin.}}(|\vec{k} - \vec{q}|)\,\left[F_2(\vec{q}, \vec{k} - \vec{q})\right]^2,$$

$$p_{13}(\vec{k}, \vec{q}) = 6\mathcal{P}_{\text{lin.}}(k)\,\mathcal{P}_{\text{lin.}}(q)\,F_3(\vec{q}, -\vec{q}, \vec{k}).$$

$\mathcal{P}_{13}(k) \equiv$



, $\mathcal{P}_{22}(k) \equiv$



two-loop DM power spectrum

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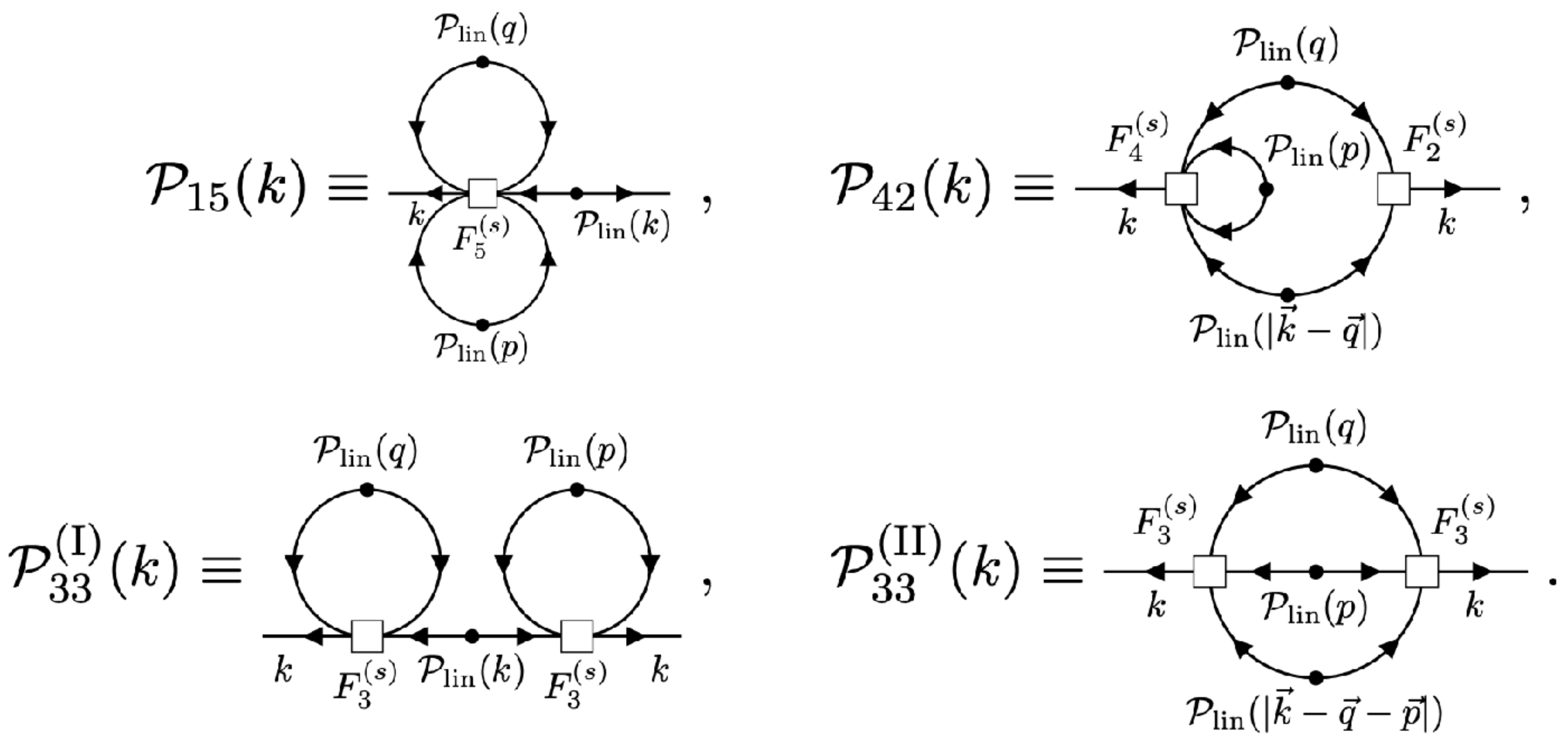
two-loop power spectrum



two-loop DM power spectrum

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two-loop power spectrum

$$p_{15}(\vec{k},\vec{p},\vec{q}) = 30\mathcal{P}_{\text{lin.}}(k)\,\mathcal{P}_{\text{lin.}}(q)\,\mathcal{P}_{\text{lin.}}(p)\,F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p})\,,$$

$$p_{42}(\vec{k},\vec{p},\vec{q}) = 24\,\mathcal{P}_{\text{lin.}}(q)\,\mathcal{P}_{\text{lin.}}(p)\,\mathcal{P}_{\text{lin.}}(|\vec{k}-\vec{q}|) \times F_2(\vec{q},\vec{k}-\vec{q})F_4(-\vec{q},\vec{q}-\vec{k},\vec{p},-\vec{p})\,,$$

$$p_{33}^{(\text{I})}(\vec{k},\vec{p},\vec{q}) = 9\,\mathcal{P}_{\text{lin.}}(k)\,\mathcal{P}_{\text{lin.}}(q)\,\mathcal{P}_{\text{lin.}}(p)\,F_3(\vec{k},\vec{q},-\vec{q})F_3(-\vec{k},\vec{p},-\vec{p})\,,$$

$$p_{33}^{(\text{II})}(\vec{k},\vec{p},\vec{q}) = 6\,\mathcal{P}_{\text{lin.}}(q)\mathcal{P}_{\text{lin.}}(p)\mathcal{P}_{\text{lin.}}(|\vec{k}-\vec{q}-\vec{p}|) \times F_3(\vec{q},\vec{p},\vec{k}-\vec{q}-\vec{p})F_3(-\vec{q},-\vec{p},-\vec{k}+\vec{q}+\vec{p})\,.$$

two-loop DM power spectrum

Anastasiou, Favorito, ML,
Senatore, Zheng 25

counterterm diagrams

$$\mathcal{P}^{\rm ct}_{13}(k,a) = 2D(a)^4 \langle \tilde\delta^{(1)}_{\rm ct}(\vec k)\tilde\delta^{(1)}(\vec k')\rangle' \ ,$$
$$\mathcal{P}^{\rm ct}_{15}(k,a) = 2D(a)^6 \left(\langle \tilde\delta^{(3)}_{\rm ct}(\vec k)\tilde\delta^{(1)}(\vec k')\rangle' + \langle \tilde\delta^{(1)}_{\partial^2{\rm ct}}(\vec k)\tilde\delta^{(1)}(\vec k')\rangle' \right) \ ,$$
$$\mathcal{P}^{\rm ct}_{33}(k,a) = D(a)^6 \left(2\langle \tilde\delta^{(1)}_{\rm ct}(\vec k)\tilde\delta^{(3)}(\vec k')\rangle' + \langle \tilde\delta^{(1)}_{\rm ct}(\vec k)\tilde\delta^{(1)}_{\rm ct}(\vec k')\rangle' \right) \ ,$$
$$\mathcal{P}^{\rm ct}_{42}(k,a) = 2D(a)^6 \langle \tilde\delta^{(2)}_{\rm ct}(\vec k)\tilde\delta^{(2)}(\vec k')\rangle' \ ,$$

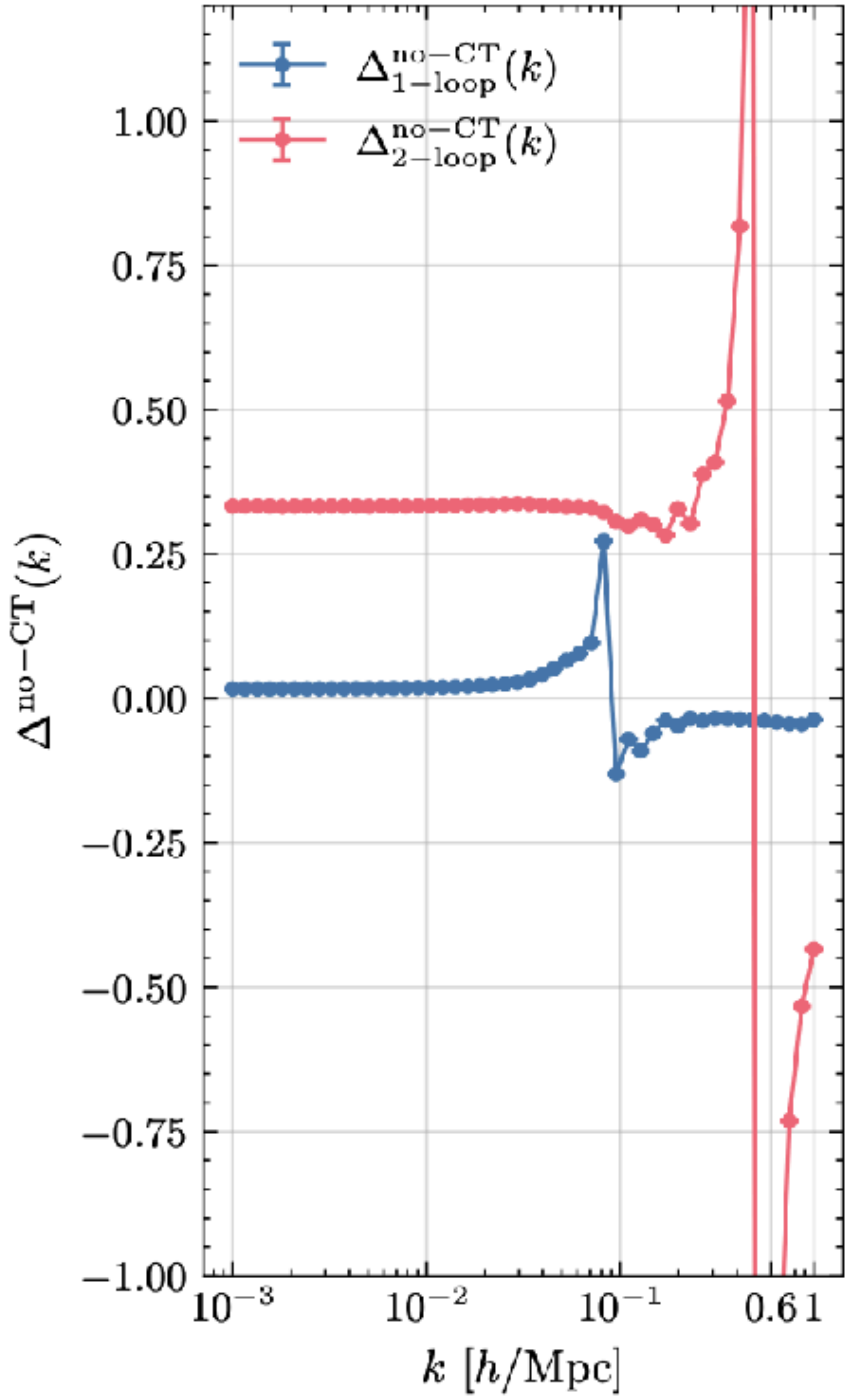
in terms of 18 EFT coefficients in τ^{ij} we found before

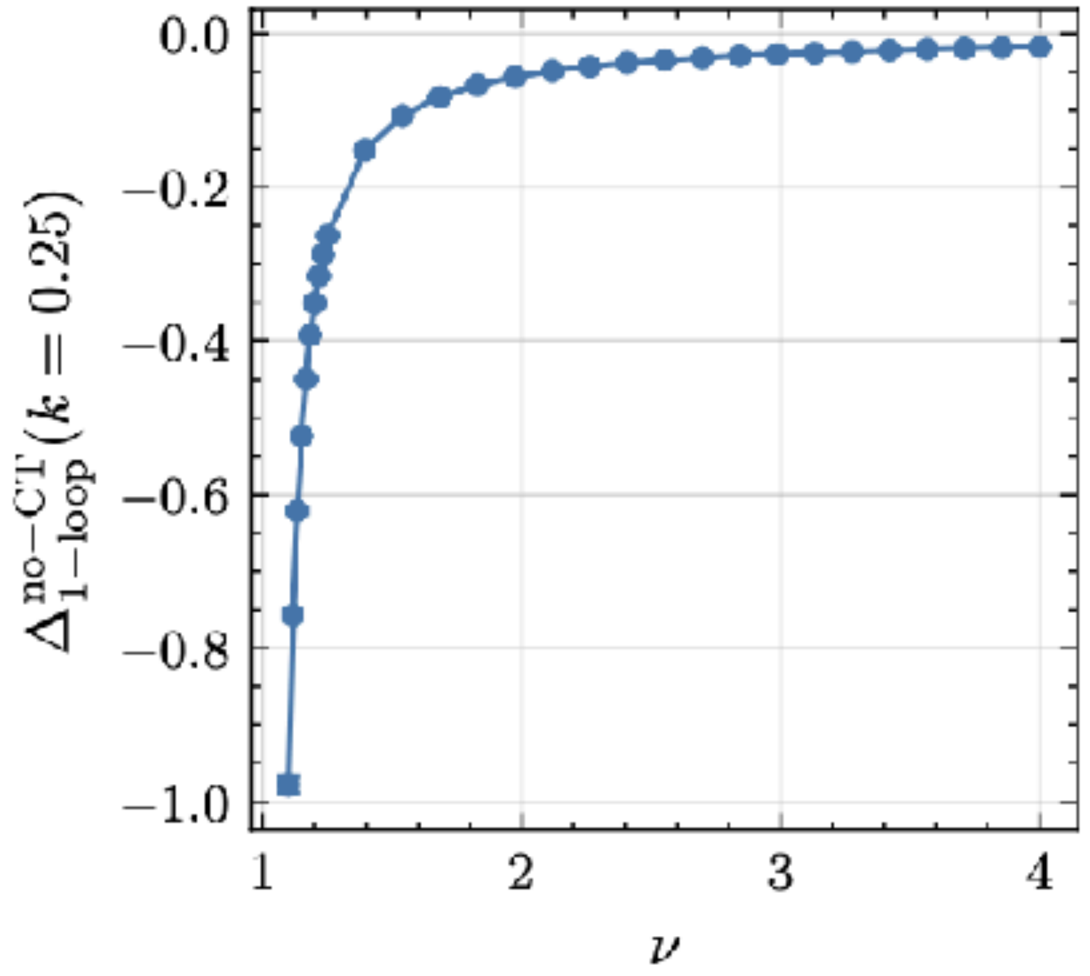
after contractions, 8 independent EFT coefficients

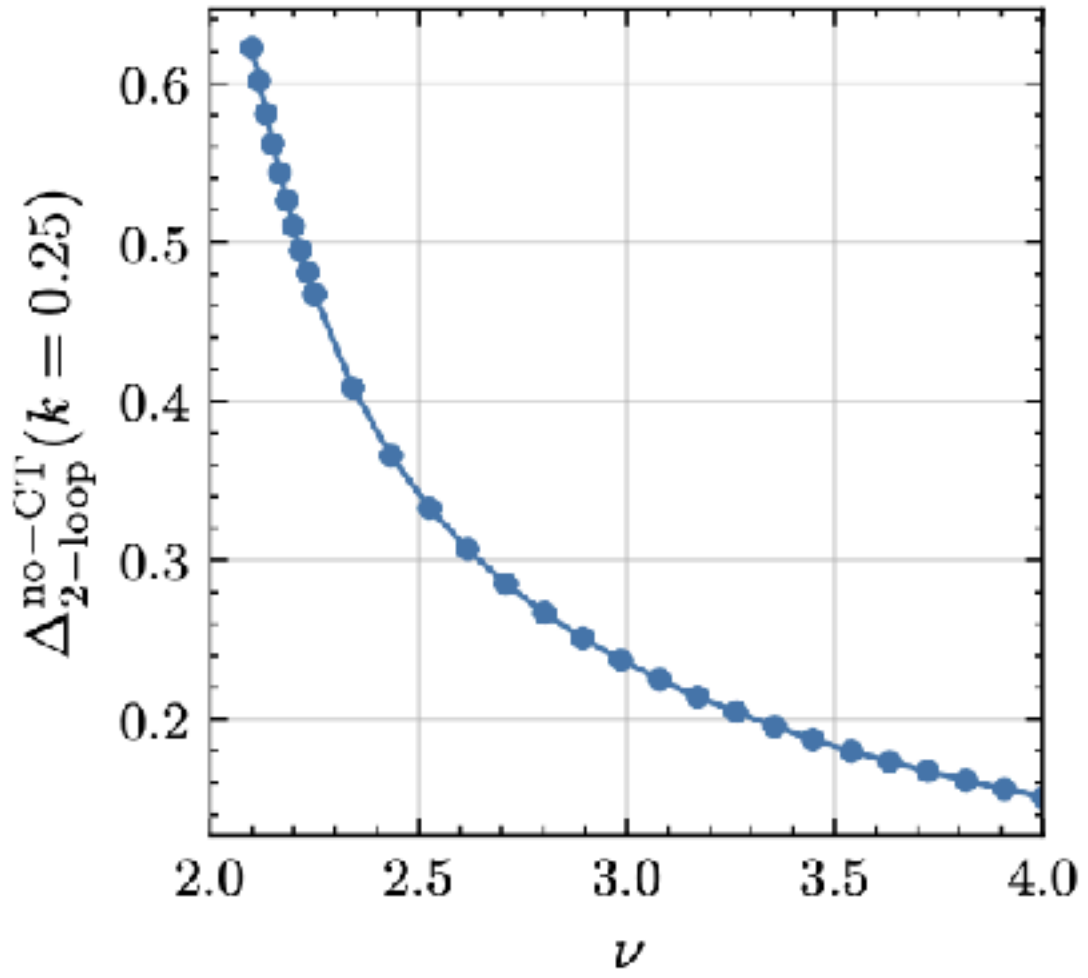
two-loop DM power spectrum

Anastasiou, Favorito, ML, Senatore, Zheng 25

UV regulation and renormalization







measure contribution from UV

$$\Delta_{n\text{-loop}} = \frac{\int_{1.3}^{\infty} p^{\text{no-}ct}_{n\text{-loop}}}{\int_{0}^{\infty} p^{\text{no-}ct}_{n\text{-loop}}}$$

not UV divergent, but slightly UV dependent

$$\mathcal{P}_{\text{lin.}}(k;\nu) = \mathcal{P}_{\text{lin.}}(k)\,\theta(q_{UV}-k) + \mathcal{P}_{\text{lin.}}(q_{UV})\left(\frac{q_{UV}}{k}\right)^{\nu}\theta(k-q_{UV}).$$

two-loop DM power spectrum

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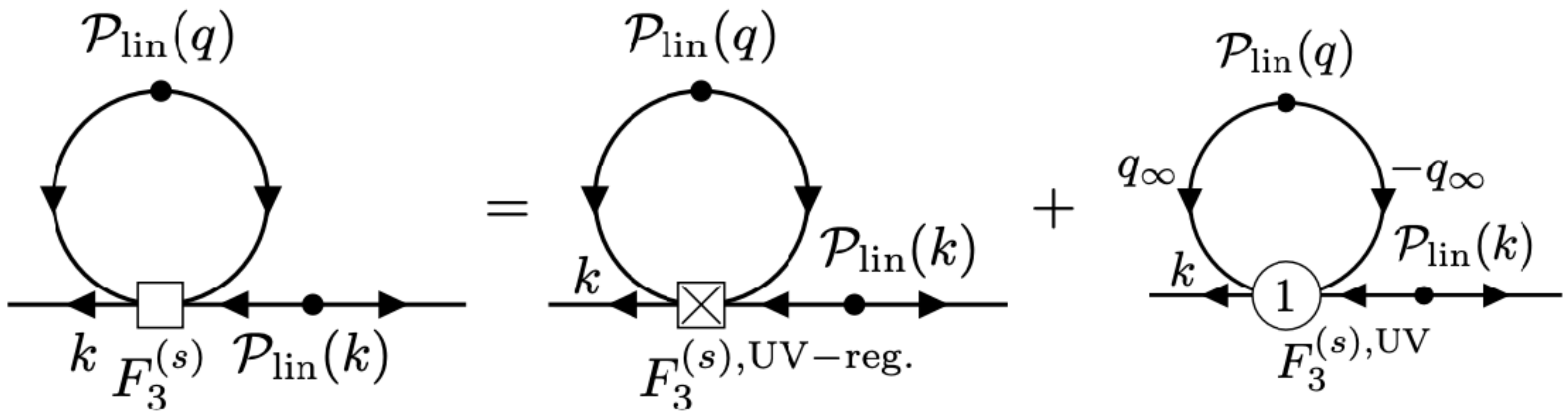
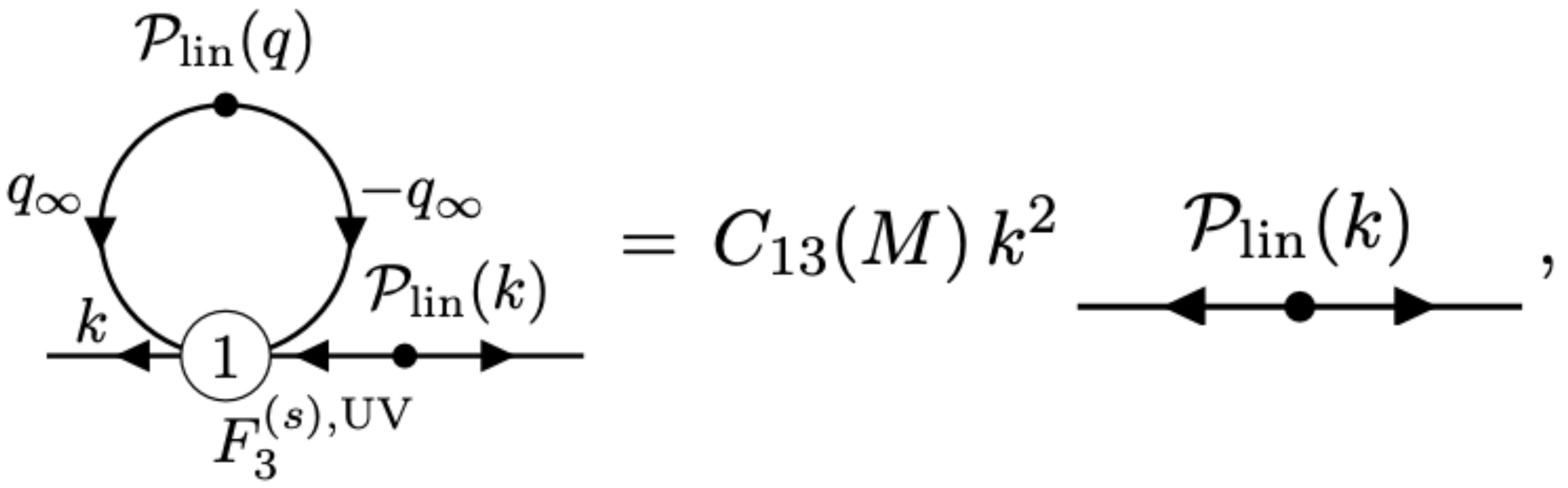
Senatore, Zheng 25

UV regulation and renormalization

one loop



UV regulated **UV limit**



two-loop DM power spectrum

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UV regulation and renormalization

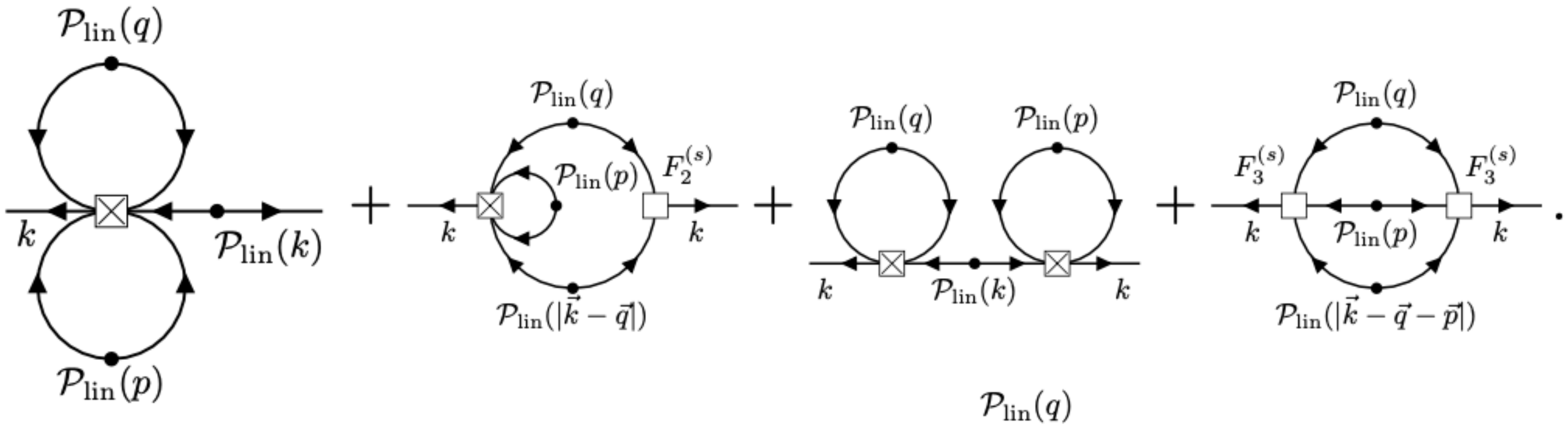
two loops

$$\mathcal{P}_{2-\text{loop}} = \mathcal{P}^{\text{UV}-\text{reg.}}_{2-\text{loop}} + \mathcal{P}^{\text{UV}-(1)}_{2-\text{loop}} + \mathcal{P}^{\text{UV}-(2)}_{2-\text{loop}}$$

single and double hard limits

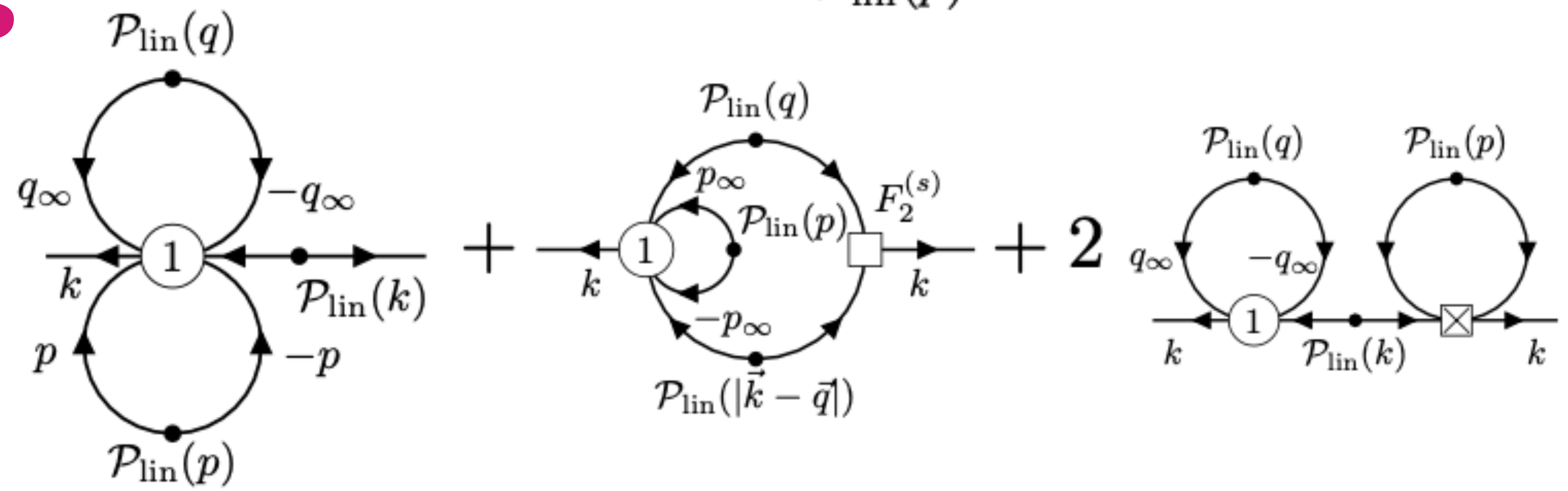
regulated, locally UV finite

$$\mathcal{P}^{\text{UV}-\text{reg.}}_{2-\text{loop}} =$$

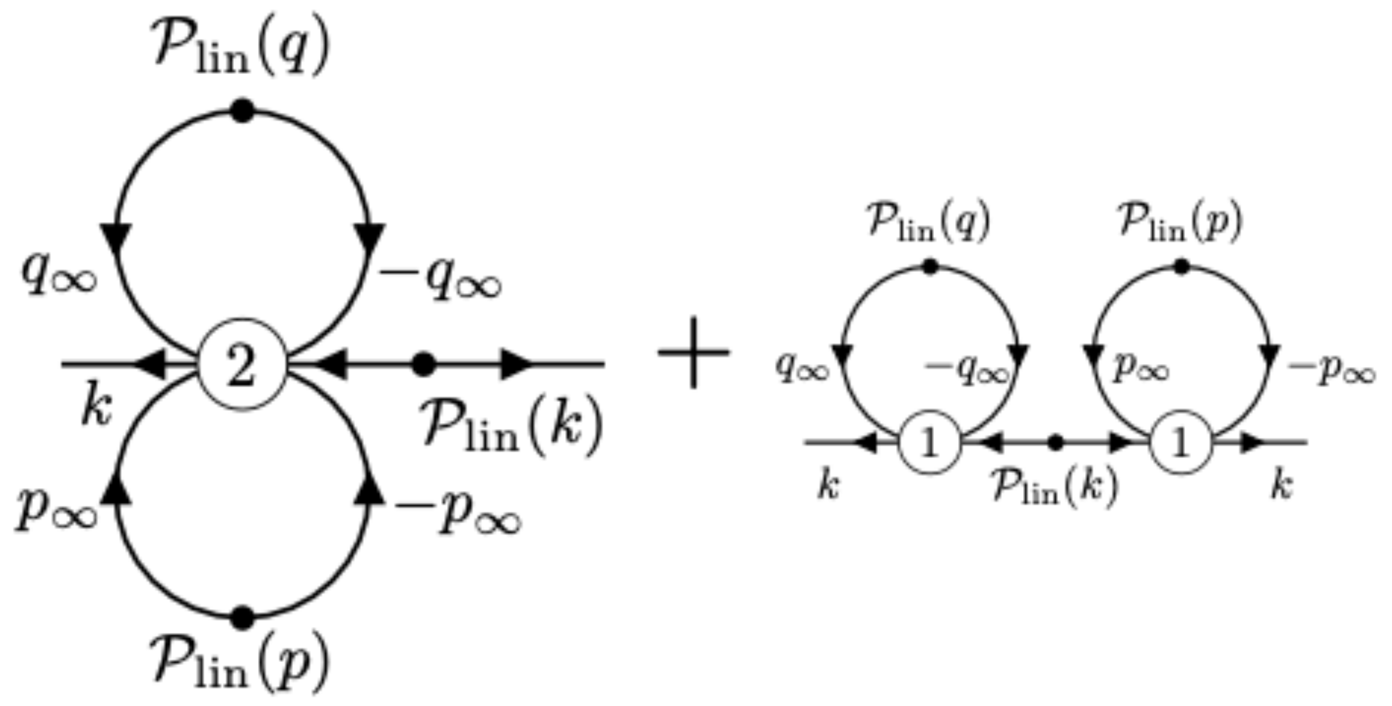


UV limits

$$\mathcal{P}^{\text{UV}-(1)}_{2-\text{loop}} = 2$$



$$\mathcal{P}^{\text{UV}-(2)}_{2-\text{loop}} =$$



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two-loop DM power spectrum

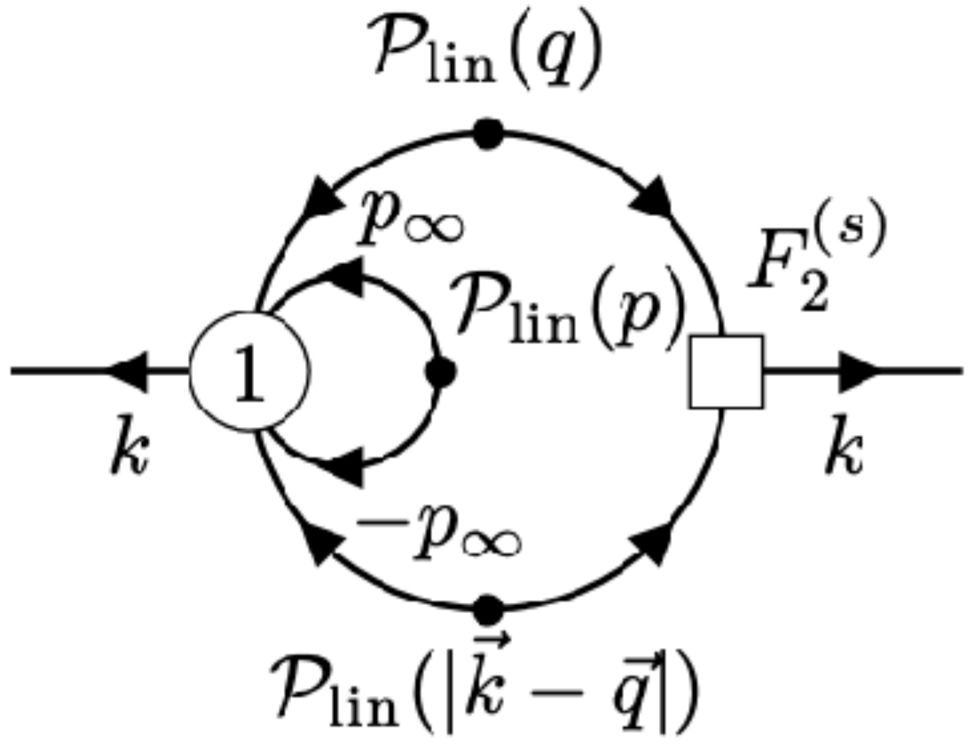
Anastasiou, Favorito, ML, Senatore, Zheng 25

UV regulation and renormalization

local UV subtractions in the integrand

$$p_{\text{2-loop}}^{\mathrm{UV-reg}} = p_{\text{2-loop}} - \mathcal{R}_{1-UV}[p_{\text{2-loop}}] - \mathcal{R}_{2-UV}\left[p_{\text{2-loop}} - \mathcal{R}_{1-UV}[p_{\text{2-loop}}]\right]$$

P_{42} eg.



$$= \left(\int_{\vec{p}} \frac{\mathcal{P}_{\rm lin.}(p)}{p^2} f_{\rm screen}(p^2)\right) \int_{\vec{q}} \frac{\mathcal{P}_{\rm lin.}(q)\mathcal{P}_{\rm lin.}(|\vec{k}-\vec{q}|)F_2(\vec{q},\vec{k}-\vec{q})}{169785 q^2 |\vec{k}-\vec{q}|^2} \times$$
$$\times \left(48096(\vec{k}\cdot\vec{q})^3 + (16892k^2 - 48096q^2)(\vec{k}\cdot\vec{q})^2 + \right.$$
$$\left. + (-32879(k^2)^2 + 35744q^2k^2)(\vec{k}\cdot\vec{q}) - 25933q^2(k^2)^2 + 6176(q^2)^2k^2\right)$$

$$f_{\rm screen}(q^2) = \theta(q^2 - (0.6\,h/\mathrm{Mpc})^2)$$

two-loop DM power spectrum

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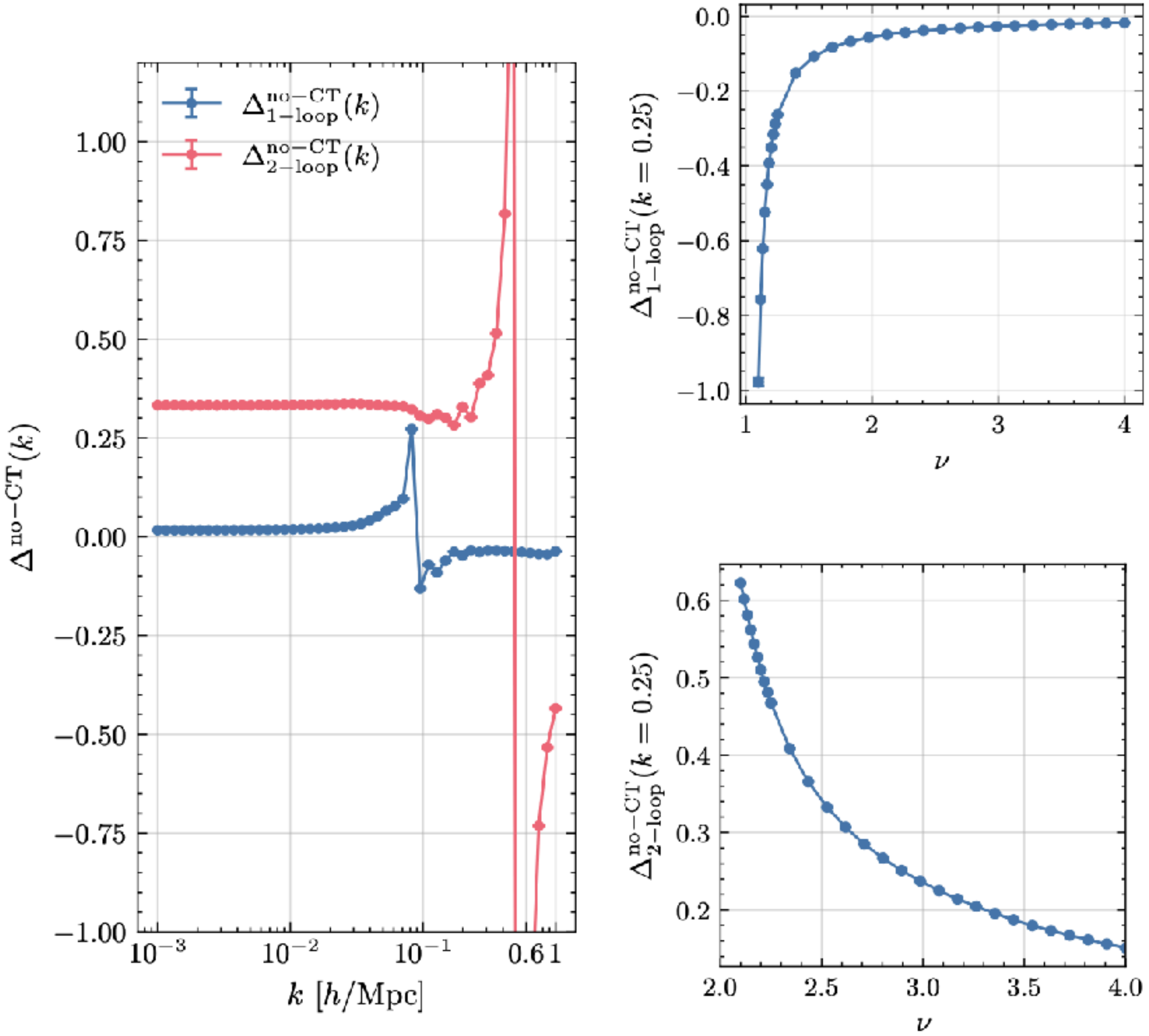
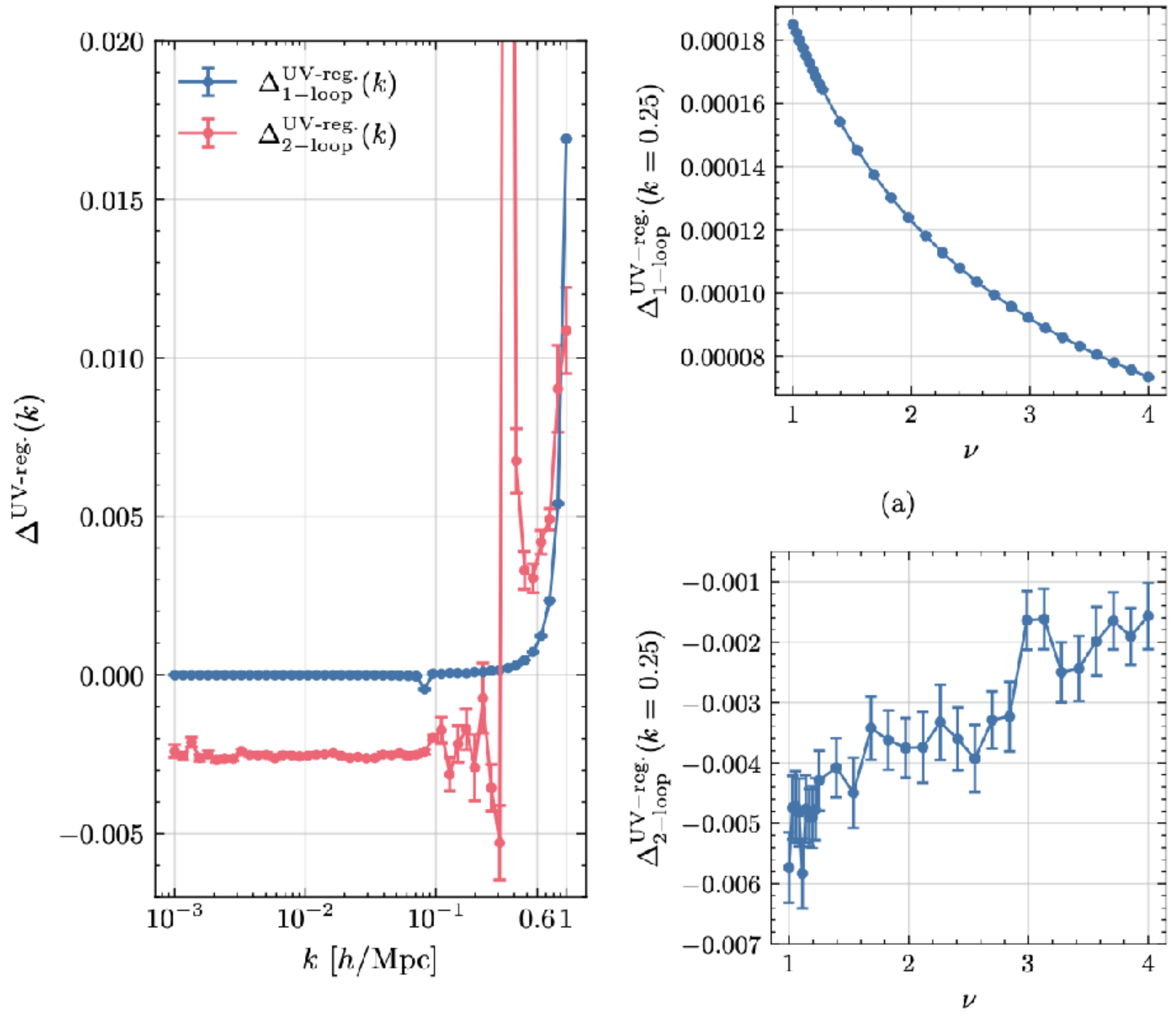
UV sensitive



UV regulated



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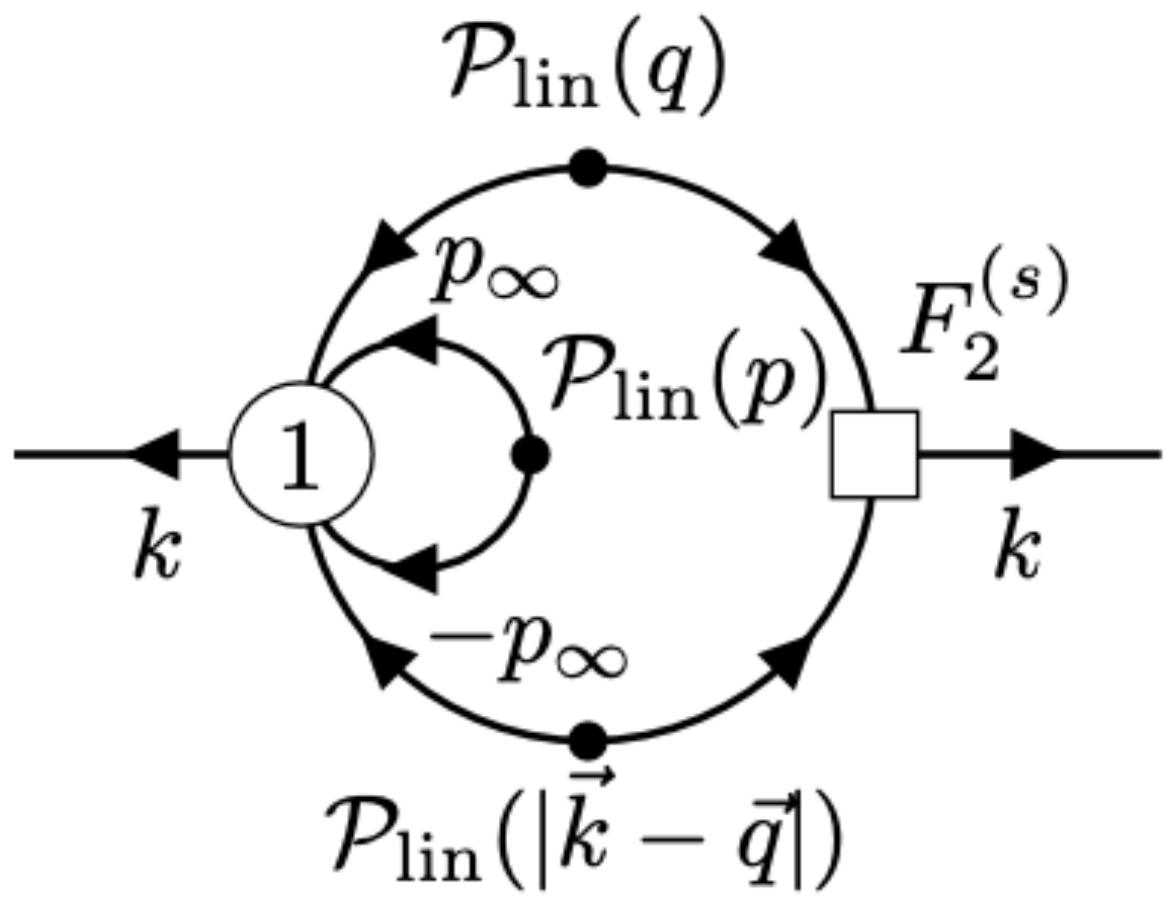
UV matching

every UV divergence should be cancelled by an EFT counterterm

eg. UV limit of P_{42}

determines 4 EFT coefficients in $\delta_{ct}^{(2)}$



$$= \quad 2\langle \tilde{\delta}_{\rm ct}^{(2)}(\vec{k})\tilde{\delta}^{(2)}(\vec{k}')\rangle$$

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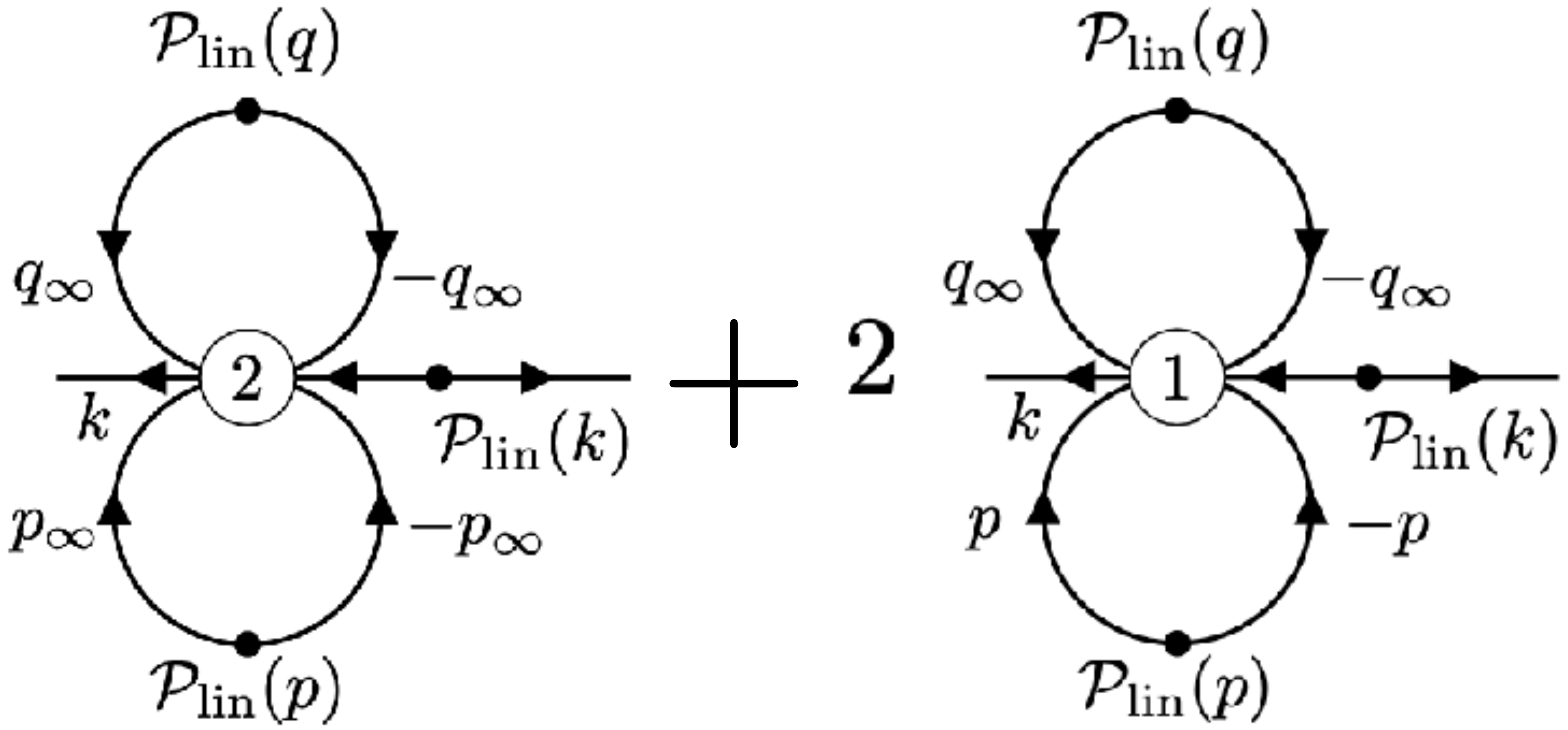
UV matching

every UV divergence should be cancelled by an EFT counterterm

similarly for P_{15}

determines 4 EFT coefficients



$$= 2\left(\langle\tilde{\delta}^{(3)}_{\rm ct}(\vec{k})\tilde{\delta}^{(1)}(\vec{k}')\rangle + \langle\tilde{\delta}^{(1)}_{\partial^2{\rm ct}}(\vec{k})\tilde{\delta}^{(1)}(\vec{k}')\rangle\right)$$

all 8 EFT coefficients are necessary and sufficient to cancel UV

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importance of EFT counterterms

how many of the 8 are needed for data?

$$\theta^i = \{c_{\delta,1}, c_{\delta,2}, c_{\delta^2,1}, c_{r\delta,1}, c_{\delta,3}, c_{r\delta,2}, c_{r\delta^2,1}, c_{\partial^2\delta,1}\}$$

$$F_{ij} = \frac{1}{2}\frac{\partial^2}{\partial\theta^i\partial\theta^j}\sum_{n,m}\mathcal{P}(k_n)C^{-1}(k_n,k_m)\mathcal{P}(k_m)\Big|_{\theta=0}$$

Fisher matrix, DESI-size covariance

diagonalizing

$$D^{-1/2} \approx \{0.03, 0.08, 0.25, 14, 180, 590, 14000,\ 27000\}$$

so a DESI-size survey would determine 3 parameters well

previous works have used only 3/4 EFT coefficients, tested UV sensitivity numerically (but integrals are actually convergent)

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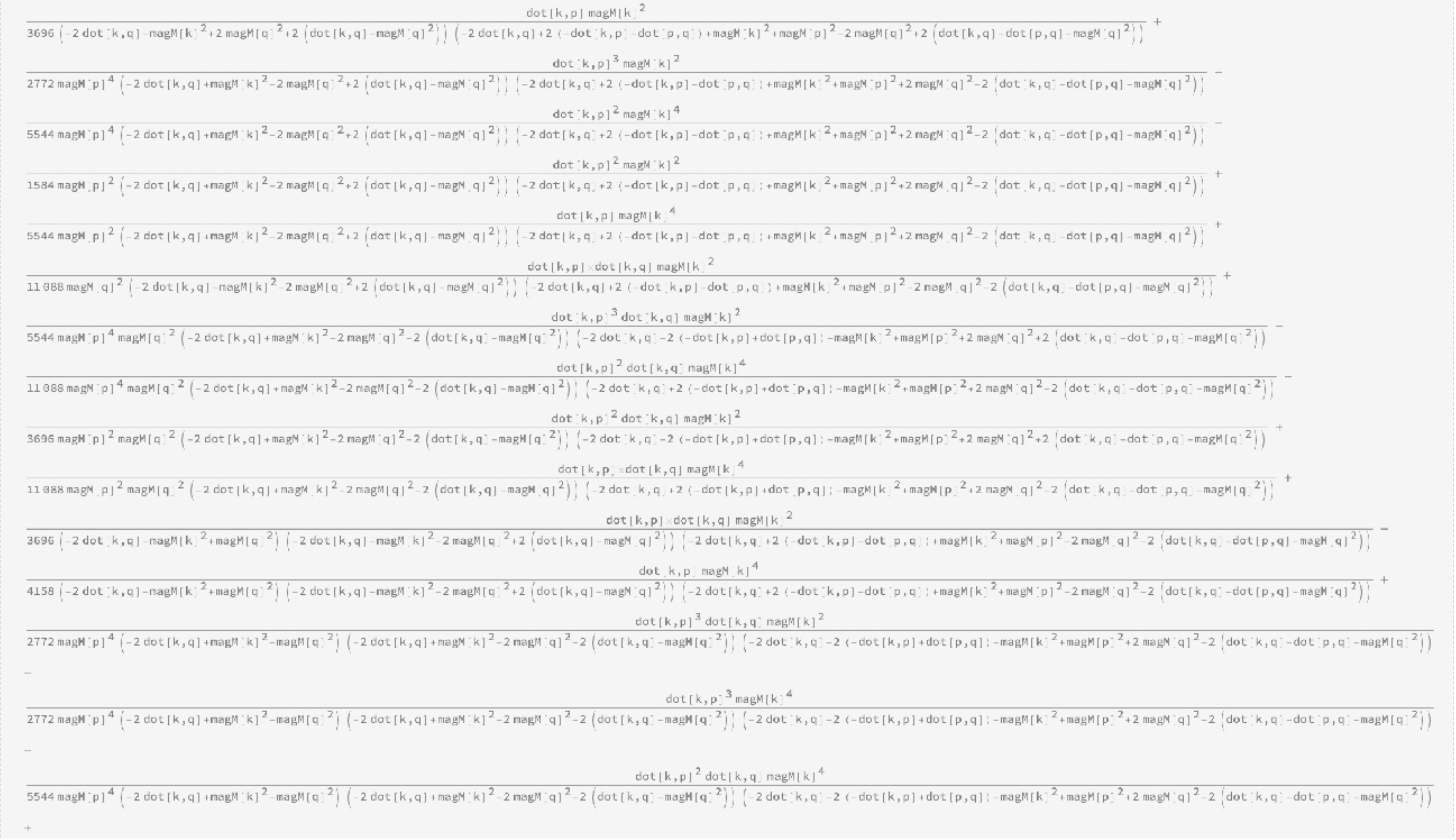
EFT counterterms

P_{42} diagram
~ 20x more terms than counterterms



integration

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IR limits

IR divergences cancel between diagrams because of the equiv. principle and Galilean invariance

but this can be a problem when numerically computing diagrams separately

particularly difficult when IR divergences are located at different points in integration domain

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IR limits

eg. P_{22}

divergence for $\vec{q} \to 0$ and $\vec{k} - \vec{q} \to 0$

can use partition of unity

$$\mathcal{P}_{22}(k) = \int_{\vec{q}} p_{22}(\vec{q}, \vec{k}) = \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k})\, (|\vec{k}-\vec{q}|^2)^2}{(q^2)^2 + (|\vec{k}-\vec{q}|^2)^2} + \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k})\, (q^2)^2}{(q^2)^2 + (|\vec{k}-\vec{q}|^2)^2}$$

$$= 2 \int_{\vec{q}} \frac{p_{22}(\vec{q}, \vec{k})\, (|\vec{k}-\vec{q}|^2)^2}{(q^2)^2 + (|\vec{k}-\vec{q}|^2)^2}$$

now only divergence for $\vec{q} \to 0$, which is actually numerically finite because of the integration measure

now we have IR and UV finite integrals at the level of the integrands

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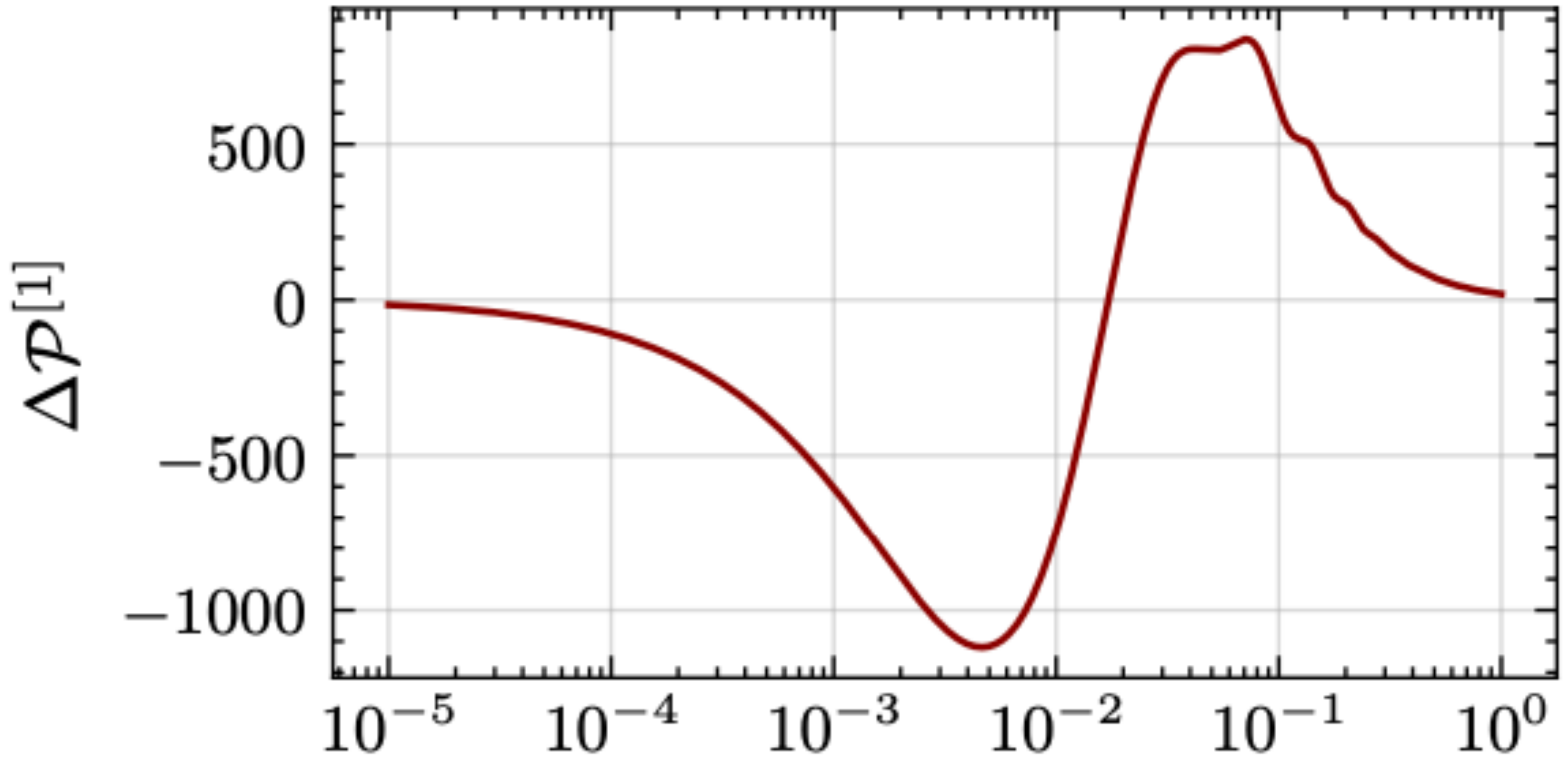
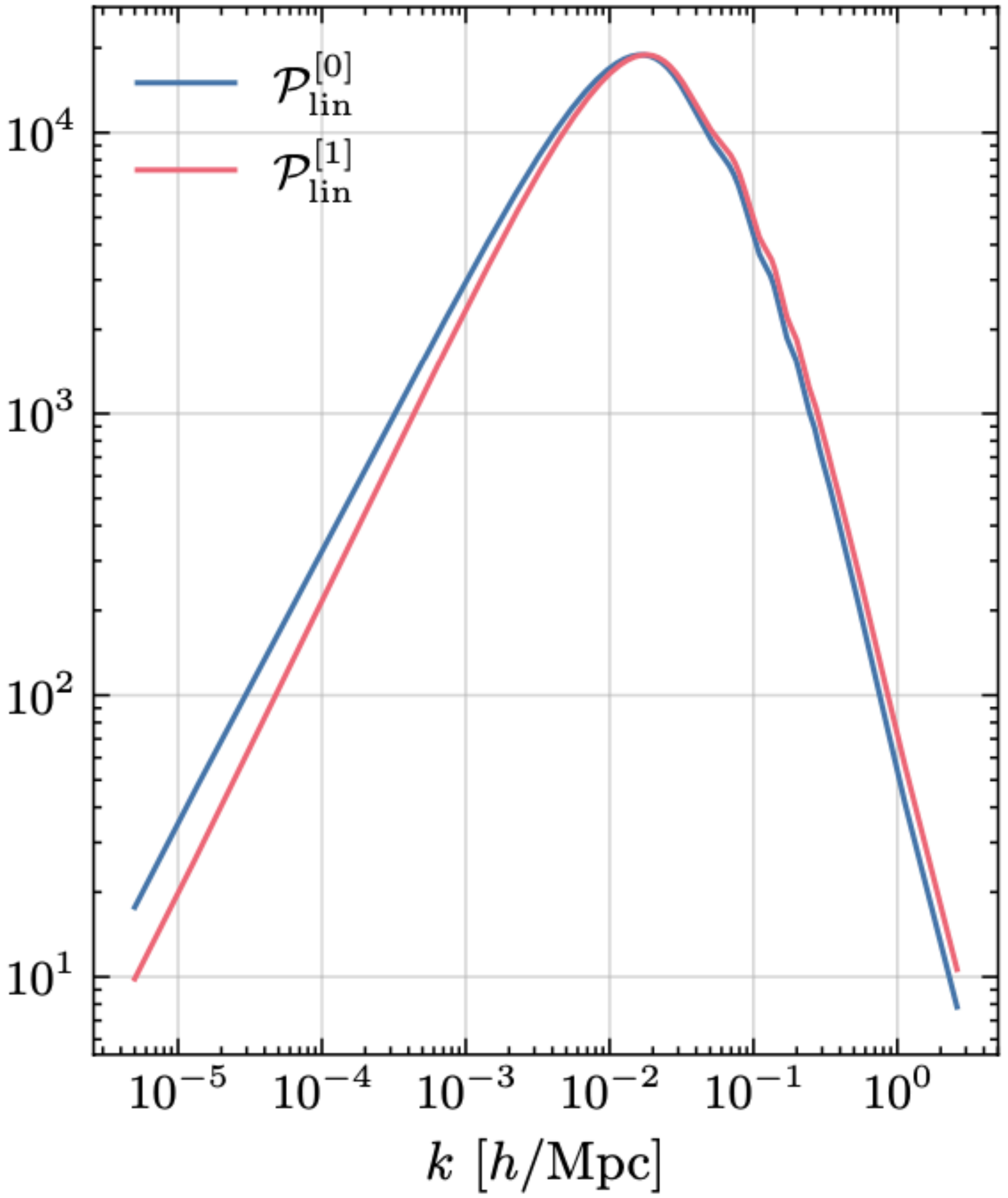
integration strategy

- use Monte Carlo integration for a base cosmology and do loops numerically
- for a general cosmology, expand the difference in a convenient basis

$$\Delta\mathcal{P}^{[j]}(k) = \mathcal{P}^{[j]}_{\text{lin.}}(k) - \mathcal{N}^{[j]}\,\mathcal{P}^{[0]}_{\text{lin.}}(k)$$





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two-loop DM power spectrum

integration strategy

decompose each linear power spectrum difference

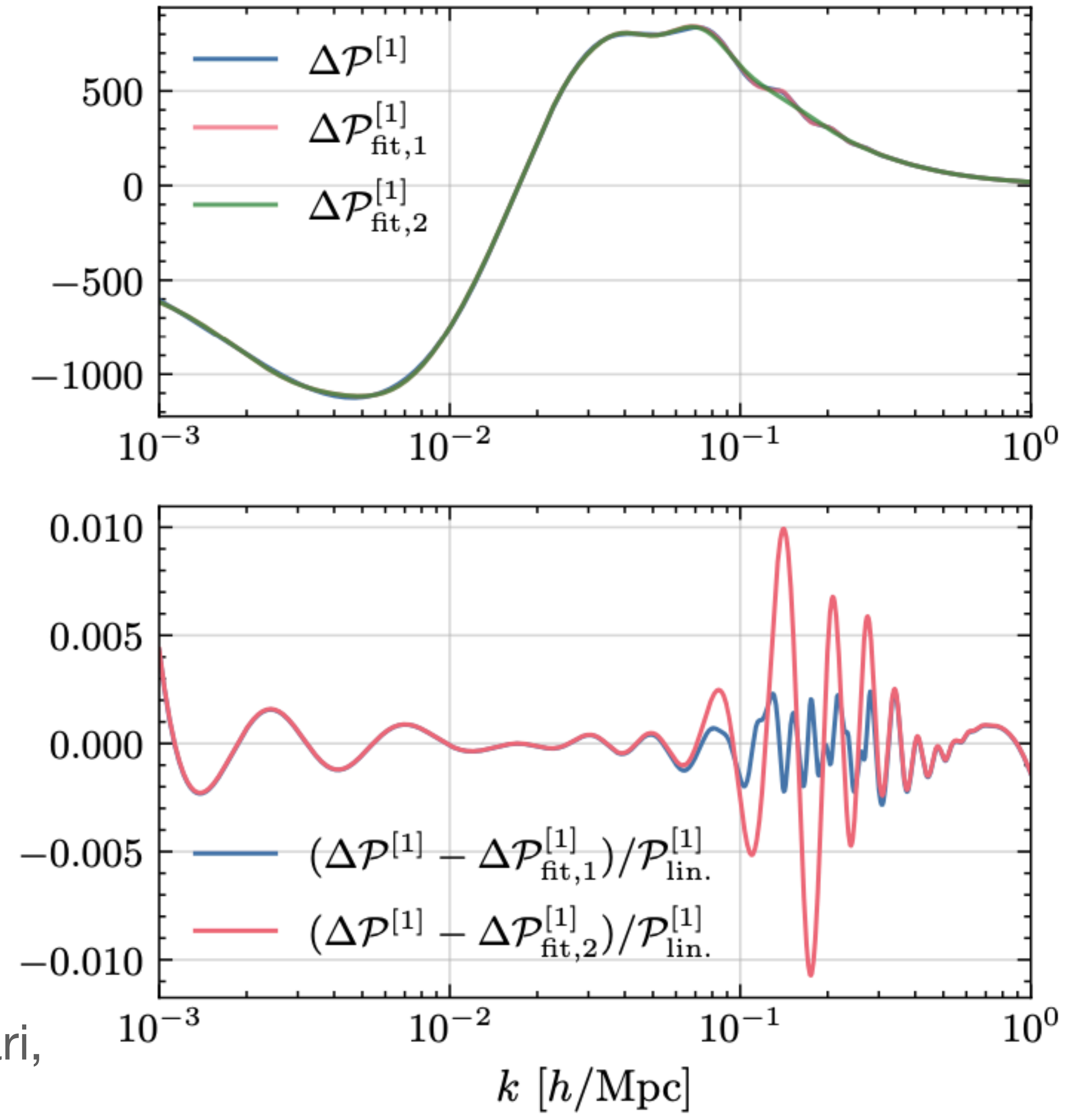
$$\Delta\mathcal{P}^{[j]}(k) = \sum_i c_i^{[j]} \frac{(k/k_0)^{\alpha_i}}{\left(1 + \frac{k^2 - k_{peak}^2}{k_{UV}^2}\right)^{\beta_i}}$$

cosmology dependent

fixed basis, ~20 functions

COBRA uses singular value decomposition

Bakx, Rubira, Chisari, Vlah 25



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integration strategy

now plug into integrals, eg. P_{15}

$$\mathcal{P}_{15}(k) \propto \mathcal{P}_{\text{lin.}}(k) \int_{\vec{p},\vec{q}} F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p}) \mathcal{P}_{\text{lin.}}(q)\, \mathcal{P}_{\text{lin.}}(p)$$

$$= \mathcal{P}_{\text{lin.}}(k) \int_{\vec{p},\vec{q}} F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p}) (\mathcal{P}^{\text{Planck}}_{\text{lin.}}(q) + \Delta\mathcal{P}(q))\, (\mathcal{P}^{\text{Planck}}_{\text{lin.}}(p) + \Delta\mathcal{P}(p))$$

$$\approx \mathcal{P}^{\text{Planck}}(k) + 2\mathcal{P}_{\text{lin.}}(k) \int_{\vec{p},\vec{q}} F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p}) \Delta\mathcal{P}(q) \mathcal{P}^{\text{Planck}}_{\text{lin.}}(p) + \ldots$$

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integration strategy

now plug into integrals, eg. P_{15}

$$\mathcal{P}_{\rm lin.}(k)\int_{\vec{p},\vec{q}} F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p})\Delta\mathcal{P}(q)\mathcal{P}^{\rm Planck}_{\rm lin.}(p) =$$

$$\sum_i c_i \boxed{\mathcal{P}_{\rm lin.}(k)\int_{\vec{p},\vec{q}} F_5(\vec{k},\vec{q},-\vec{q},\vec{p},-\vec{p}) f_i(q)\mathcal{P}^{\rm Planck}_{\rm lin.}(p)}$$

cosmology dependent

~ 3500 precomputed integrals total

two-loop DM power spectrum

results

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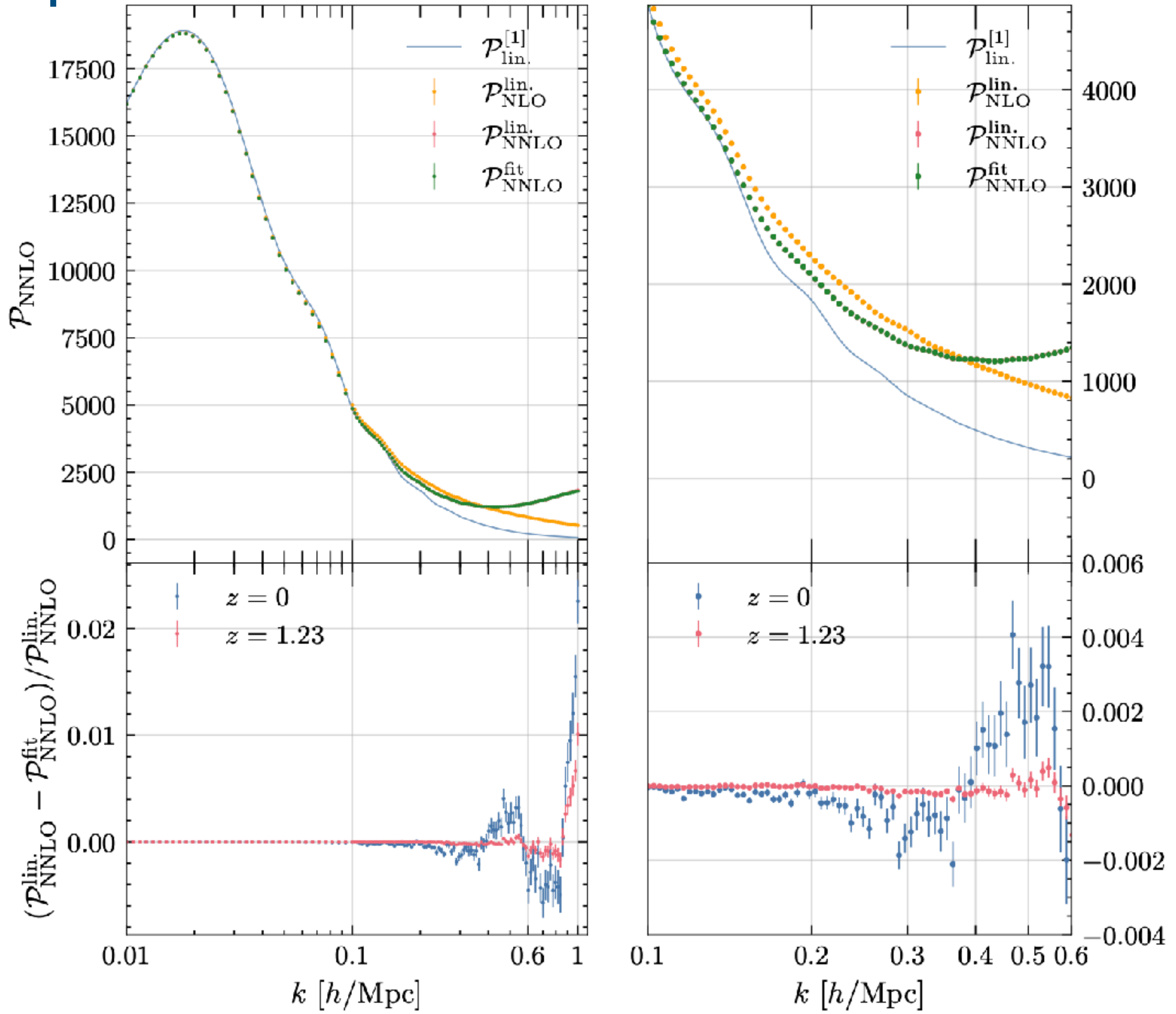
Mathematica code to compute any cosmology and any EFT parameters in <1s



exciting time for LSS!

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