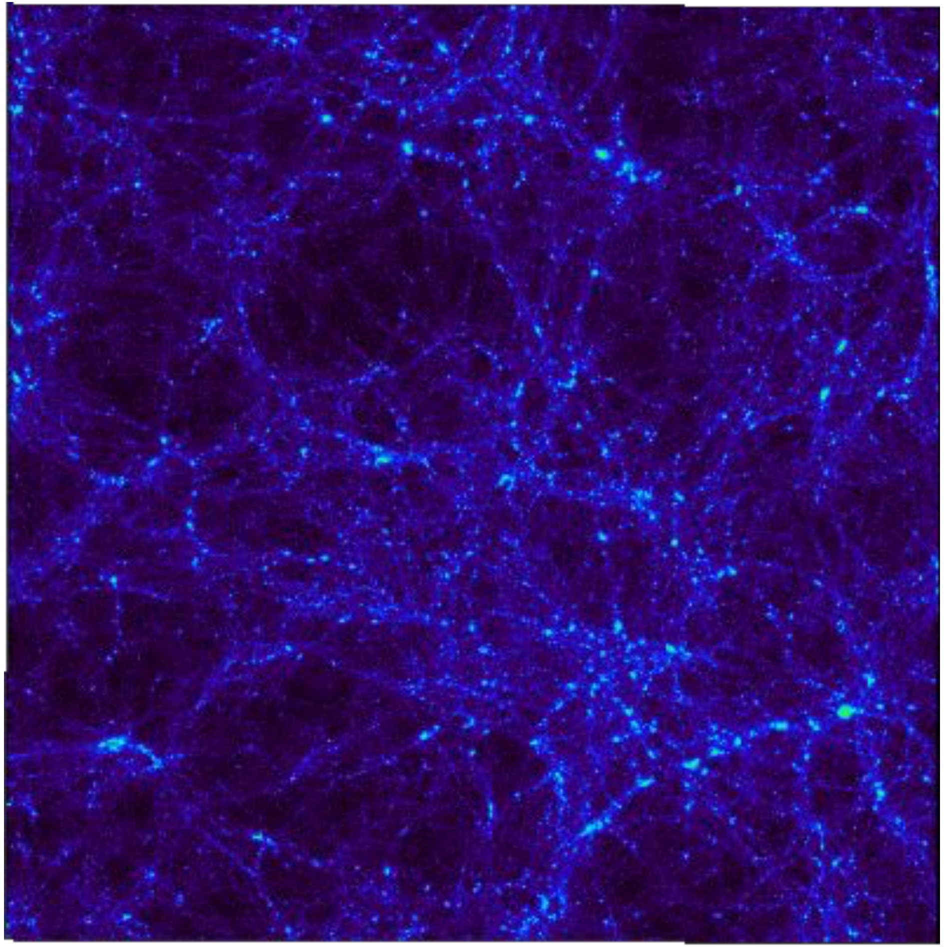
Cosmology from projected clustering: two- and three-point statistics

David Alonso (U. Oxford) - GGI, 16th September 2025

UNIVERSITY OF OXFORD

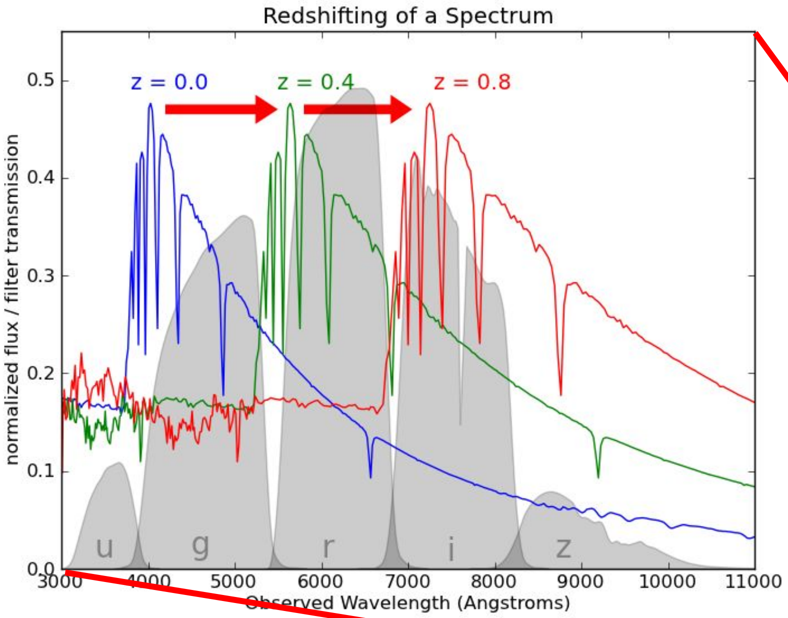
Part 1
Projected LSS tracers

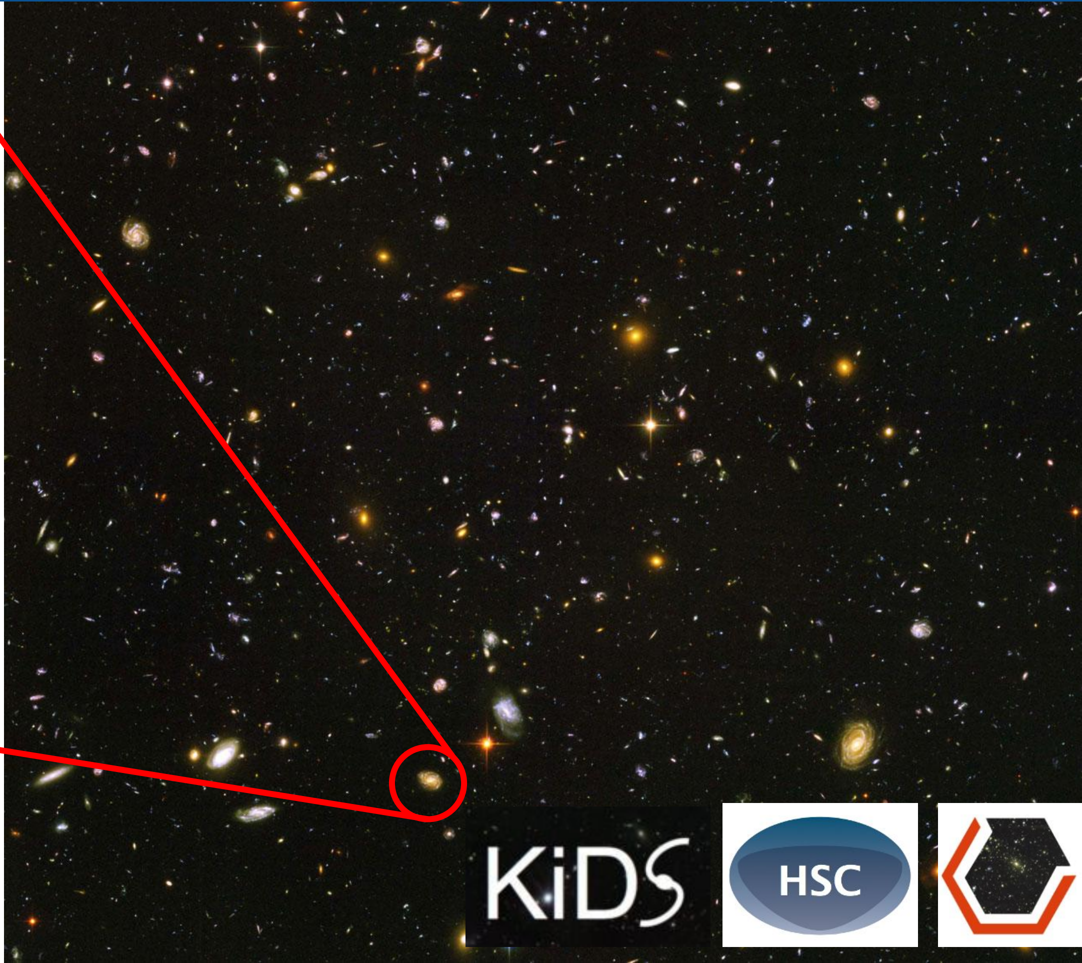
Large-Scale Structure



$$\Delta_m(\mathbf{x}) = \frac{\rho_M(x) - \bar{\rho}_M}{\bar{\rho}_M}$$

Projected LSS tracers: photometric surveys





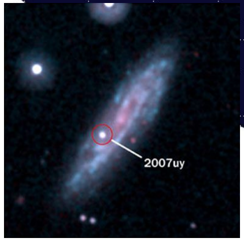
- Use all galaxies you can detect
- Good image quality
- No spectra

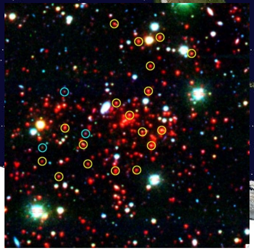
Projected LSS tracers: photometric surveys

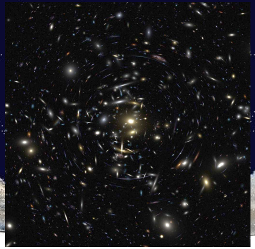
Rubin LSST

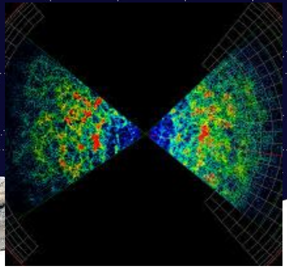
Outstanding numbers:
- World largest imager 8.4m, 9.6 deg^2 FOV
- Wide: 20k deg^2
- Deep: r~27
- Fast: ~100 visits/year
- Big data: ~15TB/day

Dark Energy Science Coll.
- Supernovae
- Cluster science
- Strong lensing
- Weak lensing
- Galaxy clustering







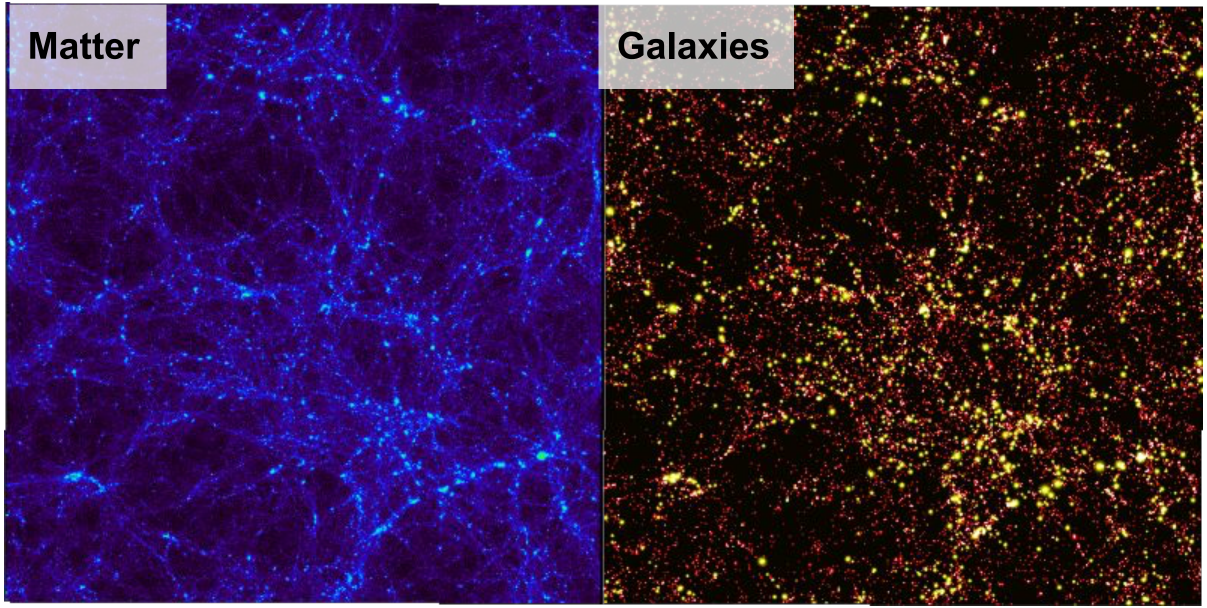


First look
June 23rd!

Projected LSS tracers: galaxy clustering

Galaxy clustering:

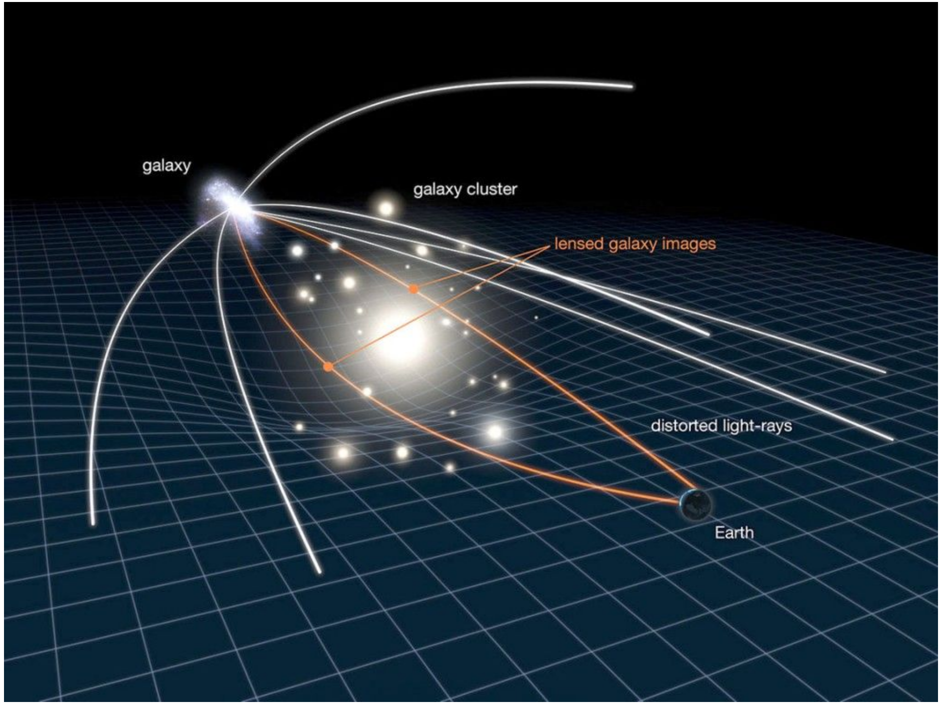
- $\delta_g = f[\delta_M] \sim b_g\, \delta_M$
- Local

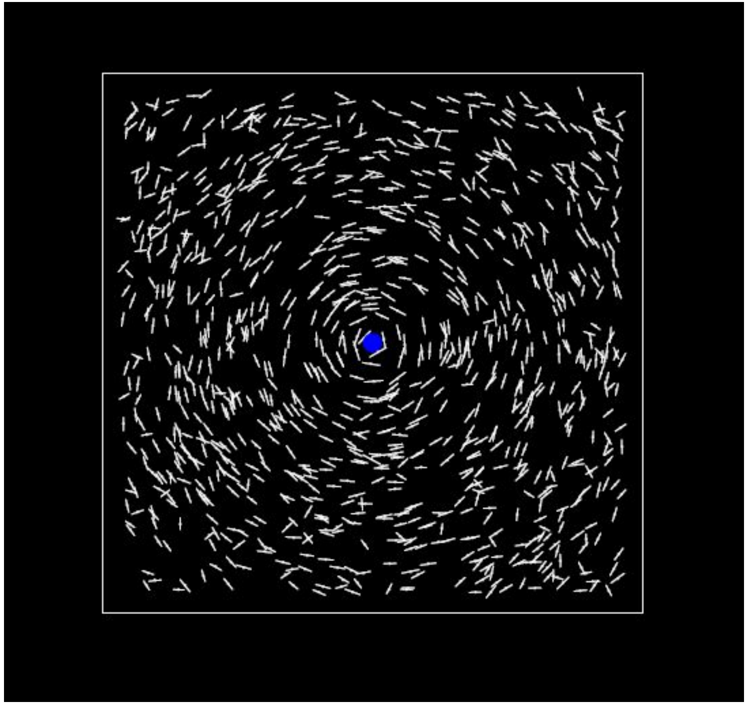


Projected LSS tracers: cosmic shear

Weak lensing:

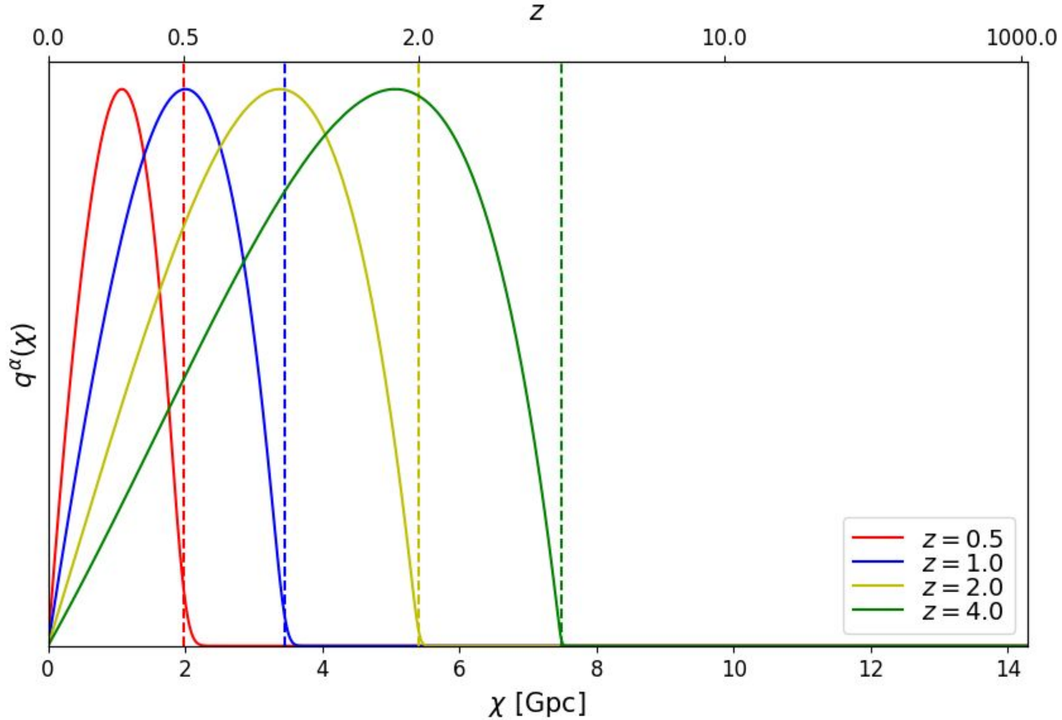
- $e_i \sim \gamma_i \sim \delta_M$
- LOS-integrated





Projected LSS tracers: cosmic shear

$$\gamma_E(\hat{\mathbf{n}}) = \int d\chi\, q_\gamma(\chi)\, \Delta_m(\chi\hat{\mathbf{n}}), z) \qquad q_\gamma(\chi) = \frac{3}{2} H_0^2 \Omega_m \frac{\chi}{a} \int_{z(\chi)}^{\infty} dz' \frac{dp}{dz} \frac{\chi(z') - \chi}{\chi(z')}$$



LSS tracers: “3x2-point”

Projected statistics

$$C_\ell^{ab} = \int \frac{d\chi}{\chi^2} q_a(\chi) q_b(\chi) P_{ab}(k \simeq \ell/\chi)$$

Radial kernels:

3D power spectrum at projected scale k~ℓ / χ

$$\delta^a_{\rm 2D}(\hat{\mathbf{n}}) = \int d\chi\, q_a(\chi)\, \delta^a_{\rm 3D}(\chi\hat{\mathbf{n}})$$

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3x2-point = shear-shear + shear-galaxy + galaxy-galaxy

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Clumpiness   Growth + shape

LSS tracers: “3x2-point”

3x2-point = **shear-shear** + shear-galaxy + galaxy-galaxy

Weak lensing:

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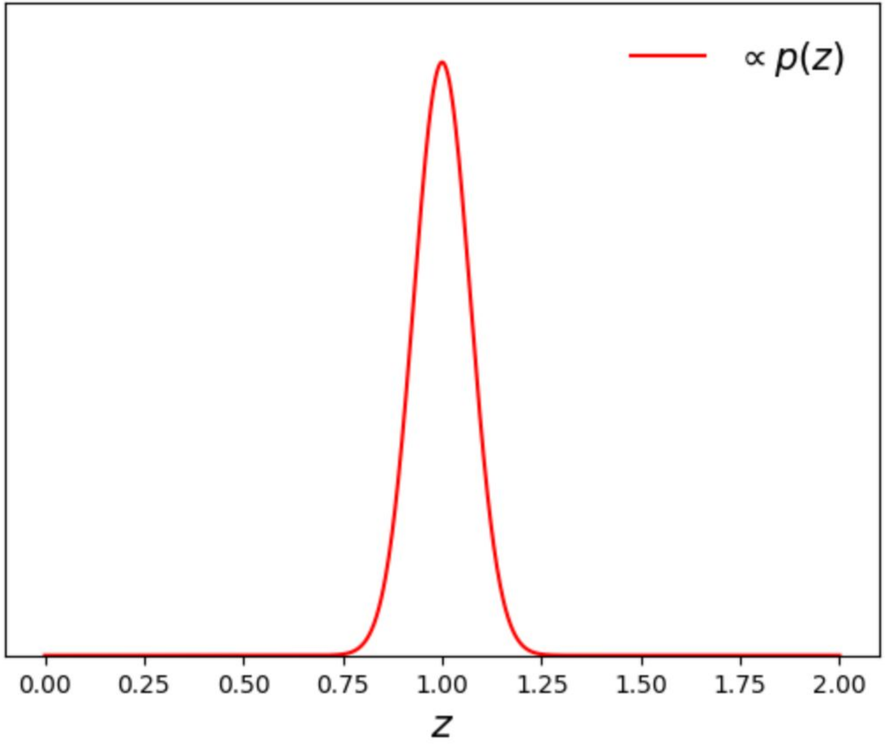

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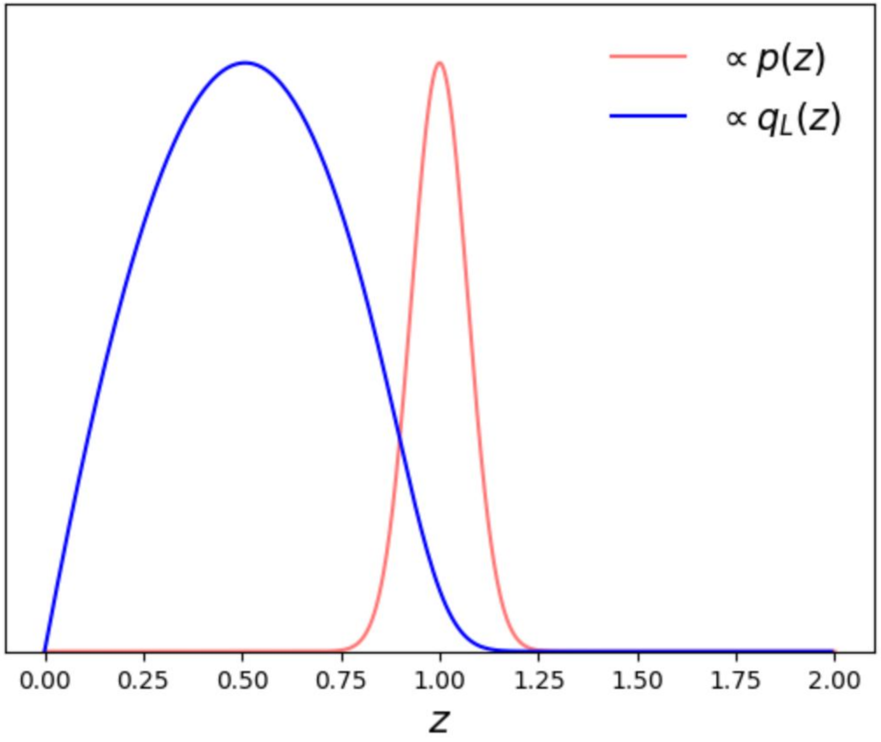
1. Less sensitive to evolution.



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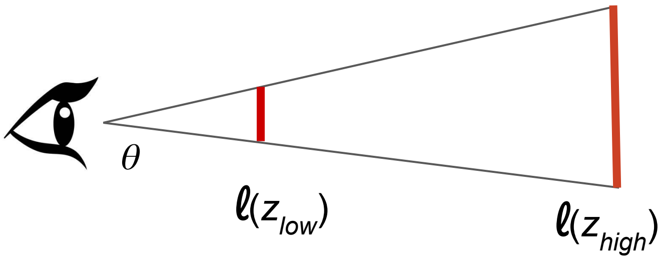
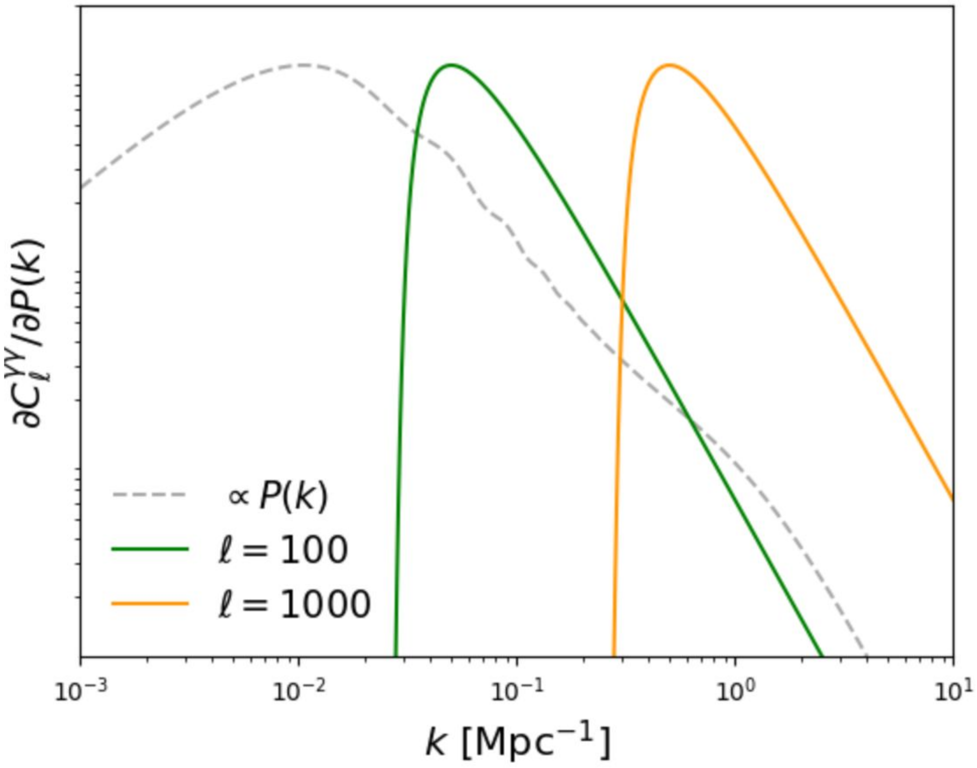
1. Less sensitive to evolution.
2. More sensitive to small scales





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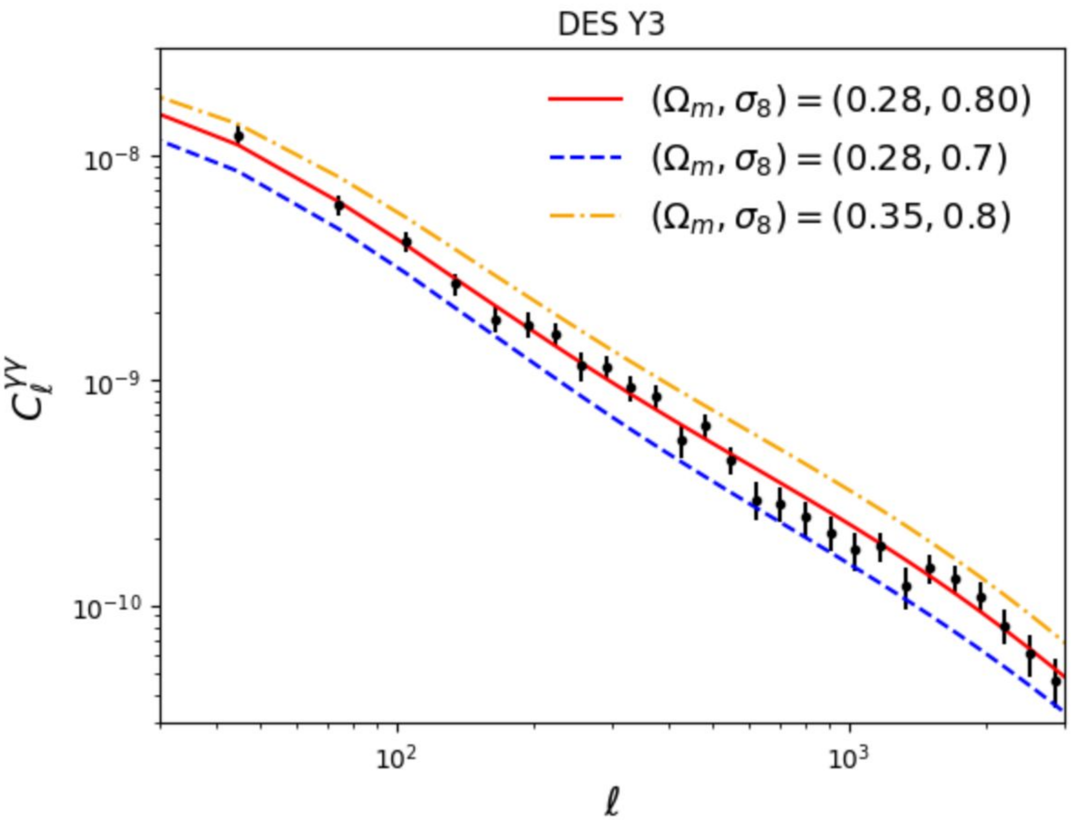
1. Less sensitive to evolution.
2. More sensitive to small scales
3. Direct measurement of "clumpiness amplitude"



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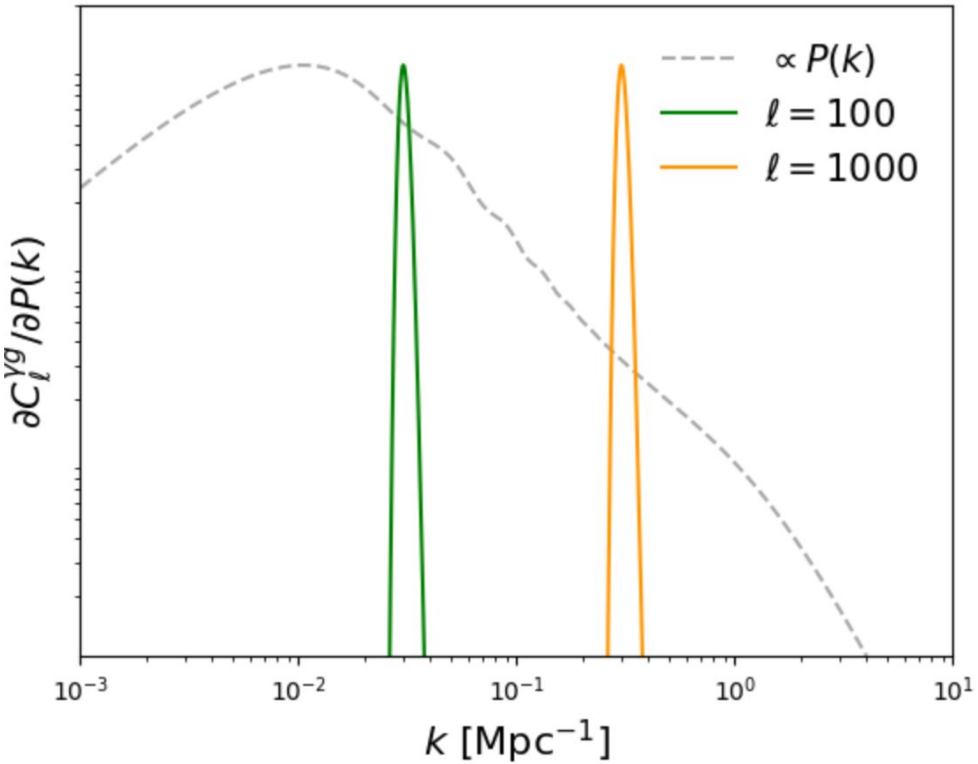
1. Sensitive to scale dependence



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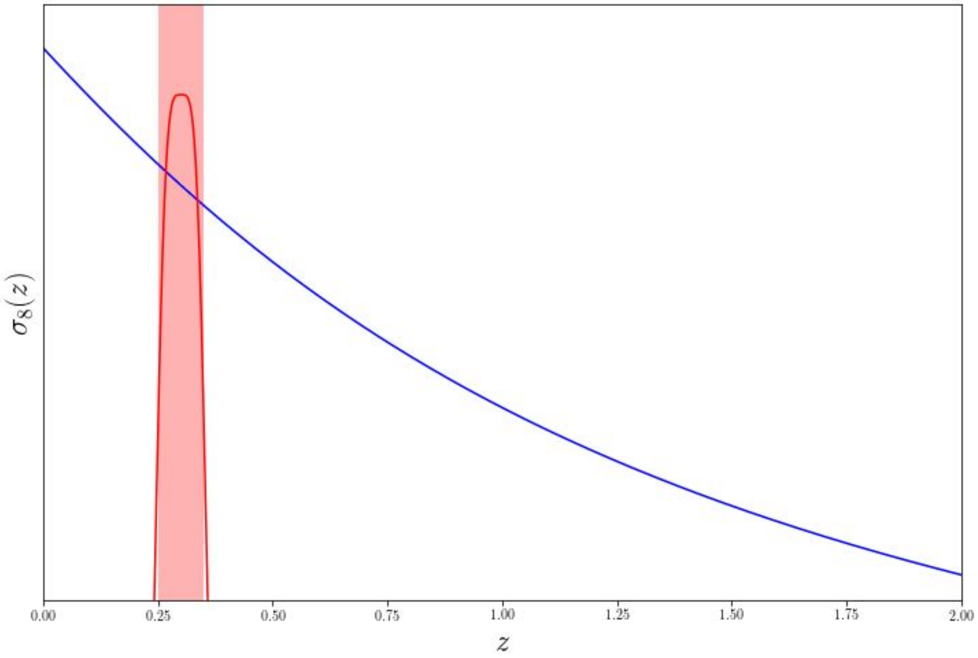
1. Sensitive to scale dependence
2. Sensitive to evolution

$$C_\ell^{g\gamma} \propto \sigma_8^2 b_g \qquad C_\ell^{gg} \propto \sigma_8^2 b_g^2$$



LSS tracers: “3x2-point”

DES 2025

Galaxy clustering:
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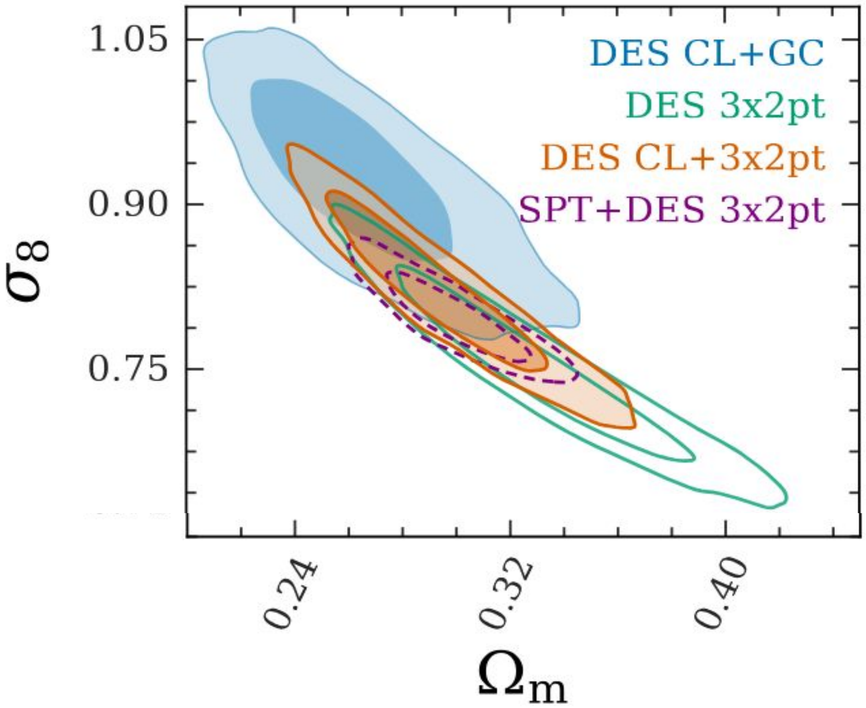
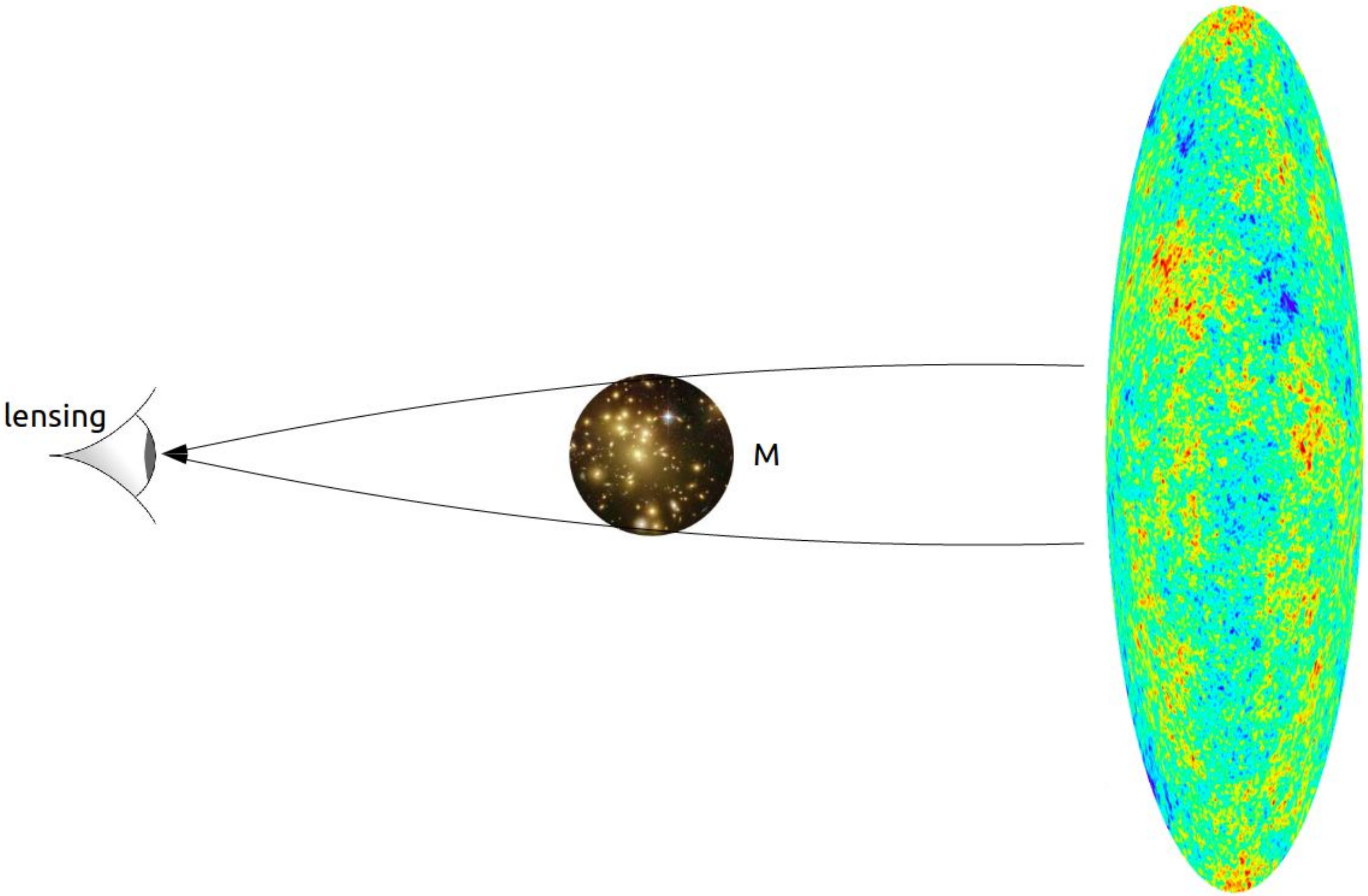


$$S_8 \equiv \sigma_8\,(\Omega_m/0.3)^{0.5}$$

Projected LSS tracers: CMB lensing



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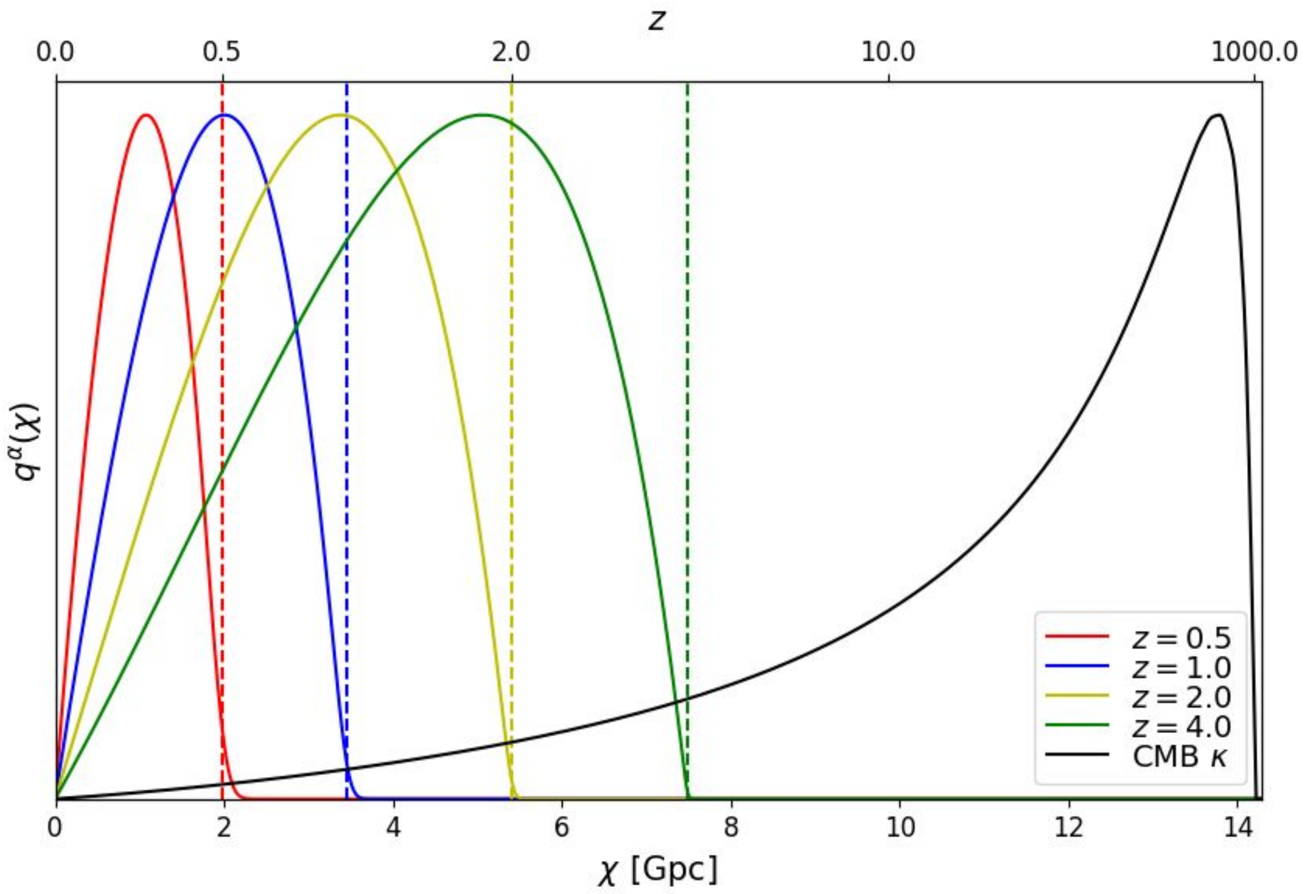
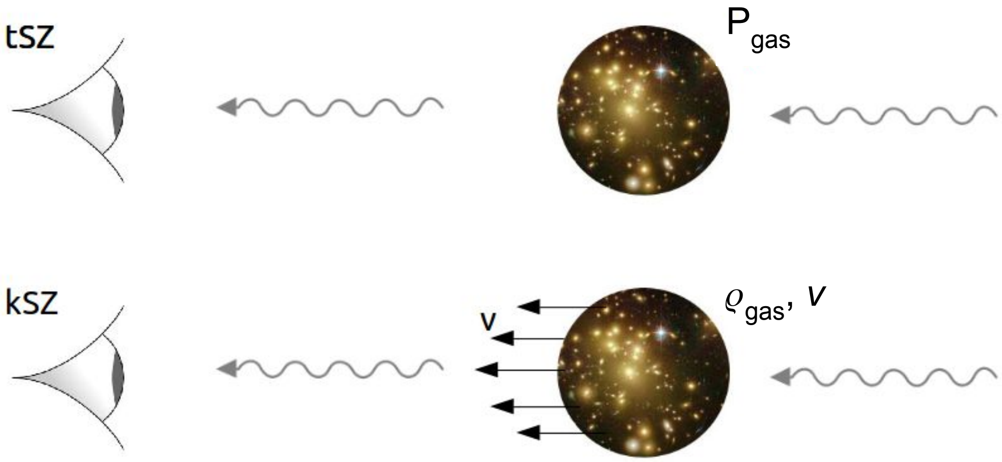
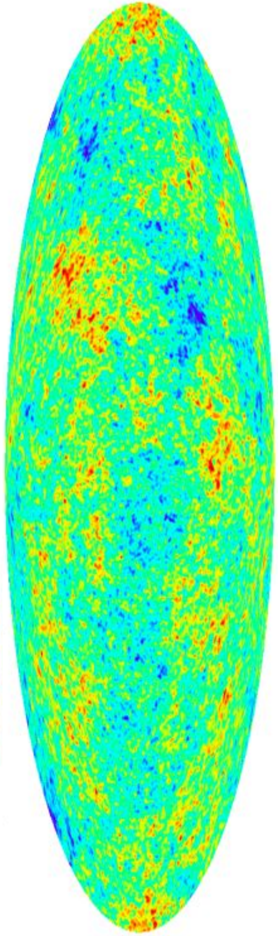


Projected LSS tracers: Sunyaev-Zel'dovich

<u>Thermal and kinematic Sunyaev-Zel'dovich effects:</u>

- Scattering of CMB photons by hot gas
- Clean probes of gas thermodynamics (and LSS)





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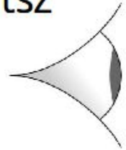
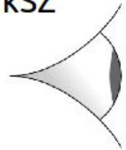
tSZ

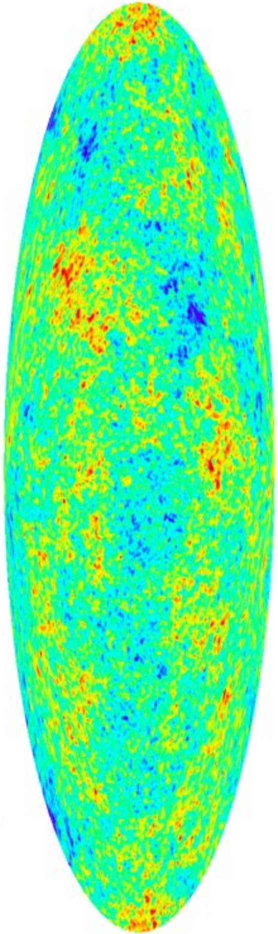


$$\left.\frac{\Delta T}{T}\right|_{\rm tSZ} = f_{\rm tSZ}(\nu)\,\frac{\sigma_T}{m_e c^2}\int \frac{d\chi}{1+z}\,\underline{(n_e\,T_e)}(\chi\hat{\mathbf{n}}, z)$$

kSZ



$$\left.\frac{\Delta T}{T}\right|_{\rm kSZ} = f_{\rm CMB}(\nu)\,\sigma_T\int \frac{d\chi}{1+z}\,\underline{n_e(\chi\hat{\mathbf{n}}, z)\,\mathbf{v}\cdot\hat{\mathbf{n}}}$$



Projected LSS tracers: CMB secondary anisotropies

Simons Observatory (SO):

- 1 Large Aperture Telescope (LAT)
  High-res. science. CMB lensing.
- 6 Small-Aperture Telescopes (SATs)
  Large-scale B-modes (gravity waves)

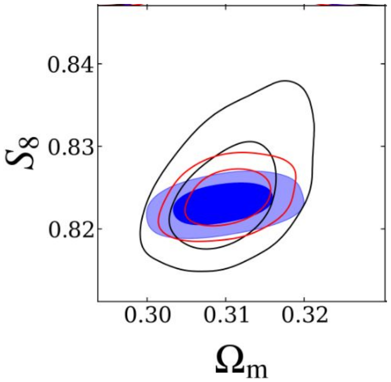
Taking data now!

Part 2
Learning about baryons

Why learn about baryons?

Why?

- Is S_8 tension real?
- Stage-IV lensing cannot avoid baryonic effects



With baryons
No baryons
Calibrated baryons

Wayland et al. 2025

Amy Wayland

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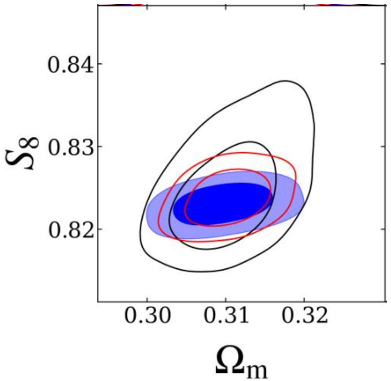

With baryons
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Amy Wayland

- Understanding feedback key for galaxy formation/evolution
- Unlock new cosmological probes:
  - kSZ: measure $Hf\sigma_8$
  - tSZ: high-sensitivity to σ_8/growth/dark energy

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How?

- Target multiple probes of the same astrophysics
  E.g. tSZ+Xray, kSZ+ FRB
- Avoid regimes requiring complex modelling
  E.g.: avoid small-scale correlations with galaxies

Example 1: tSZ tomography

Idea: estimate $\langle bP_e \rangle$ from large-scale x-corr between galaxies and tSZ maps

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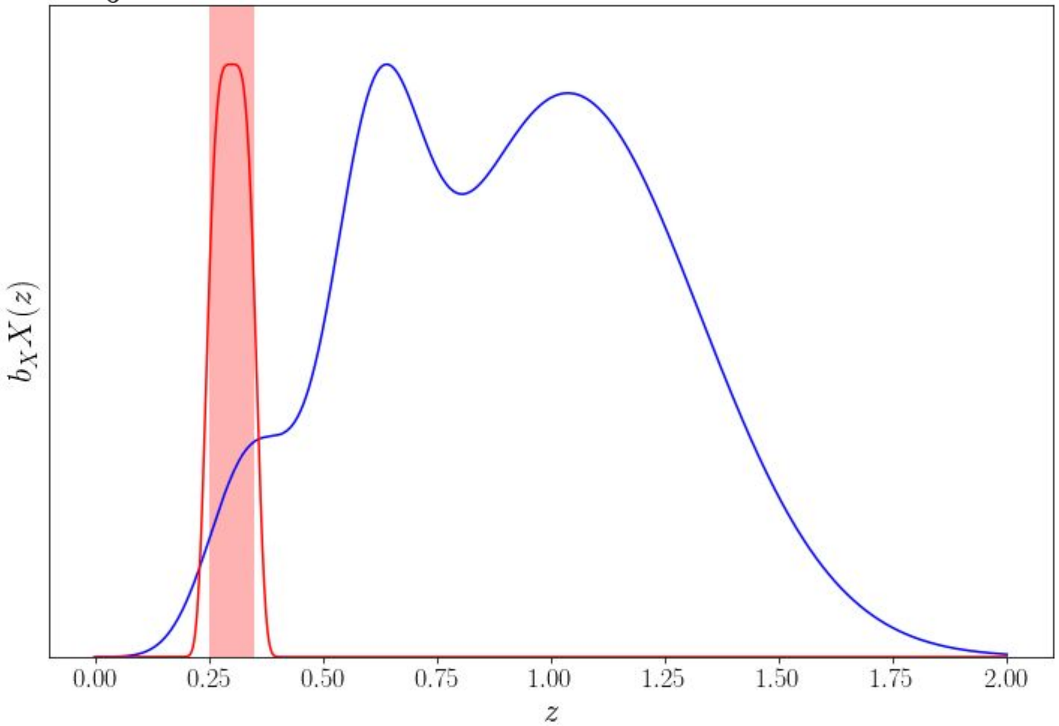
$$x(\hat{\mathbf{n}}) = \int d\chi\, q_X(\chi)\, X(\chi\hat{\mathbf{n}}) \qquad C_\ell^{gx} \propto b_g \langle bX \rangle$$

Same principle as:

- Lensing tomography
- Clustering redshifts



Example 1: tSZ tomography

Koukoufilippas et al. 2019

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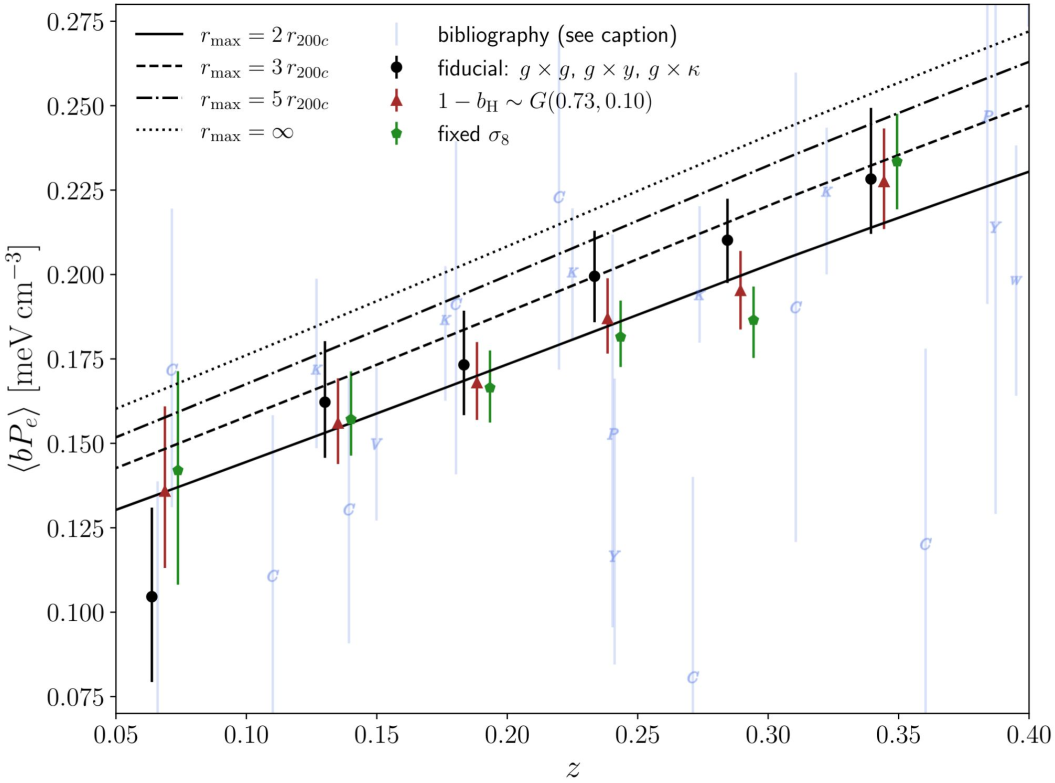
Nick Koukoufilippas - Oxford



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Maleubre et al. 2025

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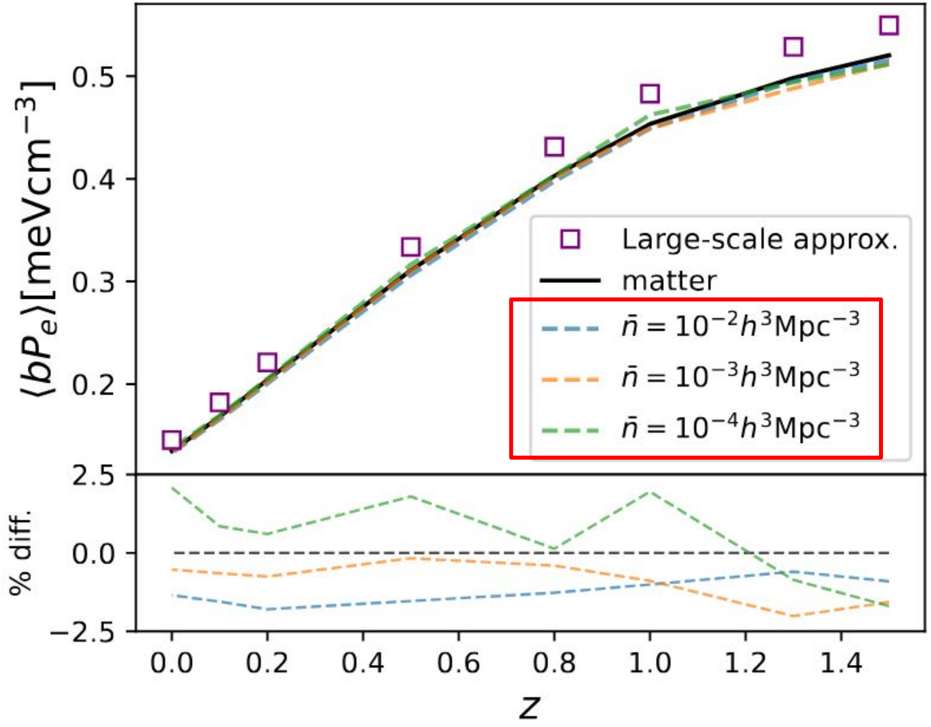
Robust to galaxy bias

Sara Maleubre



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Good agreement with hydro sims

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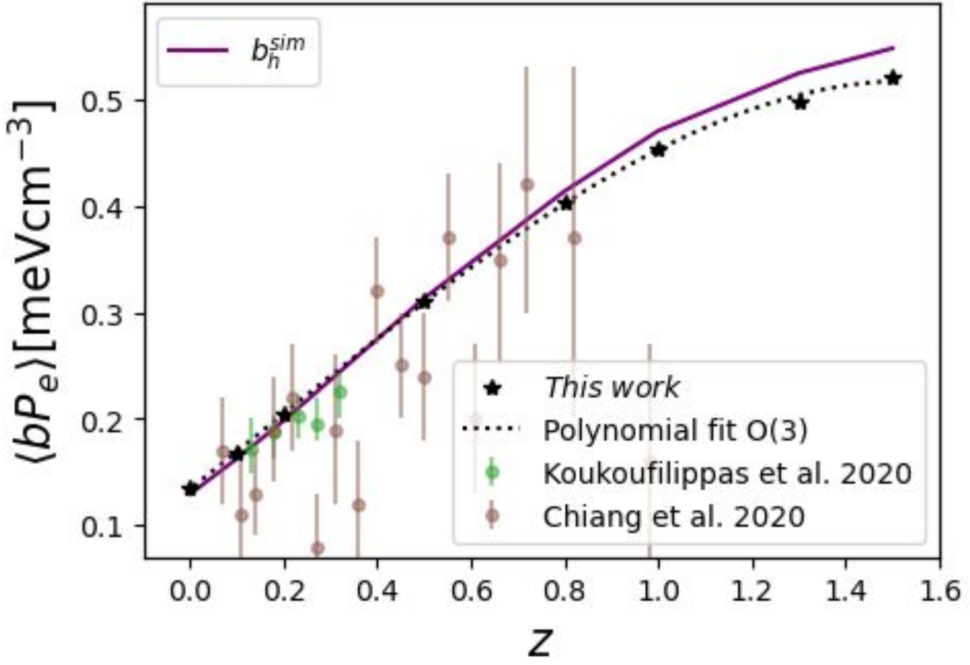

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[La Posta et al. 2025](#)

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Adrien La Posta

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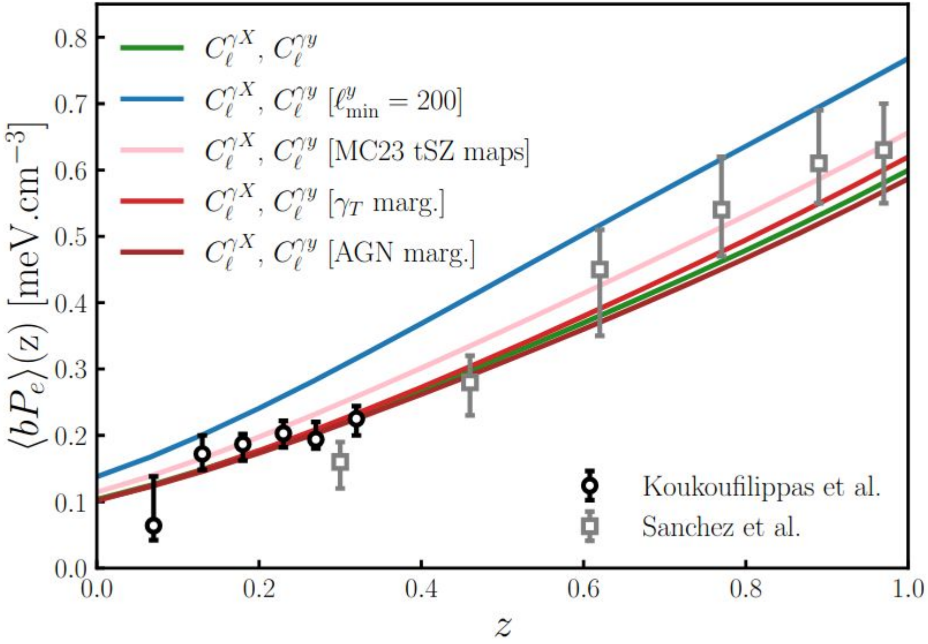
Information on gas thermodynamics and baryonic effects



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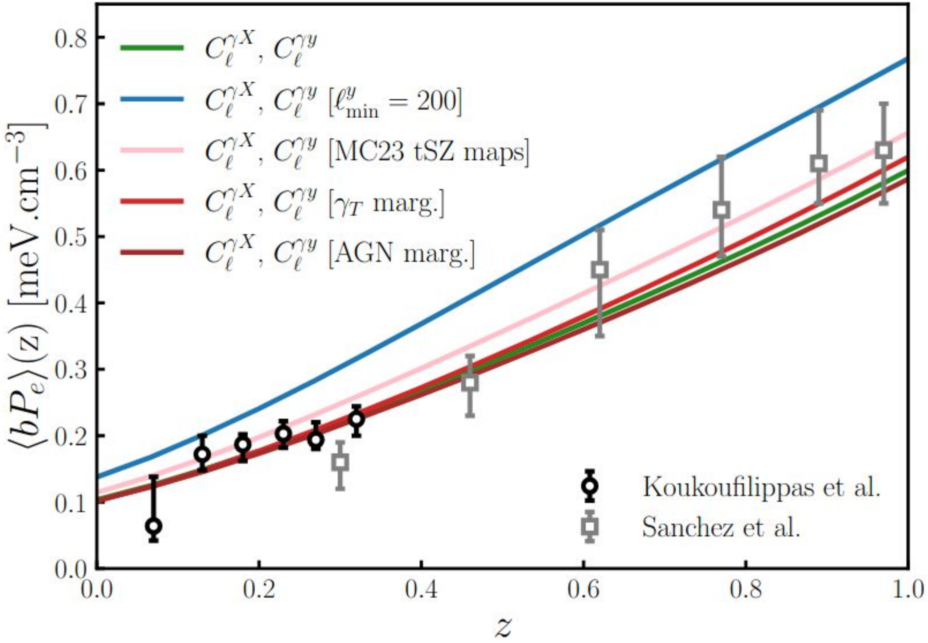
… and cosmology? Chen et al. 2024



Example 2: tSZ x shear

La Posta et al. 2025

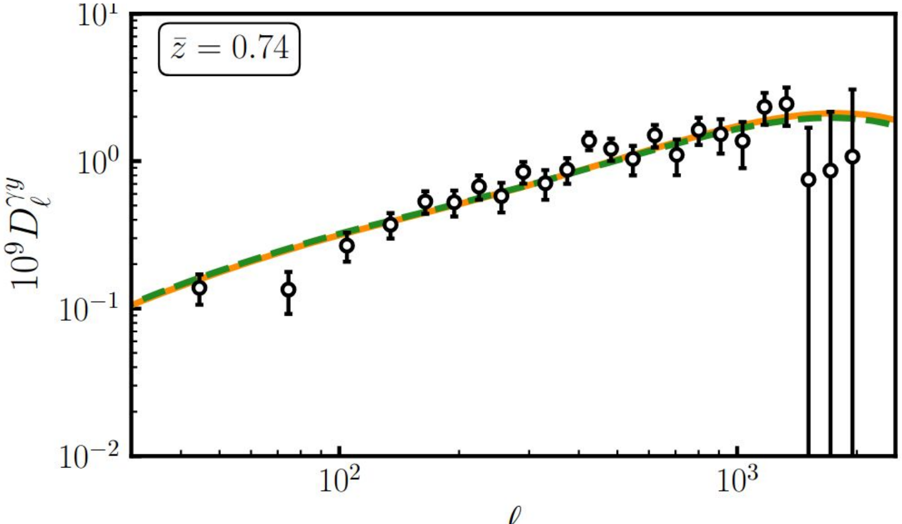
Idea: small-scale correlation between tSZ and lensing

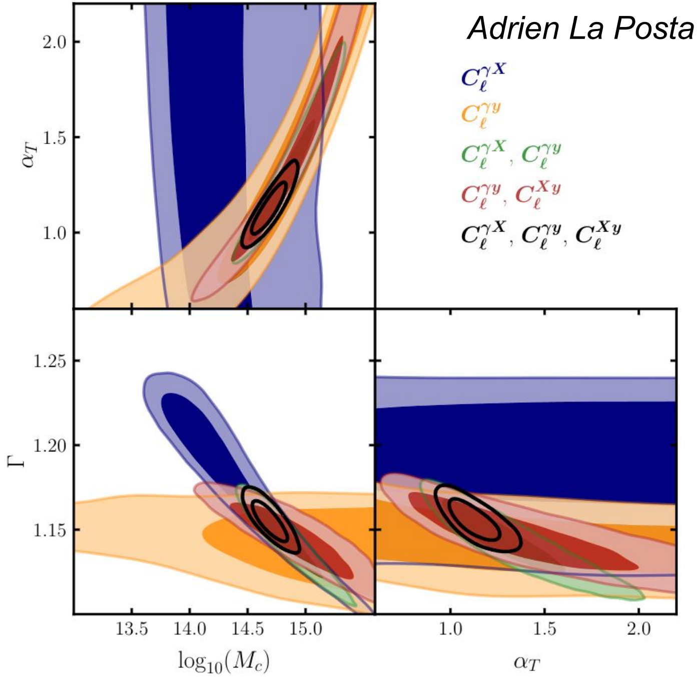
Sensitive (mostly) to purely hydro quantities:

$P_{mm}(k)$, $P_{mP}(k)$ <- no galaxies, simpler model

Constrain baryonic feedback (?)

Adrien La Posta





Example 2: tSZ x shear

Troster et al. 2022

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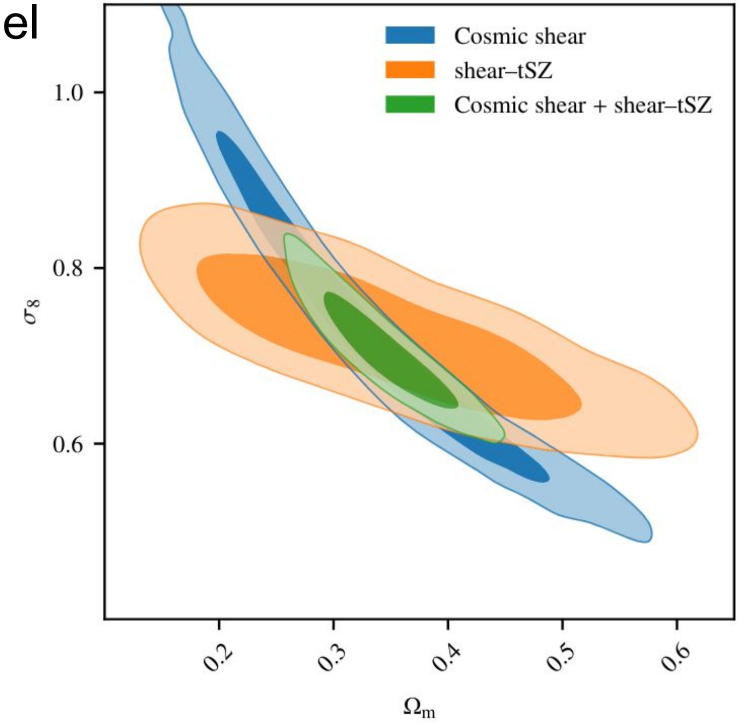
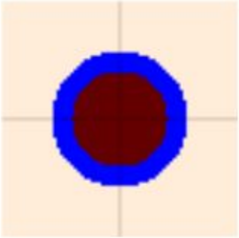
Constrain baryonic feedback (?)

Exploit sensitivity of tSZ to growth (?)



Example 3: kSZ

Idea: correlate CMB map with reconstructed galaxy momentum field $\theta_d = 3.5'$

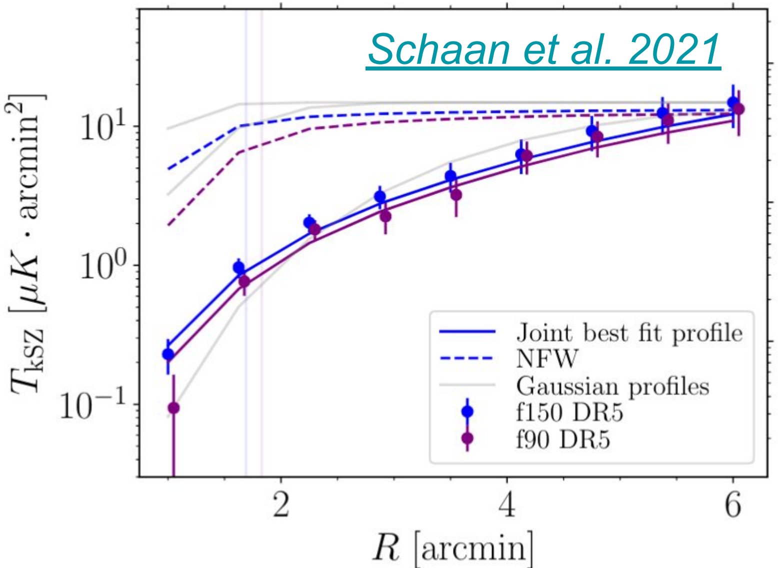


"kSZ stacking" approach:

1. Project CMB map centred on galaxies
2. Apply CAP filter to minimise CMB noise
3. Stack signal weighted by reconstructed galaxy velocity

Signal: ~ cumulative electron density profile

$$\left.\frac{\Delta T}{T}\right|_{\mathrm{kSZ}} = f_{\mathrm{CMB}}(\nu)\,\sigma_T \int \frac{d\chi}{1+z}\, n_e(\chi\hat{\mathbf{n}}, z)\,\mathbf{v}\cdot\hat{\mathbf{n}}$$



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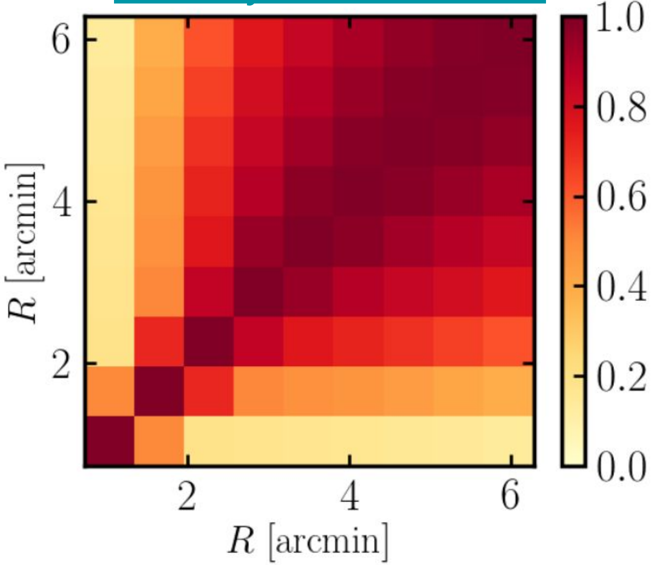
Drawbacks:

- Computationally slow (~5min on ~10 NERSC nodes)
- Highly correlated measurements. Unreliable covariance

Hadzhiyska et al. 2024



Example 3: kSZ

Harscouet et al. (in prep.)

Idea: correlate CMB map with reconstructed galaxy momentum field

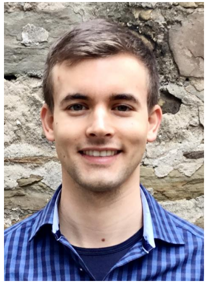
C_ℓ approach: stacking estimator can be expressed as C_ℓ weighted by CAP filter

$$\Delta\hat{T}_{\rm kSZ}(\theta_d) = \sum_\ell \frac{2\ell+1}{4\pi}\hat{C}_\ell^{\pi_g T}\, W_\ell^{\rm CAP}(\theta_d)$$

- π_g = projected galaxy momentum

$$\pi_g \equiv \frac{1}{\bar{n}_\Omega}\sum_i \delta^D(\hat{\mathbf{n}},\hat{\mathbf{n}}_i)v_{r,i} \equiv \int dz\, p(z)\,(1+\delta_g)v_r$$

Lea Harscouet



Kevin Wolz

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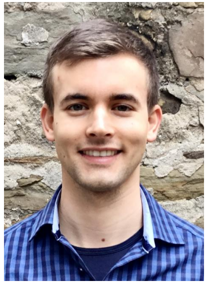


Kevin Wolz

- π_g = projected galaxy momentum

$$\pi_g \equiv \frac{1}{\bar{n}_\Omega} \sum_i \delta^D(\hat{\mathbf{n}}, \hat{\mathbf{n}}_i) v_{r,i} \equiv \int dz\, p(z)\, (1+\delta_g) v_r$$

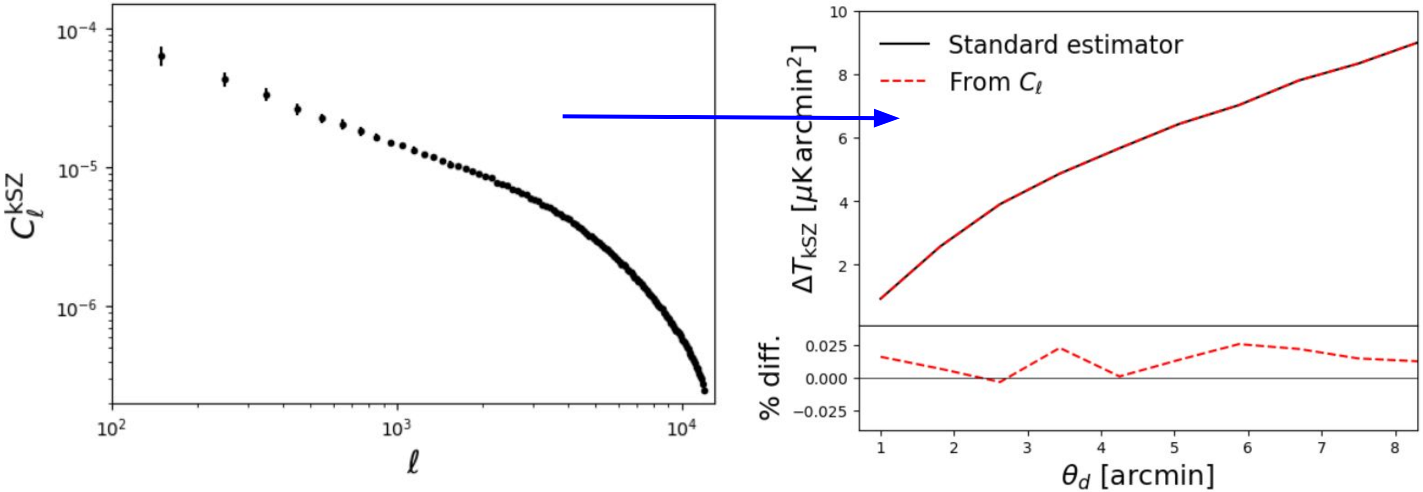
- Equivalent at the estimator level!
- All information encoded in C_ℓ
- Catalog-based method to recover small-scale signal without pixels. (Wolz et al. 2024)

Example 3: kSZ

Harscouet et al. (in prep.)

Idea: correlate CMB map with reconstructed galaxy momentum field

C_ℓ approach: stacking estimator can be expressed as C_ℓ weighted by CAP filter



- **Fast:** 30s on single node
- Quick and accurate **covariance** using C_ℓ methods (Garcia-Garcia et al. 2019) Including cross-covariances for multi-probe analyses.

Learning about baryons

Looking forward:

- Multi-probe approach vital for robust understanding of small-scale clustering and feedback.
- Develop fast and reliable estimators for baryonic probes.
  Enable e.g.: 3x2pt + γy + $\langle bP_e \rangle$ + kSZ C_ℓ

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Amy Wayland

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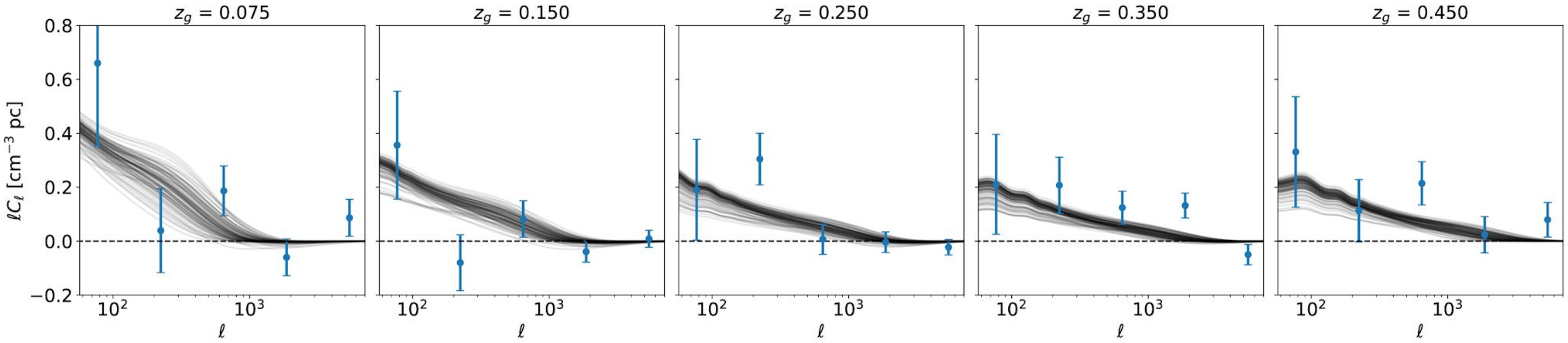
- Develop thorough models for kSZ, accounting for velocity-density correlations, satellites, 2-halo contributions (*Wayland et al. in prep.*)
- Explore novel probes: e.g. FRBs (*Wang et al. 2025*)



Amy Wayland

Part 3
2- and 3-point information

Motivation

Why study the bispectrum as a HOS?

- Well-understood theoretical framework
- Can be connected with fundamental ingredients
  no need to emulate the whole survey
- Potential to break important degeneracies (e.g. b_1 - σ_8)
- Test for self-consistency of bias model

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Why not?

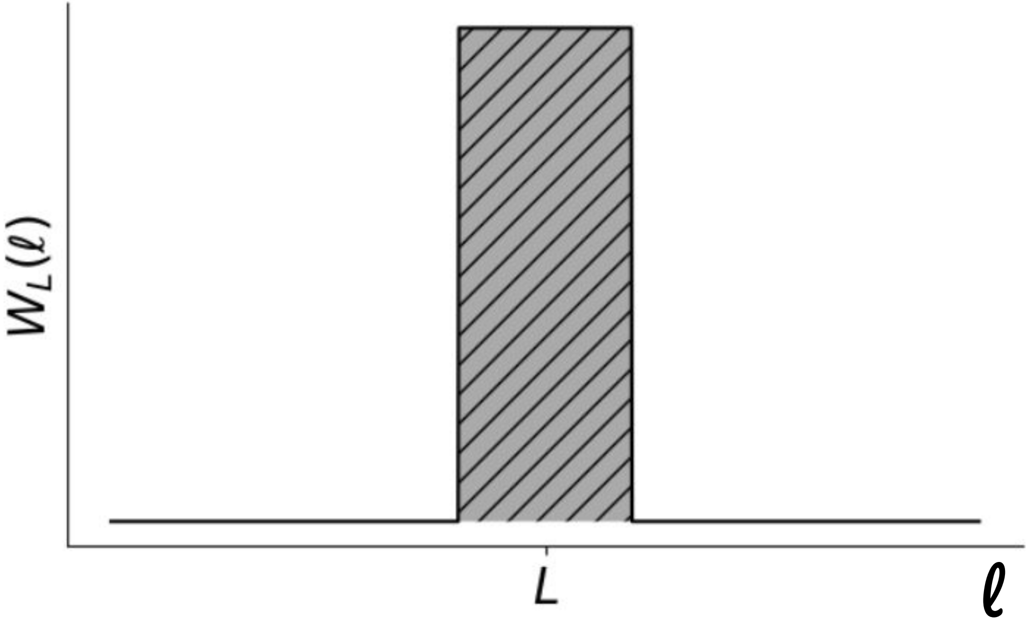
- Estimators can be very slow
- Lots of triangle configurations!
- Complicated covariance matrix

The Filter-Square Bispectrum

Harscouet et al. 2024

The FSB estimator:

1. Filter your field $\delta_L \equiv W_L \otimes \delta$



Lea Harscouet

Anze Slosar

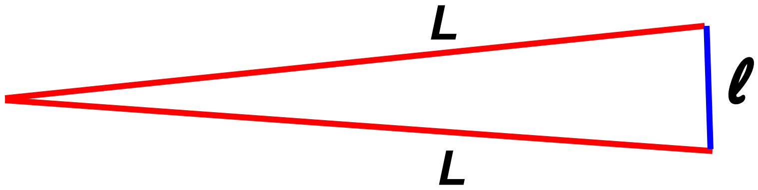
The Filter-Square Bispectrum

Harscouet et al. 2024

The FSB estimator:

1. Filter your field $\delta_L \equiv W_L \otimes \delta$
2. Square it: $s_L(\hat{\mathbf{n}}) \equiv [\delta_L(\hat{\mathbf{n}})]^2$
3. Correlate it with your original field

$$\Phi^{\delta\delta\delta}_{LL\ell} \equiv \langle \delta_{\ell m} (s_L)^*_{\ell m} \rangle \sim b_{LL\ell}$$




Lea Harscouet


Anze Slosar

The Filter-Square Bispectrum

Harscouet et al. 2024

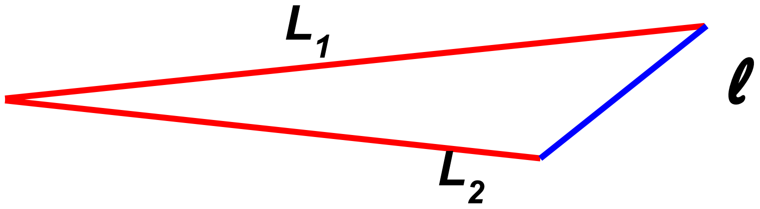
The FSB estimator:

- Sensitive to close-to-isosceles configurations

    But most triangles are close to isosceles if your bins are wide enough

    Easy to generalise to arbitrary configurations and multi-field ("Filter-Multiply" -> FMB)

$$\Phi^{abc}_{L_1 L_2 \ell} \equiv \langle a_{\ell m} (b_{L_1} c_{L_2})^*_{\ell m} \rangle \sim b^{abc}_{L_1 L_2 \ell}$$



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  - Residual mask effects from filtering are negligible.

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  - Residual mask effects from filtering are negligible.

- **Fast, accurate and data-driven covariance** matrix
  - Again using pseudo-C_ℓ methods

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The Filter-Square Bispectrum

Harscouet et al. 2024

The bispectrum covariance:

- The correlator expansion picture

$$\mathrm{Cov}(b,b) \sim \langle \delta^2 \rangle^3 + \langle \delta^3 \rangle^2 + \langle \delta^2 \rangle \langle \delta^4 \rangle_c + \langle \delta^6 \rangle_c$$

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The Filter-Square Bispectrum

Harscouet et al. 2024

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- The power spectrum picture:
  "disconnected trispectrum" of the fields involved, AKA "Gaussian covariance"

$$\mathrm{Cov}(\langle\delta\, s_L\rangle_\ell, \langle\delta\, s_L\rangle_{\ell'}) \sim \delta_{\ell\ell'}\left[\langle\delta^2\rangle\langle s_L^2\rangle + \langle\delta\, s_L\rangle^2\right]$$

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The Filter-Square Bispectrum

Harscouet et al. 2024

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These are exactly the purely "diagonal" elements of the correlator expansion!
And they can be estimated purely from the data.
Similar result for *Cov(b, C_ℓ)*

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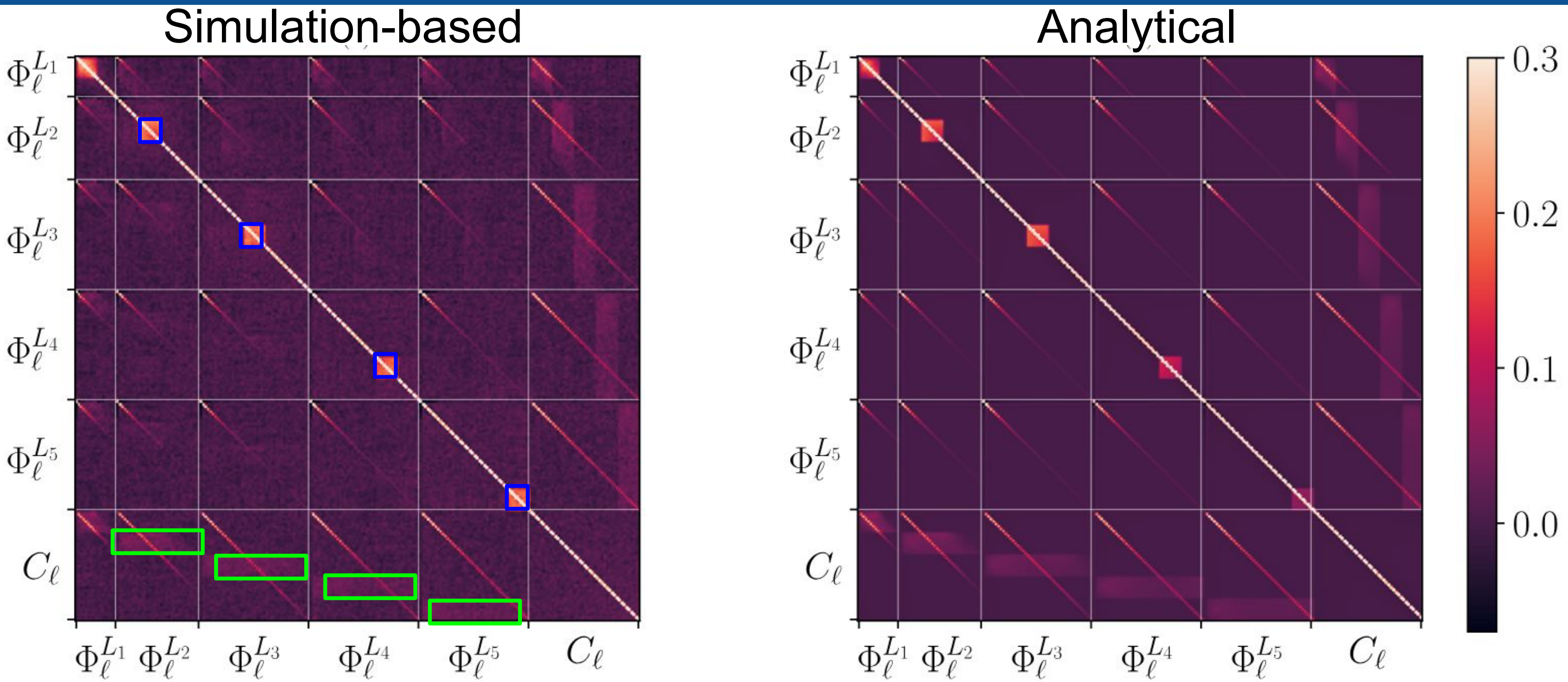
The Filter-Square Bispectrum

Harscouet et al. 2024



Dominant off-diagonal elements are relatively simple.

Can also be calculated from the data.

$$\mathrm{Cov}^{N_{222}}(\Phi_{LL\ell}, \Phi_{L'L'\ell'}) = \delta_{LL'} \frac{C_\ell C_{\ell'}}{\pi} \sum_{\ell'' \in L} (2\ell'' + 1) C_{\ell''} \begin{pmatrix} \ell & \ell' & \ell'' \\ 0 & 0 & 0 \end{pmatrix}^2$$

$$\mathrm{Cov}^{N_{32}}(\Phi_{LL\ell}, C_{\ell'}) = \delta_{\ell' \in L} \frac{4 C_{\ell'} \Phi_{L\ell\ell'}}{2\ell + 1}$$

First cosmological (2+3)-point analysis

Harscouet et al. 2025

Idea: apply FSB to CMB lensing tomography, targeting $\langle gg \rangle$, $\langle g\kappa \rangle$, $\langle ggg \rangle$, $\langle gg\kappa \rangle$

- Improve cosmological constraints adding higher-order statistics.
- Useful consistency test (predict 3pt from 2pt and vice-versa).
- Test self-consistency of bias model.

Data:

- Planck PR4 lensing maps (*Carron et al. 2022*)
- DESI photometric LRG sample (*Zhou et al. 2023*, *Sailer et al. 2024*)

Lea Harscouet

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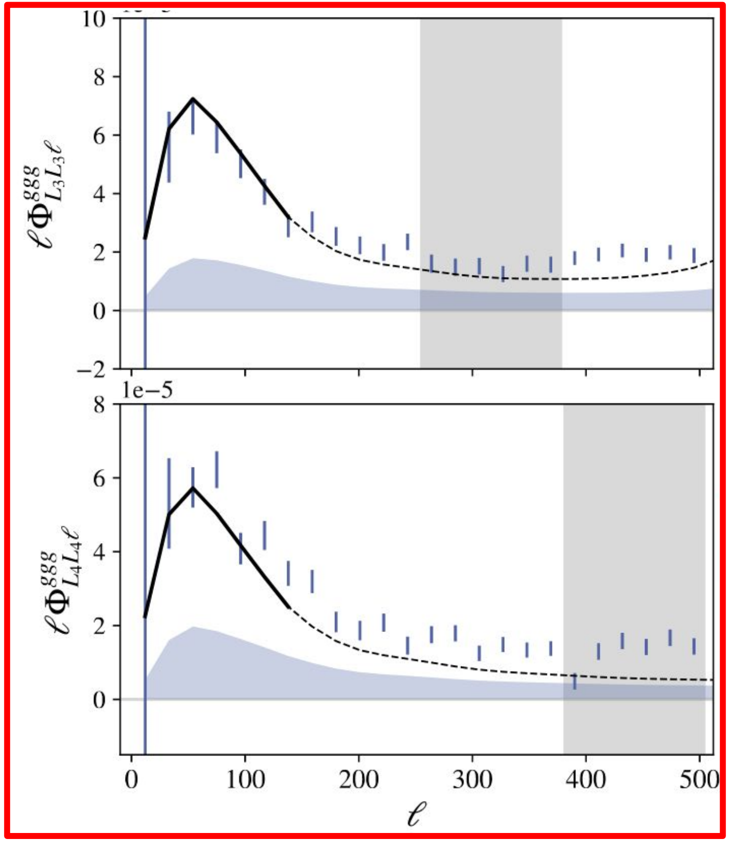

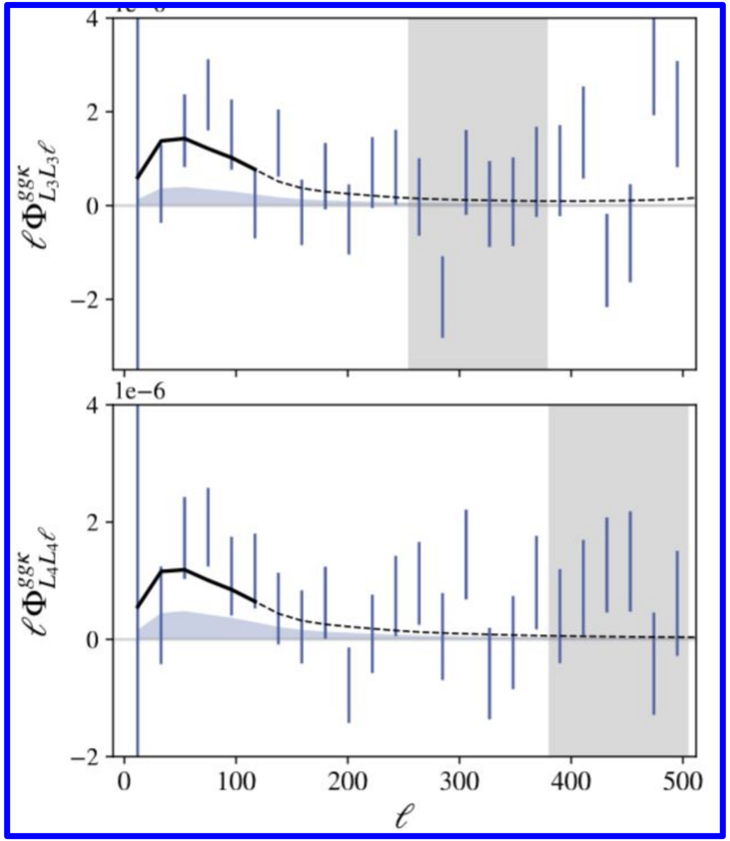

40σ 5-6σ

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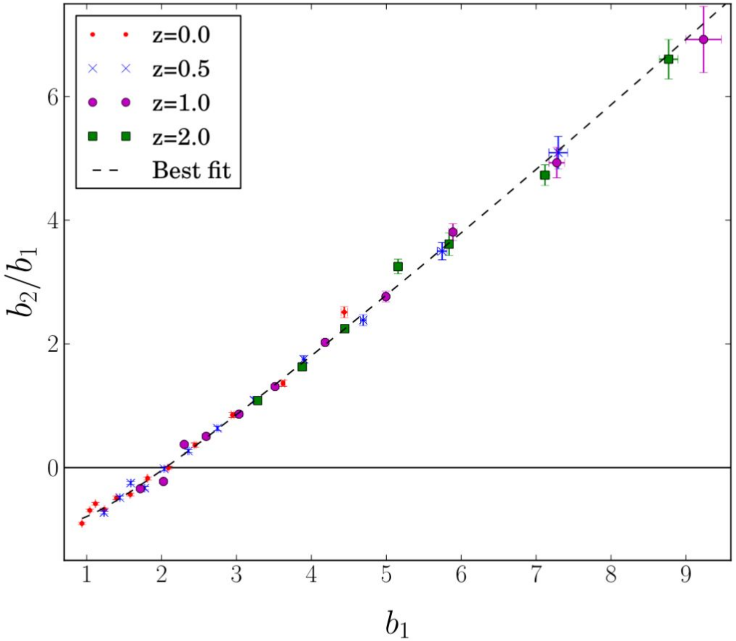
Tree-level (2+3)-point analysis

- Tree-level Cl depends only on b_1, but bispectrum depends on b_2, b_s
- Assume coevolution relations $b_2(b_1)$, $b_s(b_1)$ (*Lazeyras et al.* 2016, 2018)



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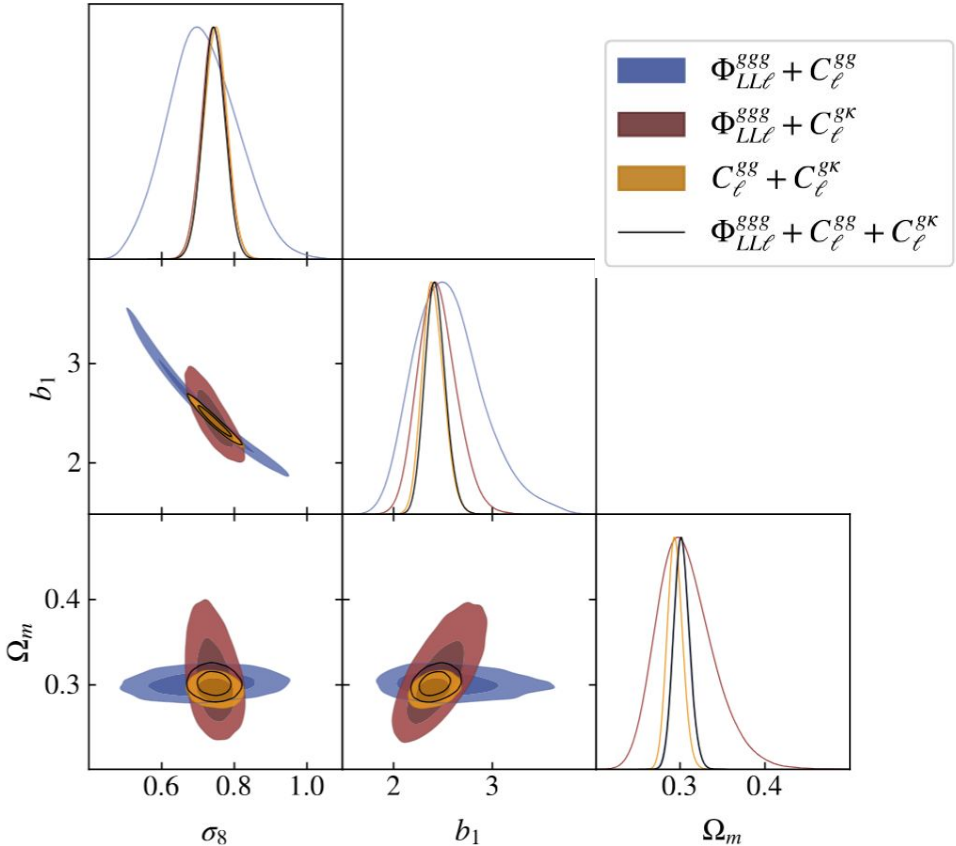

- Consistent results for different probe combinations

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Harscouet et al. 2025

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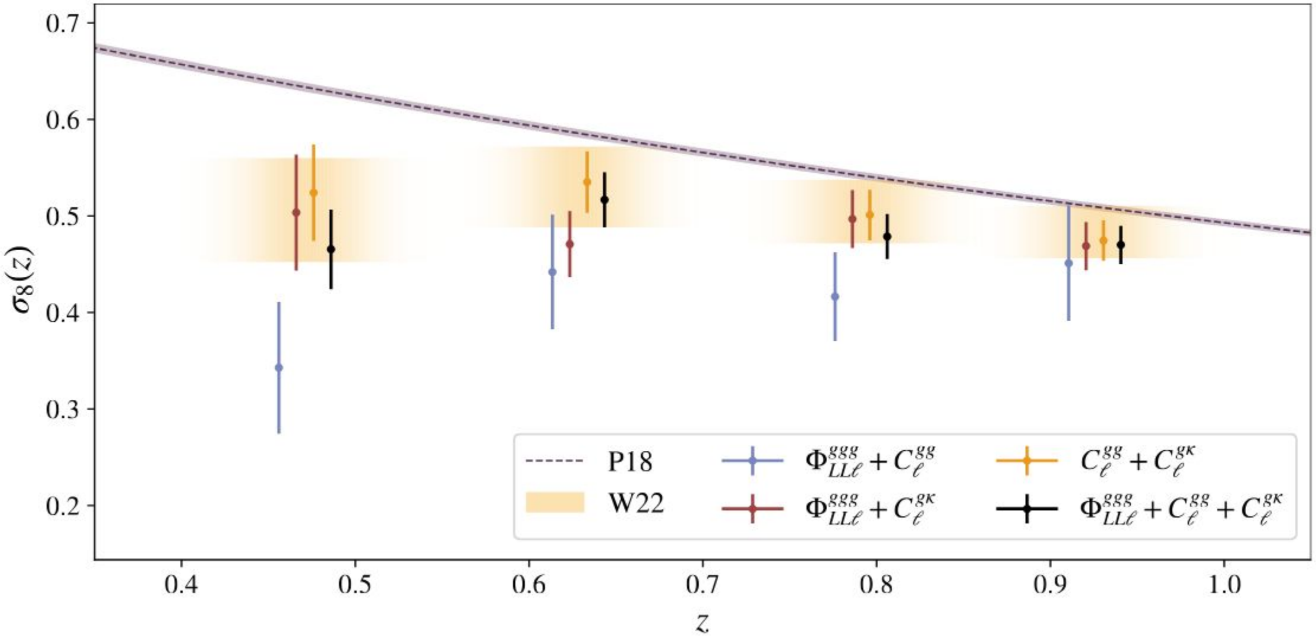
- Consistent results for different probe combinations

- 10-20% improvement over 2x2pt



Lea Harscouet

First (proper) cosmological (2+3)-point analysis *Verdiani et al. (in prep.)*

Idea: self-consistent bias and cosmology from gg+gκ (1-loop) and ggg (tree)

Francesco Verdiani

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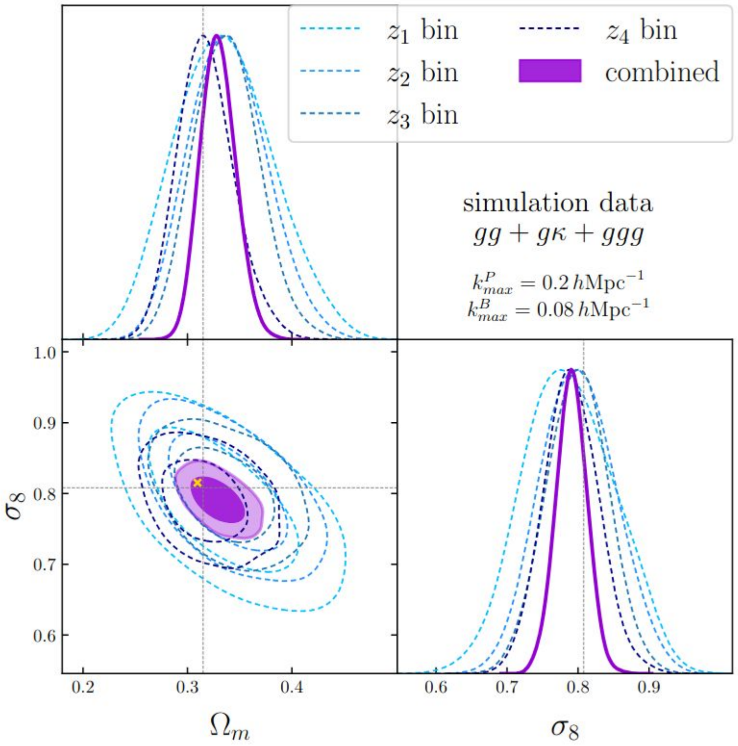
- Model and scale cuts validated against N-body sims



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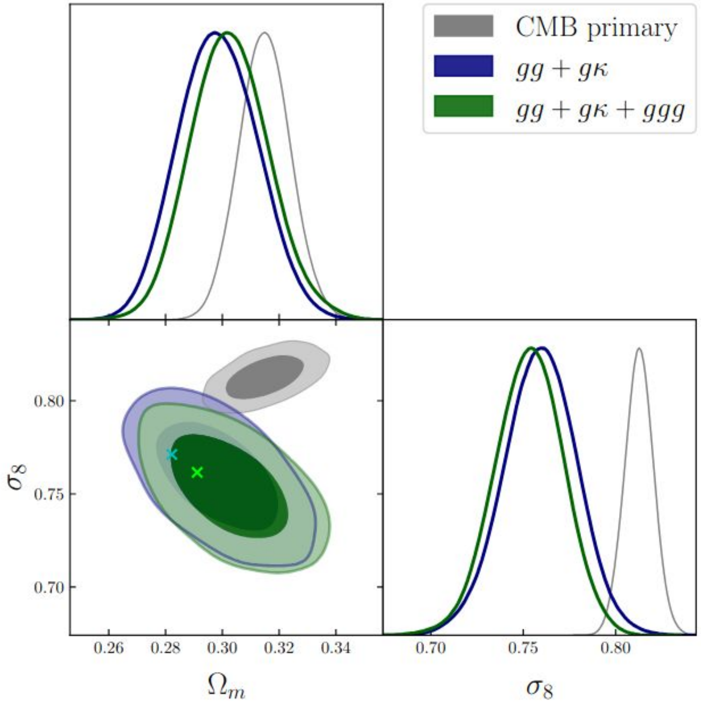

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- Model and scale cuts validated against N-body sims
- ~10-20% improvements on cosmology from 2x2pt
- Significant improvement on bias parameters
- Talk to Francesco to know more!

Francesco Verdiani

(2+3)-point

Looking forward:

- Current constraints limited by modeling uncertainties
  - Go to 1-loop bispectrum? HEFT even better.
  - Is HOS for projected galaxy clustering actually useful for cosmology?

- FSB-like approaches to other higher-order stats.
  - E.g. 1: trispectrum covariance
  - E.g. 2: parity-odd bi/tri-spectra

- FSB in the presence of very complex masks (e.g. cosmic shear)

- Is small-scale, multi-tracer bispectrum useful for baryons?

Summary

Summary

- Weak lensing, galaxy clustering and other LSS tracers are highly complementary for cosmology and astrophysics.

- Multi-tracer approach vital for robust, data-driven constraints in non-linear regime.

- Efficient approach to projected bispectrum (FSB)
  Improvements on cosmology and bias. Important consistency test.

Grazie mille!