From new methods for matter clustering to new dark sector physics

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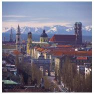
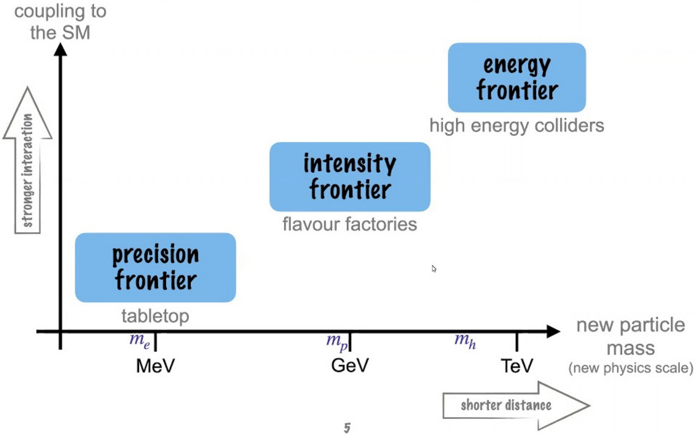
TECHNISCHE UNIVERSITÄT MÜNCHEN



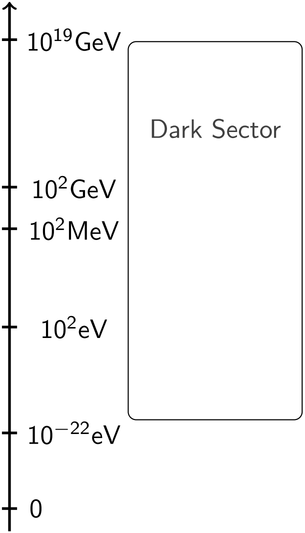
New physics from galaxy clustering, GGI, Florence, 19.09.25

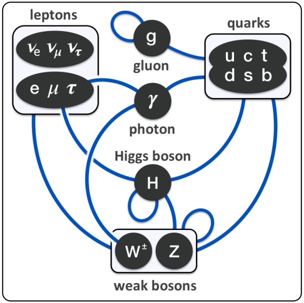
the quest for new physics

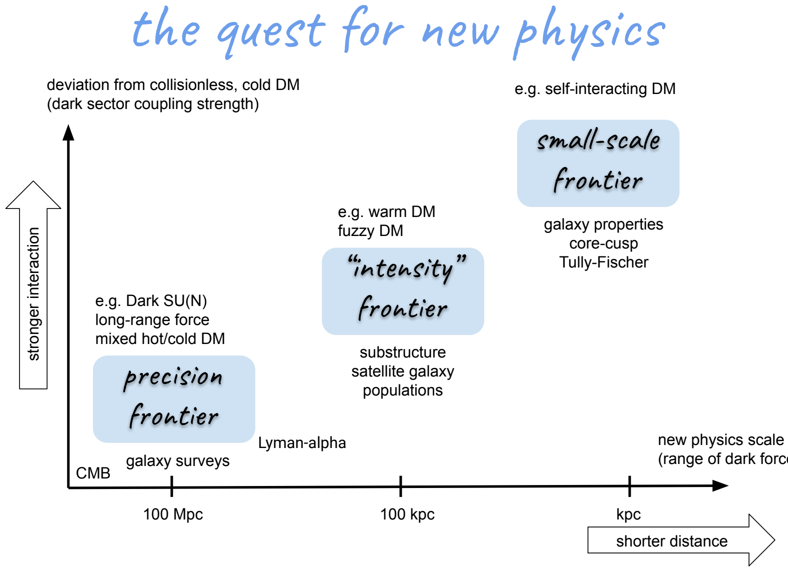


slide from Yotam Soreq

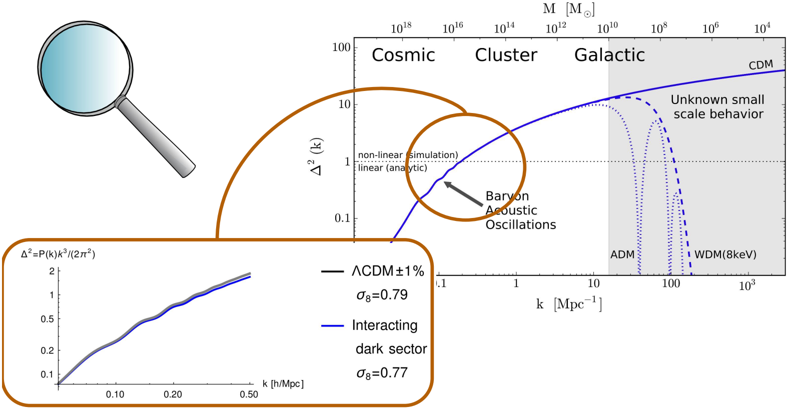
New Physics in the Dark Sector?






the quest for new physics
deviation from collisionless, cold DM
(dark sector coupling strength)
stronger interaction
e.g. self-interacting DM
small-scale frontier
galaxy properties
core-cusp
Tully-Fischer
e.g. warm DM
fuzzy DM
"intensity" frontier
substructure
satellite galaxy
populations
e.g. Dark SU(N)
long-range force
mixed hot/cold DM
precision frontier
galaxy surveys
Lyman-alpha
CMB
new physics scale
(range of dark force)
100 Mpc
100 kpc
kpc
shorter distance

“precision frontier”



Outline

- Part I: new dark sector physics (dark SU(N)/H_0)

  2508.03795, 2404.07256

  with Florian Niedermann (Nordita), Henrique Rubira (LMU/Cambridge), Martin Sloth (SDU)

  + earlier work by Asmaa Mazoun (TUM/LMU) et al 2411.19911, 2312.17622

- Part II: new methods for matter clustering (VPT)

  2505.02907, 2502.20451 + 2210.08088, 2210.08089

  with Román Scoccimarro (NYU), Dominik Laxhuber (TUM)

H_0: Systematics or Beyond-ΛCDM Physics?

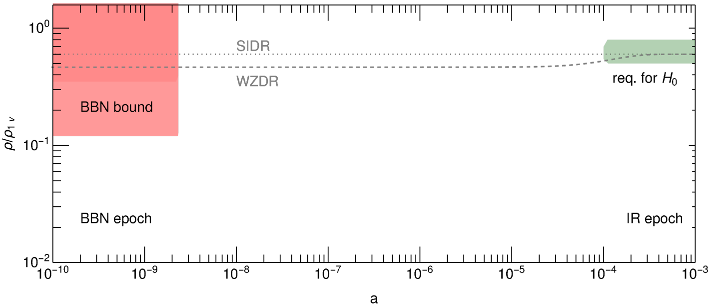
- Indirect measurements set by the ratio of the sound horizon at recombination, and the angular diameter distance

$$
\begin{aligned}
r_s(z_{rec}) &= \int_{z_{rec}}^{\infty} dz' \frac{c_s(z')}{H(z')} \propto 1/\sqrt{\rho_{\text{total}}(z \sim z_{rec})}\,, \\
d_A(z_{obs}) &= \int_0^{z_{obs}} dz' \frac{1}{H(z')} \propto 1/H_0
\end{aligned}
$$

- Increasing the inferred value of H_0 from CMB and galaxy surveys (BAO) requires to lower the sound horizon, e.g. via additional (dark sector) energy around recombination

[.. and changing ω_m, ..., see e.g. Poulin et al 2407.18292]

Extra (self-interacting) dark radiation?

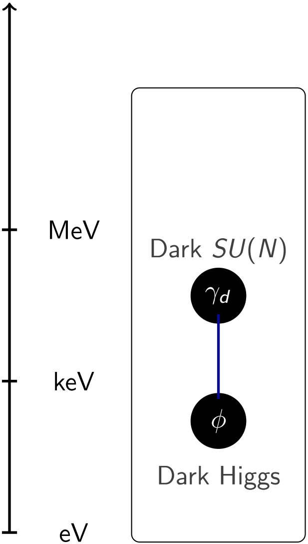


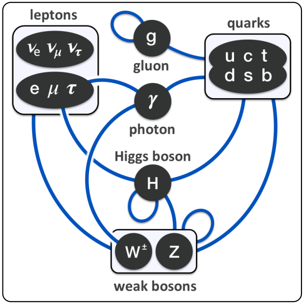
two problems: BBN-bound; CMB-perturbations

e.g. Schmaltz, Weiner, Joseph, Aloni, Berlin, Weiner

Allali, Notari Rompineve, Hertzberg, Poulin, Simon, Schöneberg ...

Dark Sector: take known physics as guideline





- Dark sector governed by known principles (gauge symm., SSB)

$$\mathcal{L} = (D\psi)^2 - V(\psi) - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

- Dark Higgs $\psi = (\psi_1, \ldots, \psi_N)$ in fundamental of $SU(N)$

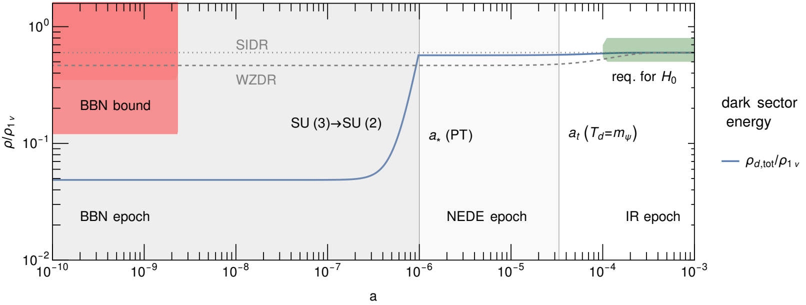
$$\boxed{SU(N) \to SU(N-1)}$$

- Supercooled first-order phase transition for (almost) scale-inv. Higgs potential via Coleman-Weinberg mechanism

Witten 1981; Levi, Opferkuch, Redigolo 22; ...

Assume Debye length $\sim 1/(gT_{DR}) \ll$ confinement length scale $1/\Lambda_c \propto e^{-1/g^2}$

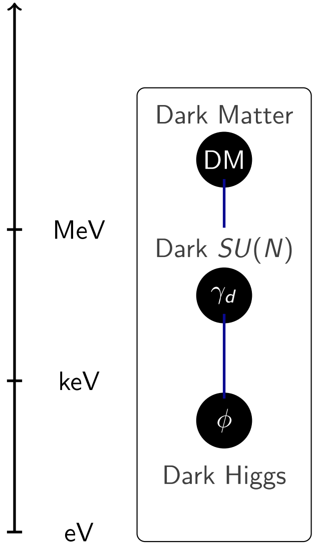
Dark $SU(N)$ + Higgs mechanism

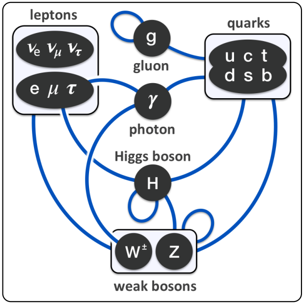


MG, Niedermann, Rubira, Sloth 2404.07256

see also Allali, Notari, Rompineve 2404.15220

DRMD


MeV
keV
eV
Dark Matter
DM
Dark $SU(N)$
γ_d
ϕ
Dark Higgs

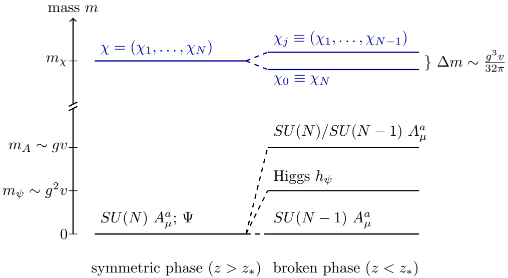

leptons
ν_e ν_μ ν_τ
e μ τ
g
gluon
quarks
u c t
d s b
γ
photon
Higgs boson
H
W±
Z
weak bosons

DRMD

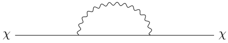
- Dark Matter $\chi = (\chi_1, \ldots, \chi_N)$ in fundamental of $SU(N)$

$$\mathcal{L} = \bar{\chi}\left(i\not{D} - m_\chi\right)\chi + (D\psi)^2 - V(\psi) - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$$

DRMD

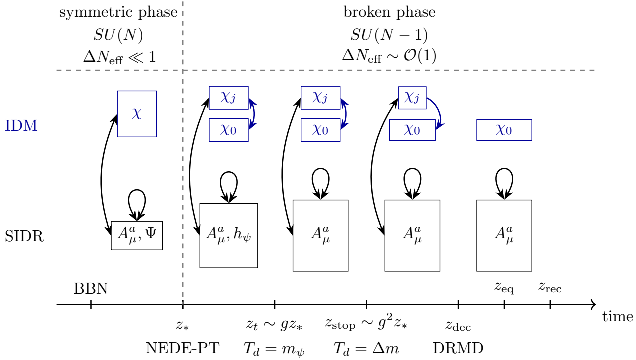


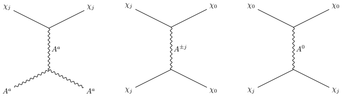
mass m
m_χ
$m_A \sim gv$
$m_\psi \sim g^2 v$
0
$\chi = (\chi_1, \ldots, \chi_N)$
$\chi_j \equiv (\chi_1, \ldots, \chi_{N-1})$
$\chi_0 \equiv \chi_N$
$\} \; \Delta m \sim \frac{g^3 v}{32\pi}$
$SU(N)/SU(N-1)\ A^a_\mu$
Higgs h_ψ
$SU(N)\ A^a_\mu;\ \Psi$
$SU(N-1)\ A^a_\mu$
symmetric phase $(z > z_*)$
broken phase $(z < z_*)$



χ
χ

DRMD





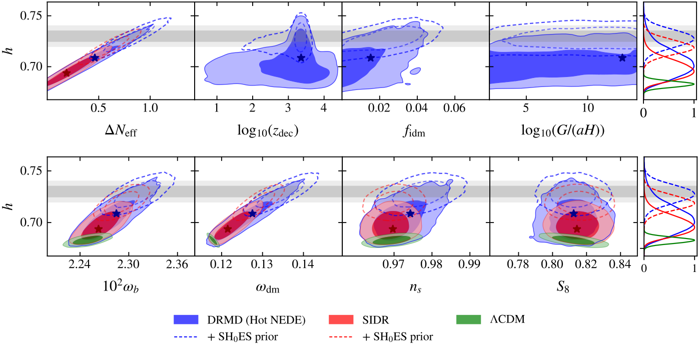
(a) t-channel Compton scattering

(b) t-channel dark matter conversion

(c) t-channel dark matter scattering

MG, Niedermann, Rubira, Sloth 2508.03795

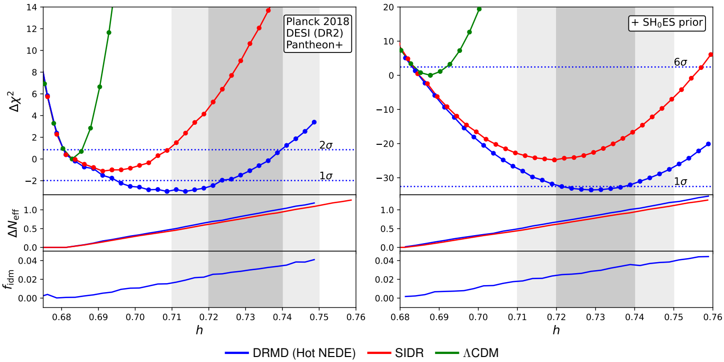
DRMD



	Base				Base + SH$_0$ES prior				$Q_{\rm DMAP}^M$	$Q_{\rm Gauss}$
	H_0	$\Delta N_{\rm eff}$	$\Delta\chi^2$	ΔAIC	H_0	$\Delta N_{\rm eff}$	$\Delta\chi^2$	ΔAIC		
ΛCDM	68.30 ± 0.29	–	–	–	68.70 ± 0.29	–	–	–	5.7σ	4.4σ
SIDR	$69.8^{+0.8}_{-1.0}$ (69.4)	< 0.525 (0.188)	-1.2	0.8	71.9 ± 0.7 (72.0)	0.61 ± 0.13 (0.610)	-24.5	-22.5	3.0σ	2.5σ
DRMD	$70.5^{+1.0}_{-1.6}$ (70.9)	< 0.811 (0.462)	-3.0	3.0	72.8 ± 0.8 (72.9)	0.80 ± 0.16 (0.829)	-33.3	-27.3	1.4σ	1.8σ

MG, Niedermann, Rubira, Sloth 2508.03795

DRMD



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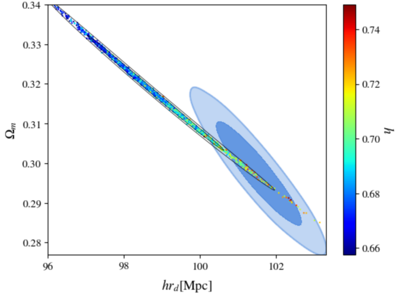
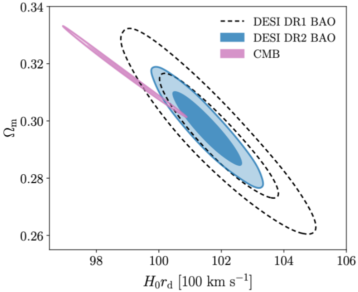
CMB vs DESI*

DRMD



ΛCDM



MG, Niedermann, Rubira, Sloth 2508.03795

DESI DR2 2503.14738

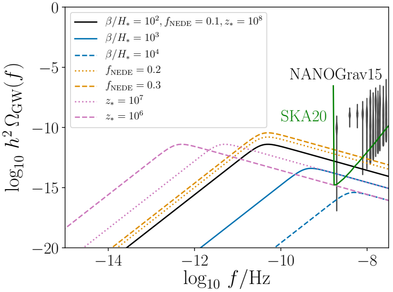
https://github.com/NEDE-Cosmo/DRMD-CLASS

(*no SN)

Further signatures

- features in C_ℓ (ACT?) and $P(k)$ (DAO related to DRMD)
- "isocurvature"/spectral dist. from stochasticity of PT

  e.g. Elor,Jinno,Kumar,McGehee,Tsai 2311.16222; Ramberg, Ratzinger, Schwaller 2209.14313
- GW background in PTAs (a bit on the tail...)



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  2505.02907, 2502.20451 + 2210.08088, 2210.08089

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$$
\begin{aligned}
\frac{\partial \delta(\tau,\mathbf{x})}{\partial \tau} + \nabla \cdot \{(1+\delta(\tau,\mathbf{x}))\mathbf{v}(\tau,\mathbf{x})\} &= 0 \quad \text{(continuity)} \\
\frac{\partial \mathbf{v}(\tau,\mathbf{x})}{\partial \tau} + \mathcal{H}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} &= -\nabla\phi - \frac{1}{\rho}\nabla_j(\sigma_{ij}\rho) \quad \text{(Euler)} \\
\nabla^2\phi(\tau,\mathbf{x}) &= \frac{3}{2}\Omega_m\mathcal{H}^2\delta(\tau,\mathbf{x}) \quad \text{(Poisson)}
\end{aligned}
$$

SPT: pressureless perfect fluid approximation

$$\sigma_{ij}(\tau, \mathbf{x}) \equiv 0$$

EFT: eff. dispersion tensor with free parameters c_a ("counterterms")

$$\sigma_{ij}(\tau, \mathbf{x}) = \sum_a c_a(\tau) \mathcal{O}_a(\tau, \mathbf{x})$$

VPT: perturbation theory for nonlinear collisionless BE + gravity

$$0 = \frac{df}{d\tau} = \left(\frac{\partial}{\partial \tau} + \frac{\mathbf{p}}{am} \frac{\partial}{\partial \mathbf{x}} - am \nabla \phi \frac{\partial}{\partial \mathbf{p}} \right) f(\tau, \mathbf{x}, \mathbf{p}) \quad \text{(Vlasov)}$$

$$\nabla^2 \phi(\tau, \mathbf{x}) \quad = \quad \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(\tau, \mathbf{x}) \quad \text{(Poisson)}$$

$$
\begin{aligned}
1+\delta &= \int d^3p\, f(\tau,\mathbf{x},\mathbf{p}) \\
v_i &= (1+\delta)^{-1}\int d^3p\, \frac{p_i}{am} f(\tau,\mathbf{x},\mathbf{p}) \\
\sigma_{ij} &= (1+\delta)^{-1}\int d^3p\, \frac{p_i p_j}{a^2m^2} f(\tau,\mathbf{x},\mathbf{p}) - v_i v_j
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}(\tau,\mathbf{x},\mathbf{l}) &\equiv \log\int d^3p\, e^{\mathbf{l}\cdot\mathbf{p}/(am)}\, f(\tau,\mathbf{x},\mathbf{p}) \\
&= \log(1+\delta) + l_i v_i + \frac{1}{2} l_i l_j \sigma_{ij} + \frac{1}{6} l_i l_j l_k \mathcal{C}_{ijk} + \ldots
\end{aligned}
$$

Pueblas, Scoccimarro '08; Uhlemann '18; ...

Vlasov equation for $f(\tau, \mathbf{x}, \mathbf{p})$

$\Rightarrow$ Coupled hierarchy of non-linear EoM for cumulants $\delta, v_i, \sigma_{ij}, \mathcal{C}_{ijk}, \ldots$

- 0th order: continuity
- 1st order: Euler
- 2nd order:

$$\partial_\tau \sigma_{ij} + 2\mathcal{H}\sigma_{ij} + v_k \nabla_k \sigma_{ij} + \sigma_{jk} \nabla_k v_i + \sigma_{ik} \nabla_k v_j = -\nabla_k \mathcal{C}_{ijk} - \mathcal{C}_{ijk} \nabla_k \ln(1+\delta)$$

- 3rd order:

$$\partial_\tau \mathcal{C}_{ijk} + \ldots = -\nabla_l \mathcal{C}_{ijkl} - \mathcal{C}_{ijkl} \nabla_l \ln(1+\delta)$$

- 4th order: ...

Truncate at some cumulant order c_{max}, assess validity by varying c_{max}

2505.02907, 2502.20451 + 2210.08088, 2210.08089 Dominik Laxhuber, MG, Román Scoccimarro

$\rightarrow$ *similar for massive neutrino pert. [Ma, Bertschinger], but in VPT we do include non-linearities*

Why VPT?

- We know pressureless perfect fluid approximation breaks down for collisionless matter due to shell crossing *e.g. Peebles '80, Buchert '97, Pueblas, Scoccimarro '08; ..., Vlasov-Poisson/velocity moments/LPT-extension/... Hahn+ 12+, Colombi 15; Seljak, Vlah 15+; Taruya, Colombi 17; Rampf, Frisch 17; McDonald, Vlah 17, Saga, Taruya, Colombi 18,21+; Rampf, Hahn 20; Rampf, Saga, Taruya, Colombi 23, ...*
- In practice, this reflects in spurious UV sensitivity of SPT loops (e.g. $P_{13}, P_{15}, \ldots$) and prevents convergence

  e.g. 3-loop Blas, MG, Konstandin '13
- Collisionless CDM via N-body; there UV sensitivity is much smaller (UV screening) *Peebles 80, Hogan+ 82, Goroff+ 86, ..., Nichimichi, Bernardeau, Taruya 1411.2970; 1708.08946,...*

  $\Rightarrow$ clustering on BAO-scales less sensitive to small-scale dynamics compared to what it seems when using SPT

Why VPT?

- VPT = perturbation theory for collisionless matter + gravity
- Superposition of many shell crossings on small scales generate (average) velocity dispersion $\langle \sigma_{ij} \rangle$ (non-perturbative)

  Buchert 97, ..., Pueblas, Scoccimarro '08, McDonald '09, ..., Erschfeld '18

- $\Rightarrow$ seeds also fluctuations $\delta\sigma_{ij}(z, \mathbf{k})$ correlated with large-scale modes (perturbative for $k < k_{nl}$), that back-react on δ and θ
- VPT captures the physics of UV screening and decoupling of small-scale modes due to shell crossing in a first principle approach

  $\Rightarrow$ improves over SPT while preserving predictivity
- Phase-space information useful for RSD (..still work in progress)

- Split cumulants into average and fluctuation part
- Example: 2nd cumulant (= velocity dispersion)

$$\epsilon_{ij}(z,\mathbf{x}) \equiv \frac{\sigma_{ij}(\tau(z),\mathbf{x})}{(\mathcal{H}(z)f(z))^2} = \epsilon(z)\delta_{ij} + \delta\epsilon_{ij}(z,\mathbf{x})$$

- Average velocity dispersion introduces a new scale

$$\boxed{k_\sigma(z) \equiv \epsilon(z)^{-1/2}}$$

(typically not far from non-linear scale $k_{nl}(z)$) *e.g. McDonald 09*

- Generated from average effect of shell-crossing dynamics on small scales; non-perturbative input for VPT
- Fluctuations on large scales, perturbative computation from Vlasov

$\delta\epsilon_{ij}$: 2 scalar, 2 vector, 2 tensor modes (svt)

- Various possibilities for how to determine dispersion scale k_σ
  - (i) self-consistently
  - (ii) halo model
  - (iii) matching $P_{\delta\delta}$ to N-body

MG, Laxhuber, Scoccimarro 25, 22

- Even infinitesimal non-zero initial value of $\epsilon(z_0)$ makes solutions deviate from SPT *Scoccimarro, Pueblas 08; McDonald 09*

- Write all fluctuations into a big vector

$$\psi_a = (\delta, \theta, (\nabla \times v), \delta\epsilon_{ij}, \dots)$$

- EoM has the familiar form

$$\begin{aligned} & \frac{d}{d \ln D(z)} \psi_a(z, \mathbf{k}) + \Omega_{ab}(z, k) \psi_b(z, \mathbf{k}) \\ &= \sum_{b,c} \int d^3p d^3q \delta_D(\mathbf{k} - \mathbf{p} - \mathbf{q}) \gamma_{abc}(\mathbf{p}, \mathbf{q}) \psi_b(z, \mathbf{p}) \psi_c(z, \mathbf{q}) \end{aligned}$$

- Vertices $\gamma_{abc}(\mathbf{p}, \mathbf{q})$
- Evolution matrix $\Omega_{ab}(z, k)$, contains dispersion scale k_σ [and average values of 4th cumulant, etc.]

Linear+non-linear terms obtained from Vlasov eq.

$$u_i = u_i^S + u_i^V = \frac{\nabla_i}{\nabla^2}\theta - \frac{\varepsilon_{ijk}\nabla_j}{\nabla^2}w_k\,,$$

$$\delta\epsilon_{ij}^S = \delta_{ij}\,\delta\epsilon + \frac{\nabla_i\nabla_j}{\nabla^2}g\,,$$
$$\delta\epsilon_{ij}^V = -\frac{\varepsilon_{ilk}\nabla_l\nabla_j}{\nabla^2}\nu_k - \frac{\varepsilon_{jlk}\nabla_l\nabla_i}{\nabla^2}\nu_k\,,$$
$$\delta\epsilon_{ij}^T = t_{ij} \equiv P_{ij,ls}^T\delta\epsilon_{ls}\,,$$

$$\psi \equiv (\delta, \theta, g, \delta\epsilon, A, w_i, \nu_i, t_{ij})\,.$$

$$\Omega = \begin{pmatrix} \Omega^S & & \\ & \Omega^V & \\ & & \Omega^T \end{pmatrix}, \qquad \Omega^S = \begin{pmatrix} & -1 & & & \\ -3/2 & 1/2 & k^2 & k^2 & k^2\epsilon \\ & -2\epsilon & 1 & & \\ & & & 1 & \\ & -1 & & & \end{pmatrix}, \qquad \Omega^V = \begin{pmatrix} 1/2 & k^2 \\ -\epsilon & 1 \end{pmatrix}, \quad \Omega^T = 1\,,$$

$$\gamma_{\delta\theta\delta}(p,q) = \frac{1}{2}\alpha_{pq} = \frac{(p+q)\cdot p}{2p^2}\,,$$
$$\gamma_{\theta\theta\theta}(p,q) = \beta_{pq} = \frac{(p+q)^2 p\cdot q}{2p^2q^2}\,,$$
$$\gamma_{A\theta A}(p,q) = \frac{1}{2}\frac{q\cdot p}{p^2}\,,$$
$$\gamma_{\theta Ag}(p,q) = -\frac{1}{2}(p+q)\cdot q\;\frac{q\cdot p}{q^2}\,,$$
$$\gamma_{\theta A\epsilon}(p,q) = -\frac{1}{2}(p+q)\cdot p\,,$$

$$\gamma_{\delta w_i\delta}(p,q) = \gamma_{Aw_iA}(p,q) = \frac{1}{2}\frac{(p\times q)_i}{p^2}\,,$$
$$\gamma_{\theta w_i\theta}(p,q) = \frac{1}{2}\left(1+\frac{2p\cdot q}{q^2}\right)\frac{(p\times q)_i}{p^2}\,,$$
$$\gamma_{\theta A\nu_i}(p,q) = \frac{1}{2}\left(1+\frac{2p\cdot q}{q^2}\right)(p\times q)_i\,,$$
$$\gamma_{\theta At_{ij}}(p,q) = -\frac{1}{2}p_ip_j\,,$$

$$\gamma_{g\theta g}(p,q) = \frac{1}{2}\frac{p\cdot q}{q^2}\left(\frac{k^2}{p^2}+\frac{1}{2}-\frac{3}{2}\frac{(k\cdot p)^2}{k^2p^2}\right),$$
$$\gamma_{g\theta\epsilon}(p,q) = \frac{1}{2}\left(3\frac{(k\cdot p)^2}{k^2p^2}-1\right),$$
$$\gamma_{\epsilon\theta g}(p,q) = \frac{1}{2}\frac{p\cdot q}{2p^2q^2k^2}\left((k\cdot p)^2-p^2k^2\right),$$
$$\gamma_{\epsilon\theta\epsilon}(p,q) = \frac{1}{2}\frac{k\cdot pk\cdot q}{k^2p^2}\,.$$

$$\gamma_{gw_ig}(p,q) = \frac{1}{2}\frac{1}{p^2}\left(\frac{3}{2}\frac{(k\cdot q)^2}{k^2q^2}+3\frac{k\cdot qp\cdot q}{k^2q^2}-\frac{p\cdot q}{q^2}-\frac{1}{2}\right)(p\times q)_i\,,$$
$$\gamma_{gw_i\epsilon}(p,q) = \frac{1}{2}3\frac{k\cdot p}{p^2k^2}(p\times q)_i\,,$$
$$\gamma_{g\theta\nu_i}(p,q) = -\frac{1}{2}\frac{1}{k^2p^2q^2}\left(p\cdot qk^2+3k\cdot pk\cdot q\right)(p\times q)_i\,,$$
$$\gamma_{g\theta t_{ij}}(p,q) = -\frac{1}{2}\frac{2k^2-3p\cdot q-6k\cdot p}{2k^2p^2}p_ip_j\,,$$

Example: 2nd cumulant truncation *a bit tedious ... but straightforward*

5 scalar + 2*2 vector [+ 1*2 tensor modes], 33 [+15] vertices

- Perturbative solution in initial density δ_0 as usual

$$\begin{aligned}\psi_a(z,\mathbf{k}) &= \sum_n \int d^3k_1 \cdots d^3k_n \, \delta_D(\mathbf{k}-\mathbf{k}_1-\cdots-\mathbf{k}_n) \\ &\quad \times F_{n,a\ldots}(\mathbf{k}_1,\ldots,\mathbf{k}_n;z)\delta_0(\mathbf{k}_1)\cdots\delta_0(\mathbf{k}_n)\end{aligned}$$

$\Rightarrow$ boils down to using F_n kernels different from SPT in loop integrals

- Example: UV limit of F_3 kernel $q \gg k_\sigma \gg k$

$$F_3^{\rm SPT}(\mathbf{k},\mathbf{q},-\mathbf{q}) \quad \to \quad -\frac{61}{1890} \times \frac{k^2}{q^2}$$

$$F_{3,a=\delta}^{\rm VPT}(\mathbf{k},\mathbf{q},-\mathbf{q};z) \quad \to \quad -\mathrm{const} \times \frac{k^2}{q^2} \times \left(\frac{k_\sigma(z)}{q}\right)^{4/\alpha}$$

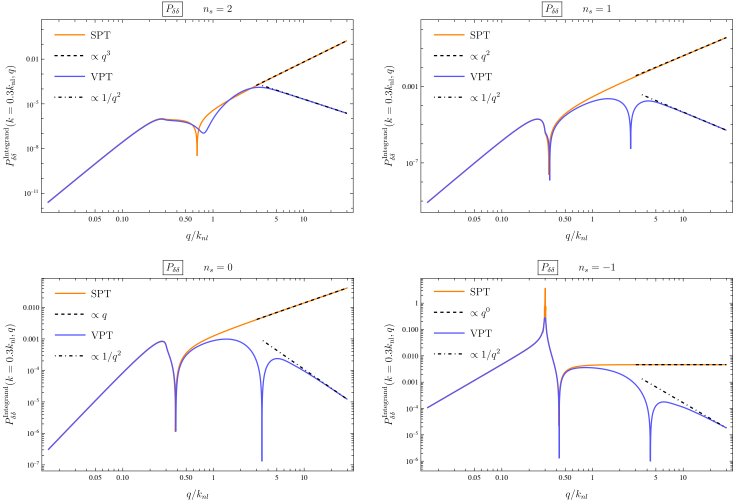
for average dispersion $\epsilon(z) \propto D(z)^\alpha$, i.e. $k_\sigma(z) \propto D(z)^{-\alpha/2}$ (*)

- UV screening captured by VPT; obtained from collisionless dynamics (1st principle)
- Power-law index $4/\alpha$ independent of truncation order

  [only important that incoming linear modes stop growing at $z = z_q$ when $k_\sigma(z) \sim q$]
- For scaling universe $\alpha = 4/(3 + n_s)$

(*): For arbitrary $\epsilon(z)$ replace $(k_\sigma(z)/q)^{4/\alpha}$ by $(D(z_q)/D(z))^2$

One-loop integrand for power law spectra $P_0 \propto k^{n_s}$



L-loop: SPT $\propto q^{(n_s+3)L-2}$; VPT $\propto q^{-2}$ indep. of n_s and L

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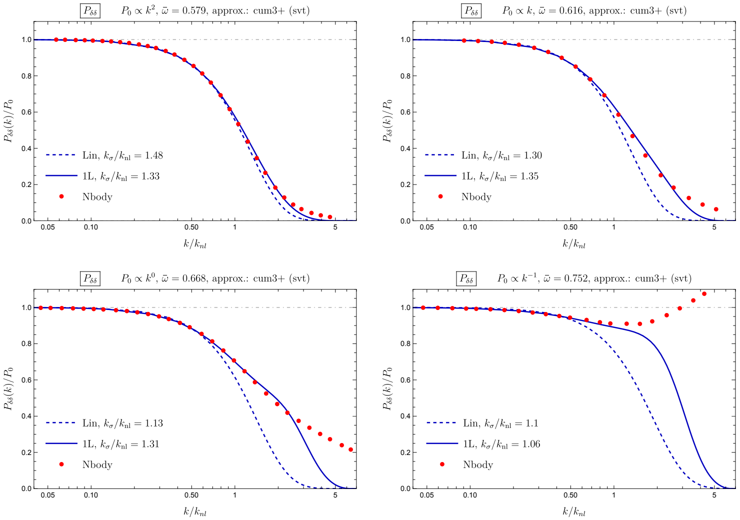
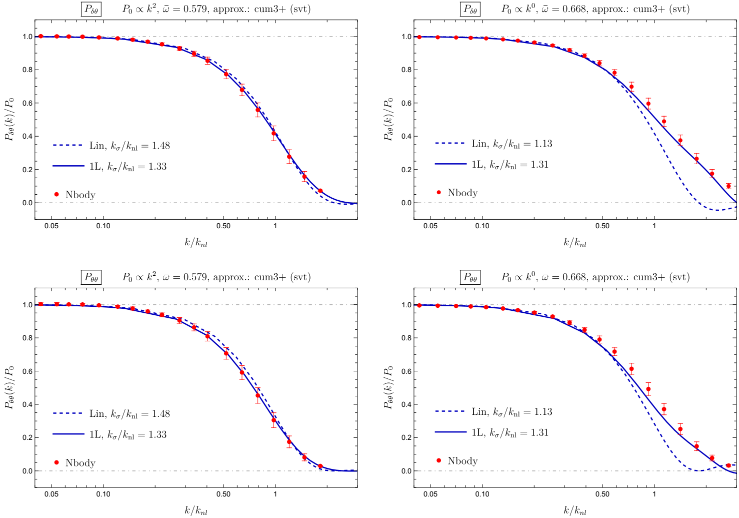
$P_{\delta\delta}$ for power law spectra $P_0 \propto k^{n_s}$



VPT with 1-param ($=k_\sigma/k_{nl}$) matched to N-body *MG, DL, Scoccimarro 2210.08089*

$P_{\delta\theta}$ and $P_{\theta\theta}$



$P_{\delta\theta}$ and $P_{\theta\theta}$ VPT with 0 free parameters [k_σ/k_{nl} already fixed from $P_{\delta\delta}$]

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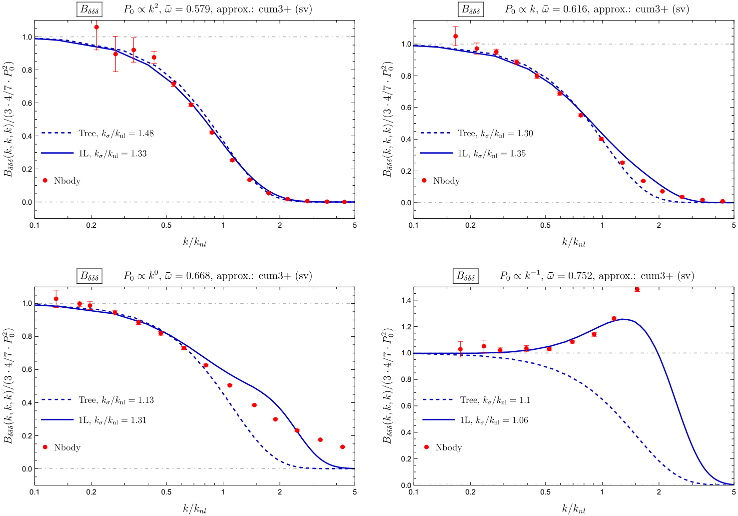
One-loop equilateral bispectrum



$B(k,k,k)$ VPT with 0 free parameters [k_σ/k_{nl} already fixed from $P_{\delta\delta}$]

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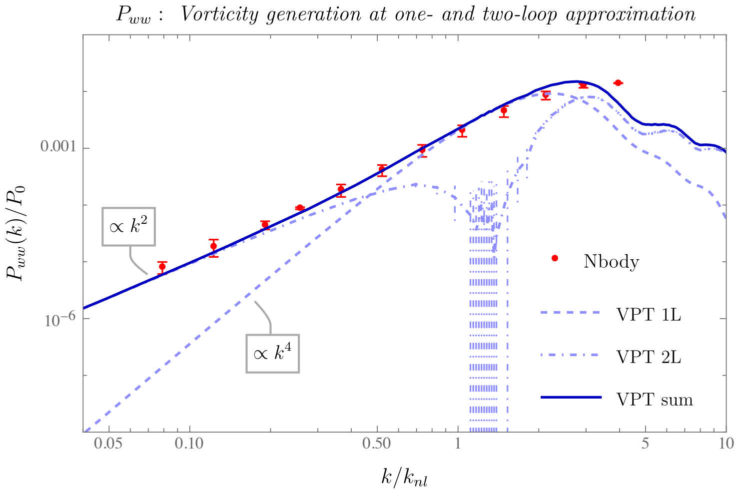
Two-loop vorticity power spectrum



P_{ww} VPT with 0 free parameters [k_σ/k_{nl} already fixed from $P_{\delta\delta}$]

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Relation to EFT?

EFT

$$F_n(k_1,\ldots,k_n;z) = F_n^{SPT}(k_1,\ldots,k_n) + \sum_a c_a(z) k^2 \Delta_a(k_1/k,\ldots,k_n/k) + \mathcal{O}(k^4)$$

with free parameters c_a ("counterterms")

VPT

- Taylor expand VPT kernels in $\epsilon(\eta)$
- match to EFT, yields predictions for c_a

$n = 1$: ("the c_s^2 counterterm")

$$c_a = I_\delta = \int^\eta d\eta' \, w(\eta,\eta')\epsilon(\eta')$$

$n = 2$ (counterterms for 1L bisp.)

$$c_a \in \{\beta_a, \beta_b, \beta_c, \beta_d\}$$

$$\begin{aligned} F_{1,\delta}(k,\eta) &= 1 - k^2 I_\delta(\eta) + \mathcal{O}(\epsilon^2)\,, \\ F_{1,\theta}(k,\eta) &= 1 - k^2 I_\theta(\eta) + \mathcal{O}(\epsilon^2)\,, \end{aligned} \qquad (40)$$

where

$$\begin{aligned} I_\delta(\eta) &\equiv \frac{2}{5}\int^\eta d\eta' \left(1 - e^{5(\eta'-\eta)/2}\right) \\ &\quad \left(\epsilon(\eta') + 2\int^{\eta'} d\eta''\, e^{2(\eta''-\eta')}\epsilon(\eta'')\right)\,, \\ I_\theta(\eta) &\equiv \frac{2}{5}\int^\eta d\eta' \left(1 + \frac{3}{2}e^{5(\eta'-\eta)/2}\right) \\ &\quad \left(\epsilon(\eta') + 2\int^{\eta'} d\eta''\, e^{2(\eta''-\eta')}\epsilon(\eta'')\right)\,. \end{aligned} \qquad (41)$$

$$\begin{aligned} F_{2,\delta}(\boldsymbol{p},\boldsymbol{q},\eta) &= F_2(\boldsymbol{p},\boldsymbol{q}) - (\boldsymbol{p}+\boldsymbol{q})^2 \sum_{k=a,b,c,d} \Delta_k(\boldsymbol{p},\boldsymbol{q})\beta_k^\delta(\eta) \\ &\quad + \mathcal{O}(\epsilon^2)\,, \\ F_{2,\theta}(\boldsymbol{p},\boldsymbol{q},\eta) &= G_2(\boldsymbol{p},\boldsymbol{q}) - (\boldsymbol{p}+\boldsymbol{q})^2 \sum_{k=a,b,c,d} \Delta_k(\boldsymbol{p},\boldsymbol{q})\beta_k^\theta(\eta) \\ &\quad + \mathcal{O}(\epsilon^2)\,, \end{aligned} \qquad (57)$$

with coefficients $\beta_k^{\delta/\theta}(\eta)$ for $k = a,b,c,d$ given by linear combinations of the $J_j^{\delta/\theta}(\eta)$ with coefficients $\beta_{j,k}$ from Table I. Using the results from App. B we find

$$\begin{aligned} \beta_a^\delta(\eta) &= \frac{2}{5}(2E_0(\eta) - 6E_1(\eta) + 5E_2(\eta) + 3E_{5/2}(\eta) \\ &\quad - 5E_3(\eta) + E_{7/2}(\eta))\,, \\ \beta_b^\delta(\eta) &= \frac{2}{5}(6E_1(\eta) - 10E_2(\eta) + 5E_3(\eta) - E_{7/2}(\eta))\,, \\ \beta_c^\delta(\eta) &= \frac{1}{35}(3E_1(\eta) - 20E_2(\eta) + 30E_{5/2}(\eta) - 15E_3(\eta) \\ &\quad + 2E_{7/2}(\eta))\,, \\ \beta_d^\delta(\eta) &= \frac{3}{35}(3E_1(\eta) - 10E_2(\eta) + 15E_3(\eta) - 8E_{7/2}(\eta))\,, \end{aligned} \qquad (58)$$

$$E_m(\eta) \equiv \int^\eta d\eta'\, e^{m(\eta'-\eta)}\epsilon(\eta')\,, \qquad (36)$$

$$\begin{aligned} \Delta_a(\boldsymbol{p},\boldsymbol{q}) &\equiv (p^2+q^2)F_2(\boldsymbol{p},\boldsymbol{q})/(\boldsymbol{p}+\boldsymbol{q})^2\,, \\ \Delta_b(\boldsymbol{p},\boldsymbol{q}) &\equiv F_2(\boldsymbol{p},\boldsymbol{q})\,, \\ \Delta_c(\boldsymbol{p},\boldsymbol{q}) &\equiv (p^2+q^2)K(\boldsymbol{p},\boldsymbol{q})/(\boldsymbol{p}+\boldsymbol{q})^2\,, \\ \Delta_d(\boldsymbol{p},\boldsymbol{q}) &\equiv K(\boldsymbol{p},\boldsymbol{q}) \equiv (\boldsymbol{p}\cdot\boldsymbol{q})^2/(p^2q^2) - 1\,, \end{aligned} \qquad (56)$$

[6] Using the basis functions $E_{1,2,3}$ and Γ in the convention of [31] one has $E_1 = k^2(-\Delta_a + \Delta_b - \frac{5}{7}\Delta_c - \frac{2}{7}\Delta_d)$, $E_2 = k^2(-\frac{2}{3}\Delta_a + \frac{2}{3}\Delta_b - \frac{10}{21}\Delta_c + \frac{17}{21}\Delta_d)$, $E_3 = k^2(-\frac{1}{3}\Delta_a + \frac{1}{3}\Delta_b + \frac{11}{42}\Delta_c - \frac{2}{21}\Delta_d)$, $\Gamma = k^2(\frac{2}{11}\Delta_a + \frac{9}{11}\Delta_b - \frac{1}{7}\Delta_c - \frac{6}{77}\Delta_d)$, where $k^2 \equiv (\boldsymbol{p}+\boldsymbol{q})^2$.

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Dependence on truncation?

- Hierarchy of evolution equations for cumulants $\mathcal{C}_{i_1 i_2 \dots i_c}$
- Evolution of cth cumulant depends on $c' \leq c+1$
- Truncation at c_{max}

Dependence on truncation?

- VPT including all cumulants up to order $c_{\rm max}$
- Use spherical harmonic basis *cf. Seljak, McDonald,... '11; MG Scoccimarro '25*

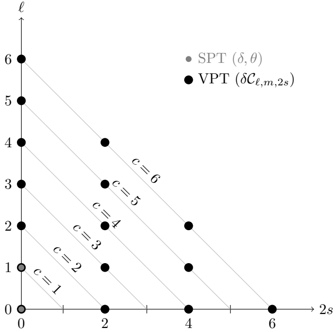
$$\begin{aligned}\mathcal{C}(z,\mathbf{k},\mathbf{L}) &\equiv \log\int d^3p\, e^{\mathbf{L}\cdot\mathbf{p}/(am)}\, f(z,\mathbf{k},\mathbf{p}) \\ &= \log(1+\delta) + L_i v_i + \frac{1}{2}L_i L_j \sigma_{ij} + \frac{1}{6}L_i L_j L_k \mathcal{C}_{ijk} + \ldots \\ &= \sum_{\ell,m,s} |\mathbf{L}|^{2s+\ell}\, Y_{\ell m}(\hat{\mathbf{L}})\, \mathcal{C}_{\ell,m,2s}(z,\mathbf{k})\end{aligned}$$

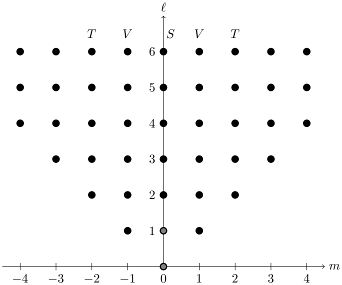
- Cumulants at order c:

$$\mathcal{C}_{\ell,m,2s}(z,\mathbf{k})$$

with $c = \ell + 2s$

Dependence on truncation?



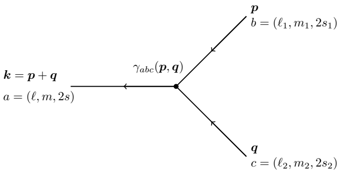


- e.g first cumulant v_i: $\ell = 1, 2s = 0$; $m = 0, \pm 1$ (θ, vorticity)
- e.g. second cumulant σ_{ij}:
  $\ell = 0, 2s = 2$; (one scalar mode)
  $\ell = 2, 2s = 0, m = 0, \pm 1, \pm 2$ (1S+2V+2T modes)

Linear+non-linear terms obtained from Vlasov eq.

$$
\begin{aligned}
\psi_a \;=\; & \Big(\delta(\eta,\boldsymbol{k}), \delta C_{000}(\eta,\boldsymbol{k}), \delta C_{1m0}(\eta,\boldsymbol{k})|_{m=0,\pm1}, \\
& \delta C_{002}(\eta,\boldsymbol{k}), \delta C_{2m0}(\eta,\boldsymbol{k})|_{m=0,\pm1,\pm2}, \\
& \delta C_{1m2}(\eta,\boldsymbol{k})|_{m=0,\pm1}, \delta C_{3m0}(\eta,\boldsymbol{k})|_{m=0,\pm1,\pm2,\pm3}, \\
& \delta C_{004}(\eta,\boldsymbol{k}), \delta C_{2m2}(\eta,\boldsymbol{k})|_{m=0,\pm1,\pm2}, \\
& \qquad \delta C_{4m0}(\eta,\boldsymbol{k})|_{m=0,\pm1,\pm2,\pm3,\pm4}, \dots \Big),
\end{aligned}
$$

$$
\begin{aligned}
\partial_\eta \psi_a(\eta,\boldsymbol{k}) \;+\; & \Omega_{ab}(\eta,k)\psi_b(\eta,\boldsymbol{k}) \\
& = \int_{\boldsymbol{pq}} \gamma_{abc}(\boldsymbol{p},\boldsymbol{q})\psi_b(\eta,\boldsymbol{p})\psi_c(\eta,\boldsymbol{q}) \,, \quad (41)
\end{aligned}
$$



$$
\Omega_{ab} = \left(\delta^K_{\ell\ell'} A^{ss'}_{\ell m} + \delta^K_{\ell-1,\ell'} B^{ss'}_{\ell m} + \delta^K_{\ell+1,\ell'} C^{ss'}_{\ell m}\right) \delta^K_{mm'} \,, \quad (44)
$$

with

$$
\begin{aligned}
A^{ss'}_{\ell m} &= \left(\frac{3}{2}\frac{\Omega_m}{f^2} - 1\right)(\ell + 2s)\delta^K_{ss'} \,, \\
B^{ss'}_{\ell m} &= \frac{\{1, k^2\}}{2\ell+1}\sqrt{\ell^2 - m^2}\Bigg(\frac{\delta^K_{s+1,s'}}{(2s+1)} \\
&\qquad + \frac{(2s)!\,\mathcal{E}_{2s-2s'+2}(\eta)}{(2(s-s')+1)!(2s')!}\Theta_{s-s'}\Bigg) \,, \\
C^{ss'}_{\ell m} &= -\frac{\{1, k^2\}}{2\ell+1}\sqrt{(\ell+1)^2 - m^2}\Bigg((2\ell+3+2s)\,\delta^K_{ss'} \\
&\qquad + \frac{(2s)!\,\mathcal{E}_{2s-2s'}(\eta)}{(2(s-s')-1)!(2s')!}\Theta_{s-1-s'}\Bigg) \,, \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\gamma}_{abc}(\boldsymbol{p},\boldsymbol{q}) \;=\; & (-1)^{\frac{\ell_1+\ell_2-1-\ell}{2}} \frac{(2s)!}{(2s_1)!(2s_2)!} \delta^K_{\ell_1+2s_1+\ell_2+2s_2,\ell+2s+1} \\
& \times \frac{pk^{a_\ell}}{p^{a_{\ell_1}} q^{a_{\ell_2}}} (2\ell_1+1)(-1)^{m'} [D^{m'}_{\ell m}((R^{\boldsymbol{p}})^{-1}R^{\boldsymbol{k}})]^* D^{m'_2}_{\ell_2 m_2}((R^{\boldsymbol{p}})^{-1}R^{\boldsymbol{q}}) \\
& \times \Bigg[(2\ell_2+1+2s_2)\begin{pmatrix} \ell & \ell_1 & \ell_2-1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell & \ell_1 & \ell_2-1 \\ -m' & m_1 & m'_2 \end{pmatrix}\sqrt{\ell_2^2 - (m'_2)^2} \\
& + 2s_2 \begin{pmatrix} \ell & \ell_1 & \ell_2+1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell & \ell_1 & \ell_2+1 \\ -m' & m_1 & m'_2 \end{pmatrix}\sqrt{(\ell_2+1)^2 - (m'_2)^2}\Bigg] .
\end{aligned}
$$

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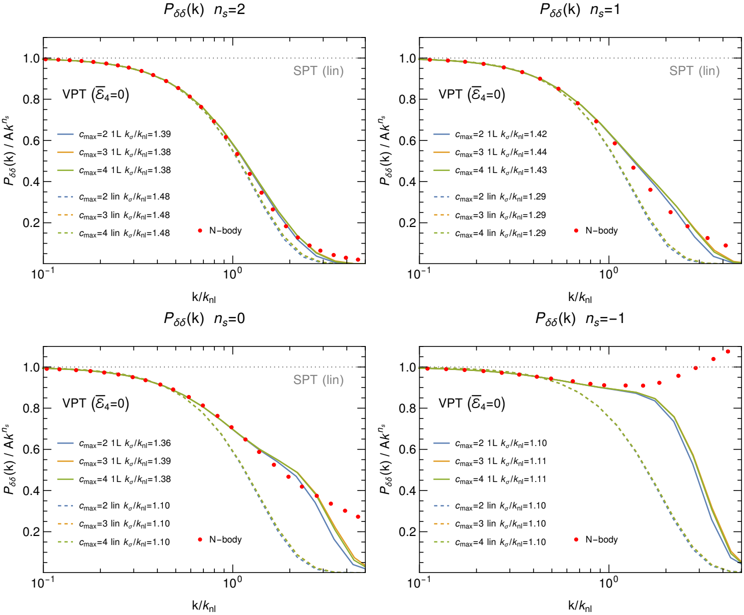
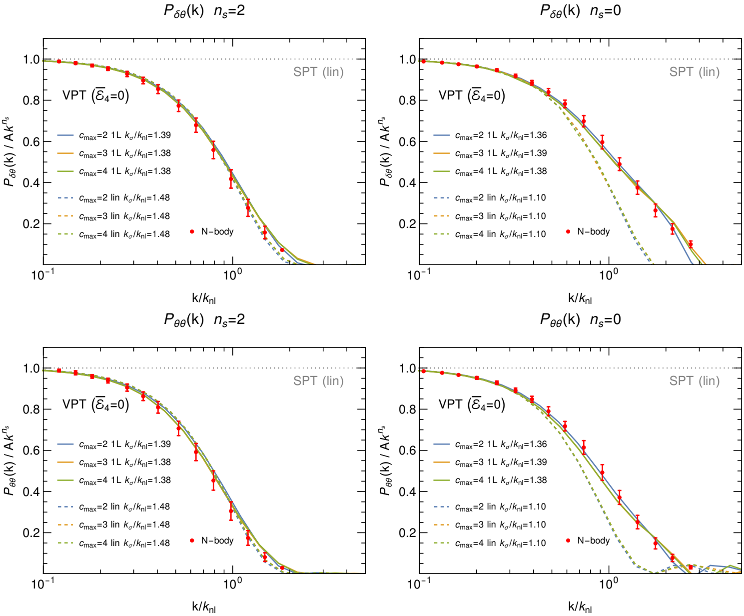
Dependence on truncation?



Dependence on truncation?



Application to ΛCDM cosmology

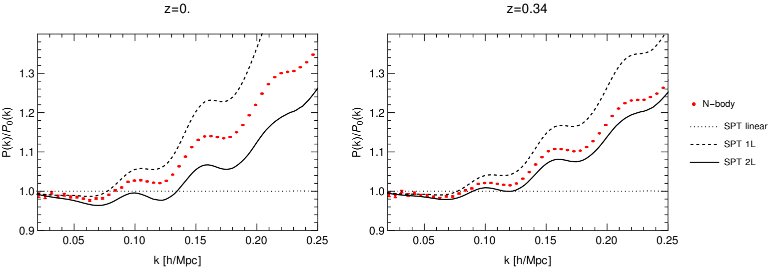
VPT input: average dispersion $\epsilon(z) = \langle \sigma_{ii} \rangle / 3(f\mathcal{H})^2$

We consider two possibilities:

- (1) Estimated from simulation (using halo finder and estimate of dispersion in and around halos based on NFW); yields $k_\sigma(z) = \epsilon(z)^{-1/2}$ comparable to nonlinear scale
  $\Rightarrow$ 0 free parameters
- (2) Treating $k_\sigma(z) = \epsilon(z)^{-1/2}$ as free parameter
  $\Rightarrow$ 1 free parameter

Application to ΛCDM cosmology: case (1)

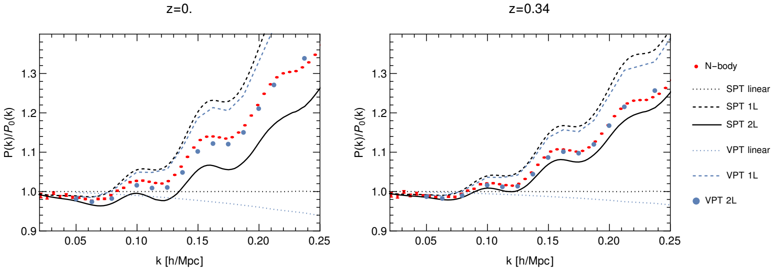
$$P_{\delta\delta}(z,k)$$



red = N-body, black = SPT,

Application to ΛCDM cosmology: case (1)

$$P_{\delta\delta}(z,k)$$



red = N-body, black = SPT, blue = VPT

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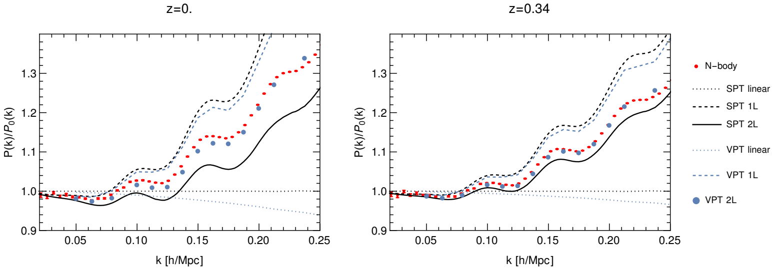
Application to ΛCDM cosmology: case (1)

$P_{\delta\delta}(z, k)$



red = N-body, black = SPT, blue = VPT

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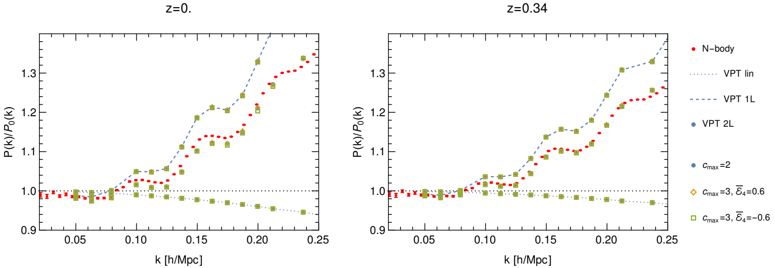
Residual difference at $z = 0$ consistent with missing 3-loop contributions

$k_\sigma(z = 0) = 0.36h/\mathrm{Mpc}$, $\quad k_\sigma(z = 0.34) = 0.47h/\mathrm{Mpc}$

$k_{\mathrm{nl}}(z = 0) = 0.27h/\mathrm{Mpc}$, $\quad k_{\mathrm{nl}}(z = 0.34) = 0.37h/\mathrm{Mpc}$

Application to ΛCDM cosmology: case (1)



Truncation $c_{\text{max}} = 2$ (circles) versus $c_{\text{max}} = 3$ (diamonds, boxes)

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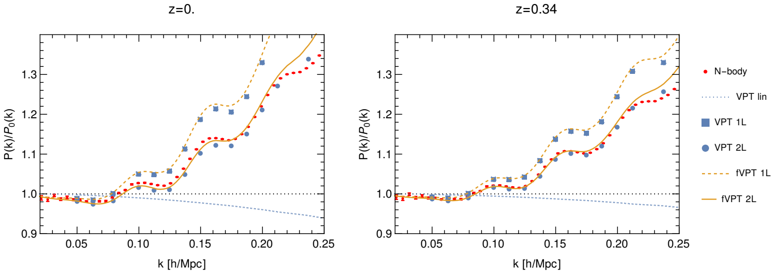
Application to ΛCDM cosmology: case (1)



Full VPT versus simplified scheme fVPT (includes main features of VPT; computational effort equal to SPT)

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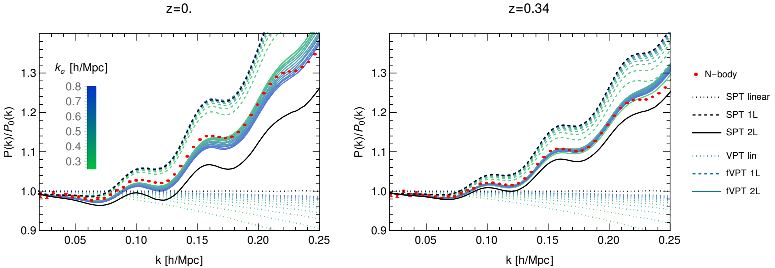
Application to ΛCDM cosmology: case (2)



Variation with k_σ; partial cancellation in 2-loop result

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Vlasov Perturbation Theory

- Conceptually simple idea:
  VPT = perturbation theory for collisionless matter
  SPT = pert. theory for perfect fluid
- based on first principles
- physical new non-pert. scale $k_\sigma(z)$ related to shell crossing
- includes known features of structure formation that SPT misses (decoupling of UV modes; vorticity/dispersion/...)
- demonstrated for $P_{\delta\delta}, P_{\delta\theta}, P_{\theta\theta}, B(k,k,k), P_{ww}$
- future: RSD; could be used in EFT pipeline instead of SPT (better motivated priors on EFT corrections)