Simulation-based inference for galaxy clustering with LEFTfield

Beatriz Tucci

MPA (→ Stanford LITP soon)

with Fabian Schmidt, Nhat-Minh Nguyen, Ivana Nikolac, Ivana Babić, Julia Stadler, Andrija Kostić, Martin Reinecke

New Physics from Galaxy Clustering at GGI

MAX-PLANCK-INSTITUT FÜR ASTROPHYSIK

Current cosmological inference analysis

Observation

Compression

Summary statistics data vector

$$\mathbf{D}$$

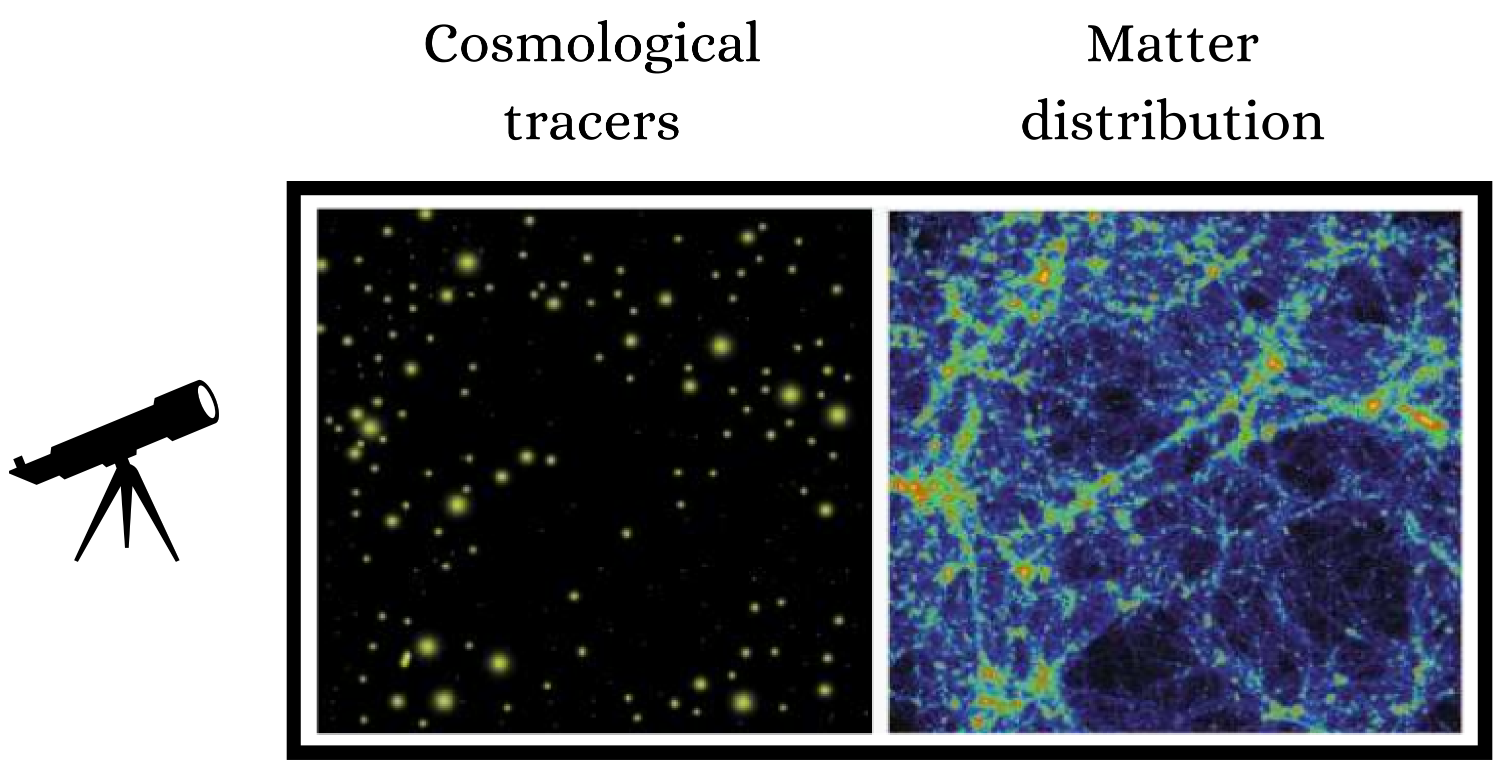
$$\underbrace{\mathcal{P}(\boldsymbol{\theta}|\mathbf{D})}_{\text{Posterior}} \propto \underbrace{\mathcal{L}(\mathbf{D}|\boldsymbol{\theta})}_{\text{Likelihood}}\underbrace{\pi(\boldsymbol{\theta})}_{\text{Prior}}$$

E.g., assuming that the data vector is Gaussian distributed:

$$\boxed{-2\ln\mathcal{L}(\underbrace{\mathbf{D}}_{\text{Data vector}}|\boldsymbol{\theta}) = \big(\mathbf{D} - \mathbf{T}(\boldsymbol{\theta})\big)\cdot \underbrace{\mathbf{C}}_{\text{Covariance of the data vector}}^{-1}\cdot \big(\mathbf{D} - \underbrace{\mathbf{T}(\boldsymbol{\theta})}_{\text{Theoretical prediction of data vector}}\big)}$$

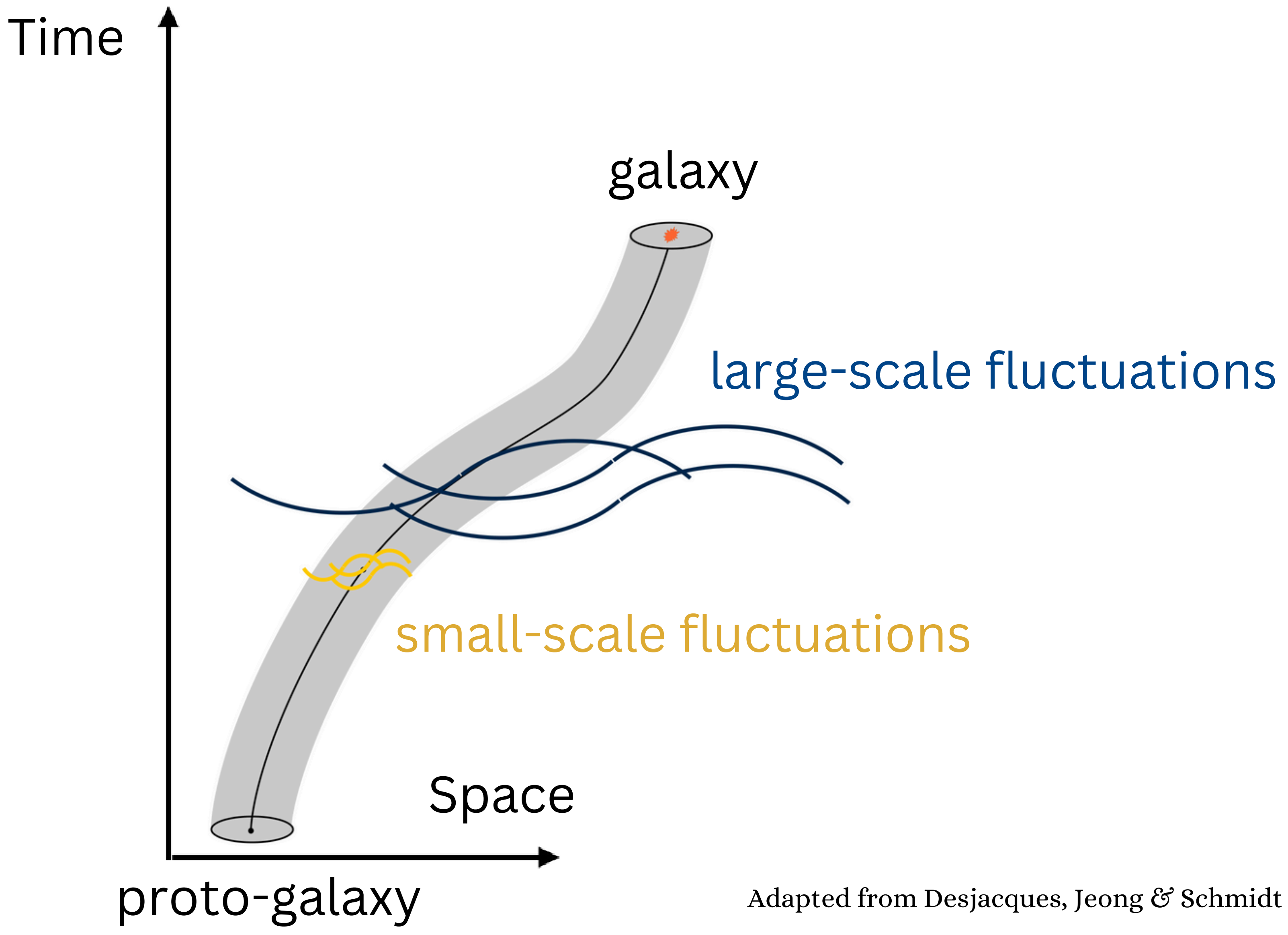
The bias expansion

$$\delta_g(\boldsymbol{k}, z) = \delta_{g,\mathrm{det}}(\boldsymbol{k}, z) + \delta_{g,\mathrm{stoch}}(\boldsymbol{k}, z)$$



$\boldsymbol{\theta}$

Cooray & Sheth (2002)



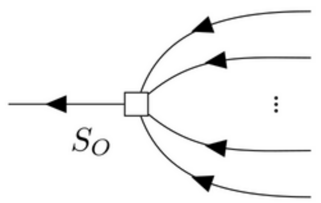
Adapted from Desjacques, Jeong & Schmidt (2016)

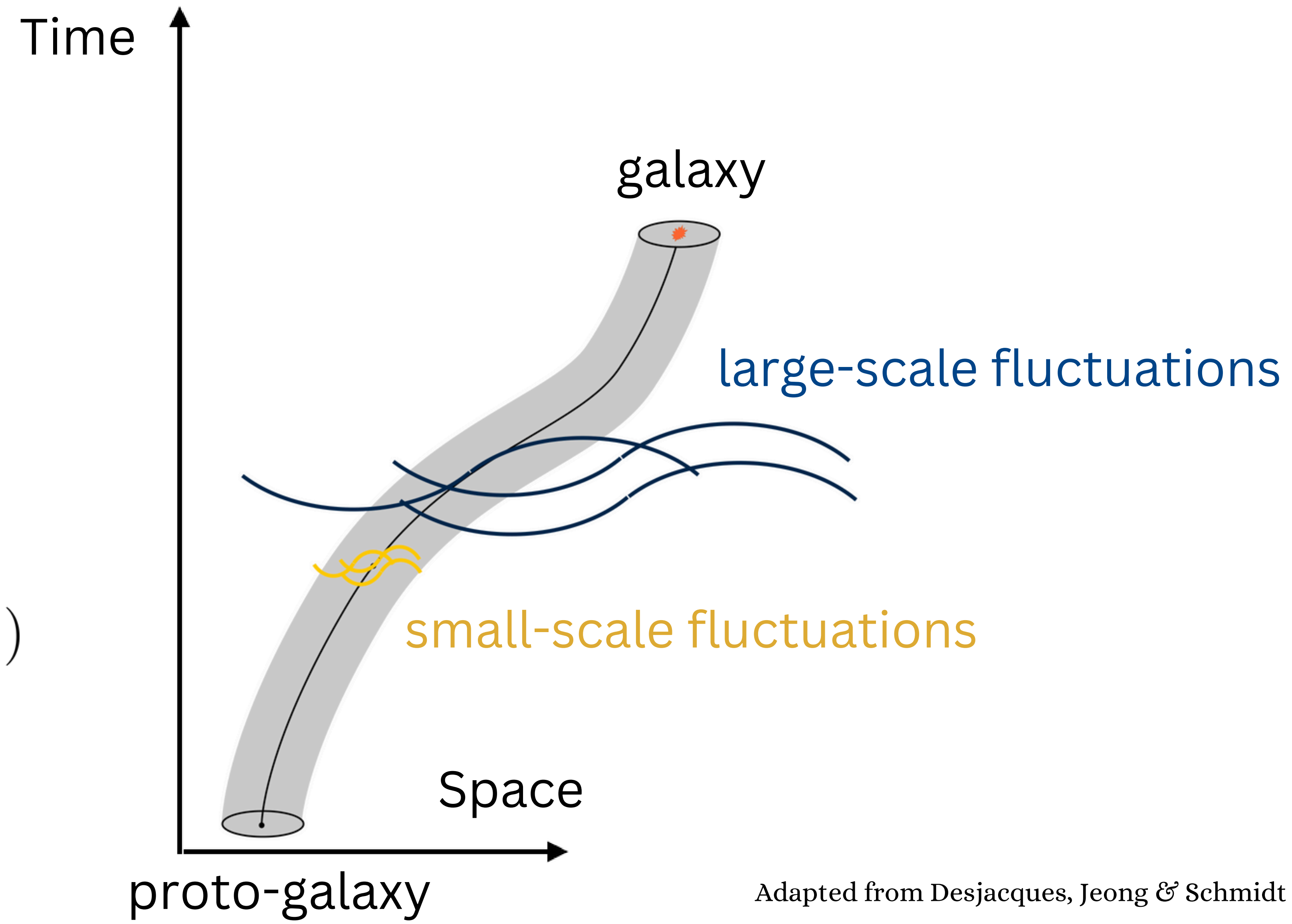
The bias expansion

$$\delta_g(\boldsymbol{k}, z) = \boxed{\delta_{g,\mathrm{det}}(\boldsymbol{k}, z)} + \delta_{g,\mathrm{stoch}}(\boldsymbol{k}, z)$$
$$= \boxed{\sum_O b_O(z) O(\boldsymbol{k}, z)} + \varepsilon(\boldsymbol{k}, z)$$

$\boxed{\{b_O\}}$ **Free bias parameters**

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\mathrm{D}}\left(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}\right) \underbrace{S_O\left(\boldsymbol{p}_1, \ldots \boldsymbol{p}_n\right)}_{\text{operator “convolution”}} \delta\left(\boldsymbol{p}_1\right) \cdots \delta\left(\boldsymbol{p}_n\right)$$





Adapted from Desjacques, Jeong & Schmidt (2016)

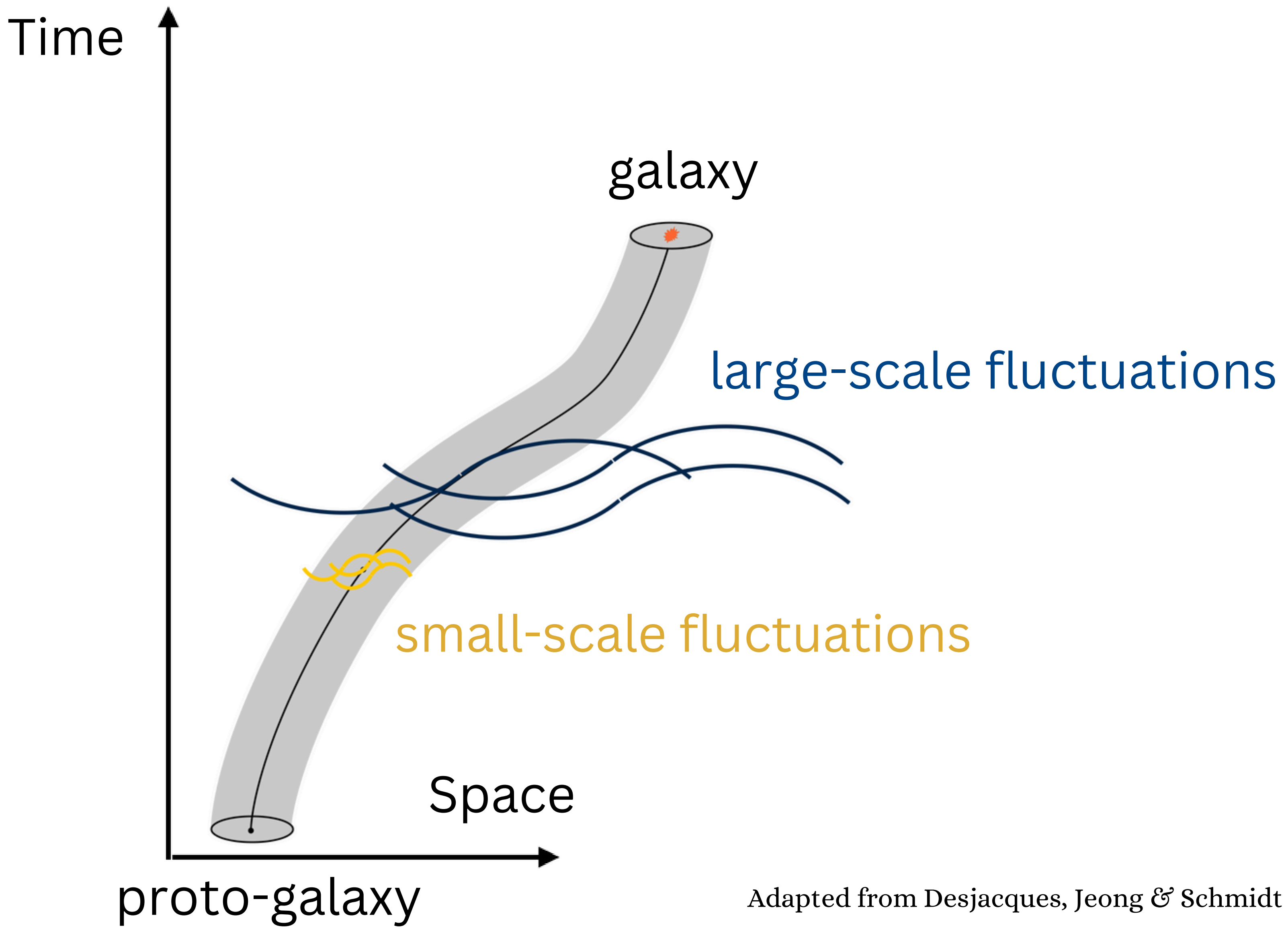
The bias expansion

$$\delta_g(\boldsymbol{k}, z) = \delta_{g,\mathrm{det}}(\boldsymbol{k}, z) + \boxed{\delta_{g,\mathrm{stoch}}(\boldsymbol{k}, z)}$$
$$= \sum_O b_O(z) O(\boldsymbol{k}, z) + \boxed{\varepsilon(\boldsymbol{k}, z)}$$

$$\langle \varepsilon(\boldsymbol{k}, z)\varepsilon(\boldsymbol{k}', z)\rangle' \propto \sigma_\varepsilon^2(k)$$

$$\sigma_\varepsilon(k) = \boxed{\sigma_{\varepsilon,0}}\left[1 + \boxed{\sigma_{\varepsilon,k^2}} k^2\right]$$

Free stochastic parameters $\boxed{\{\sigma_\varepsilon\}}$

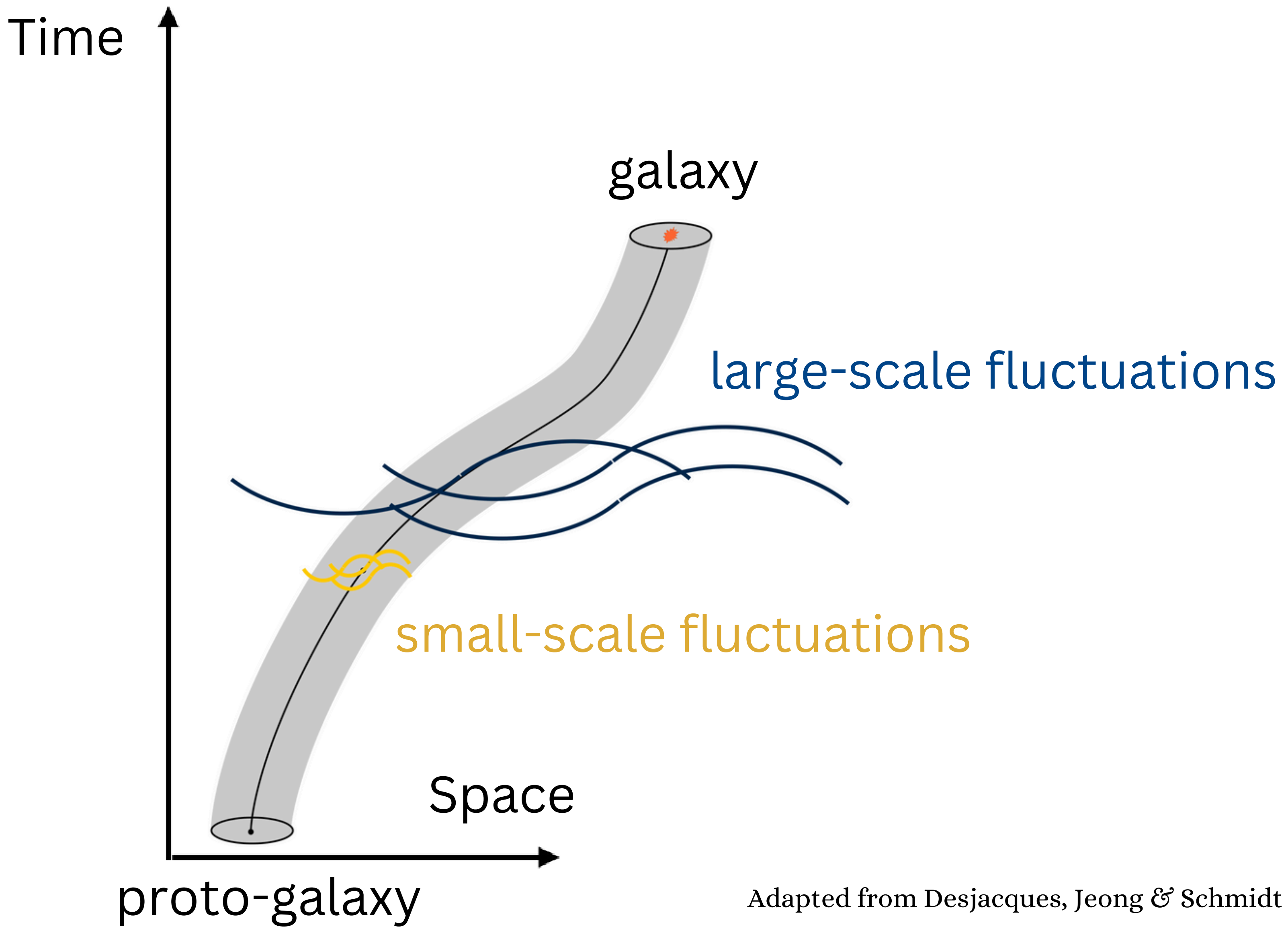


Adapted from Desjacques, Jeong & Schmidt (2016)

The bias expansion

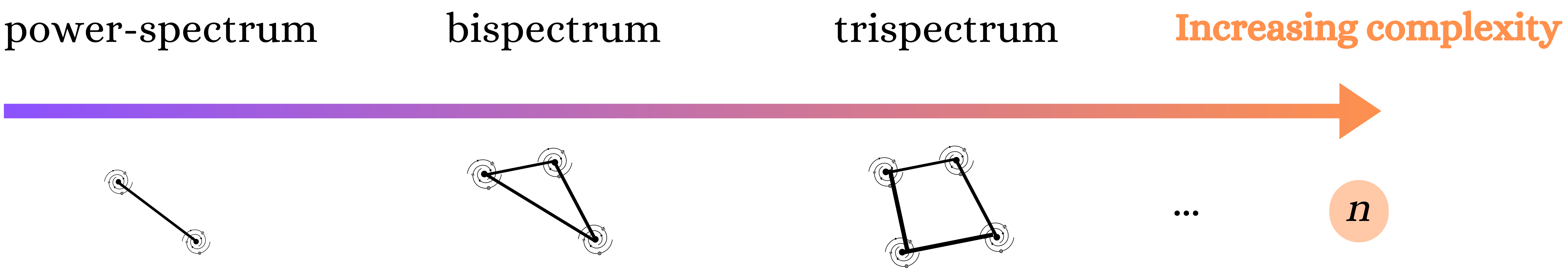
$$\delta_g(\boldsymbol{k}, z) = \delta_{g,\mathrm{det}}(\boldsymbol{k}, z) + \delta_{g,\mathrm{stoch}}(\boldsymbol{k}, z)$$
$$= \sum_O b_O(z) O(\boldsymbol{k}, z) + \varepsilon(\boldsymbol{k}, z)$$

$$\boxed{\{\boldsymbol{\theta}, \{b_O\}, \{\sigma_\varepsilon\}\}}$$



Adapted from Desjacques, Jeong & Schmidt (2016)

Inferring the cosmological parameters: **challenges**



Inferring the cosmological parameters: **challenges**

power-spectrum bispectrum trispectrum **Increasing complexity**

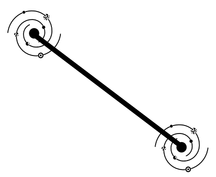
$D = 18$

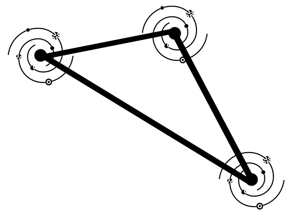
$D = 714$

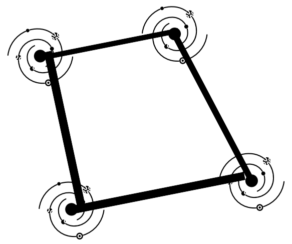
$$k_{\max} = 0.12h\,\mathrm{Mpc}^{-1}$$
$$\Delta k = 2k_f$$
$$L = 2000h^{-1}\mathrm{Mpc}$$

Inferring the cosmological parameters: **challenges**

power-spectrum | bispectrum | trispectrum | **Increasing complexity**







... n

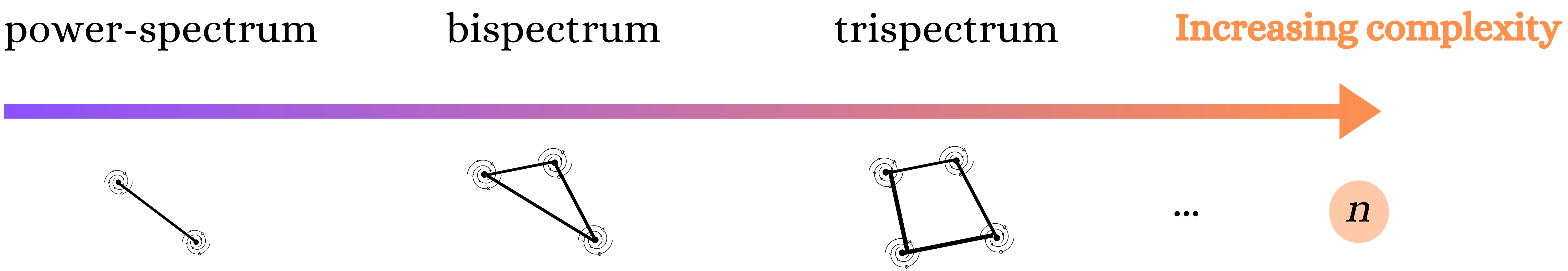
$D = 18$ | $D = 714$ | $D = 15\,093$

$$k_{\max} = 0.12h\,\mathrm{Mpc}^{-1}$$

$$\Delta k = 2k_f$$

$$L = 2000h^{-1}\mathrm{Mpc}$$

Inferring the cosmological parameters: **challenges**



Analytical approximations

Estimation

Modelling

$$-2\ln\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \big(\mathbf{D}-\mathbf{T}(\boldsymbol{\theta})\big)\cdot\mathbf{C}^{-1}\cdot\big(\mathbf{D}-\mathbf{T}(\boldsymbol{\theta})\big)$$

Measurements

Part I

Simulation-based inference

(SBI)

SBI: the main idea

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \cancel{\mathcal{L}(\mathbf{D}|\boldsymbol{\theta})}\pi(\boldsymbol{\theta})$$

$$\boxed{\mathbf{T}(\boldsymbol{\theta}) \sim \mathrm{simulator}(\boldsymbol{\theta})}$$

Explicit likelihood → implicit likelihood

SBI: the main idea

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \cancel{\mathcal{L}(\mathbf{D}|\boldsymbol{\theta})}\pi(\boldsymbol{\theta})$$

$$\boxed{\mathbf{T}(\boldsymbol{\theta}) \sim \mathrm{simulator}(\boldsymbol{\theta})}$$

Explicit likelihood → implicit likelihood

- ✓ Higher-order moments of the distribution beyond the covariance
- ✓ Parameter-dependent covariance
- ✓ Use data with non-tractable likelihood

Cosmological inference with SBI

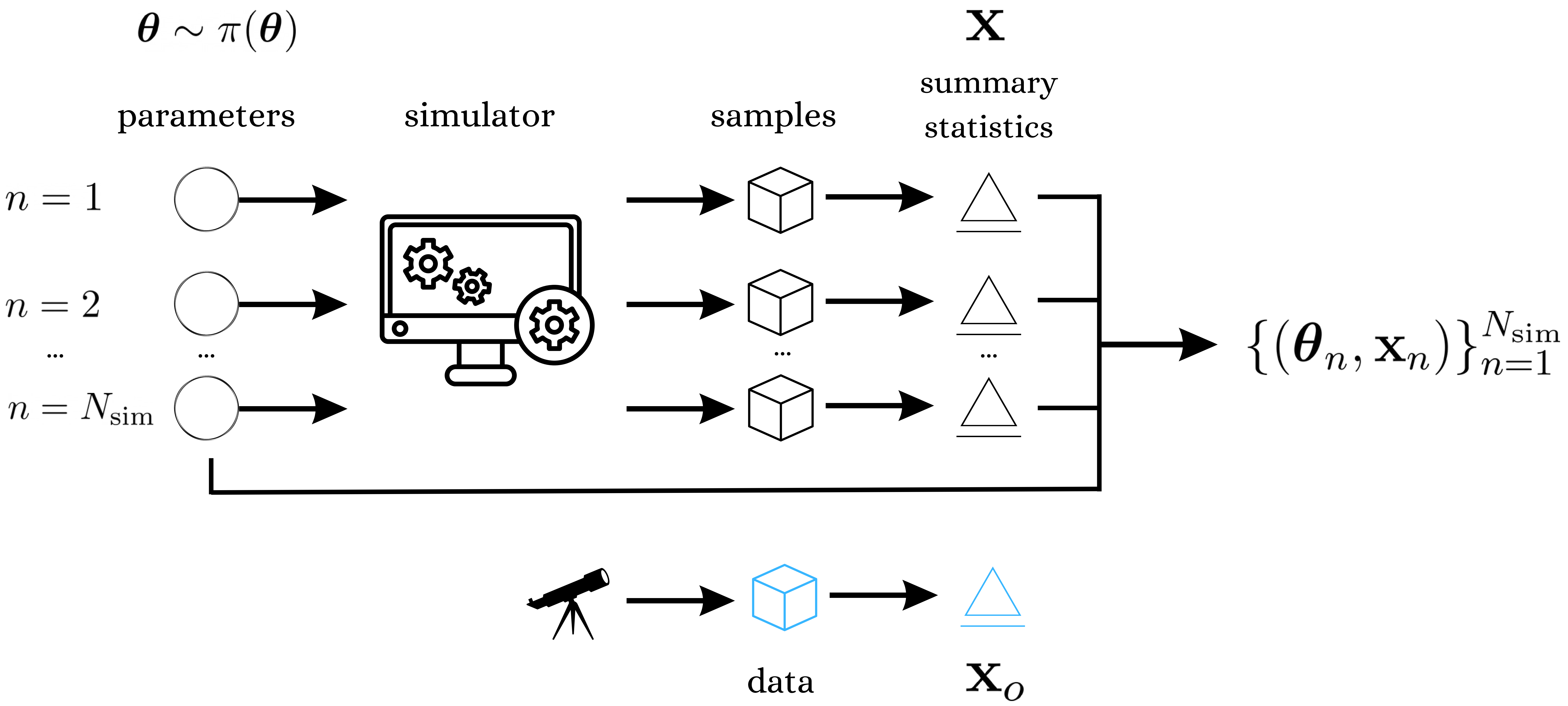
$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{D}) \propto \mathcal{L}\cancel{(\mathbf{D}|\boldsymbol{\theta})}\pi(\boldsymbol{\theta})$$

$$\boxed{\mathbf{T}(\boldsymbol{\theta}) \sim \text{simulator}(\boldsymbol{\theta})}$$

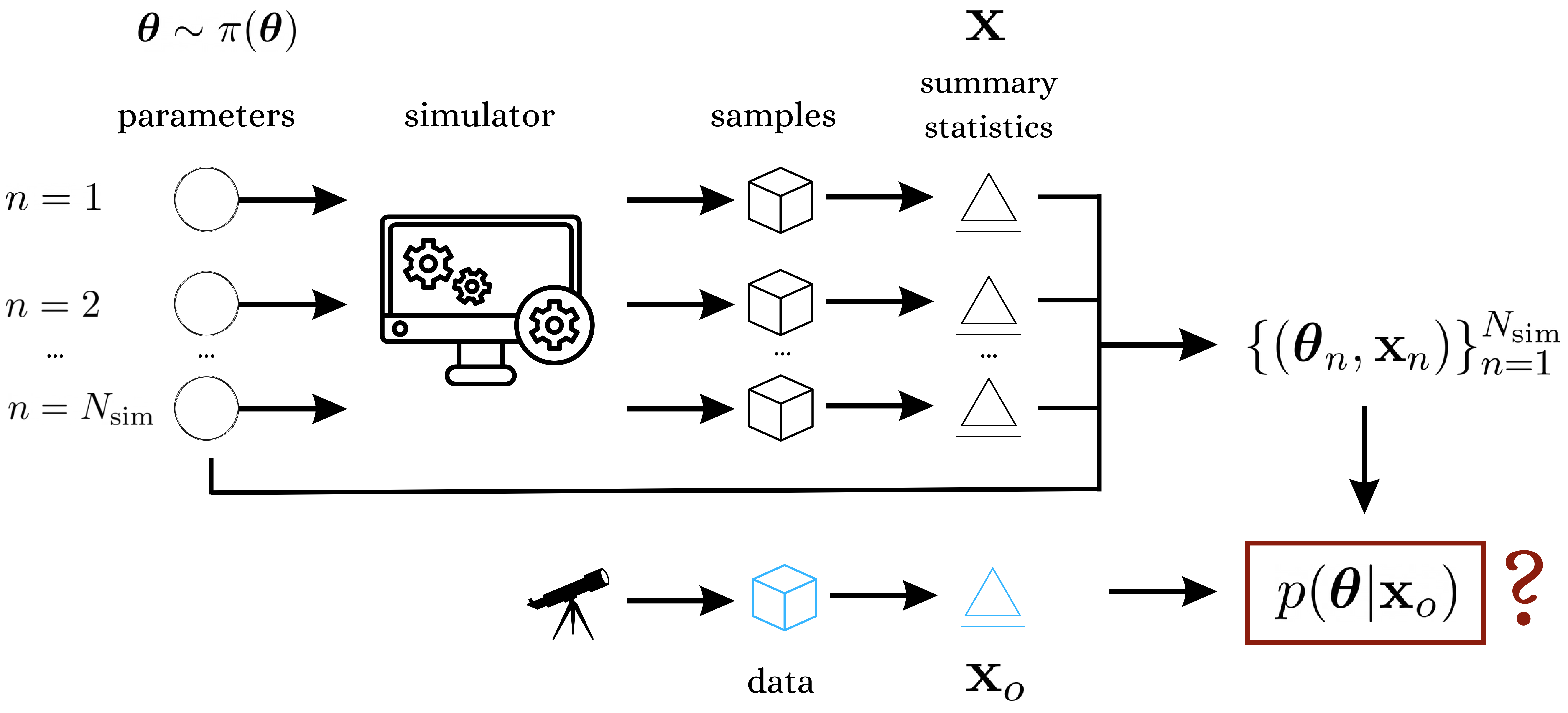
Explicit likelihood → implicit likelihood

- ✓ We are free to choose more informative summary statistics, independently of their distribution
- ✓ No need of cumbersome covariance calculations
- ✓ Modelling of observational systematics and survey mask at the simulation level

Simulation-based inference

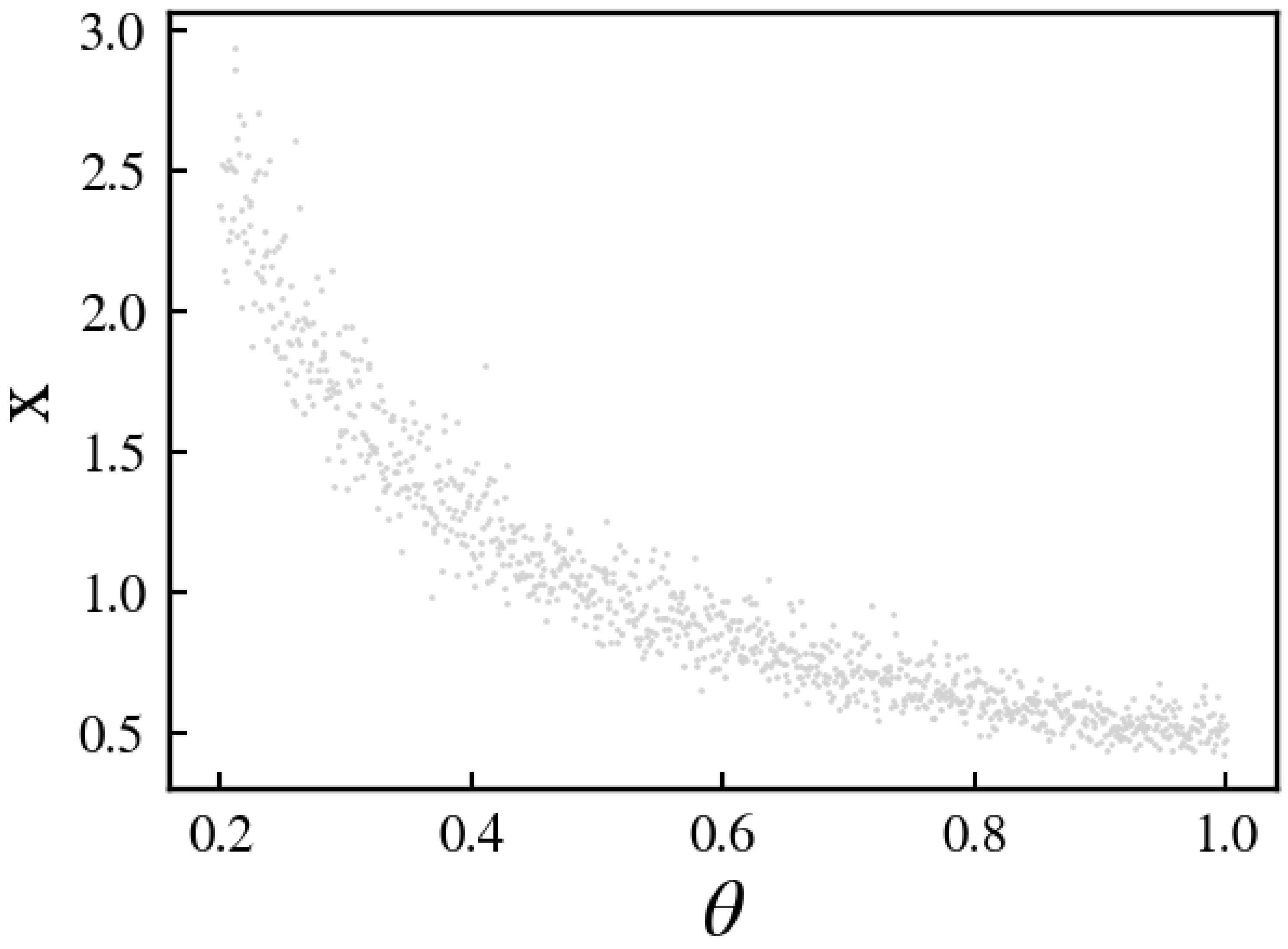


Simulation-based inference

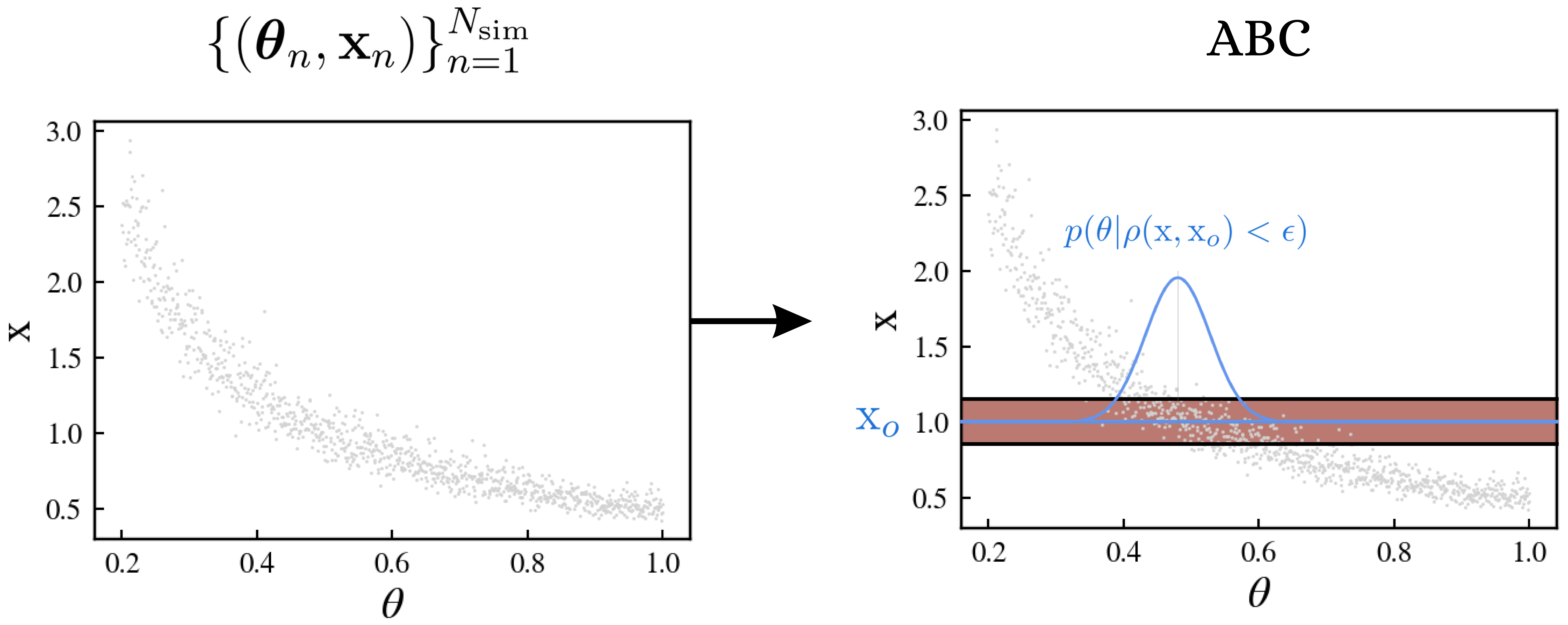


Simulation-based inference

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$$



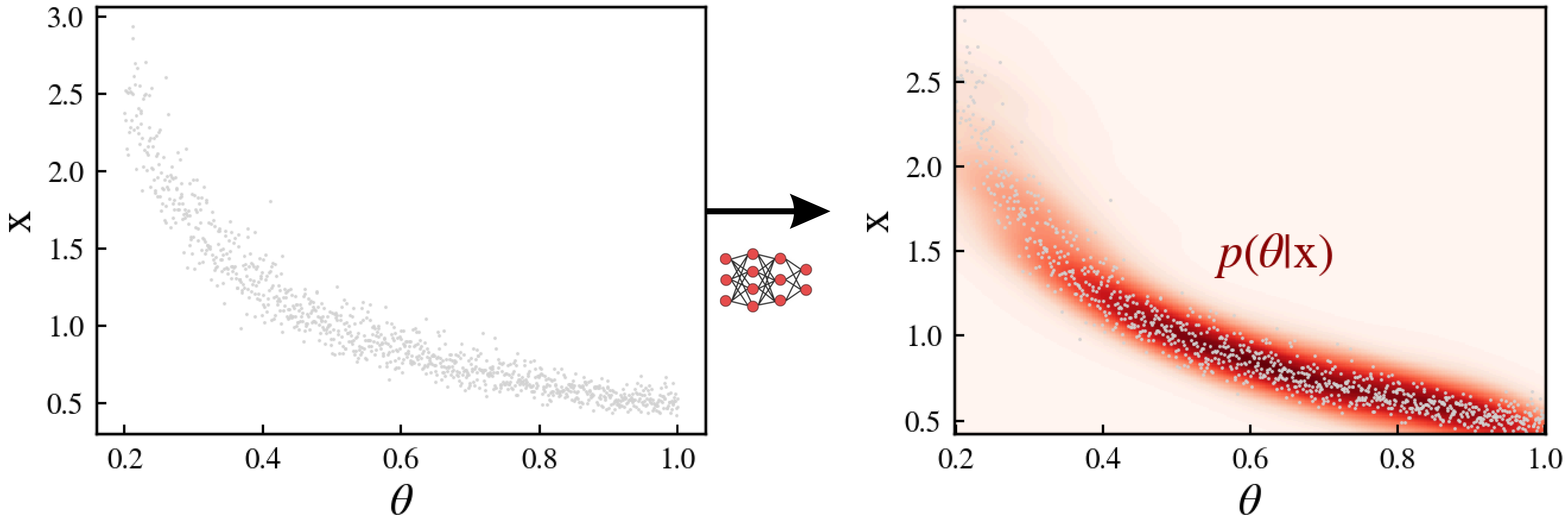
Approximate Bayesian Computation (ABC)



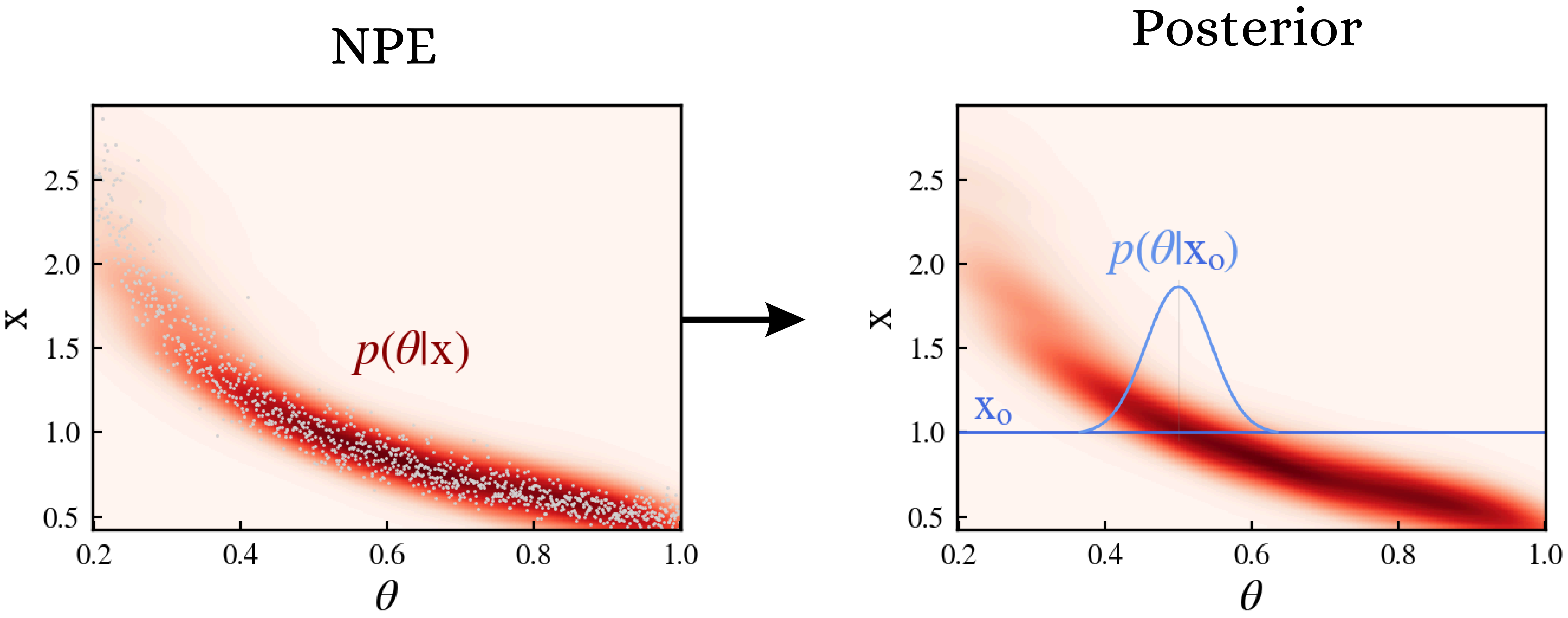
Neural Posterior Estimation (NPE)

$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$

NPE

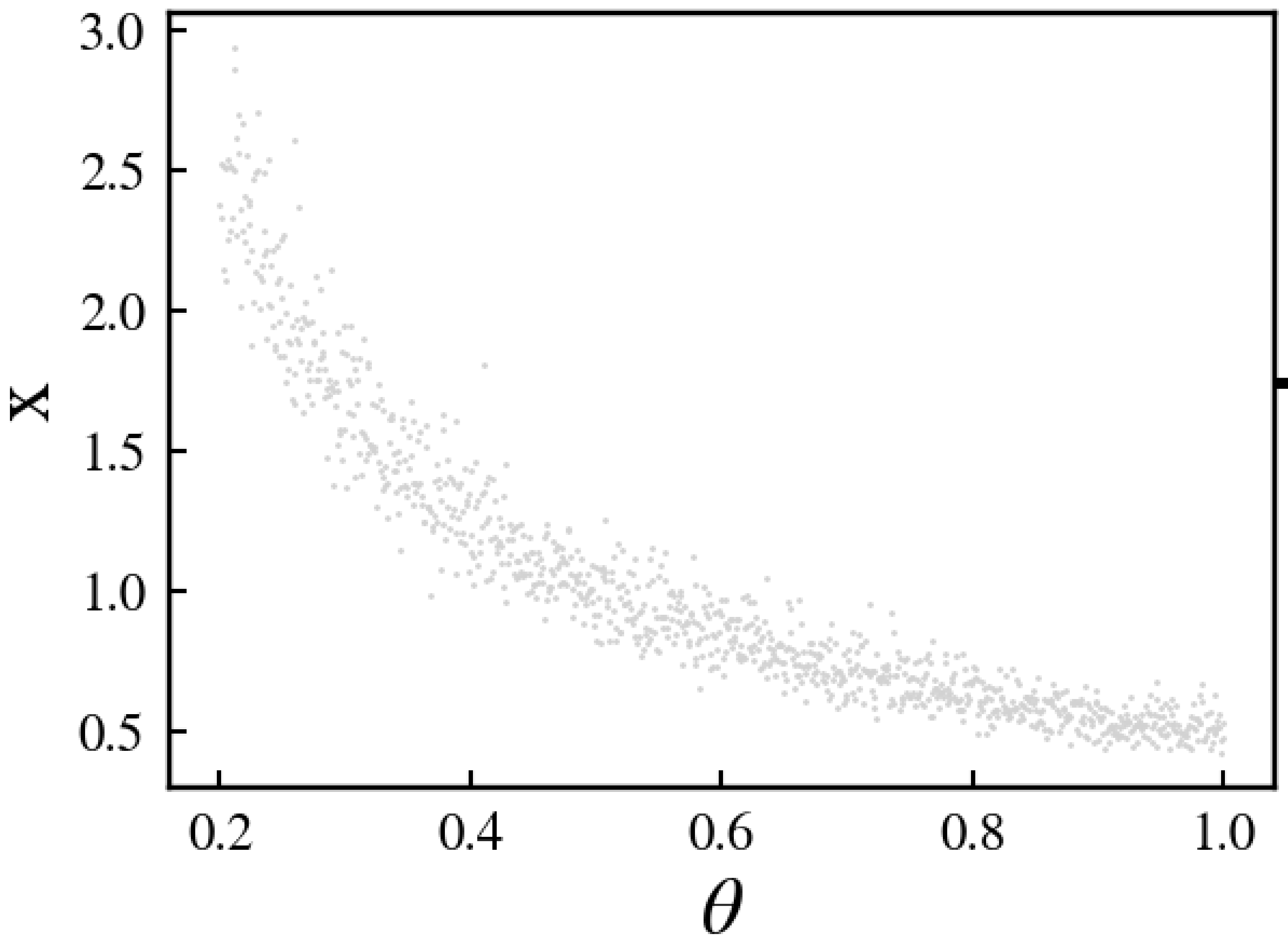


Neural Posterior Estimation (NPE)



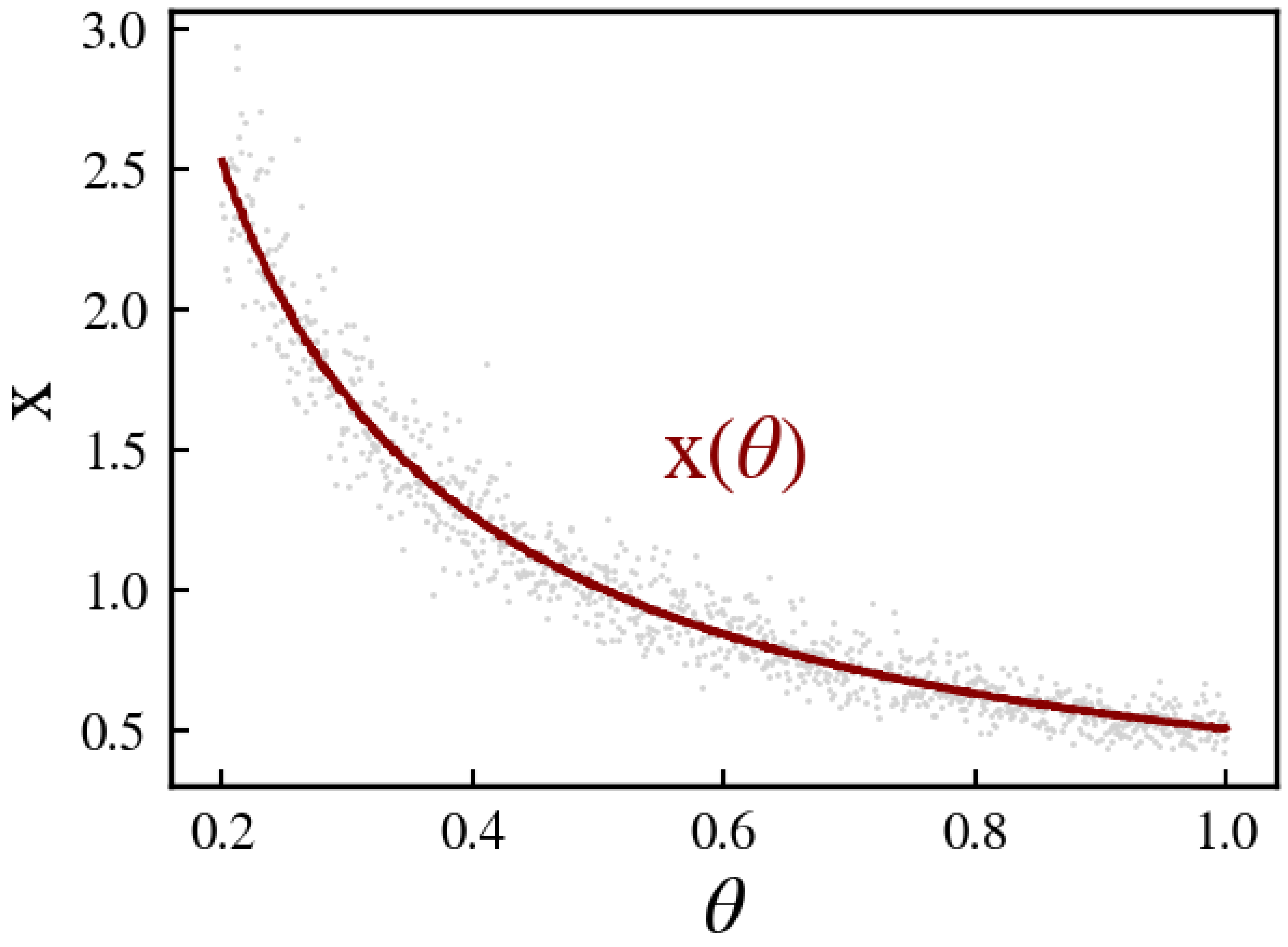
A parenthesis on emulators

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\rm sim}}$$



Explicit likelihood inference

$$\ln \mathcal{L} \propto \frac{(\mathrm{x}_o - \mathrm{x}(\theta))^2}{\mathrm{Cov}[\mathrm{x}]}$$



Summary statistics emulators

Neural Density Estimators

How do NDEs work in practice?

Neural Density Estimation

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$$

Neural Posterior Estimation (NPE)

$$q_\phi(\boldsymbol{\theta}|\mathbf{x}) \longrightarrow \hat{p}(\boldsymbol{\theta}|\mathbf{x}_o) = q_\phi(\boldsymbol{\theta}|\mathbf{x}_o)$$

Neural Likelihood Estimation (NLE)

$$q_\phi(\mathbf{x}|\boldsymbol{\theta}) \longrightarrow \hat{p}(\boldsymbol{\theta}|\mathbf{x}_o) \propto q_\phi(\mathbf{x}_o|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

Neural Density Estimation

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\mathrm{sim}}}$$

Neural Posterior Estimation (NPE)

$$q_{\phi}(\boldsymbol{\theta}|\mathbf{x}) \longrightarrow \hat{p}(\boldsymbol{\theta}|\mathbf{x}_o) = q_{\phi}(\boldsymbol{\theta}|\mathbf{x}_o)$$

Neural Likelihood Estimation (NLE)

$$q_{\phi}(\mathbf{x}|\boldsymbol{\theta}) \longrightarrow \hat{p}(\boldsymbol{\theta}|\mathbf{x}_o) \propto q_{\phi}(\mathbf{x}_o|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

neural network
trainable parameters

Neural Density Estimation

How to train the model? (For example, NLE)

$$\mathcal{L}_{\text{loss}} = \mathbb{E}_{p(\boldsymbol{\theta})}\left[D_{\text{KL}}\left[p(\mathbf{x}|\boldsymbol{\theta}) \,||\, q_{\phi}(\mathbf{x}|\boldsymbol{\theta})\right]\right]$$

loss function

target density

?

neural network
trainable parameters

Neural Density Estimation

How to train the model? (For example, NLE)

$$\mathbb{E}_{p(\boldsymbol{\theta})}\left[D_{\mathrm{KL}}\left[\underbrace{p(\mathbf{x}|\boldsymbol{\theta})}_{\text{target density}} \,||\, q_{\underbrace{\phi}_{\text{neural network trainable parameters}}}(\mathbf{x}|\boldsymbol{\theta})\right]\right] = \int d\boldsymbol{\theta}\, p(\boldsymbol{\theta}) \int d\mathbf{x}\, p(\mathbf{x}|\boldsymbol{\theta}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_{\phi}(\mathbf{x}|\boldsymbol{\theta})}\right)$$

?

Neural Density Estimation

How to train the model? (For example, NLE)

$$\mathbb{E}_{p(\boldsymbol{\theta})}\left[D_{\mathrm{KL}}\left[\underbrace{p(\mathbf{x}|\boldsymbol{\theta})}_{\text{target density}} \,||\, \underbrace{q_{\phi}}_{\substack{\text{neural network}\\ \text{trainable parameters}}}(\mathbf{x}|\boldsymbol{\theta})\right]\right] = \int d\boldsymbol{\theta}\, p(\boldsymbol{\theta}) \int d\mathbf{x}\, p(\mathbf{x}|\boldsymbol{\theta}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_{\phi}(\mathbf{x}|\boldsymbol{\theta})}\right)$$

$$= \int d\boldsymbol{\theta}\, d\mathbf{x}\, p(\boldsymbol{\theta}, \mathbf{x}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_{\phi}(\mathbf{x}|\boldsymbol{\theta})}\right)$$

?

$$\boxed{p(\boldsymbol{\theta}, \mathbf{x}) = p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})}$$

Neural Density Estimation

How to train the model? (For example, NLE)

$$\mathbb{E}_{p(\boldsymbol{\theta})}\left[D_{\mathrm{KL}}\left[\underbrace{p(\mathbf{x}|\boldsymbol{\theta})}_{\text{target density}} \,||\, q_{\underbrace{\phi}_{\text{neural network trainable parameters}}}(\mathbf{x}|\boldsymbol{\theta})\right]\right] = \int d\boldsymbol{\theta}\, p(\boldsymbol{\theta}) \int d\mathbf{x}\, p(\mathbf{x}|\boldsymbol{\theta}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_\phi(\mathbf{x}|\boldsymbol{\theta})}\right)$$

$$= \int d\boldsymbol{\theta}\, d\mathbf{x}\, p(\boldsymbol{\theta}, \mathbf{x}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_\phi(\mathbf{x}|\boldsymbol{\theta})}\right)$$

$$= -\mathbb{E}_{p(\boldsymbol{\theta},\mathbf{x})}\left[\log q_\phi(\mathbf{x}|\boldsymbol{\theta})\right] + \text{const.}$$

?

the loss function we wish to minimize is independent of the target density form!

Neural Density Estimation

How to train the model? (For example, NLE)

$$\mathbb{E}_{p(\boldsymbol{\theta})}\left[D_{\mathrm{KL}}\left[\underbrace{p(\mathbf{x}|\boldsymbol{\theta})}_{\text{target density}} \,||\, q_{\underbrace{\phi}_{\text{neural network trainable parameters}}}(\mathbf{x}|\boldsymbol{\theta})\right]\right] = \int d\boldsymbol{\theta}\, p(\boldsymbol{\theta}) \int d\mathbf{x}\, p(\mathbf{x}|\boldsymbol{\theta}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_\phi(\mathbf{x}|\boldsymbol{\theta})}\right)$$

$$= \int d\boldsymbol{\theta}\, d\mathbf{x}\, p(\boldsymbol{\theta}, \mathbf{x}) \log\left(\frac{p(\mathbf{x}|\boldsymbol{\theta})}{q_\phi(\mathbf{x}|\boldsymbol{\theta})}\right)$$

$$= -\mathbb{E}_{p(\boldsymbol{\theta},\mathbf{x})}\left[\log q_\phi(\mathbf{x}|\boldsymbol{\theta})\right] + \text{const.}$$

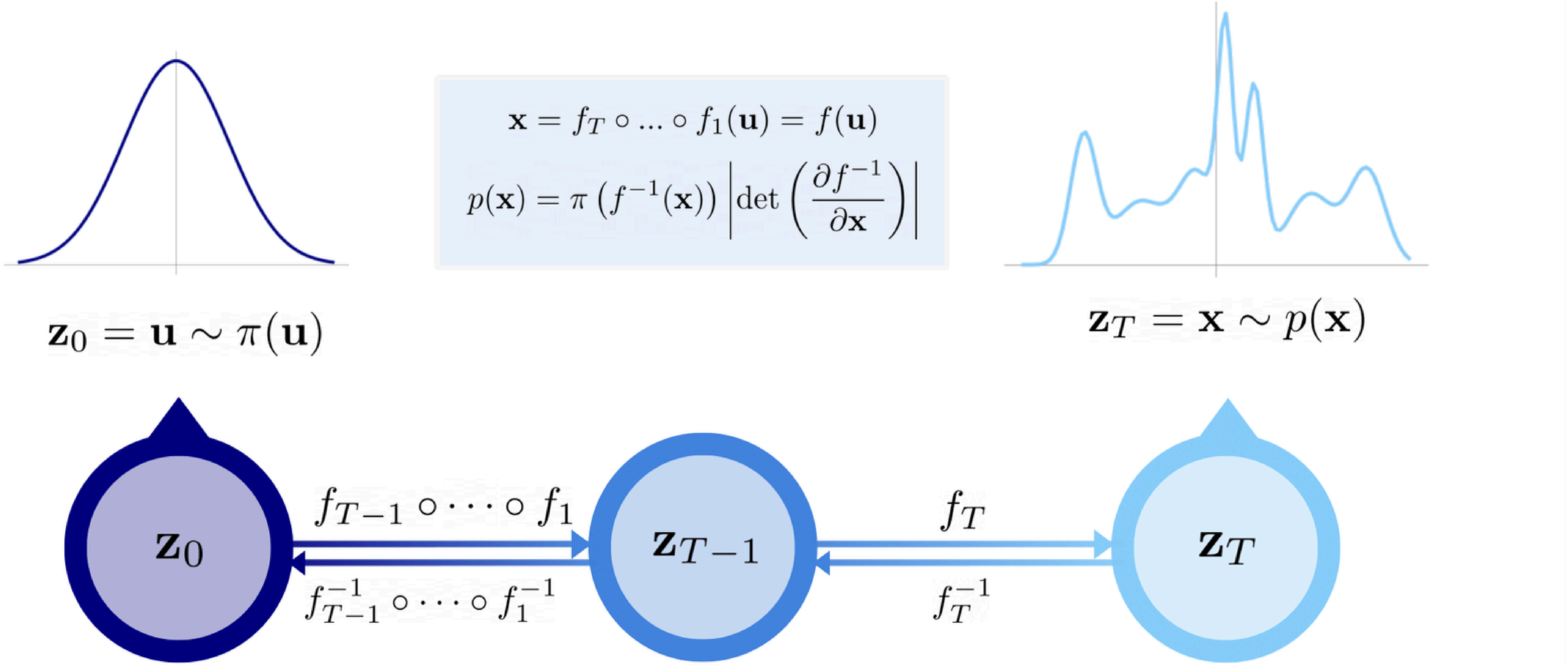
$$\approx -\frac{1}{N_{\text{sim}}} \sum_{n=1}^{N_{\text{sim}}} \log q_\phi(\mathbf{x}_n|\boldsymbol{\theta}_n) + \text{const.}\,,$$

$$\boxed{\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}}$$

Normalizing Flows

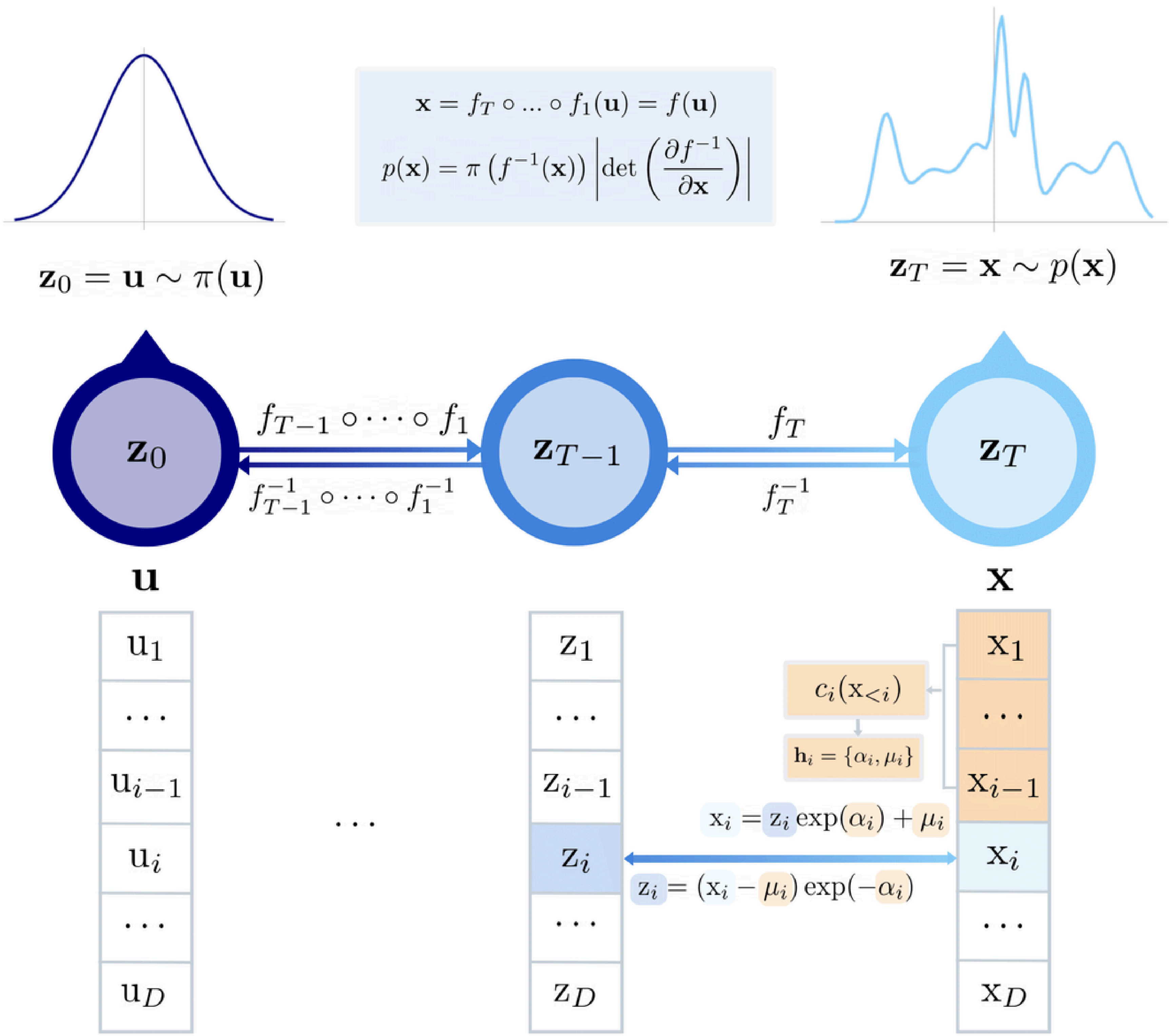
f is a *diffeomorphism*:

- f is invertible
- f and its inverse are differentiable



Tucci & Schmidt (2023)

Normalizing Flows

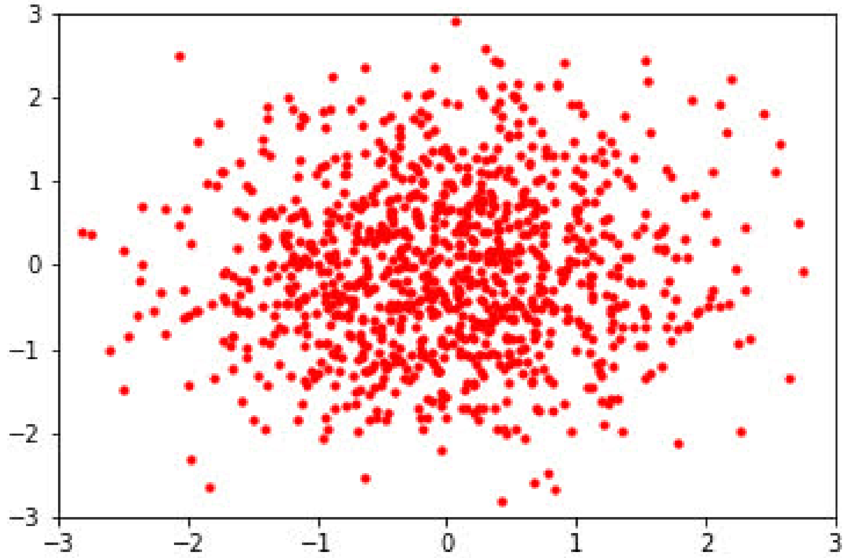


Tucci & Schmidt (2023)

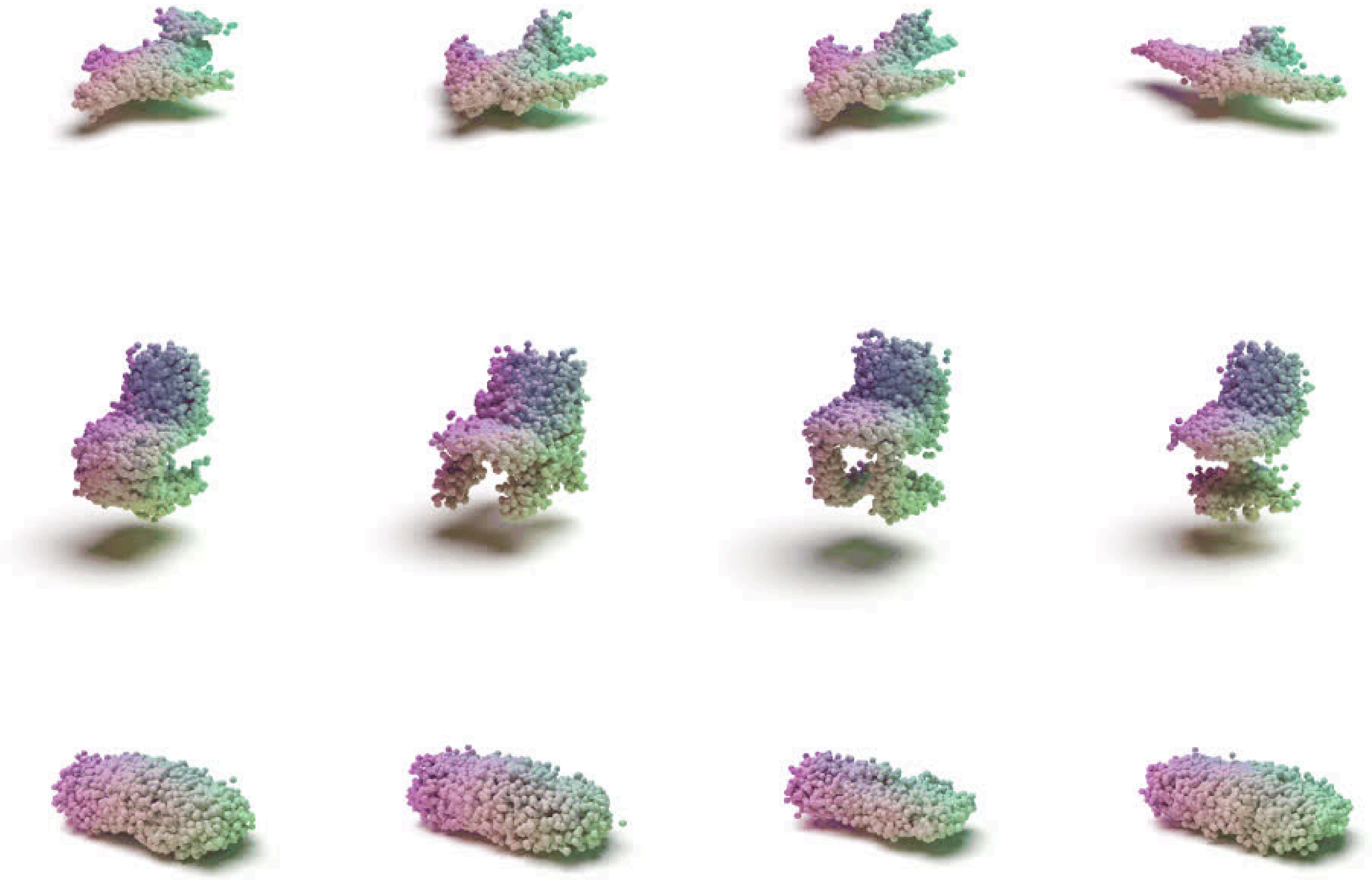
$$\boxed{q_\phi(\mathbf{x}|\boldsymbol{\theta}) = \mathcal{N}(\mathbf{z}_0|\mathbf{0}, \mathbf{I}) \prod_{t=1}^{T} \left|\det\left(\frac{\partial f_t}{\partial \mathbf{z}_{t-1}}\right)\right|^{-1}}$$

- Easy to evaluate: triangular Jacobian
- Expressive: composition of transforms

Normalizing Flows

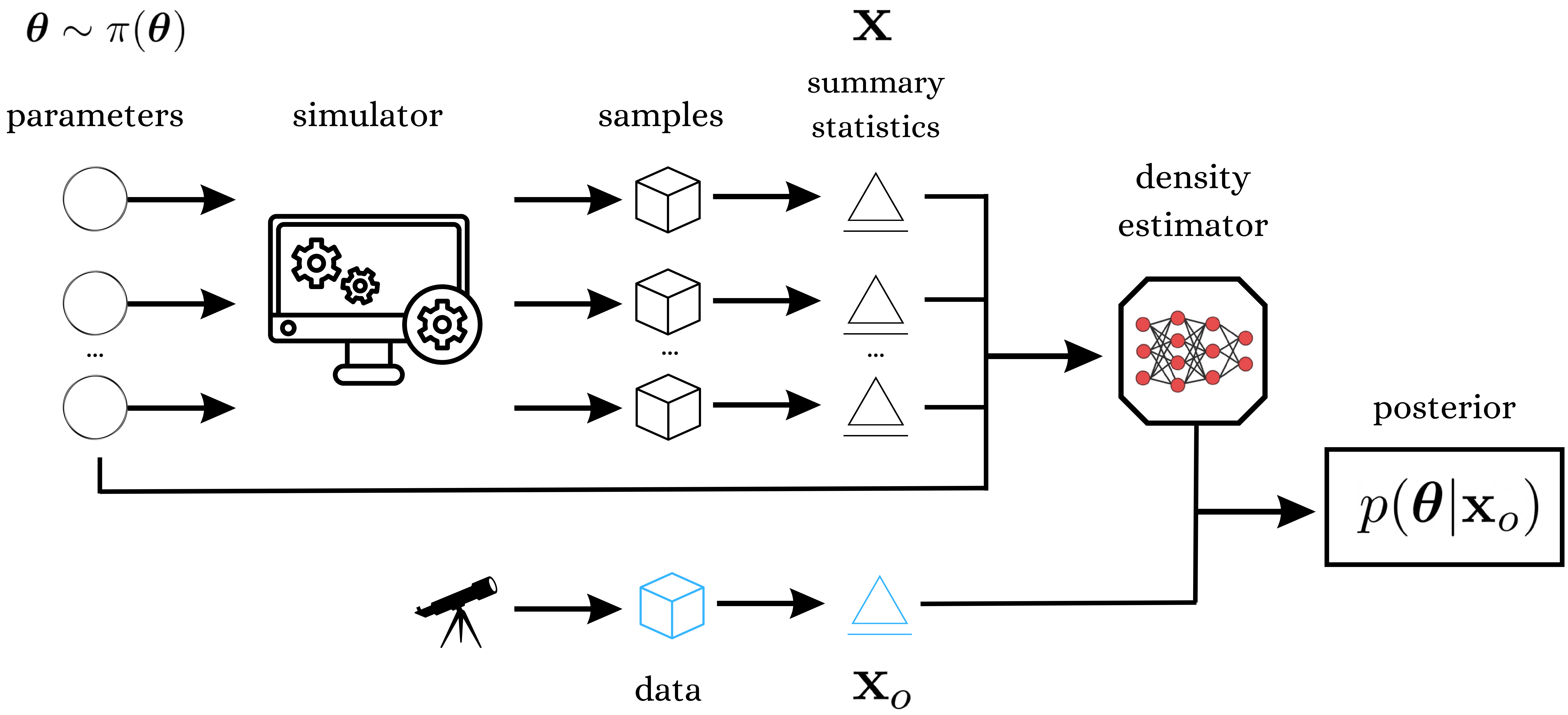


Credits: Eric Jang



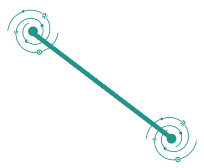
Credits: [PointFlow]() (Yang+19)

Simulation-based inference

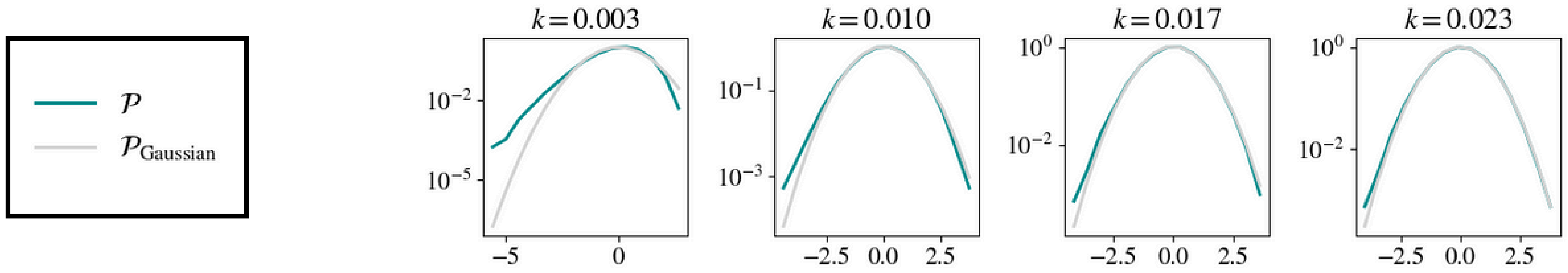


On the Gaussianity assumption of the n-point functions

power-spectrum

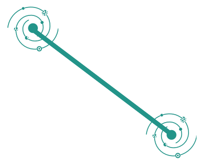


large scales

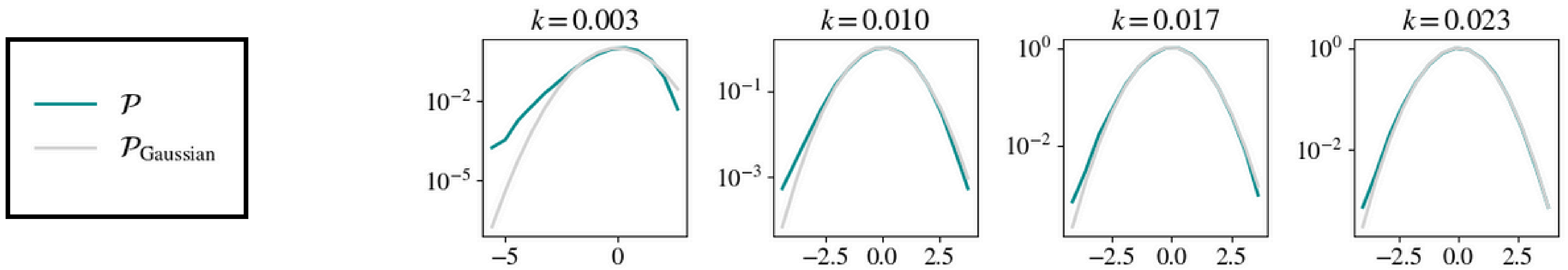


On the Gaussianity assumption of the n-point functions

power-spectrum



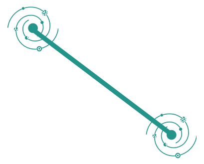
large scales

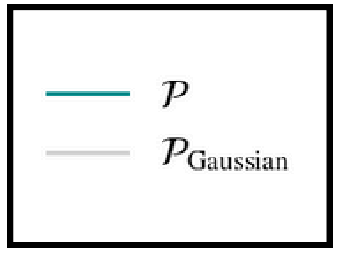


breaking of central limit theorem on large scales induces
deviation from the Gaussian likelihood assumption!

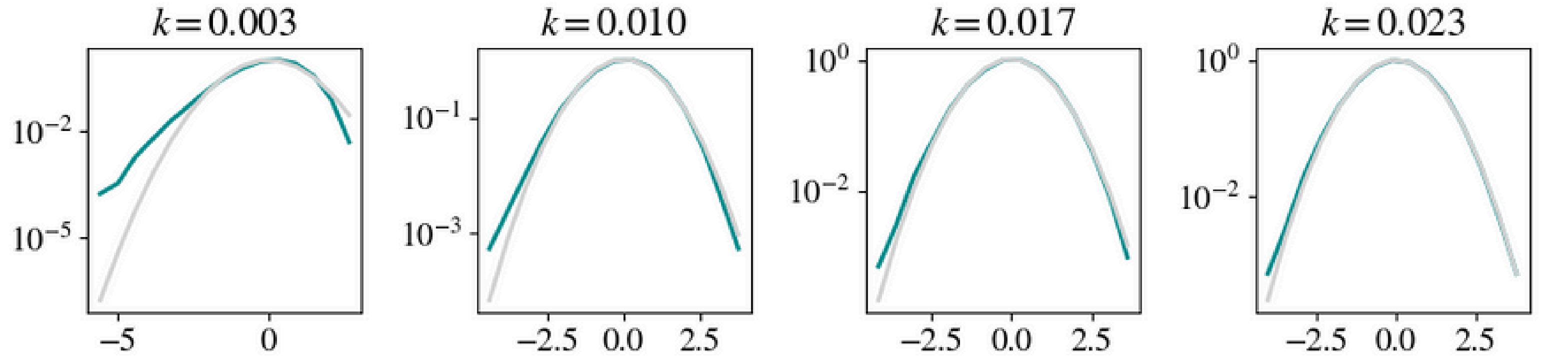
On the Gaussianity assumption of the n-point functions

power-spectrum





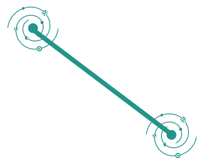
large scales

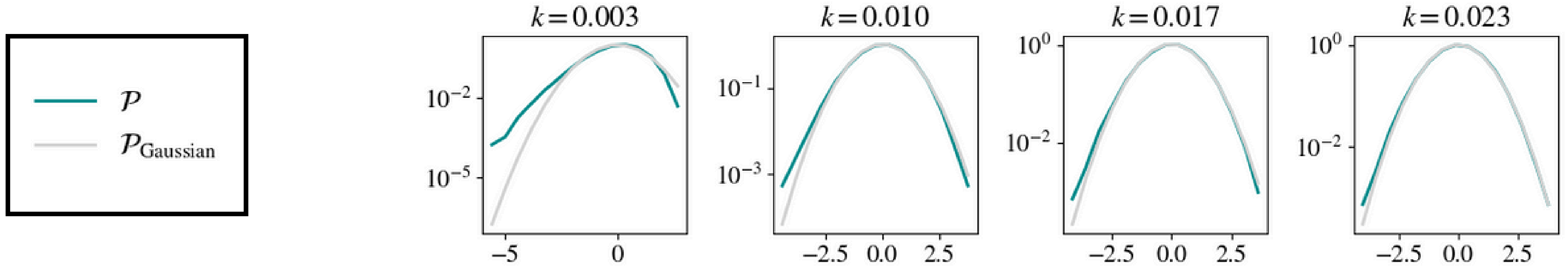


breaking of central limit theorem on large scales induces deviation from the Gaussian likelihood assumption!

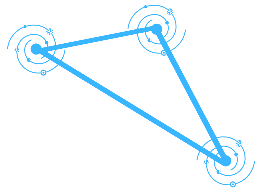
On the Gaussianity assumption of the n-point functions

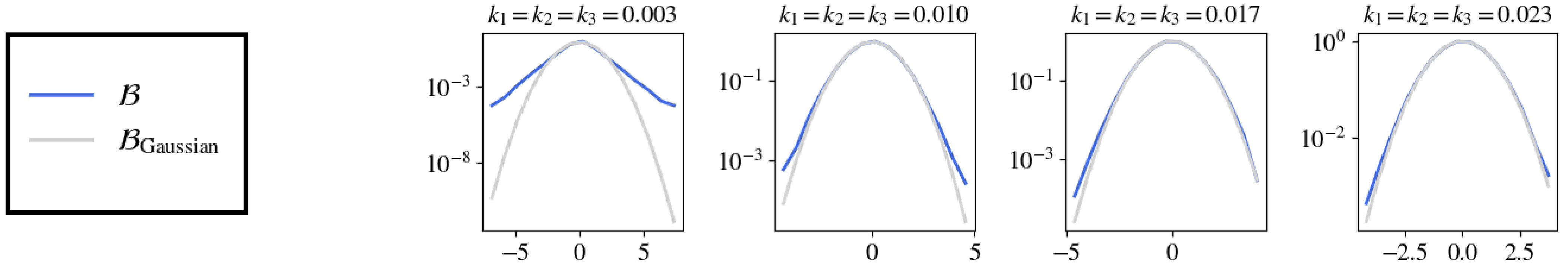
power-spectrum



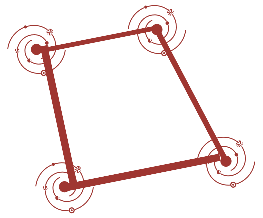


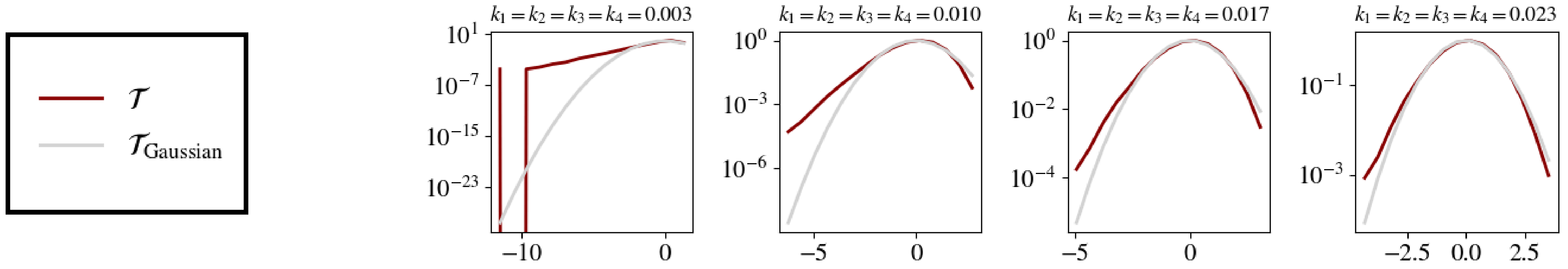
bispectrum





trispectrum





LEFTfield

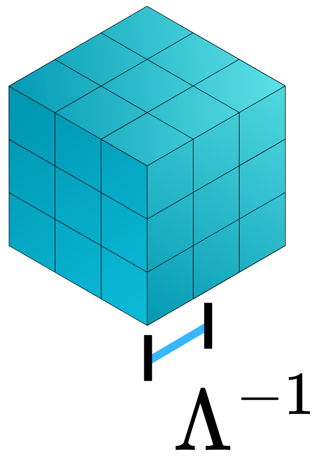
The forward model

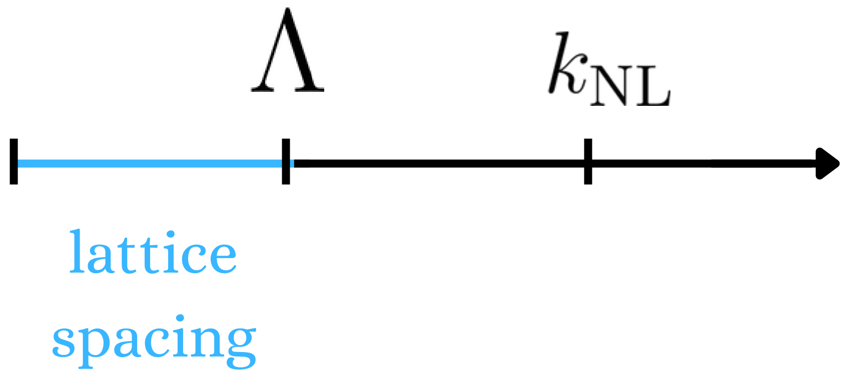
based on the EFTofLSS & the bias expansion

The forward model

$$\boldsymbol{\theta} \sim \mathcal{P}(\boldsymbol{\theta})$$

$$\delta_\Lambda^{(1)} \sim \mathcal{N}(0, P_L\boxed{(\boldsymbol{\theta})})$$

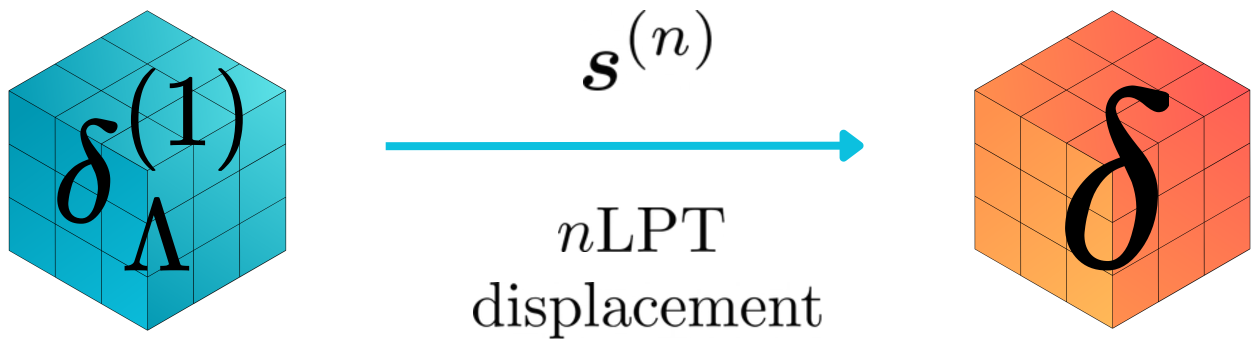




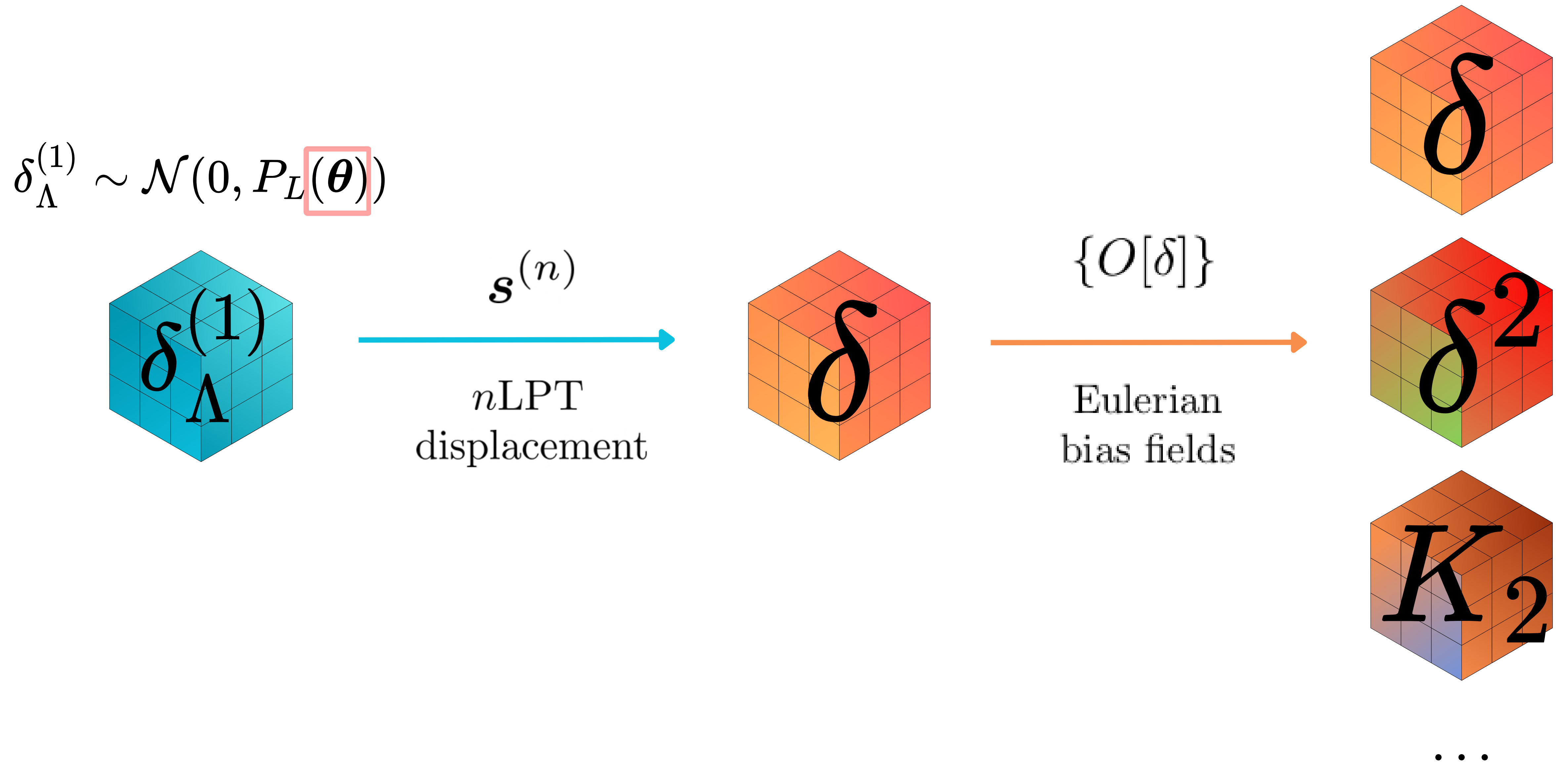
The forward model

$$\boldsymbol{\theta} \sim \mathcal{P}(\boldsymbol{\theta})$$

$$\delta_\Lambda^{(1)} \sim \mathcal{N}(0, P_L\boxed{(\boldsymbol{\theta})})$$

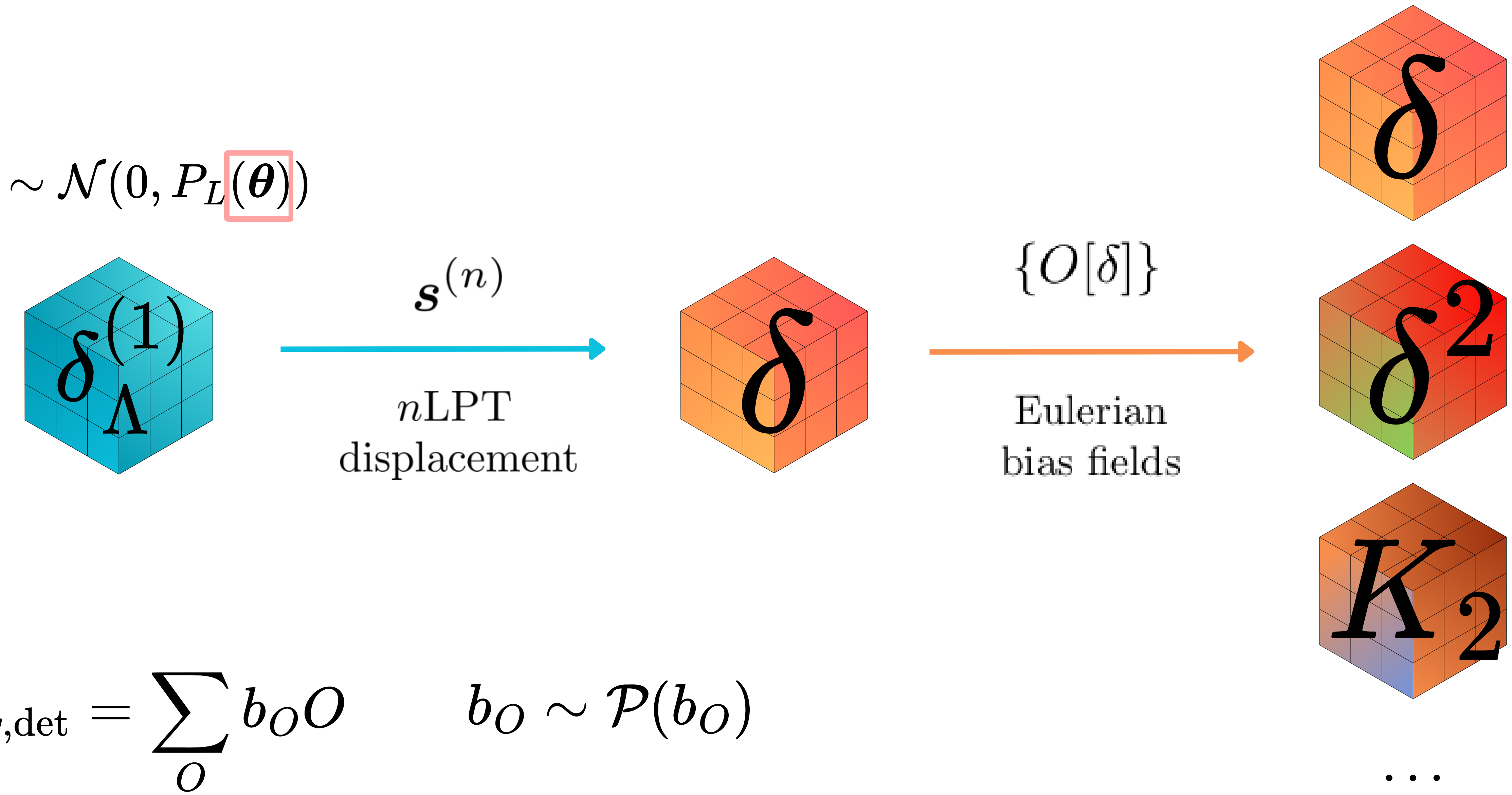


The forward model



The forward model

$$\delta_\Lambda^{(1)} \sim \mathcal{N}(0, P_L(\boxed{\boldsymbol{\theta}}))$$

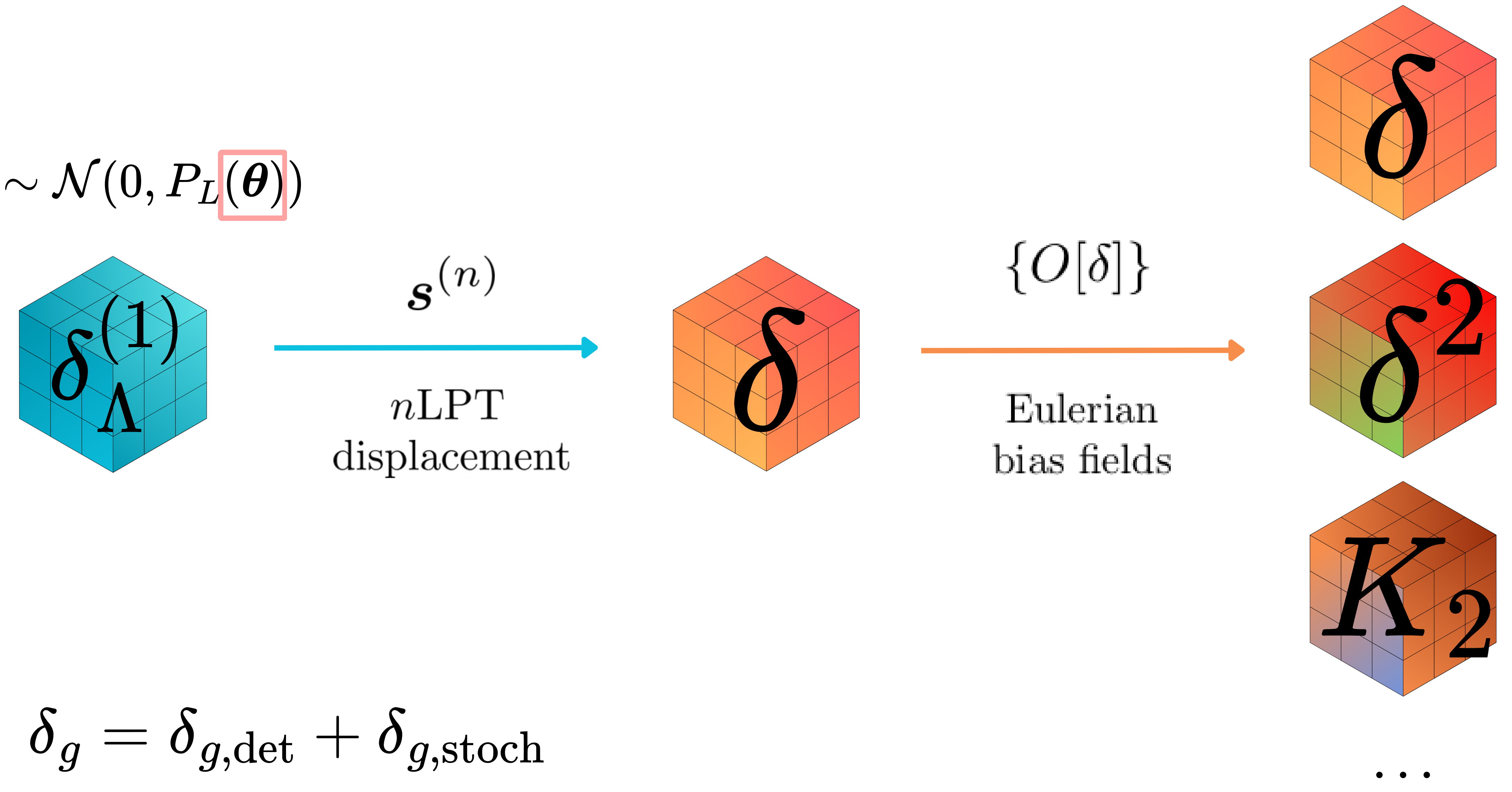


$$\delta_{g,\mathrm{det}} = \sum_O b_O O \qquad b_O \sim \mathcal{P}(b_O)$$

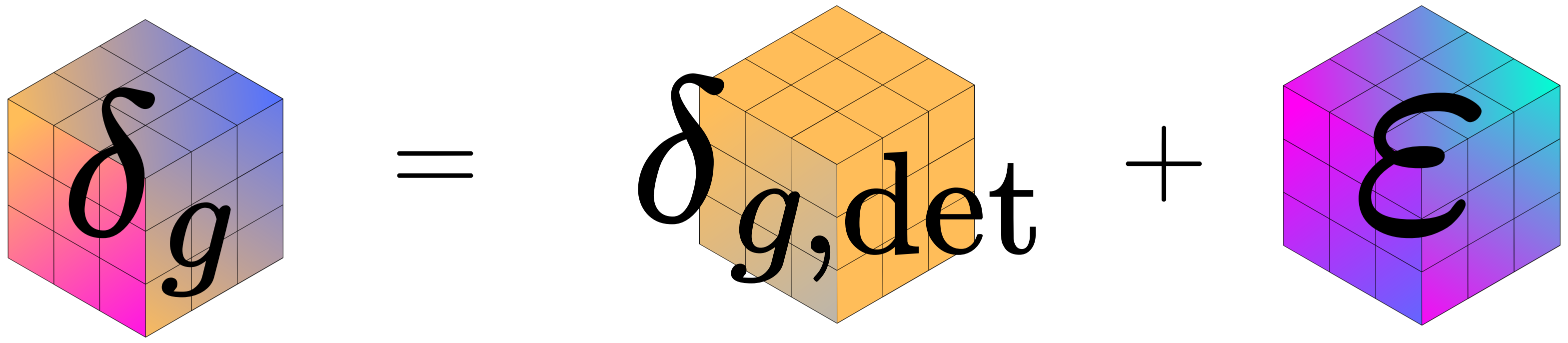
$$\delta_{g,\mathrm{det}} = b_1\, \delta + b_{\delta^2}\, \delta^2 + b_{K_2}\, K_2 + \cdots$$

The forward model

$\delta_\Lambda^{(1)} \sim \mathcal{N}(0, P_L(\boxed{\boldsymbol{\theta}}))$

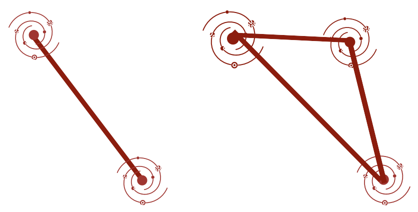


$\delta_g = \delta_{g,\mathrm{det}} + \delta_{g,\mathrm{stoch}}$



$P_\varepsilon \sim \mathcal{P}(P_\varepsilon)$

$\varepsilon \sim \mathcal{N}(0, P_\varepsilon)$



Testing SBI on Euclid-like mock data

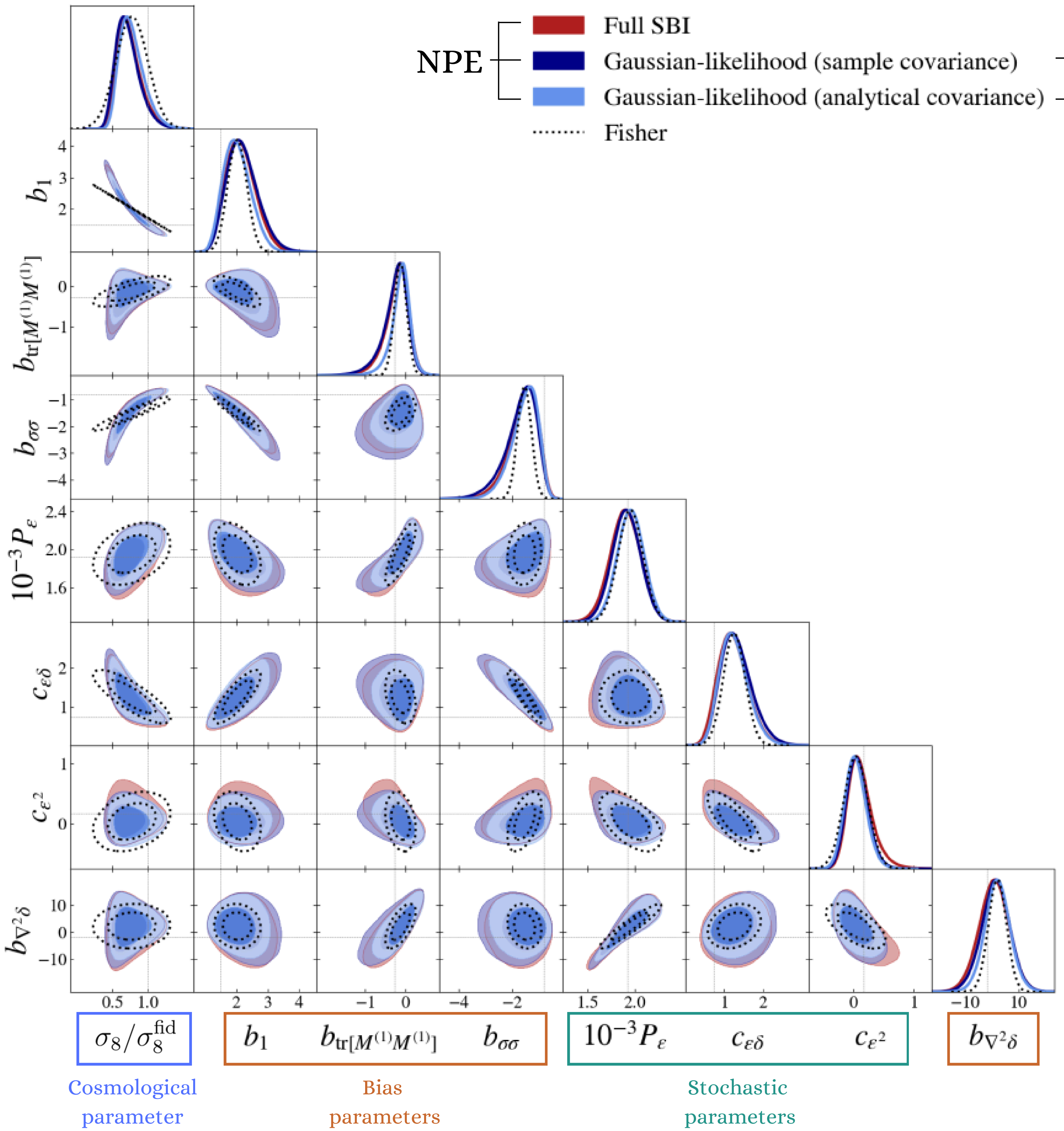
Breaking degeneracy between σ_8 and bias parameters with the galaxy power-spectrum and bispectrum

Tucci & Schmidt (2024)

JCAP

Cosmological constraints

$$N_{\rm sim} = 10^5$$
$$k_{\rm max} = \Lambda = 0.1 h\mathrm{Mpc}^{-1}$$
$$D = N_{\rm bin} + N_{\rm tri} = 33$$

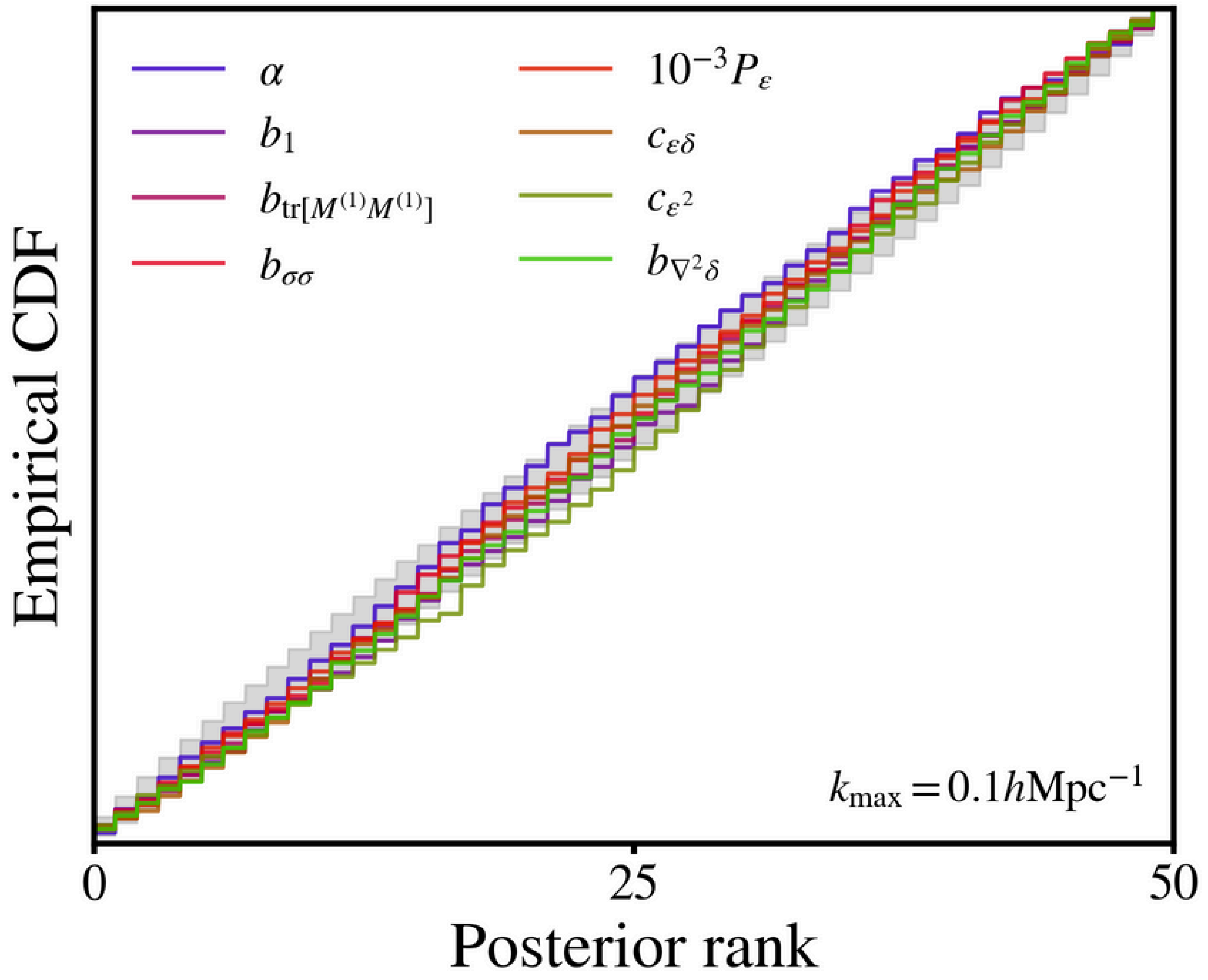


<u>Gaussian-likelihood</u>

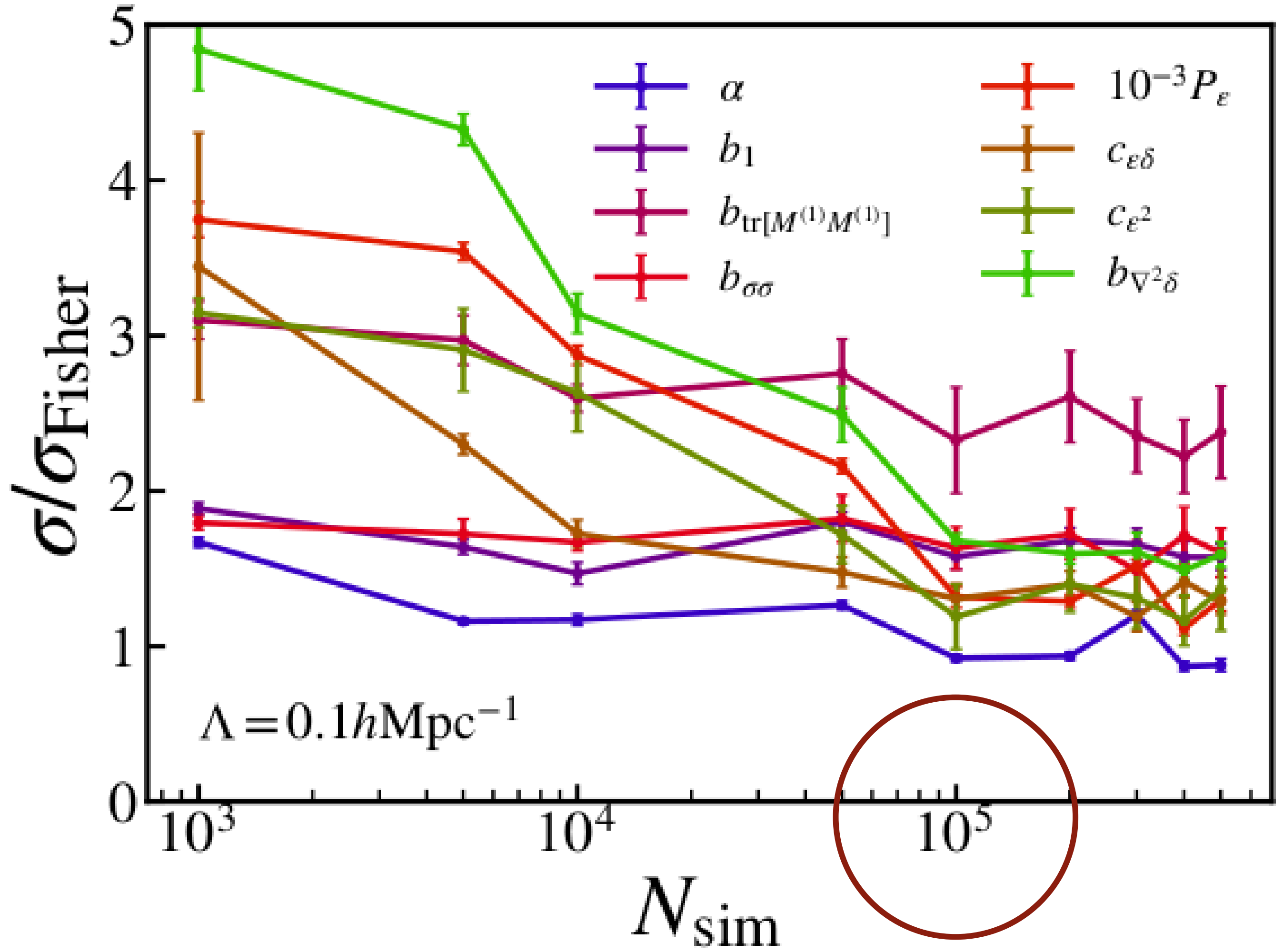
$$\mathbf{x}_n \sim \mathcal{N}\left(\langle \mathbf{x}_n \rangle, \mathrm{Cov}[\mathbf{x}_o]\right)$$

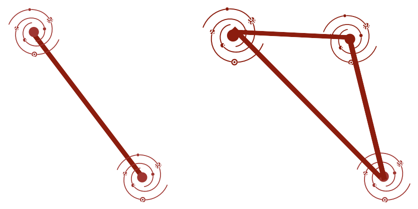
Tests of inference

Simulation-based calibration



Convergence





SBI on dark-matter halos

Breaking degeneracy between σ_8 and bias parameters with the galaxy power-spectrum and bispectrum

Nguyen, Schmidt, **Tucci** et al. (2024)

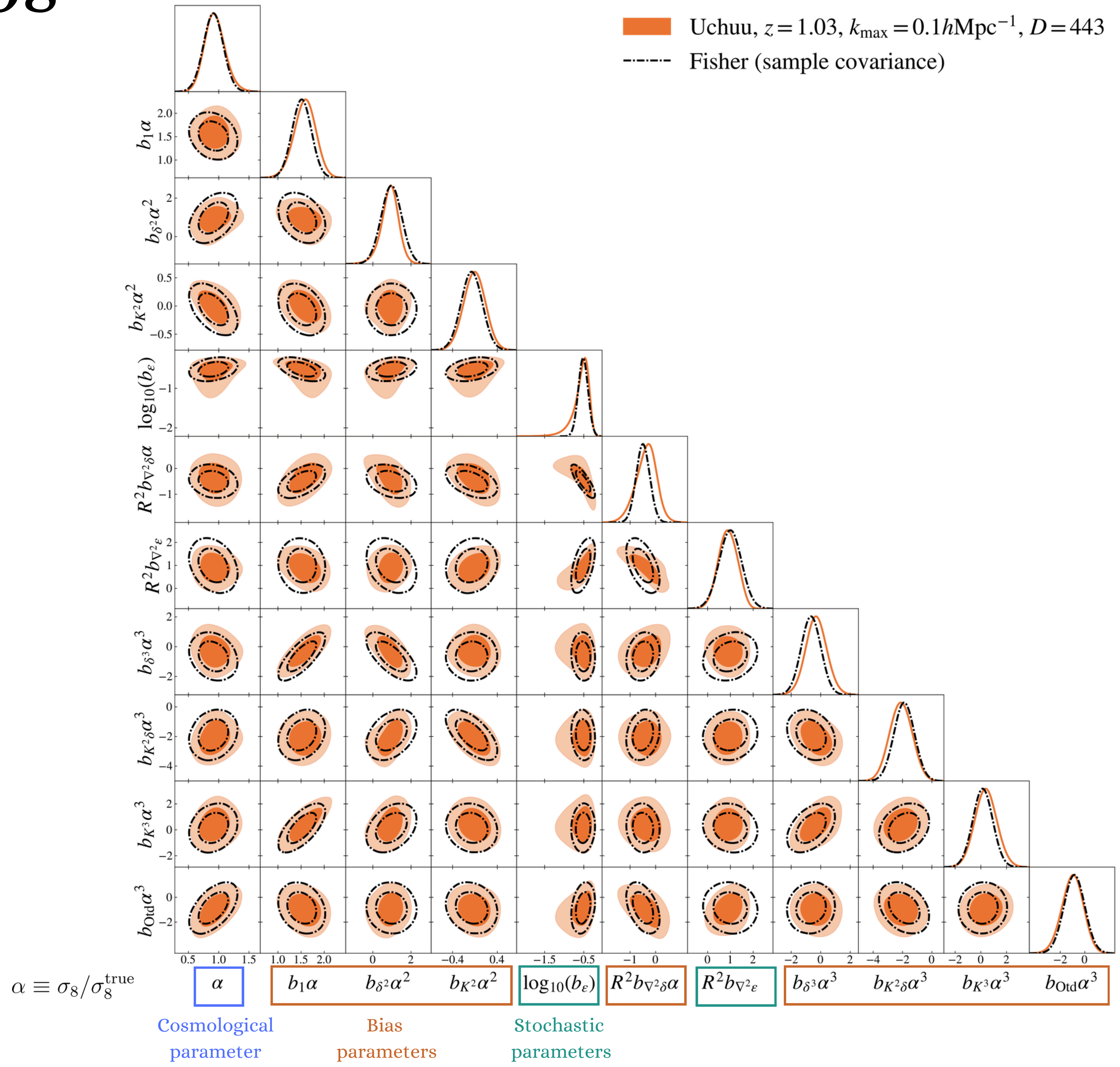
Inference setup: halo samples

	SNG	Uchuu
Redshift	$z = 0.50$	$z = 1.03$
$V\,[h^{-3}\mathrm{Mpc}^3]$	2000^3	2000^3
$\bar{n}_g\,[h^3\mathrm{Mpc}^{-3}]$	1.3×10^{-3}	3.6×10^{-3}

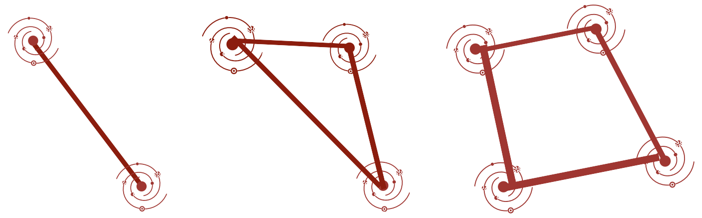
Two scale cuts:

$k_{\mathrm{max}} = 0.1h\mathrm{Mpc}^{-1}$ & $k_{\mathrm{max}} = 0.12h\mathrm{Mpc}^{-1}$

SBI on halos



Uchuu, $z=1.03$, $k_{\max}=0.1h\mathrm{Mpc}^{-1}$, $D=443$
Fisher (sample covariance)
$b_1\alpha$
$b_{\delta^2}\alpha^2$
$b_{K^2}\alpha^2$
$\log_{10}(b_\varepsilon)$
$R^2 b_{\nabla^2\delta}\alpha$
$R^2 b_{\nabla^2\varepsilon}$
$b_{\delta^3}\alpha^3$
$b_{K^2\delta}\alpha^3$
$b_{K^3}\alpha^3$
$b_{\mathrm{Otd}}\alpha^3$
$\alpha \equiv \sigma_8/\sigma_8^{\mathrm{true}}$
α
Cosmological parameter
Bias parameters
Stochastic parameters

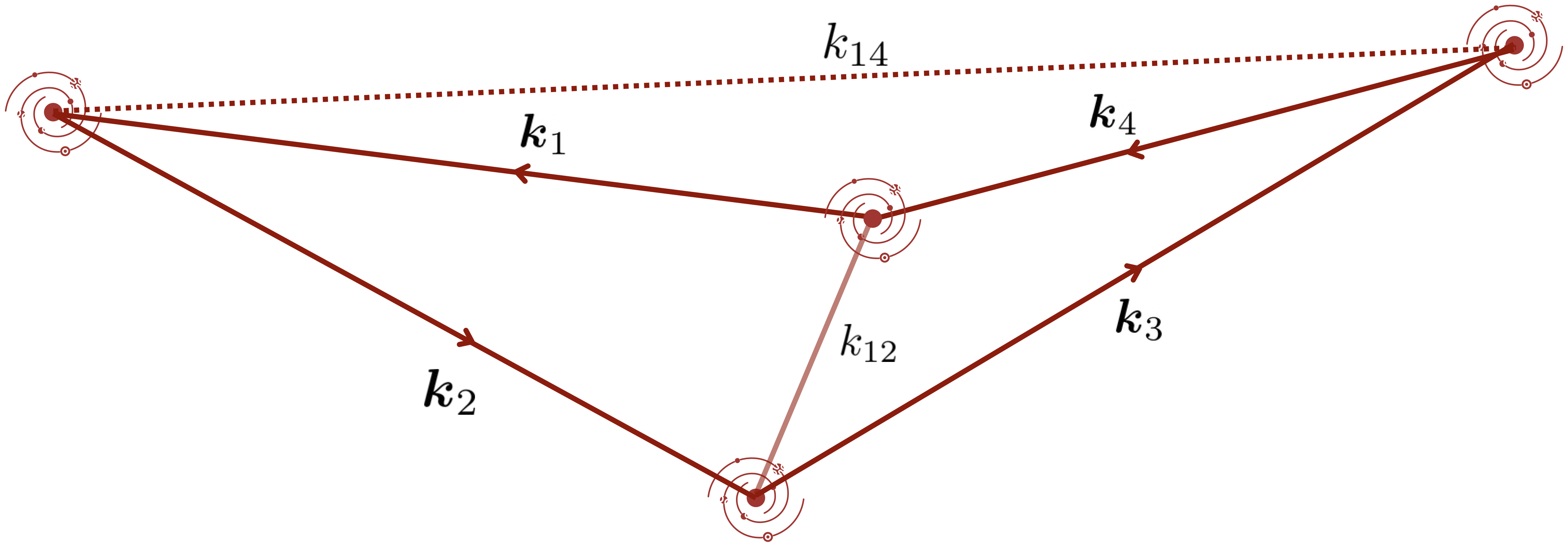


What if we add the galaxy **trispectrum**?

Breaking degeneracy between σ_8 and bias parameters with power-spectrum, bispectrum and trispectrum on dark-matter halos

Tucci & Schmidt (in prep.)

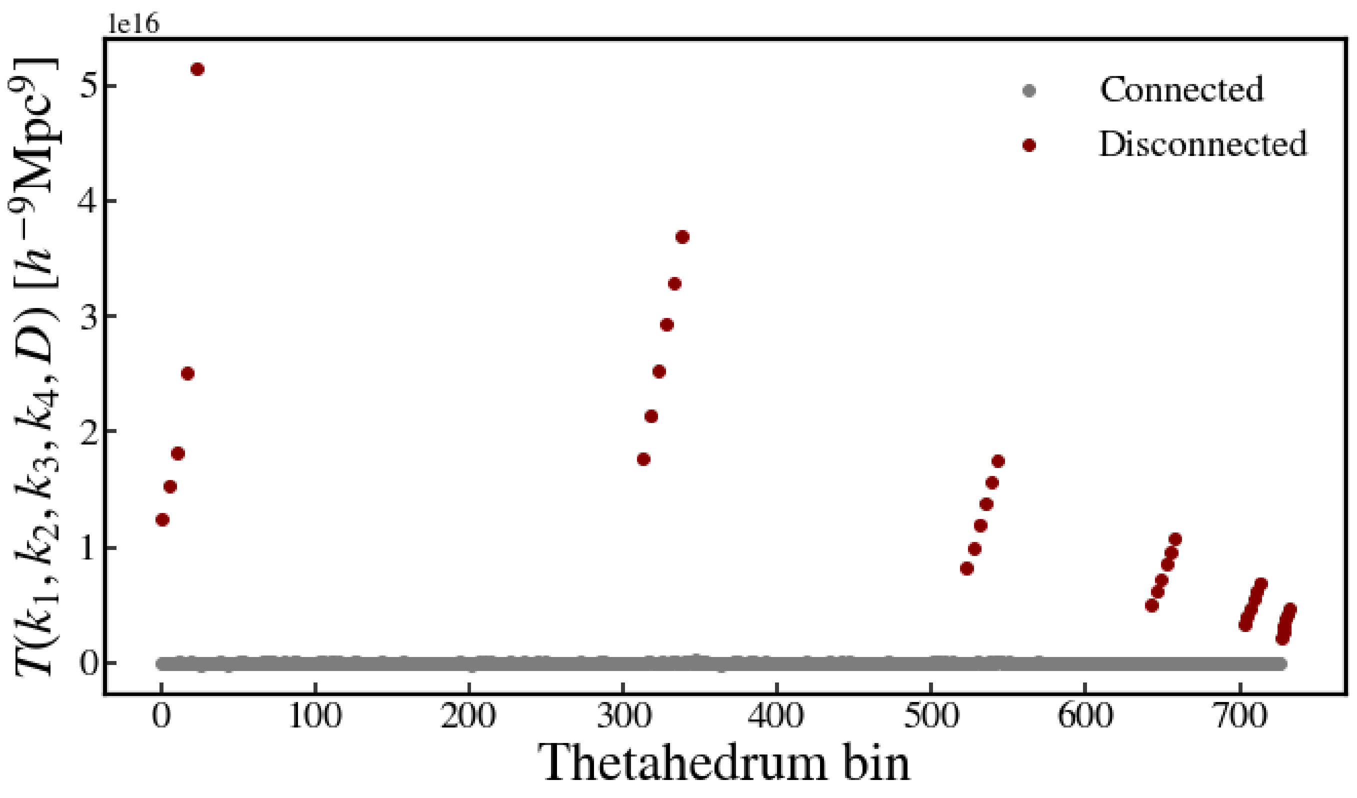
Trispectrum: the estimator



Jung+23, Coulton+23, Goldstein+24

Trispectrum Validation

$$\delta_g(\boldsymbol{k}) = b_1\,\delta^{(1)}(\boldsymbol{k})$$



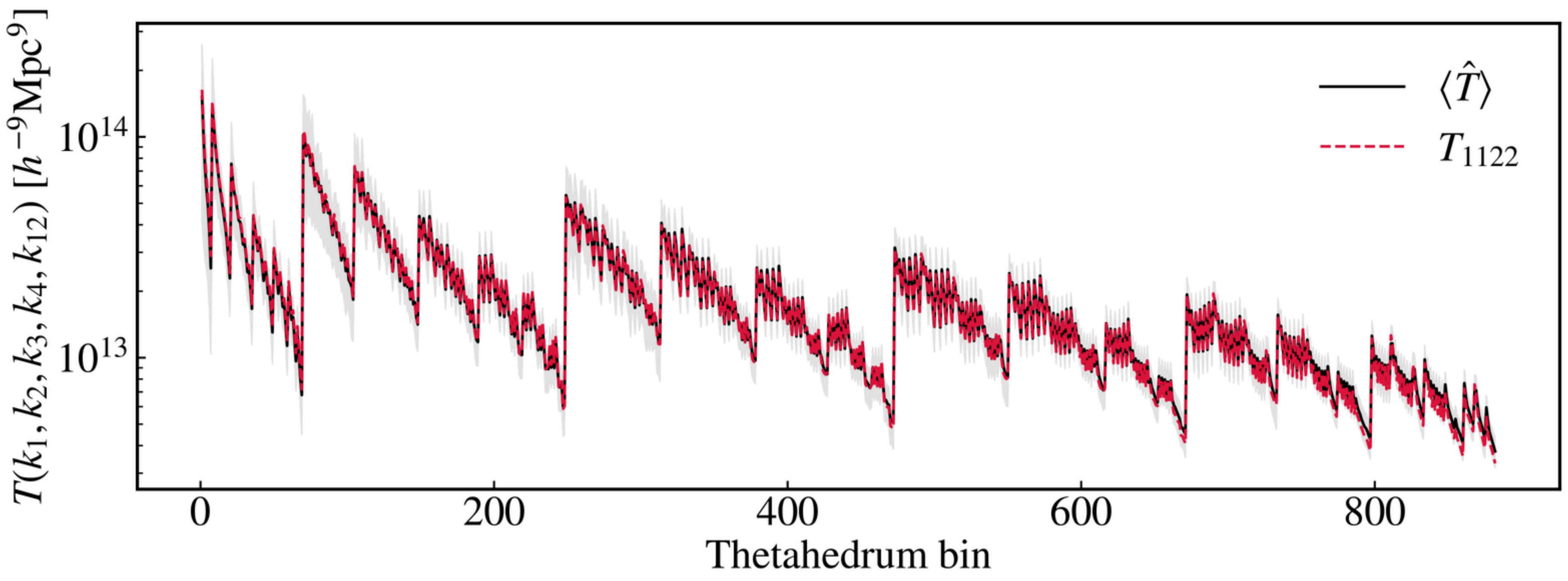
Trispectrum Validation

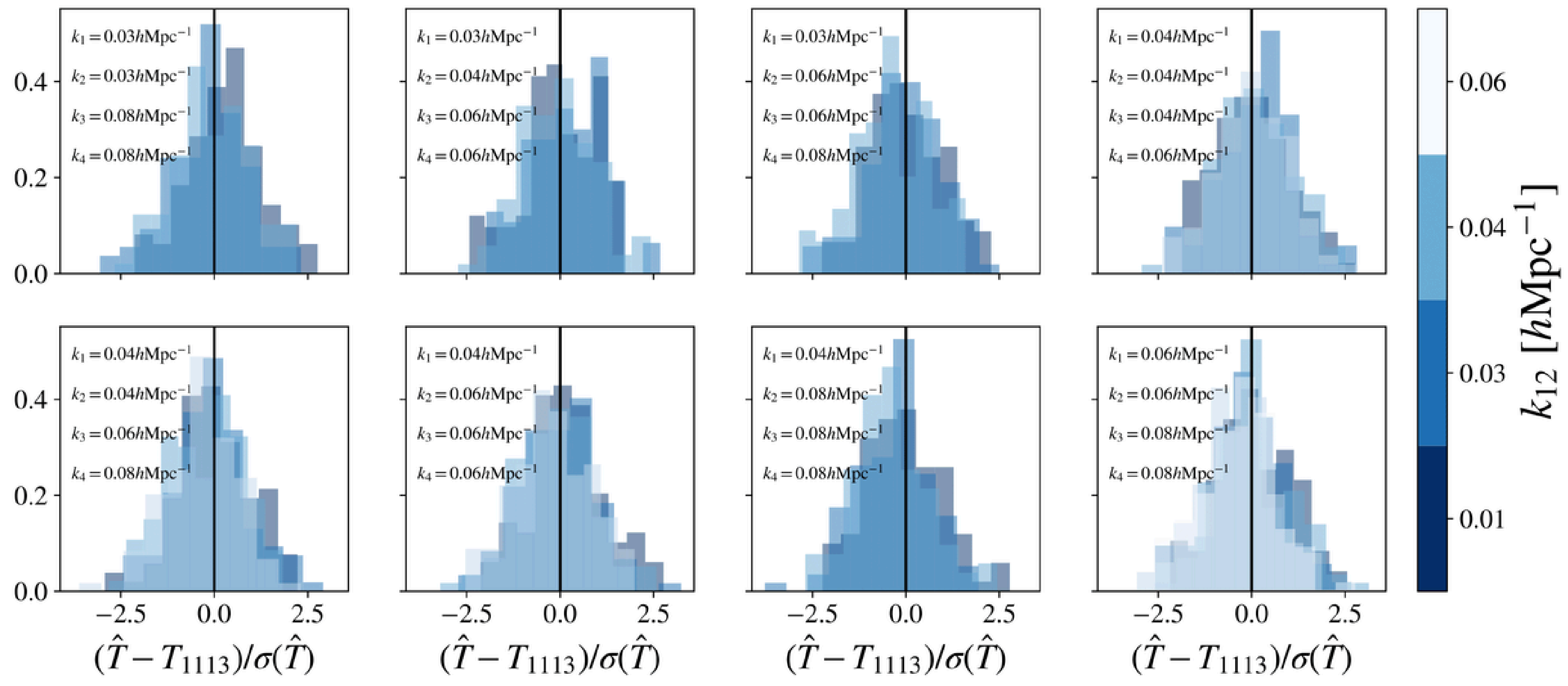
$$\delta_g(\boldsymbol{k}) = b_1\,\delta^{(1)}(\boldsymbol{k}) + b_2\,\left([\delta^{(1)}]^2(\boldsymbol{k}) - \langle[\delta^{(1)}]^2\rangle\right) + b_3\,[\delta^{(1)}]^3(\boldsymbol{k})$$

$$T_g^{\mathrm{LO}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) = b_1^2\,b_2^2\,T_{1122}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) + b_1^3\,b_3\,T_{1113}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$

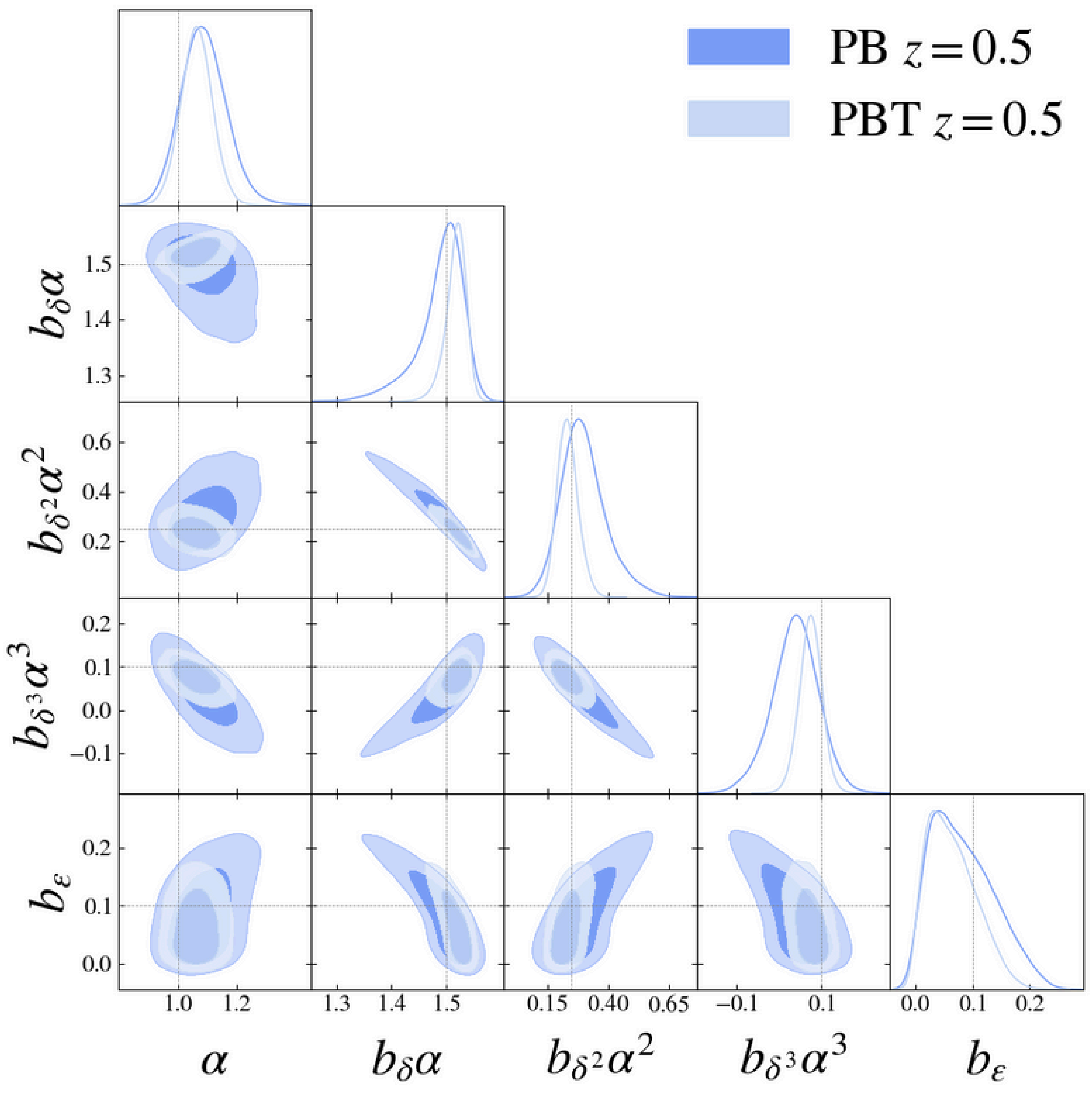
T_{1122}

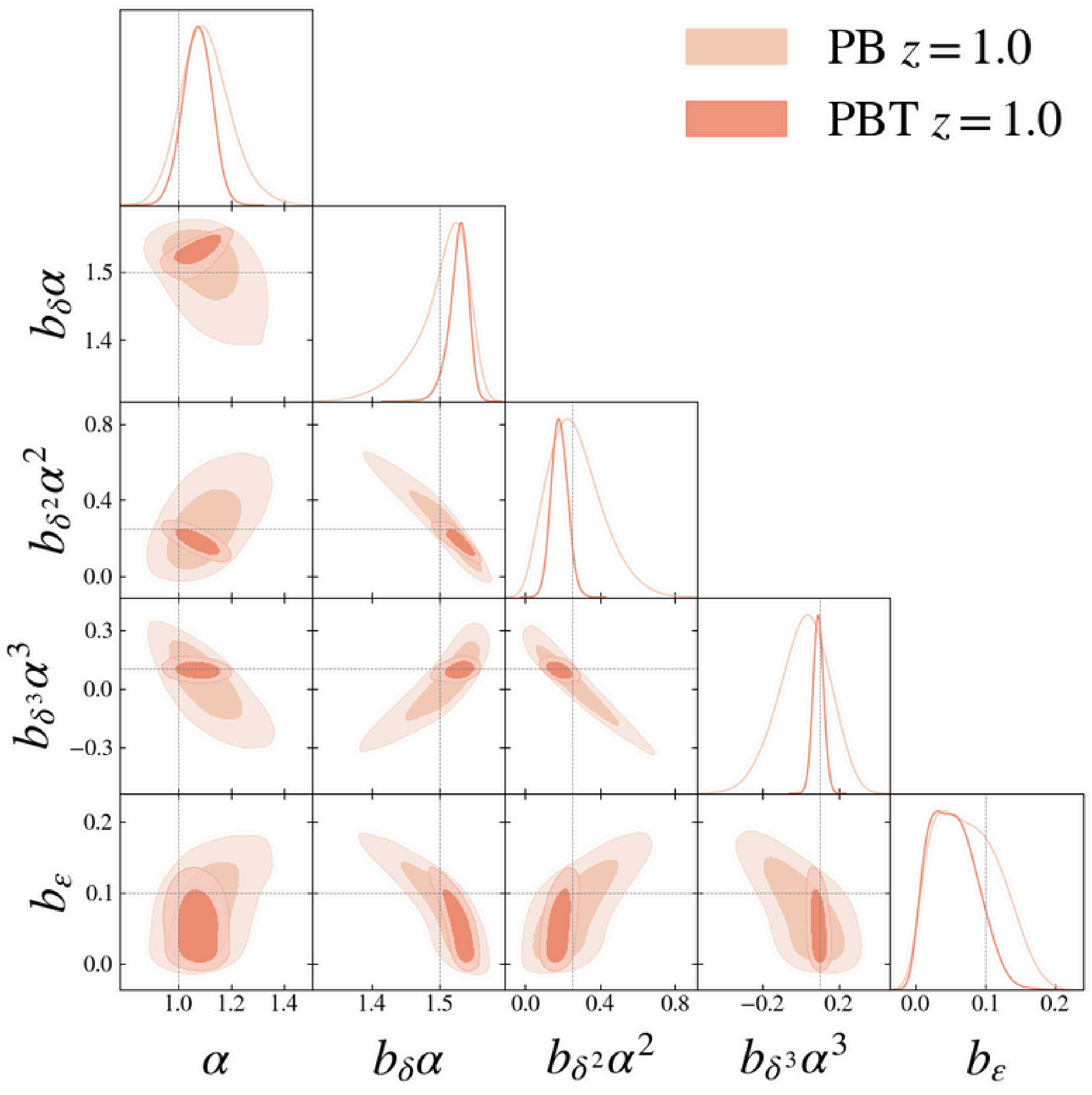
T_{1113}



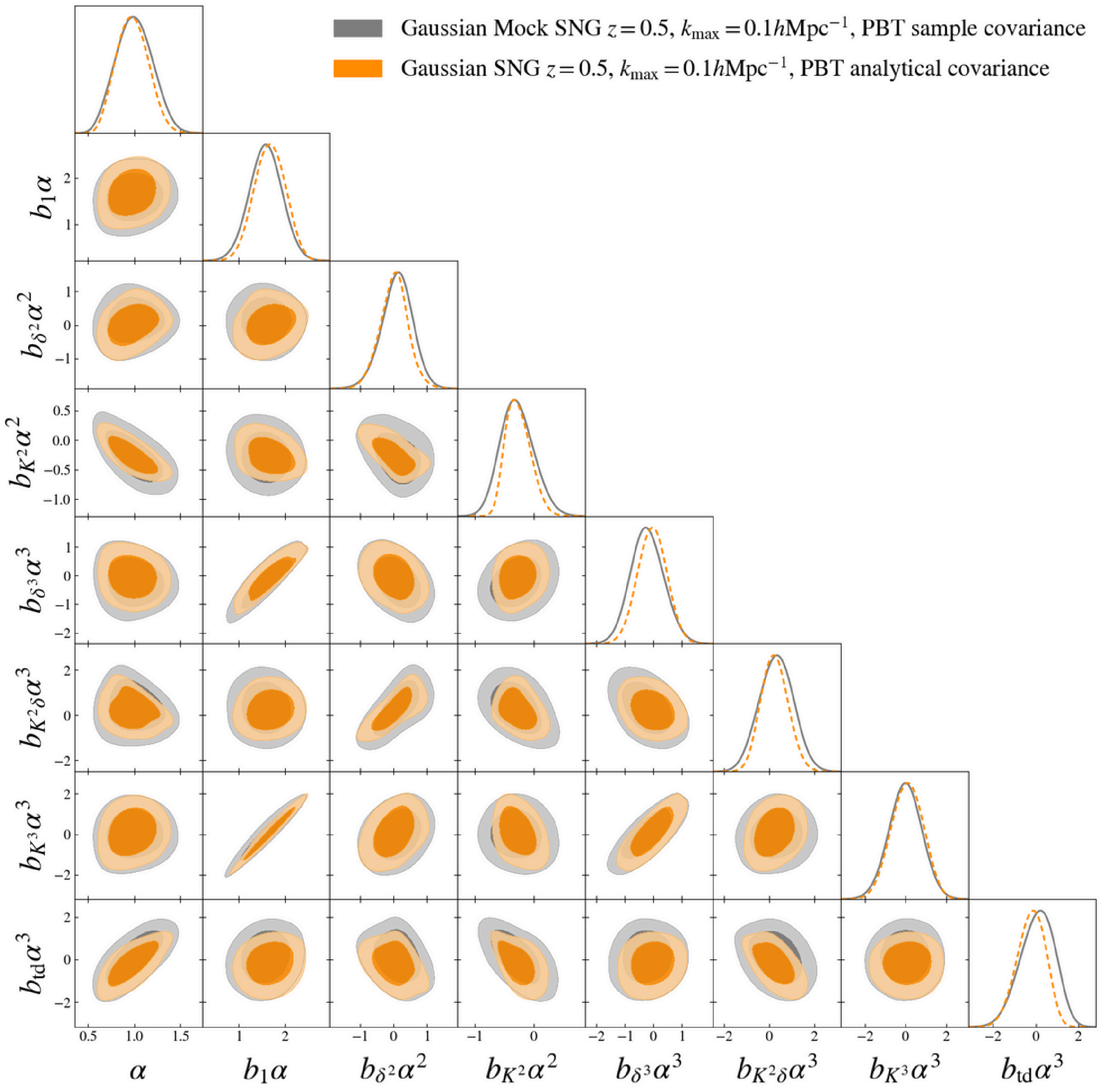


Trispectrum Information Content



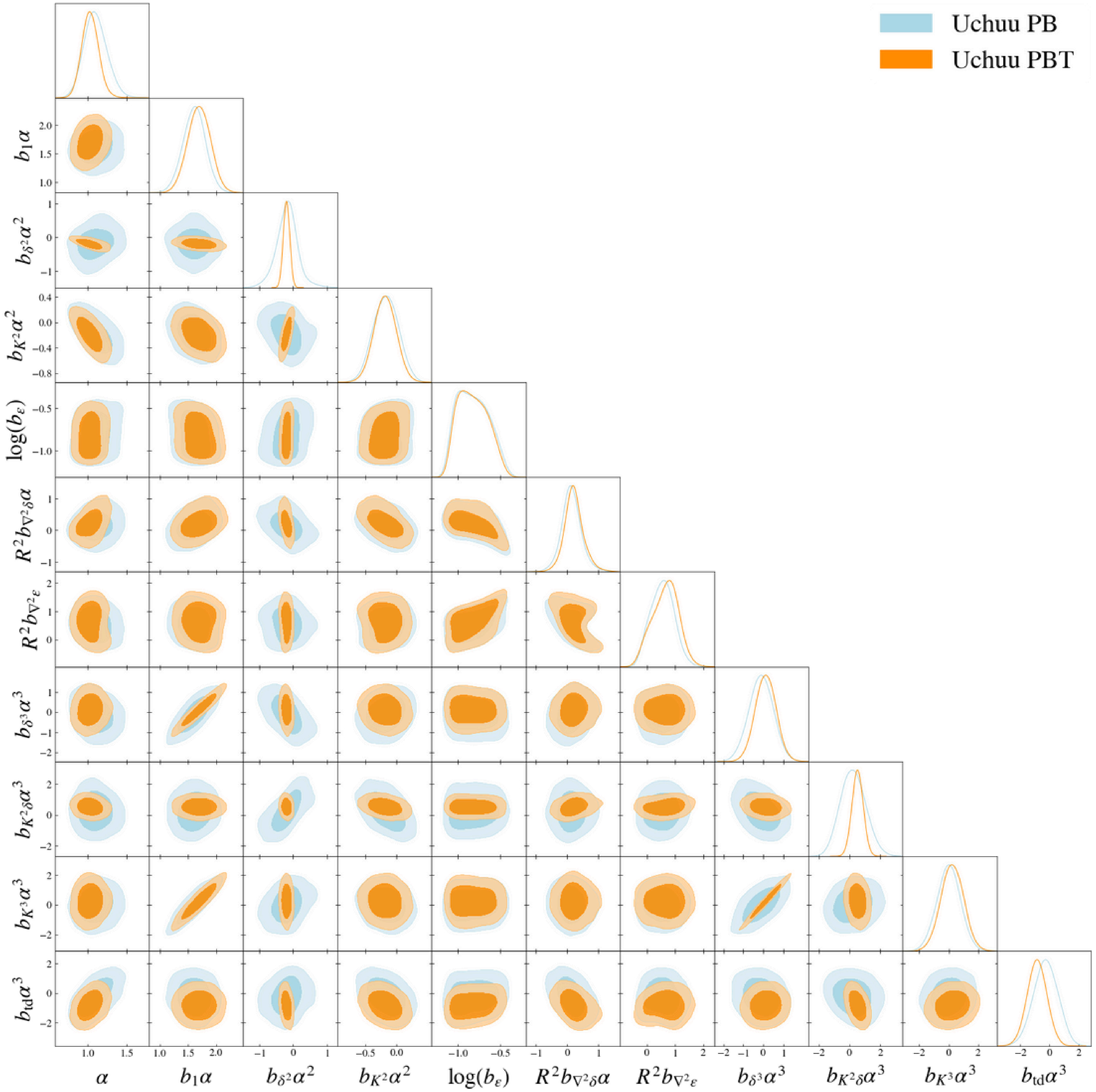


Trispectrum Information Content



Trispectrum Information Content

$$\frac{\sigma_{\mathrm{PB}}(\alpha)}{\sigma_{\mathrm{PBT}}(\alpha)} \sim 1.4$$



SBI with LEFTfield: Conclusions

- Robust analysis with EFTofLSS and bias expansion
- LEFTfield allows for **fast** analysis in **cosmological volumes** with **convergence** and posterior **diagnostics** tests
- Need order of 10^5 simulations for convergence (investigating how we can improve that)
- SBI allows for cosmological inference using **trispectrum**, which is **unfeasible** with standard inference techniques
- No need to assume Gaussian likelihood, explicit loop or covariance calculations

Inferring the cosmological parameters: **challenges**

power-spectrum bispectrum trispectrum **Increasing complexity**

... n

Analytical approximations Estimation Modelling

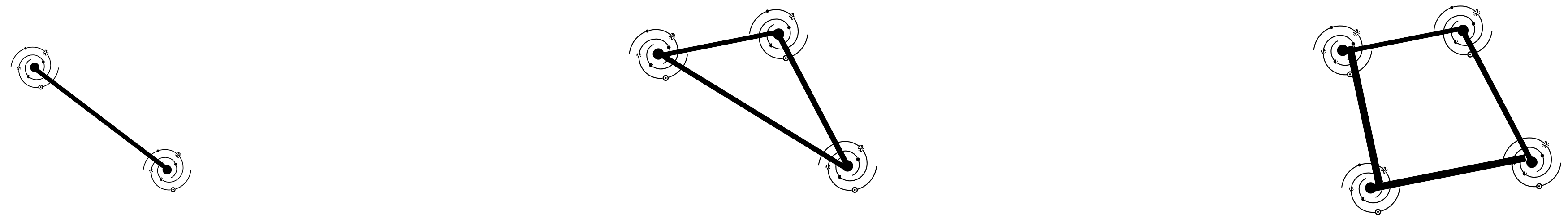
$$-2\ln \mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = (\mathbf{D}-\mathbf{T}(\boldsymbol{\theta}))\cdot \mathbf{C}^{-1}\cdot(\mathbf{D}-\mathbf{T}(\boldsymbol{\theta}))$$

Measurements

SBI

Inferring the cosmological parameters: **challenges**

power-spectrum bispectrum trispectrum **Increasing complexity**



... n

where to stop?
is there a better way?

Analytical approximations Estimation Modelling

$$-2\ln \mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \left(\mathbf{D}-\mathbf{T}(\boldsymbol{\theta})\right)\cdot \mathbf{C}^{-1}\cdot \left(\mathbf{D}-\mathbf{T}(\boldsymbol{\theta})\right)$$

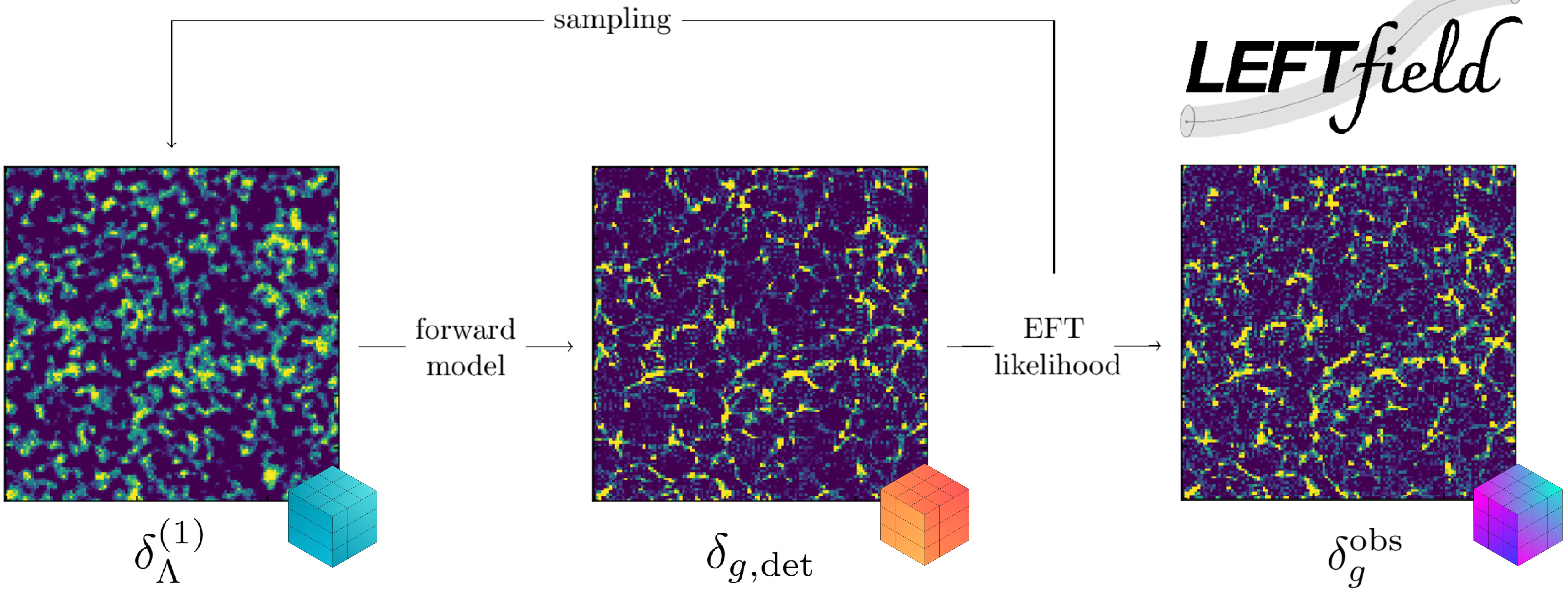
Measurements

SBI

Part II

Field-level Bayesian inference

(FBI)

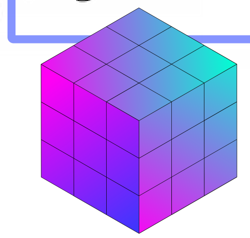


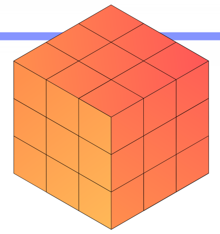
Credits: Julia Stadler

Field level Likelihood

Mode by mode
data and theory
comparison!

$$\ln \mathcal{L}\left(\delta_g^{\mathrm{obs}} \middle| \delta_{g,\mathrm{det}}[\boldsymbol{\theta}, \delta_\Lambda^{(1)}, \{b_O\}], \{\sigma_\varepsilon\}\right) = -\frac{1}{2} \sum_{k<k_{\max}} \left[\frac{1}{\sigma_\varepsilon^2(k)} \boxed{\left|\delta_g^{\mathrm{obs}}(\boldsymbol{k}) - \delta_{g,\mathrm{det}}[\boldsymbol{\theta}, \delta_\Lambda^{(1)}, \{b_O\}](\boldsymbol{k})\right|^2} + \ln[2\pi\sigma_\varepsilon^2(k)] \right]$$

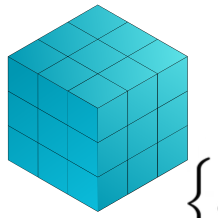




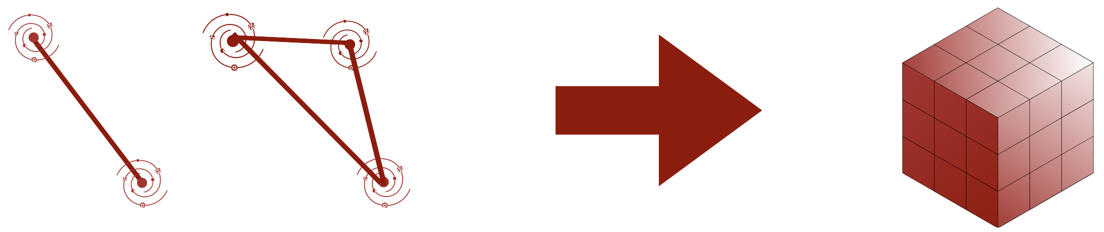
$\downarrow$ HMC

$$\boxed{\mathcal{P}\left(\boldsymbol{\theta}, \boxed{\delta_\Lambda^{(1)}}, \{b_O\}, \{\sigma_\varepsilon\} \middle| \delta_g^{\mathrm{obs}}\right)}$$

Full posterior
including initial
conditions!



$\left\{\delta_{\Lambda,i}^{(1)}\right\}_{i=1}^{N_g^\Lambda}$



How much information is retained at the galaxy density field?

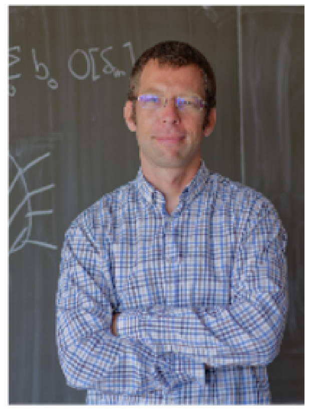
Breaking degeneracy between σ_8 and bias parameters on dark-matter halos

Nguyen, Schmidt, **Tucci** et al. (2024)

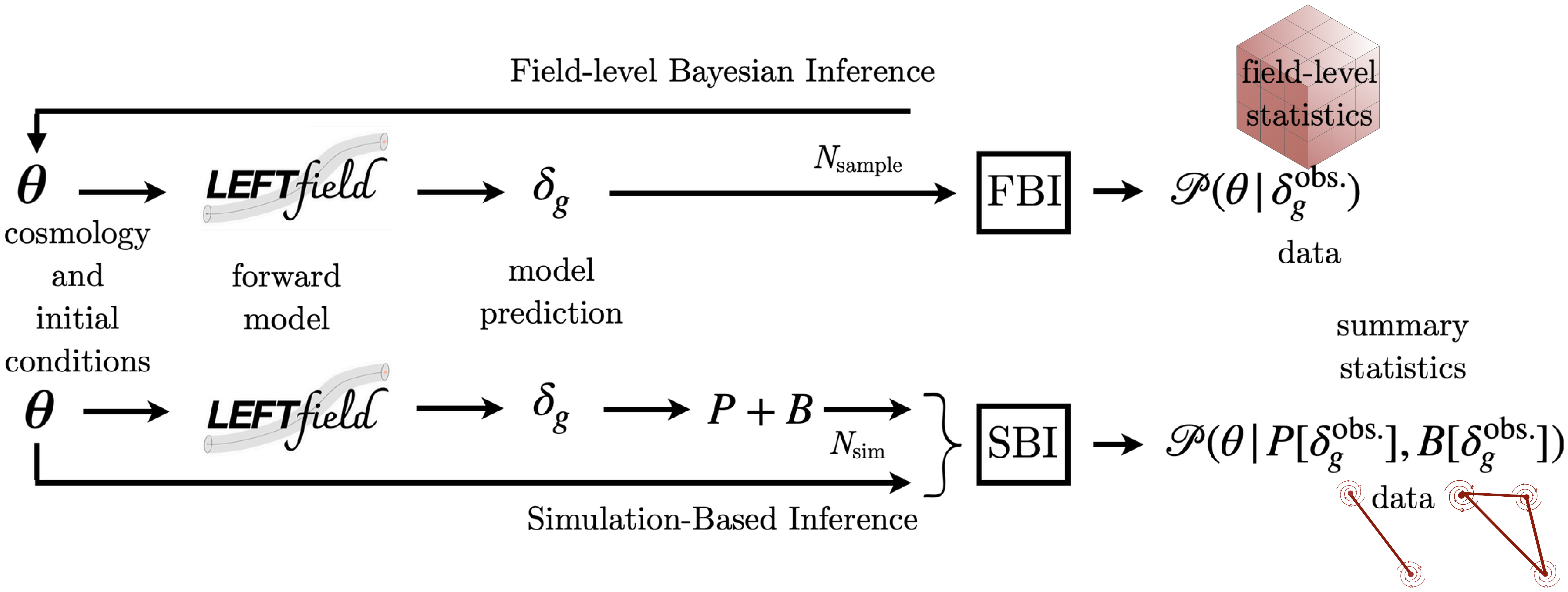
3rd order bias expansion

$$\boxed{\begin{aligned} O_{\mathrm{det}} &\in \left[\delta, \delta^2, K^2, \delta^3, K^3, \delta K^2, O_{\mathrm{td}}, \nabla^2\delta\right] \\ O_{\mathrm{stoch}} &\in \left[\varepsilon, \nabla^2\varepsilon\right] \end{aligned}}$$

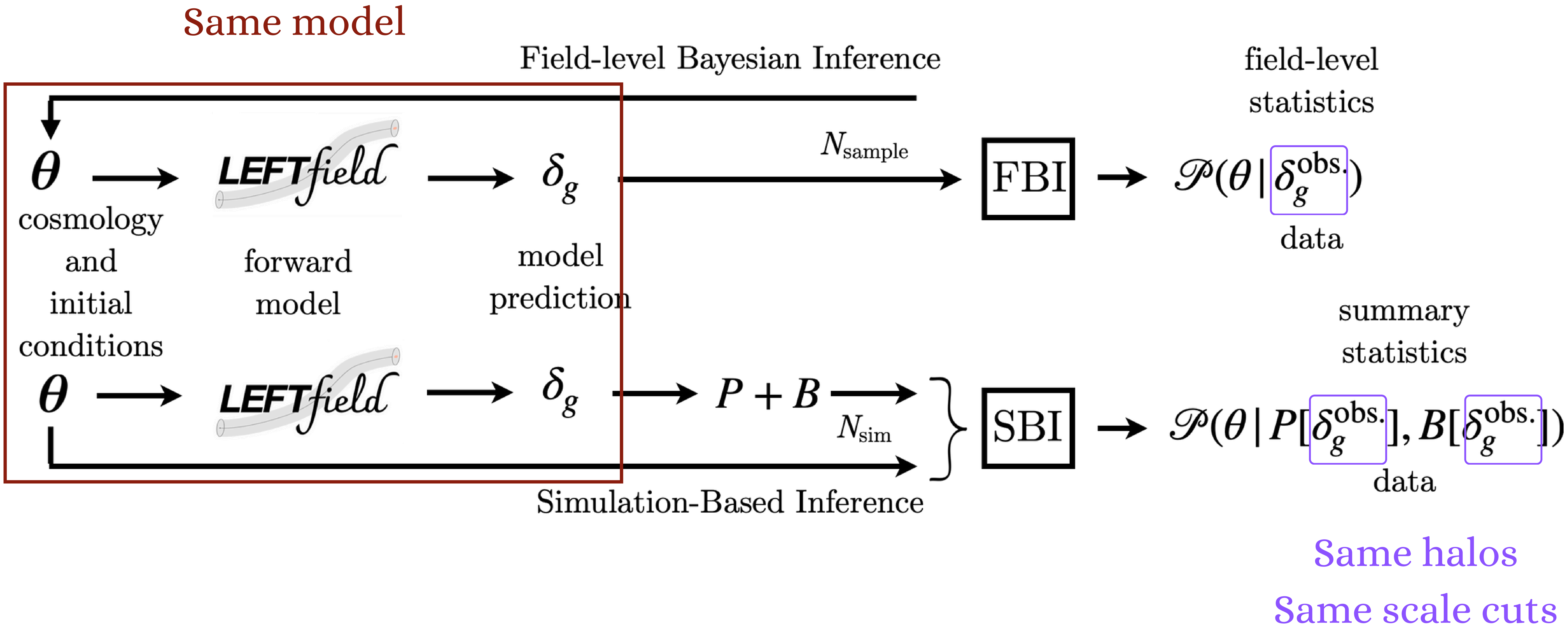
Nhat-Minh Nguyen
(IPMU)



Fabian Schmidt
(MPA)

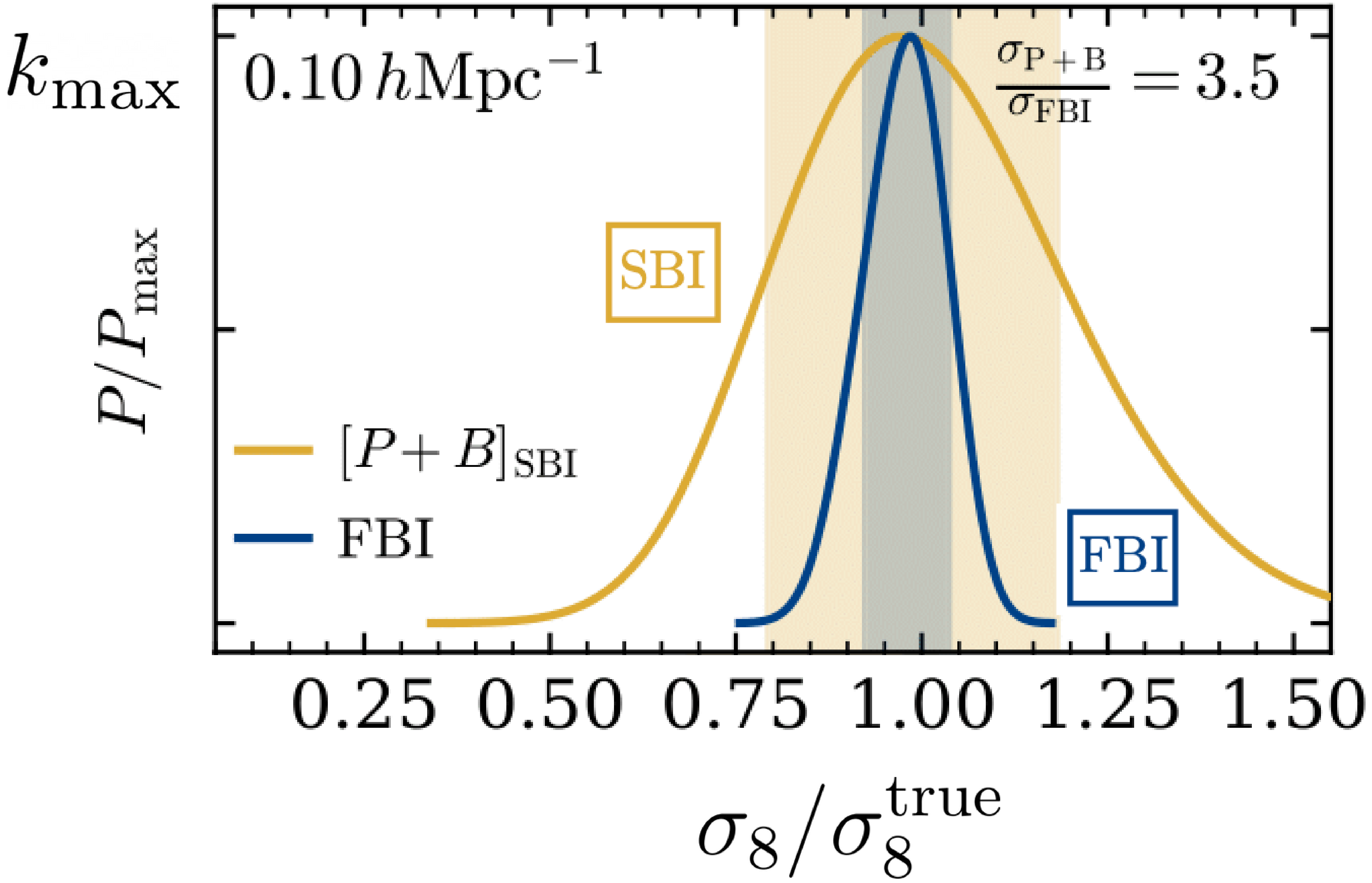


Apples-to-apples comparison



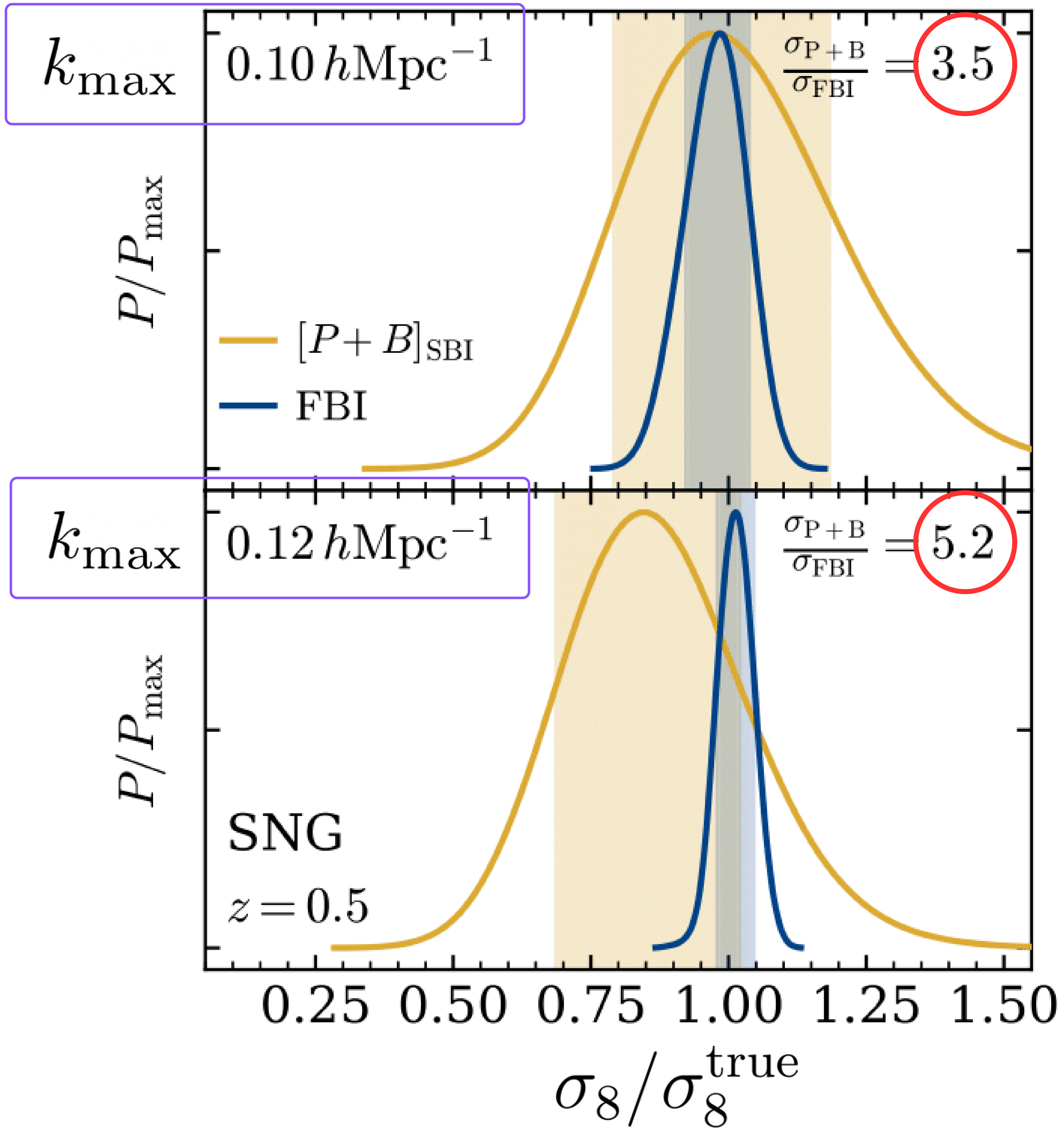
A lot of reliable information at the field-level!

SNG halos

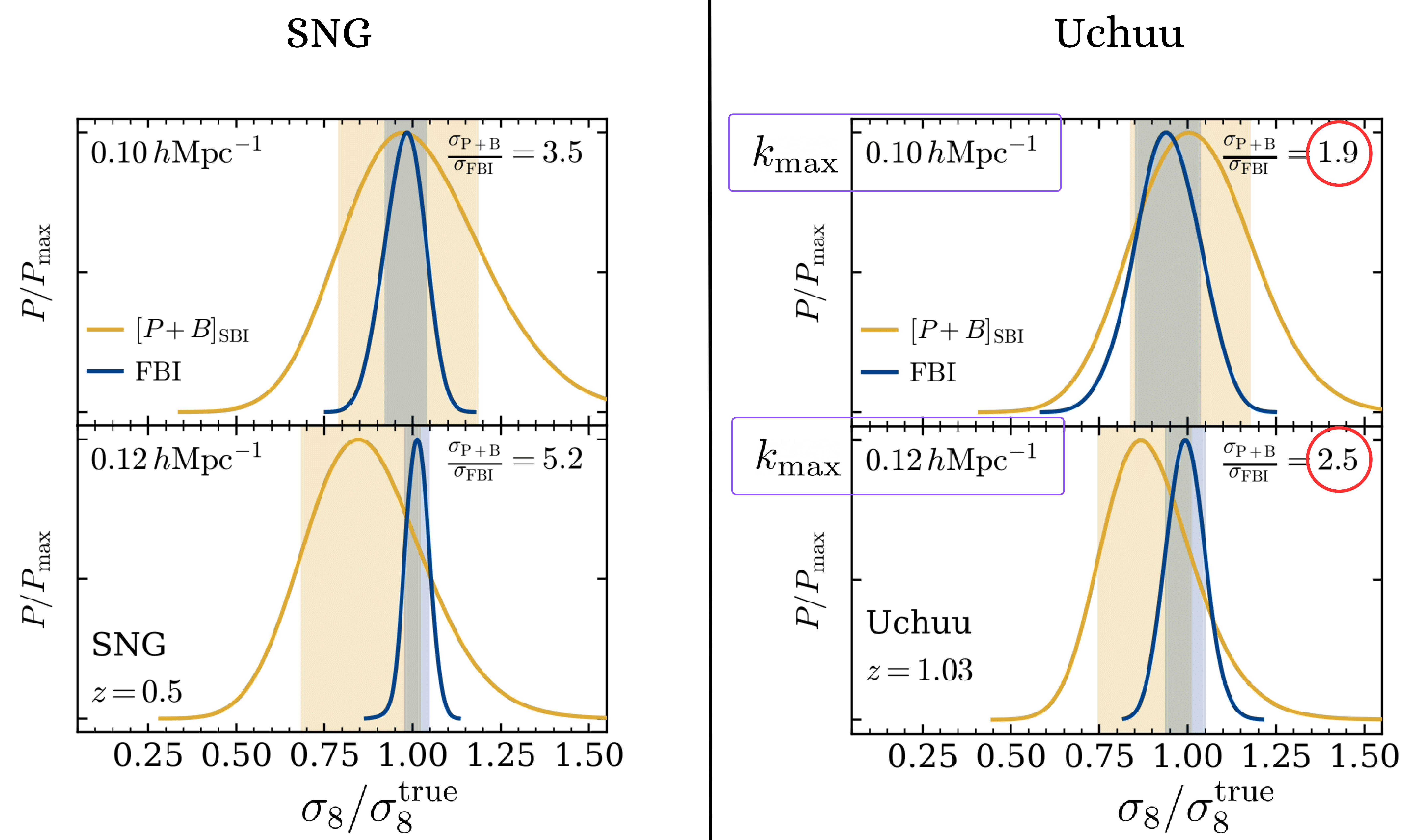


3.5 improvement factor!

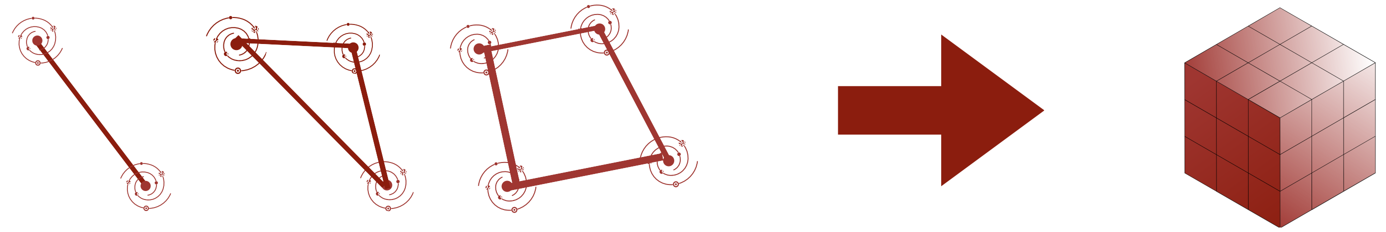
SNG



$k_{\rm max}$
$0.10\,h{\rm Mpc}^{-1}$
$\frac{\sigma_{\rm P+B}}{\sigma_{\rm FBI}} = 3.5$
$[P+B]_{\rm SBI}$
FBI
$k_{\rm max}$
$0.12\,h{\rm Mpc}^{-1}$
$\frac{\sigma_{\rm P+B}}{\sigma_{\rm FBI}} = 5.2$
SNG
$z = 0.5$
$P/P_{\rm max}$
0.25
0.50
0.75
1.00
1.25
1.50
$\sigma_8/\sigma_8^{\rm true}$



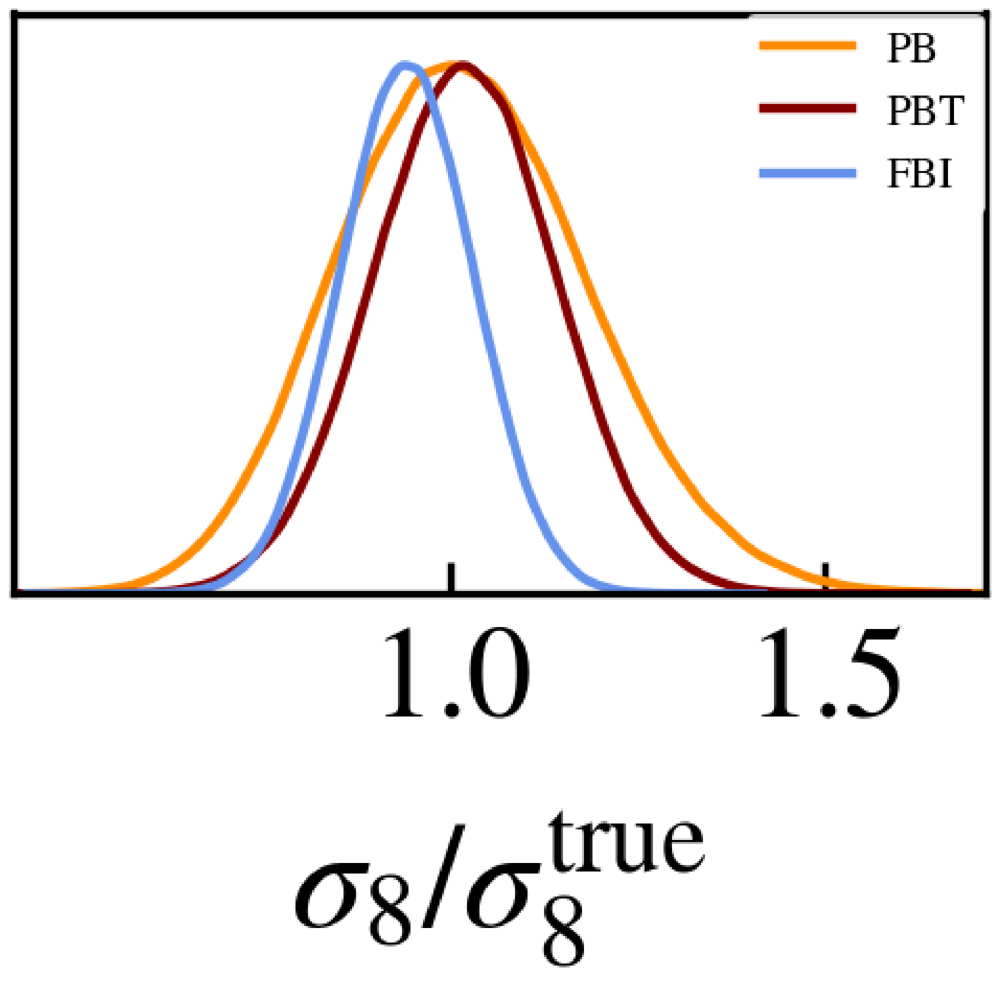
SNG
Uchuu
0.10 hMpc^-1
σP+B / σFBI = 3.5
0.12 hMpc^-1
σP+B / σFBI = 5.2
[P+B]SBI
FBI
SNG
z = 0.5
kmax
0.10 hMpc^-1
σP+B / σFBI = 1.9
kmax
0.12 hMpc^-1
σP+B / σFBI = 2.5
Uchuu
z = 1.03
P/Pmax
0.25 0.50 0.75 1.00 1.25 1.50
σ8/σ8^true

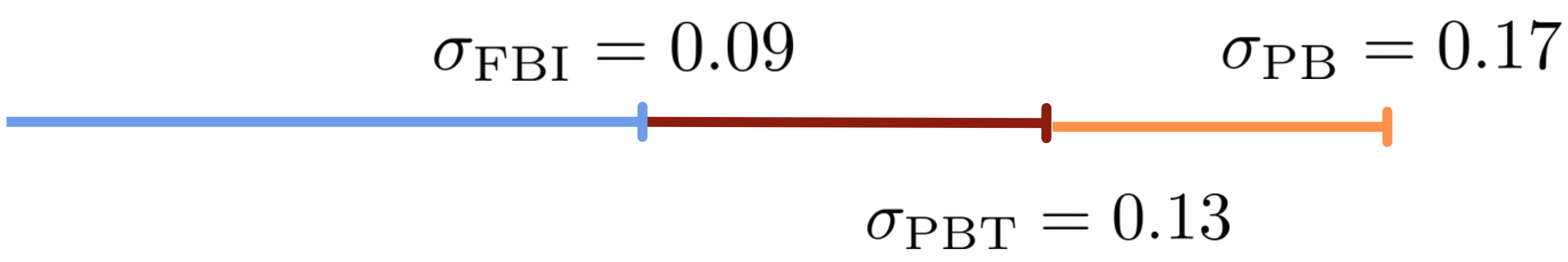


What if we add the **trispectrum**?

Tucci & Schmidt (in prep.)

Trispectrum: **preliminary** results





$k_{\rm max} = 0.1 h\,{\rm Mpc}^{-1}$

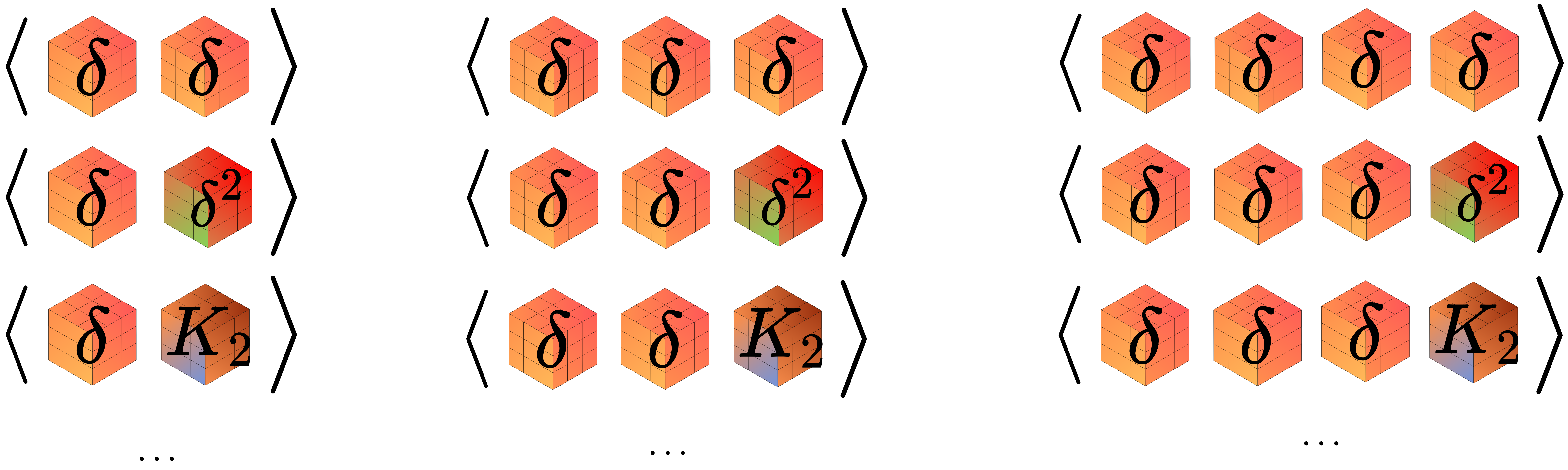
Uchuu halos at z=1

Brute force approach:
10^6 simulations

SBI with LEFTfield:
final thoughts

MCMC

$$\langle O(\boldsymbol{k}) \dots O'(\boldsymbol{k}')\rangle(\boldsymbol{\theta})$$



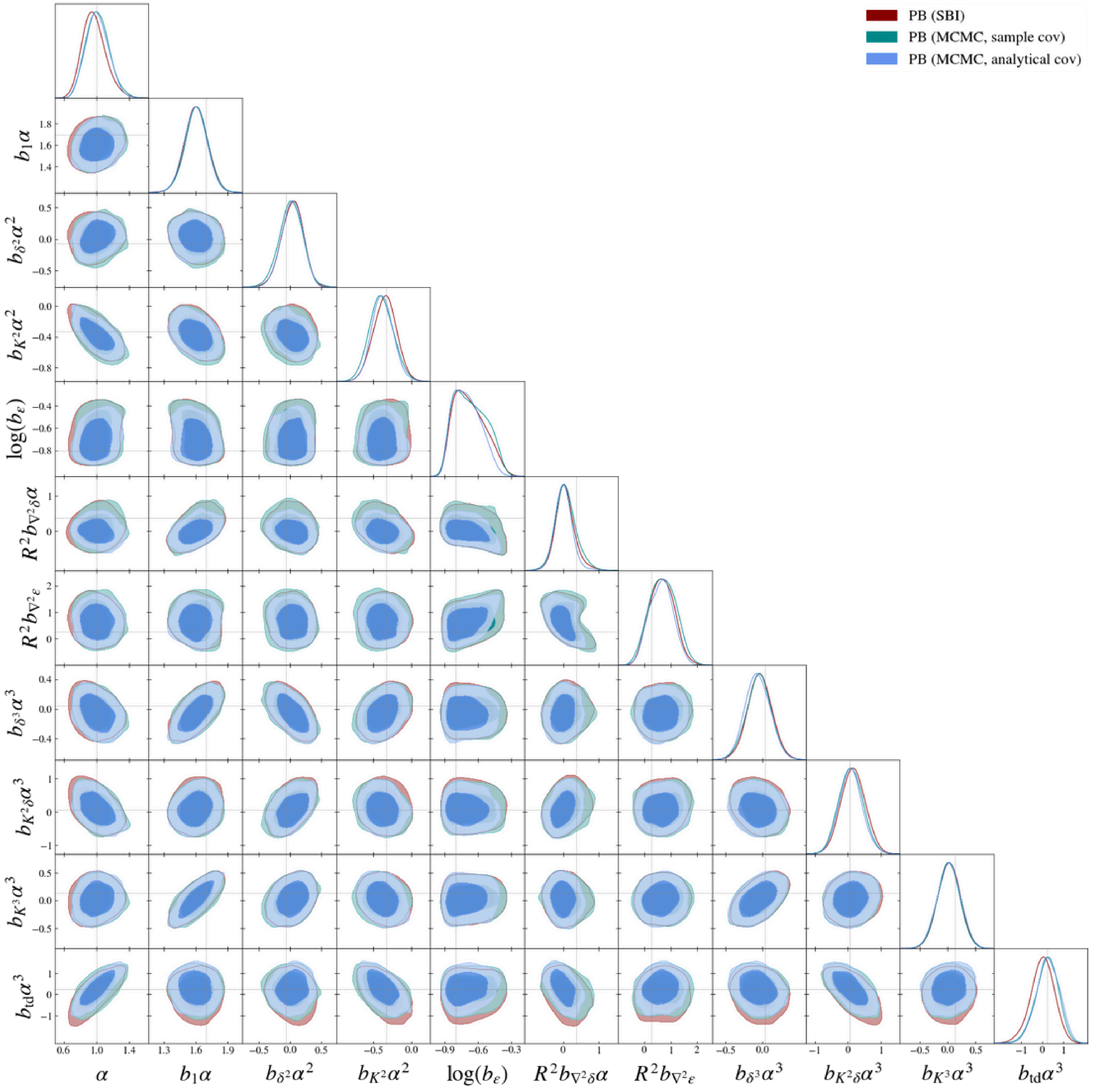
$$P_g(k) = \sum_{O_\alpha, O_\beta} b_{O_\alpha} b_{O_\beta} \langle O_\alpha(k) O_\beta(k)\rangle$$

$$B_g(k_1, k_2, k_3) = \sum_{O_\alpha, O_\beta, O_\gamma} b_{O_\alpha} b_{O_\beta} b_{O_\gamma} \langle O_\alpha(k_1) O_\beta(k_2) O_\gamma(k_3)\rangle$$

$$T_g(k_1, k_2, k_3, k_4, k_D) = \sum_{O_\alpha, O_\beta, O_\gamma, O_\sigma} b_{O_\alpha} b_{O_\beta} b_{O_\gamma} b_{O_\sigma} \langle O_\alpha(k_1) O_\beta(k_2) O_\gamma(k_3) O_\sigma(k_4)\rangle|_{k_D}$$

Comparison to MCMC

SNG (z=0.5) halos

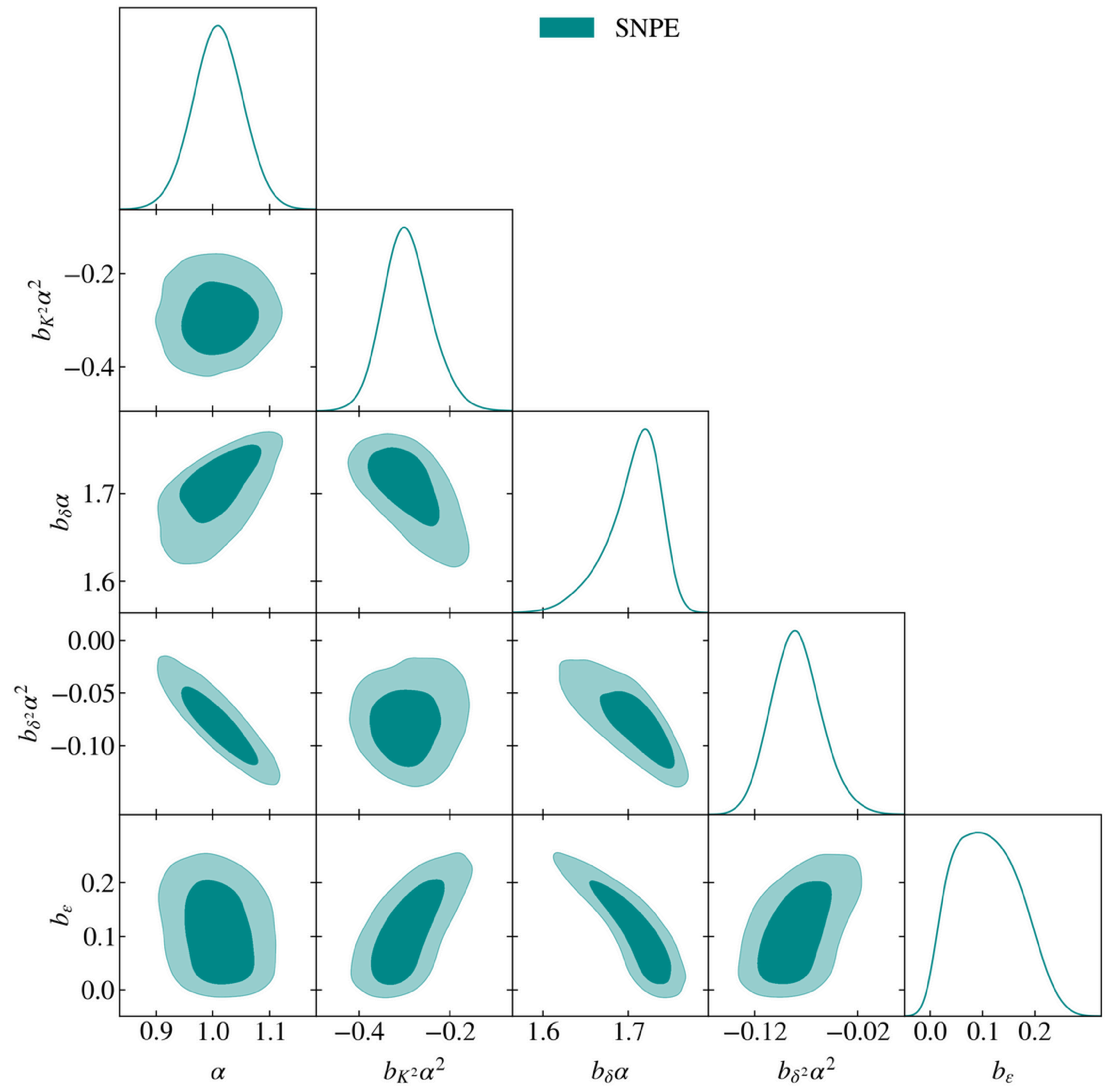


SBI and MCMC do match

Comparison to Spezzati, Marinucci, Simonović (2025)

$\sigma(\alpha) = 0.2$	$\sigma(\alpha) = 0.06$
Nguyen et al.	**Spezzati et al.**
PB (tree + parts of loops)	P (tree + 1loop) + B (tree)
SBI on halos	Fisher
Third order bias	Second order bias + bg3
Non-Gaussian likelihood	Gaussian likelihood
Full covariance	Analytical, diagonal cov.
...	...

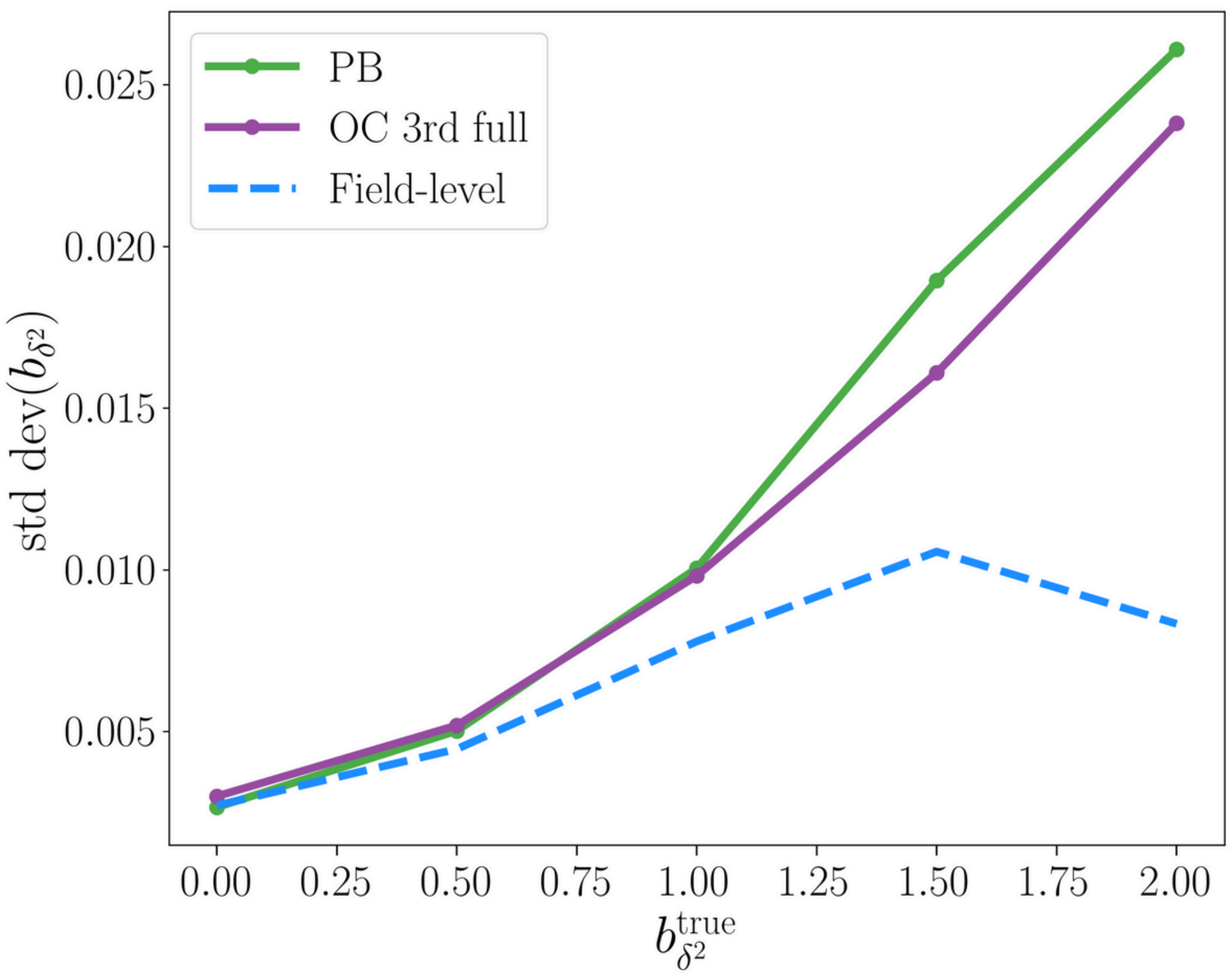
Comparison to Spezzati, Marinucci, Simonović (2025)



PB on SNG (z=0.5)-like mock data

$$\sigma(\alpha) = 0.05$$

Fiducial values of bias parameters matter



Nikolac, Schmidt, Tucci (in prep.)

Talk by Ivana Nikolac
next week

Conclusion & Next Steps

- We demonstrated to have **unbiased** and **accurate** results from halo catalogs using LEFTfield for SBI and FBI
- **Apple-to-apple comparison** of field-level inference and SBI shows that there is a lot of **reliable** information beyond 2+3(+4)-point functions in the 3D maps of galaxies

Next steps to connect with observations:

- Include more observational effects
- Expand the cosmological parameter space
- Explore summaries in SBI

MAX-PLANCK-INSTITUT FÜR ASTROPHYSIK

LEFTfield

Beatriz Tucci

tucci@mpa-garching.mpg.de