How things change with scale?
or... the running bias
or... the Renormalization group for LSS

Based on:
2307.15031, 2404.16929, 2405.21002, 2507.13905, 2508.00611

Henrique Rubira

(Cambridge/LMU)

LMU

UNIVERSITY OF CAMBRIDGE

with: Fabian Schmidt, Charalampos Nikolis, Mathias Garny, Thomas Bakx, Zvonimir Vlah

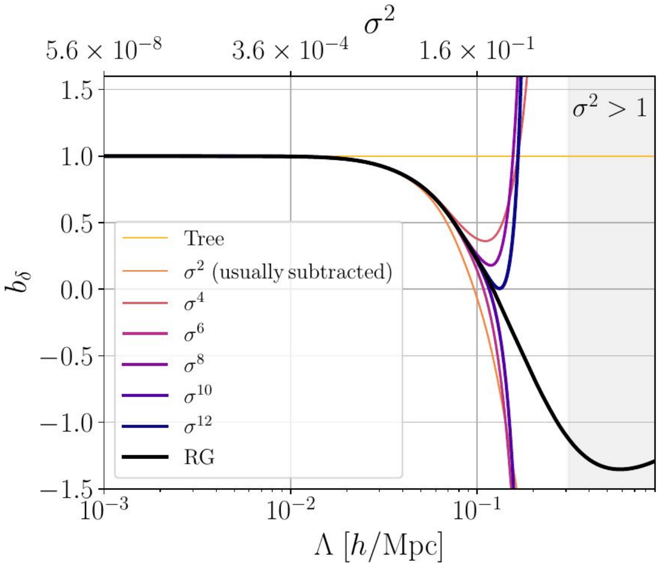
GGI, September 2025

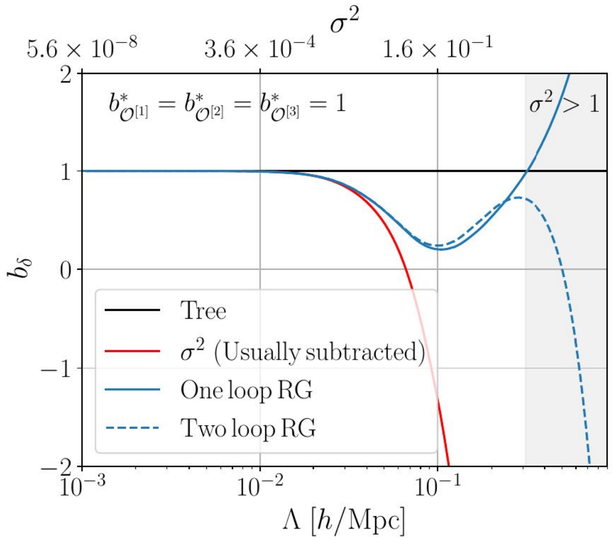
henrique.rubira@lmu.de

Message to take home

Callan-Symanzik equation:

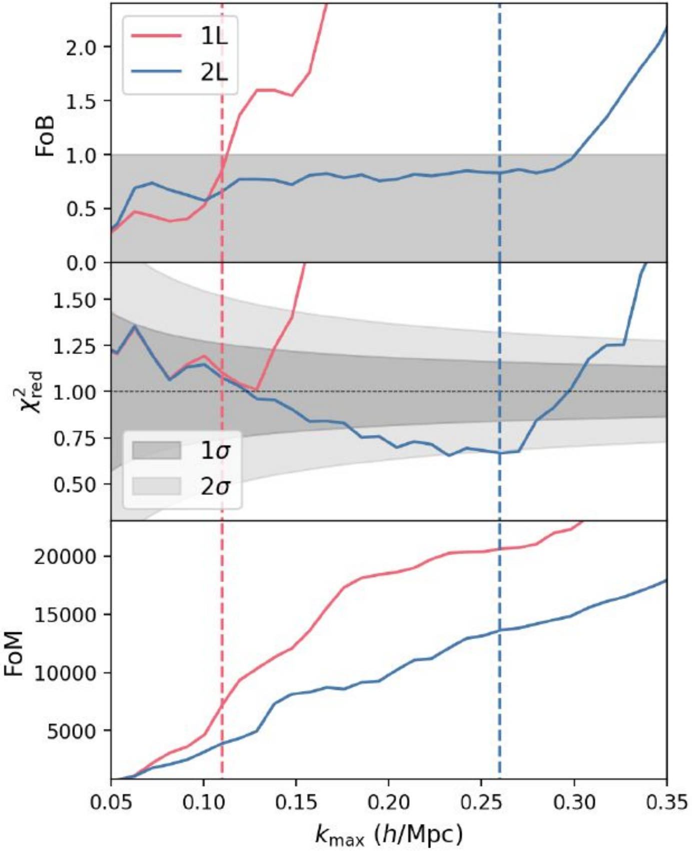
$$\frac{\partial g}{\partial \ln \Lambda} = \beta(g) \quad \longrightarrow \quad \boxed{\left.\frac{db_a}{d\Lambda}\right|_{1L} = -b_b s_{ba} \frac{d\sigma^2_\Lambda}{d\Lambda}\,,}$$

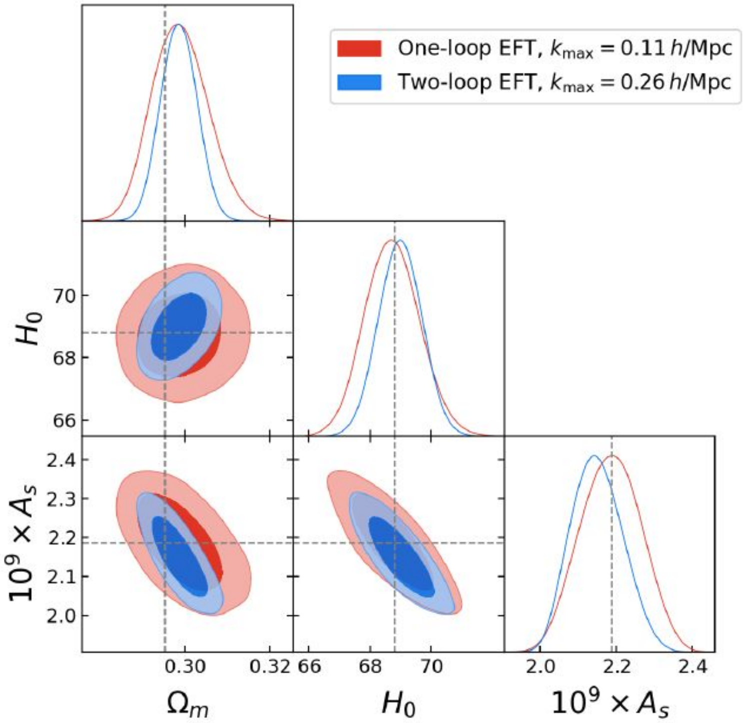




Fast two-loop evaluation

See Thomas's slides from last week

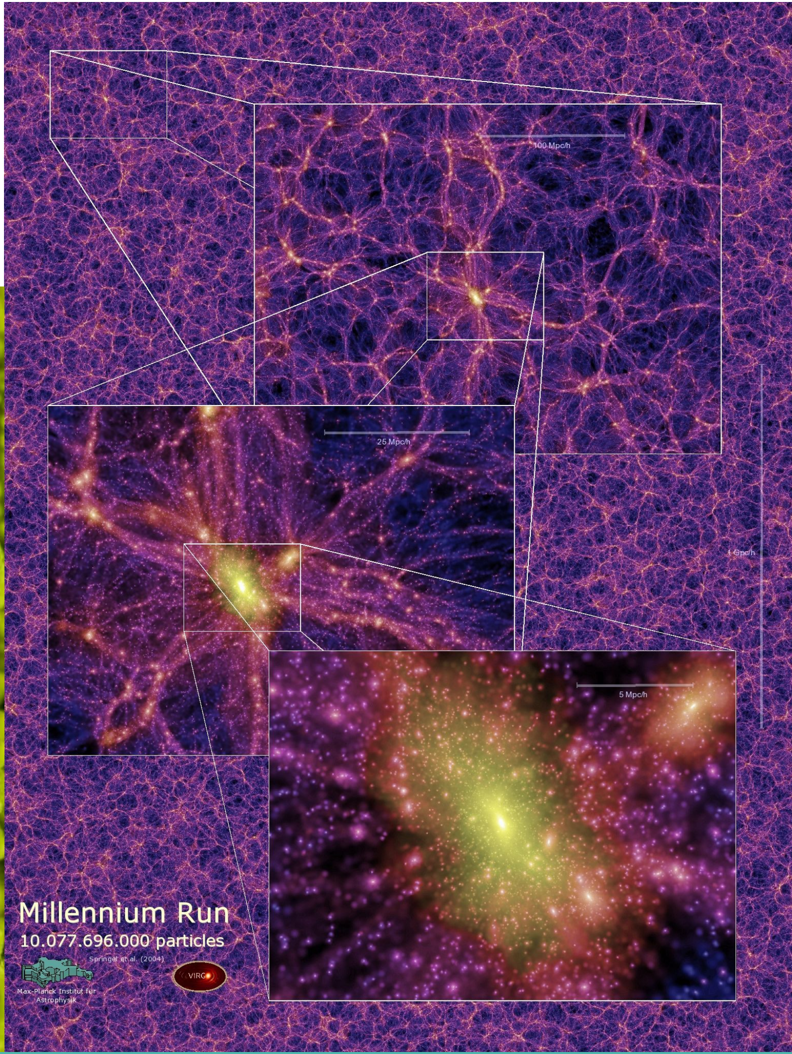




Bakx, **HR**, Chisari, Vlah 2025;

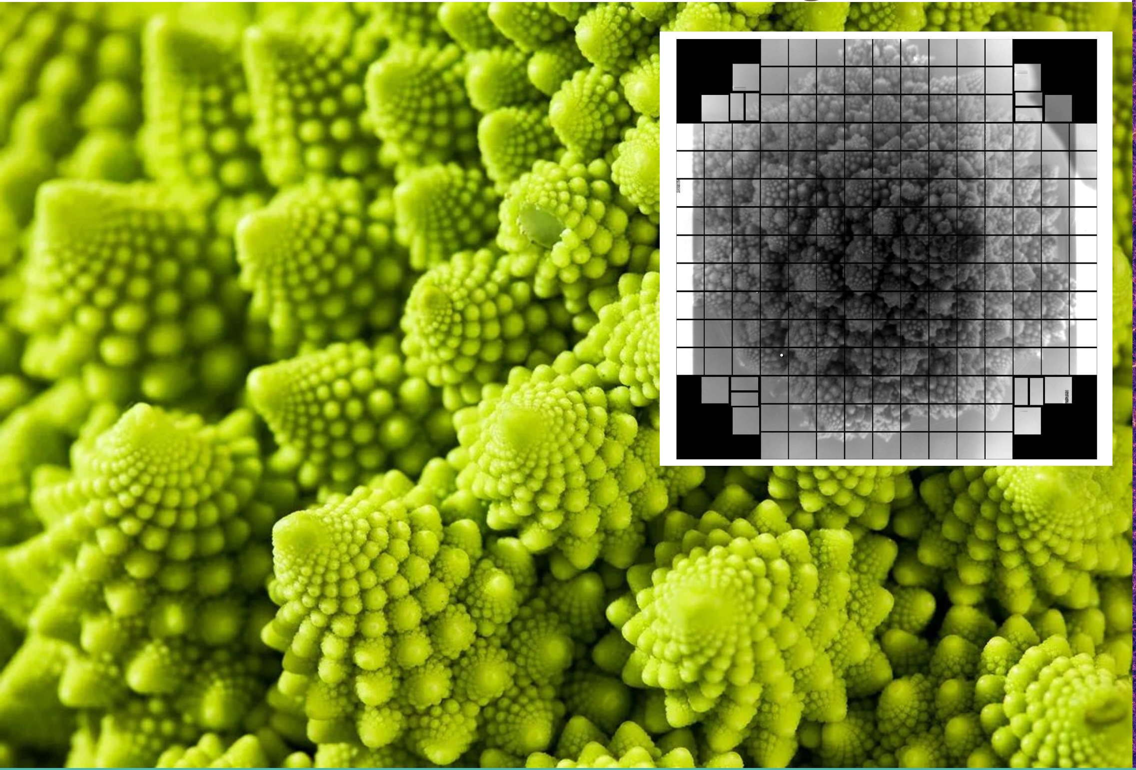
Intro

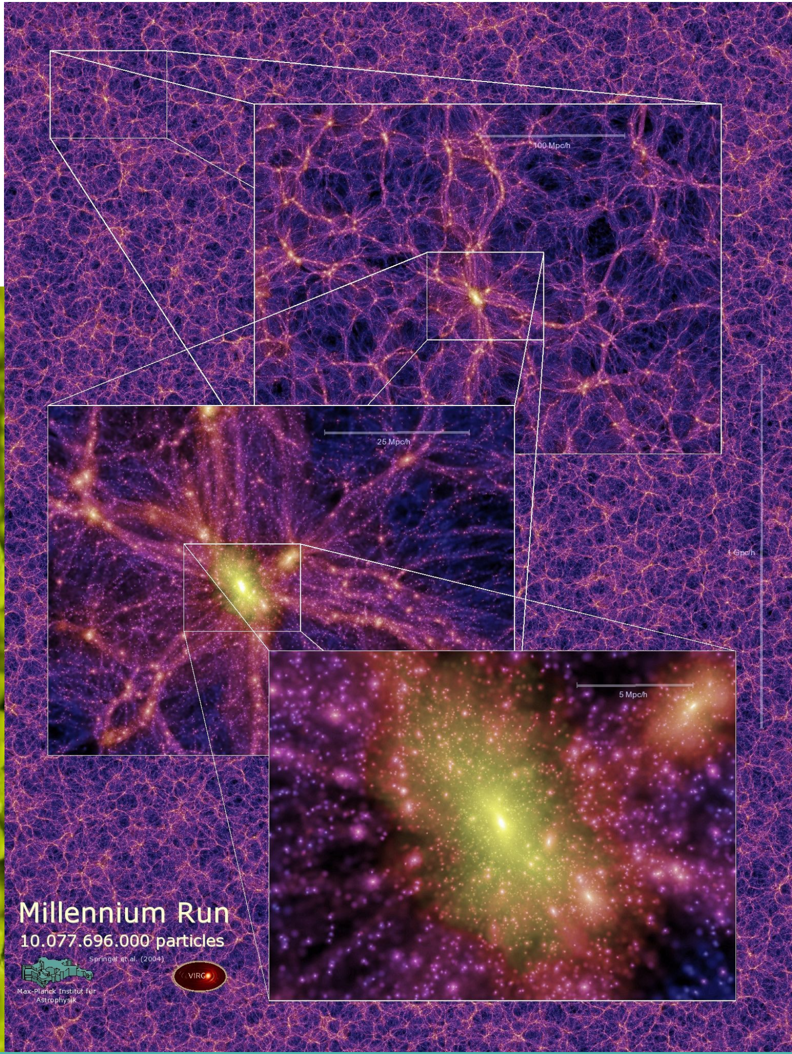
How things change with scale?
(from food to galaxies)



How things change with scale? (from food to galaxies)

First images of Rubin





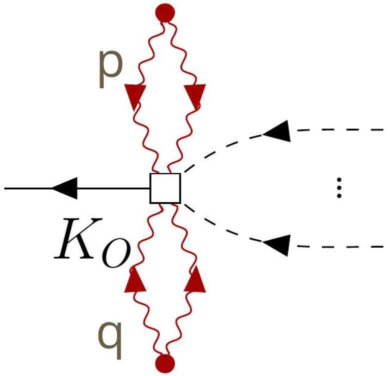
How things change with scale?

... Or on how to use a **one-loop** (renormalization group) to get **information about higher-loop** terms 'for free'

How things change with scale?

... Or on how to use a **one-loop** (renormalization group) to get **information about higher-loop** terms 'for free'

Intuition: **(1loop)^n ~ n-loop**
(for some part of the integrals domain)

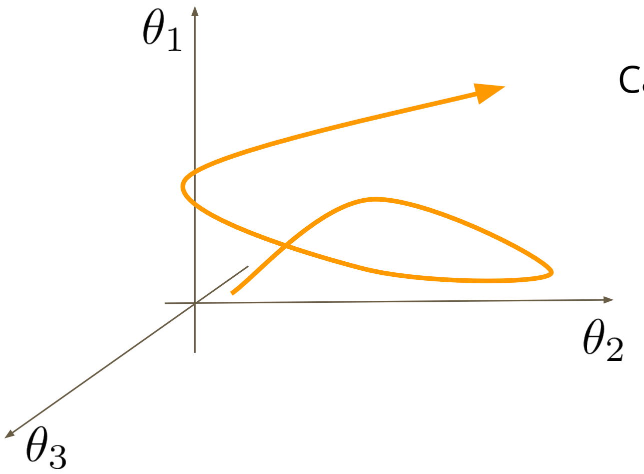


Method of regions (Beneke and Smirnov):

- p>>q (or q<<p): Absorbed by (1loop)^2
- p~q: Intrinsic 2-loop

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

Bias review: Desjacques, Jeong, Schmidt

Renormalizing the bias parameters

In a nutshell, it is an **Operator Product Expansion (OPE)**

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

Bias review: Desjacques, Jeong, Schmidt

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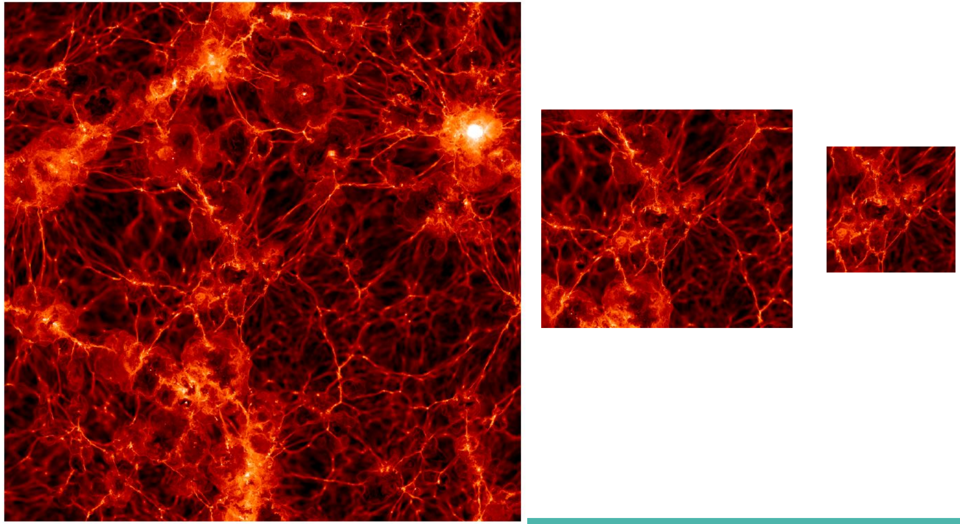
$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O^{\Lambda}(\tau) + c_{\epsilon,O}^{\Lambda}(\tau)\,\epsilon^{\Lambda}(\boldsymbol{x},\tau) \right] O^{\Lambda}(\boldsymbol{x},\tau) + \epsilon^{\Lambda}(\boldsymbol{x},\tau)$$

\+ counter-terms (Λ)



First order: δ ;

Second order: $\delta^2, \ \mathcal{G}_2$;

Third order: $\delta^3, \ \delta\,\mathcal{G}_2, \ \Gamma_3, \ \mathcal{G}_3$;

Contribution from arbitrarily small scales!

Option 1: Renormalize bias

McDonald 09
Assassi, Baumann, Green, Zaldarriaga
D'amico+ 22, for 4th-order

Renormalized bias: $$[\mathcal{O}_a] \equiv \mathcal{O}_a + Z_{ab}\mathcal{O}_b$$

RG conditions (to match n-pt functions):

$$\langle [\mathcal{O}_a]\delta_L(\boldsymbol{k}_1)\cdots\delta_L(\boldsymbol{k}_n)\rangle\Big|_{\boldsymbol{k}_i\to 0} = \langle \mathcal{O}_a\delta_L(\boldsymbol{k}_1)\cdots\delta_L(\boldsymbol{k}_n)\rangle_{\rm tree}\Big|_{\boldsymbol{k}_i\to 0},$$

We find:

$$Z^{1L}_{ab} = -\sigma^2_\Lambda s^{1L}_{ab},$$

$$Z^{2L}_{ab} = -Z^{1L}_{ac}\sigma^2_\Lambda s^{1L}_{cb} - \frac{1}{2}\int_{p,q<\Lambda} s^{2L}_{ab}(p/q)P^{\rm lin}(p)P^{\rm lin}(q)\,.$$

Order sigma^2

$$[\mathcal{O}_a]\Big|_{1L} = \mathcal{O}_a - \sigma^2_\Lambda s^{1L}_{ab}\mathcal{O}_b\,,$$

$$[\mathcal{O}_a]\Big|_{2L} = \mathcal{O}_a - \sigma^2_\Lambda s^{1L}_{ab}[\mathcal{O}_b]\Big|_{1L} - \frac{1}{2}\left(\int_{p,q<\Lambda} s^{2L}_{ab}(q/p)P^{\rm lin}(p)P^{\rm lin}(q)\right)\mathcal{O}_b\,.$$

Bakx, Garny, **HR**, Vlah

5th-order bias and 2-loop

Option 1: Ren

McDonald 09
i, Baumann, Green, Zaldarriaga
o+ 22, for 4th-order

Renormalized bias:

RG conditions (to ma

$$\langle[\mathcal{O}_a]\delta_L(\boldsymbol{k}_1)\cdots \quad {}_L(\boldsymbol{k}_n)\rangle_{\rm tree}\Big|_{\boldsymbol{k}_i\to 0}\,,$$

We find:

$$Z^{1\rm L}_{ab} = -\sigma^2_\Lambda \quad s^{2\rm L}_{ab}(p/q)P^{\rm lin}(p)P^{\rm lin}(q)\,.$$

$${}_{q<\Lambda}$$

$$[\mathcal{O}_a]\Big|_{1\rm L} = \mathcal{O}_a - \sigma^2_\Lambda s^{1\rm L}_{ab}\mathcal{O}_b\,,$$

$$[\mathcal{O}_a]\Big|_{2\rm L} = \mathcal{O}_a - \sigma^2_\Lambda s^{1\rm L}_{ab}[\mathcal{O}_b]\Big|_{1\rm I}$$

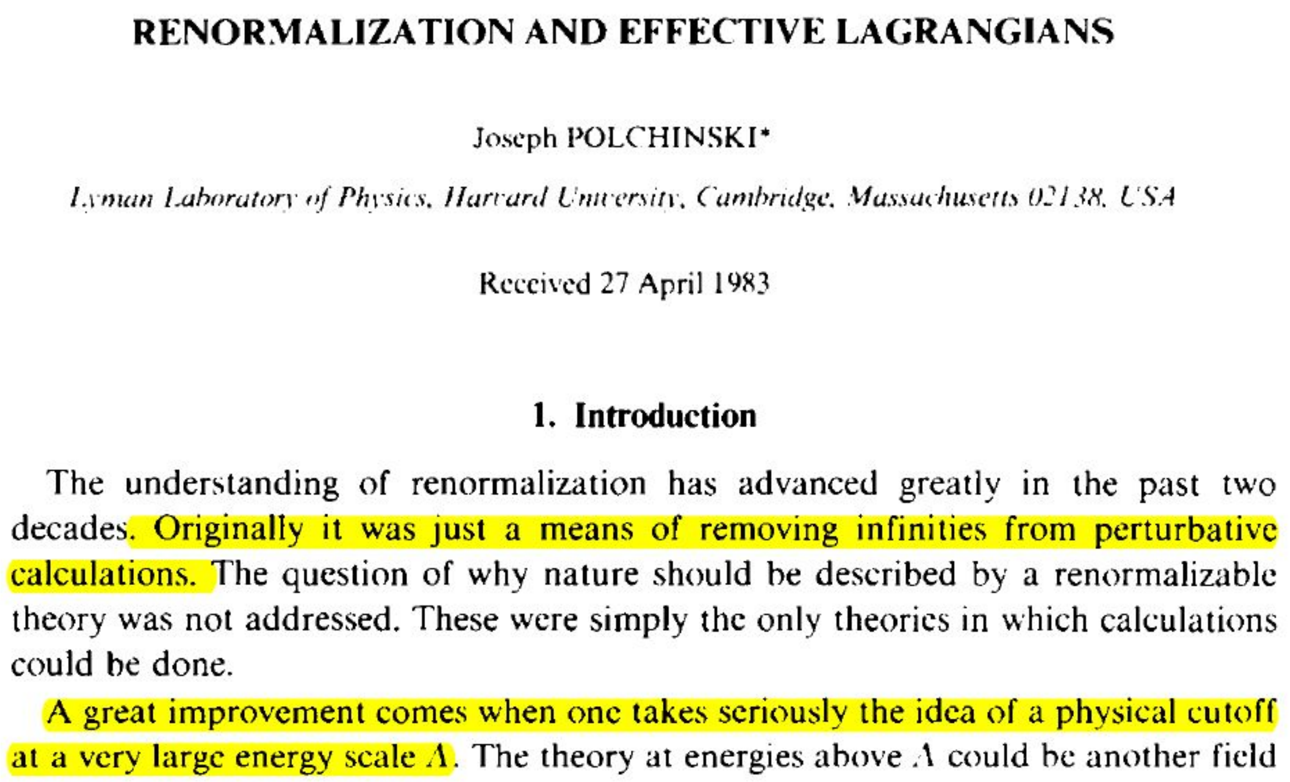
$c^{(5)}_{ab}$ / a \ b	$\mathrm{tr}[\Pi^{[1]}]$	$\mathrm{tr}[(\Pi^{[1]})^2]$	$(\mathrm{tr}[\Pi^{[1]}])^2$	$(\mathrm{tr}[\Pi^{[1]}])^3$	$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\mathrm{tr}[(\Pi^{[1]})^3]$	$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$
$\mathrm{tr}[\Pi^{[1]}]$	0	0	0	0	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]$	$\frac{34}{105}$	$\frac{127}{11025}$	$\frac{1312}{3675}$	$\frac{70832}{509355}$	$\frac{2928}{94325}$	$\frac{228808}{12733875}$	$-\frac{1322}{67375}$
$(\mathrm{tr}[\Pi^{[1]}])^2$	$\frac{34}{105}$	$\frac{127}{11025}$	$\frac{1312}{3675}$	$\frac{70832}{509355}$	$\frac{2928}{94325}$	$\frac{228808}{12733875}$	$-\frac{1322}{67375}$
$(\mathrm{tr}[\Pi^{[1]}])^3$	$\frac{3}{10}$	0	$\frac{34}{35}$	$\frac{1312}{1225}$	$\frac{127}{3675}$	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\frac{1}{6}$	$\frac{58}{525}$	$\frac{88}{175}$	$\frac{877}{1715}$	$\frac{23129}{77175}$	$\frac{892}{8575}$	$-\frac{5}{63}$
$\mathrm{tr}[(\Pi^{[1]})^3]$	$\frac{1}{10}$	$\frac{29}{175}$	$\frac{47}{175}$	$\frac{3971}{17150}$	$\frac{2224}{5145}$	$\frac{1338}{8575}$	$-\frac{5}{42}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	$\frac{41}{210}$	$\frac{38}{315}$	$\frac{134}{245}$	$\frac{2489609}{5093550}$	$\frac{30616}{94325}$	$\frac{1631026}{12733875}$	$-\frac{19231}{173250}$
$(\mathrm{tr}[\Pi^{[1]}])^4$	0	0	$\frac{3}{5}$	$\frac{68}{35}$	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[\Pi^{[1]}]$	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{47}{175}$	$\frac{76}{175}$	$\frac{29}{175}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{1}{10}$	$\frac{7}{30}$	$\frac{358}{525}$	$\frac{286}{525}$	0	0
$(\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{19}{75}$	$\frac{2}{75}$	$\frac{16}{245}$	$\frac{3092}{3675}$	$\frac{464}{3675}$	0
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{103}{1050}$	$\frac{17}{150}$	$\frac{64334}{231525}$	$\frac{10862}{25725}$	$\frac{40009}{231525}$	$\frac{11}{525}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{2}{35}$	$\frac{9}{35}$	$\frac{162353}{231525}$	$\frac{5416}{15435}$	$\frac{103736}{1157625}$	$\frac{197}{7875}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{134}{1575}$	$\frac{17}{75}$	$\frac{940493}{1697850}$	$\frac{30455}{67914}$	$\frac{605653}{4244625}$	$-\frac{3307}{86625}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{233}{3675}$	$\frac{1084}{3675}$	$\frac{174028}{231525}$	$\frac{1012}{3087}$	$\frac{4822}{46305}$	$\frac{758}{11025}$
$(\mathrm{tr}[\Pi^{[1]}])^5$	0	0	0	1	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^3](\mathrm{tr}[\Pi^{[1]}])^2$	0	0	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^3$	0	0	0	$\frac{3}{10}$	$\frac{3}{10}$	0	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	0	0	0	$\frac{7}{50}$	$\frac{9}{50}$	0
$\mathrm{tr}[\Pi^{[1]}](\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	0	0	$\frac{2}{75}$	$\frac{29}{75}$	0	0
$(\mathrm{tr}[\Pi^{[1]}])^2\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	0	0	$\frac{67}{210}$	$\frac{4}{35}$	0	$\frac{1}{10}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	0	0	$\frac{19}{735}$	$\frac{1819}{7350}$	$\frac{116}{3675}$	$\frac{19}{150}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	0	0	$\frac{17}{150}$	$\frac{143}{1050}$	$\frac{1}{21}$	$\frac{1}{15}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}\Pi^{[2]}]$	0	0	0	$\frac{211}{1715}$	$\frac{1037}{10290}$	$\frac{1493}{51450}$	$\frac{64}{525}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	0	0	$\frac{428}{1225}$	$\frac{313}{3675}$	0	$\frac{2}{21}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[3]}]$	0	0	0	$\frac{6407}{66150}$	$\frac{1189}{7350}$	$\frac{5371}{66150}$	$\frac{1}{50}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	0	0	$\frac{2579}{9450}$	$\frac{10}{63}$	$\frac{136}{3375}$	$\frac{41}{2250}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[3]}]$	0	0	0	$\frac{71047}{231525}$	$\frac{9997}{77175}$	$\frac{2411}{92610}$	$\frac{229}{3150}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[4]}]$	0	0	0	$\frac{261944}{2546775}$	$\frac{98489}{1131900}$	$\frac{1332383}{50935500}$	$\frac{373}{28875}$

(B.3)

akx, Garny, **HR**, Vlah

h-order bias and 2-loop

Option 2 (this talk): live well with the Lambda-dep



RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 27 April 1983

1. Introduction

The understanding of renormalization has advanced greatly in the past two decades. Originally it was just a means of removing infinities from perturbative calculations. The question of why nature should be described by a renormalizable theory was not addressed. These were simply the only theories in which calculations could be done.

A great improvement comes when one takes seriously the idea of a physical cutoff at a very large energy scale Λ. The theory at energies above Λ could be another field

How to relate the renormalization schemes?

N-point function renormalized bias
(McDonald and Assassi, Baumann, Green, Zaldarriaga)

$$[\![O']\!](\boldsymbol{k}')$$

How to connect both?

Finite cutoff bias
(This work)

$$O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$

How to relate the renormalization schemes?

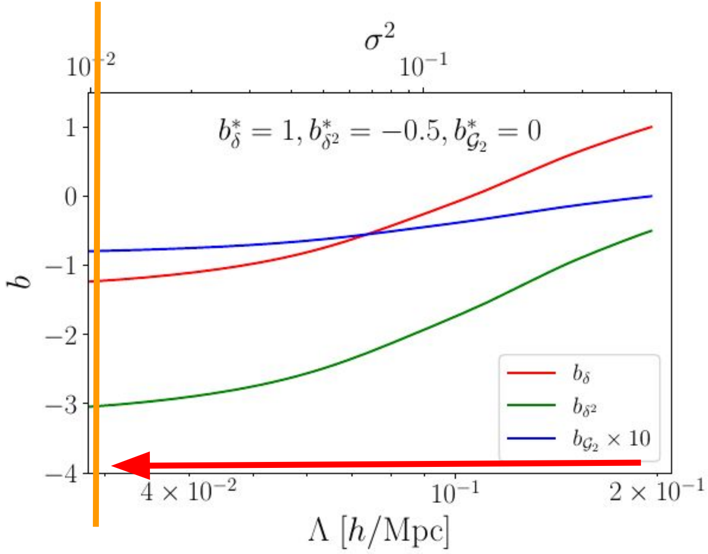
N-point function renormalized bias
(McDonald and Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

How to connect both?

Separate Universe

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$



Solution: Run the bias towards

$$\Lambda \to 0$$

HR, Schmidt 23

The equations

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

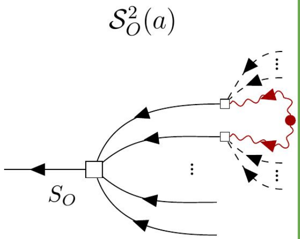
$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1L} + \frac{db_a}{d\Lambda}\Big|_{2L} + \ldots$$

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1L} + \frac{db_a}{d\Lambda}\Big|_{2L} + \dots$$



one-loop: $$\frac{db_a}{d\Lambda}\Big|_{1L} = -b_b s^{1L}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$

HR, Schmidt, 23

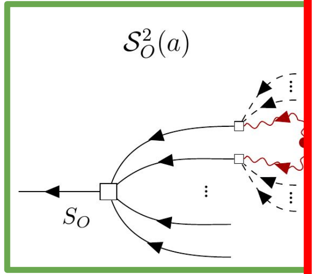
$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} + \frac{db_a}{d\Lambda}\Big|_{2\mathrm{L}} +$$



a	$c^{(3)}_{a\delta}$	$d^{(5)}_a$	$\tilde{d}^{(5)}_a$
$\mathrm{tr}[\Pi^{[1]}]$	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^2$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^3$	1	$\frac{70739}{33075}$	$\frac{4}{105}$
$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\frac{5}{9}$	$\frac{2917}{2205}$	$\frac{716}{1323}$
$\mathrm{tr}[(\Pi^{[1]})^3]$	$\frac{1}{3}$	$\frac{30263}{33075}$	$\frac{1748}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	$\frac{41}{63}$	$\frac{134957}{99225}$	$\frac{148}{441}$
$(\mathrm{tr}[\Pi^{[1]}])^4$	0	$\frac{272}{105}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[\Pi^{[1]}]$	0	$\frac{82}{105}$	$\frac{5}{21}$
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{6352}{4725}$	$\frac{4}{21}$
$(\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{592}{675}$	$\frac{8}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{16112}{19845}$	$\frac{3706}{6615}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{18166}{14175}$	$\frac{373}{735}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{12814}{11025}$	$\frac{401}{2205}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{27784}{19845}$	$\frac{232}{315}$
$(\mathrm{tr}[\Pi^{[1]}])^5$	0	1	0
$\mathrm{tr}[(\Pi^{[1]})^3](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{11}{45}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^3$	0	$\frac{7}{15}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	$\frac{31}{225}$	0
$\mathrm{tr}[\Pi^{[1]}](\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{163}{675}$	0
$(\mathrm{tr}[\Pi^{[1]}])^2\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{47}{105}$	$\frac{2}{21}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{173}{675}$	$\frac{2}{63}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{391}{1575}$	$\frac{5}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{197}{735}$	$\frac{82}{441}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{5057}{11025}$	$\frac{110}{441}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{643}{2835}$	$\frac{104}{945}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{5477}{14175}$	$\frac{4}{35}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[3]}]$	0	$\frac{8629}{19845}$	$\frac{491}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[4]}]$	0	$\frac{33403}{198450}$	$\frac{74}{6615}$

one-loop:

$$\frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} = -b_b s^{1\mathrm{L}}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$

HR, Schmidt, 23

$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	68/21	-	3	-	-	$-4/3$
δ^2	-	8126/2205	-	68/7	-	-	$-376/105$
$\mathcal{G}_2$	-	254/2205	-	-	-	-	116/105

two-loop:

$$\frac{db_\delta}{d\Lambda}\Big|_{2\mathrm{L}} = -30 b_b \tilde{d}^{(5)}_b \frac{d\sigma^2_\Lambda}{d\Lambda}\int_0^\Lambda dq \frac{q^2 P^{\mathrm{lin}}(q)}{2\pi^2} g(q/\Lambda)\,,$$

Bakx, Garny, **HR**, Vlah

Single-hard and double-hard limits

For the 1-loop:

$$K_b^{(n+2)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n, \boldsymbol{p}, -\boldsymbol{p})_{\mathrm{av}_{\hat{\mathbf{p}}}}^{p\to\infty} = c_{ba}^{(n+2)} K_a^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)$$

Bakx, Garny, **HR**, Vlah

Single-hard and double-hard limits

For the 1-loop:

$$K_b^{(n+2)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n, \boldsymbol{p}, -\boldsymbol{p})_{\mathrm{av}_{\hat{\mathbf{p}}}}^{p \to \infty} = c_{ba}^{(n+2)} K_a^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)$$

For the 2-loop:

$$K_b^{(n+4)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n, \boldsymbol{q}, -\boldsymbol{q}, \boldsymbol{p}, -\boldsymbol{p})_{\mathrm{av}_{\hat{\mathbf{p}}, \hat{\mathbf{q}}}}^{p,q \to \infty}\Big|_{r \equiv p/q = \mathrm{fixed}} = d_{ba}^{(n+4)}(r)\, K_a^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)$$

Bakx, Garny, **HR**, Vlah

Single-hard and double-hard limits

For the 1-loop:

$$K_b^{(n+2)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n,\boldsymbol{p},-\boldsymbol{p})^{p\to\infty}_{\mathrm{av}_{\hat{\mathbf{p}}}} = c_{ba}^{(n+2)}\, K_a^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$

For the 2-loop:

$$K_b^{(n+4)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n,\boldsymbol{q},-\boldsymbol{q},\boldsymbol{p},-\boldsymbol{p})^{p,q\to\infty}_{\mathrm{av}_{\hat{\mathbf{p}},\hat{\mathbf{q}}}}\Big|_{r\equiv p/q=\mathrm{fixed}} = d_{ba}^{(n+4)}(r)\, K_a^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$

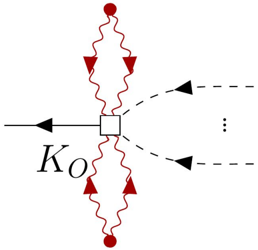
'one-loop squared'

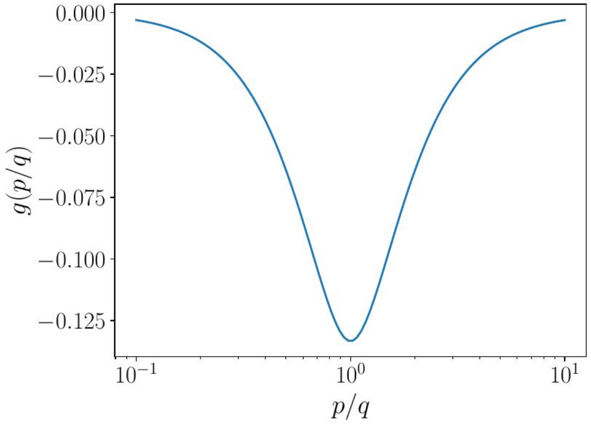
p

$$d_{b\delta}^{(5)}(p/q) = d_b^{(5)} + \tilde{d}_b^{(5)} \times g(p/q)$$

'Intrinsic' two-loop

q





Bakx, Garny, **HR**, Vlah

The (one-loop) solutions

Solutions (one-loop)

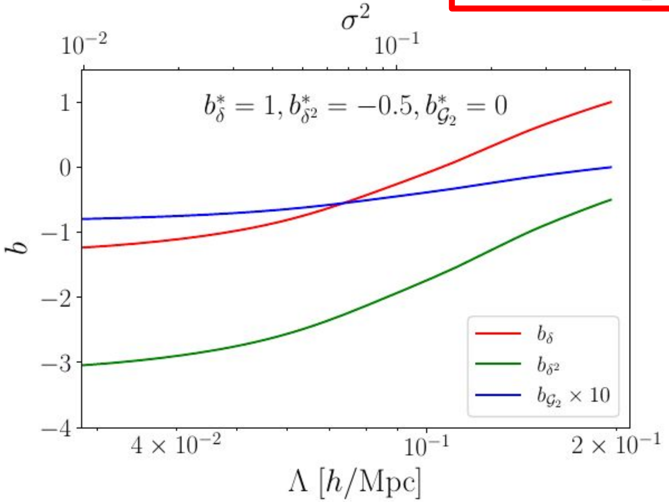
Wilson-Polchinski RG-equations

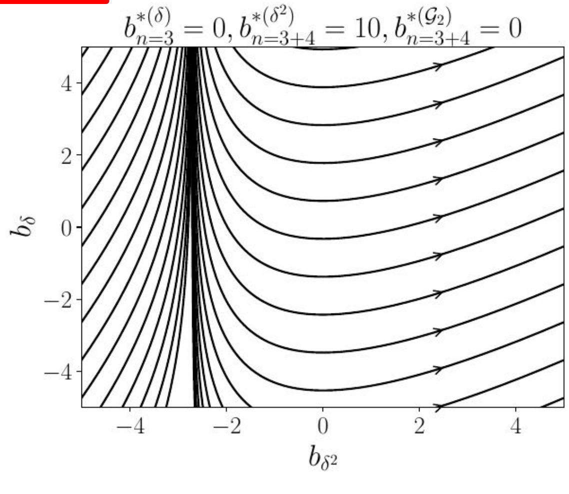
$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

HR, Schmidt 23





Why should you care?

What do the solutions of the RG tell us?

Bakx, Garny, **HR**, Vlah

We can always diagonalize the bias basis

$$\frac{db_i^{\rm diag}}{d\sigma^2} = \lambda_i b_i^{\rm diag}$$

$$b_a(\sigma^2) = p_{ai} e^{\lambda_i(\sigma^2 - \sigma_*^2)} c_i$$

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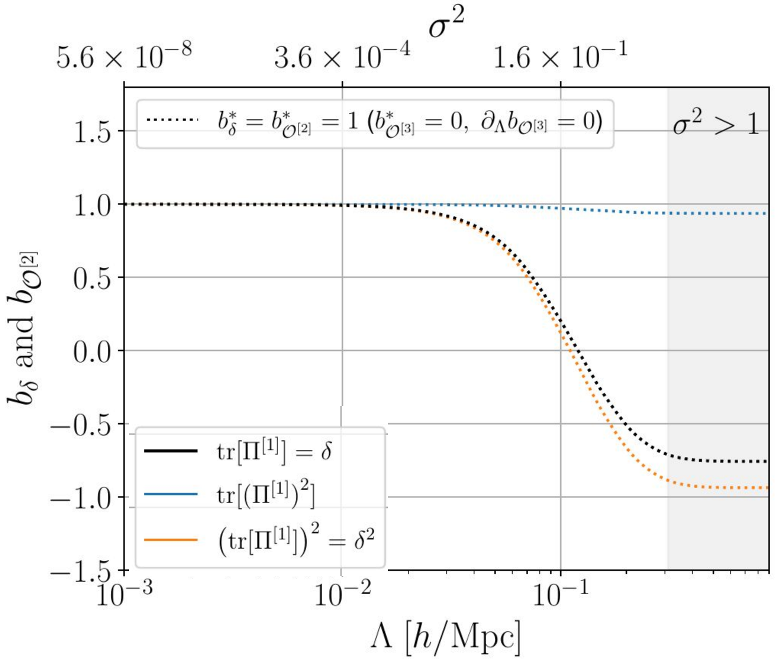
If we stop at second-order, we find:

$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \{0, 0, -3.69\}$$

Marginal *Relevant*



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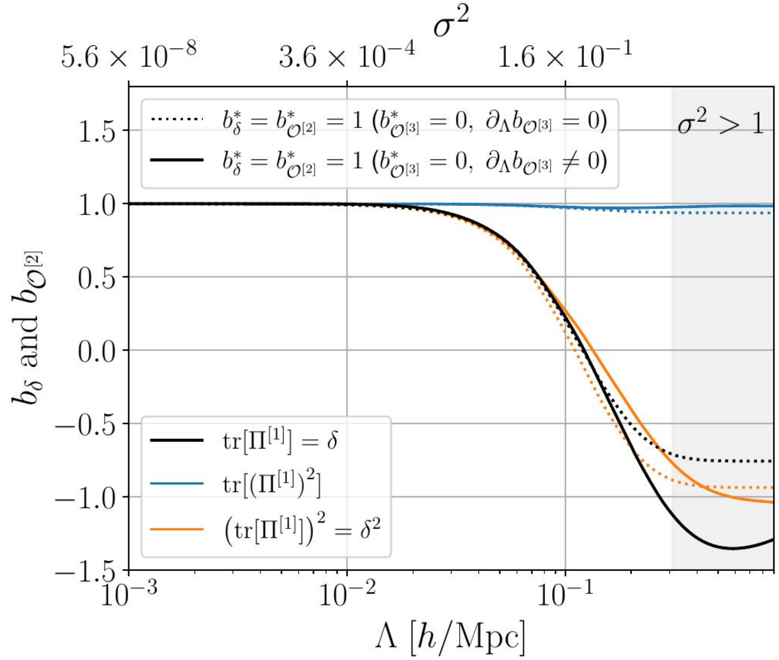
Marginal *Relevant*

Extending to third-order:

Irrelevant

$$\{0, 0, 0, -12.6, -3.44, -2.01, 0.220\}$$



~~What I want to say~~ vs. what I can say

We can always diagonalize the bias basis

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~~What I want to say~~ vs. what I can say

We can always diagonalize the bias basis

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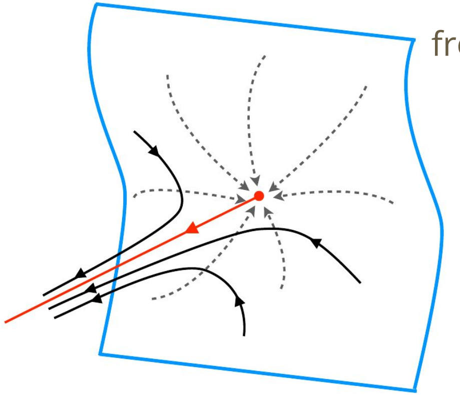


from Skinner notes

What happens when going to higher order?

If we stop at second-order, we find:

$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \{0, 0, -3.69\}$$

Marginal *Relevant*

~~Some parameters are more important than others~~

~~Reduce dimensionality of the problem~~

Extending to third-order:

Irrelevant

$$\{0, 0, 0, -12.6, -3.44, -2.01, 0.220\}$$

Some parameters are supersensitive to the UV

Why should you care II?

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$

Solution

$$b_a(\sigma^2) =$$

$$= b^*_a - (\sigma^2 - \sigma^2_*)\bar{s}^{1\mathrm{L}}_{ac} b^*_c + \frac{1}{2}(\sigma^2 - \sigma^2_*)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b^*_c - \frac{1}{6}(\sigma^2 - \sigma^2_*)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b^*_c + \ldots$$

1-loop

(1-loop)^2

(1-loop)^3

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$

Solution

$$b_a(\sigma^2) = \left[e^{-\bar{s}^{1\mathrm{L}}\times(\sigma^2-\sigma_*^2)}\right]_{ac} b_c^*$$

RG resums the series!

$$= b_a^* - (\sigma^2 - \sigma_*^2)\bar{s}^{1\mathrm{L}}_{ac} b_c^* + \frac{1}{2}(\sigma^2 - \sigma_*^2)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b_c^* - \frac{1}{6}(\sigma^2 - \sigma_*^2)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b_c^* + \dots$$

1-loop

(1-loop)^2

(1-loop)^3

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$

Solution

$$b_a(\sigma^2) = \left[e^{-\bar{s}^{1\mathrm{L}}\times(\sigma^2-\sigma^2_*)}\right]_{ac} b^*_c$$

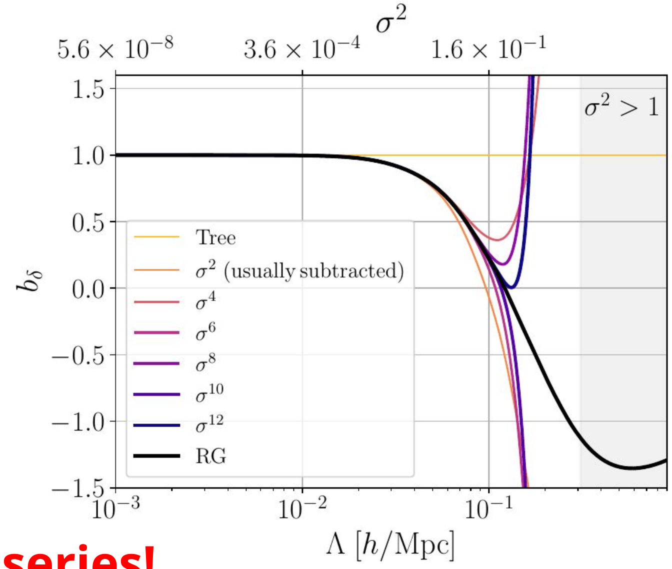
$$= b^*_a - (\sigma^2-\sigma^2_*)\bar{s}^{1\mathrm{L}}_{ac} b^*_c + \frac{1}{2}(\sigma^2-\sigma^2_*)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b^*_c - \frac{1}{6}(\sigma^2-\sigma^2_*)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b^*_c + \ldots$$

RG resums the series!

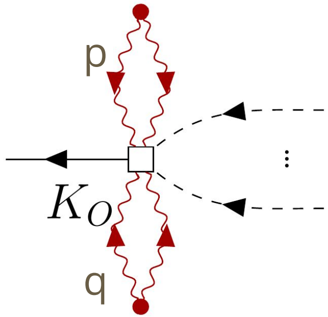
1-loop

(1-loop)^2

(1-loop)^3



Partial conclusion



Method of regions (Beneke, Smirnov):

- p>>q (or q<<p): Absorbed by (1loop)^
- p~q: Intrinsic 2-loop

1-loop RG resums part of higher-loop contributions (p>>q or q<<p regions)

...But is the other part ('intrinsic 2-loop', p~q region) small?

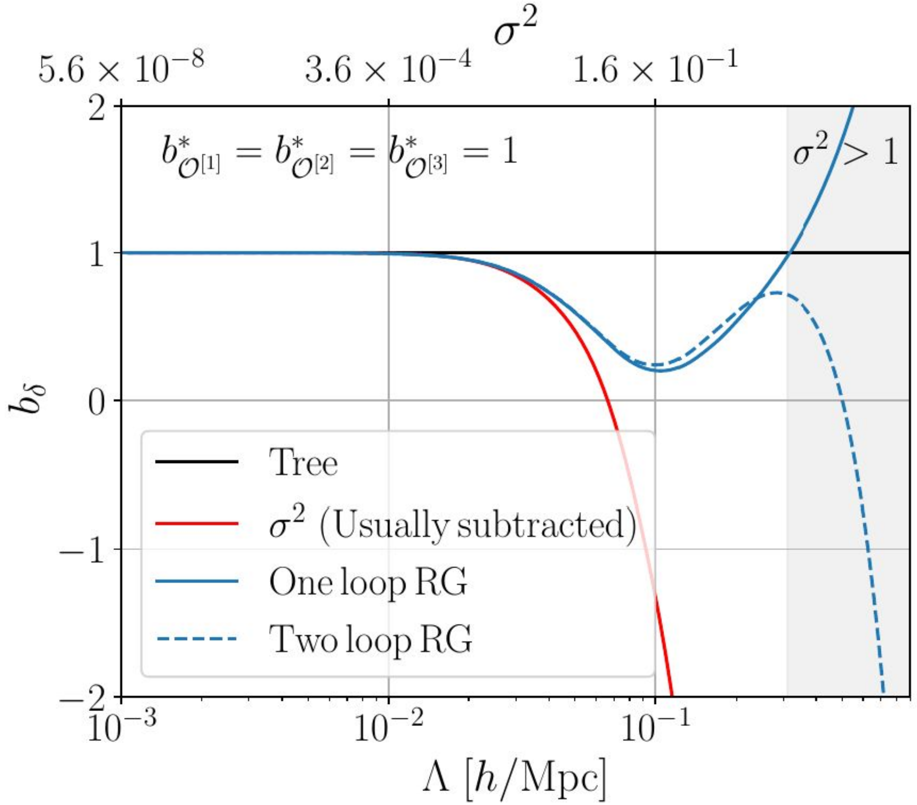
If YES: amaaaazing, 1-loop RG is doing something

If NO: out, I have to calculate the loops anyway to get most of info

The (two-loop) solutions

Solutions (two-loop)

Bakx, Garny,
HR, Vlah



'Intrinsic' two-loop part is small!

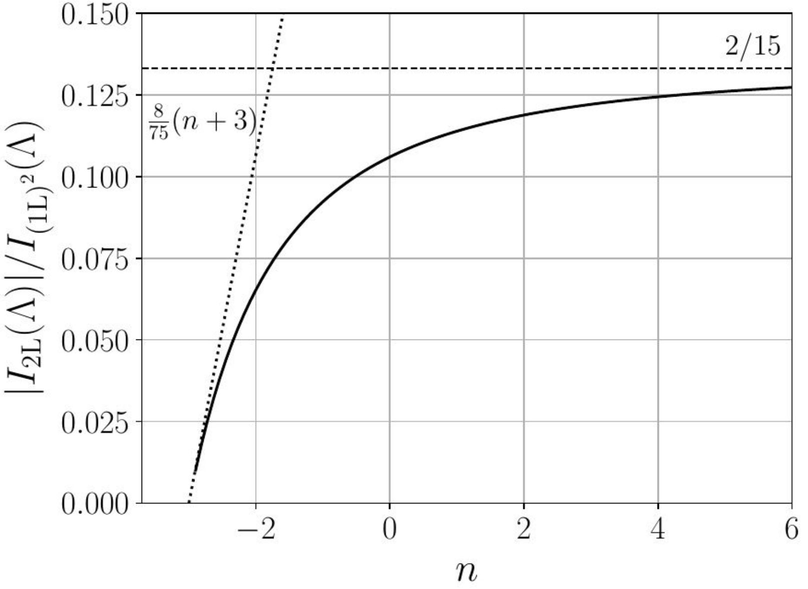
Not just small, it is PARAMETRICALLY small

Bakx, Garny, **HR**, Vlah

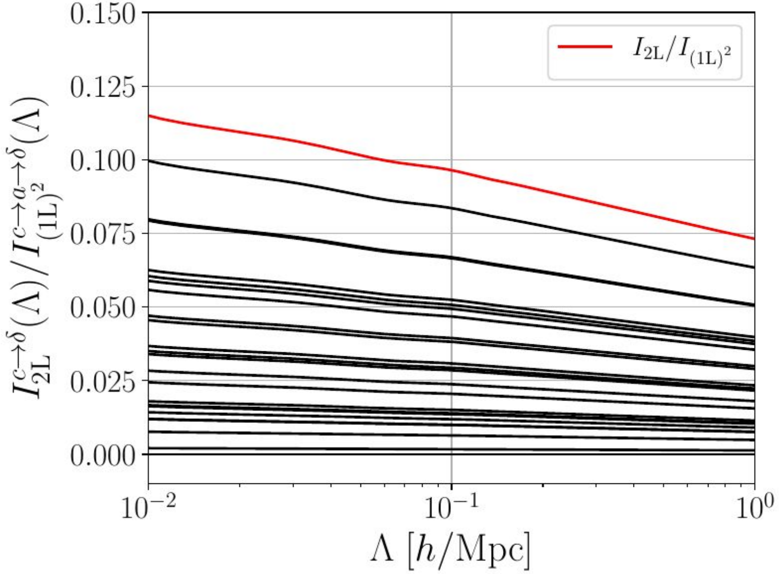
Good news: 1Loop RG takes care of most of the information

Ratio 2Loop over (1Loop)^2

Power law



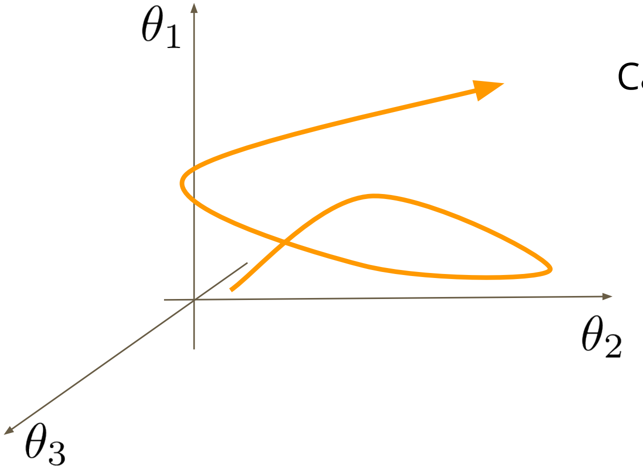
LCDM



QFT101

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

QFT101

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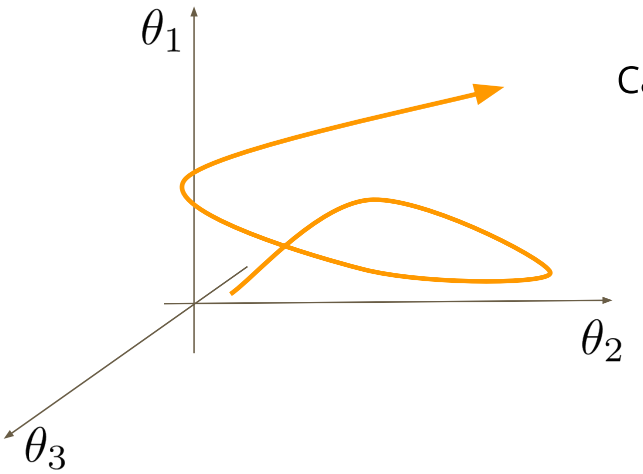

Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

For the fine-structure constant (QED):

$$\frac{d\alpha}{d\ln\mu} = \beta_{1\mathrm{L}}\alpha^2 + \beta_{2\mathrm{L}}\alpha^3 + O(\alpha^4)$$

$$\beta_{1\mathrm{L}} = 2/(3\pi)$$

$$\beta_{2\mathrm{L}} = 1/(4\pi^2)$$

Solution to the RG

$$\alpha(\mu)\big|_{\mathrm{LL}} = \frac{\alpha}{1 - \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*)}$$

$$= \alpha\left[1 + \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*) - \beta_{1\mathrm{L}}^2\alpha^2\ln^2(\mu/\mu_*) + \ldots\right]$$

QFT101

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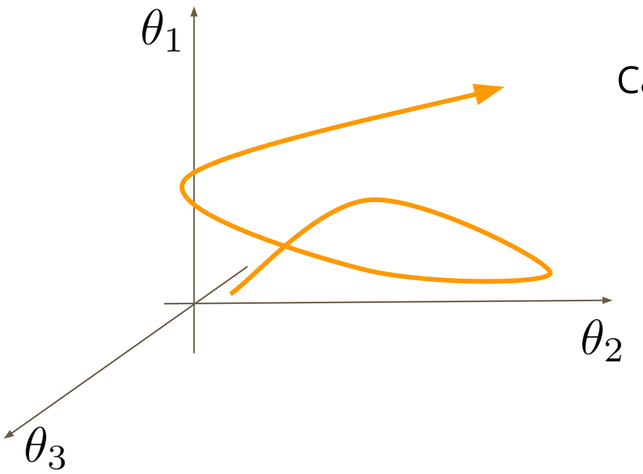

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$$= \alpha\left[1 + \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*) - \beta_{1\mathrm{L}}^2\alpha^2\ln^2(\mu/\mu_*) + \ldots\right]$$

Suppose you have an amplitude

$$\frac{\sigma_{\ell\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^\ell\left[c^{(\ell,\ell)}\ln^\ell(\mu/\mu_*) + c^{(\ell,\ell-1)}\ln^{\ell-1}(\mu/\mu_*) + \ldots\right]$$

$$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0[c^{(0,0)}\ln^0]$$

$$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1[c^{(1,1)}\ln^1 + c^{(1,0)}\ln^0]$$

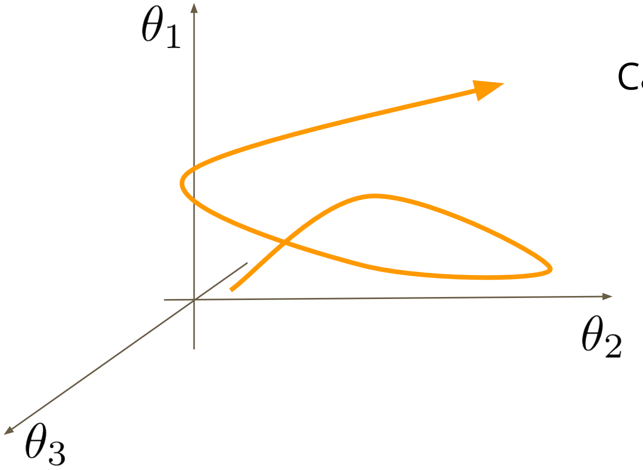
$$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2[c^{(2,2)}\ln^2 + c^{(2,1)}\ln^1 + c^{(2,0)}\ln^0]$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

QFT101

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

For the fine-structure constant (QED):

$$\frac{d\alpha}{d\ln\mu} = \beta_{1L}\alpha^2 + \beta_{2L}\alpha^3 + O(\alpha^4)$$

$$\beta_{1L} = 2/(3\pi)$$

$$\beta_{2L} = 1/(4\pi^2)$$

Solution to the RG

$$\alpha(\mu)\big|_{LL} = \frac{\alpha}{1 - \beta_{1L}\alpha\ln(\mu/\mu_*)}$$

$$= \alpha\left[1 + \beta_{1L}\alpha\ln(\mu/\mu_*) - \beta_{1L}^2\alpha^2\ln^2(\mu/\mu_*) + \ldots\right]$$

Suppose you have an amplitude

$$\frac{\sigma_{\ell L}}{\sigma_{\text{tree}}} = \alpha^\ell\left[c^{(\ell,\ell)}\ln^\ell(\mu/\mu_*) + c^{(\ell,\ell-1)}\ln^{\ell-1}(\mu/\mu_*) + \ldots\right]$$

	LL (1loop RG)	NLL (2loop RG)	N^{2}LL (3loop RG)
$\frac{\sigma_{\text{tree}}}{\sigma_{\text{tree}}} = \alpha^0$	$[c^{(0,0)}\ln^0]$		
$\frac{\sigma_{1L}}{\sigma_{\text{tree}}} = \alpha^1$	$[c^{(1,1)}\ln^1$	$+c^{(1,0)}\ln^0]$	
$\frac{\sigma_{2L}}{\sigma_{\text{tree}}} = \alpha^2$	$[c^{(2,2)}\ln^2$	$+c^{(2,1)}\ln^1$	$+c^{(2,0)}\ln^0]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

What for galaxies?

Where are the large logs?

$$\ln(\mu/\mu_*) \mapsto \frac{1}{n+3}\,.$$

$$\ln(\mu/\mu_*) \sim \frac{1}{3}\ln(\Lambda/k_{\rm eq})\,,$$

	LL (1loop RG)	NLL (2loop RG)	N²LL (3loop RG)
$\frac{P^{\rm lin}_{ab}}{P^{\rm lin}} = (\Delta^2_\Lambda)^0$	$\left[c^{(0,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$		
$\frac{P^{\rm 1L}_{ab}}{P^{\rm lin}} = (\Delta^2_\Lambda)^1$	$\left[c^{(1,1)}_{ab}\left(\frac{1}{n+3}\right)^1\right.$	$\left.+c^{(1,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$	
$\frac{P^{\rm 2L}_{ab}}{P^{\rm lin}} = (\Delta^2_\Lambda)^2$	$\left[c^{(2,2)}_{ab}\left(\frac{1}{n+3}\right)^2\right.$	$+c^{(2,1)}_{ab}\left(\frac{1}{n+3}\right)^1$	$\left.+c^{(2,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

	LL (1loop RG)	NLL (2loop RG)	N²LL (3loop RG)
$\frac{\sigma_{\rm tree}}{\sigma_{\rm tree}} = \alpha^0$	$\left[c^{(0,0)}\ln^0\right]$		
$\frac{\sigma_{\rm 1L}}{\sigma_{\rm tree}} = \alpha^1$	$\left[c^{(1,1)}\ln^1\right.$	$\left.+c^{(1,0)}\ln^0\right]$	
$\frac{\sigma_{\rm 2L}}{\sigma_{\rm tree}} = \alpha^2$	$\left[c^{(2,2)}\ln^2\right.$	$+c^{(2,1)}\ln^1$	$\left.+c^{(2,0)}\ln^0\right]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

~~What I want to say~~ vs. what I can say

~~RG allows for resummation of part of the information from higher loops~~

In High-Energy, RG is powerful: resumming information from integrating in between different scales

We still have to better understand which scales can the RG for LSS help

Finally:
PNG and stochasticity

PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b^{*\{\delta\}\mathrm{G}}_{n=3}\right]\frac{d\sigma^2_\Lambda}{d\Lambda} - a_0 f_{\mathrm{NL}}\left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b^{*\{\delta\}\mathrm{NG}}_{n=3}\right]\left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)}\frac{d\sigma^2_\Lambda}{d\Lambda};$$

Rederivation of Dalal+ 07 (in an elegant way)

Now a coupled set of ODEs

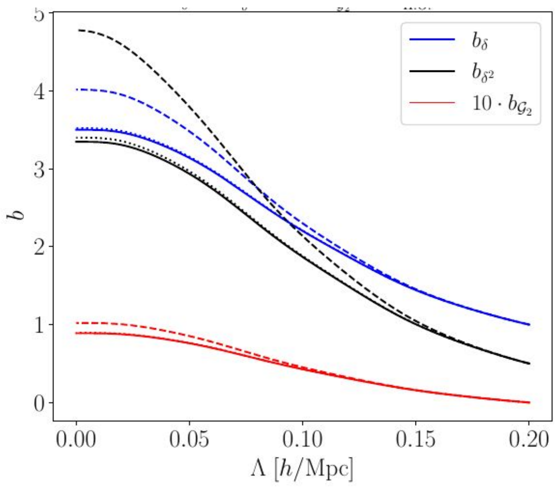
$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\mathrm{NL}} b^{*\{\Psi\}\mathrm{NG}}_{n=3}\frac{d\sigma^2_\Lambda}{d\Lambda} - 4 a_0 f_{\mathrm{NL}} b_{\delta^2}\frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\mathrm{NL}}\left[\frac{272}{21} b_{\delta^2} + b^{*\{\Psi\delta\}\mathrm{G}}_{n=3+4} + b^{*\{\Psi\delta\}\mathrm{NG}}_{n=3+4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

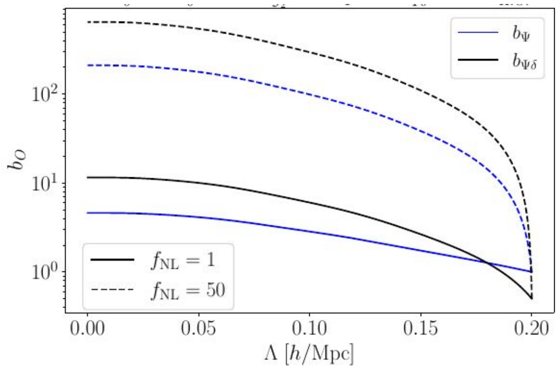
$s^O_{O'}$	δ^2	δ^3	$\delta\mathcal{G}_2$	Ψ	$\Psi\delta$	$\Psi\delta^2$	$\Psi\mathcal{G}_2$	$\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\delta\,\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\mathrm{Tr}\,\Psi\Pi^{[2]}$
δ	68/21	3	−4/3	−13/21	13/21	2	−4/3	34/21	1	34/21
δ^2	8126/2205	68/7	−376/105	43/135	478/135	47/21	−31/21	124/315	178/105	14347/6027
$\mathcal{G}_2$	254/2205	-	116/105	−1699/13230	79/2205	-	−1/21	−661/4410	4/35	−241/735
Ψ	4	-	-	-	-	1	-	-	-	-
$\delta\Psi$	272/21	12	−8/3	-	-	68/21	-	-	-	-
$\mathrm{Tr}\,\Psi\Pi^{[1]}$	64/105	-	16/15	-	-	-	-	-	8/105	58/305

Nikolis, **HR**, Schmidt

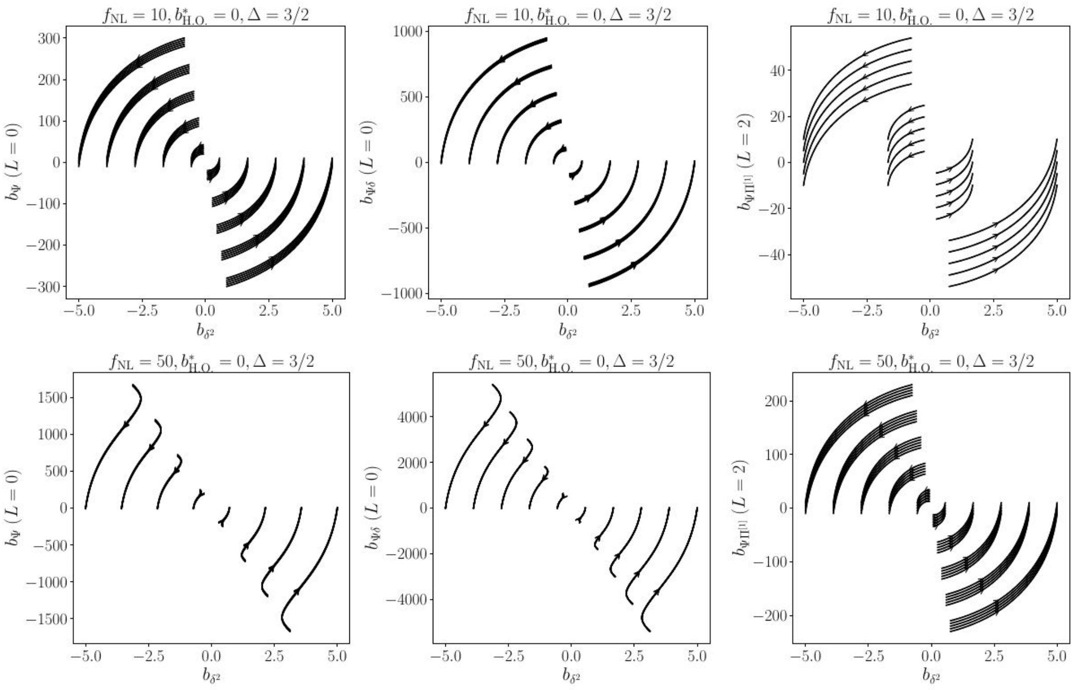
PNGs



b_δ
b_{δ^2}
$10 \cdot b_{\mathcal{G}_2}$
b
Λ [h/Mpc]



b_Ψ
$b_{\Psi\delta}$
$f_{\rm NL} = 1$
$f_{\rm NL} = 50$
b_O
Λ [h/Mpc]



$f_{\rm NL} = 10, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 50, b^*_{\rm H.O.} = 0, \Delta = 3/2$
b_Ψ $(L = 0)$
$b_{\Psi\delta}$ $(L = 0)$
$b_{\Psi\Pi^{[1]}}$ $(L = 2)$
b_{δ^2}

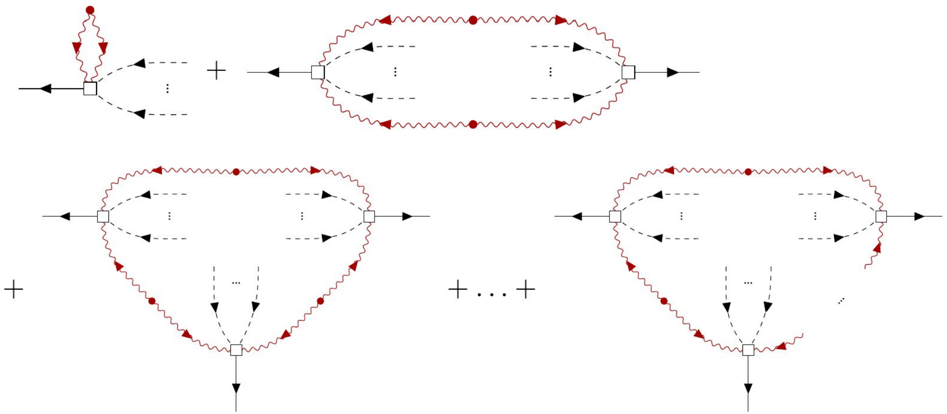
Stochasticity

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

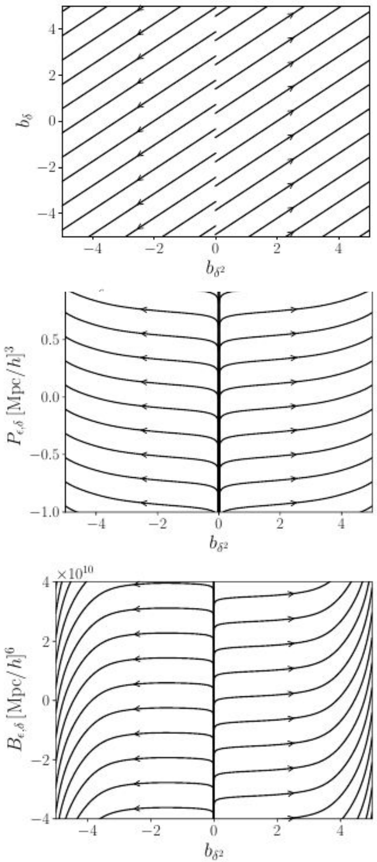
$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,O} O(\boldsymbol{k}_{m+1})$$

Simple expression for how stochastic terms talk to each other

$$\frac{d}{d\Lambda} C^{(m)}_O(\Lambda) \propto -\left[P_{\rm L}(\Lambda)\right]^{p-1} \frac{d\sigma^2_\Lambda}{d\Lambda} \sum_{O_1,O_2,\ldots O_m} s^O_{O_1 O_2 \ldots O_m} C^{(i_1)}_{O_1}(\Lambda)\ldots C^{(i_p)}_{O_p}(\Lambda)$$



Simple diagrammatic interpretation



HR, Schmidt, 24

Conclusions

- Cross-check for EFT inference;

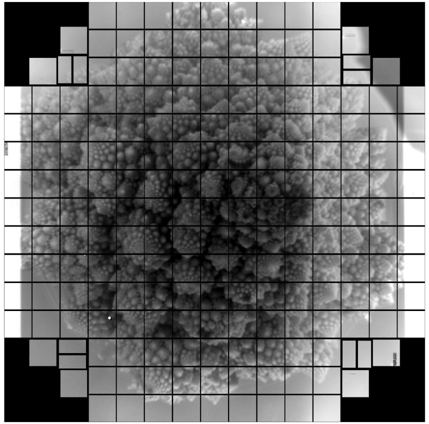
- Systematic renormalization (+ stochastic +PNG);

- More information from resummation? TBD!

- Still to be understood:

    1) RG stability when going to higher-order
    2) scales in between which RG can operate

First images of Rubin



Thank you!