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Relativistic effects on the past light-cone

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Introduction

Nowadays we have a good understand of our Universe composition. Although some very important problems remains to be answered.

- As the nature of the Dark Matter and Dark Energy.
- Tensions on H_o and σ_8 .
- Is inflationary theory the correct description of the primordial universe? Which inflationary model is the correct description?

Precision cosmology may open new prospects of advances in these problems. As well expectations are set in the possibility of probing new features as primordial gravitational waves, primordial black holes, primordial non-Gaussianity...

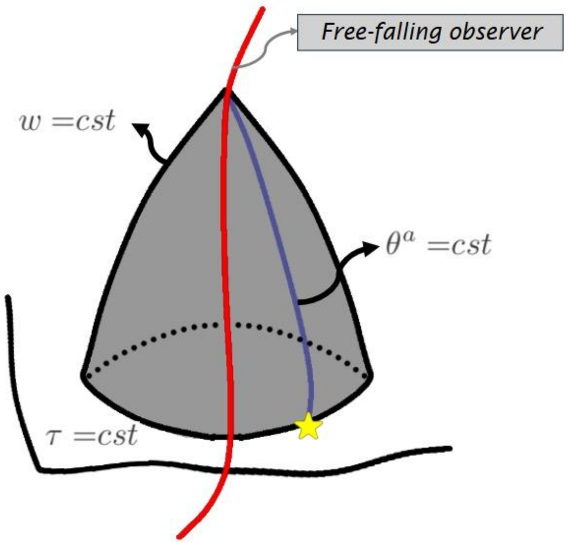
Very interesting possibilities. However, precision cosmology means not only accurate observations, but also accurate theoretical predictions.

What does we measure?

Measurements of the luminosity distance - redshift → Expansion rate

Light reach us through our past light-cone and perturbations modifies the information that we receive.

In order to fully account for these perturbative effects one must to perform theoretical predictions direct aligned with how observations are performed.



Observations are performed in our past light-cone and in terms of observed angles "apparent angular position of the source" and observed redshift (see Bonvin and Durrer PRD 2012).

Cosmological Perturbation Theory on the past light-cone

1. The Cosmological Perturbation Theory on the Geodesic Light-Cone background. *In collaboration with: Giuseppe Fanizza, Giovanni Marozzi and Gloria Schiaffino* ***JCAP (2021)***.
2. Gauge Invariance on the Light-Cone: curvature perturbations and radiative degrees of freedom.. *In collaboration with: Giuseppe Fanizza and Giovanni Marozzi* ***JCAP (2023)***.
3. Second Order Cosmological Perturbation Theory on the Geodesic Light-Cone background. *In collaboration with: Pierre Béchaz, Giuseppe Fanizza and Giovanni Marozzi* ***To appear soon.***
4. Iterative method to non-linear relations between observables. *In collaboration with: Giuseppe Fanizza, Giovanni Marozzi and Tiziano Schiavone* ***To appear soon.***

Motivations

- *Gauge invariance of cosmological observables*

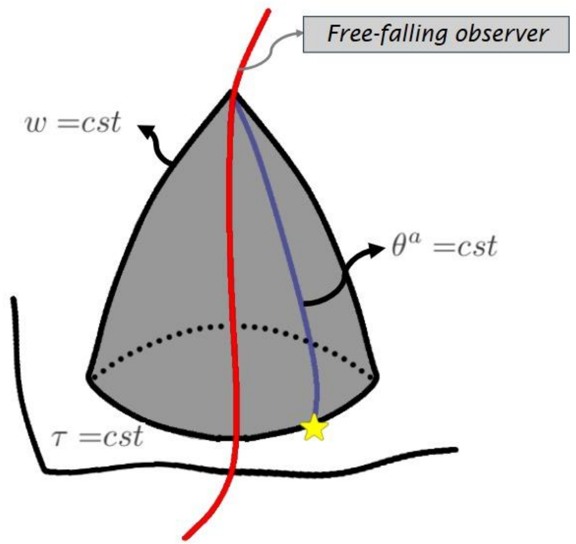
Regarding late-time cosmological observables, the approach of the pioneering works (see *Bonvin, Durrer and Gasparini* ***PRD 2006***) are resumed in

Solving perturbatively the geodesics equation → **Solving perturbatively the Jacobi Map** → **Using the Jacobi Map to compute the observable** → **Express it in terms of a fiducial model (e.g. redshift, luminosity distance,..)**

Instead, the GLC gauge provides covariant equations (see Fanizza, Gasperini, Marozzi and Veneziano ***JCAP 2013***) for cosmological observables

Linearizing the covariant expression → **Express it in terms of a fiducial model** → **Gauge invariant cosmological observable**

GLC Gauge



$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}\left(d\theta^a - U^a dw\right)\left(d\theta^b - U^b dw\right)$$

$$(1+z) = \frac{\Upsilon_o}{\Upsilon_s}$$

$$u_\mu = -\,\partial_\mu \tau = -\delta^\tau_\mu$$
$$k_\mu = -\,\partial_\mu w = -\delta^w_\mu$$

$$d_A^2 = \frac{\sqrt{\gamma}}{\left(\frac{det\partial_\tau \gamma_{ab}}{4\sqrt{\gamma}}\right)_o}$$

Prescription to observables

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}\left(d\theta^a - U^a dw\right)\left(d\theta^b - U^b dw\right)$$

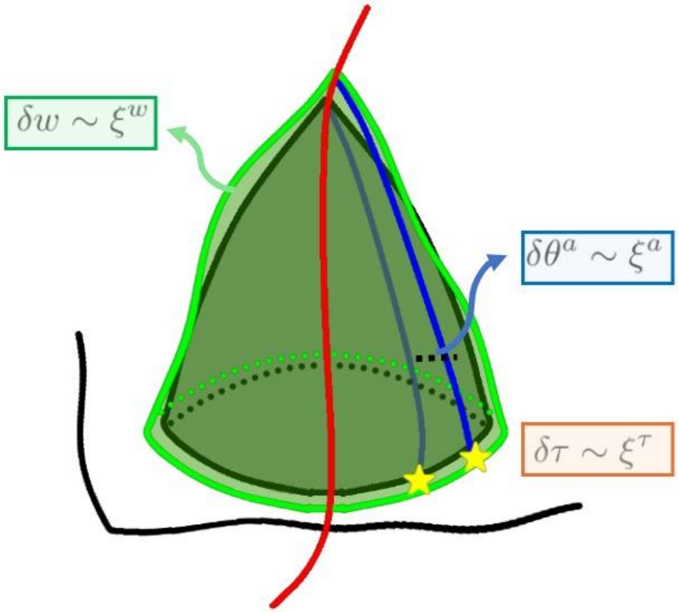
Gauge Invariant

$$\mathcal{O}\left(\Upsilon, U_a, \gamma_{ab}\right) \to \mathcal{O}\left(\Upsilon, U_a, \gamma_{ab}\right) + \delta\tau_z \partial_\tau \mathcal{O} \dots$$
$$\to \mathcal{O}_z\left(\Upsilon, U_a, \gamma_{ab}\right)|_{PG} + f\left(\xi^\tau, \xi^w, \xi^a\right) \dots$$

Gauge Invariant Bardeen Potentials

$$\tilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} - 2\nabla_{(\mu}\xi_{\nu)} + \dots ,$$

Different perspectives on cosmological perturbation theory



Gauge Invariant Velocity Potential

$\partial_w \xi^w$ → Integrated and Local Sachs-Wolfe effect

$$\xi^\tau = \int_{\eta_{in}}^{\eta} d\eta' \left[a\Phi + \partial_\eta (a\sigma)\right] ,$$

$$\xi^w = -\left(S_r + \sigma\right)_s^o - \int_{\eta}^{\eta_o} d\eta' \left(\Phi + \Psi - \sigma_r^{GI} - h_{rr}\right) + w_o$$

$$= -\left(S_r + \sigma\right)_s^o + \delta w_{GI} + w_o ,$$

$$\xi^a = -\left(\bar\gamma^{ab} S_b\right)_s^o + \int_{\eta}^{\eta_o} d\eta' \left\{\bar\gamma^{ab}\left[\left(\sigma_b^{GI} + 2h_{rb} - D_b \delta w_{GI}\right) - D_b \left(\sigma + S_r + w\right)^o\right]\right\}$$

$$D_a \xi^a = -D^a \left(\frac{S_a}{r^2}\right)_s^o + \int_{\eta}^{\eta_o} \frac{d\eta'}{r^2} \left[D^a \left(\sigma_a^{GI} + 2h_{ra} - D_a \delta w_{GI}\right) - D^2 \left(\sigma + S_r + w\right)^o\right] \quad (4)$$

Lensing

General Recipe

- *Roadmap to gauge invariant observables*

General Recipe:

1. Linearize the GLC expression for the chosen observable.

$$\mathcal{O} = \bar{\mathcal{O}}\left(\tau\right) + \delta\mathcal{O}\left(\tau,\, w,\, \tilde{\theta}^a\right)$$

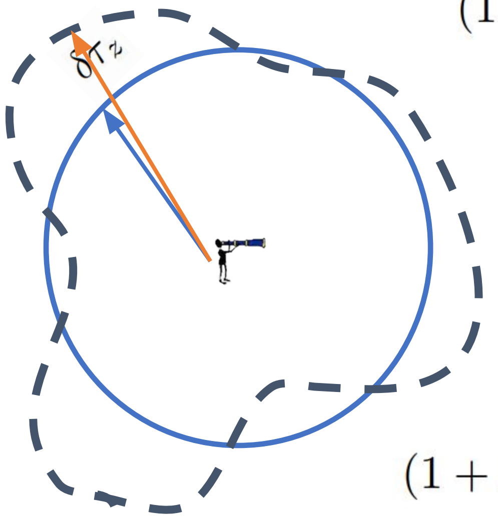
1. Express the time coordinate in terms of the redshift of the source.

$$\begin{aligned}\bar{\mathcal{O}}\left(\tau\right) + \delta\mathcal{O}\left(\tau,\, w,\, \tilde{\theta}^a\right) =& \bar{\mathcal{O}}\left(\tau_z\right) + \delta\mathcal{O}\left(\tau_z,\, w,\, \tilde{\theta}^a\right) + \delta\tau_z\partial_\tau\bar{\mathcal{O}} \\ =& \bar{\mathcal{O}}\left(\tau_z\right) + \delta\mathcal{O}_z\left(\tau_z,\, w,\, \tilde{\theta}^a\right)\end{aligned}$$

General Recipe

- *Roadmap to gauge invariant observables*

2. Express the time coordinate in terms of the redshift of the source.



$$(1+z)=\frac{\Upsilon_o}{\Upsilon_s}=\frac{a_o}{a_s}\left(1+\frac{1}{2}N|_s^o\right)$$

$$\delta\tau_z\equiv\tau-\tau_z \qquad (1+z)=\frac{a_o}{a(\tau_z)}$$

$$(1+z)=\frac{a_o}{a_s}\left(1-H_z\delta\tau_z+\frac{1}{2}N|_s^o\right)$$

$$\delta\tau_z=\frac{1}{2H_z}N|_s^o$$

Relation between observables and gauge invariance

- *Building the fiducial coordinates at non-linear order*

$$f^A\left(x^\mu\right) = 0$$

$$A = 0,\, 1,\, 2,\, 3$$

$$Q^A_{(n)}\left(\bar{x}^\mu\right) \to \delta x^\nu \partial_\nu \delta f^A + h.o.$$

$$f^A\left(x^\mu\right) = \bar{f}^A\left(\bar{x}^\mu\right) + \delta f^A\left(\bar{x}^\mu\right) + \delta x^\nu \partial_\nu \bar{f}^A\left(\bar{x}^\mu\right) + Q^A_{(n)}\left(\bar{x}^\mu\right) + h.o.$$

$$\bar{f}^A\left(\bar{x}^\mu\right) = 0$$

$$\delta x^\mu_{(n)}\left(\partial_\mu \bar{f}^A\right)_{x^\nu = \bar{x}^\nu} = -\delta f^A_{(n)}\left(\bar{x}^\nu\right) - Q^A_{(n)}\left(\bar{x}^\nu\right)$$

Relation between observables and gauge invariance

- *Building the fiducial coordinates at non-linear order*

$$f^A\left(x^\mu\right)=0$$

$$A=0,\,1,\,2,\,3$$

↓

$$\delta\eta_z^{(n)}=\frac{\delta z_{(n)}}{\mathcal{H}\left(z_{obs}\right)}+\frac{Q^\eta_{(n)}}{\left(1+z_{obs}\right)\mathcal{H}\left(z_{obs}\right)}$$

$$\delta r_z^{(n)}=-\delta w_{(n)}-\frac{\delta z_{(n)}}{\mathcal{H}\left(z_{obs}\right)}-Q^r_{(n)}-\frac{Q^\eta_{(n)}}{(1+z_{obs})\mathcal{H}(z_{obs})}$$

$$\delta\theta^a_{z(n)}=-\delta\Theta^a_{(n)}-Q^a_{(n)}$$

Conclusion and ongoing developments

1. The formalism of light-cone perturbations allows for a simple interpretation of relativistic effects as light-cone perturbations.
2. It simplifies the task of calculating cosmological observables.
3. Cosmological observables are evaluated directly in terms of observed angles and past light-cone.
4. Neat description of observer terms (**See Pierre's talk**)

- Second order generalization:

Second Order Cosmological Perturbation Theory on the Geodesic Light-Cone background. *In collaboration with: Pierre Béchaz, Giuseppe Fanizza and Giovanni Marozzi.* **To appear soon**

- Number counts in luminosity distance space

An iterative method to build non-linear relations between cosmological observables. *In collaboration with: Giuseppe Fanizza, Giovanni Marozzi and Tiziano Schiavone.* **To appear soon**

Thank you!!!

Backup slides

Different perspectives on cosmological perturbation theory

$$T^{\mathbf{E}} \equiv \frac{\bar{\eth}^s T_{+\ldots s} + \eth^s T_{-\ldots s}}{2}\,, \qquad T^{\mathbf{B}} \equiv -i\frac{\bar{\eth}^s T_{+\ldots s} - \eth^s T_{-\ldots s}}{2}$$

		ν	N	M	L	u	v	$\hat{u}$	$\hat{v}$	μ	$\hat{\mu}$
Rank 0	$\mathcal{T} \to \mathcal{C}$	✓									
	$\mathcal{T}_{\vert\vert} \to \mathcal{C}_{rr}$		✓								
	$\mathcal{V}_{\vert\vert} \to \mathcal{B}_r$		✓	✓							
	$\mathcal{S} \to \phi$		✓	✓	✓						
Rank 1	$\mathcal{T}^{\mathbf{E}}_{\vert\vert\pm} \to \mathcal{C}^{\mathbf{E}}_r$					✓					
	$\mathcal{V}^{\mathbf{E}}_{\pm} \to \mathcal{B}^{\mathbf{E}}$					✓	✓				
	$\mathcal{T}^{\mathbf{B}}_{\vert\vert\pm} \to \mathcal{C}^{\mathbf{B}}_r$							✓			
	$\mathcal{V}^{\mathbf{B}}_{\vert\vert\pm} \to \mathcal{B}^{\mathbf{B}}$							✓	✓		
Rank 2	$\mathcal{T}^{\mathbf{E}}_{\pm\pm} \to \mathcal{C}^{\mathbf{E}}$									✓	
	$\mathcal{T}^{\mathbf{B}}_{\pm\pm} \to \mathcal{C}^{\mathbf{B}}$										✓

Why it does work so well?

$$\Omega = \int_{\Sigma_s} d\theta^1 \wedge d\theta^2$$

$$\Omega = \frac{1}{(k_\mu u^\mu)_o^2 \, |det J^\mu_\nu|} \int_{\Sigma} e^1_s \wedge e^2_s$$

$$A_s = \int_{\Sigma_s} e^1_s \wedge e^2_s$$

$$d_A^2 = (k_\mu u^\mu)^2 \, |det J^\alpha_\beta|$$