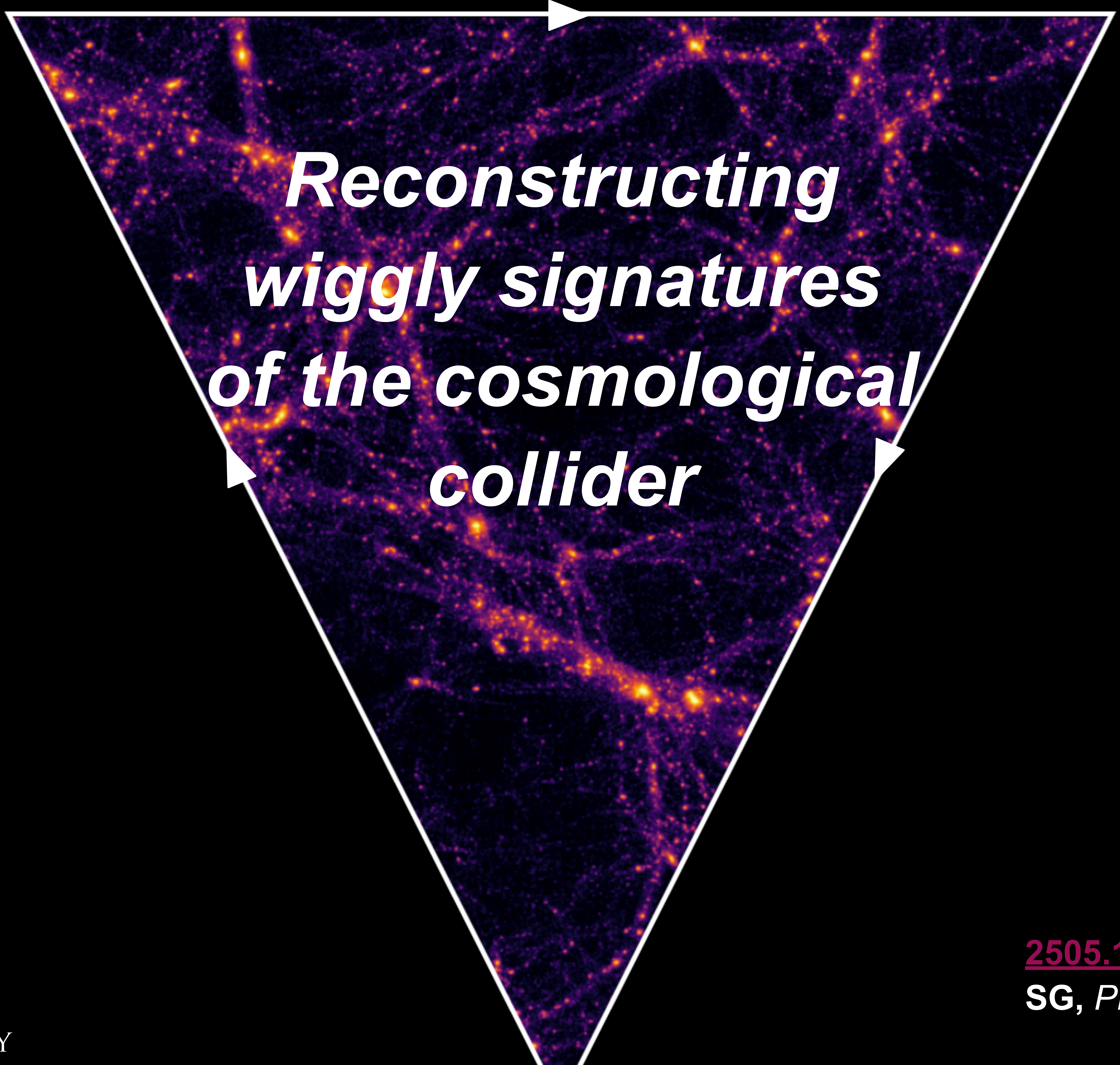
Reconstructing wiggly signatures of the cosmological collider



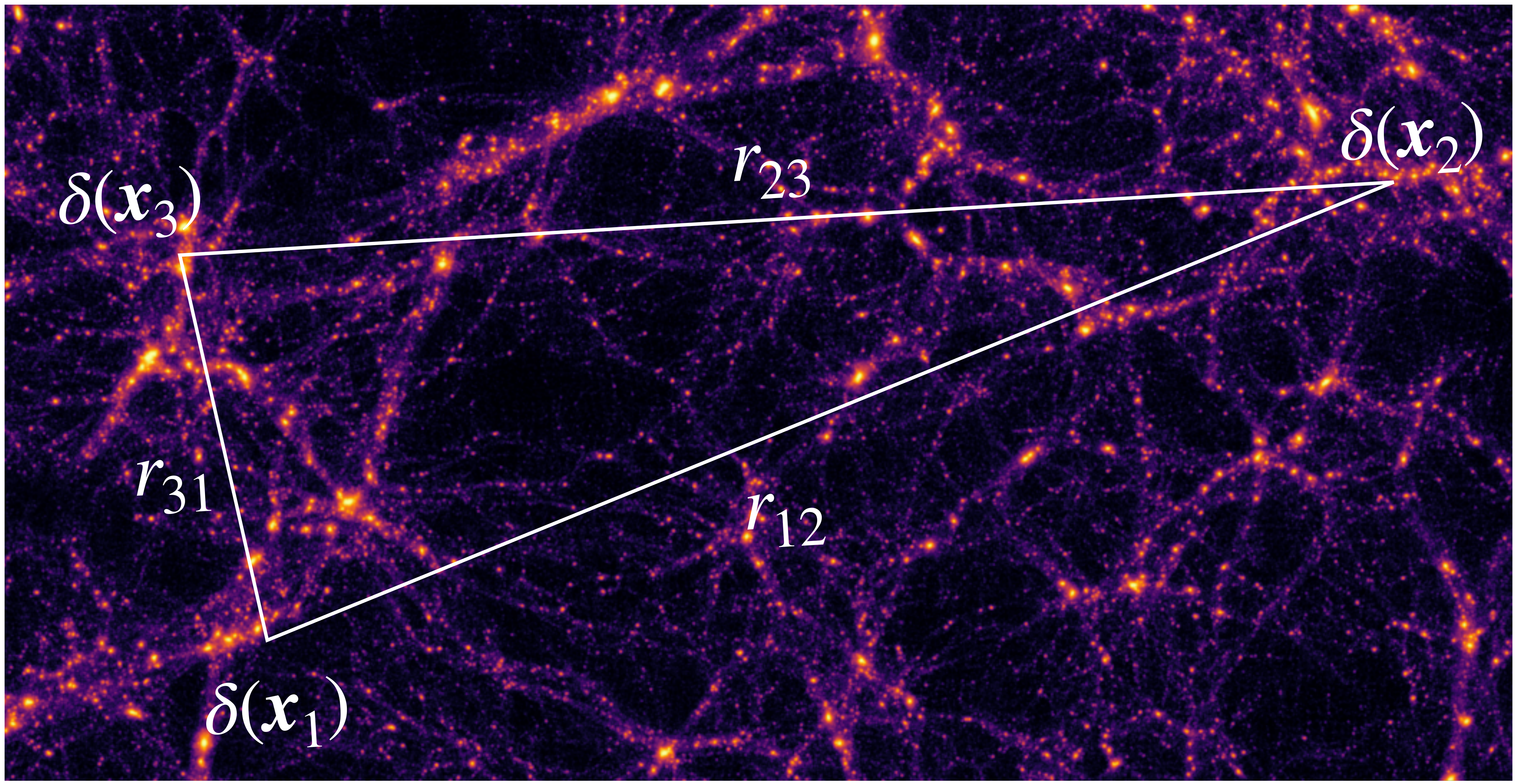
Sam Goldstein

Columbia University

2505.13443

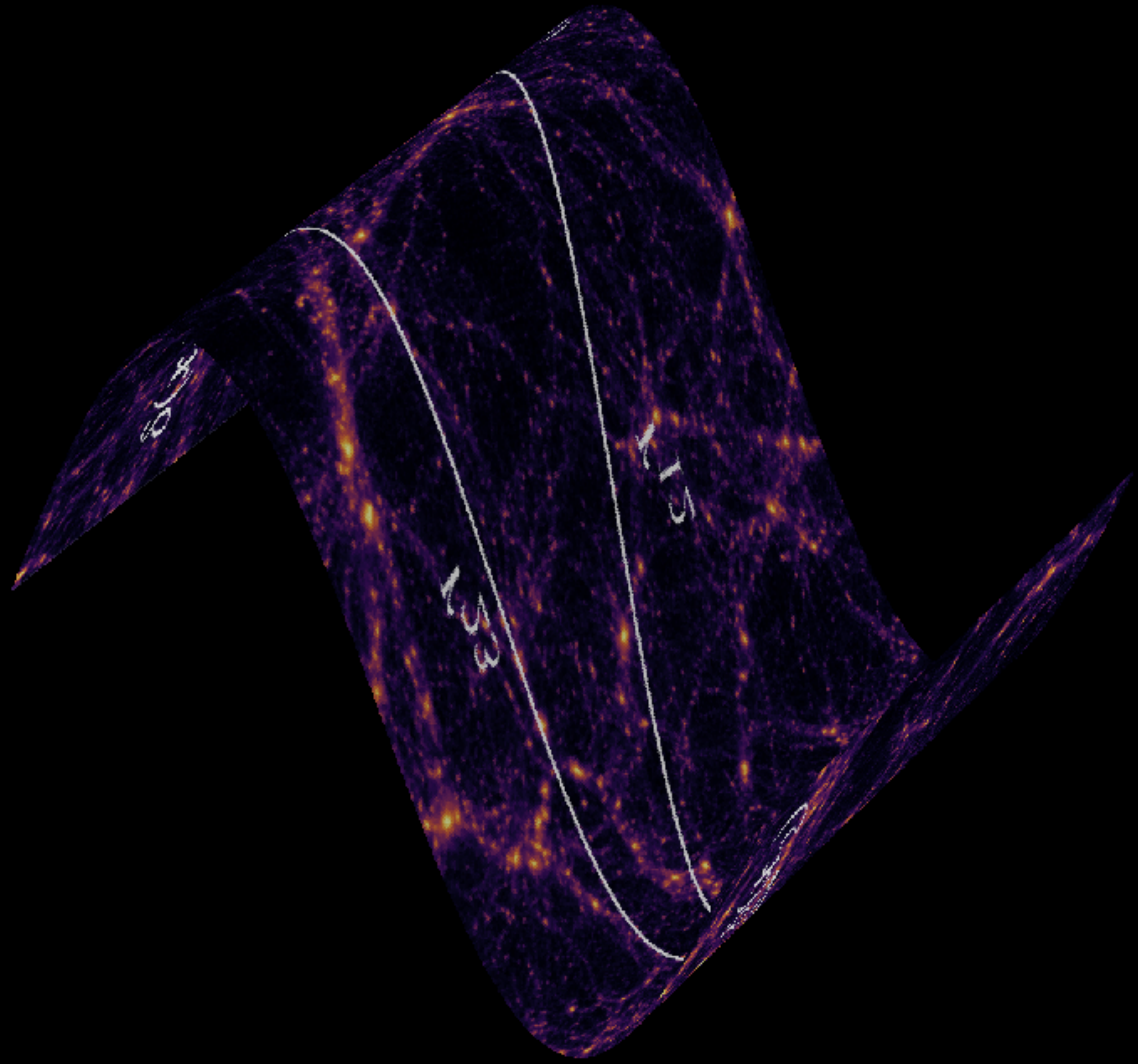
SG, *Philcox, Fondi, Coulton*

The bispectrum



$$B(k_1, k_2, k_3) \equiv \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle'$$

Should we care about wiggly bispectra?



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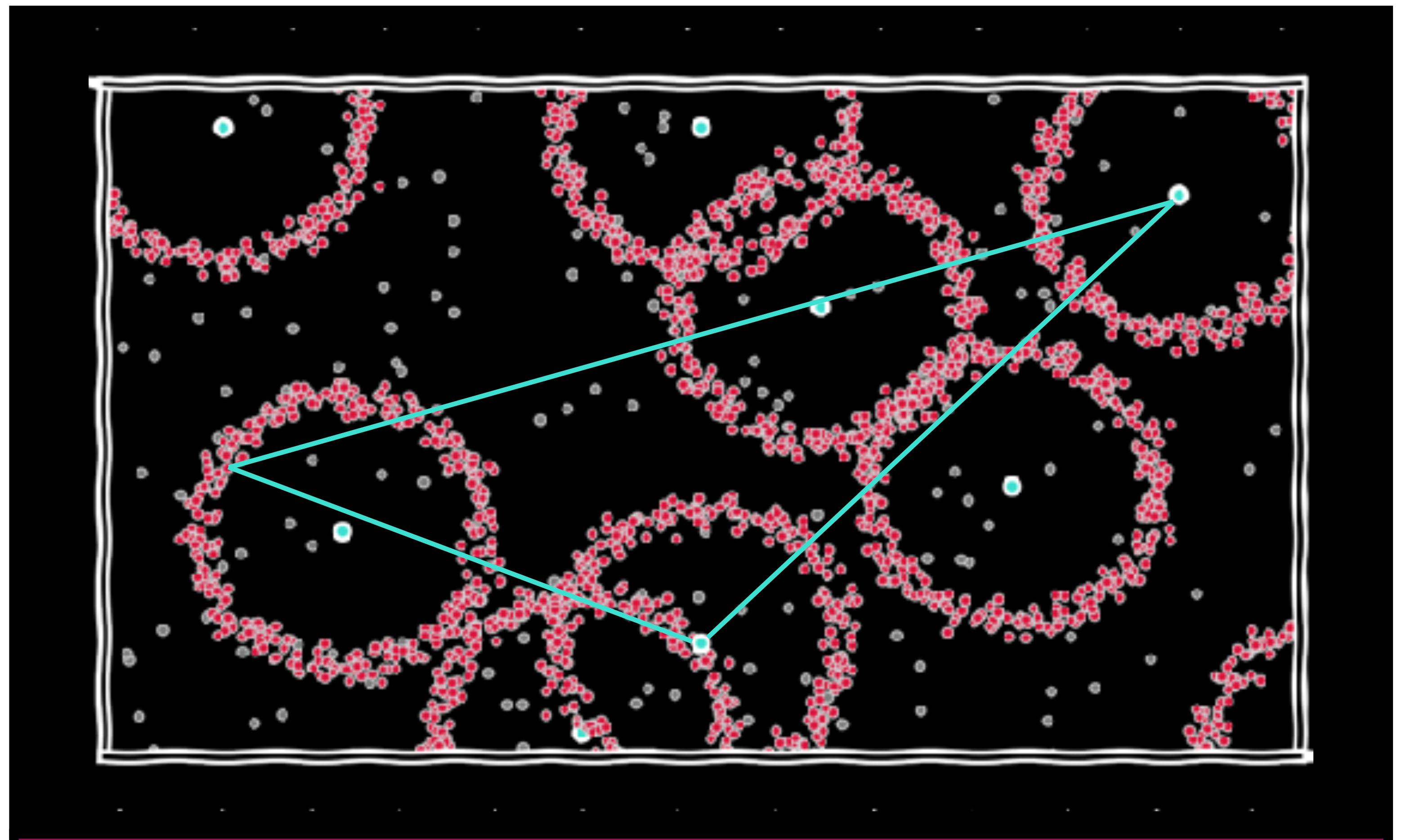

BAO

E.g., Pearson & Samushia 2017, Child et al. 2018, Behera et al. 2023, Chen, Vlah, White 2024

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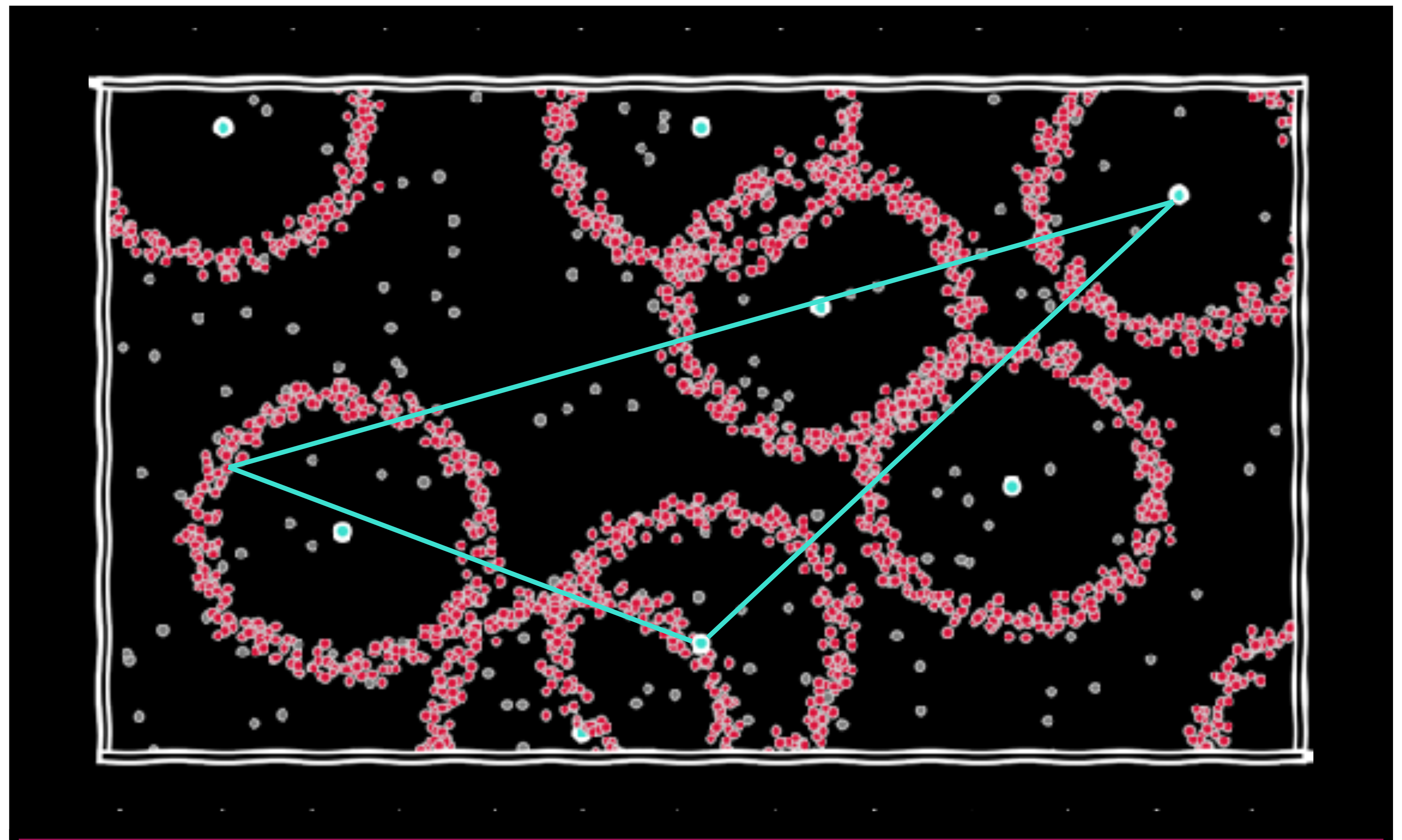

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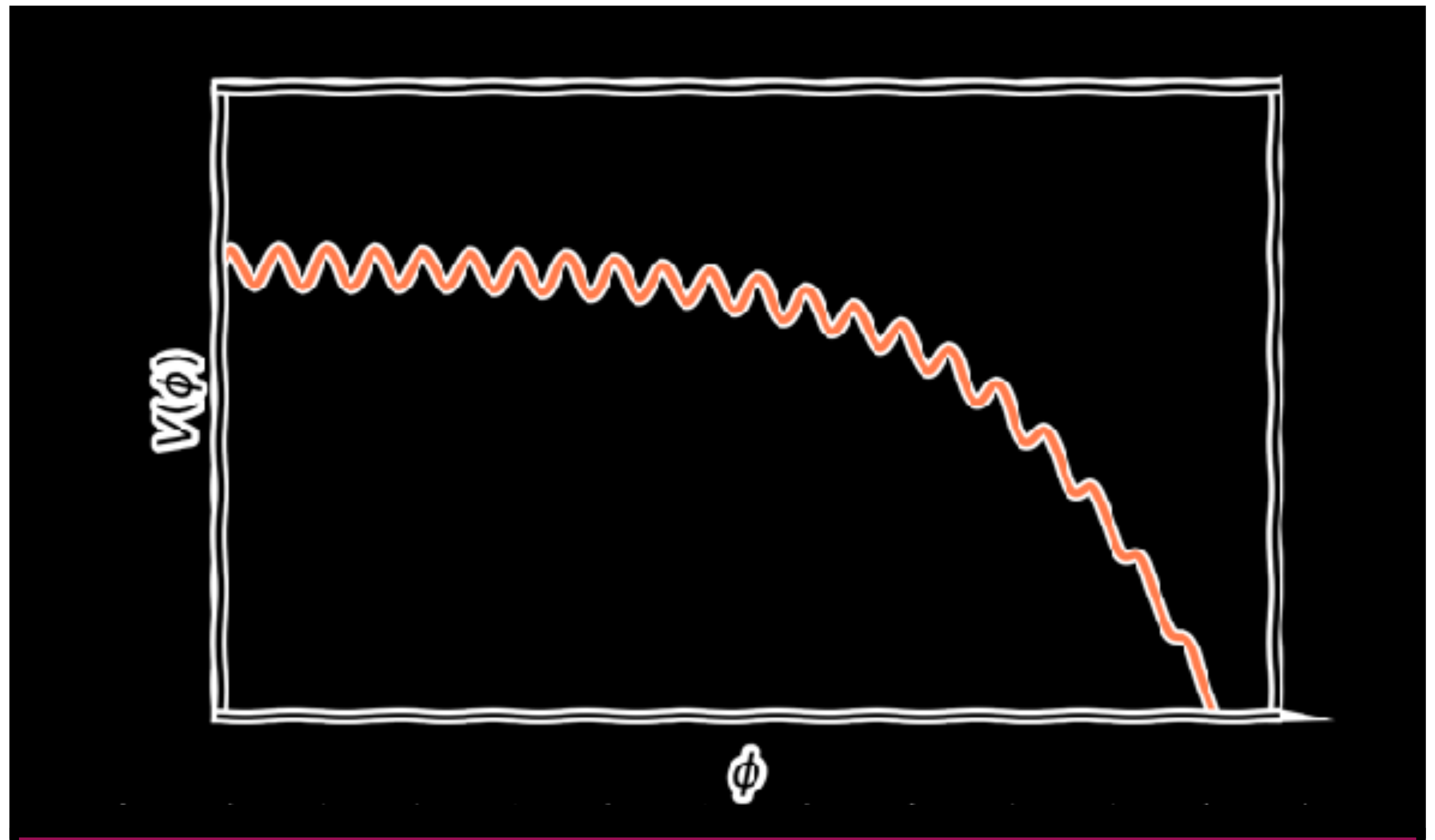

Primordial features

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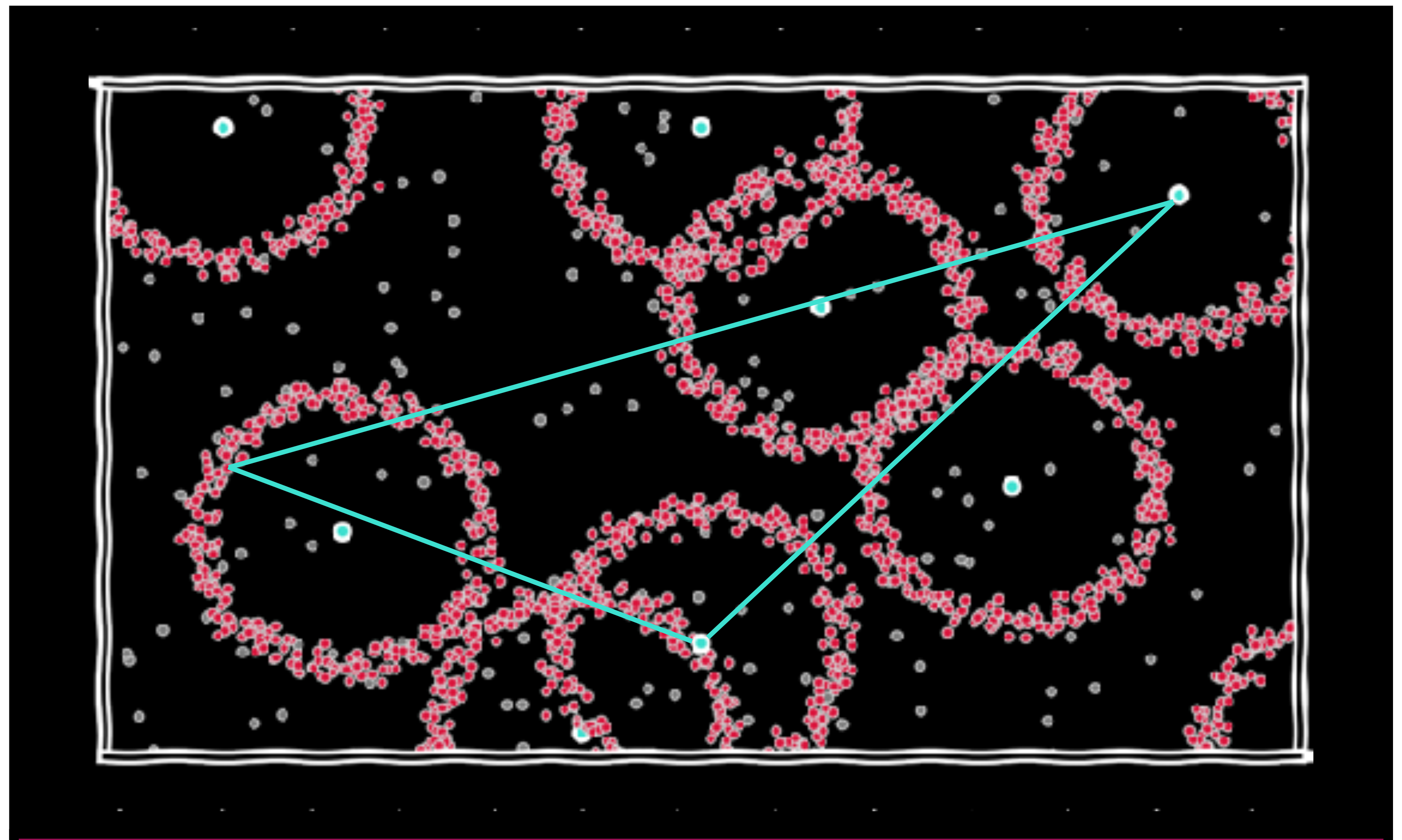

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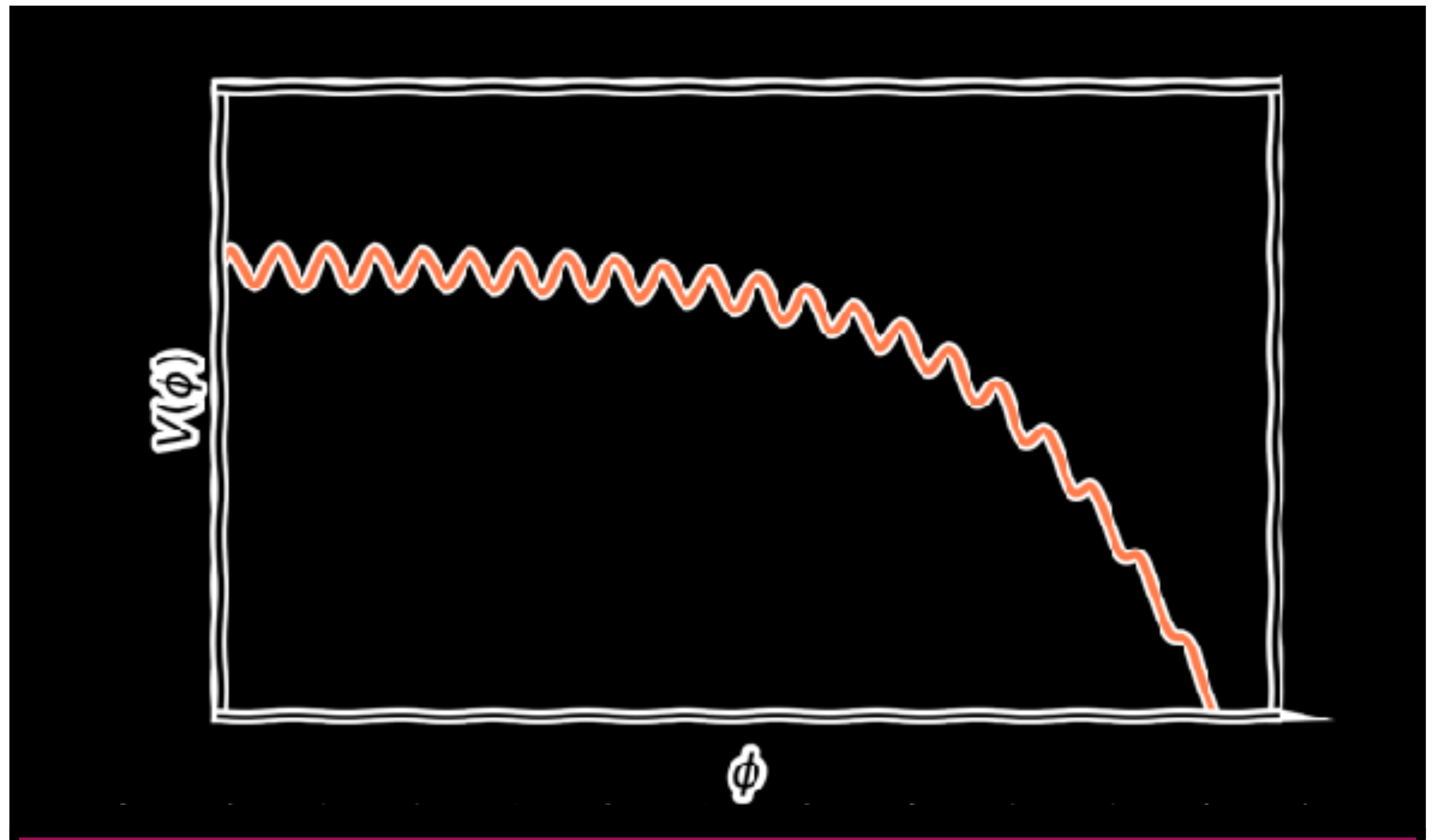

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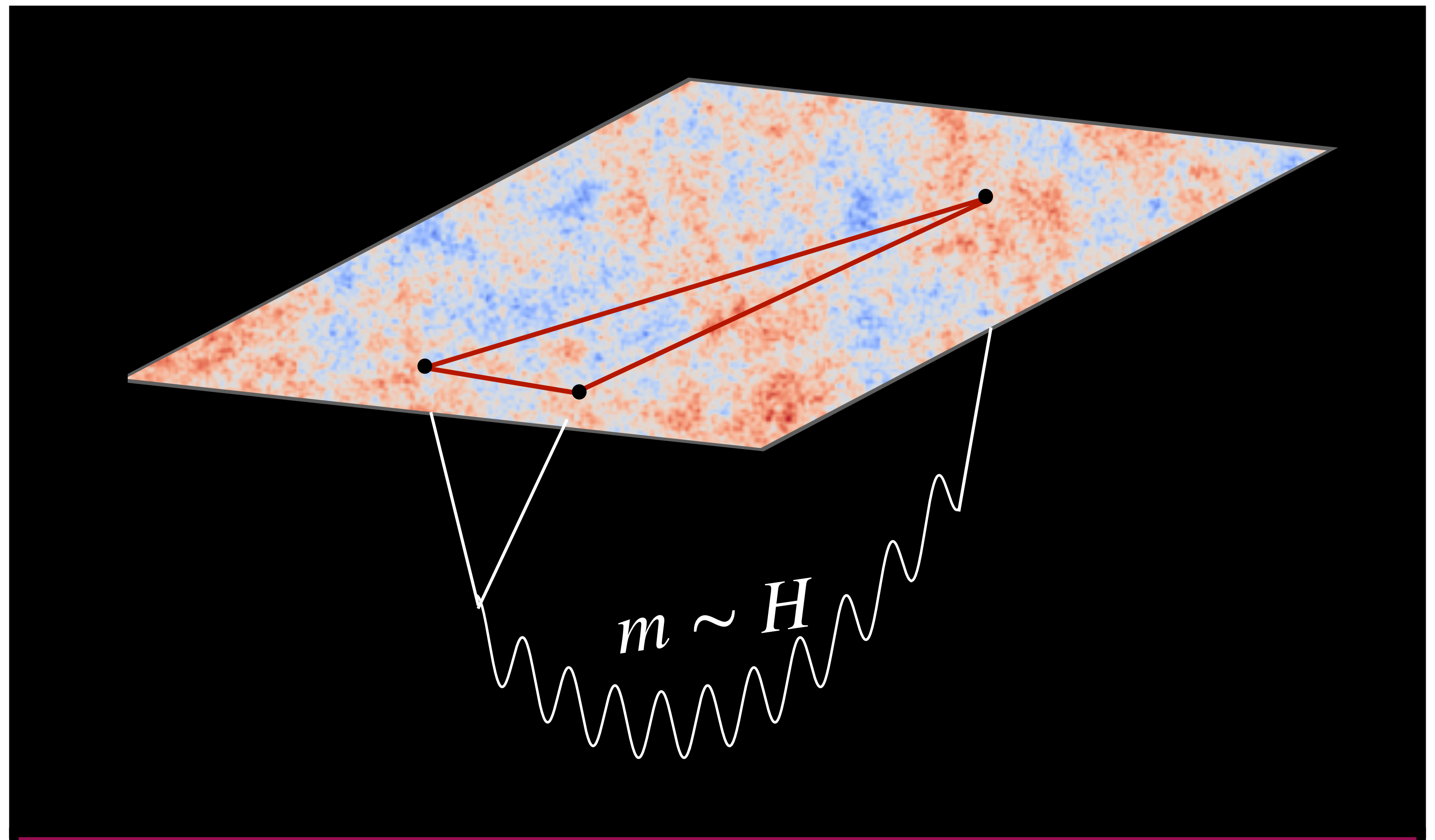

Cosmological collider

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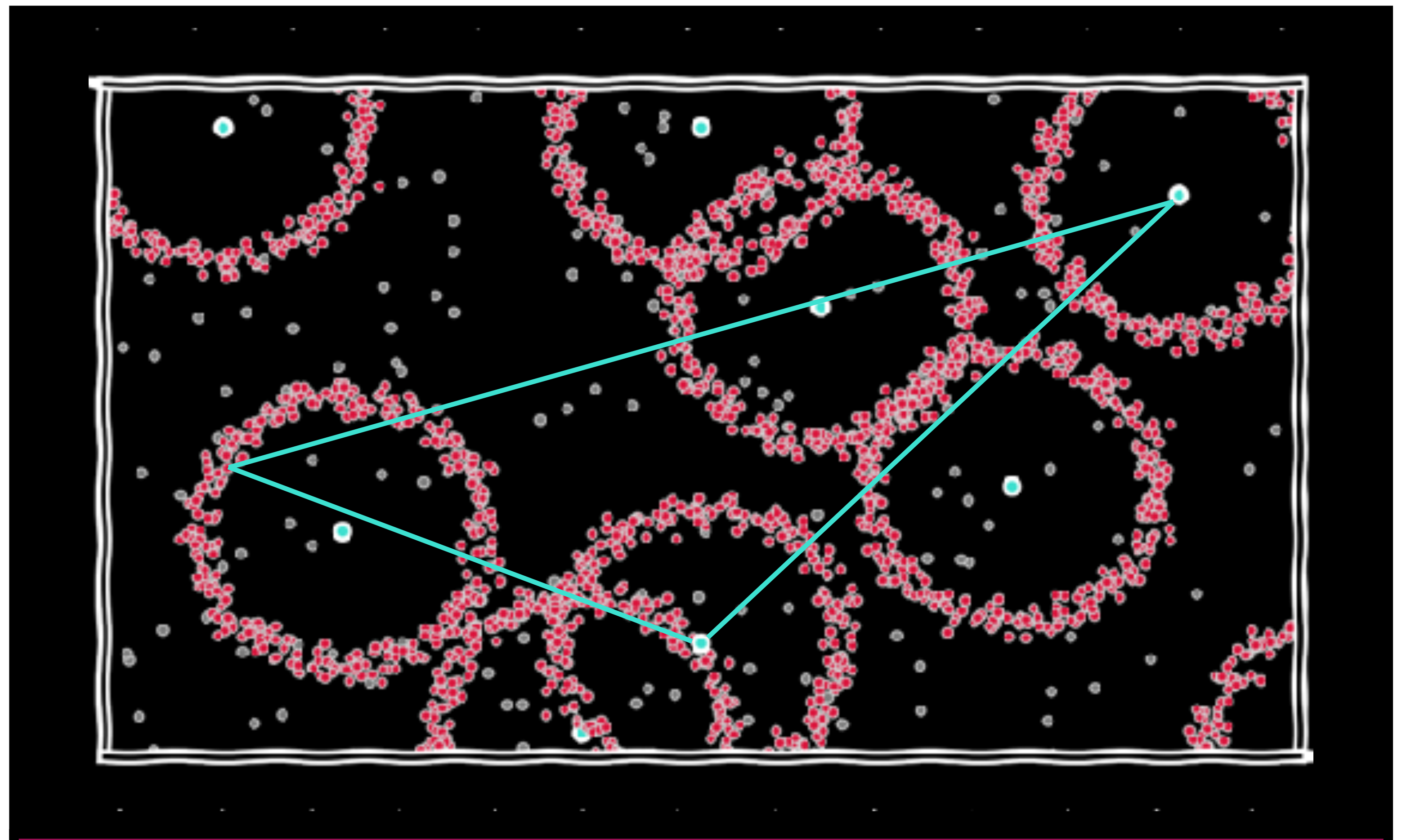

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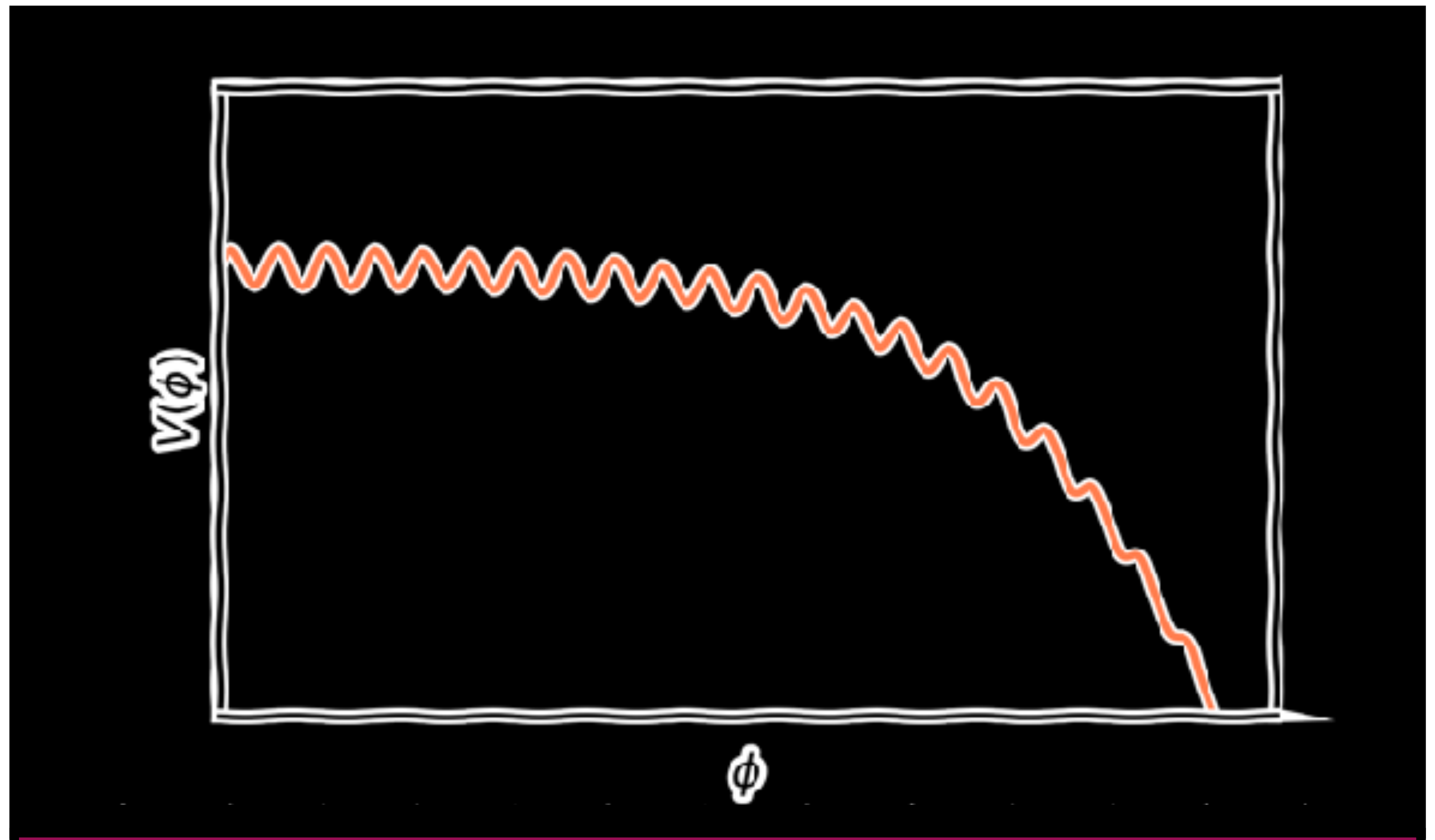

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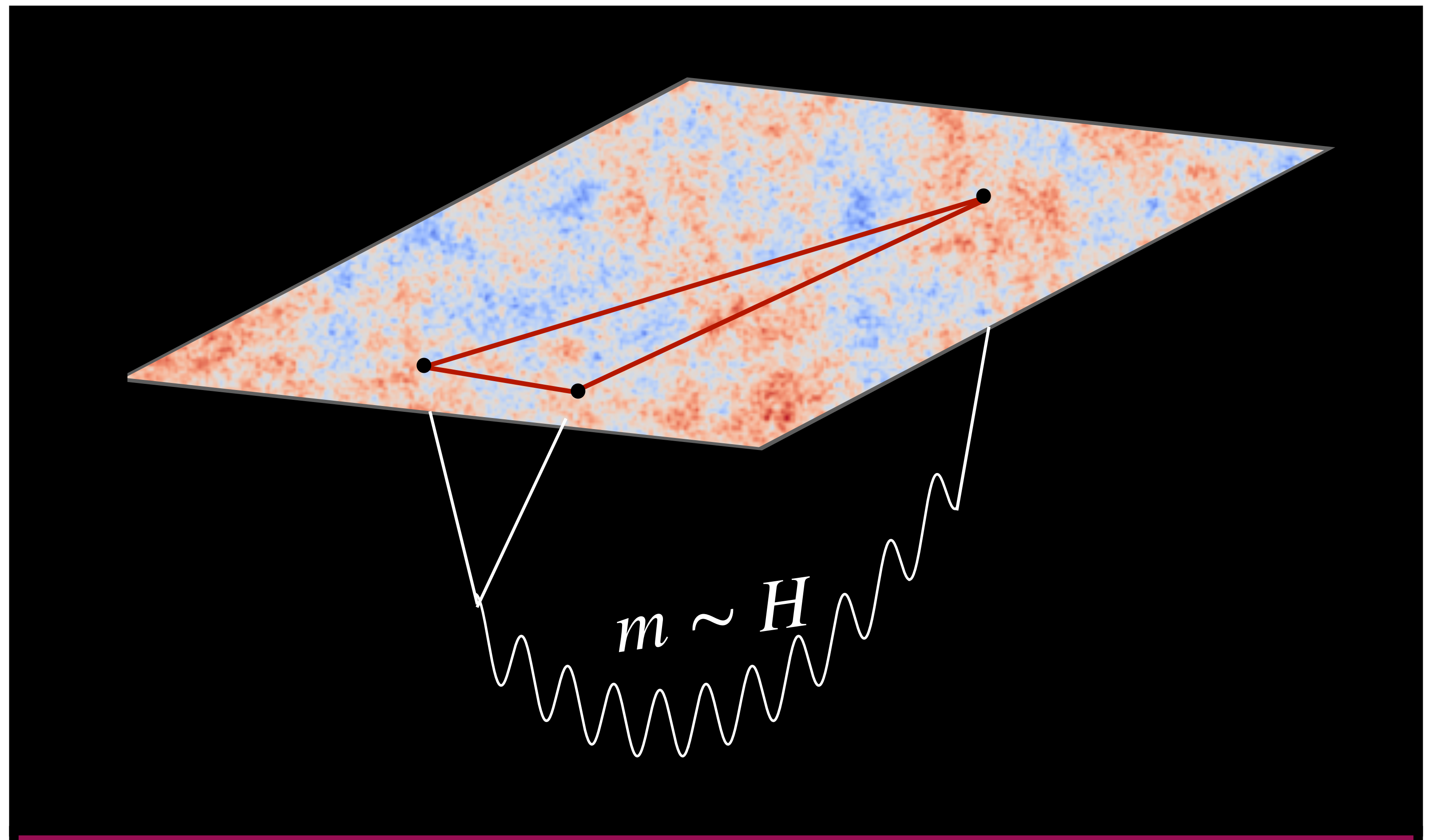

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(+substantial theory/observation progress in bispectra!
+DESI(-II), Euclid, SphereX,etc.)

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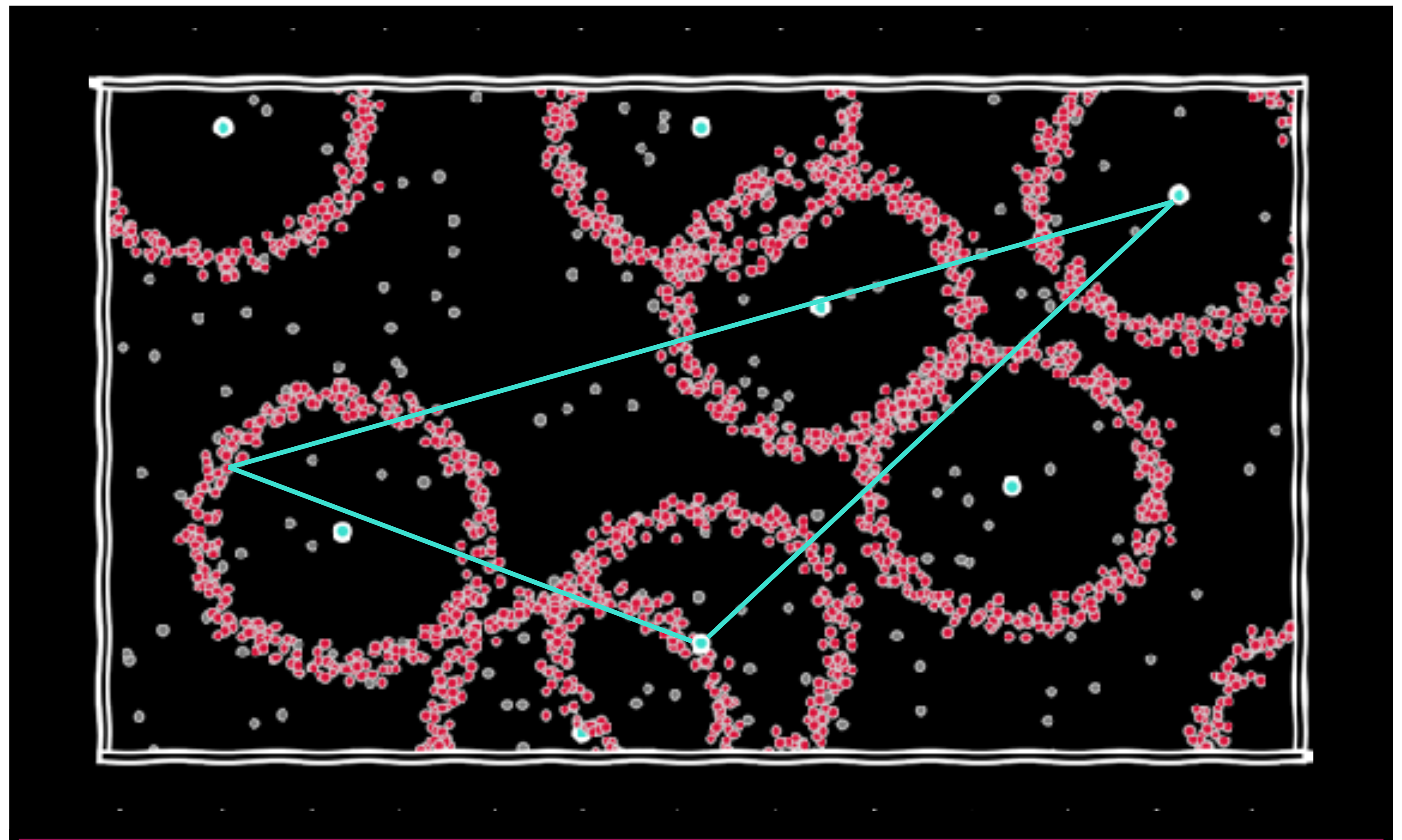

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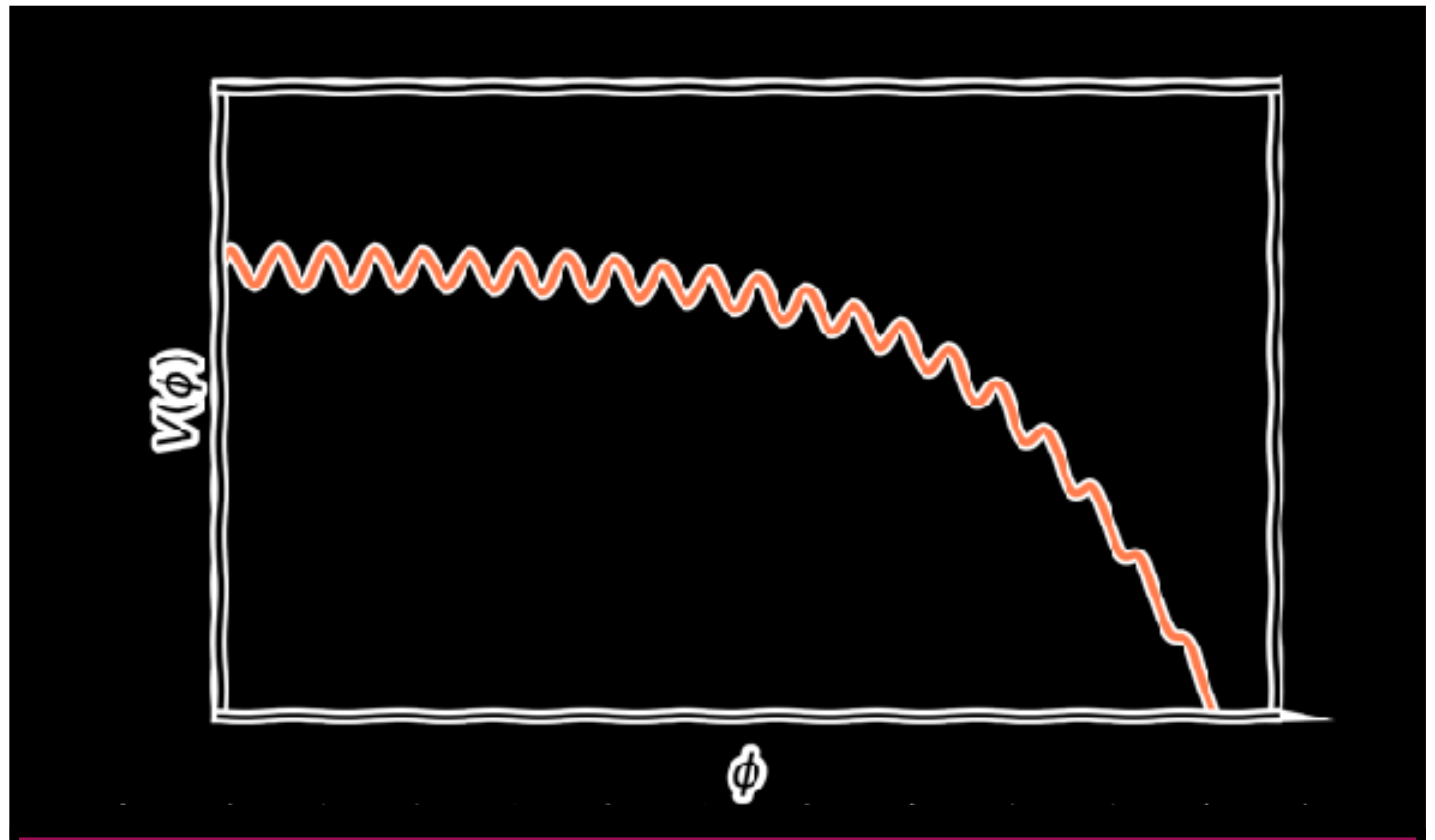

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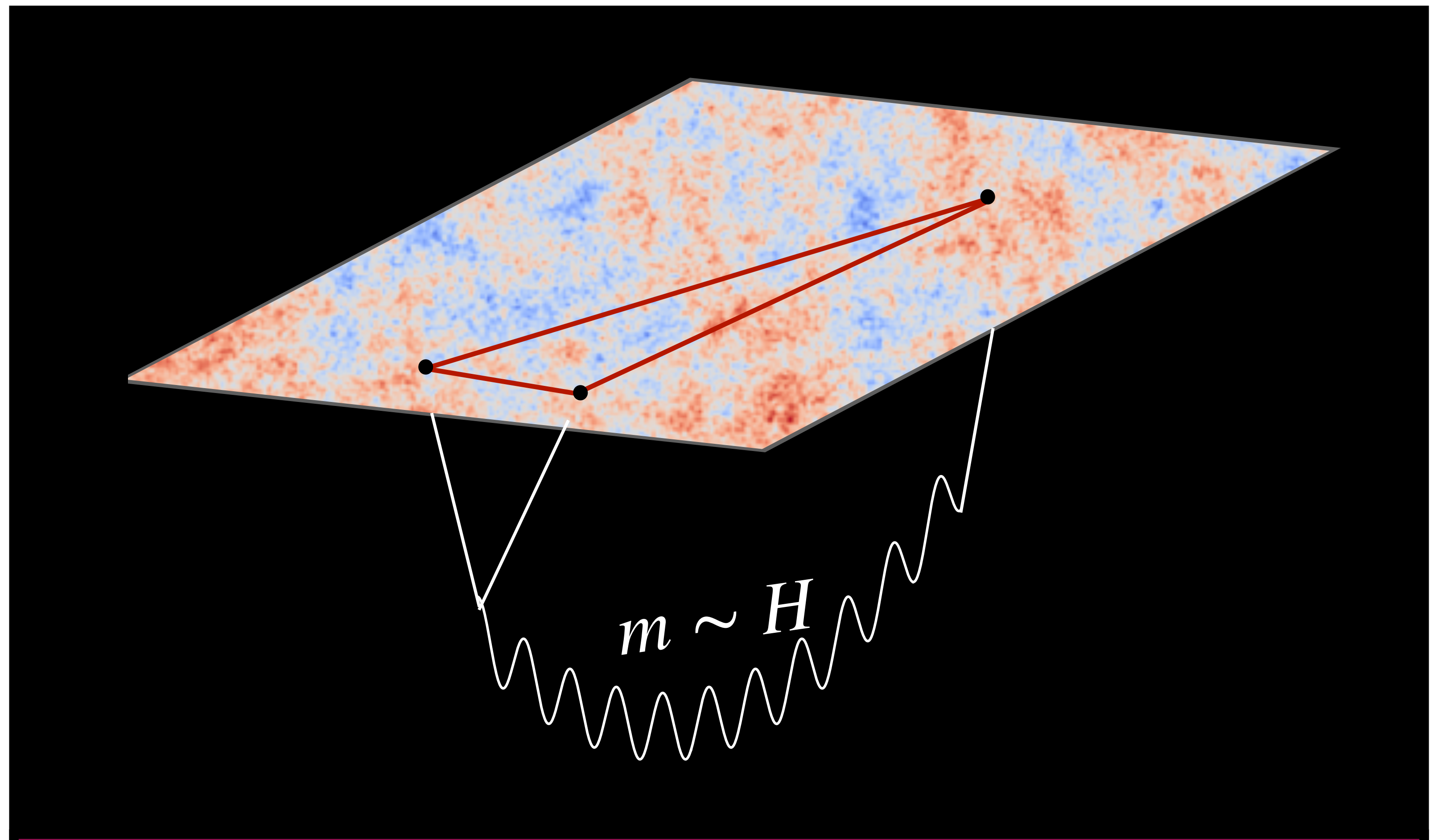

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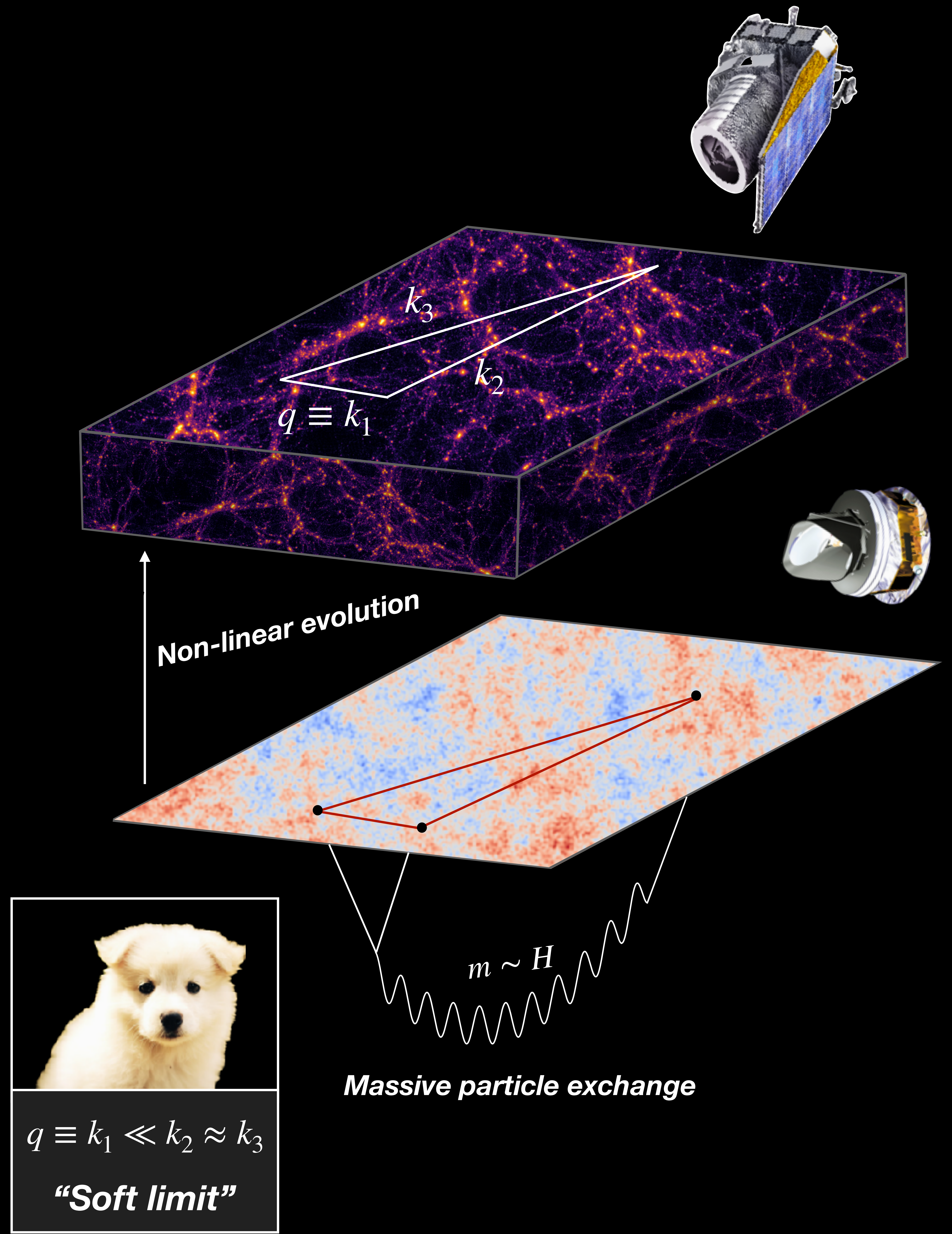
The cosmological collider

- Massive scalars during inflation have characteristic squeezed bispectrum *(Arkani-Hamed & Maldacena, 2015)*

$$\lim_{k_1 \ll k_2 \approx k_3} B_\Phi(k_1, k_2, k_3) = 4 f_{\rm NL}^{\Delta} \left(\frac{k_1}{k_2}\right)^{\Delta} P_\Phi(k_1) P_\Phi(k_2)$$



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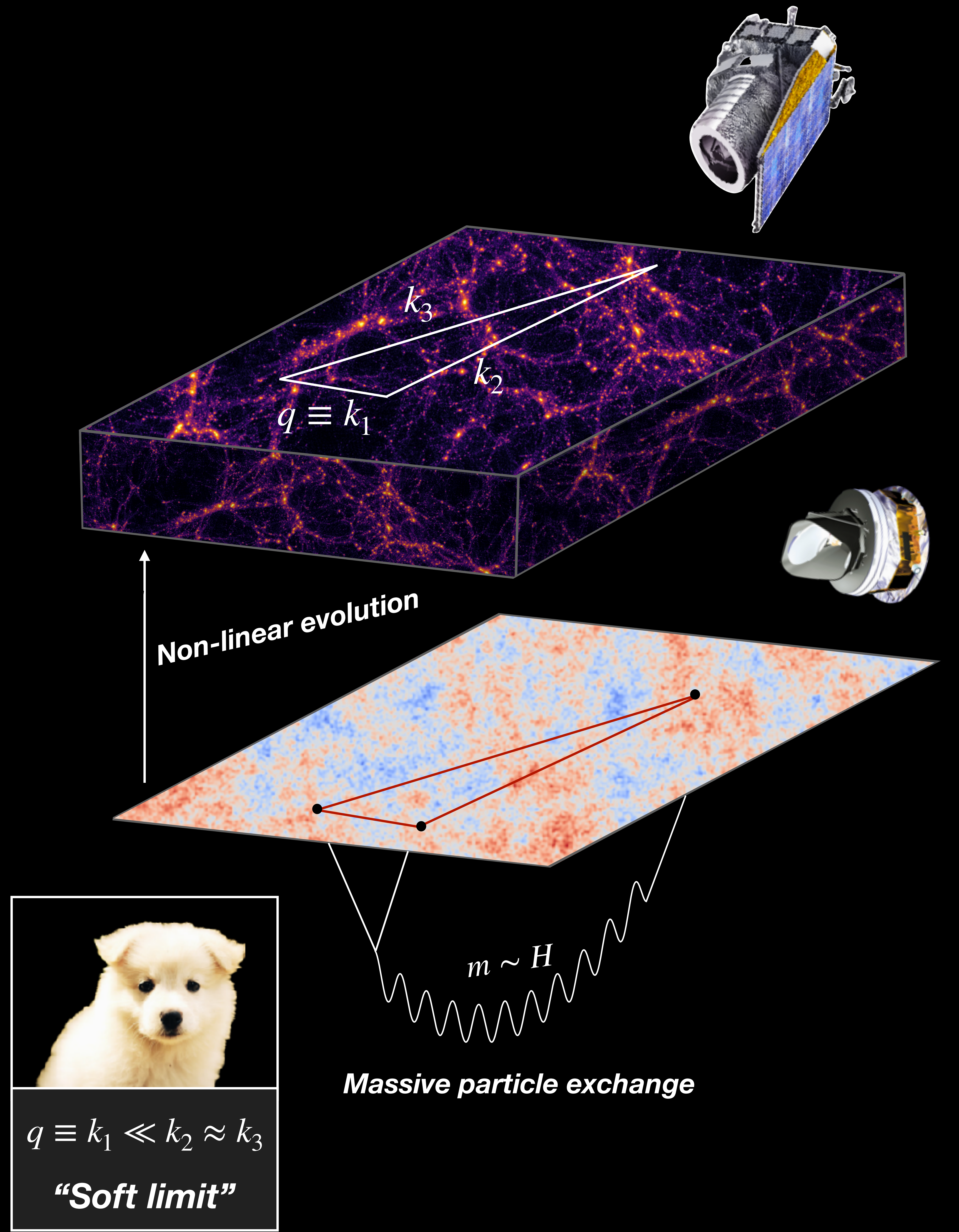
- Power depends on **mass:** $\Delta = 3/2 - \sqrt{9/4 - m^2/H^2}$



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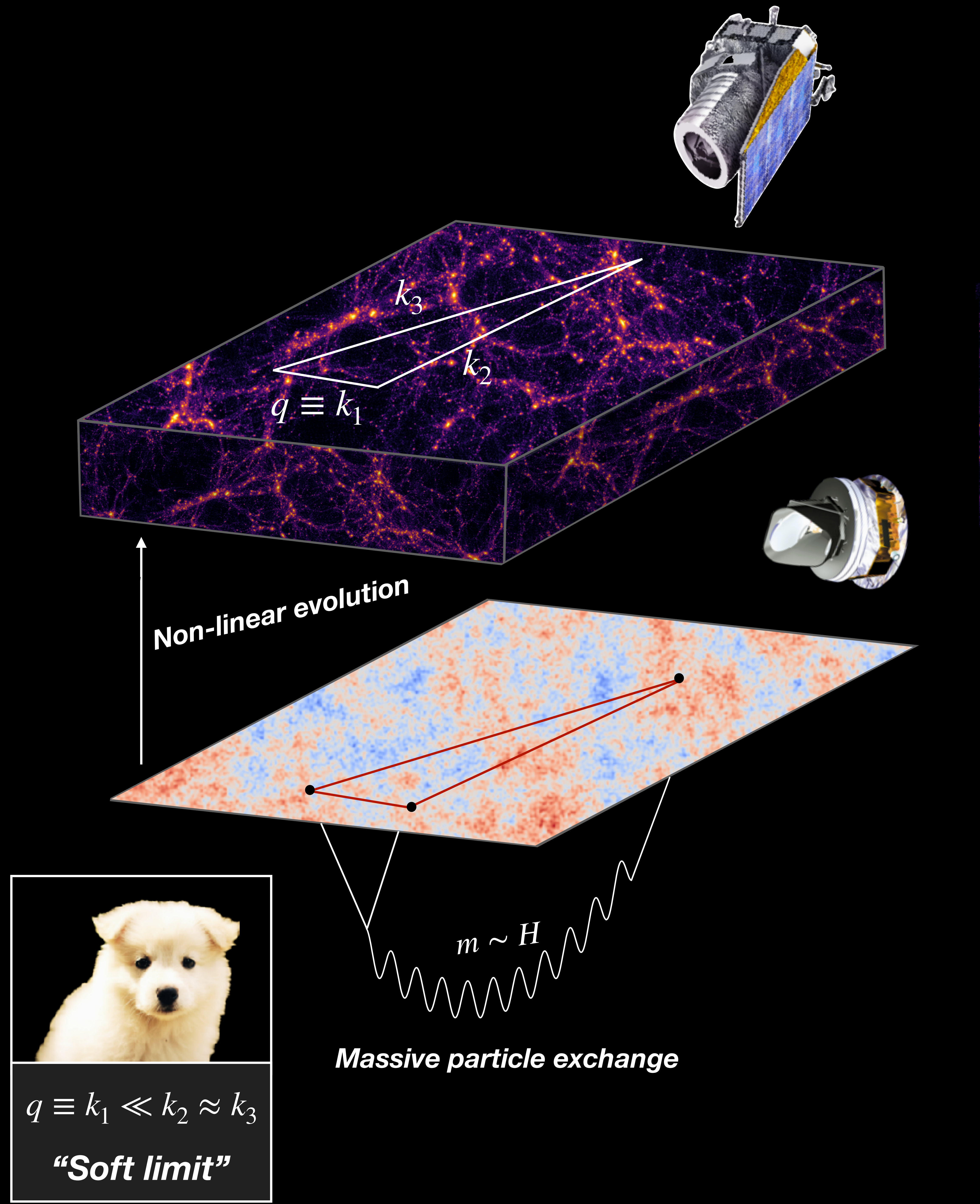
- Power depends on **mass:** $\Delta = 3/2 - \sqrt{9/4 - m^2/H^2}$
    - Wiggles when $m > 3H/2$

$$\lim_{q \ll k} B_\Phi(\boldsymbol{q}, \boldsymbol{k}) = 4 f_{\rm NL} \left(\frac{q}{k}\right)^{3/2} \cos[\mu \ln(q/k) + \delta] P_\Phi(q) P_\Phi(k)$$



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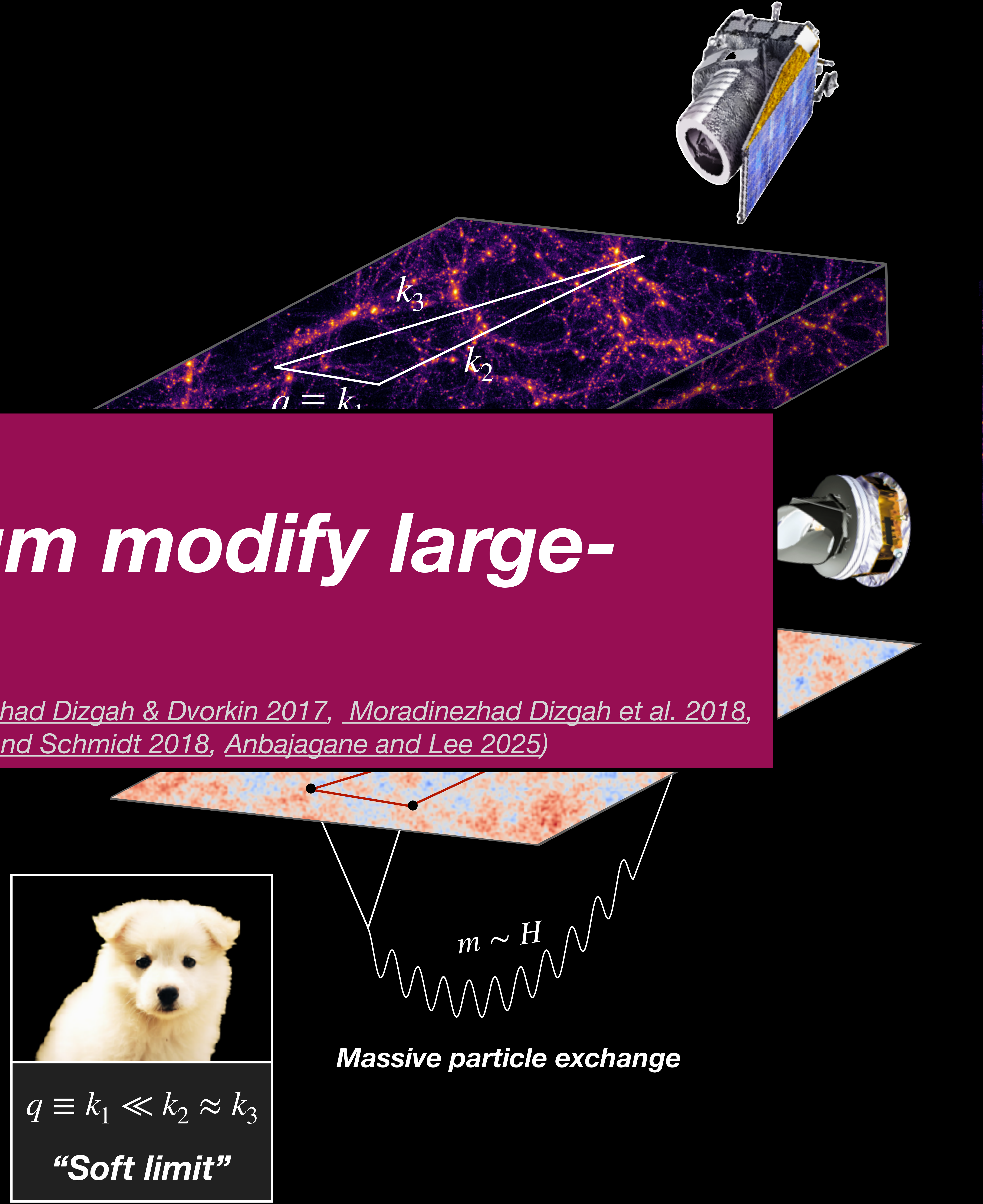
$$\lim_{k_1\ll} B\ (k\ \ k\ \ k\ ) = 4f^{\Delta} \left(\frac{k_1}{\ }\right)^{\Delta} P\ (k\ )P\ (k\ )$$

- Pow
  - W

How does this bispectrum modify large-scale structure?

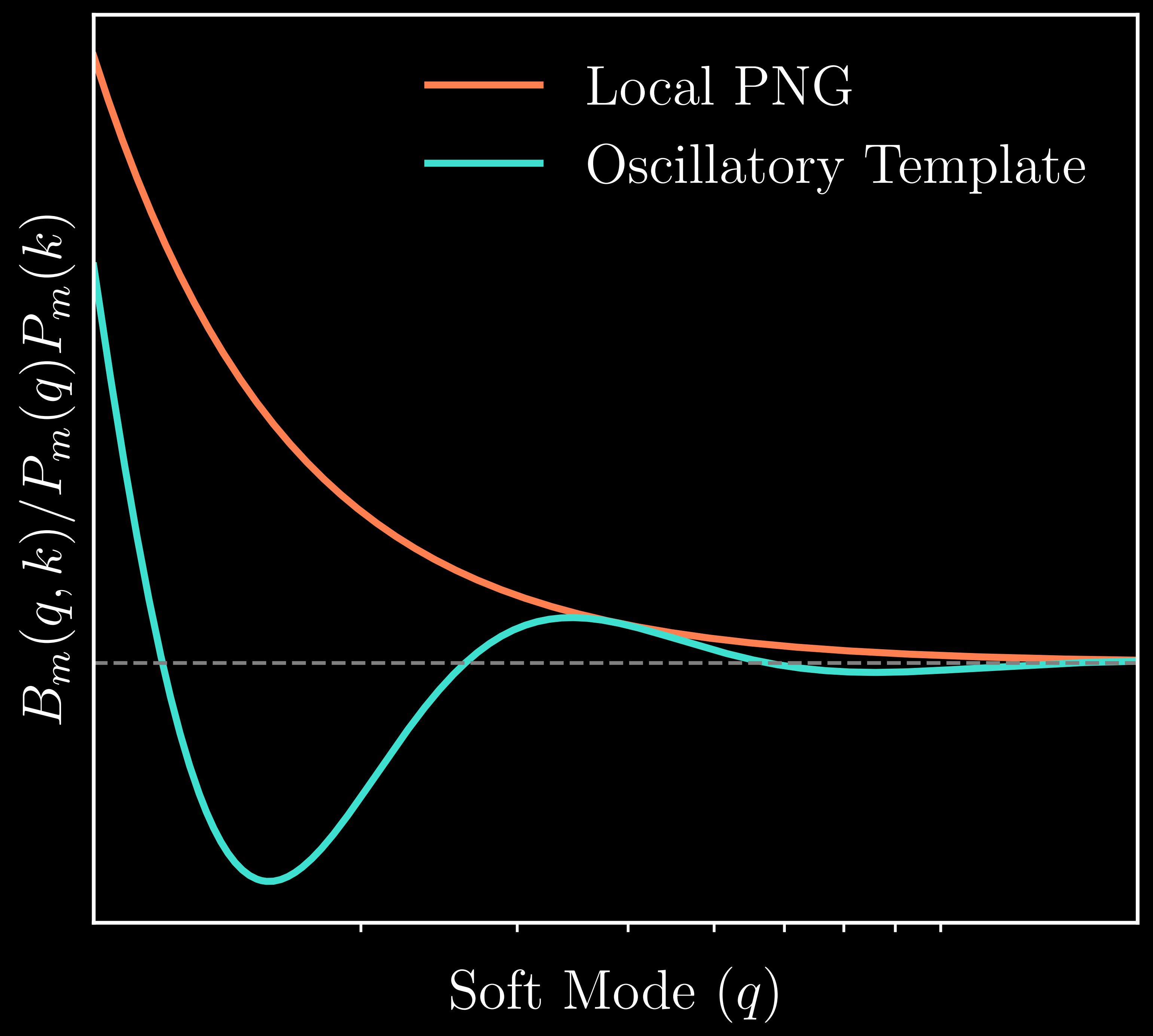
(See Moradinezhad Dizgah & Dvorkin 2017, Moradinezhad Dizgah et al. 2018, Cabass, Pajer, and Schmidt 2018, Anbajagane and Lee 2025)

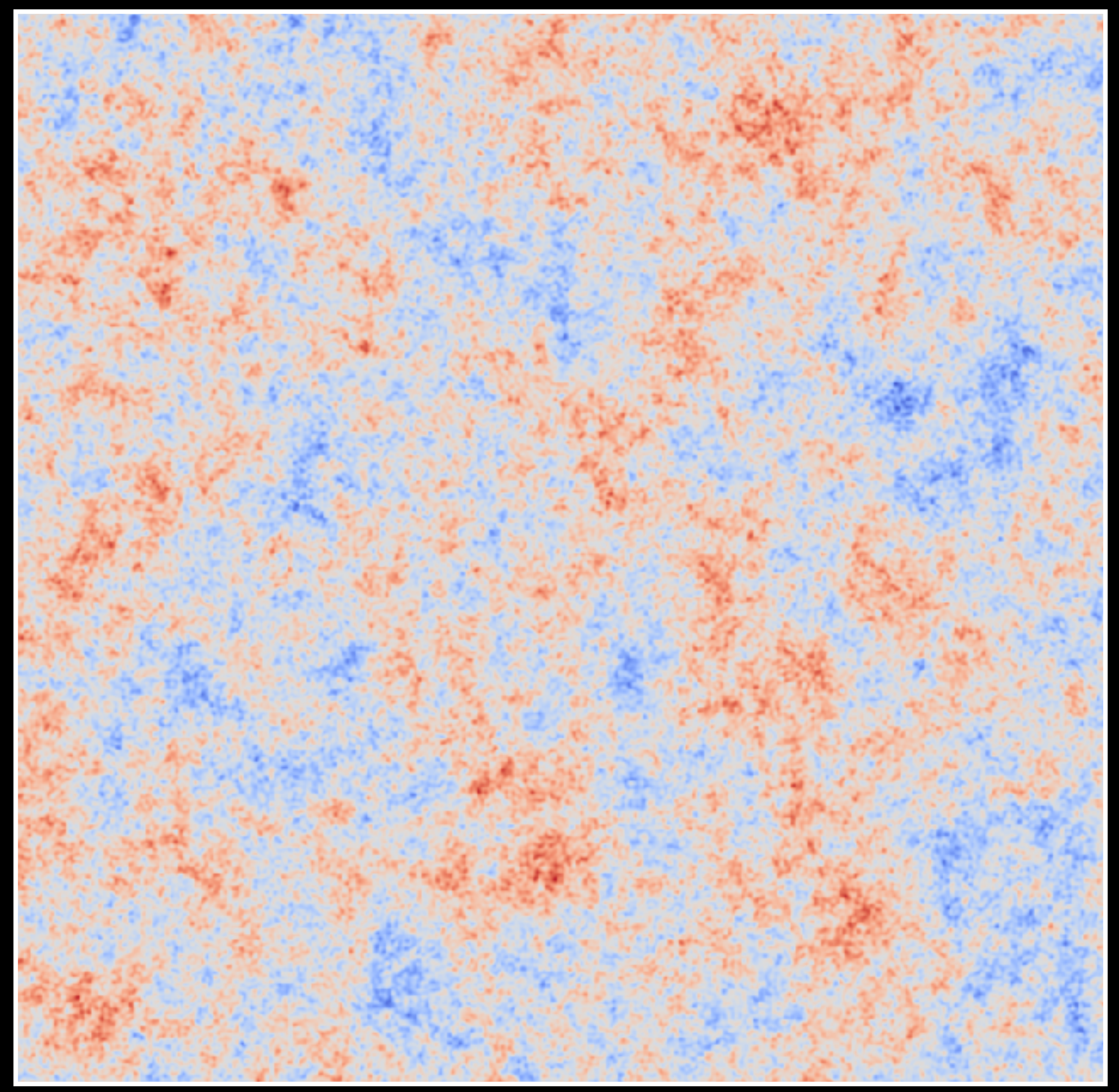
$$\boxed{\lim_{q\ll k} B_{\Phi}(\boldsymbol{q},\boldsymbol{k}) = 4f_{\rm NL}\left(\frac{q}{k}\right)^{3/2}\cos[\mu\ln(q/k)+\delta]P_{\Phi}(q)P_{\Phi}(k)}$$

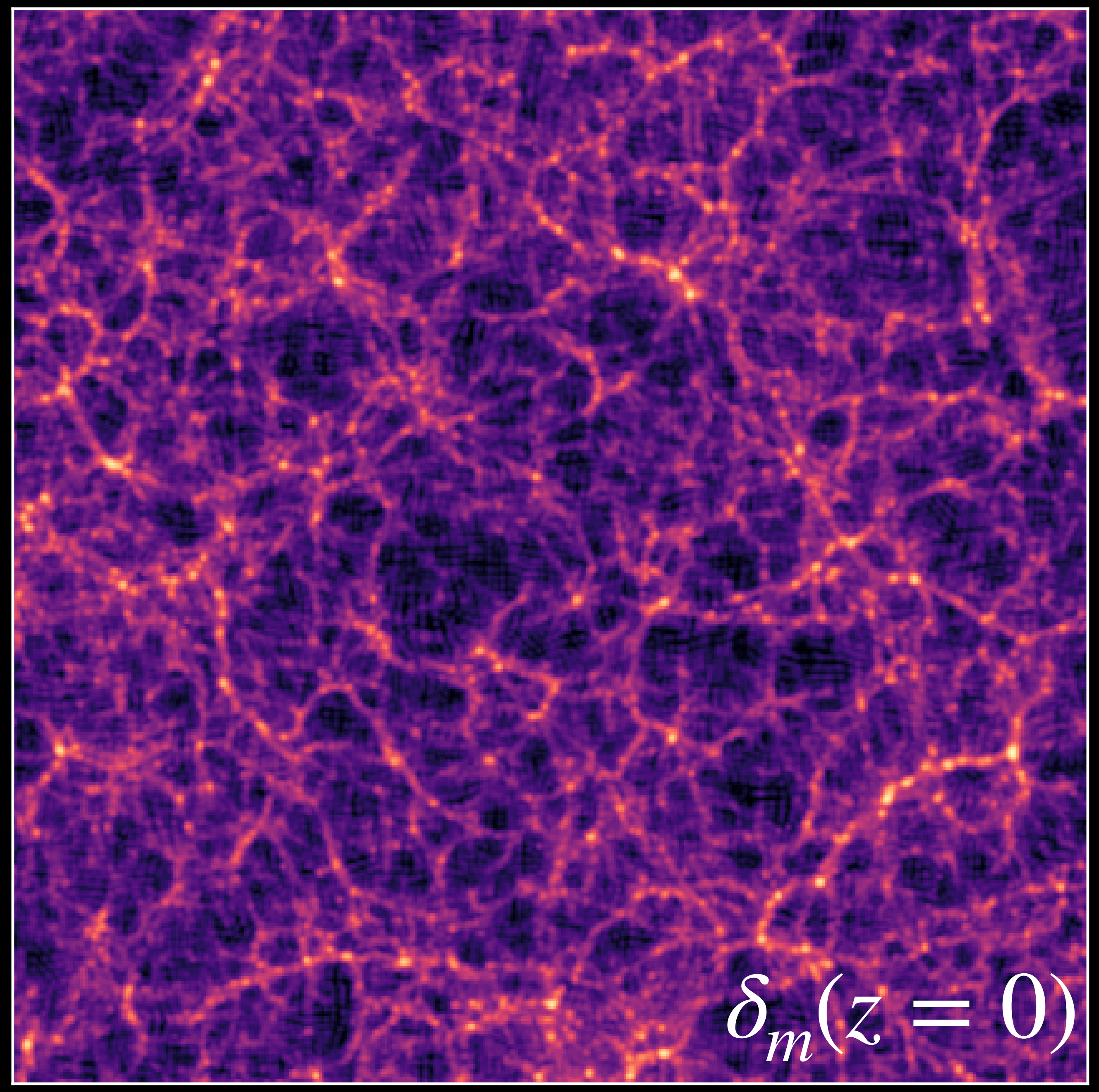


N-body simulations with oscillatory bispectra

$$\lim_{q \ll k} B_\Phi(\boldsymbol{q}, \boldsymbol{k}) = 4 f_{\rm NL} \cos[\mu \ln(q/k) + \delta] P_\Phi(q) P_\Phi(k)$$







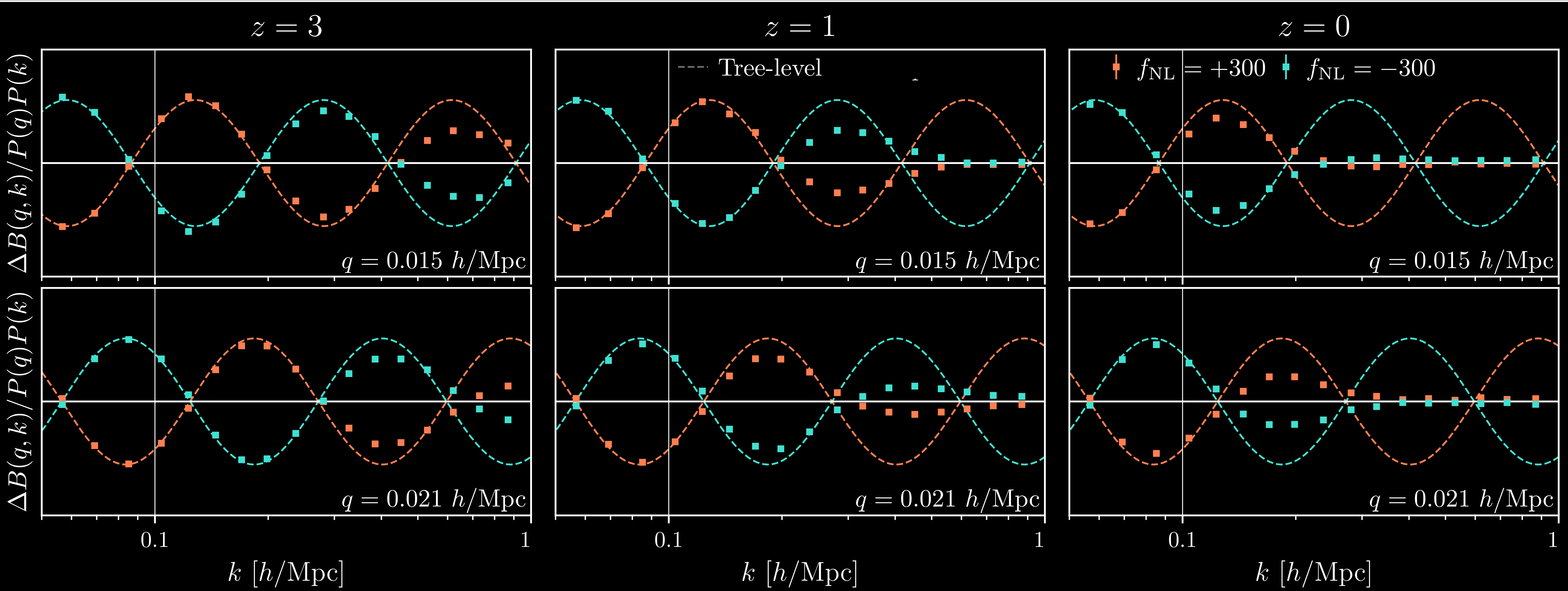
Bispectrum model → ***Initial conditions*** → ***Density field***

Modified 2LPTic

GADGET

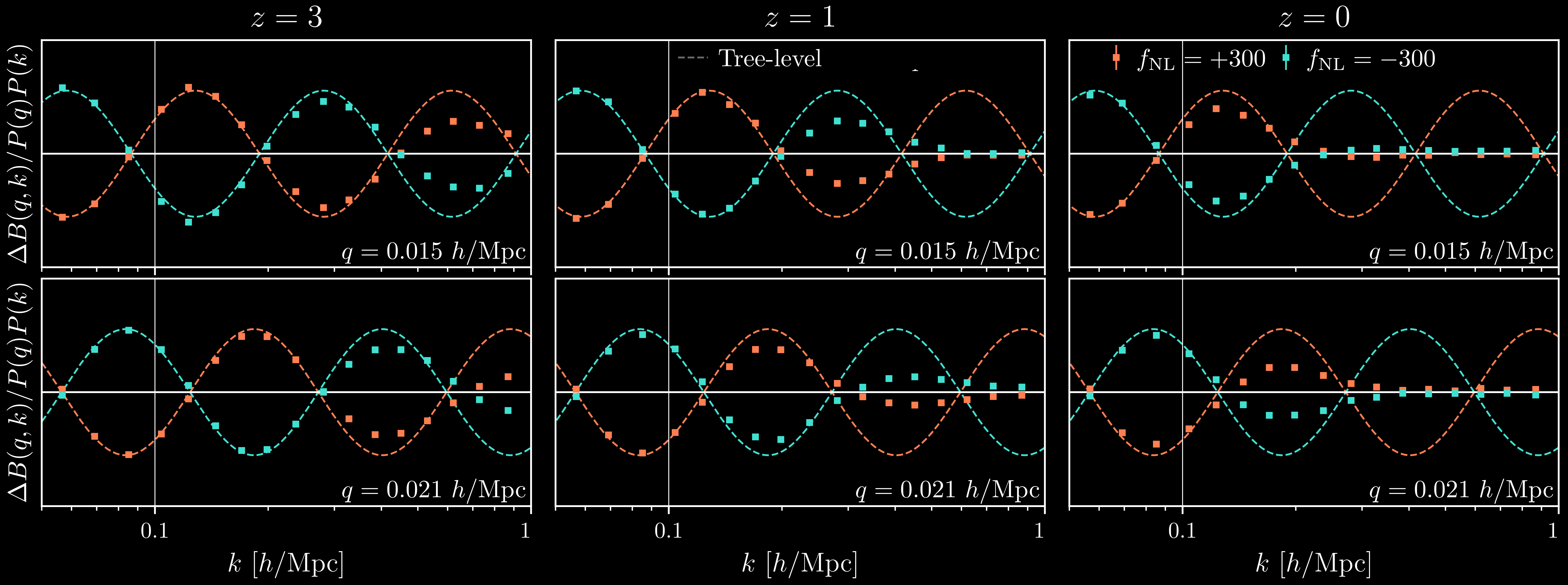
$f_{\rm NL} = \pm 300;\ \mu = 4;\ \delta = 3\pi/4$
Quijote-like sims with modified IC's

Wiggles in the squeezed bispectrum



- *Wiggles damped at low redshifts and small scales (high k)*

Wiggles in the squeezed bispectrum



- *Wiggles damped at low redshifts and small scales (high k) -> **two questions***
    1. Can we model the damping
    2. Can we undo the damping

1. Can we model the damping?

Squeezed bispectra

- Long-wavelength **modulation** of small-scale power spectrum

$$\lim_{q \ll k_{\rm NL}, k} B_m(q, k, k') = \langle \delta_m(\boldsymbol{q})\, \delta_m(\boldsymbol{k}) \delta_m(\boldsymbol{k}') \rangle'$$

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$$= P_{\Phi_L m}(q) \frac{\partial P_m(k)}{\partial \Phi_L(q)},$$

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- Non-linearities described by **“response function”**

Many papers have studied squeezed limits in similar way, e.g., Galilei 1632, Einstein 1907, Slosar et al. 2008, Schmidt and Kamionkowski 2010, Scoccimarro et al. 2011, Smith, LoVerde, Zaldarriaga 2011, Lewis, Challinor, and Hanson 2011, Seljak 2012, Schmidt, Jeong, Desjacques 2012, Kehagias and Riotto 2013, Peloso and Pietroni 2013, Creminelli et al. 2013, Chiang et al. 2014, Schmittfull, Baldauf, and Seljak 2014, Chiang 2017, Cabass, Pajer, and Schmidt 2018, Schmittfull & Moradinezhad Dizgah 2020, Goldstein et al. 2022, Giri, Münchmeyer, Smith 2023, Goldstein et al. 2024

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- Non-linearities described by **"response function"**
  - Can be large with primordial non-Gaussianity/EP violation
  - Can estimate non-perturbatively with separate universe sims

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Separate universe/peak background split

- Bispectrum model

$$B_\Phi(q,k) = 4 f_{\rm NL} \cos[\mu \ln(q/k) + \delta] P_\Phi(q) P_\Phi(k)$$

- Mode coupling ~ modification of small-scale power spectrum

$$P_m(\boldsymbol{k} \,|\, \Phi_L(\boldsymbol{q})) = \big(1 + 4 f_{\rm NL} \cos[\mu \ln(q) + \delta] \Phi_L(\boldsymbol{q}) \, \cos(\mu \ln(k))\big) \, P_m(k)$$

$$\equiv \big(1 + 2\epsilon_c \, \cos(\mu \ln(k))\big) \, P_m(k)$$

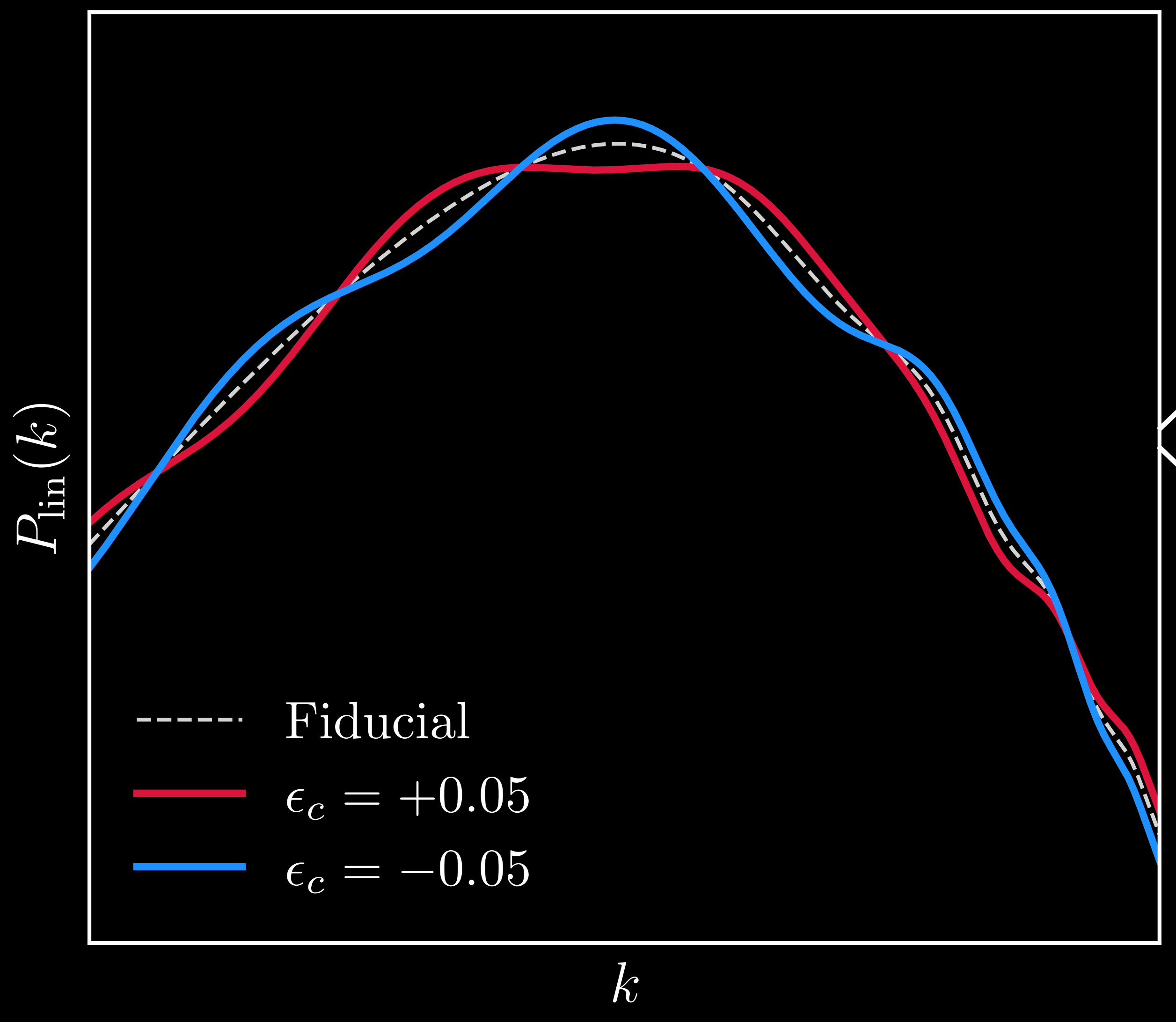
- PNG is equivalent to modification of initial conditions

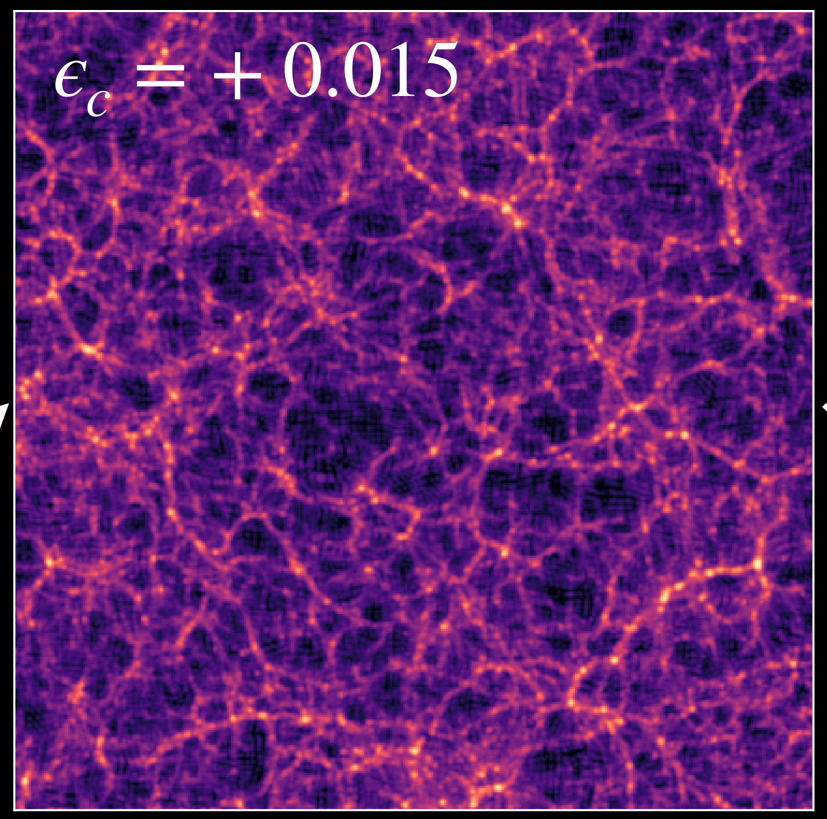
$$\frac{\partial P_m(k)}{\partial \Phi_L(\boldsymbol{q})} = 2 f_{\rm NL} \cos(\mu \ln(q) + \delta) \frac{\partial P_m(k)}{\partial_{\epsilon_c}} \bigg|_{\epsilon_c = 0}$$

Separate universe simulations (in practice)

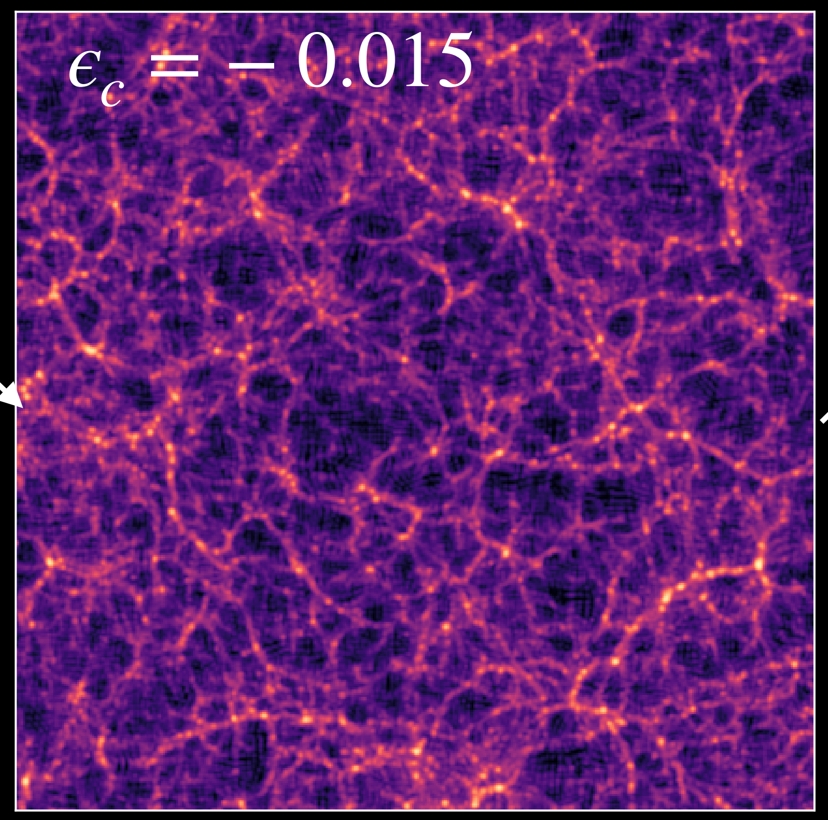
Modify $P_{\rm lin}(k)$

$$P_m(k) \to \left(1 + 2\epsilon_c \cos(\mu \ln(k))\right) P_m(k)$$

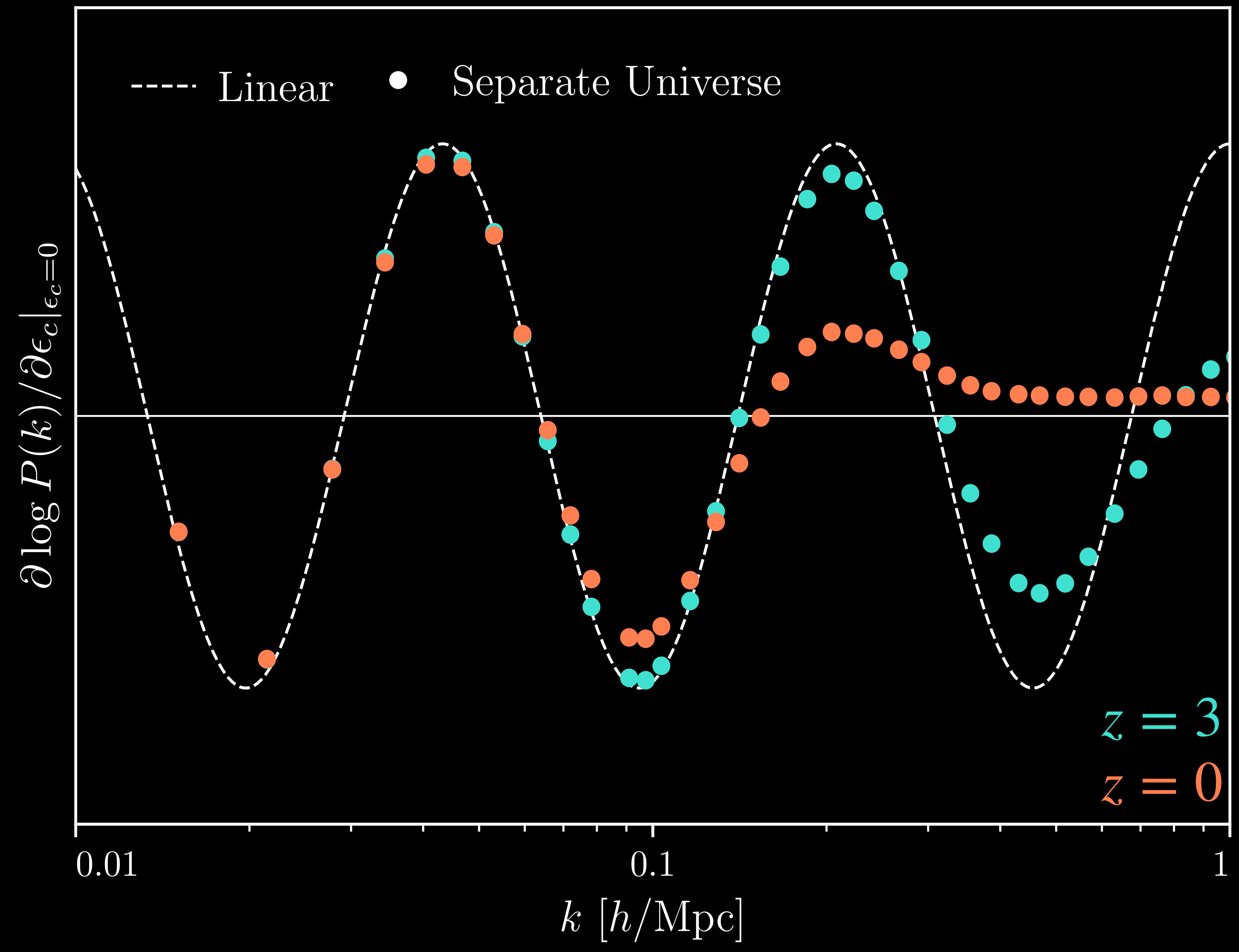




Run simulations (Gaussian IC)



Finite difference $P_m(k)$



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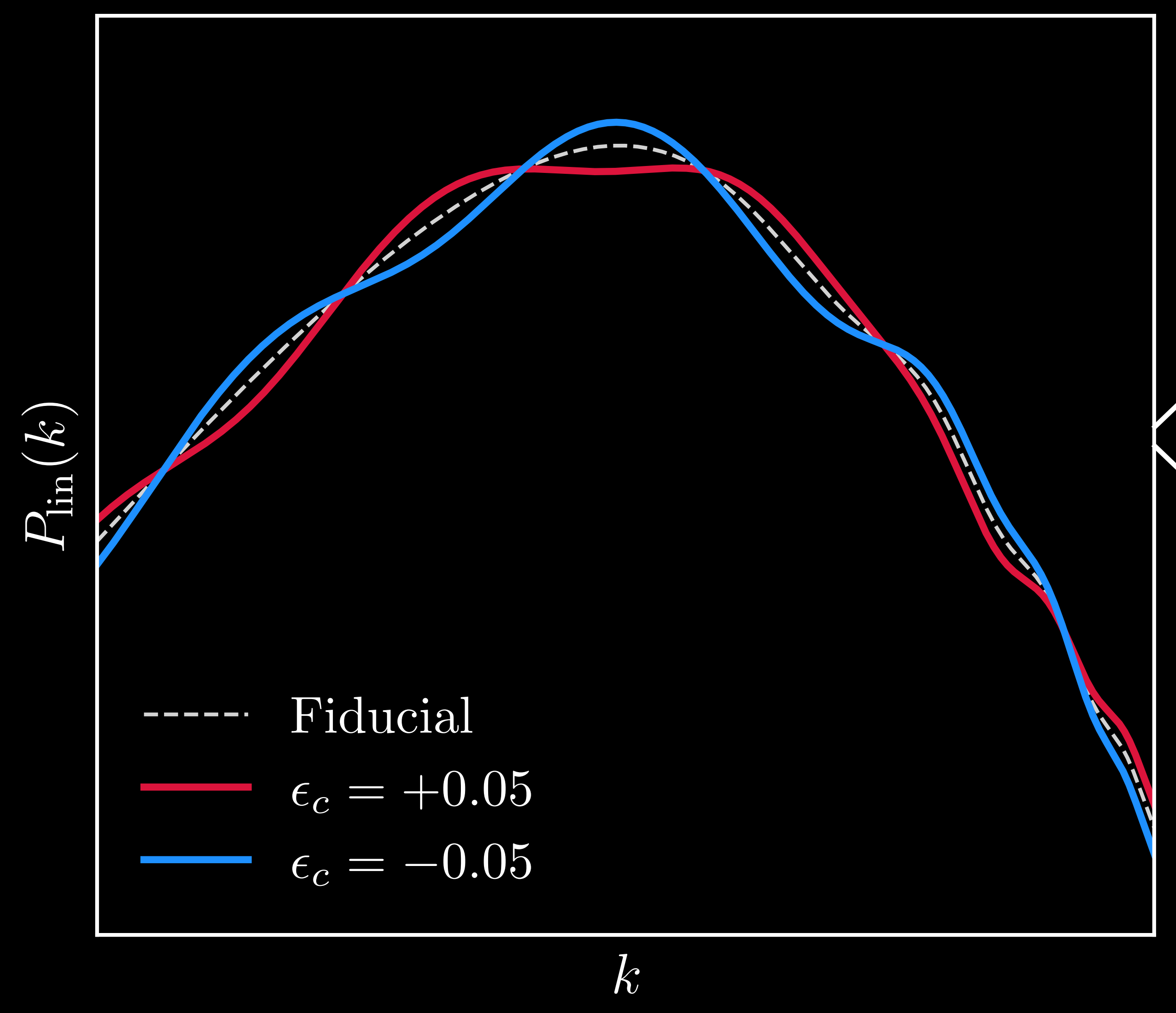

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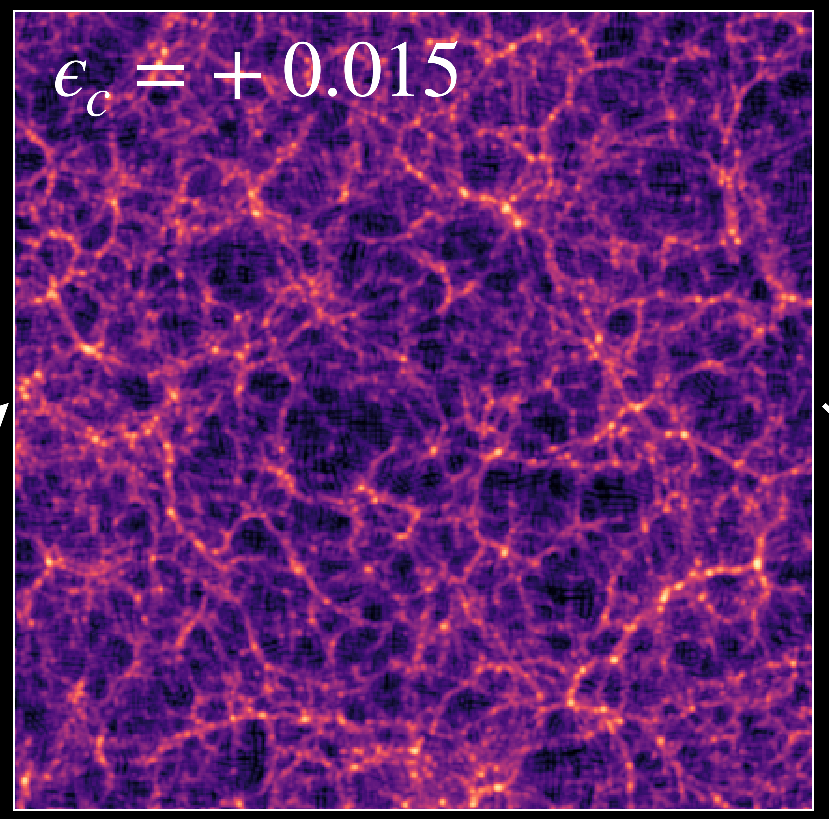

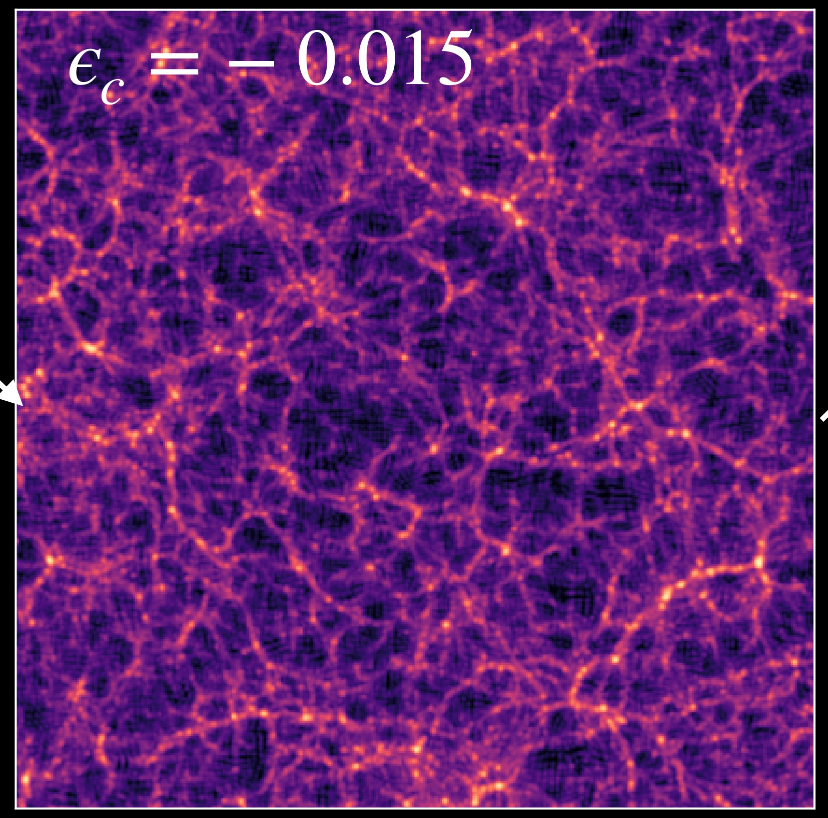

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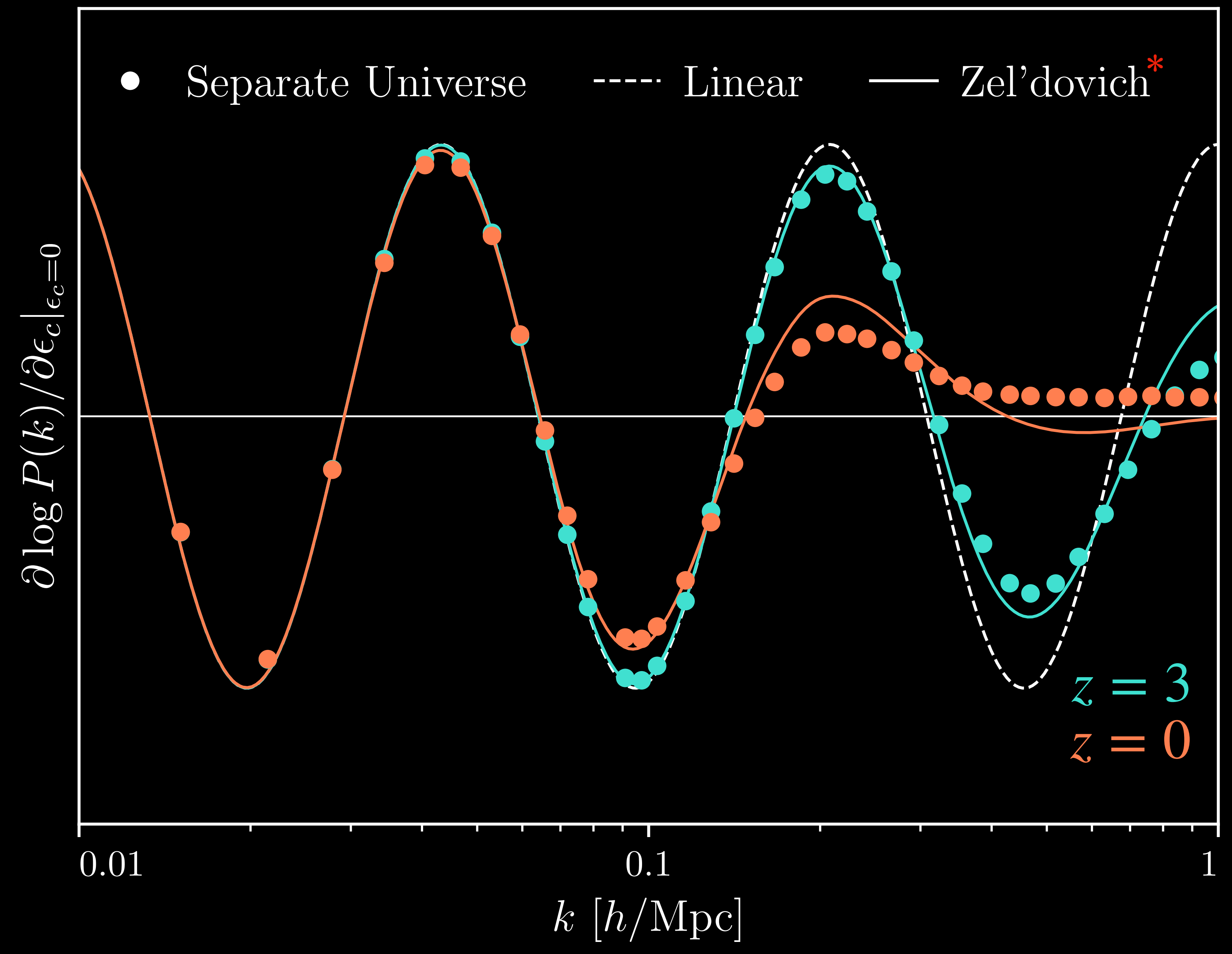
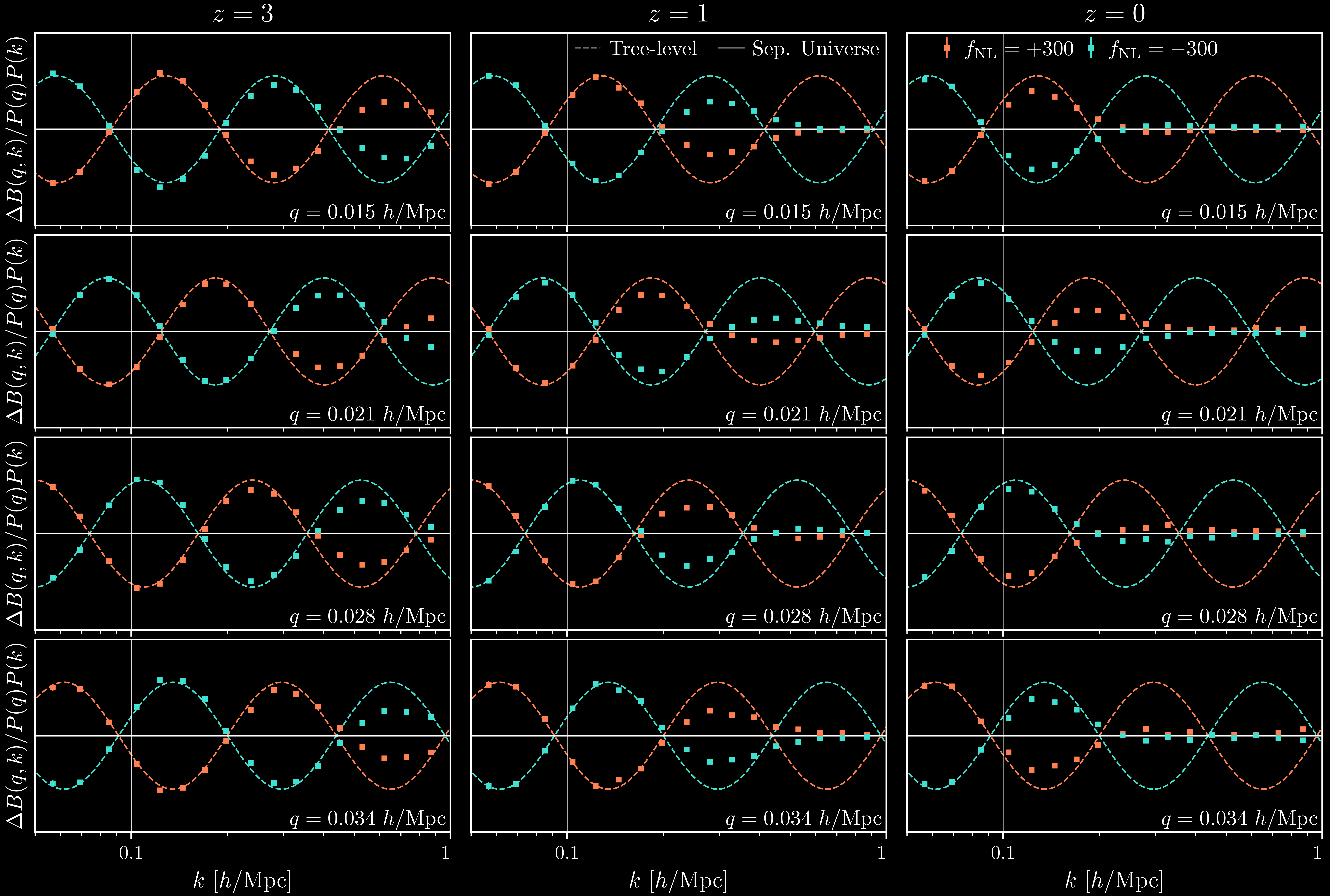
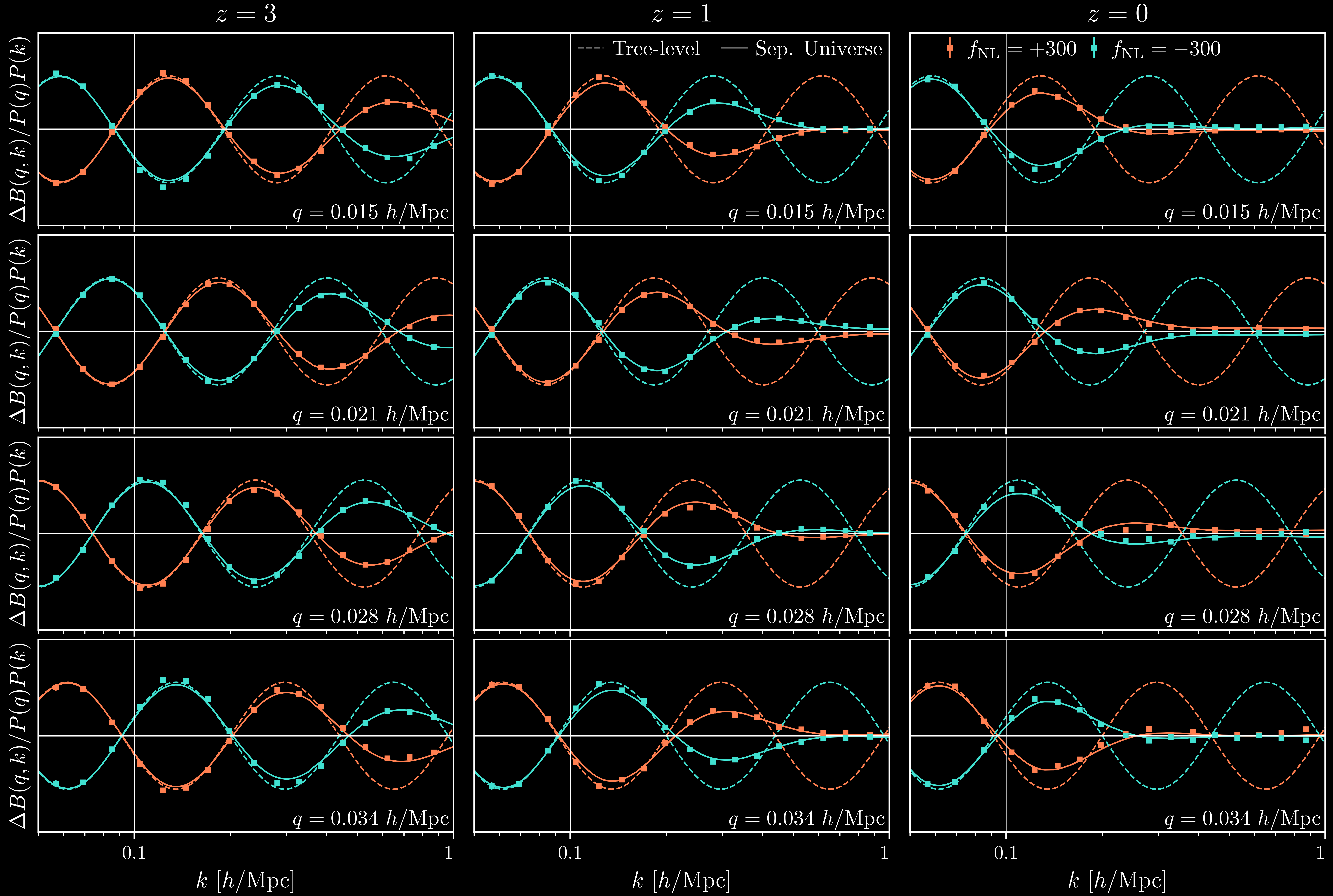


* *Computed using* velociletors

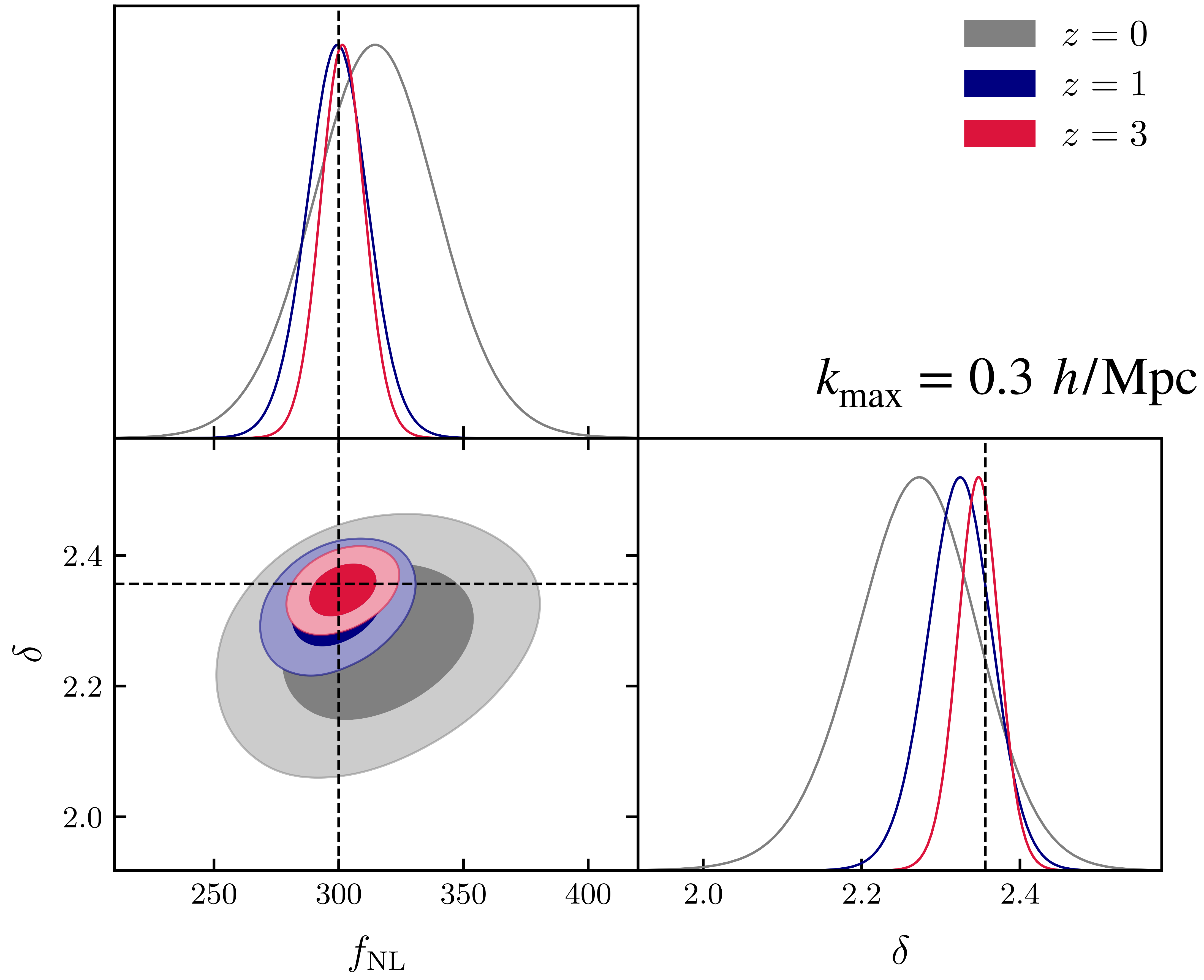


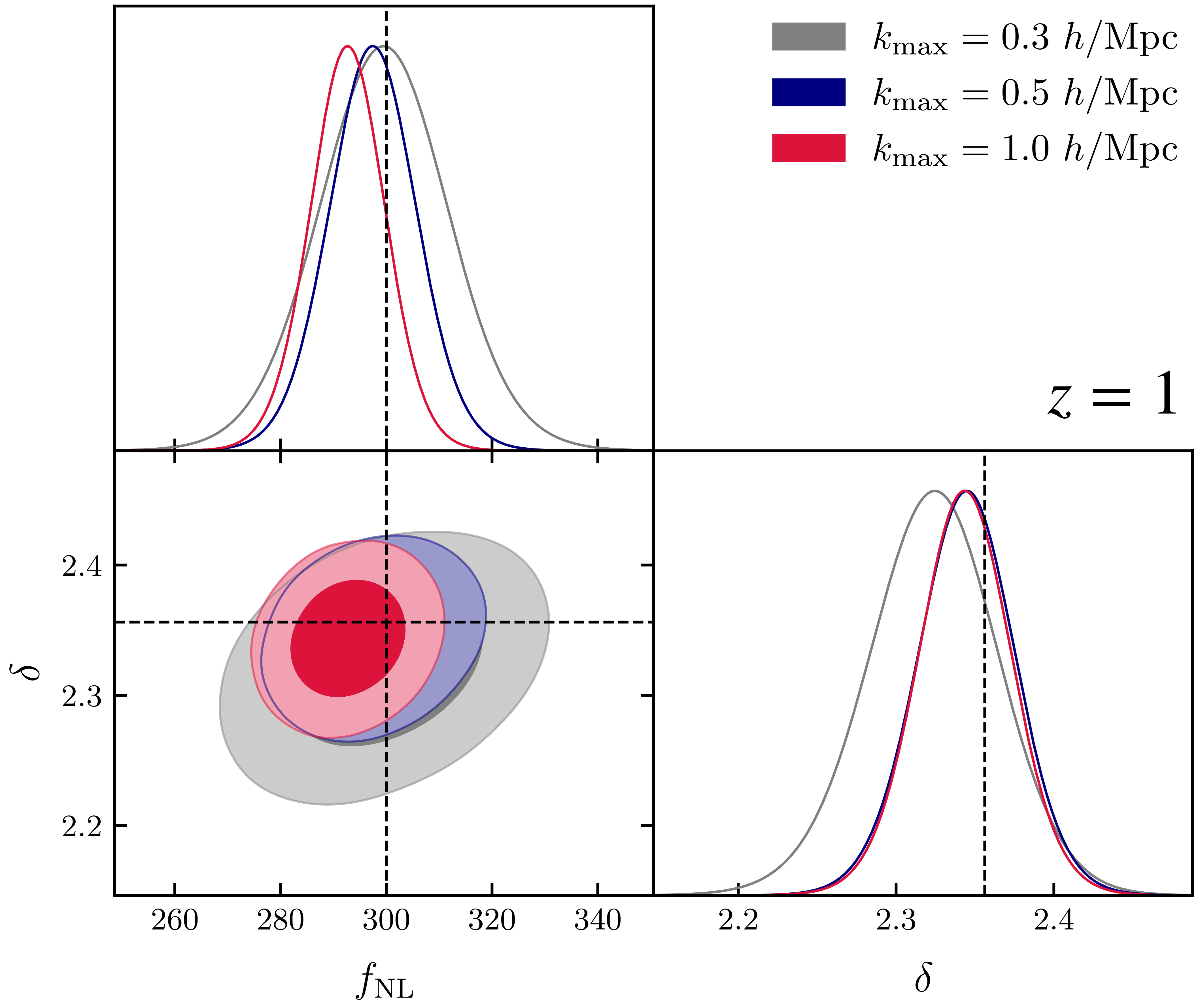
$z = 3$
$z = 1$
$z = 0$
Tree-level
Sep. Universe
$f_{\rm NL} = +300$
$f_{\rm NL} = -300$
$\Delta B(q,k)/P(q)P(k)$
$q = 0.015\ h/\mathrm{Mpc}$
$q = 0.021\ h/\mathrm{Mpc}$
$q = 0.028\ h/\mathrm{Mpc}$
$q = 0.034\ h/\mathrm{Mpc}$
0.1
1
$k\ [h/\mathrm{Mpc}]$



$z = 3$
$z = 1$
$z = 0$
Tree-level
Sep. Universe
$f_{\rm NL} = +300$
$f_{\rm NL} = -300$
$\Delta B(q,k)/P(q)P(k)$
$q = 0.015\ h/\mathrm{Mpc}$
$q = 0.021\ h/\mathrm{Mpc}$
$q = 0.028\ h/\mathrm{Mpc}$
$q = 0.034\ h/\mathrm{Mpc}$
0.1
1
$k\ [h/\mathrm{Mpc}]$

Constraints on $f_{\rm NL}$





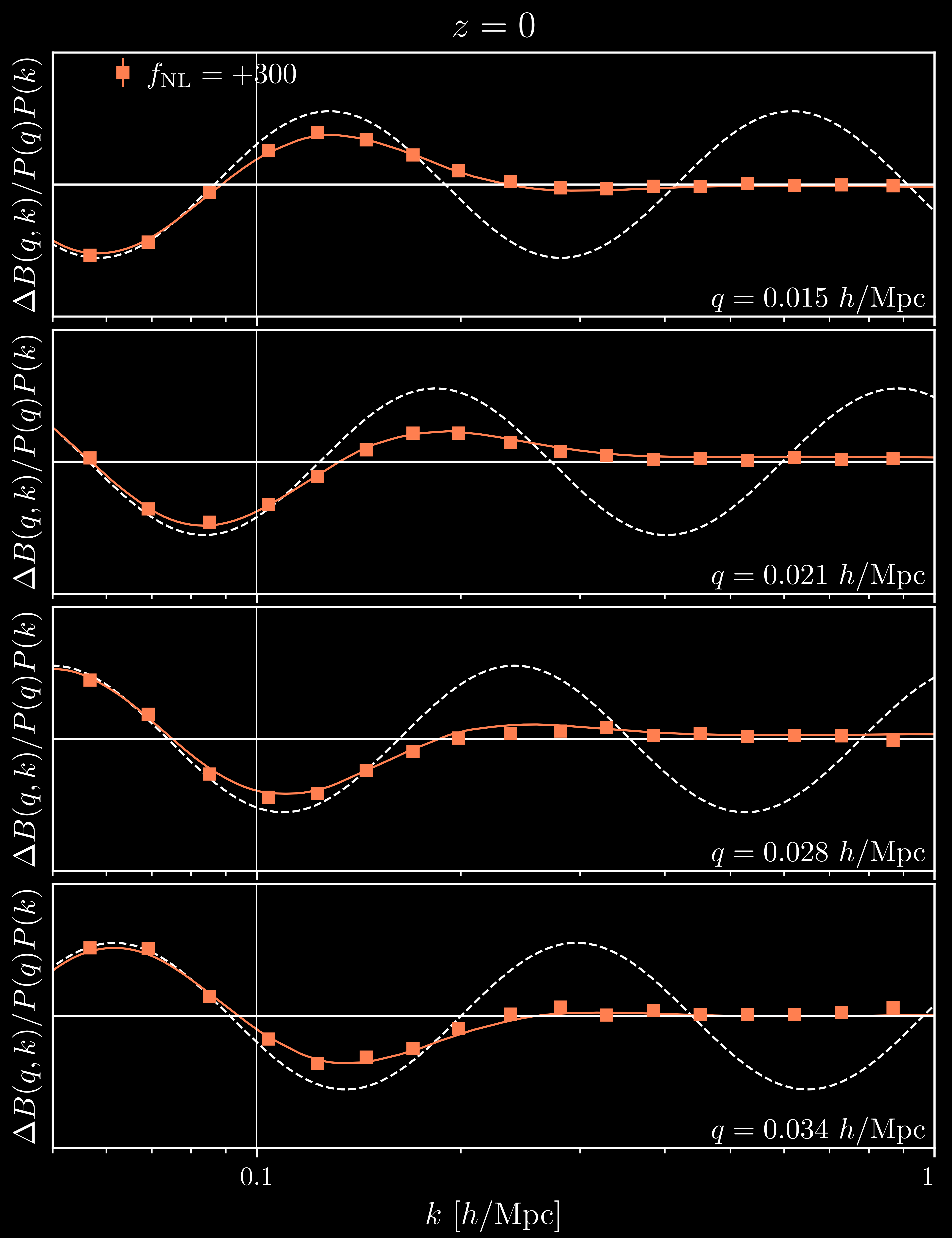
i). Information is lost at low redshifts

ii). Limited info. beyond quasi-linear scales

2. Can we undo the damping?

Reconstructing oscillatory bispectra

- Significant work has been done studying damping in power spectrum (BAO and features)
  - Mostly from long-wavelength displacements
- Reconstruction algorithms can "undo" damping
  - `iterrec`* (**Schmittfull++2017**)



*https://github.com/mschmittfull/iterrec

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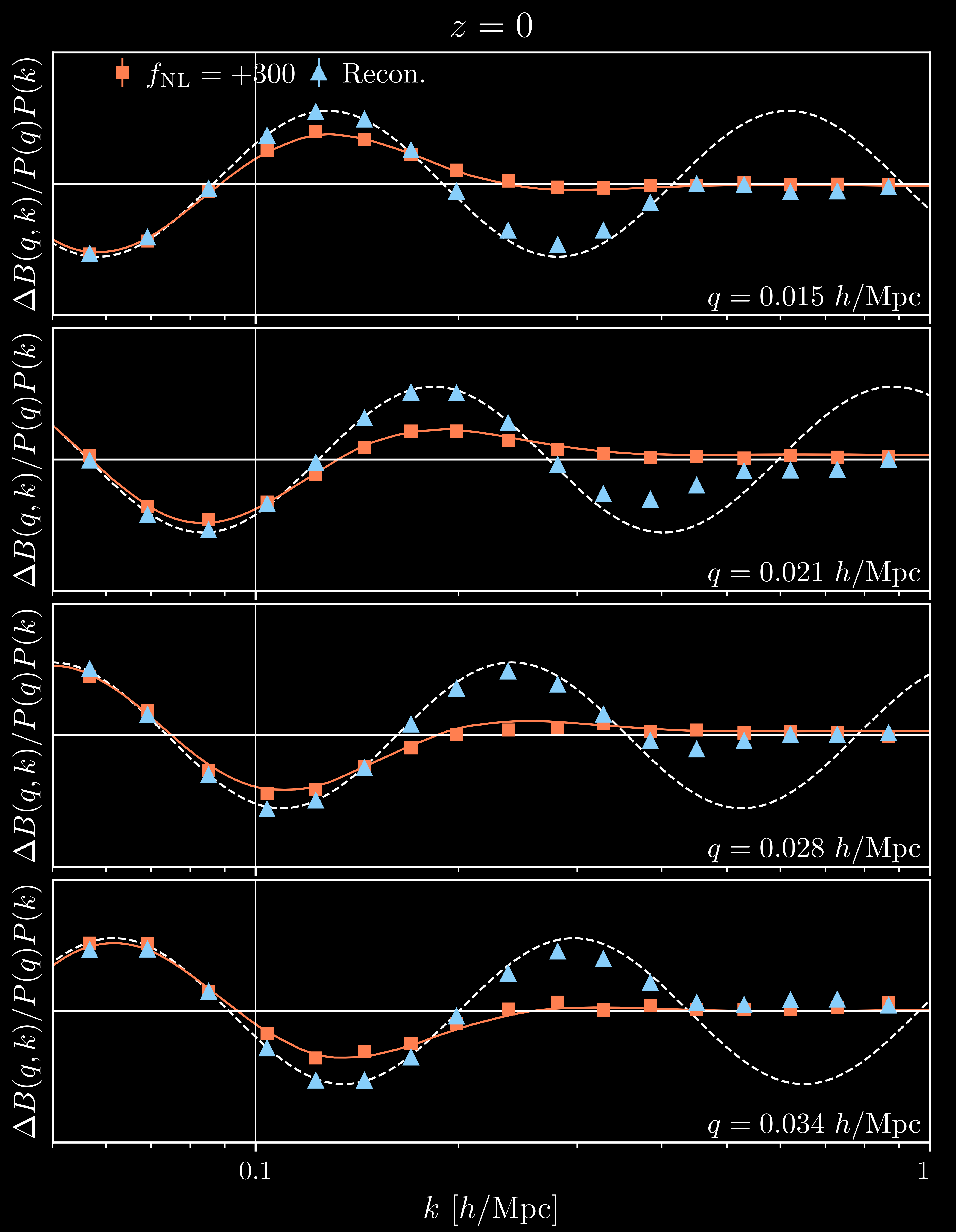

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Reconstructing oscillatory bispectra

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    - Mostly from long-wavelength displacements
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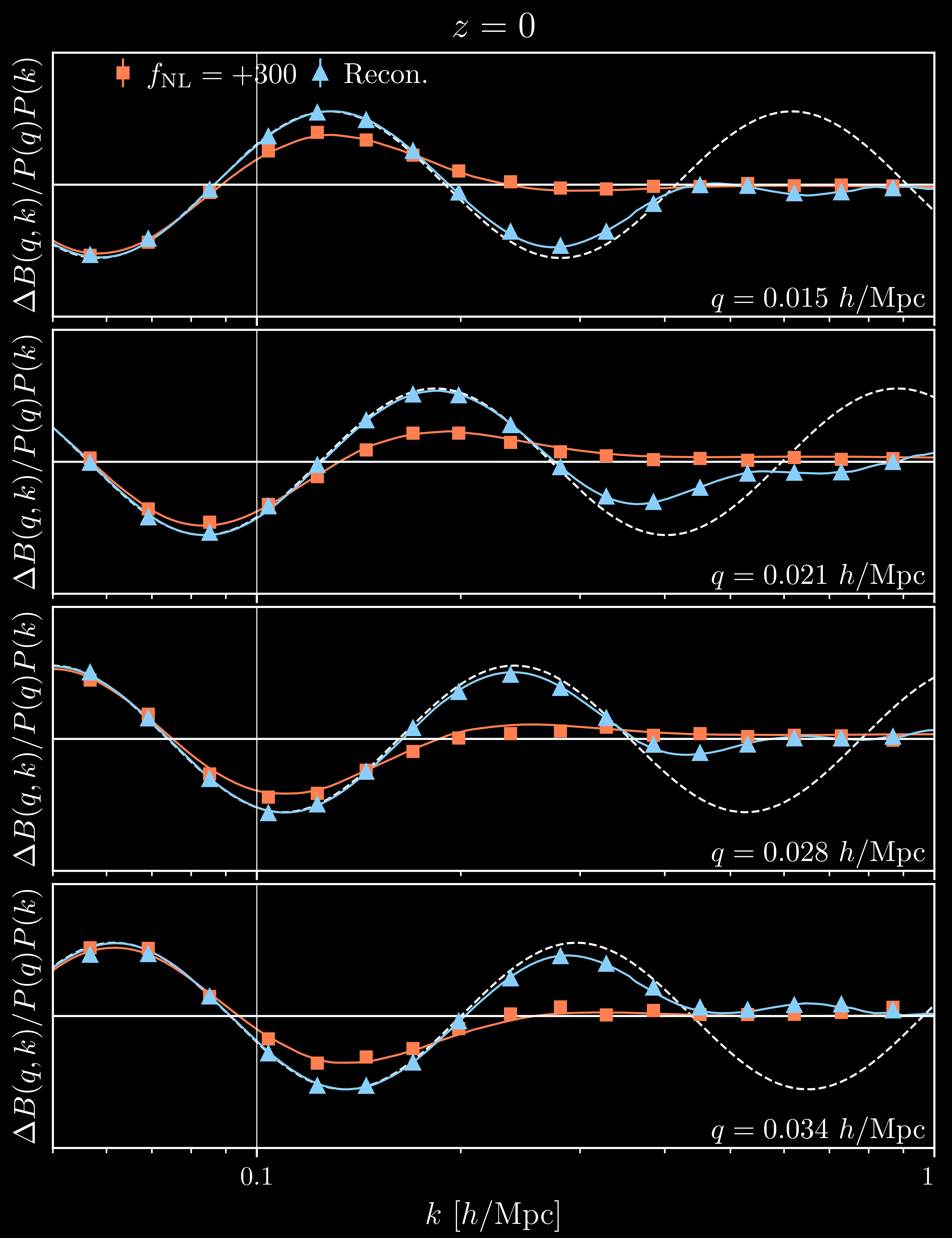
- Reconstruction largely restores oscillations
    - Post-reconstruction bispectrum can be modeled by reconstructing separate universe simulations



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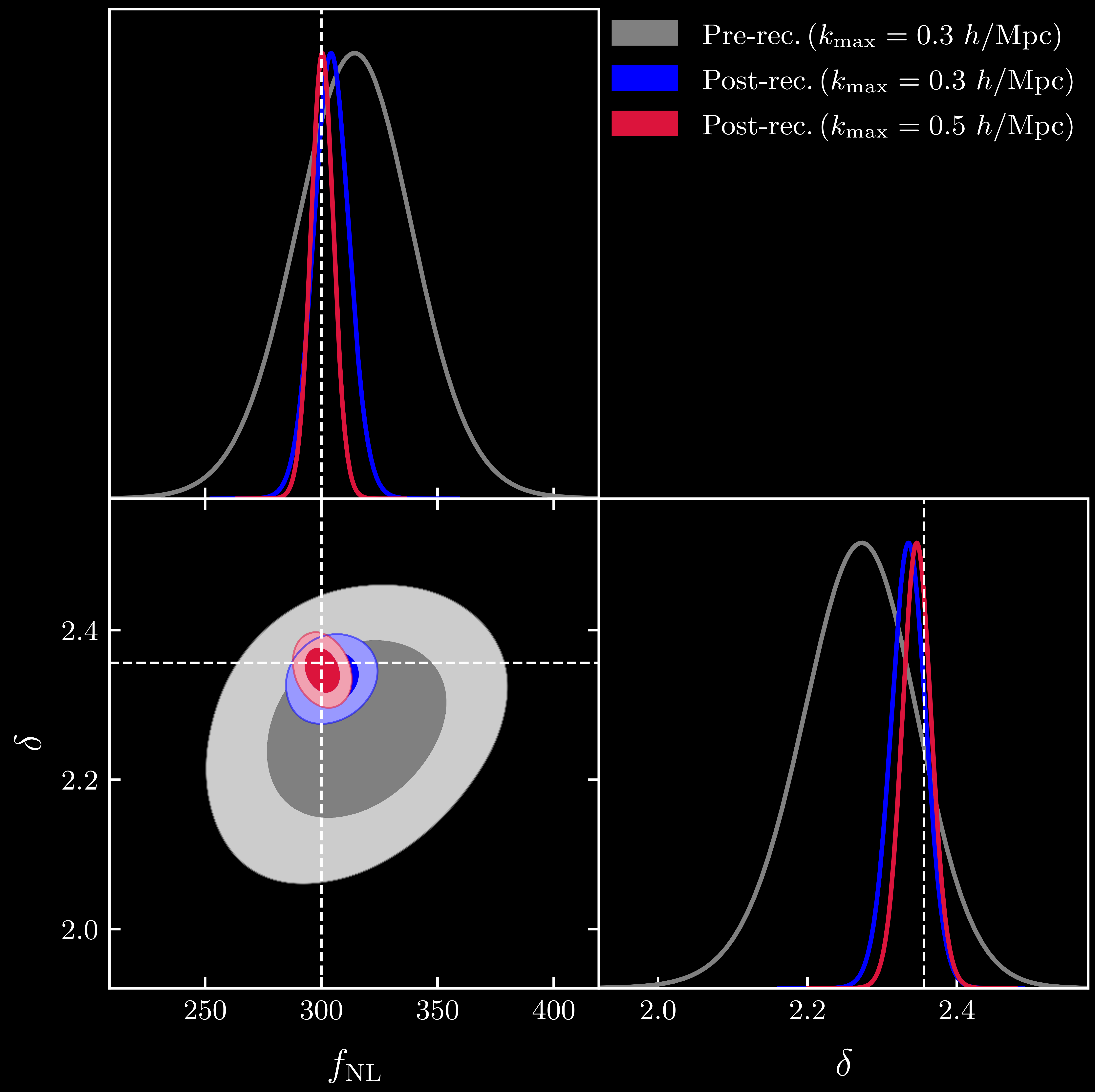
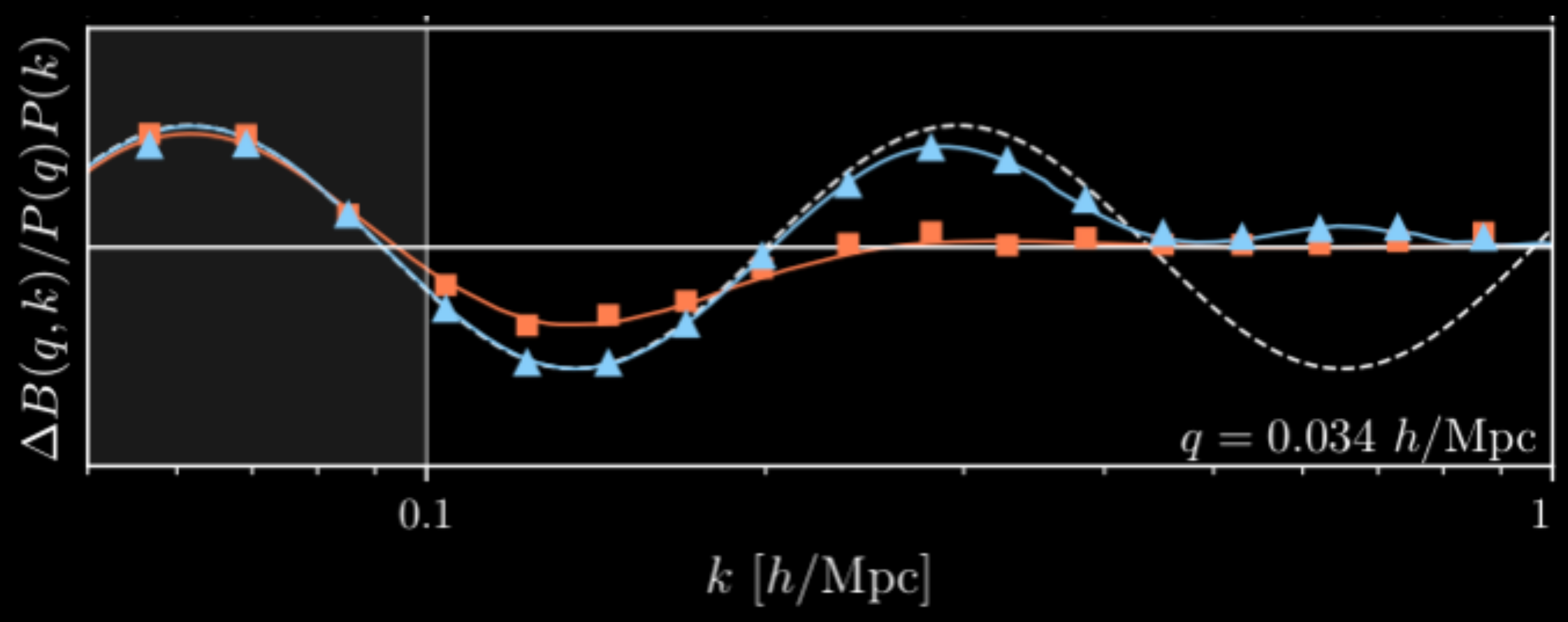
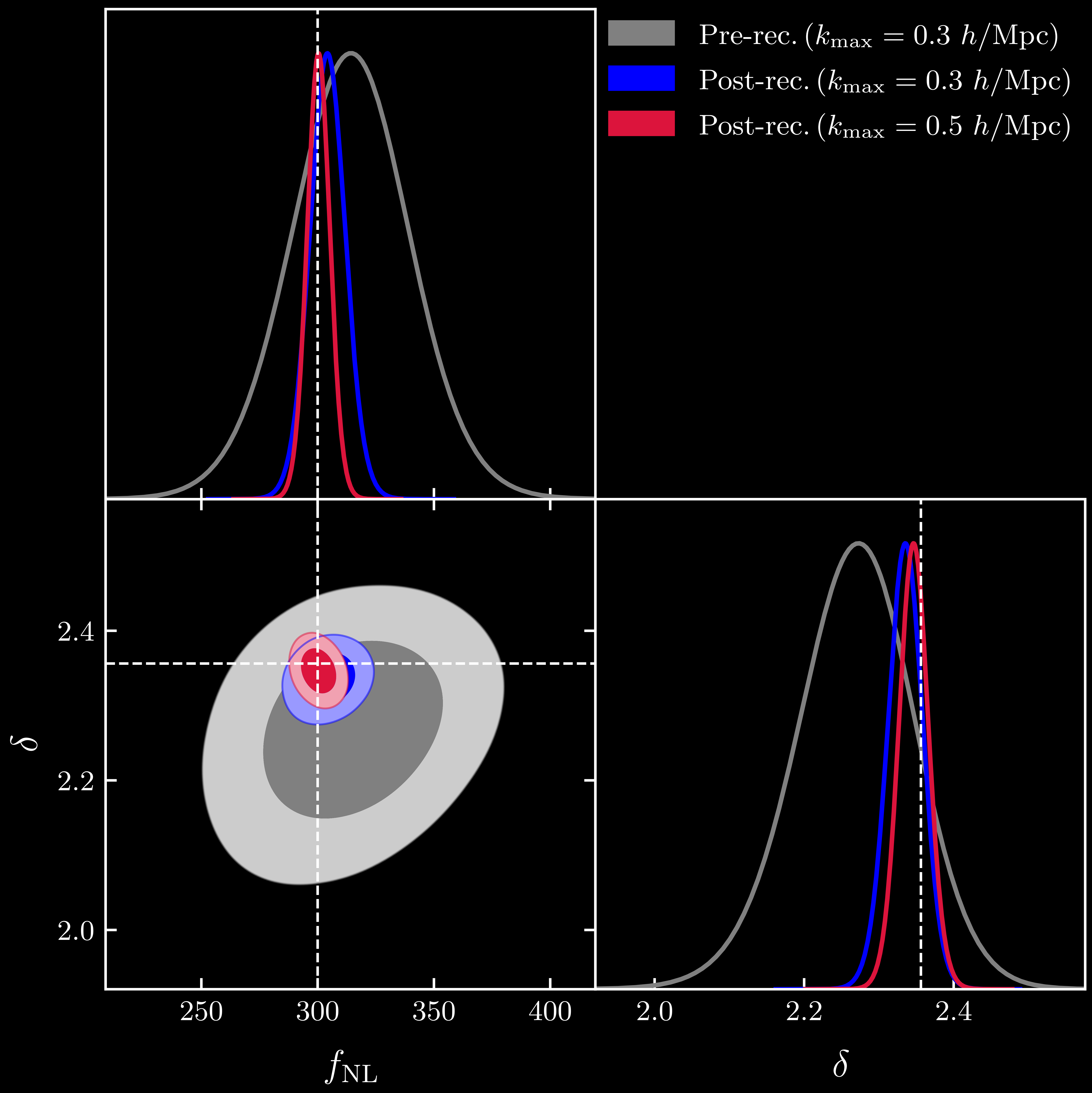
- **Factor of 5*** improvement on $f_{\rm NL}$ constraint at $z = 0$



* *Real-space matter is fairytale land! Post-recon redshift space biased tracers is nightmare land*

- **Factor of 5*** improvement on $f_{\rm NL}$ constraint at $z = 0$
- Where is the info coming from?
    1. Boosts signal





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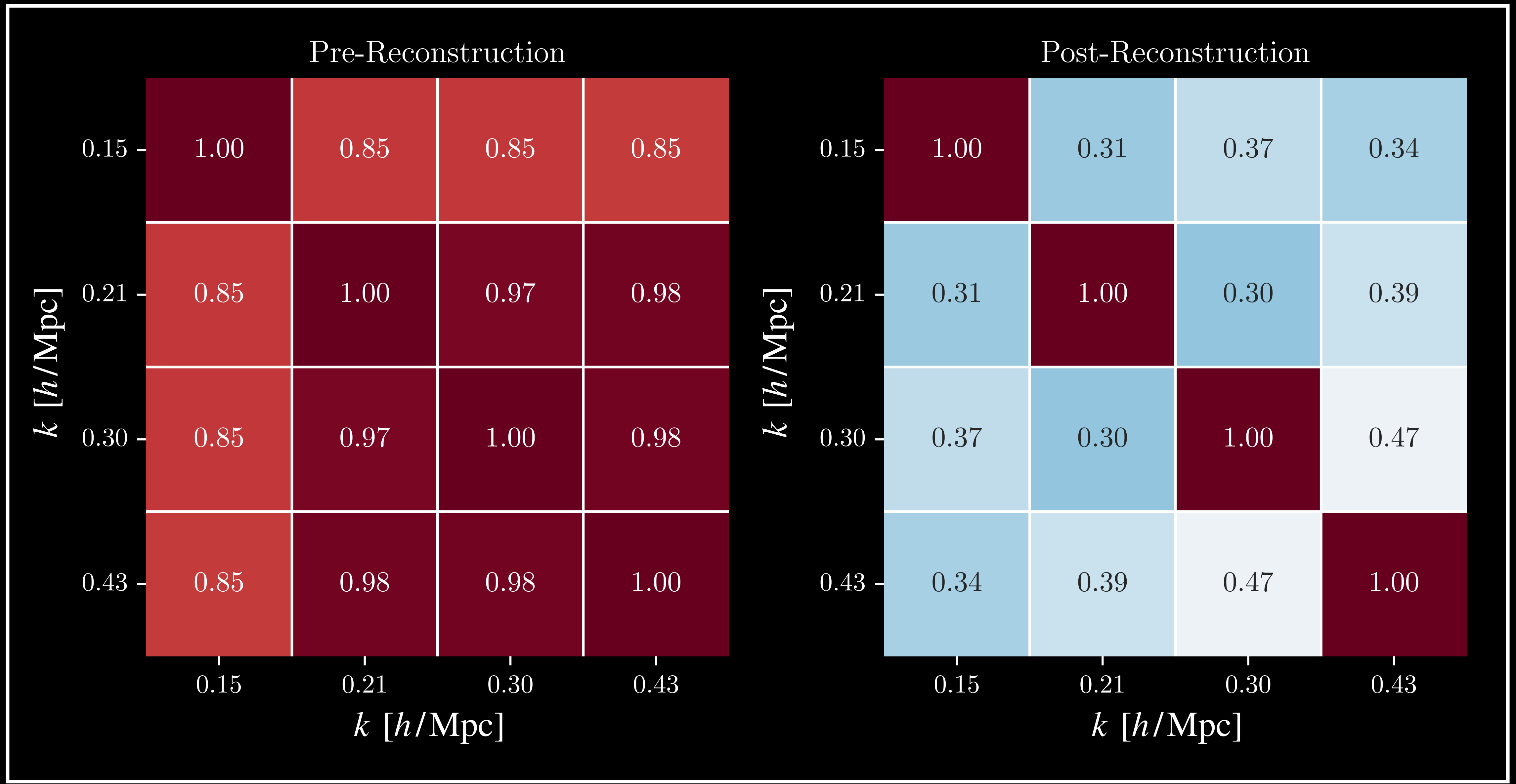
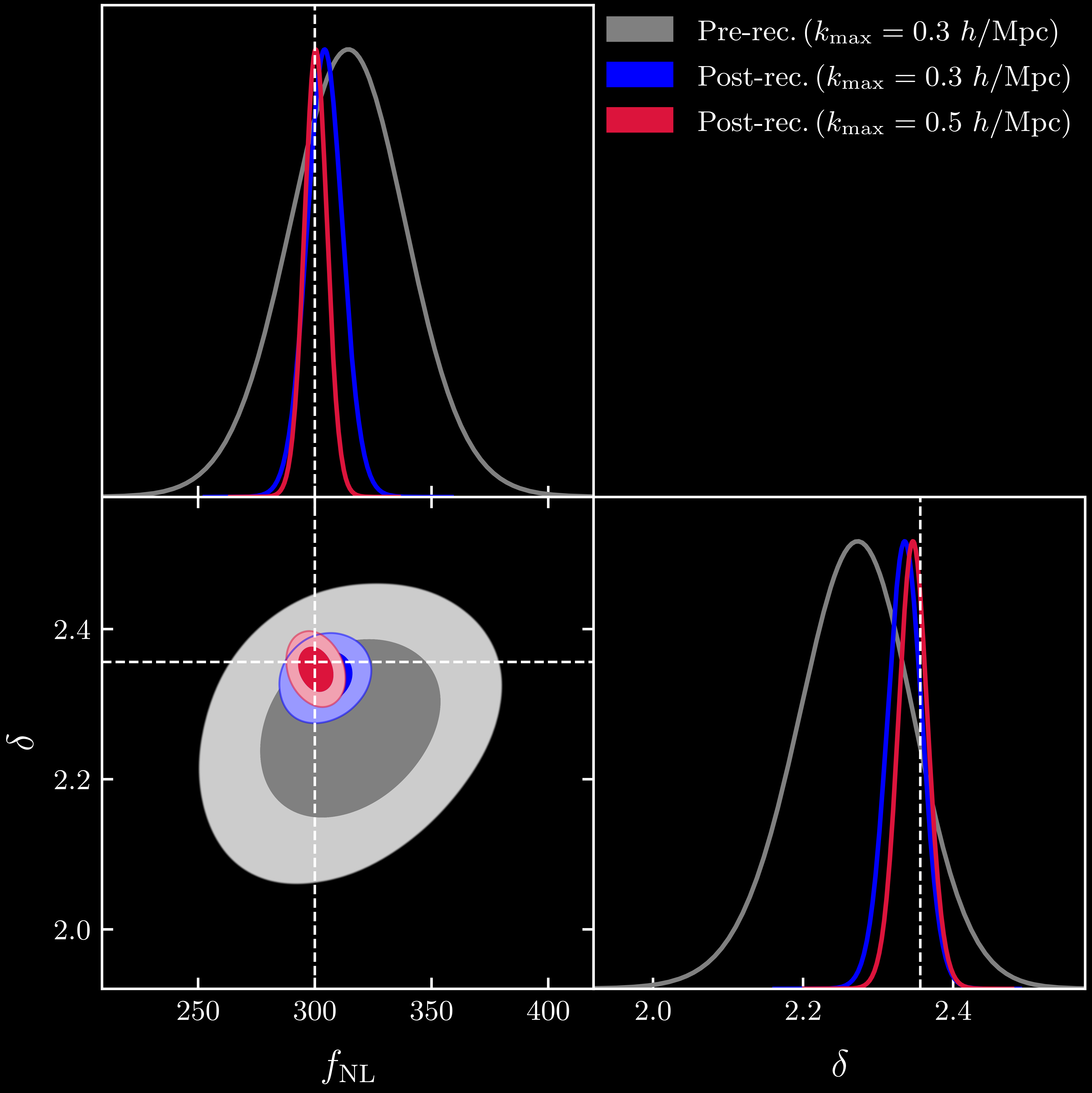
- **Factor of 5*** improvement on f_{NL} constraint at $z = 0$

- Where is the info coming from?

  1. Boosts signal

  2. Decreases noise ($B(q, k)$ has *large* non-Gaussian cov [Biagetti et al. 2021](#))





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Conclusion

Post-reconstruction squeezed bispectrum is potentially a useful probe of new physics

Backup

How to generate non-Gaussian IC's

- Given a bispectrum template, how do we get non-Gaussian ICs?
- Construct field that is quadratic in Gaussian field (ϕ_G)

$$\Phi(\mathbf{k}) = \phi_G(\mathbf{k}) + f_{\rm NL}\left[\Psi(\mathbf{k}) - \langle\Psi(\mathbf{k})\rangle\right]$$

$$\Psi(\mathbf{k}) = \int_{\mathbf{k}_1,\mathbf{k}_2} (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_{12})\, K(\mathbf{k}_1,\mathbf{k}_2)\, \phi_G(\mathbf{k}_1)\, \phi_G(\mathbf{k}_2).$$

- Kernel is (partially) fixed by bispectrum. For example, $K_{\rm loc} = 1$. For collider can use

$$\begin{aligned} K(\mathbf{k}_1,\mathbf{k}_2) &= \left\{\left(\frac{k_1}{k_{12}}\right)^{\Delta}\cos\left(\mu\ln\left(\frac{k_1}{k_{12}}\right)+\delta\right) + \left(\frac{k_2}{k_{12}}\right)^{\Delta}\cos\left(\mu\ln\left(\frac{k_2}{k_{12}}\right)+\delta\right) - \cos(\delta)\right\}, \\ &= \frac{1}{2}\left\{e^{i\delta}\left[\left(\frac{k_1}{k_{12}}\right)^{\Delta+i\mu} + \left(\frac{k_2}{k_{12}}\right)^{\Delta+i\mu} - 1\right] + e^{-i\delta}\left[\left(\frac{k_1}{k_{12}}\right)^{\Delta-i\mu} + \left(\frac{k_2}{k_{12}}\right)^{\Delta-i\mu} - 1\right]\right\}, \end{aligned}$$

Separate universe prediction for $\partial P_m(k)/\partial \Phi_L(q)$

- Split perturbation into long (L) and short (S)

$$\phi(\boldsymbol{x}) = \Phi_L(\boldsymbol{x}) + \Phi_S(\boldsymbol{x})$$

~Constant=background

- For local PNG, $\Phi_L(\boldsymbol{x})$ modulates $\Phi_S(\boldsymbol{x})$

- Same as local rescaling of amplitude of $P^{\mathrm{lin}}(k)$

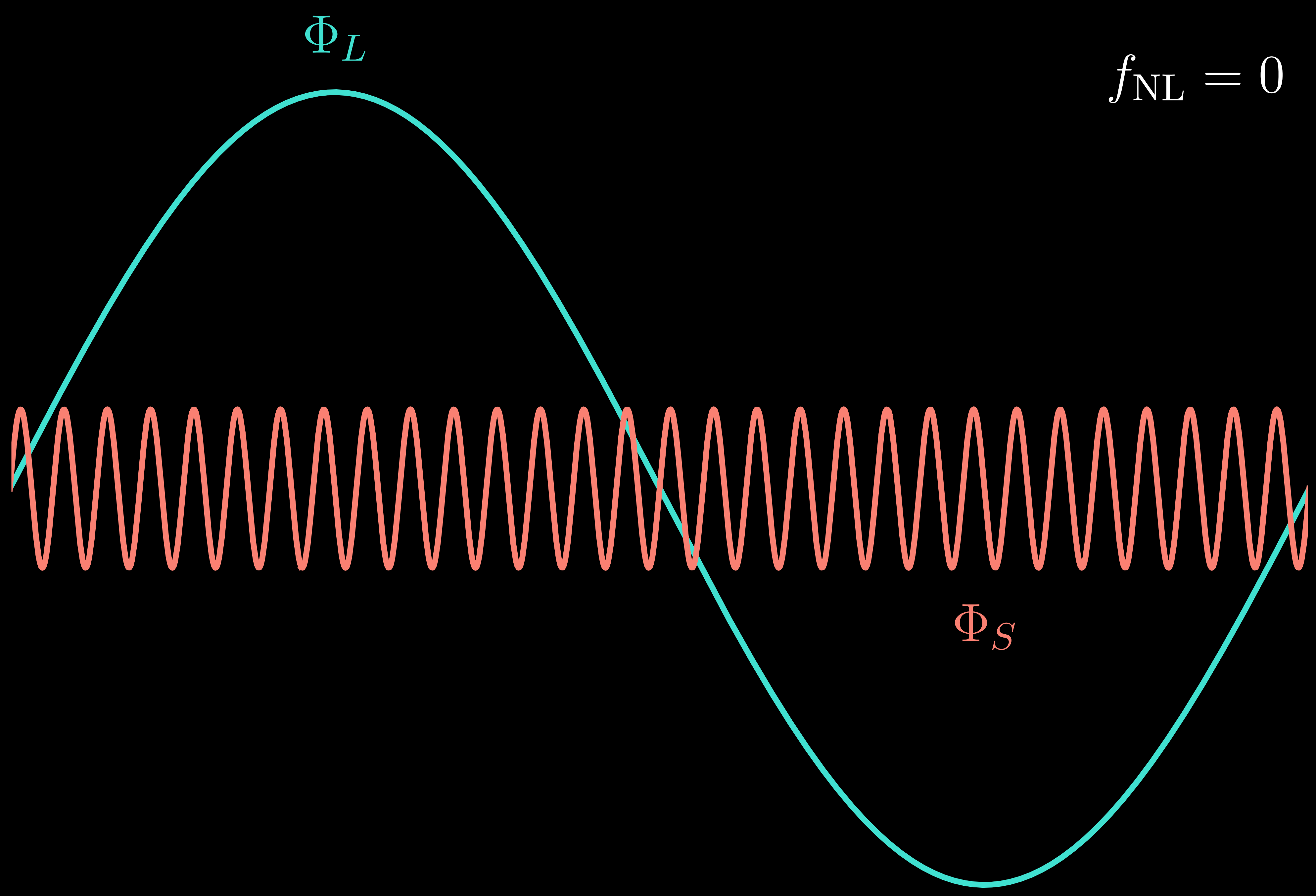
$$\sigma_8^{\mathrm{loc.}}(\boldsymbol{x}) = (1 + 2f_{\mathrm{NL}}\Phi_L(\boldsymbol{x}))\sigma_8$$

(e.g., Giri, Münchmeyer, Smith 2305.03070)

- PNG is just a modification of the ICs!

$$\frac{\partial P_m(k)}{\partial \Phi_L(q)} = 2f_{\mathrm{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8}\bigg|_{\mathrm{SU}}$$

(Dalal+07, Slosar+08, Desjacques+08)



$$\Phi = \Phi_L + f_{\mathrm{NL}}(\Phi_L^2 - \langle \Phi_L \rangle^2)$$
$$+ (1 + 2f_{\mathrm{NL}}\Phi_L)\Phi_S + f_{\mathrm{NL}}(\Phi_S^2 - \langle \Phi_S^2 \rangle)$$

Mode Coupling

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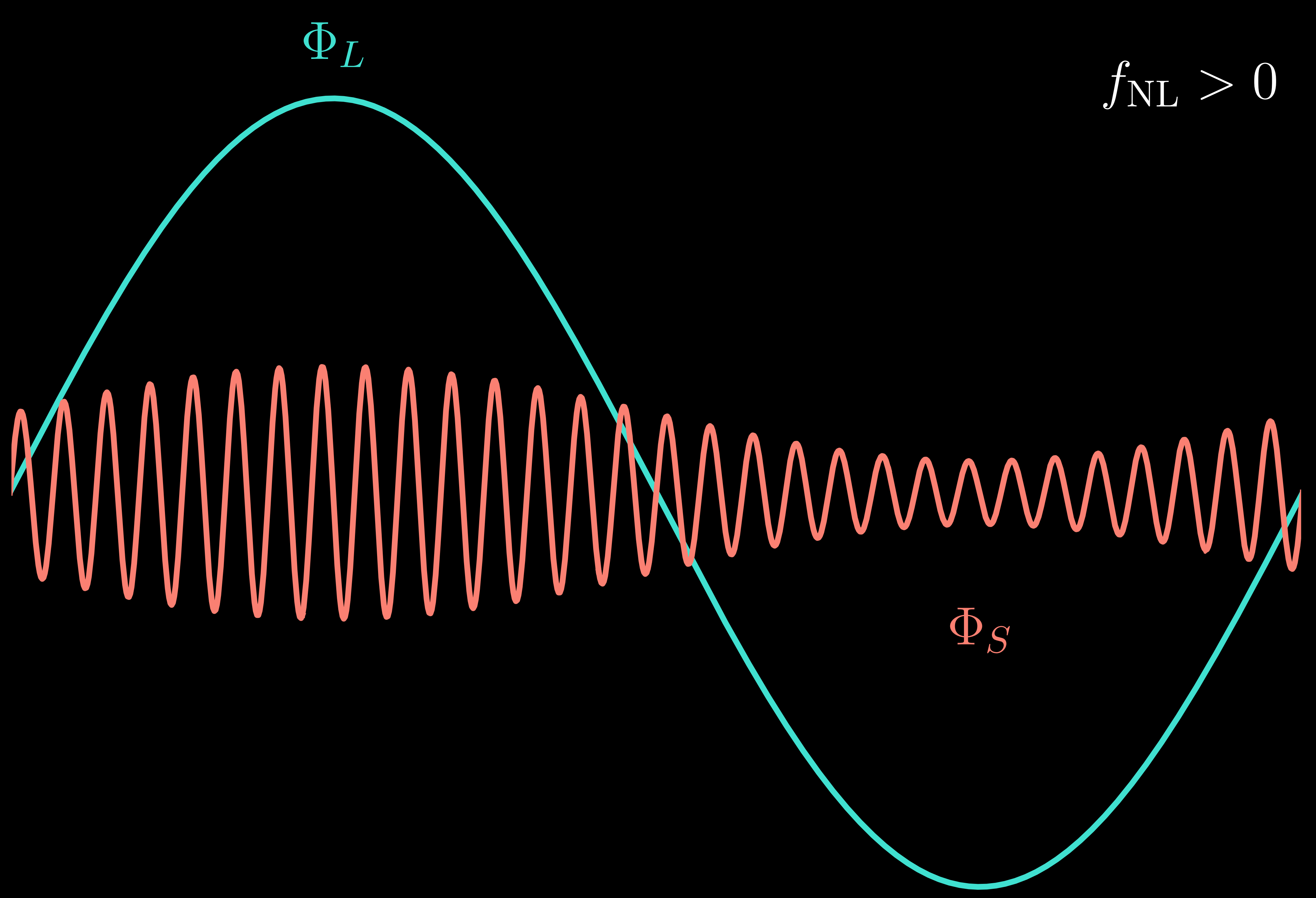

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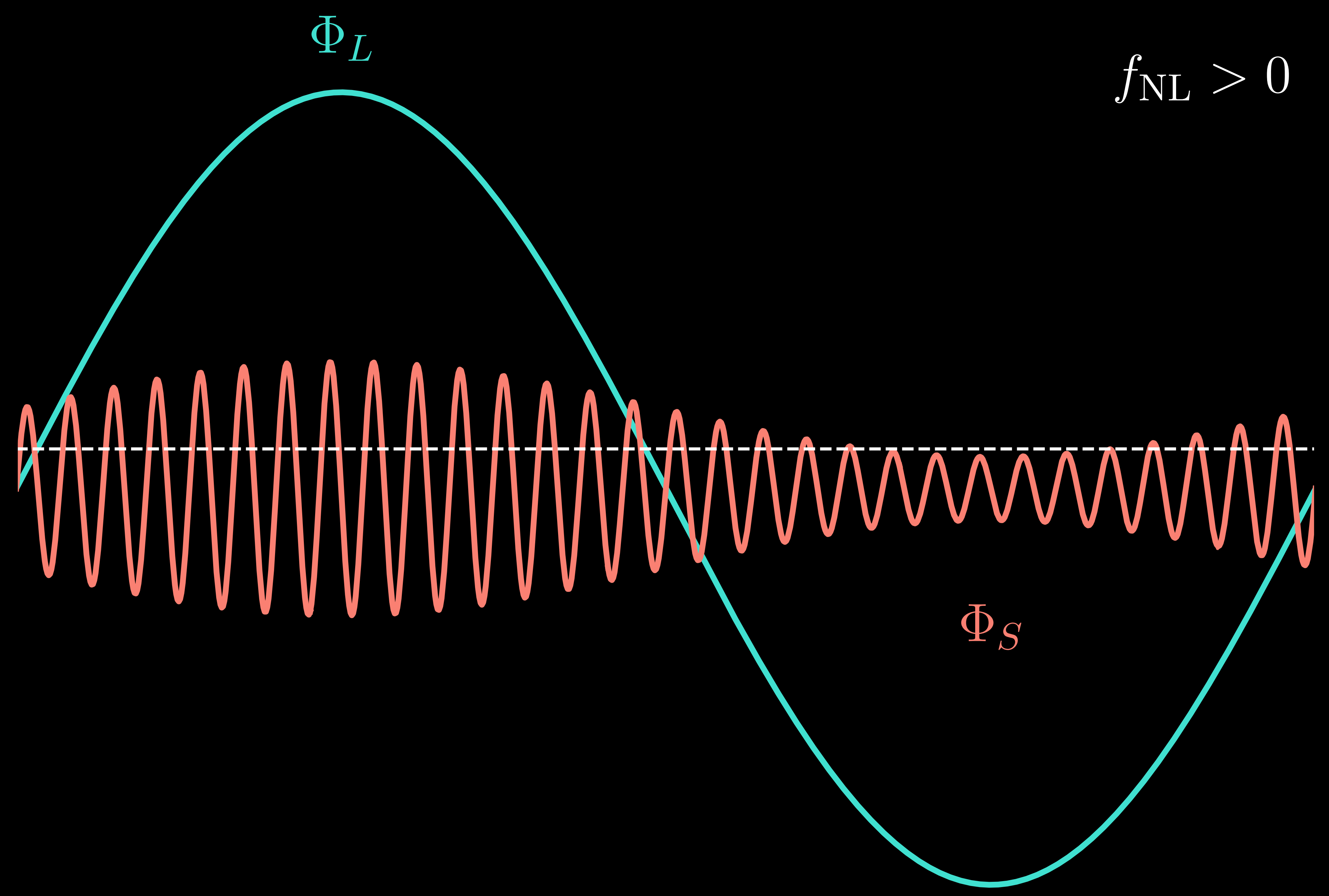

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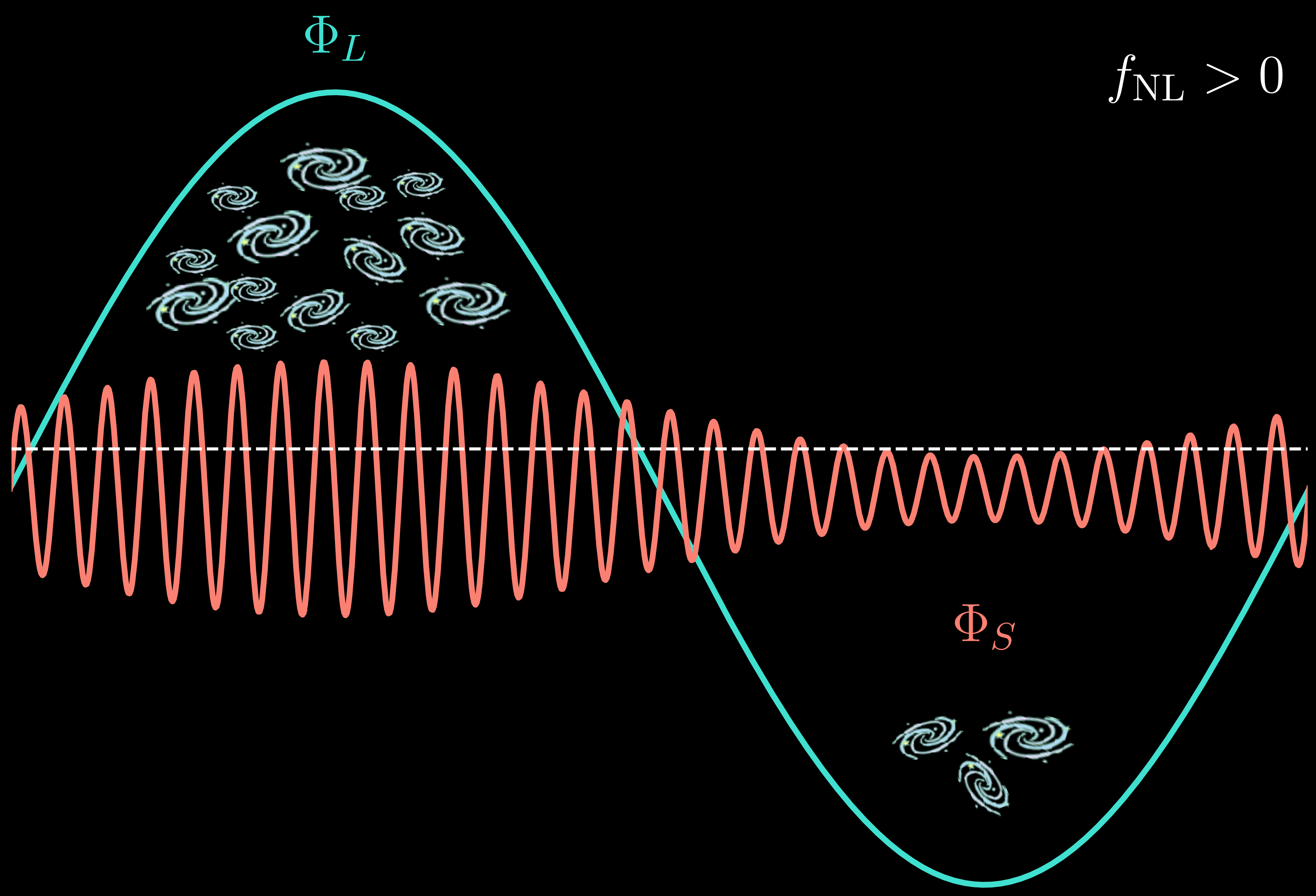


"Scale-dependent" bias
Dalal+08./Matarrese+08/
Slosar+08/Desjacques+08

Generalizing separate universe to massive-ish fields

- Consider small-scale modes $\delta_m(\boldsymbol{k}_1)$ and $\delta_m(\boldsymbol{k}_2)$ with **fixed amplitude,** but $\boldsymbol{k}_1 < \boldsymbol{k}_2$

- Add in **background** (~const.) potential fluctuation $\Phi_L(\boldsymbol{q})$

- Collider models have a **scale-dependent** response

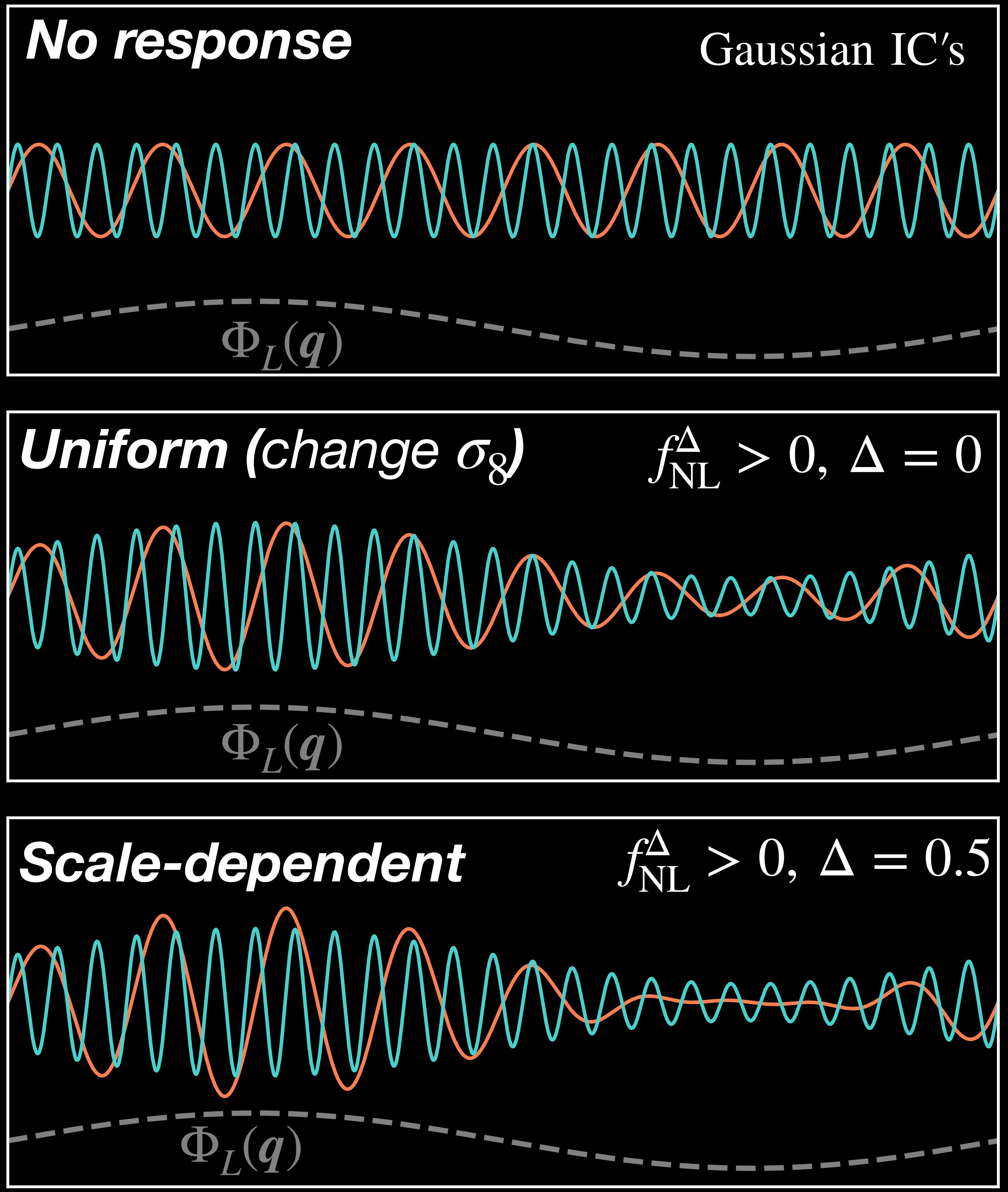
$$\delta_m(\boldsymbol{k}\,|\,\Phi_L(\boldsymbol{q})) = \left(1 + 2f_{\rm NL}^{\Delta}\left(\frac{q}{k}\right)^{\Delta}\Phi_L(\boldsymbol{q})\right)\delta_m(\boldsymbol{k})$$

Schmidt, Jeong, Desjacques, 2012

- Can compute $\partial P(k)/\partial \Phi_L(q)$ from sims with modified $P_m^{\rm lin.}$

$$P_m^{\rm lin.}(k\,|\,\epsilon,\Delta) \equiv \left(1 + 2\epsilon k^{-\Delta}\right)P_m^{\rm lin.}(k)\,.$$

$$\frac{\partial P_m(k)}{\partial\,\Phi_L(\mathbf{q})} = 2f_{\rm NL}^{\Delta}q^{\Delta}\frac{\partial P_m(k\,|\,\epsilon,\Delta)}{\partial\epsilon}\bigg|_{\epsilon=0}\,.$$



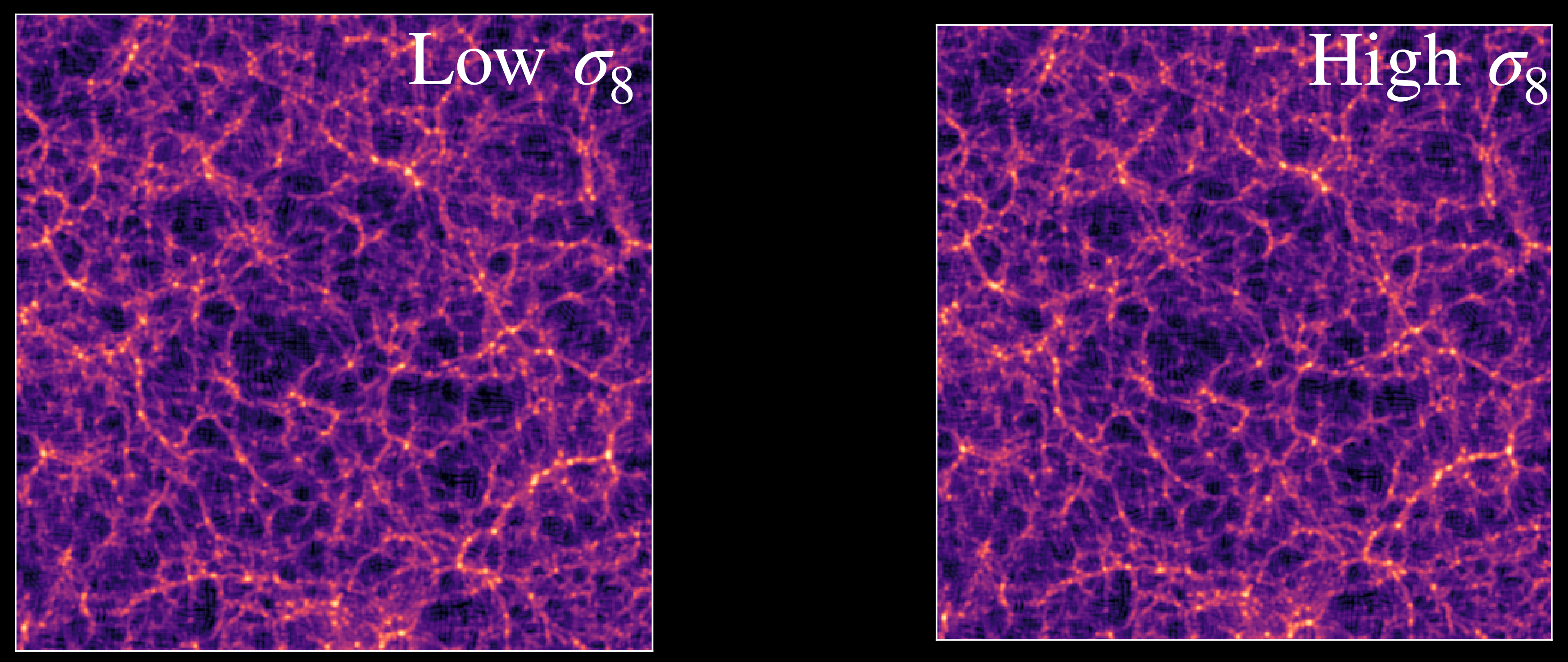
Separate universe (*continued*)

- For local PNG, we have

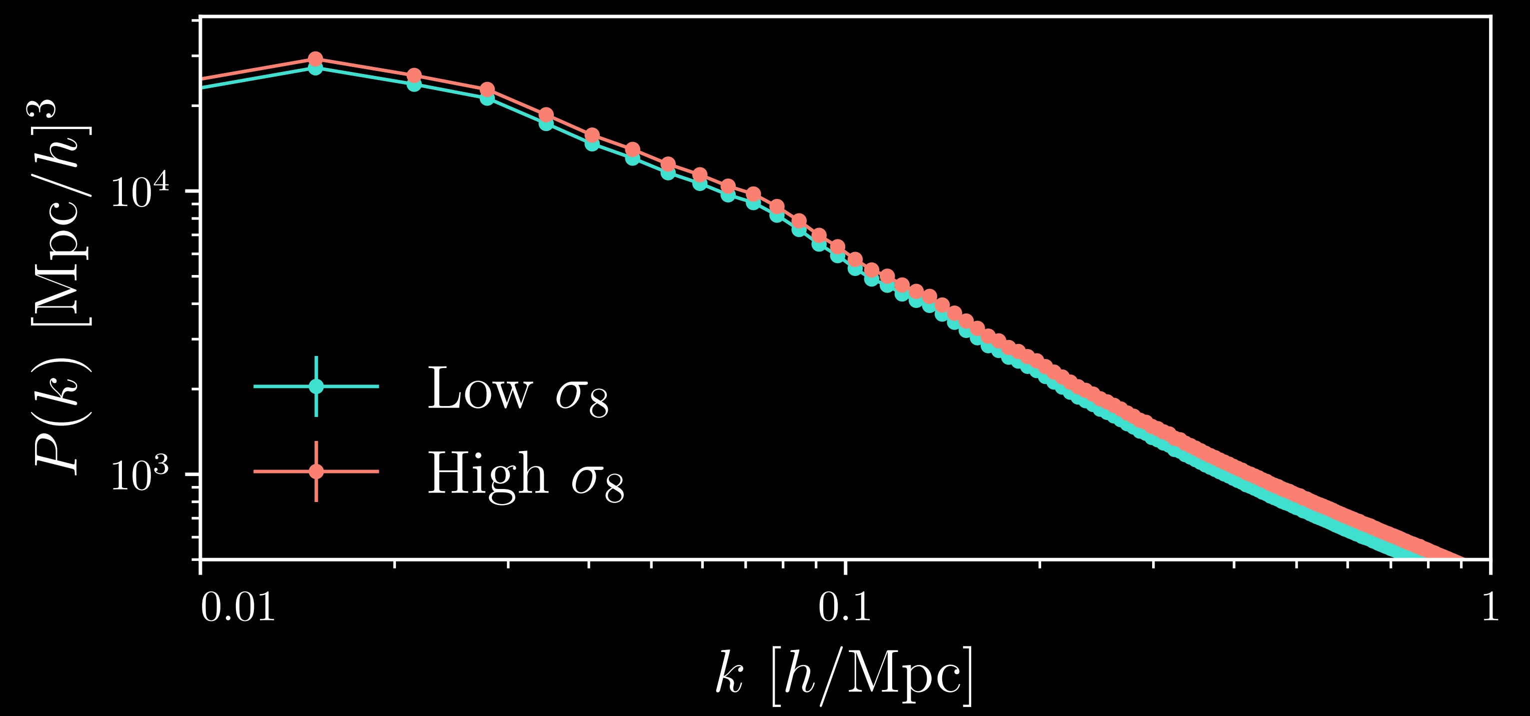
$$\frac{\partial P_m(k)}{\partial \Phi_L(q)} = 2f_{\mathrm{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8}\bigg|_{\mathrm{SU}}$$

- **a): How do we compute this?**
  - Run sims with modified $P^{\mathrm{lin.}}(k)$ and finite difference the late-time power spectra
- **b): Why is this useful?**
  - Relatively cheap to estimate from sims

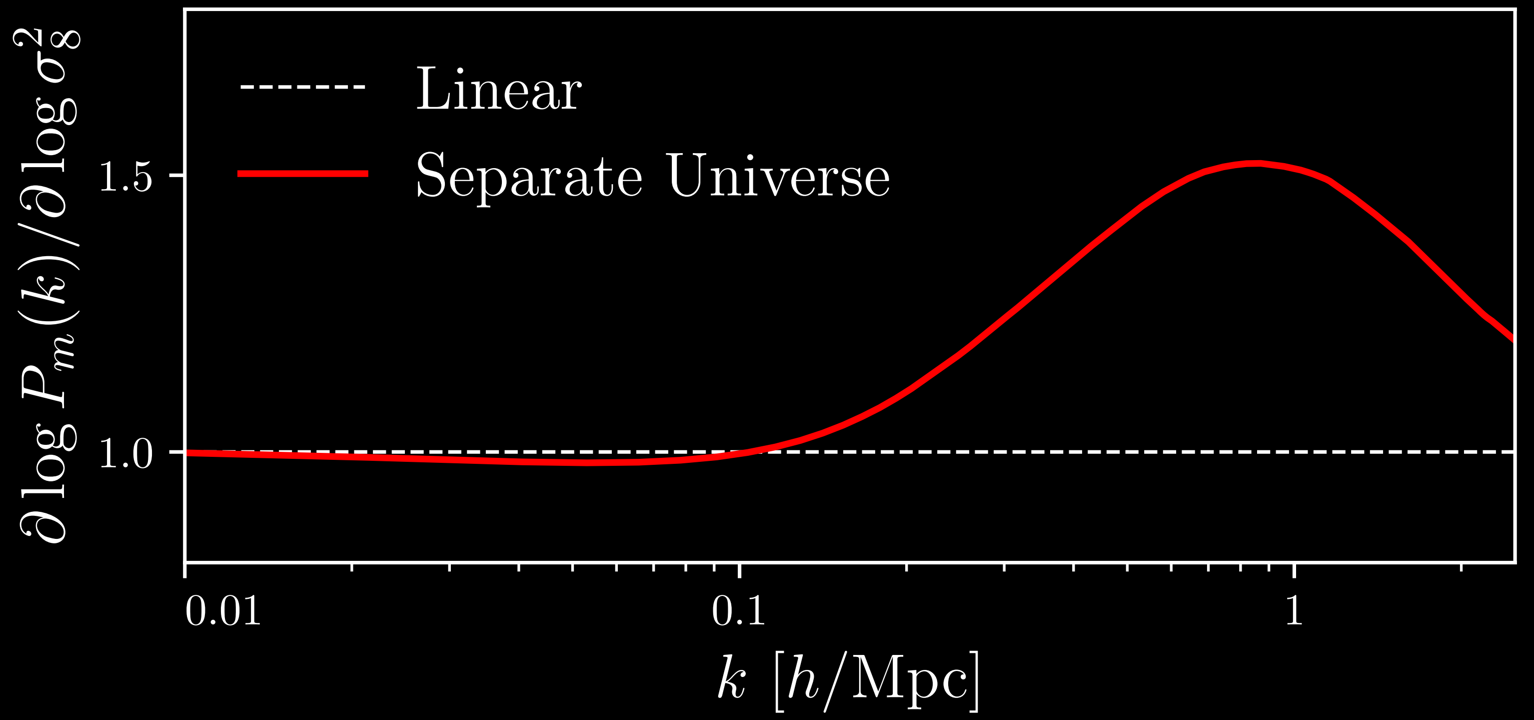
(i) Run sims with modified ICs



(ii) Compute $P_{mm}(k)$ from simulation output



(iii) Finite difference to get logarithmic derivative



Full bispectrum model

$$\lim_{q \ll k} B_m(\mathbf{q}, \mathbf{k}, z) = \frac{3 f_{\mathrm{NL}} \Omega_m H_0^2}{D_{\mathrm{md}}(z)} \frac{P_m(q, z)}{q^{2-\Delta} T(q)} \left[\cos(\mu \ln q + \delta) \left. \frac{\partial P_m(k, z|\mu, \Delta)}{\partial \epsilon_c} \right|_{\epsilon_c = 0} + \sin(\mu \ln q + \delta) \left. \frac{\partial P_m(k, z|\mu, \Delta)}{\partial \epsilon_s} \right|_{\epsilon_s = 0} \right]$$

$$+ \left(a_0(k) + a_2(k) \frac{q^2}{k^2} + \cdots \right) P_m(q, z) P_m(k, z).$$