4 November 2025

Women in Theoretical Physics

Premio Nazionale “Milla Baldo Ceolin” 2024

Title of the Thesis:

Determination of the Reionization optical depth through a combined analysis of Planck and WMAP data

Master’s Degree in Physics LM-17

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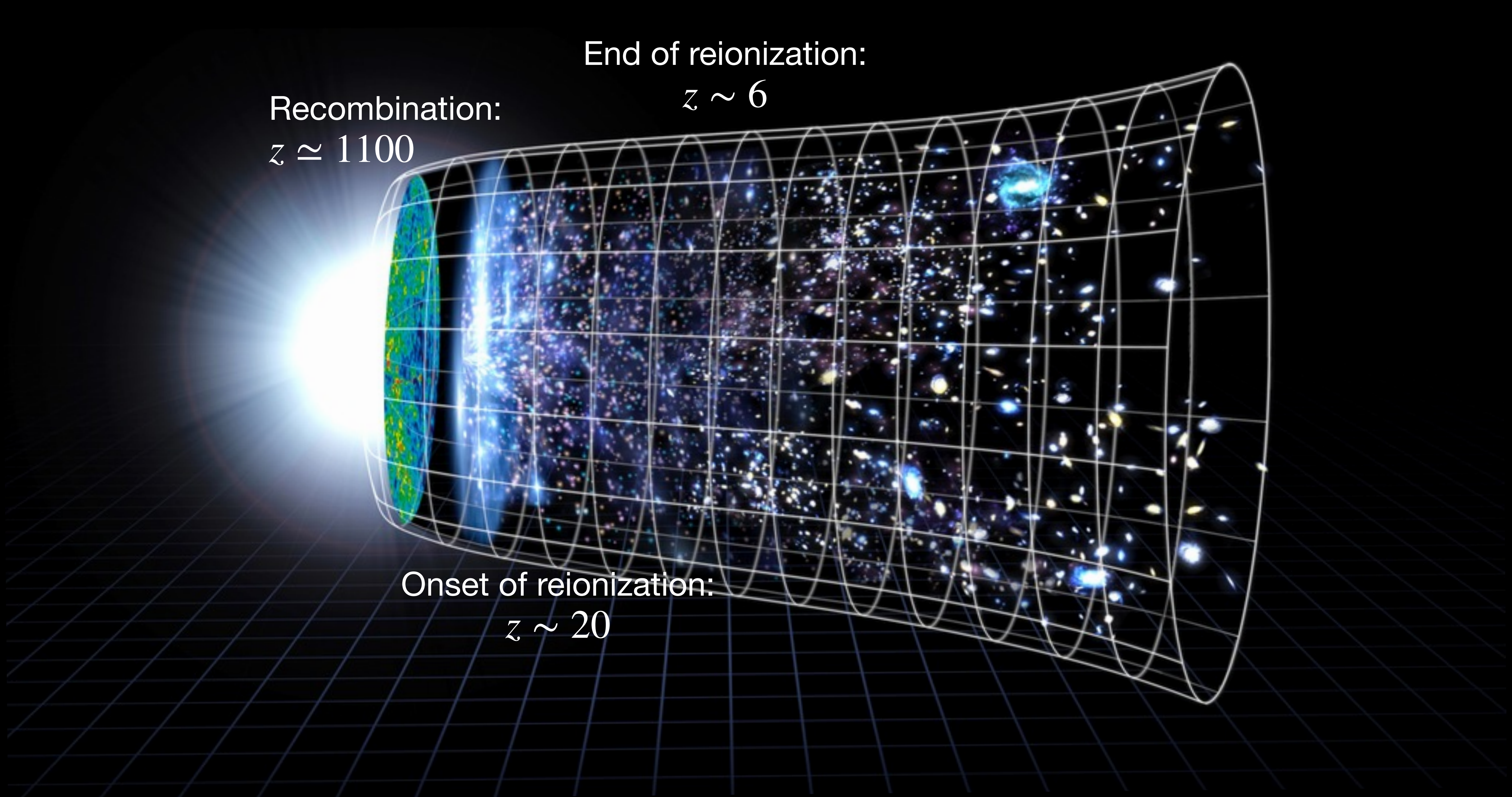
Supervisor: Prof Luca Pagano

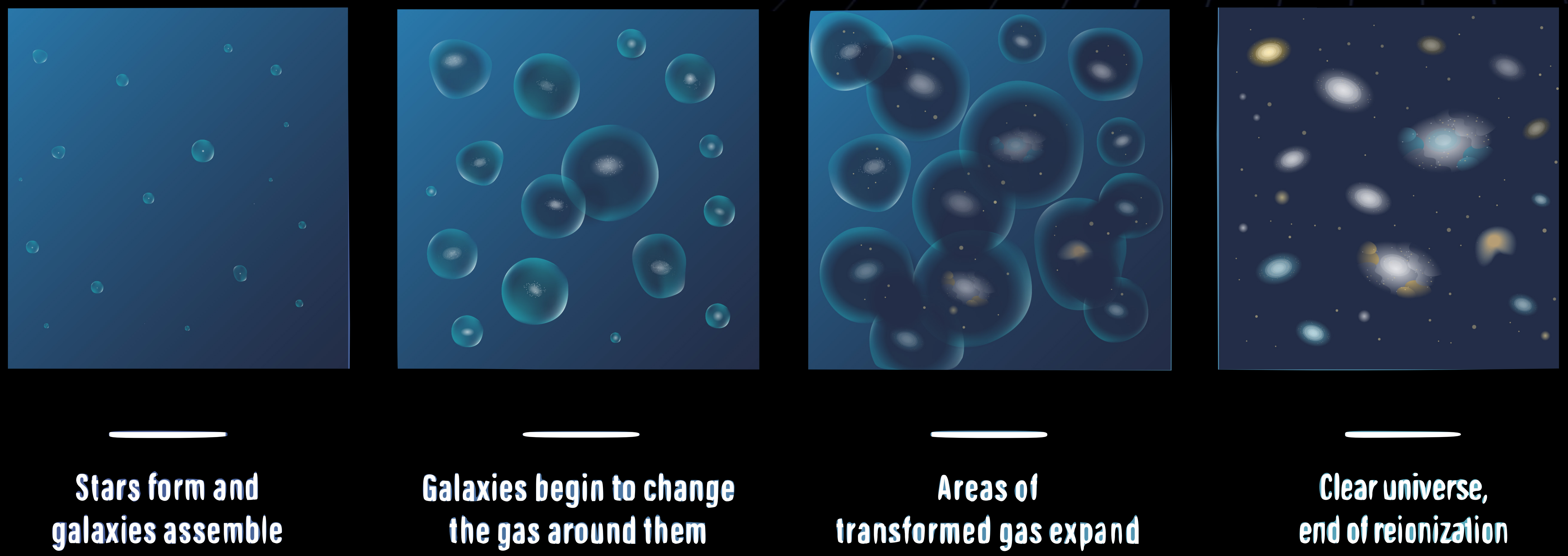
Co-supervisor: Dott. Giacomo Galloni

Outline of the Presentation:

- Thermal history of the Universe and the epoch of reionization
- Polarization of the Cosmic Microwave Background (CMB)
- Statistical analysis of CMB maps
- Methods for CMB detection
- The challenge of astrophysical foregrounds
- Introduction to the ELiCA project

What is cosmic Reionization?





The most common physical quantity used to characterize reionization is the Thomson scattering optical depth

$$\tau = \int_{t(z)}^{t_0} n_e \sigma_T c dt'$$

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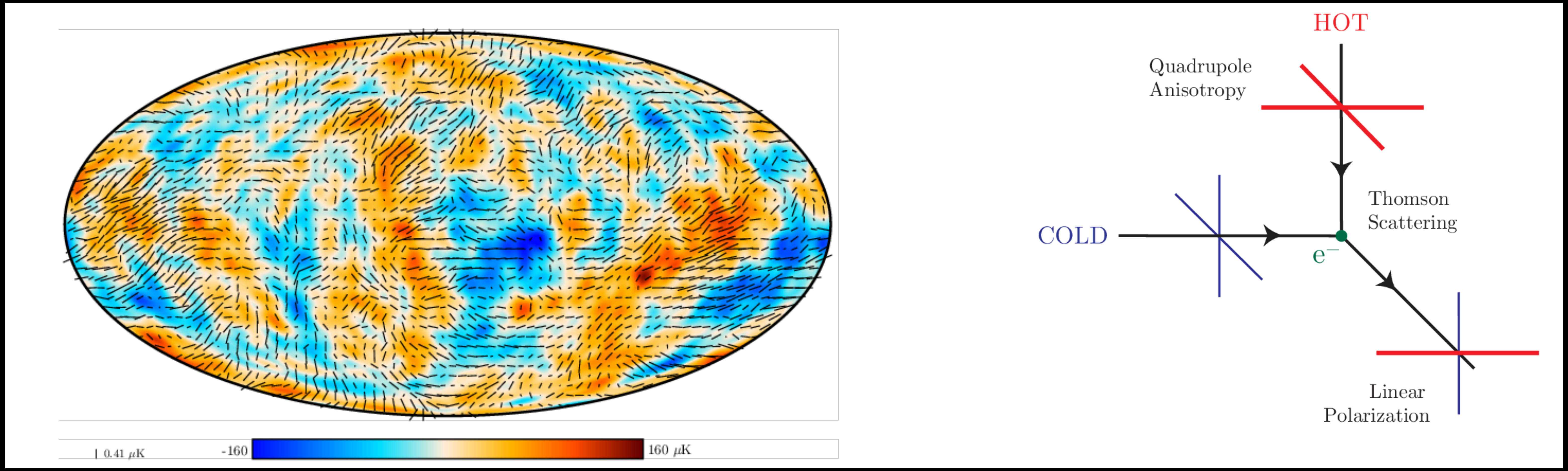
CMB Polarization

CMB is linearly polarized due to Thomson scattering of photons off free electrons in the surface of last scattering.



$$E(\hat{\mathbf{n}}) \equiv -\frac{1}{2}[\bar{\eth}^2(Q+iU) + \eth^2(Q-iU)]$$

$$B(\hat{\mathbf{n}}) \equiv \frac{i}{2}[\bar{\eth}^2(Q+iU) - \eth^2(Q-iU)]$$

While the Stokes parameters **Q** and **U** are useful for describing the linear polarization of the CMB, they depend on the coordinate system chosen and change under rotation. To make the analysis coordinate-independent, we instead use the **E-mode** and **B-mode** formalism.

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For a certain frequency A of observation, the polarization maps $\mathbf{m}^A = [Q^A(\hat{n}), U^A(\hat{n})]$ describe the linear polarization of the CMB.

Q and U can be expanded in spin-2 spherical harmonics:

$$\left(Q^{(A)} \pm i\,U^{(A)}\right)(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{(A),\,\pm 2}\ {}_{\pm 2}Y_{\ell m}(\hat{n})\,.$$

The expansion $a_{\ell m}^{\pm 2}$ coefficients are used to define the E-modes:

$$a_{\ell m}^{(A),E} = -\frac{1}{2}\left(a_{\ell m}^{(A),2} + a_{\ell m}^{(A),-2}\right).$$

The E-mode power spectrum quantifies the variance of the polarization field at different angular scales:

$$C_\ell^{AA,\,EE} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left\langle a_{\ell m}^{(A),E\,*} a_{\ell m}^{(A),E} \right\rangle.$$

Auto-spectrum.

$C_\ell^{AA,EE}$ encodes information about the early Universe.

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In practice, CMB analyses are often done in multi-frequency framework, for example, for two generic channels A and B:

$$\left(Q^{(A)} \pm i\, U^{(A)}\right)(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{(A),\,\pm 2} \;{}_{\pm 2}Y_{\ell m}(\hat{n})\,.$$

$$\left(Q^{(B)} \pm i\, U^{(B)}\right)(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{(B),\,\pm 2} \;{}_{\pm 2}Y_{\ell m}(\hat{n})\,.$$

$$a_{\ell m}^{(A),E} = -\frac{1}{2}\left(a_{\ell m}^{(A),2} + a_{\ell m}^{(A),-2}\right).$$

$$a_{\ell m}^{(B),E} = -\frac{1}{2}\left(a_{\ell m}^{(B),2} + a_{\ell m}^{(B),-2}\right).$$

The expansion $a_{\ell m}^{\pm 2}$coefficients are used to define the cross-spectrum of E-modes:

$$C_{\ell}^{AB,\,EE} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left\langle a_{\ell m}^{(A),E\,*} a_{\ell m}^{(B),E} \right\rangle.$$

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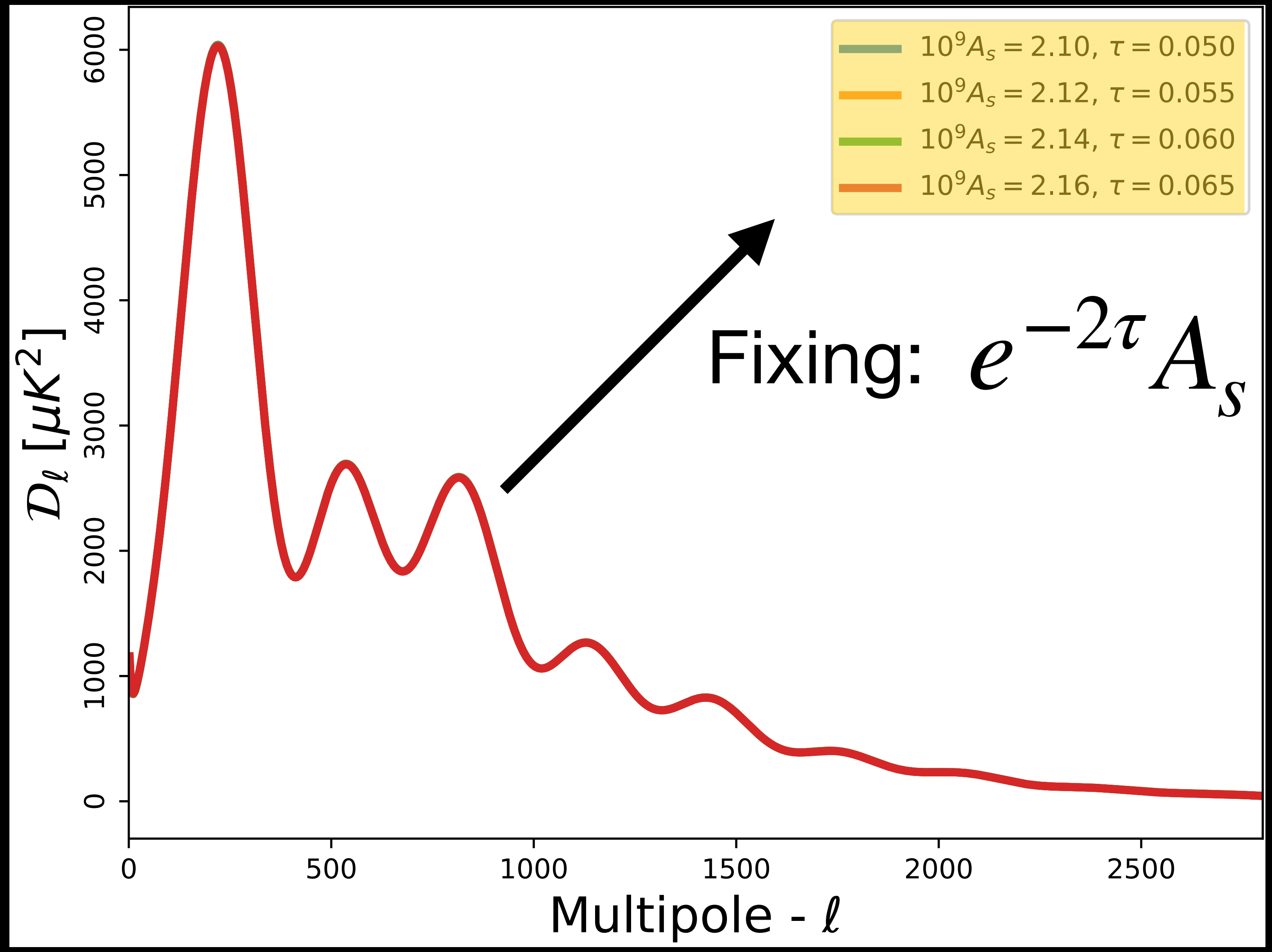
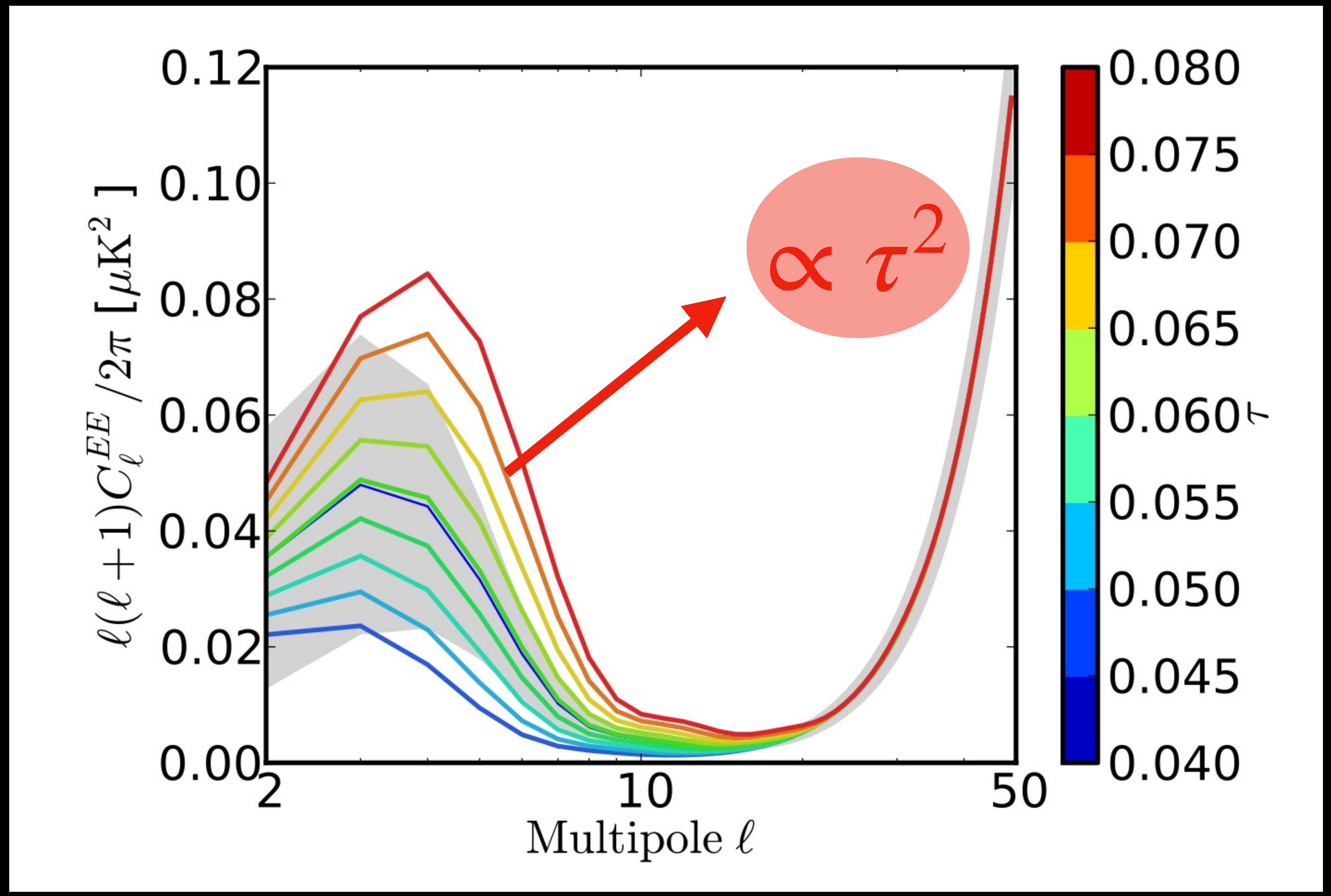
How can we extract information about Reionization from the CMB power spectra?

Free electrons during reionization scatter CMB photons via **Thomson scattering**, much like what happened during decoupling.



The temperature power spectrum remains unchanged if we modify the optical depth and the amplitude of scalar perturbation.

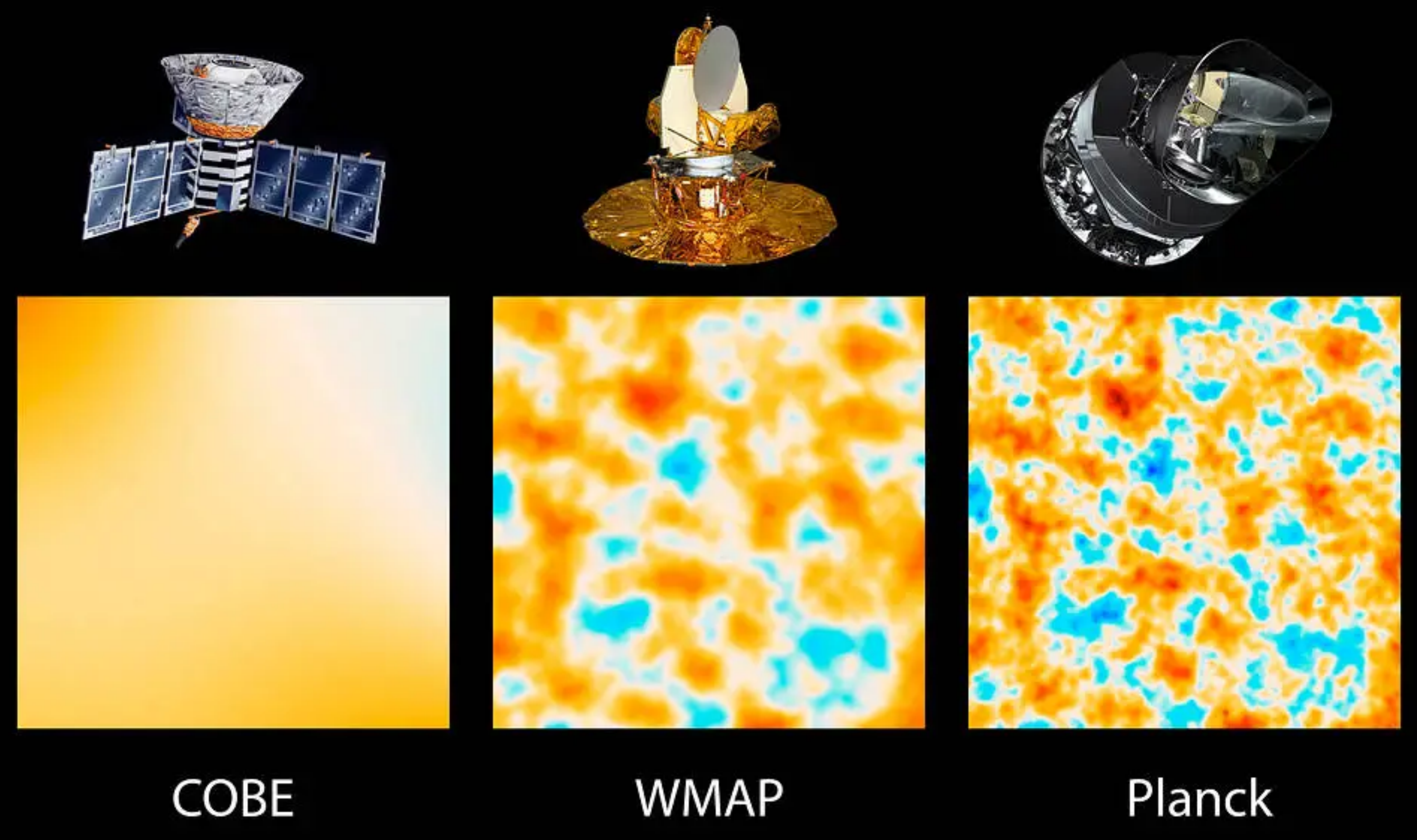
Breaking → **the degeneracy**



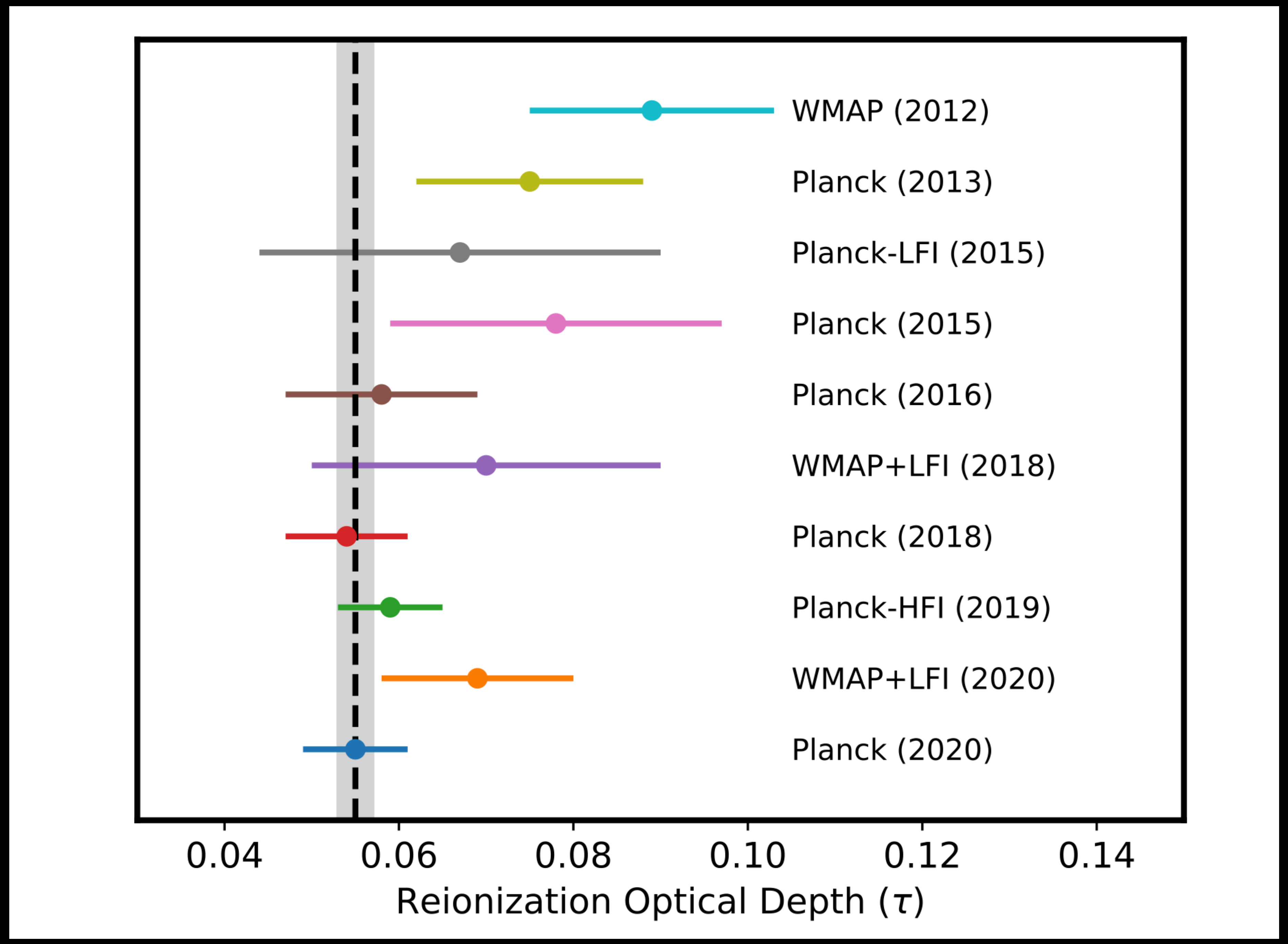
Christian L. Reichardt. *Observing the Epoch of Reionization with the Cosmic Microwave Background* (2016).

Detection of Signals on Large Scales:

Only satellite experiments can access the entirety of the sky and therefore the information stored on the large angular scales, which makes them crucial in shedding light on the Epoch of Reionization.



History of the τ constraint:

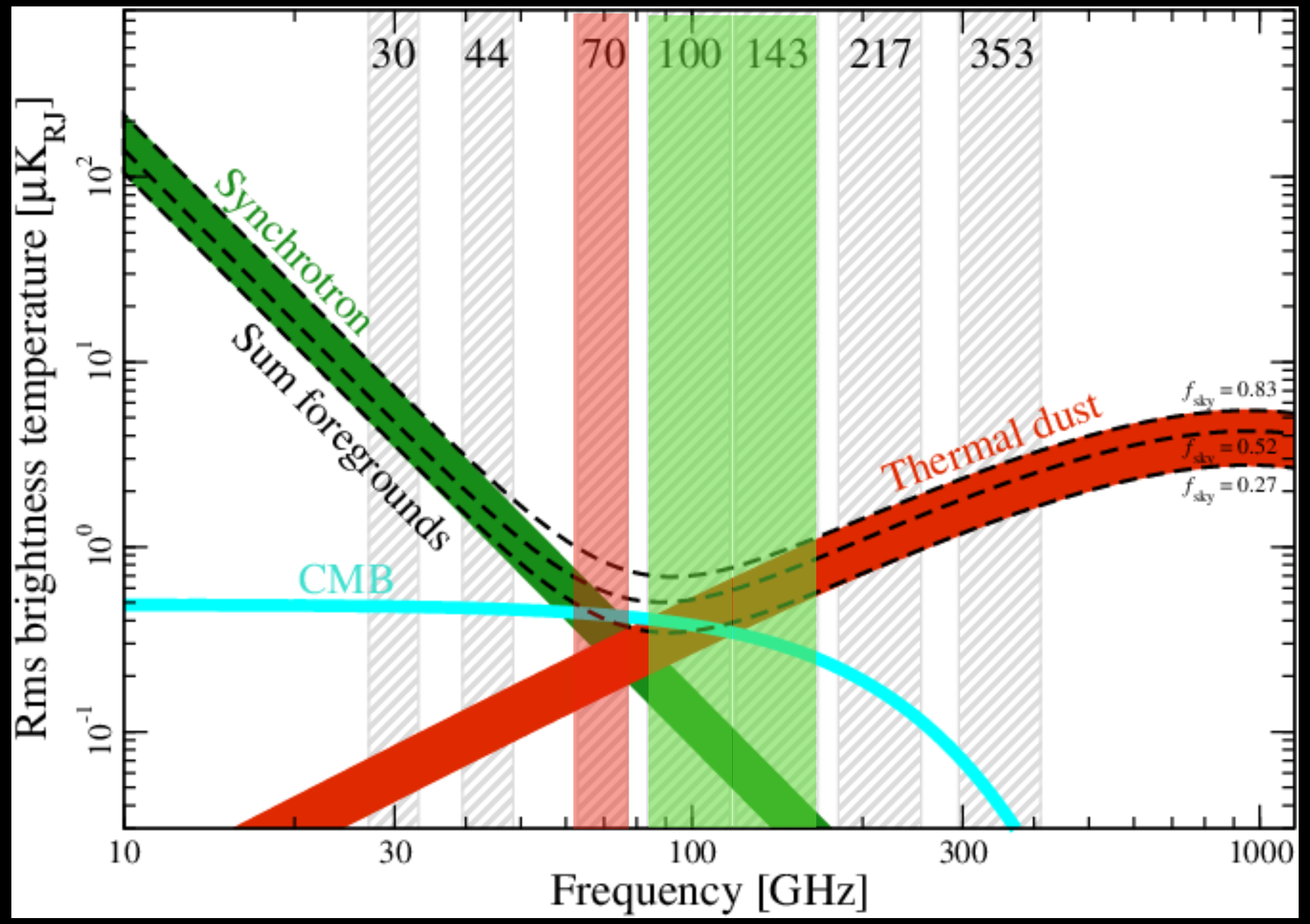


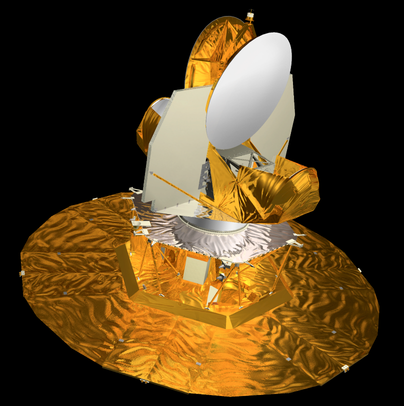
LiteBIRD collaboration. Probing cosmic inflation with the litebirdcosmic microwave background polarization survey (2022).

Detection of Signals on Large Scales:

Planck

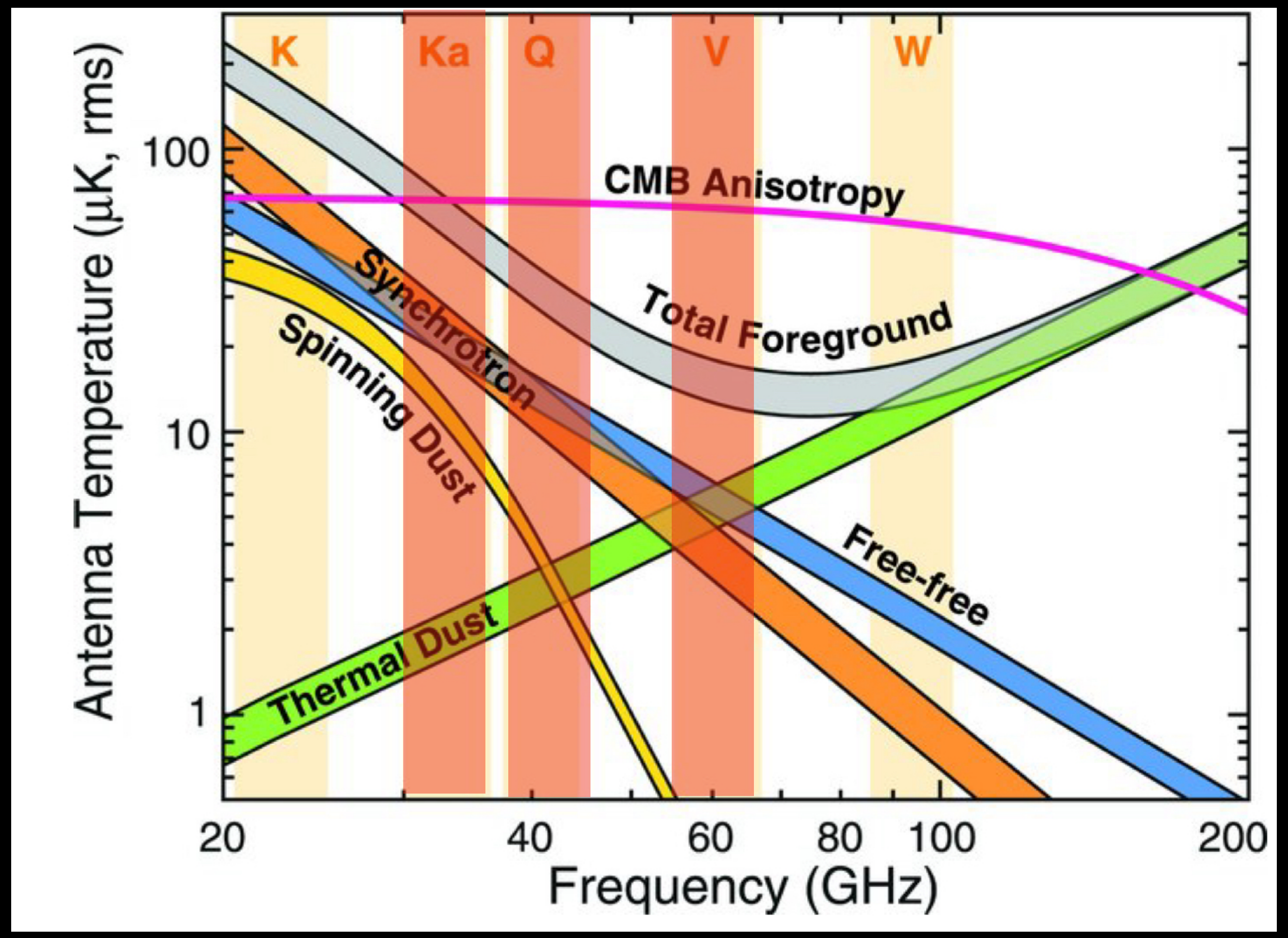
Planck collaboration-Planck 2018 results. I. Overview and the cosmological legacy of Planck (2019)





WMAP

WMAP collaboration- Nine-year wilkinson microwave anisotropy probe (wmap) observations: Final maps and results. *(2013)*



In my work, I improve our knowledge of τ through the first joint analysis of datasets from the two most recent space-based CMB experiments: Planck and WMAP.

WMAP +LFI (2020)

U. Natale-A novel CMB polarization likelihood package for large angular scales built from combined WMAP and Planck LFI legacy maps (2020).

This dataset provides a single clean map, which I will refer to as 'WL'.

Planck HFI (2019)

L. Pagano-Reionization optical depth determination from Planck HFI data with ten percent accuracy (2019)

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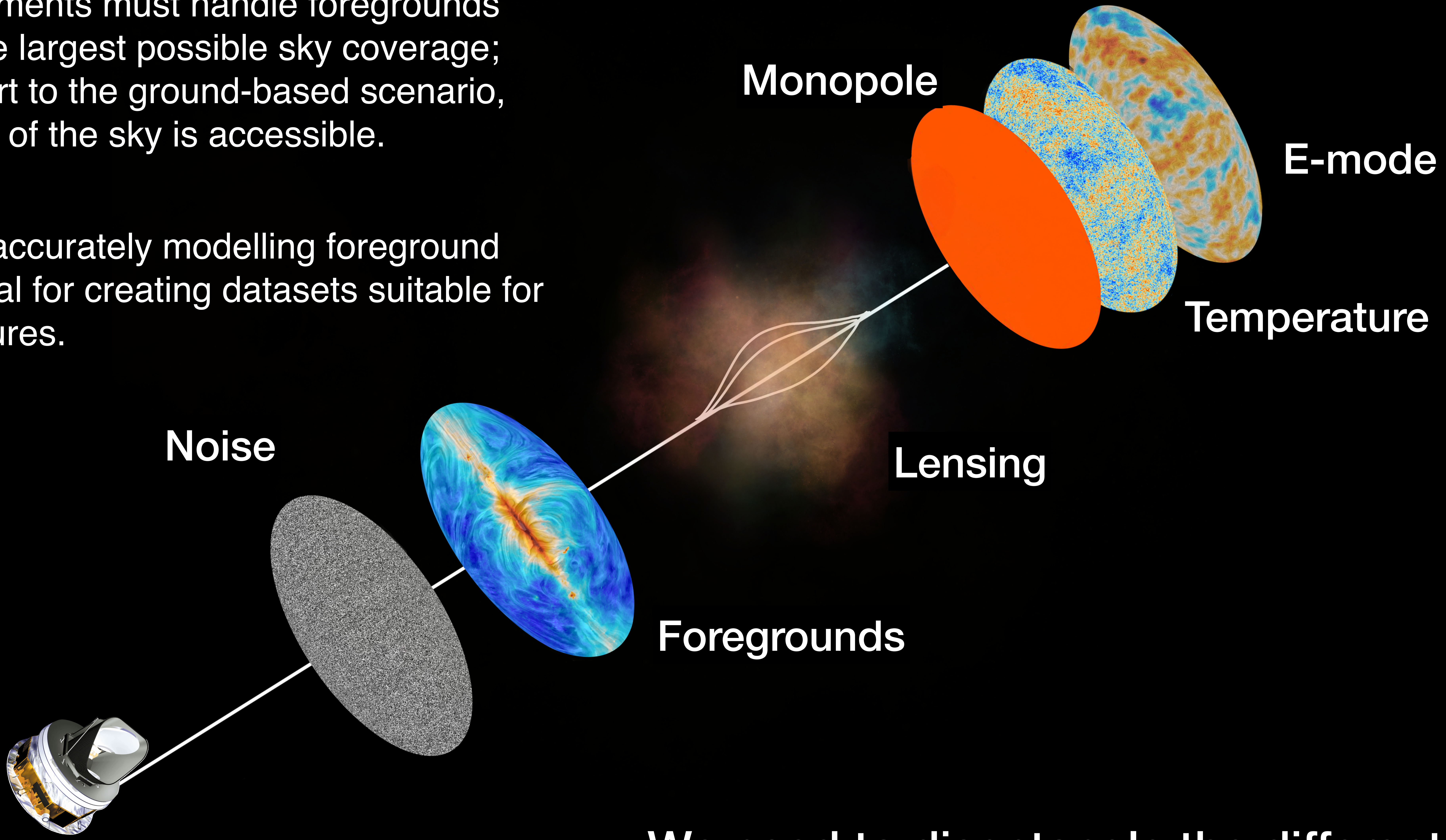
Space-based experiments must handle foregrounds carefully to retain the largest possible sky coverage; otherwise, they revert to the ground-based scenario, where only a portion of the sky is accessible.

Understanding and accurately modelling foreground emissions is essential for creating datasets suitable for analysing CMB features.



We need to disentangle the different signals.

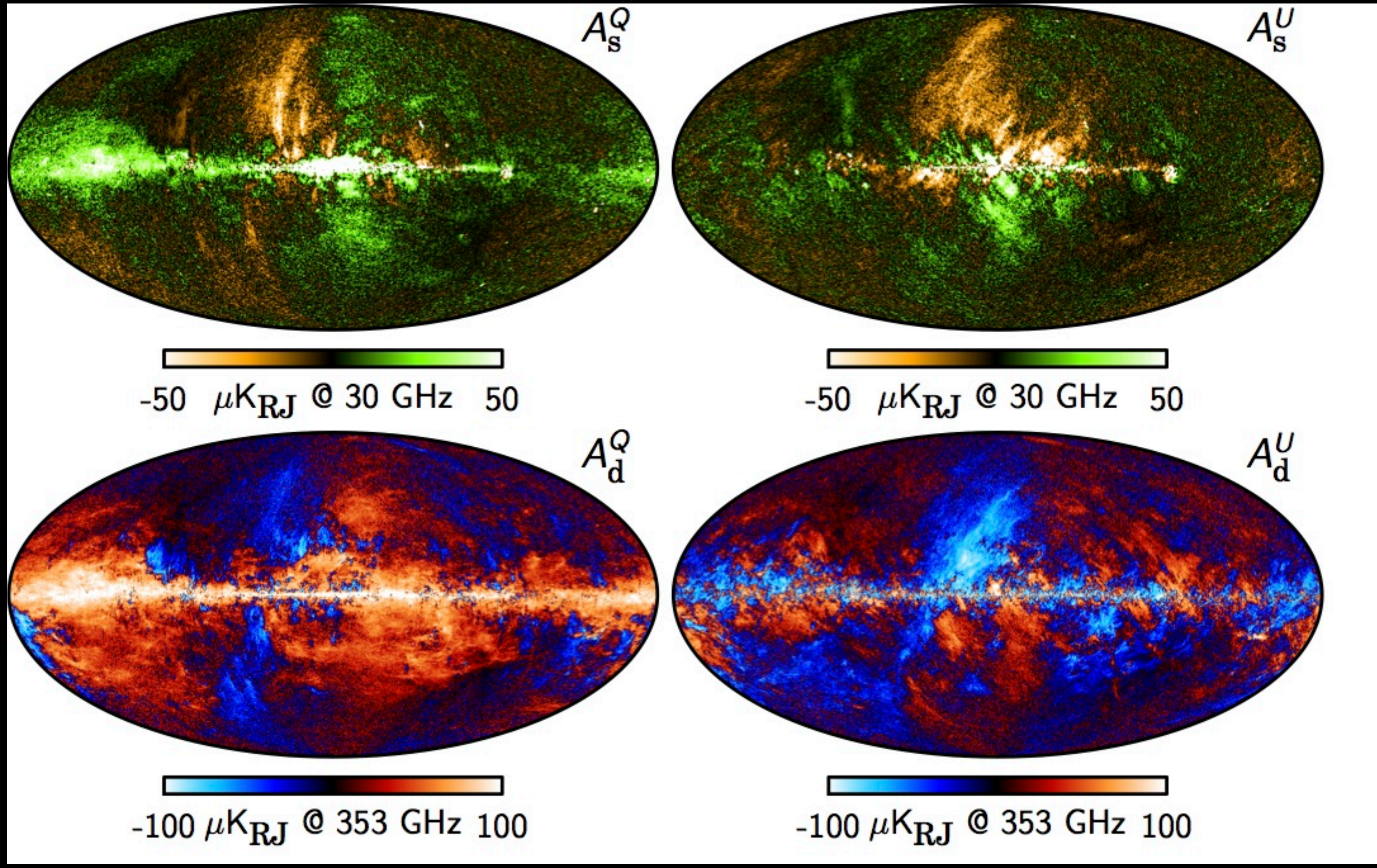
Foregrounds

Synchrotron radiation

$S_\nu \propto \nu^\alpha$

Thermal dust

$S_\nu \propto \nu^\beta B(\nu, T_d)$



Clive Dickinson. Cmb foregrounds - a brief review (2016)

The starting point of this method is the assumption that all the maps employed can be modelled as a linear combination of CMB, Dust and Synchrotron. Thus any set of three maps can be combined for obtaining a CMB map. So, selecting a map as the main channel $\mathbf{m}_\nu^P = [Q_\nu, U_\nu]$ and two maps $\mathbf{t}_d$ and $\mathbf{t}_s$ as tracers of respectively Dust and Synchrotron, the foreground-cleaned map is provided by:

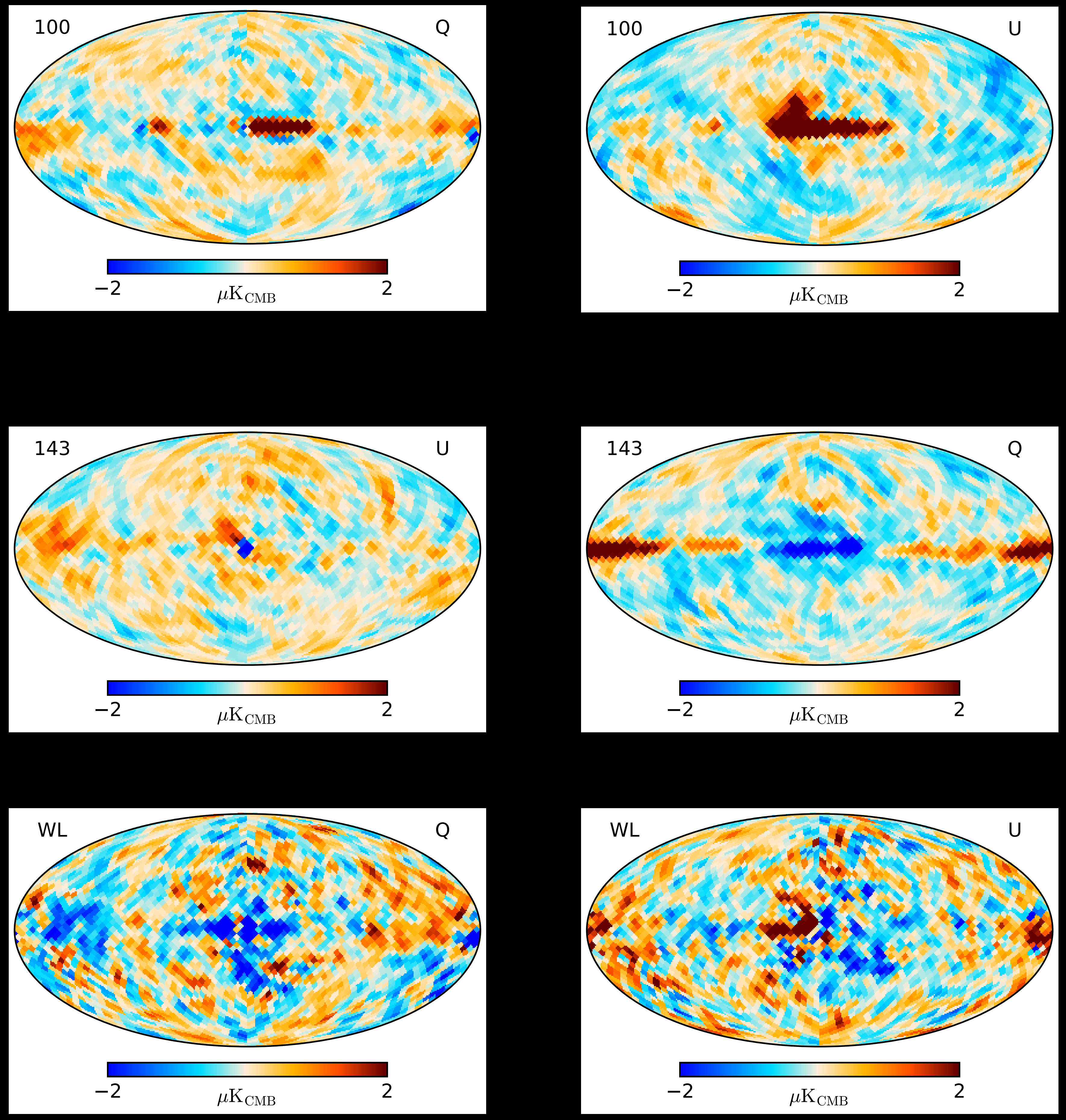
Synchrotron scaling coefficient

Thermal dust scaling coefficient

$$\tilde{\mathbf{m}}_\nu^{P,\mathrm{fc}} = \frac{\mathbf{m}_\nu^P - \alpha_\nu \mathbf{t}_s - \beta_\nu \mathbf{t}_d}{1 - \alpha_\nu - \beta_\nu}$$

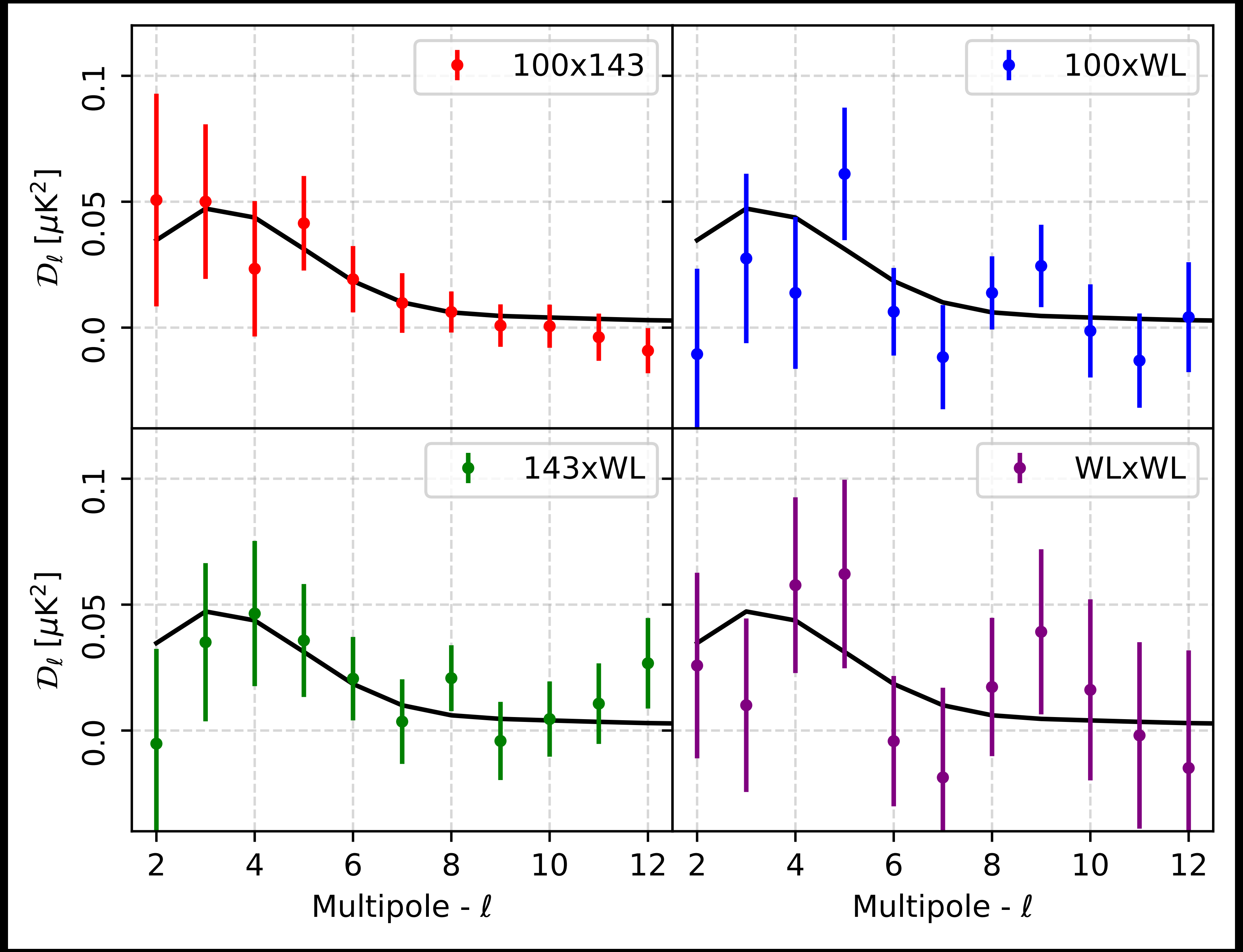
Once this quantity is minimized, then the coefficients α_ν and β_ν are estimated.

One algorithm that allows for moving back to the harmonic space is the Quadratic Maximum Likelihood Estimator (QML), which provides a method for reconstructing the CMB power spectrum with minimal error bars.



$$\hat{y}_\ell^{AB} = \mathbf{d}^{A^T}\mathbf{E}_\ell\mathbf{d}^B - b_\ell^{AB}$$

$$\hat{C}_\ell \equiv \sum_{\ell'} [W^{-1}]_{\ell\ell'}\hat{y}_{\ell'}^{AB}$$



The four power spectra are compared with the fiducial power spectrum at large scales.

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Likelihood algorithm:

We need to compare theory and data.

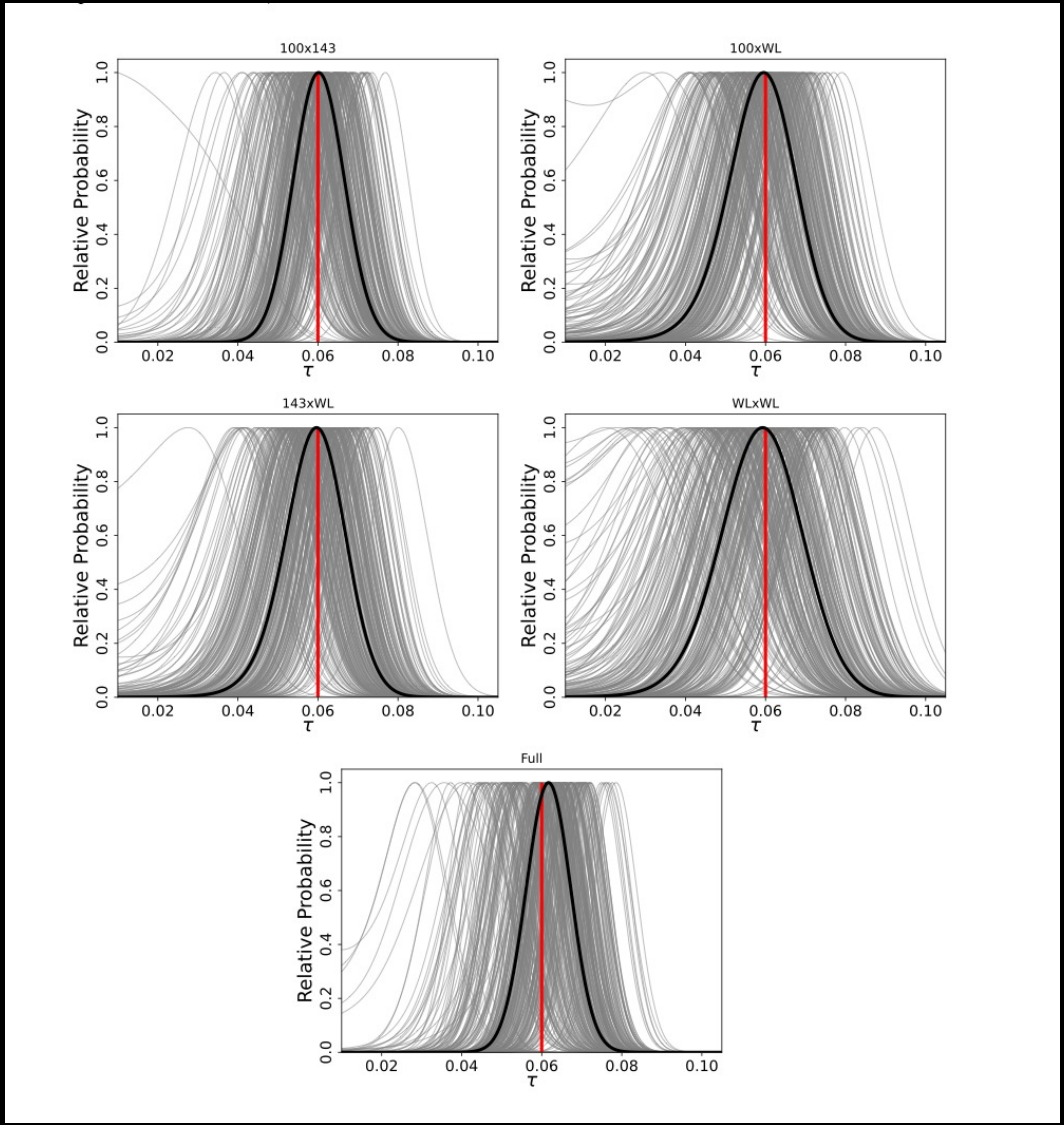
The distribution at large scales is non-Gaussian, but we can gaussianize it .

In this likelihood, the main ingredients are:

$$\mathscr{L}(C_\ell^{data} \mid C_\ell^{theory}) = -\frac{N}{2}\left[\ln\left(1+\frac{\mathbf{X}_\ell \cdot \boldsymbol{\Sigma}^{-1}_{\ell\ell'} \cdot \mathbf{X}_\ell}{N-1}\right)\right].$$

This equation marginalize over the extra variance induced by the limited number of simulations.

Empirical covariance matrix built on simulations, which are also used to validate the pipeline.



Validation

Aim: To get as output the value of τ given as input.

The grey lines represent the single posteriors of the simulations, the mean distribution is reported in black, while the red vertical line corresponds on the input optical depth.

The full datasets is defined as:

$$100 \times 143 + 100 \times WL + 143 \times WL + WL \times WL$$

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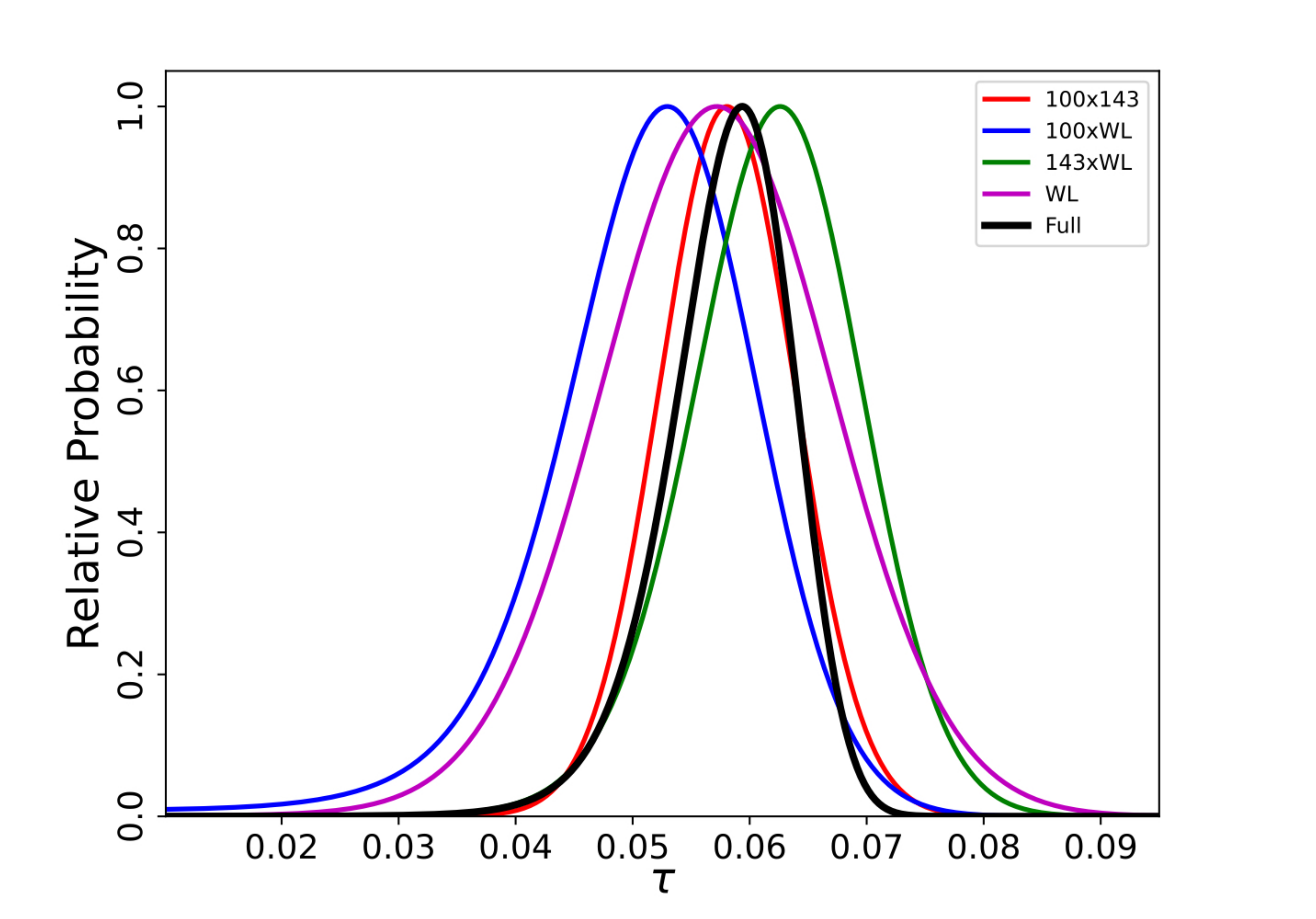
Constraint on the optical depth



Here I present the posterior for the datasets chosen.

Case	τ Constraint
100×143	0.0581 ± 0.0060
$100 \times WL$	0.0530 ± 0.0083
$143 \times WL$	0.0626 ± 0.0074
$WL \times WL$	0.057 ± 0.011
Full	0.0609 ± 0.0045

ELiCA algorithm: E-mode Likelihood for Cross-correlation Analysis

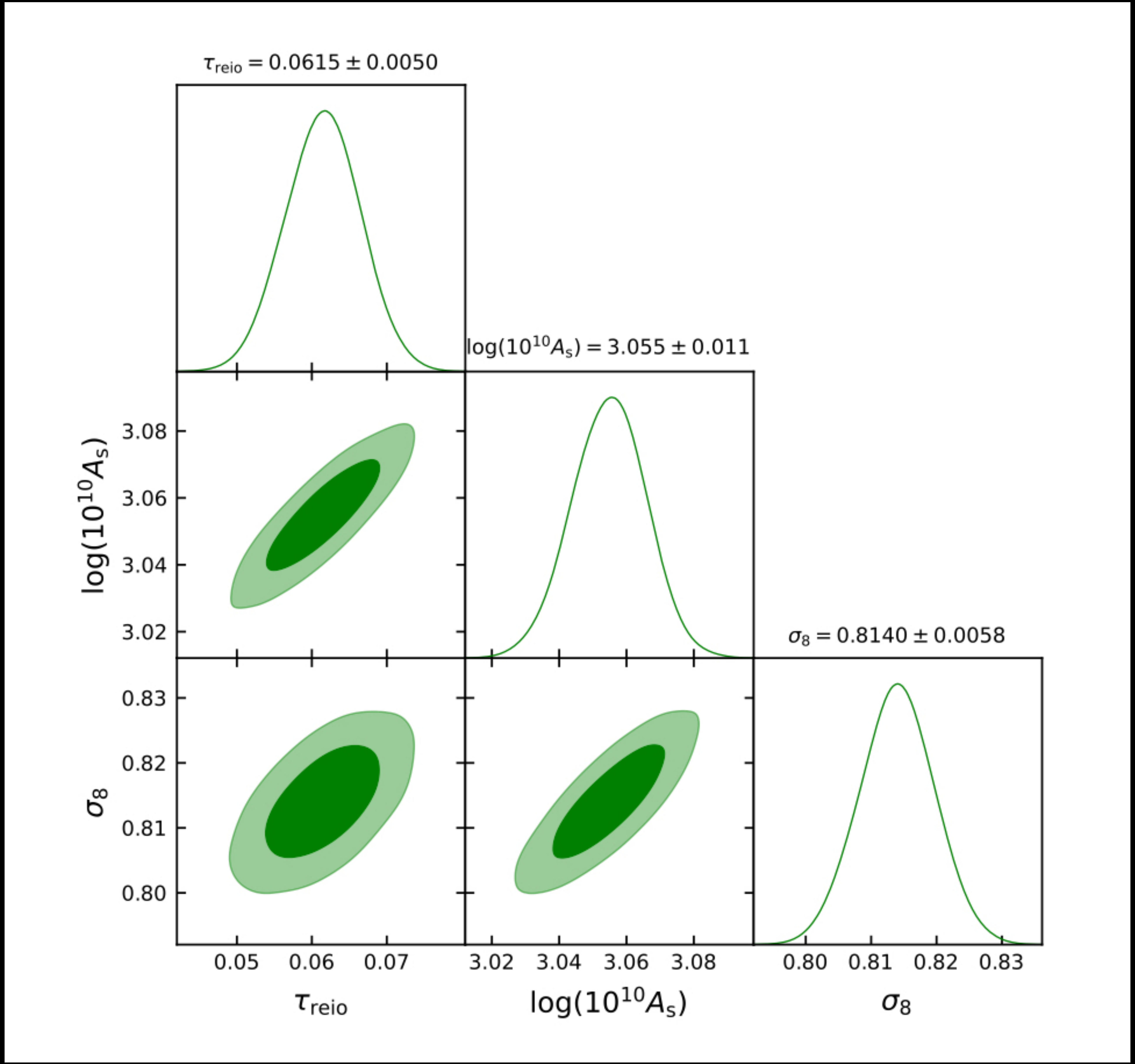
Amplitude of scalar perturbations	A_s
Power law index of scalar perturbations	n_s
Baryon energy density	$\Omega_b h^2$
Dark matter energy density	$\Omega_c h^2$
Hubble parameters	H_0
Optical depth	τ

After passing validation, we explore the impact of Elica on the parameters of the Λ Cold Dark Matter model by combining the likelihood with other CMB data.

In order to explore this larger sample space, I employ Markov Chain Monte Carlo techniques.

Matter spectrum normalization	σ_8

To handle a larger sample space, this requires Markov Chain Monte Carlo simulations.



In this picture, you see the 1-d and 2-d marginalized posterior.

Constraint on the amplitude of scalar perturbations A_S from the planck NPIPE high-ℓ TT, TE and EE spectra, combined with ELiCA low-ℓ EE spectrum, and low TT Planck (2018). Contours contain 68 % and 95 % of the probability.

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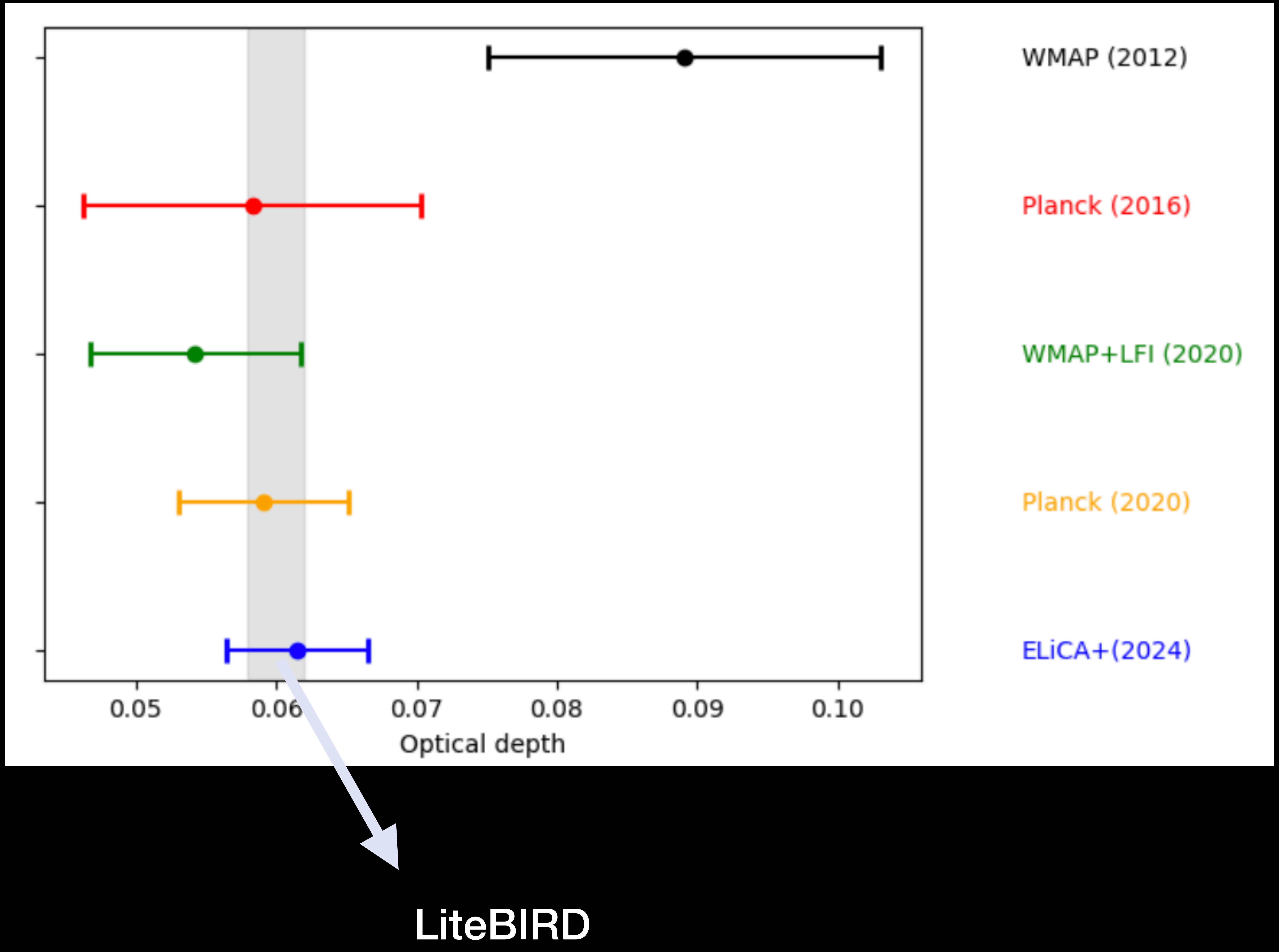
Why is this achievement significant?



With my analysis I get this constraint

$$\tau = 0.0615 \pm 0.0050$$

This is the tighest constraint on the optical depth and this will have an impact in the **neutrino sector** and will stay the state of the art up until **LiteBIRD (early 2030s).**

Impact in the neutrino sector:

Amplitude of scalar perturbations	A_s
Optical depth	τ
Sum of neutrino masses	$\sum m_\nu$
Relativistic degrees of freedom	N_{eff}

Given the new bound on As, I will now explore the impact of this constraint on extensions to the Λ-CDM model, particularly in the neutrino sector.

The neutrino sector is affected because there is a degeneracy between the amplitude of scalar perturbations As, over which we have a new constraint, and the parameter $\sum m_\nu$, as they are both related to structure formation.

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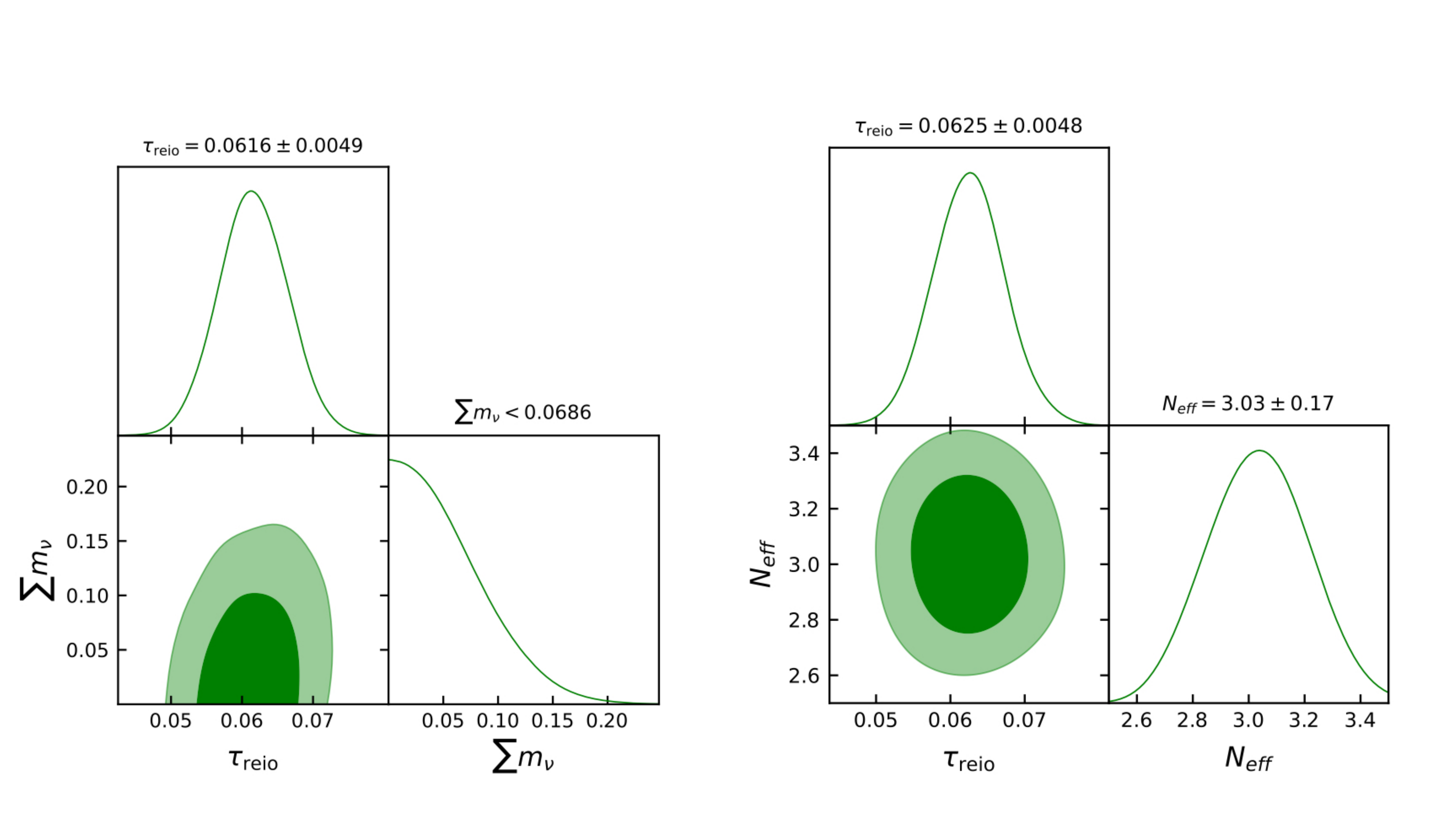
In order to do so we have added information carried by BAO and CMB lensing.

In this picture, you see the 1-d and 2-d marginalized posterior.



Contours indicate the regions containing 68% and 95% of the probability. The left panel shows constraints for the neutrino mass scenario, where this parameter is free to vary in the ΛCDM + $\sum m_\nu$ scenario.

The right panel illustrates the effective number of relativistic degrees of freedom, where this parameter is free to vary in the ΛCDM + N_{eff} scenario.

In conclusion:

- This work exploited all available data on large scales for E-mode polarization, thus this will be the state of the art up until LiteBIRD.

- A new likelihood algorithm, based on Quadratic Maximum Likelihood (QML) estimation, was developed and is now publicly accessible.

- This research has achieved the tightest bound on τ to date, and explored its implications for the neutrino sector.

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Updates:

ELiCA algorithm: E-mode Likelihood for Cross-correlation Analysis

G. Galloni-Accurate and efficient likelihood modeling for large-scale CMB data (2025)

$$-2\ln \mathscr{L}(\hat{C}_\ell \,|\, C_\ell) = \sum_{\ell\ell'} X_\ell^{\mathrm{T}} \left(M_{\ell\ell'}^{-1}\right)_{\mathrm{fid}} X_{\ell'} .$$

- C_ℓ : True power spectra
- $\hat{C}_\ell$: estimator
- $M_{\ell\ell'}$: Simulation-based covariance matrix

We need to estimate these elements:

$$C_\ell = \begin{bmatrix} C_\ell^{100\times100} + N_\ell^{100} & C_\ell^{100\times143} & C_\ell^{100\times WL} \\ C_\ell^{100\times143} & C_\ell^{143\times143} + N_\ell^{143} & C_\ell^{143\times WL} \\ C_\ell^{100\times WL} & C_\ell^{143\times WL} & C_\ell^{WL\times WL} + N_\ell^{WL} \end{bmatrix}$$

In the case of auto-spectra, we also account for the noise bias, whereas for cross-spectra the noise is assumed to be uncorrelated.

***Paper in preparation**

ELiCA algorithm: E-mode Likelihood for Cross-correlation Analysis

G. Galloni-Accurate and efficient likelihood modeling for large-scale CMB data (2025)

$$-2\ln \mathscr{L}(\hat{C}_\ell \,|\, C_\ell) = \sum_{\ell\ell'} X_\ell^{\mathrm{T}} \left(M_{\ell\ell'}^{-1}\right)_{\text{fid}} X_{\ell'}$$

We start by building this object: $C_\ell^{-\frac{1}{2}} \hat{C}_\ell C_\ell^{-\frac{1}{2}}$.

Then you need to evaluate its eigenvalues and eigenvectors

D_ℓ :diagonal matrix of eigenvalues

U_ℓ :diagonalization matrix

$$C_{g\ell} = C_{f\ell}^{1/2} U_\ell g(D_\ell) U_\ell^{\top} C_{f\ell}^{1/2}.$$

$$\mathbf{g}(\mathbf{D}_\ell) = [\mathbf{g}(\mathbf{D}_\ell)]_{ij} = \mathbf{g}(\mathbf{D}_{\ell,ii})\delta_{ij} = \mathrm{sign}(\mathbf{D}_{\ell,ii} - 1)\sqrt{2(\mathbf{D}_{\ell,ii} - \ln \mathbf{D}_{\ell,ii} - 1)}\,\delta_{ij}.$$

The independent elements of this matrix are indeed our X. However, since we do not have a reliable noise estimate for the 100 GHz and 143 GHz channels, we instead marginalize over them, obtaining:

$$X_\ell = \left[X_\ell^{100\times143}, X_\ell^{100\times WL}, X_\ell^{143\times WL}, X_\ell^{WL\times WL}\right].$$

***Paper in preparation**

One last step:

E. Sellentin, A. Heavens-Parameter inference with estimated covariance matrices (2015)

$$-2\ln\mathscr{L}(\hat{C}_\ell\,|\,C_\ell) = \sum_{\ell\ell'} X_\ell^{\mathrm{T}}\left(M_{\ell\ell'}^{-1}\right)_{\text{fid}} X_{\ell'}\,.$$

We must take into account that the covariance matrix is simulation-based.
Since we are limited by the number of simulations, we marginalize over the true covariance matrix, obtaining:

$$-2\ln\mathscr{L}(\hat{C}_\ell\,|\,C_\ell) = N\ln\left[1+\frac{\sum_{\ell\ell'} X_\ell^{\mathrm{T}}\left(M_{\ell\ell'}^{-1}\right)_{\text{fid}} X_{\ell'}}{N-1}\right]\,.$$

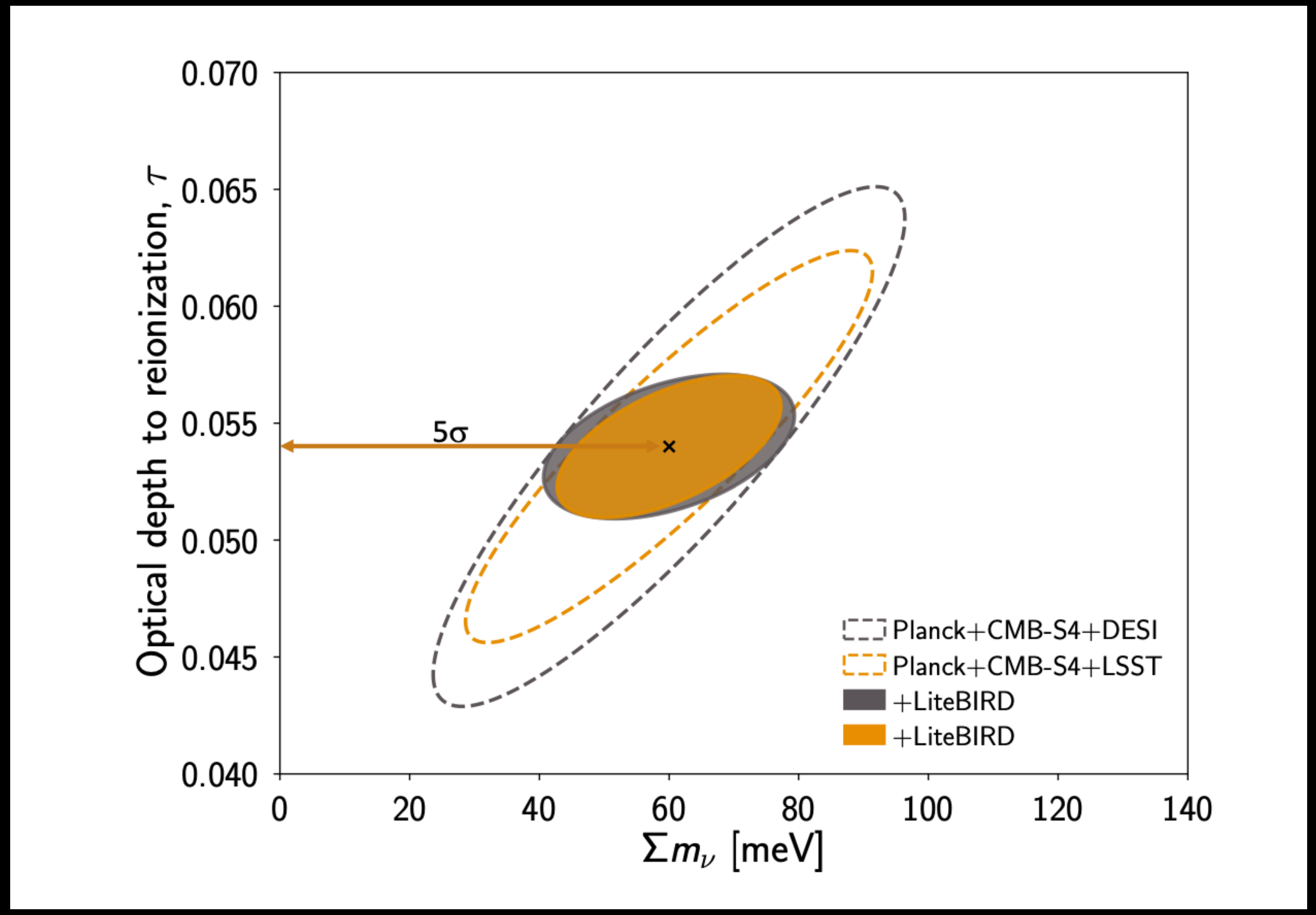
The number of simulations in my work is N=500

***Paper in preparation**

What is the impact of the new τ constraint on extensions of the ΛCDM model?

Improving the constraint on the optical depth impacts the neutrino sector.

Example:



LiteBIRD collaboration. Probing cosmic inflation with the LiteBird cosmic microwave background polarization survey. (2022)

Next steps:

- Updating τ parameter
- Investigation within the ΛCDM Cosmological Model
- Implications for Neutrino Physics

***Paper in preparation**

Thanks for the attention