THE HYPERON-NUCLEON INTERACTION IN LOW-ENERGY EFFECTIVE FIELD THEORY

Margherita Sagina (University of Pisa, INFN - Pisa)

margherita.sagina@phd.unipi.it

In collaboration with

Laura Elisa Marcucci (University of Pisa, INFN - Pisa)

Domenico Logoteta (University of Pisa, INFN - Pisa)

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Aim of the project

Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

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Hyperons

- Baryons containing at least one strange quark
- Fermions
- Lifetimes of the order of 10^{-10} s

INTRODUCTION AND MOTIVATIONS

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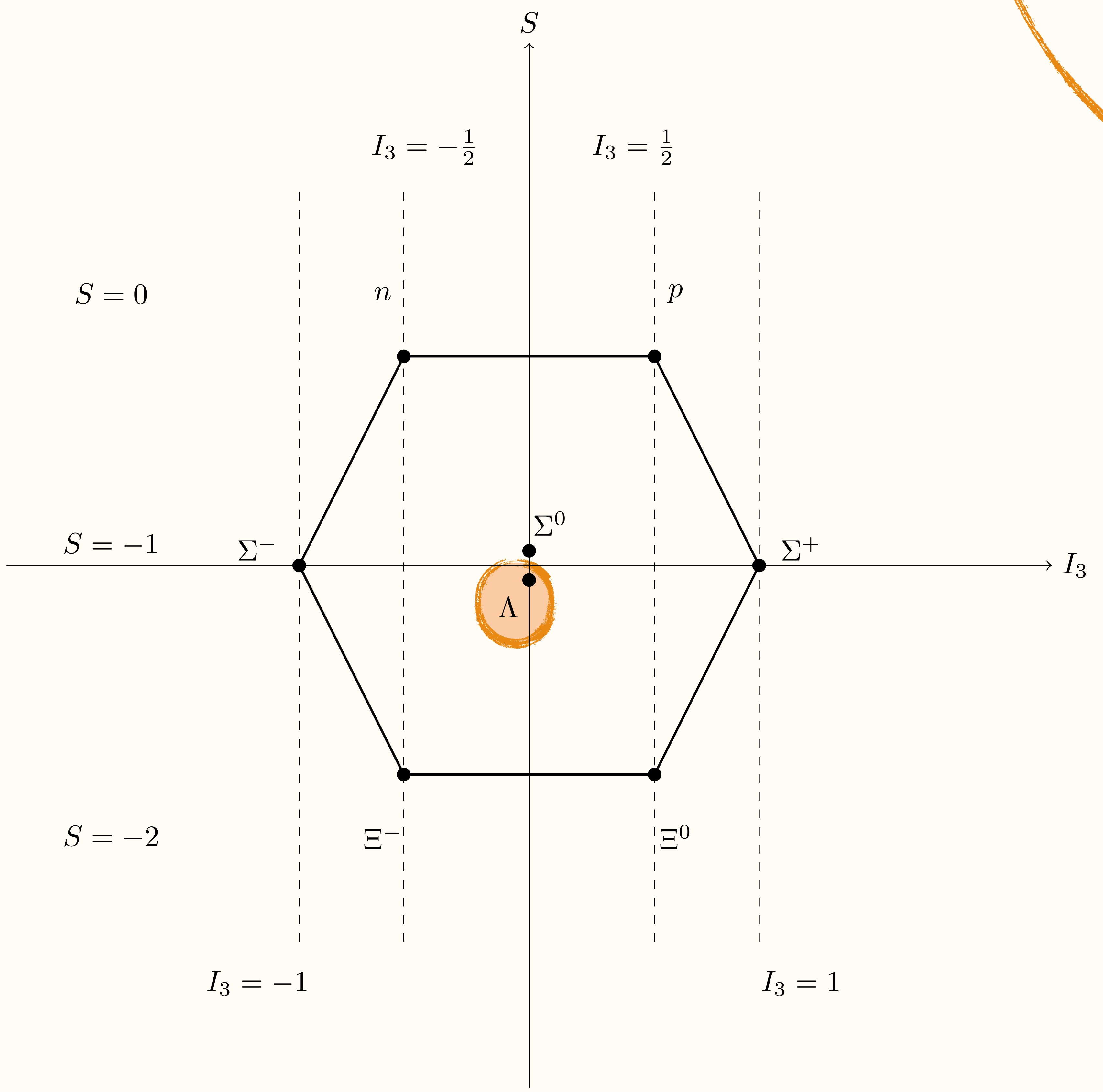

Hyperons

- Baryons containing at least one strange quark
- Fermions
- Lifetimes of the order of 10^{-10} s

Λ hyperon

- Mass 1115.683 ± 0.006 MeV
- Lifetime $(2.617 \pm 0.010) \times 10^{-10}$ s
- Isospin, charge $= 0$

INTRODUCTION AND MOTIVATIONS

Aim of the project

Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

depending only on relative distance between the two particles

terms depending on constants that need to be fixed through a fitting procedure

- Consistent with a well tested theory (QCD)
- EFT $\longrightarrow$ perturbative expansion = organization in leading order (LO), subleading order (NLO), ...

$\downarrow$

allows theoretical error estimation

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Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

Simple model

↓

Easy to handle numerically

(Also suitable for computationally expensive tasks)

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allows theoretical error estimation

Many applications to different research fields

- Hypernuclei studies
- Hyperon puzzle in Neutron Stars

POTENTIAL MODEL

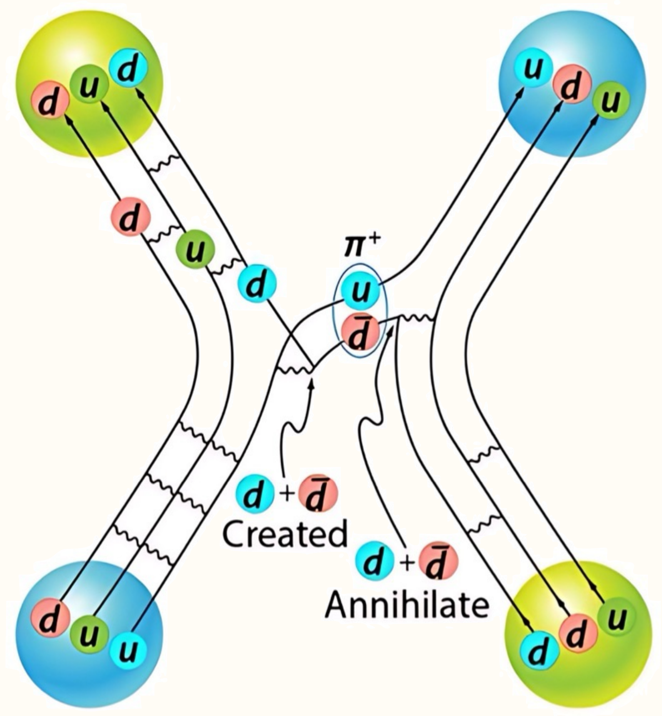
Contact Effective Field Theory

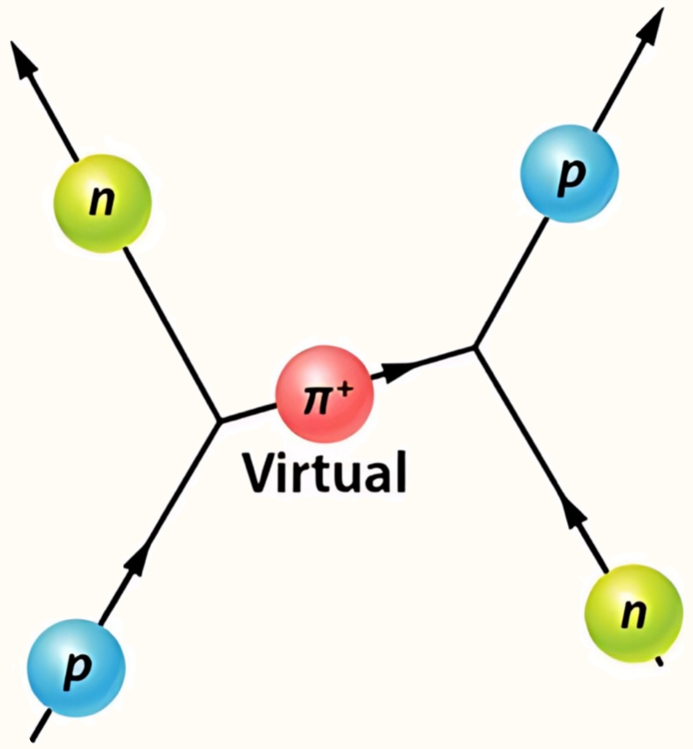
- Effective Field Theory = low energy realization of Quantum Chromodynamics

QCD
quarks, gluons
$\gtrsim$ 1 GeV

⟹

χEFT
nucleons, pions
$\sim$ 100 MeV





Adapted from APS/Alan Stonebraker

POTENTIAL MODEL

Contact Effective Field Theory

- In $\pi\!\!\!/$**EFT** even **pions** can be considered **high-energy dof** $\Rightarrow$ interaction described only by **contact terms**

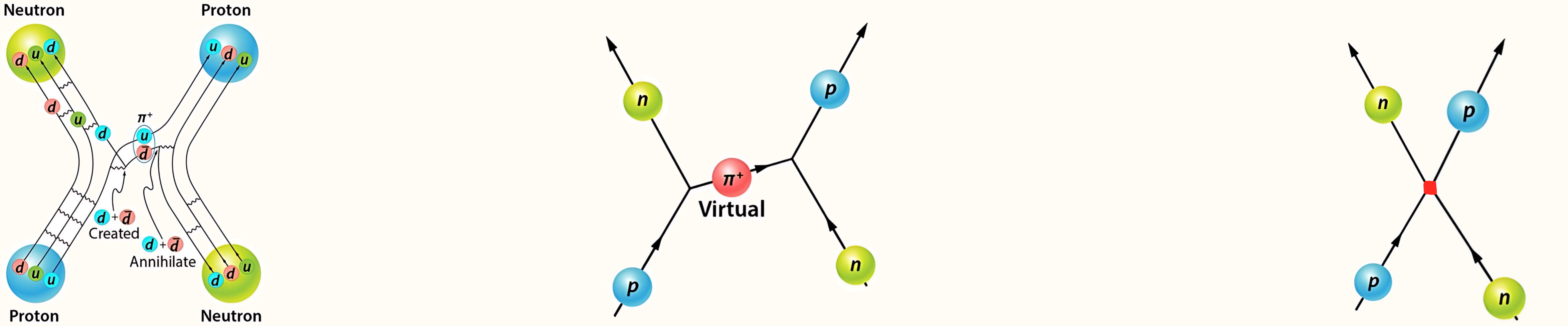
QCD
quarks, gluons
$\gtrsim$ 1 GeV

$\Rightarrow$

χEFT
nucleons, pions
$\sim$ 100 MeV

$\Rightarrow$

$\pi\!\!\!/$EFT
nucleons
$\sim$ 10 MeV



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POTENTIAL MODEL

Expression in coordinate space

- Potential model depends on **radial functions** containing combinations of $F(r), F^{(1)}(r), F^{(2)}(r)$ and **Low Energy Constants (LECs)**

$$V_{\Lambda N}^{LO} = [C_S + C_T(\sigma_\Lambda \cdot \sigma_N)]F(r),$$

$$V_{\Lambda N}^{NLO} = [C_1 + C_2(\sigma_\Lambda \cdot \sigma_N)]\left(-F^{(2)}(r) - \frac{2}{r}F^{(1)}(r)\right)$$

$$-C_3\frac{F^{(1)}(r)}{r}\mathbf{L}\cdot\mathbf{S} - C_4\left(F^{(2)}(r) - \frac{1}{r}F^{(1)}(r)\right)S_{\Lambda N}(\hat{\mathbf{r}}) - C_5\frac{F^{(1)}(r)}{r}\mathbf{L}\cdot\mathbf{D}.$$

$$F(r) = \frac{1}{\pi^{3/2}R_0^3}\exp\left(-\frac{r^2}{R_0^2}\right)$$

Model depends on **cutoff parameter** $R_0 \in [1.5, 2.5]$ **fm**

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$$V_{\Lambda N}^{LO} = [C_S + C_T(\sigma_\Lambda \cdot \sigma_N)]F(r), \longrightarrow F(r) = \frac{1}{\pi^{3/2}R_0^3}\exp\left(-\frac{r^2}{R_0^2}\right)$$

$$V_{\Lambda N}^{NLO} = [C_1 + C_2(\sigma_\Lambda \cdot \sigma_N)]\left(-F^{(2)}(r) - \frac{2}{r}F^{(1)}(r)\right)$$
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Model depends on **cutoff parameter $R_0 \in [1.5, 2.5]$ fm**

$$\mathbf{S} = \frac{(\sigma_\Lambda + \sigma_N)}{2} \qquad S_{\Lambda N}(\hat{\mathbf{r}}) = 3\,\sigma_\Lambda\cdot\hat{\mathbf{r}}\,\sigma_N\cdot\hat{\mathbf{r}} - \sigma_\Lambda\cdot\sigma_N \qquad \mathbf{D} = \frac{(\sigma_\Lambda - \sigma_N)}{2}$$

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$$F(r) = \frac{1}{\pi^{3/2}R_0^3}\exp\left(-\frac{r^2}{R_0^2}\right)$$

Model depends on **cutoff parameter** $R_0 \in [1.5, 2.5]$ **fm**

LECs fixed through fitting procedure to experimental data (Λp **elastic scattering** cross section)

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- χ^2 function using a constraint on **scattering length** → $a_j = \pm \lim_{q\to 0} \frac{\sin \delta_j}{q}$

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th}(C_S, C_T, C_1, \ldots, C_5) - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

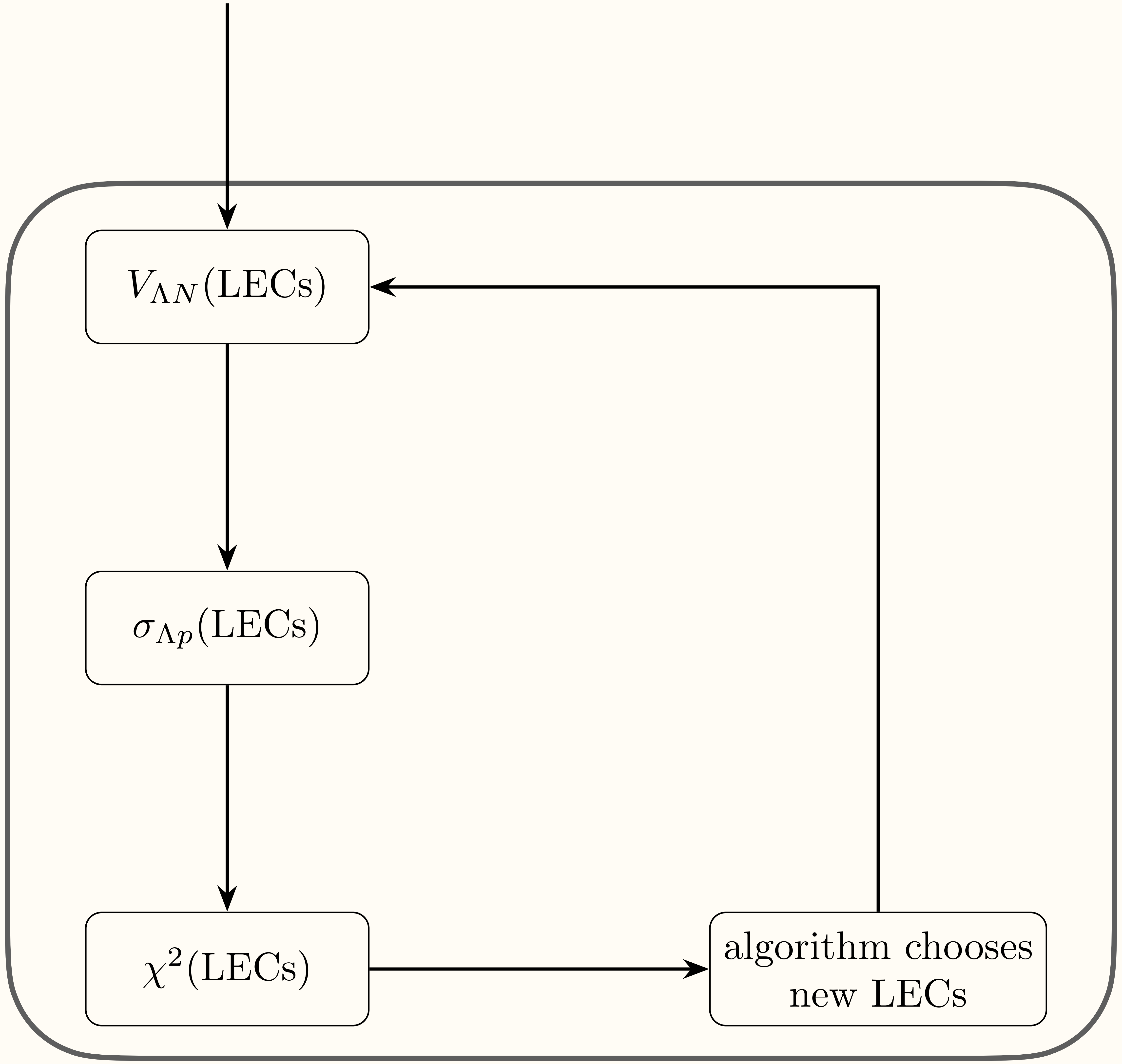
Cross section experimental data from *Radboud University, NN online archive*

“Experimental” data for Λp scattering length from *Mihaylov et al. (2024)*

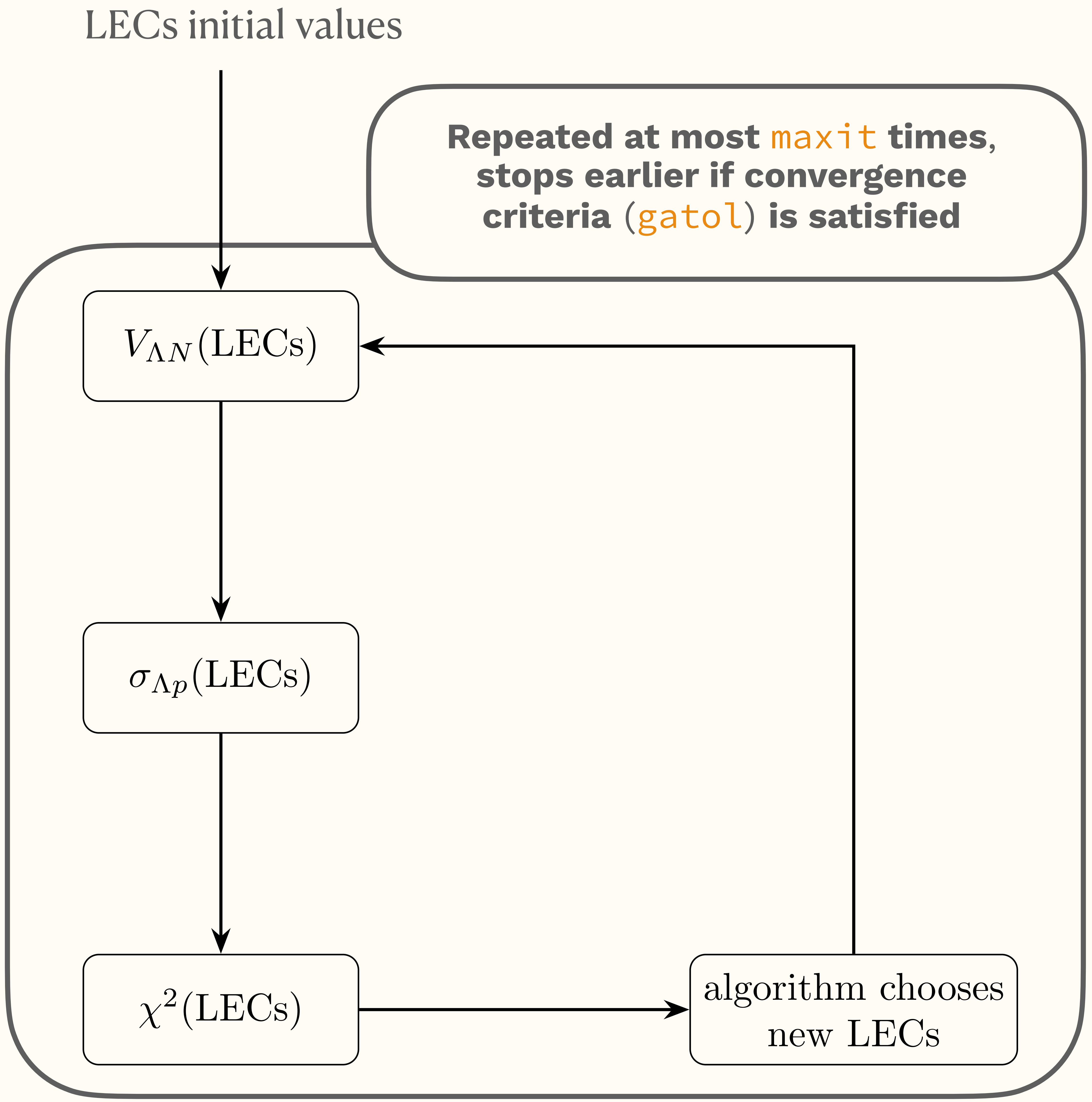
$a_s^{exp}, a_t^{exp} = (2.1,1.56)$ to $(3.34,1.18)$ fm

Minimization algorithm

LECs initial values

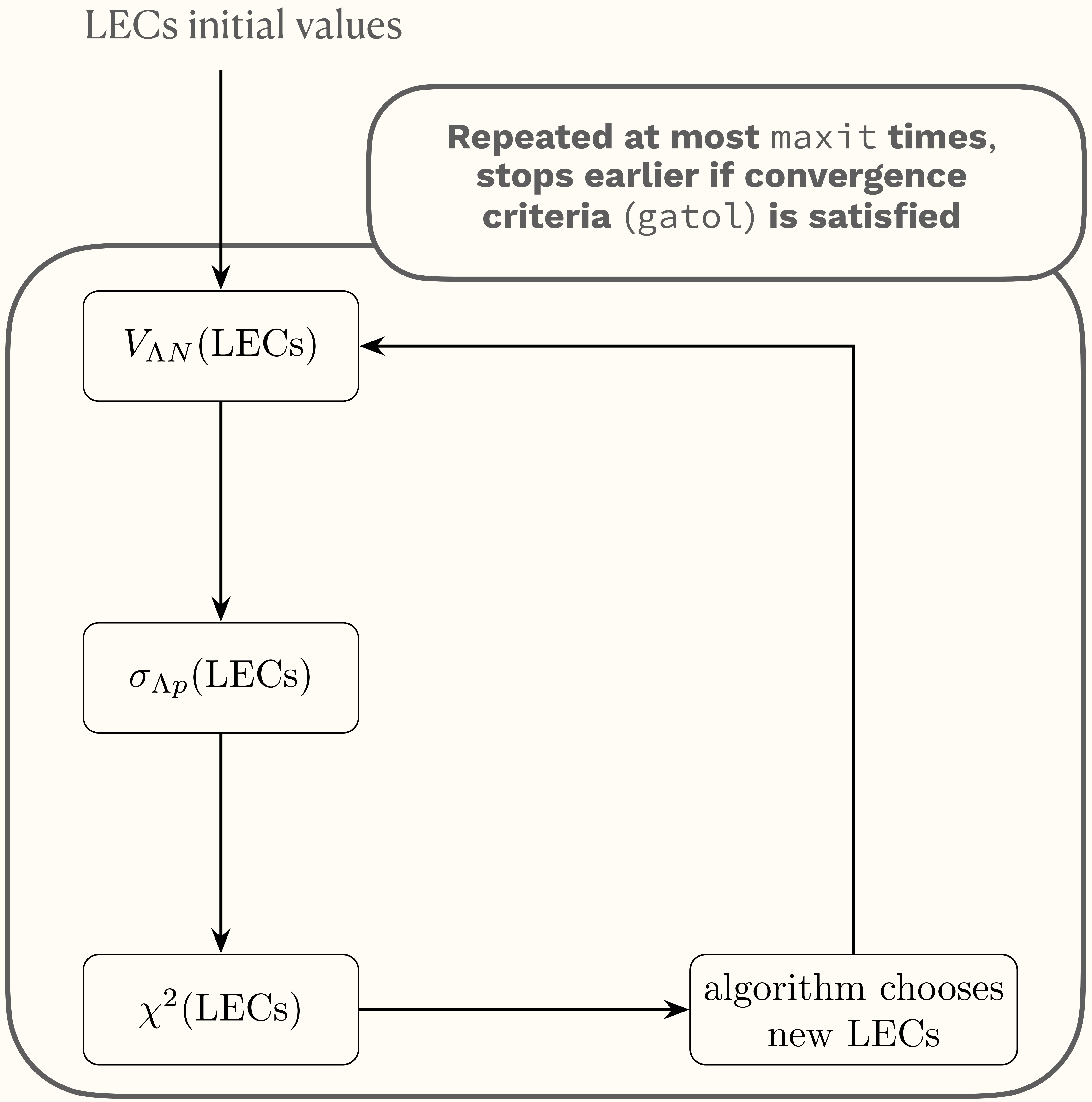


- TAOPOUNDERS, from PETSc + MPI to parallelise
- Adjustable parameters of the fitting procedure:
  - Cutoff parameter R_0
  - **Grid for initial values** of the LECs (`max, min, step`)



- TAOPOUNDERS, from PETSc + MPI to parallelise
- Adjustable parameters of the fitting procedure:
  - Cutoff parameter R_0
  - Grid for initial values of the LECs (`max, min, step`)
  - **Maximum number of calls** to optimization algorithm (`maxit`)
  - Parameter to define the threshold for **converged optimization** (`gatol`)
- Chosen values for algorithm parameters:
  - $R_0 \in [1.5, 2.5]$ fm
  - `gatol` $= 10^{-8}$
  - `maxit` $= 2 \times 10^3$

FITTING PROCEDURE
Minimization algorithm



- Fit performed at LO and NLO separately for different cutoff R_0 values

$$V_{\Lambda N}^{LO} = [C_S + C_T(\sigma_\Lambda \cdot \sigma_N)]F(r),$$

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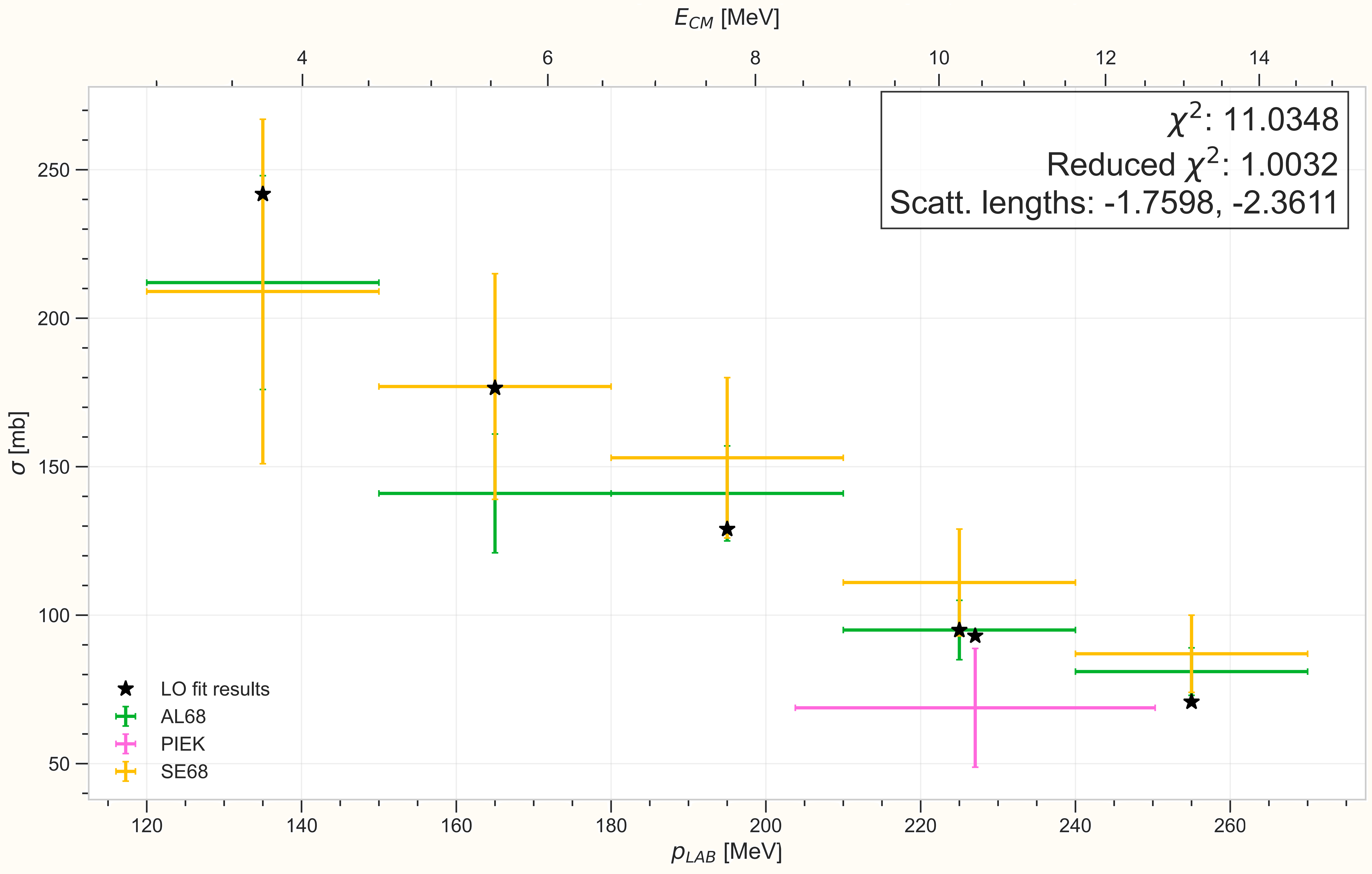
$$-C_5\frac{F^{(1)}(r)}{r}\mathbf{L}\cdot\mathbf{D}\,.$$

- Total angular momentum, energy and parity constraints:
  - **LO**: $E_{CM} < 15$ MeV, $J = 0, 1$, only positive parity
  - **NLO**: $E_{CM} < 80$ MeV, $J = 0, 1$

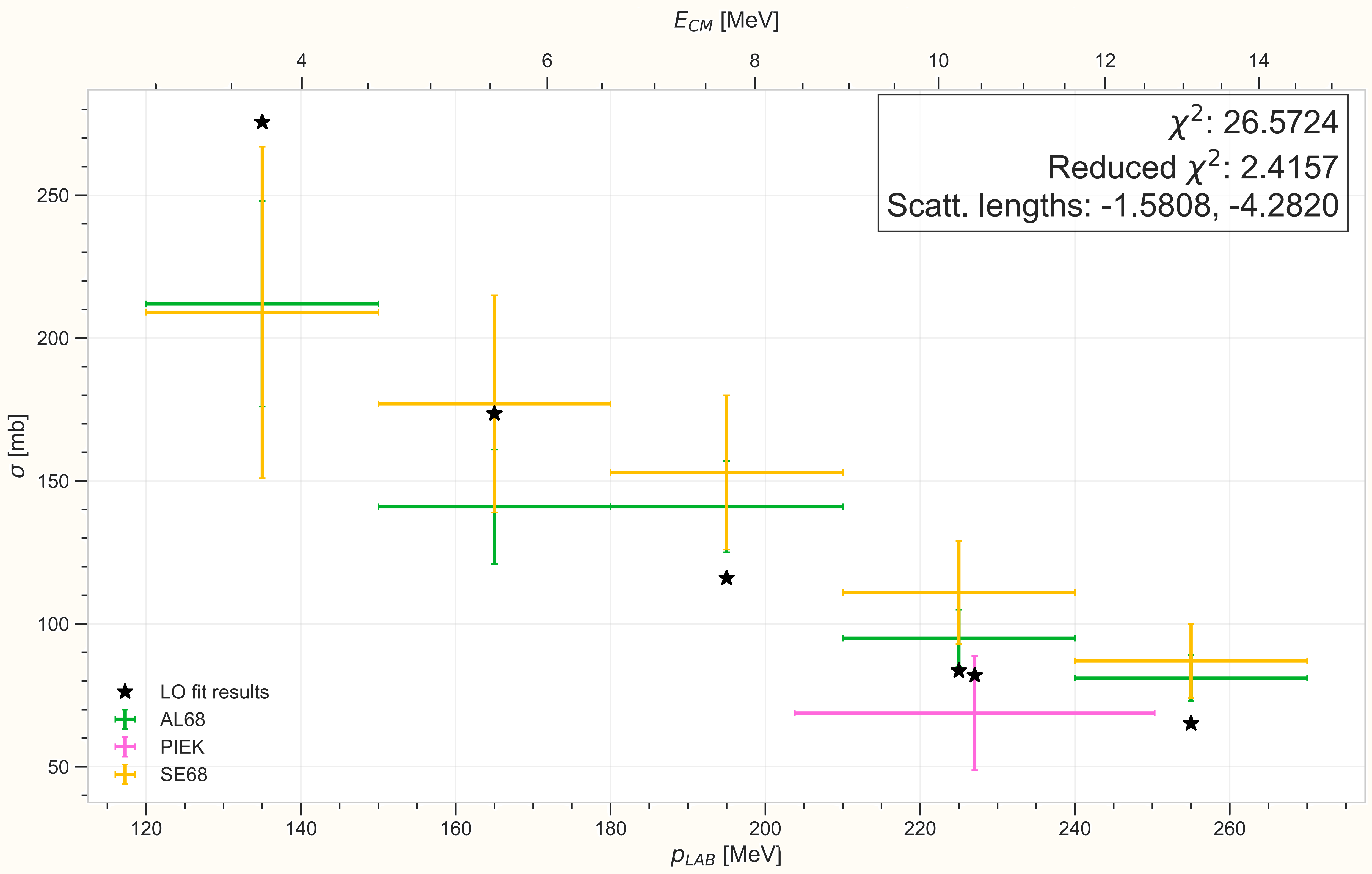
RESULTS

Leading order - Best and worst results

$R_0 = 1.5$ fm



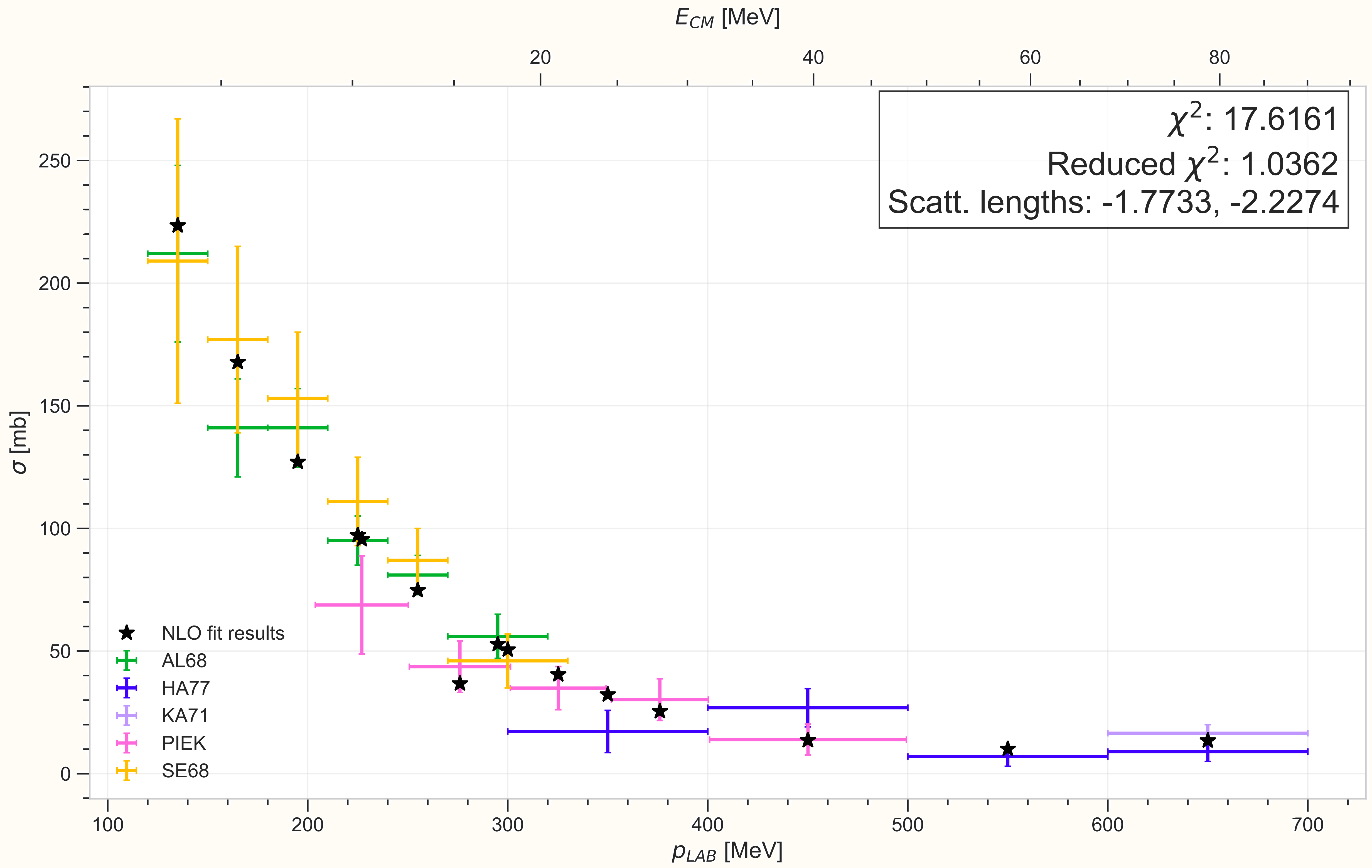
$R_0 = 2.5$ fm



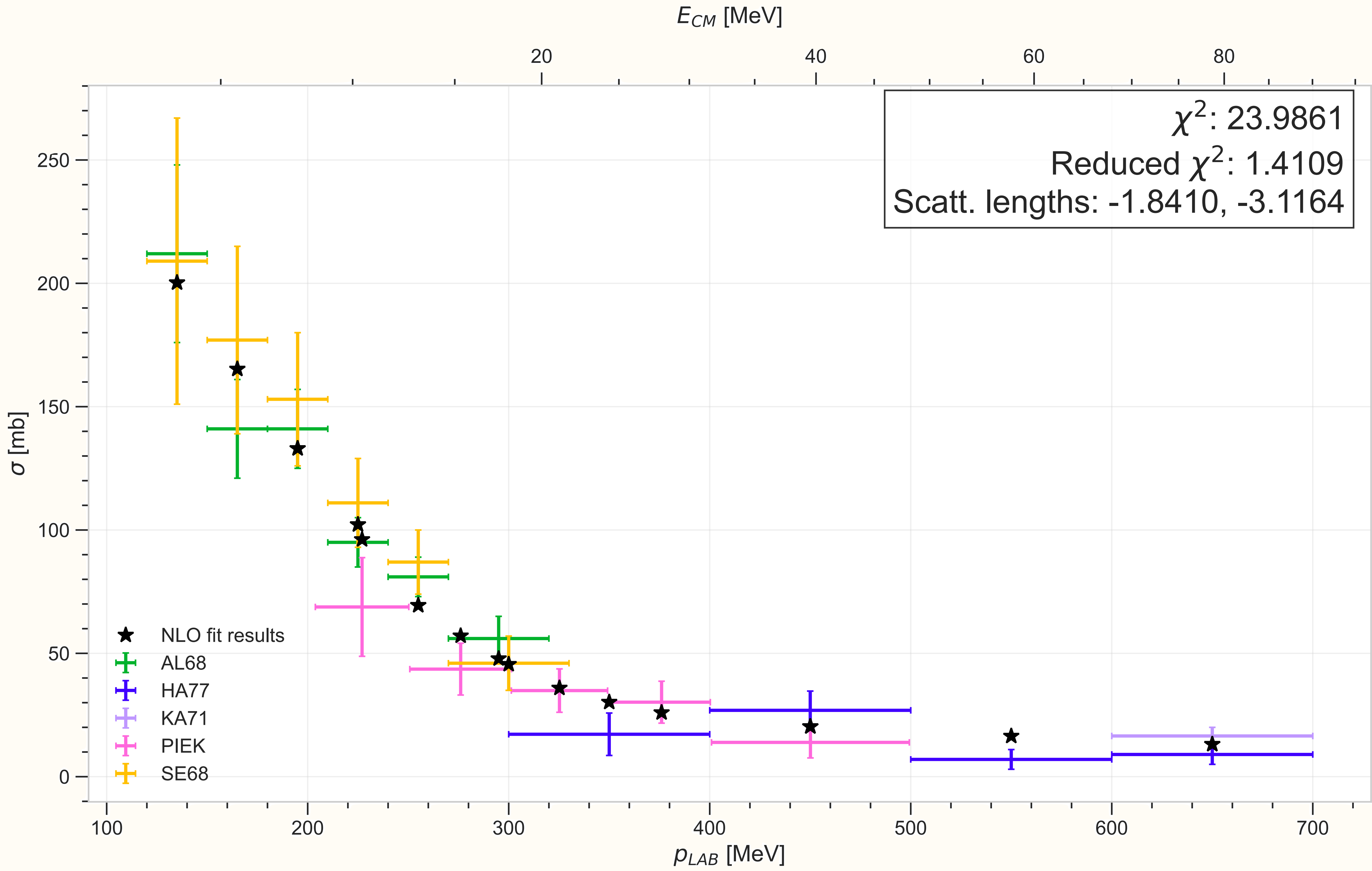
RESULTS

Next to leading order - Best and worst results

$R_0 = 1.5$ fm



$R_0 = 2.5$ fm



CONCLUSIONS

In **summary**:

- Developed a local contact potential model for the ΛN interaction up to NLO
  - Sophisticated fitting procedure
  - Compatibility with scattering data and scattering lengths

Current work and future developments

→ Further refinements of this model

→ Applications to light hypernuclei

- Develop a local χEFT potential model, with $\Lambda N - \Sigma N$ coupling
- Three-body forces (YNN, YYN, YYY)
- Hypernuclei studies and $pp\Lambda$ correlation functions
- Studies on Neutron Stars' Equation of State

THANK YOU FOR YOUR ATTENTION !

Margherita Sagina (University of Pisa, INFN - Pisa)

margherita.sagina@phd.unipi.it

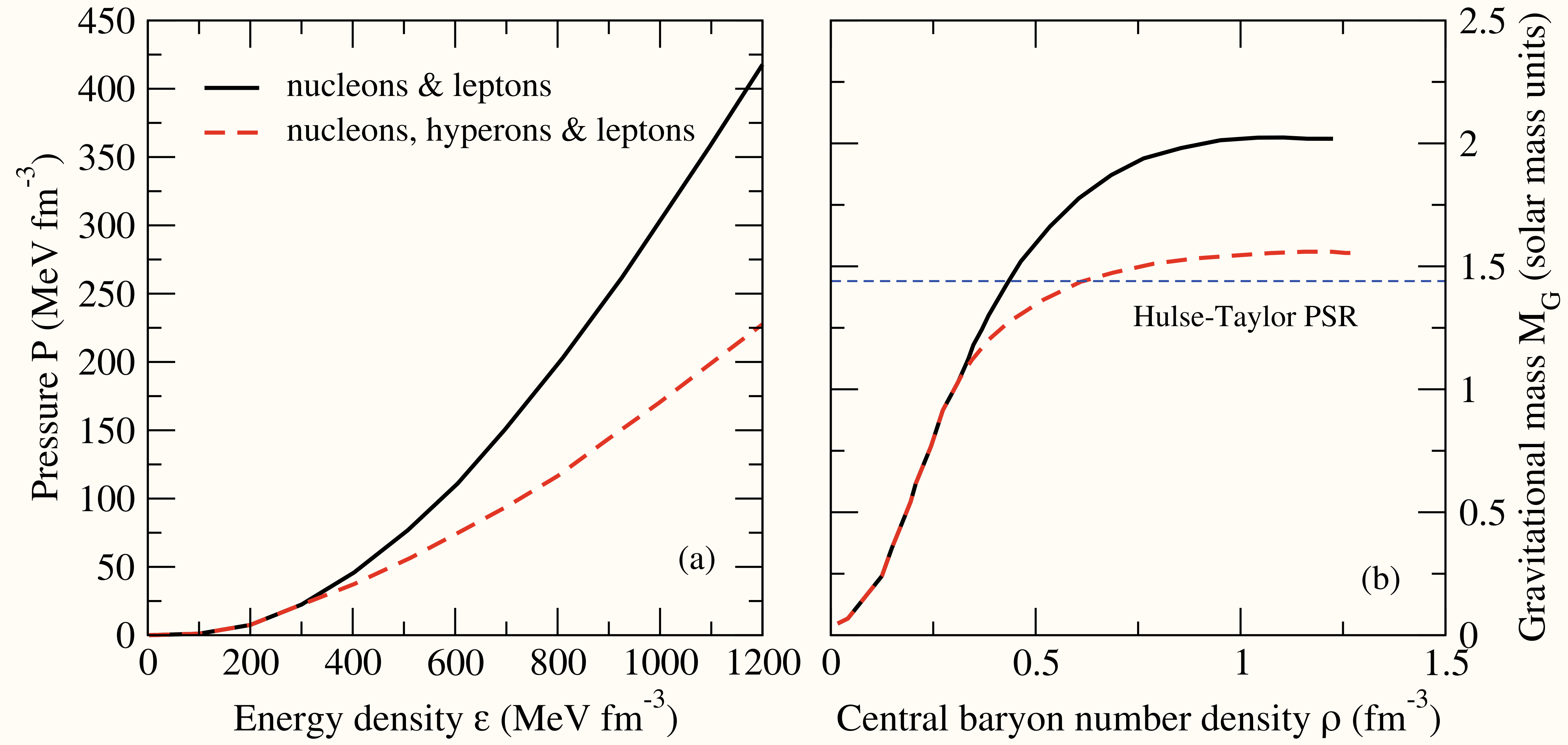
IN SUPREMÆ DIGNITATIS 1343

Dipartimento di Fisica "E. Fermi"
UNIVERSITÀ DI PISA

INFN PISA

INTRODUCTION AND MOTIVATIONS

Inside a neutron star (NS), hyperons can become stable particles $\Rightarrow$ changes in the NS equation of state (EoS)



I. Vidaña, (2016)

High density conditions in NS interior [$\rho = (2-3) \times 10^{13}$ g/cm^3]

$\Downarrow$

Increase in Fermi energy level of nucleons (Pauli exclusion principle)

$\Downarrow$

Conversion of nucleons into hyperons energetically favourable

$\Downarrow$

Decrease in energy

$\Downarrow$

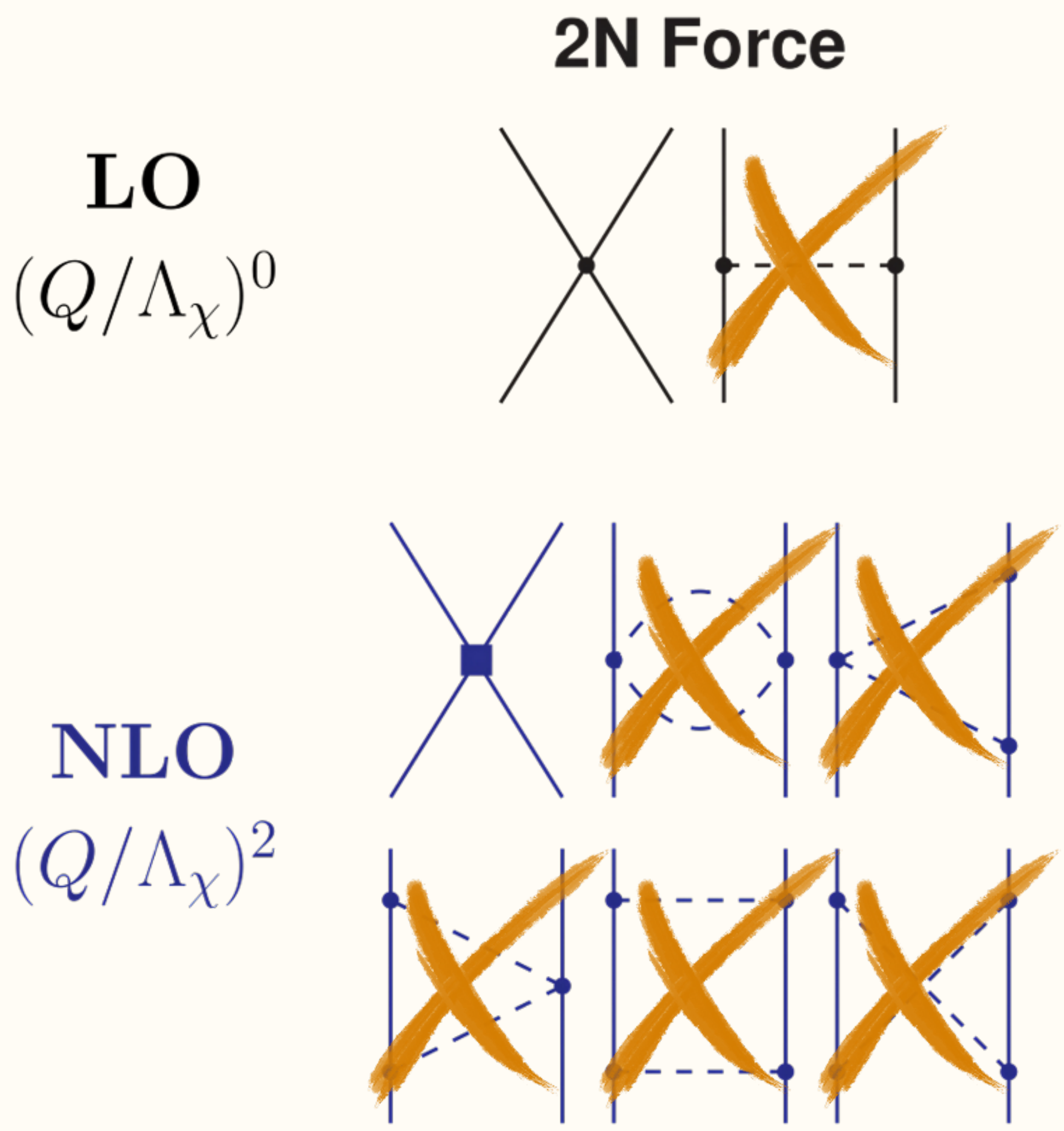
Decrease in pressure $\longrightarrow$ softening of the EoS

$\Downarrow$

Underestimation of **maximum mass** that can be reached in NS, which **contradicts experimental evidence ($M_{NS} \sim 2.1$ M$_\odot$)**

POTENTIAL MODEL DERIVATION

“Weinberg-ized” Pionless Effective Field Theory



- Keep same power counting as χEFT
- Remove diagrams that involve pion exchanges → only contact terms
- Approach used in *Schiavilla et al. (2021)*

POTENTIAL MODEL DERIVATION

Chiral Effective Field Theory

- Interaction described by most general Lagrangian that respects symmetries of QCD

- Cutoff scale (Λ_χ) $\rightarrow$ defines range of applicability of the theory

$\downarrow$

High energy effects included in contact terms that depend on low energy constants (LECs)

$\searrow$ LECs determined through fit to experimental data

- Lagrangian expanded in powers of $Q/\Lambda_\chi < 1$

  $\Rightarrow$ Organization in leading and sub-leading terms (LO, NLO, ...)

QCD

$\Lambda_\chi \sim 1$ GeV

χEFT

$Q \sim 100$ MeV

POTENTIAL MODEL DERIVATION

Hyperon-nucleon potential model – Momentum space

Literature: – *Haidenbauer et al. (2023)*: YN interaction in χEFT up to N2LO, momentum space
– *Schiavilla et al. (2021)*: NN interaction in contact EFT up to N3LO, coordinate space

↓

- Hyperon-nucleon interaction in momentum space, up to NLO, only contact terms:

$$V_{\Lambda N}^{\mathrm{LO}} = C_S + C_T\,(\sigma_\Lambda \cdot \sigma_N)\,,$$
$$V_{\Lambda N}^{\mathrm{NLO}} = C_1 \mathbf{q}^2 + C_2 \mathbf{q}^2 (\sigma_\Lambda \cdot \sigma_N) + iC_3 \mathbf{S} \cdot (\mathbf{k} \times \mathbf{q}) + C_4 S_{\Lambda N}(\mathbf{q}) + \boxed{iC_5 \mathbf{D} \cdot (\mathbf{k} \times \mathbf{q})}$$

↓

7 **LECs**

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}\,,$$
$$\mathbf{k} = (\mathbf{p} + \mathbf{p}')/2$$
$$\mathbf{S} = (\sigma_\Lambda + \sigma_N)/2\,,$$
$$\mathbf{D} = (\sigma_\Lambda - \sigma_N)/2$$
$$S_{\Lambda N}(\mathbf{q}) = 3\,\sigma_\Lambda \cdot \mathbf{q}\,\sigma_N \cdot \mathbf{q} - q^2 \sigma_\Lambda \cdot \sigma_N\,.$$

POTENTIAL MODEL DERIVATION

Hyperon-nucleon potential model – Coordinate space

- To regularize the interaction $\longrightarrow$ multiply $V_{\Lambda N}$ by a regulator function $\tilde{F}(k)$, as done in *Schiavilla et al. (2021)*

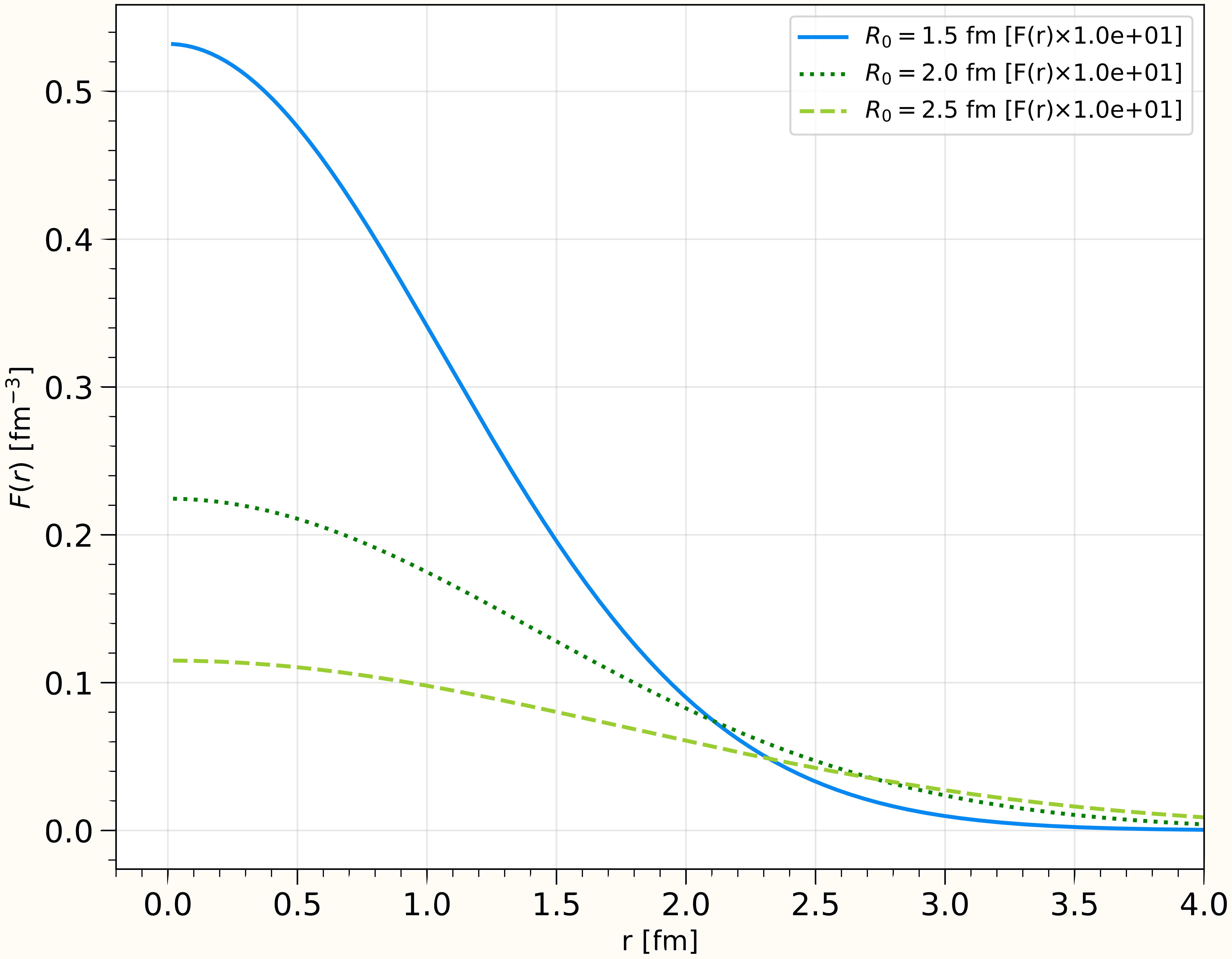
$$\tilde{F}(k) = \exp\left(-\frac{{R_0}^2 k^2}{4}\right)$$

Fourier Transform

$$F(r) = \frac{1}{\pi^{3/2}{R_0}^3}\exp\left(-\frac{r^2}{{R_0}^2}\right)$$

- Investigated cutoff parameter values $R_0 \in [1.5, 2.5]$ fm

$\Lambda_0 \in [158, 263]$ MeV



CROSS SECTION CALCULATION

Available experimental data to perform the fit: Λp **elastic scattering** cross section

- Particles are not created or destroyed
  - Unchanged amplitude of the wave function of the system
    - Only a change in phase of the outgoing wave $\longrightarrow$ phase shift δ_α

- Initial and final scattering states expanded in partial waves $\longrightarrow J^\pi$

J^π	α	L	S	$^{2S+1}L_J$
0^+	1	0	0	$^1S_0$
0^-	1	1	1	$^3P_0$
1^+	1	0	1	$^3S_1$
	2	2	1	$^3D_1$
1^-	1	1	0	$^1P_1$
	2	1	1	$^3P_1$

- Scattering effects included in the ***T*-matrix**

$\longrightarrow$ in case of single channel:

$$\sum_\beta T_{\alpha\beta} = \frac{i}{2}\left[1 - e^{2i\delta_\alpha}\right] = \sin\delta_\alpha e^{i\delta_\alpha}$$

Depends on potential model (LECs)

Total unpolarised cross section

$$\sigma = \frac{4\pi}{q^2}\sum_{J,\alpha,\beta}\left|T_{\alpha\beta}\right|^2 \frac{(2J+1)}{(2s_\Lambda+1)(2s_N+1)}$$

FITTING PROCEDURE

LO vs NLO fitting strategy

LO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 15$ MeV
  - $J = 0, 1$
  - Only positive parity
- All LECs chosen on a grid ⟶ **2 LECs**
- Parameters for grid of initial values:
  - min $= -15$
  - max $= 15$ $\quad\Rightarrow\quad \sim 900$ points
  - step $= 1$

NLO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 80$ MeV
  - $J = 0, 1$
  - Both parities
- LO LECs (i.e. C_s, C_T) fixed at LO best results ⟶ **7 LECs**
  NLO LECs (i.e. $C_1, \ldots, C_5$) chosen on a grid
- Parameters for grid of initial values:
  - min $= -15$
  - max $= 15$ $\quad\Rightarrow\quad \sim 10^5$ points
  - step $= 3$

FITTING PROCEDURE
χ^2 function

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- First attempt: $$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2}$$

  ☒ Does not work: $V_{\Lambda N} = 0$ at LO when $S = 1$

- Second attempt: **constraint on scattering length**

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th}(C_S, C_T, C_1, \ldots, C_5) - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

☑ Solves issue in $S = 1$ case

"Experimental" data for Λp scattering length from *Mihaylov et al. (2024)*

FITTING PROCEDURE
Defining energy limits

- Partial wave projection of the potential, in momentum space

$$V(^1S_0) = 4\pi(C_S - 3C_T) + \pi(4C_1 + C_2 - 12C_3 - 3C_4 - 4C_6 - C_7)(p^2 + p'^2)\,,$$
$$V(^3S_1) = 4\pi(C_S + C_T) + \pi\frac{3}{2}(12C_1 + 3C_2 + 12C_3 + 3C_4 + 4C_6 + C_7)(p^2 + p'^2)\,.$$

LO LECs

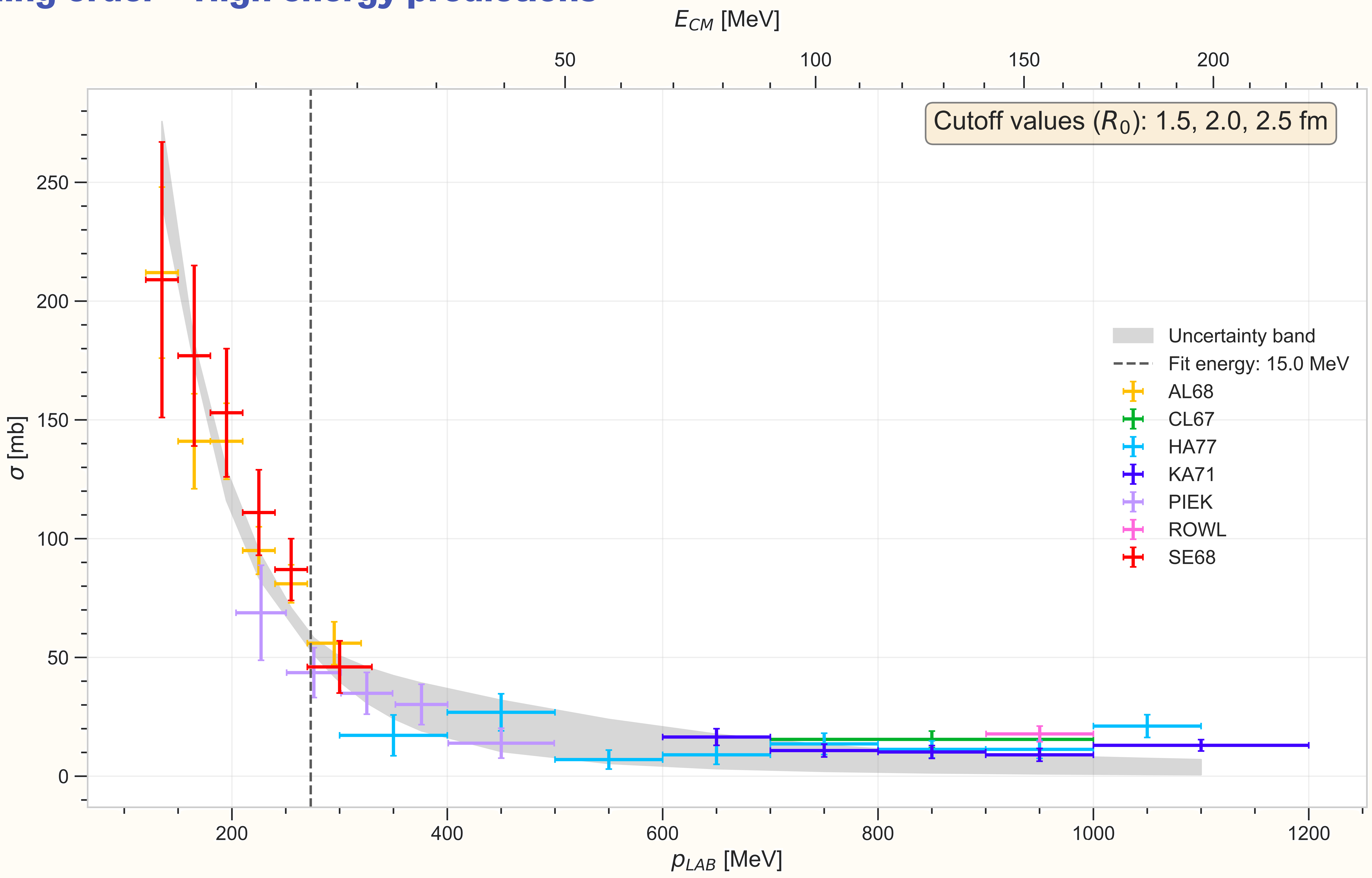
- Semi classical approach to compute p_{LAB} threshold for LO $\qquad \ell\,\hbar = p_{LAB}\,b$

- $\ell = 1$ to exclude *P*-waves and higher-order partial waves
- $b \sim 1$ fm, $\hbar c = 197.33$ MeV fm

$\Big\}\quad p_{LAB} \sim 200$ MeV $\Rightarrow E_{CM} \sim 15$ MeV

RESULTS

Leading order – High energy predictions



RESULTS

Next to leading order - High energy predictions