

RINGDOWN TESTS OF GENERAL RELATIVITY WITH FUTURE SPACE GRAVITATIONAL WAVE DETECTORS

Women in Theoretical Physics – Premio Nazionale "Milla Baldo Ceolin" 2024

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Master's Degree in Astrophysics

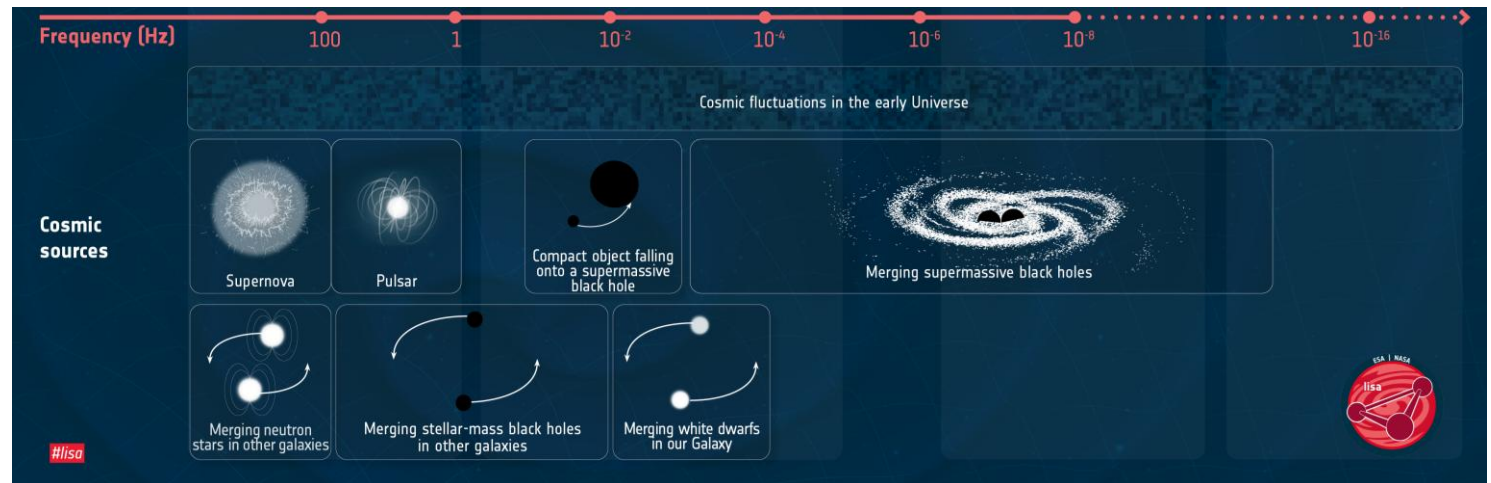
Advisors: Paolo Pani, Andrea Maselli

Sapienza University of Rome

Context: gravitational waves (GWs)

- Wave solution of Einstein's equations, studied as spacetime perturbations

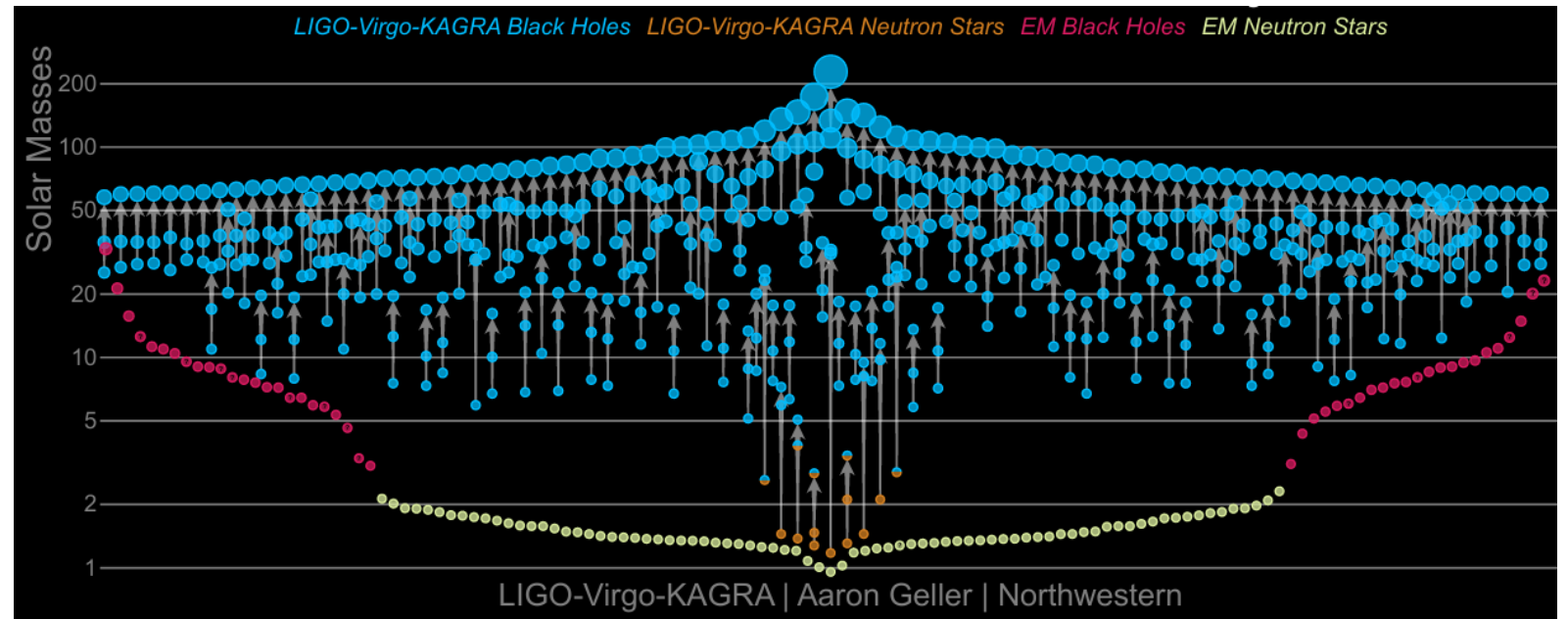
- Hard to detect ($\sim 10^{-21}$)
→ need for astrophysical sources



- First detection: GW150914 – merger of two stellar-mass black holes (BHs)

Context: gravitational waves (GWs)

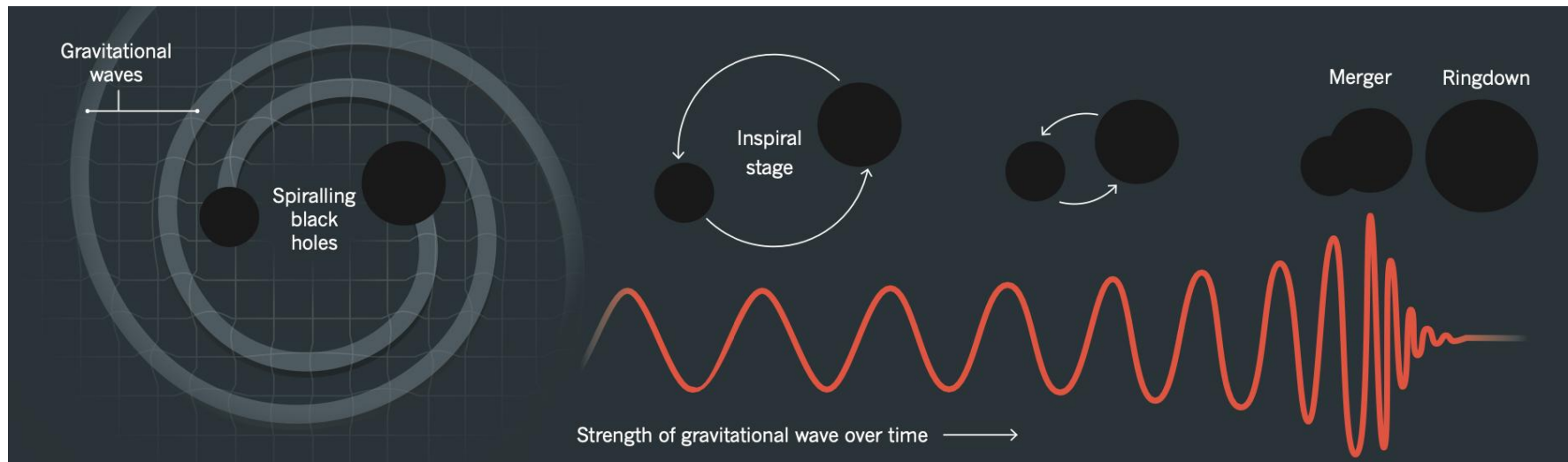
- So far ~ 200 events detected



- Current LIGO-Virgo-KAGRA observing run: O4c (until Nov 18, 2025)

Context: ringdown & quasi-normal modes (QNMs) of a BH

- Stages of a merger and its gravitational waveform



Credit: Lucy Reading-Ikkanda

Context: ringdown & quasi-normal modes (QNMs) of a BH

- Ringdown: superposition of damped sinusoids - QNMs

$$h \sim \sum_{lmn} \mathcal{A}_{lmn} e^{-t/\tau_{lmn}} \cos(2\pi f_{lmn} t + \phi_{lmn}) Y_{lmn}$$



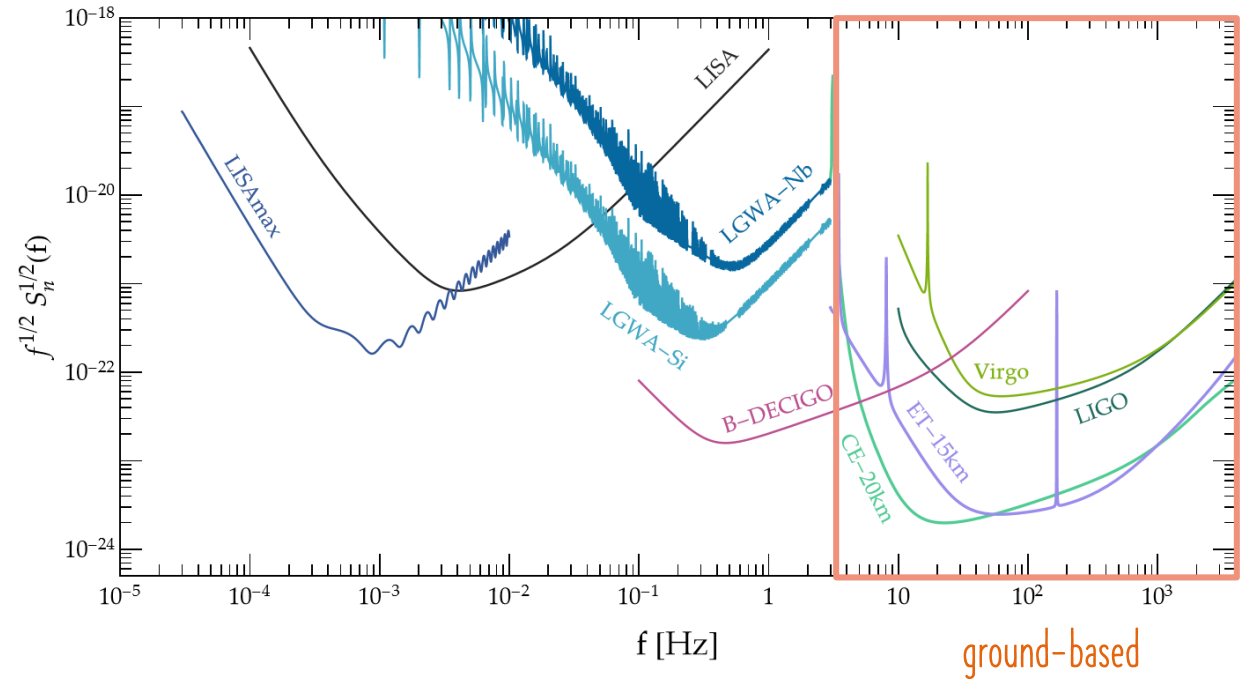
- No-hair theorem \rightarrow $f_{lmn} \equiv f_{lmn}(M_f, \chi_f), \quad \tau_{lmn} \equiv \tau_{lmn}(M_f, \chi_f)$
 - i. measurement of 1 mode (1 frequency + 1 damping time) \rightarrow mass, spin
 - ii. measurement of ≥ 1 modes \rightarrow test of the theory

Context: space-based GW detectors

- Expected high signal-to-noise ratios (SNRs) at low frequencies

characteristic strain

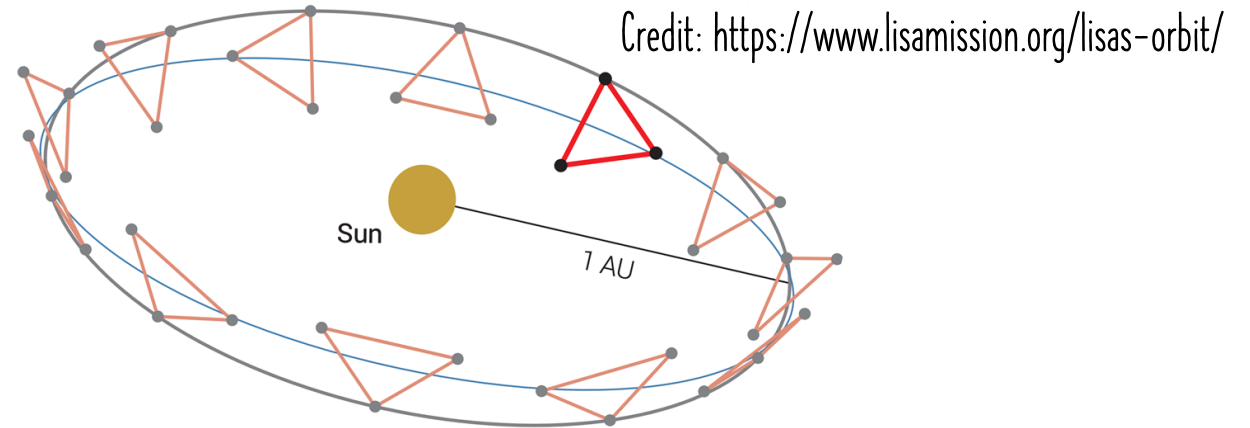
- Thesis goals: assess
 1. QNM detectability
 2. potential for tests of GR



- In this presentation: LISA (mHz) & B-DECIGO (dHz)

Context: space-based GW detectors

- Laser Interferometer Space Antenna
 - i. triangular, helio-centric orbit
 - ii. arm length: $2,5 \cdot 10^6 \text{ km}$

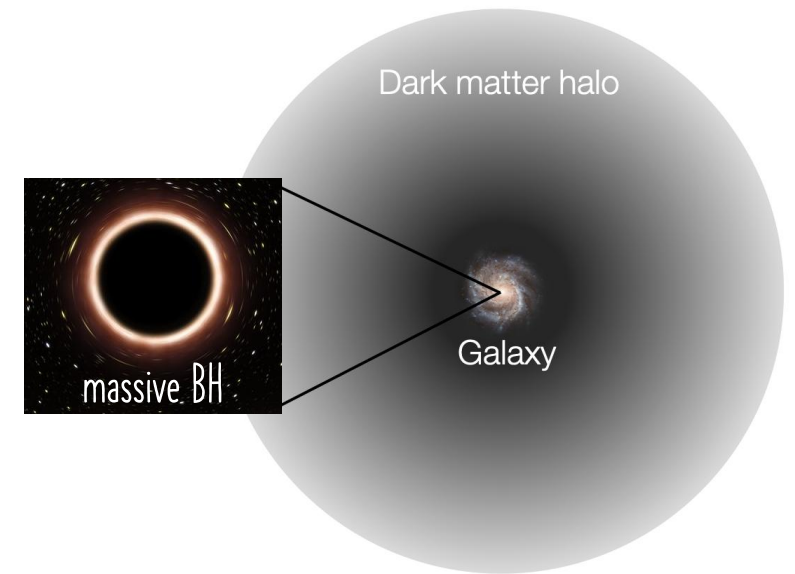


- B-DECihertz Interferometer Gravitational wave Observatory (precursor of DECIGO)
 - i. triangular, geo-centric orbit
 - ii. arm length: 100 km

Data analysis approach: two complementary paths

1. Exploration of the parameter space of remnant BH's mass and spin
 - i. independent of specific astrophysical populations
 - ii. amplitude and SNR (ρ) as functions of M_f, χ_f

2. Population inference based on astrophysically motivated catalogs of binary BHs
 - i. semi-analytic galaxy formation model
 - ii. merger between BHs at center of two galaxies



Credit: M. Powell, ESO/M. Kornmesser (adapted)

Data analysis approach: QNM model

- For a given mode $(\omega_{\ell m}, \tau_{\ell m})$

$$\begin{cases} h_+(t) = \frac{M \mathcal{A}_{\ell m} Y_+^{\ell m}}{r} \operatorname{Re}[e^{-t/\tau_{\ell m} + i(\omega_{\ell m} t + \phi_{\ell m} - m\varphi)}] & \text{"plus"} \\ h_\times(t) = \frac{M \mathcal{A}_{\ell m} Y_\times^{\ell m}}{r} \operatorname{Im}[e^{-t/\tau_{\ell m} + i(\omega_{\ell m} t + \phi_{\ell m} - m\varphi)}] & \text{"cross"} \end{cases} \quad \text{polarization}$$

↓ Fourier transform

$$\tilde{h}(f) = F_+ \tilde{h}_+(f) + F_\times \tilde{h}_\times(f)$$

sky location, polarization angle

$$\rho^2 = 4 \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}(f) \tilde{h}^*(f)}{S_n(f)} df$$

Power Spectral Density (PSD)

- Most excited modes

i. (22) "primary"

ii. (33), (21) "secondary"

Data and methods: parameter space exploration

- 100x100 generated primary BH mass and spin values
 - i. mass range fitting the PSD function
 - ii. mass ratio $q = 2, 5, 10$
 - iii. equal-spin initial BHs, with $\chi \in [0.5, 0.9]$

Detector	Mass range (M_{\odot})	Frequency range (Hz)
LISA	$10^5 - 10^8$	$10^{-4} - 1$
B-DECIGO	$10^3 - 10^5$	$10^{-1} - 10^2$

- Fixed luminosity distance of 10 Gpc

Data and methods: population inference

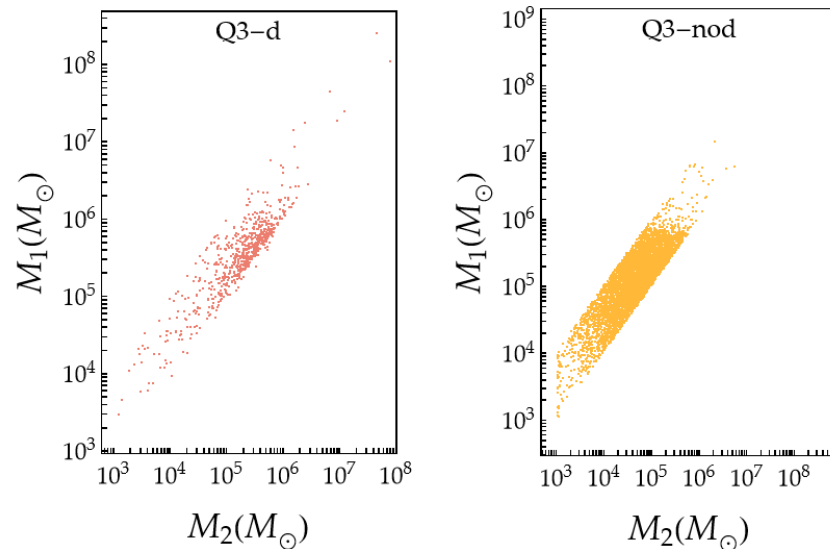
- 10 simulated populations of binary BHs, varying by
 - a. BH seed mass (light/heavy)
 - b. inclusion of SN feedback
 - c. galaxy merger delays
 - d. dark matter merger tree mass resolution ("raw"/"rec")

- Mass ratio: $q \leq 10$

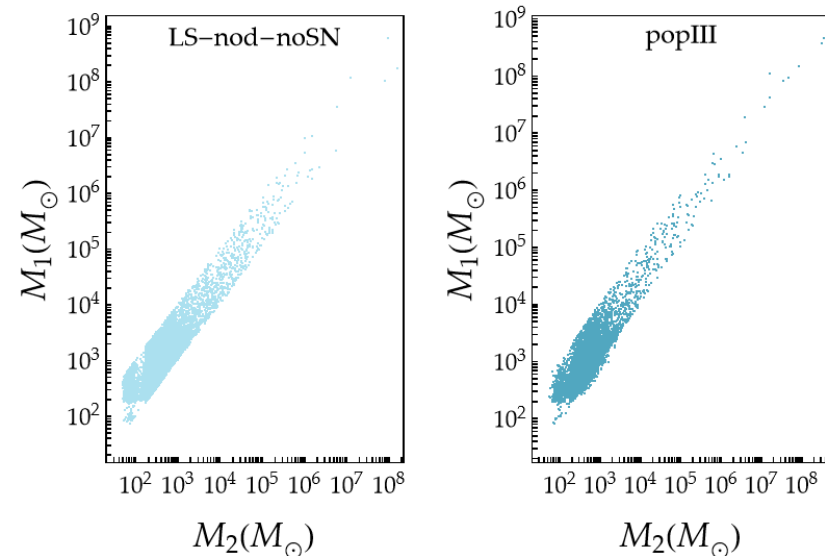
Data and methods: population inference

- Primary vs secondary BH mass distribution

Heavy seed models



Light seed models



Data and methods: pipeline (in Mathematica)

From initial masses and spins, compute

1. mass and spin of the remnant BH (M_f, χ_f)
2. frequency ω and damping time τ for (22), (33) and (21) QNMs
3. radiation efficiency $\varepsilon_{\ell m}$ and corresponding amplitude $A_{\ell m}$
4. SNR $\rho_{\ell m}$

Data and methods: pipeline (in Mathematica)

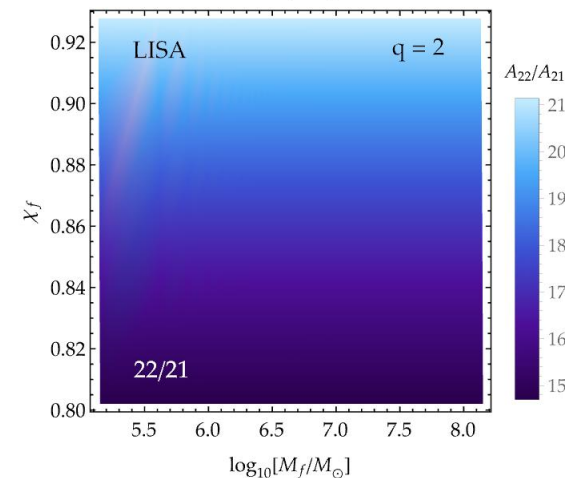
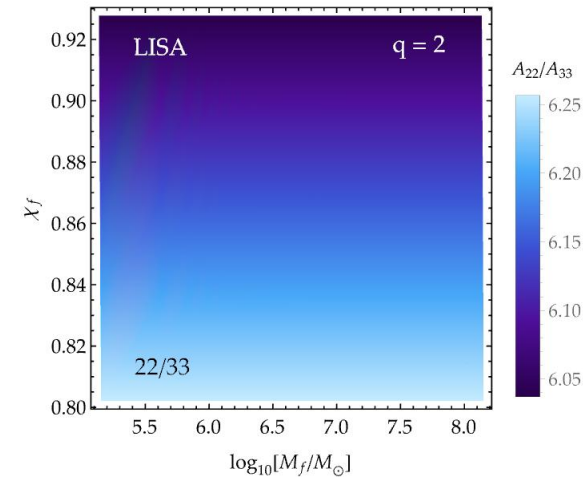
Additional steps for population inference:

5. Compute QNM parameter uncertainties $(\delta\omega, \delta\tau)$
6. Sample $(\delta\omega, \delta\tau)$ with MCMC to get posterior distribution

Results: parameter space exploration

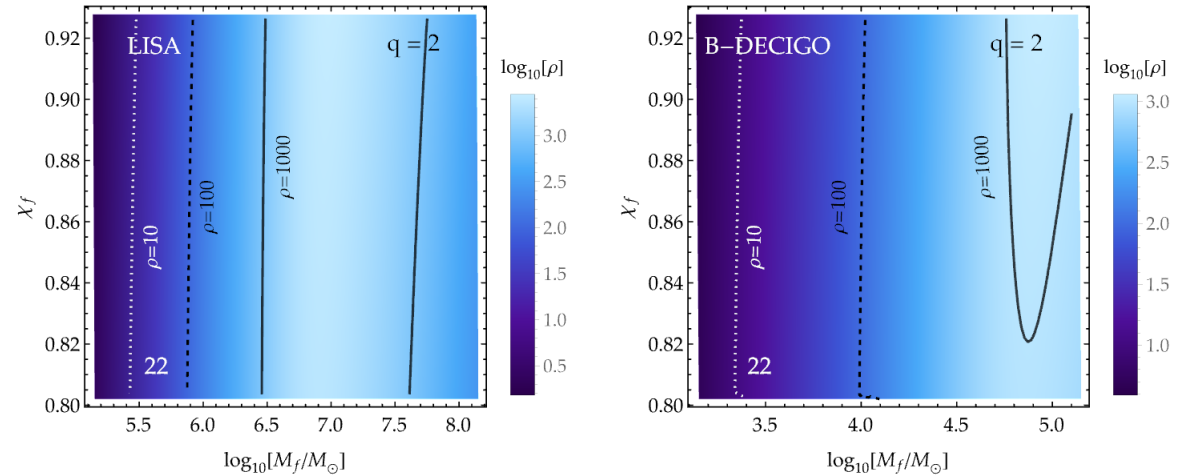
Relative amplitudes of the secondary modes as functions of (M_f, χ_f)

- Vertical gradient \rightarrow spin-dependent
- $A_{22}/A_{33} < A_{22}/A_{21}$
 - \rightarrow (33) mode stronger
 - \rightarrow (21) mode active for asymmetric binaries



Results: parameter space exploration

SNR of the primary mode as a function of (M_f, χ_f)



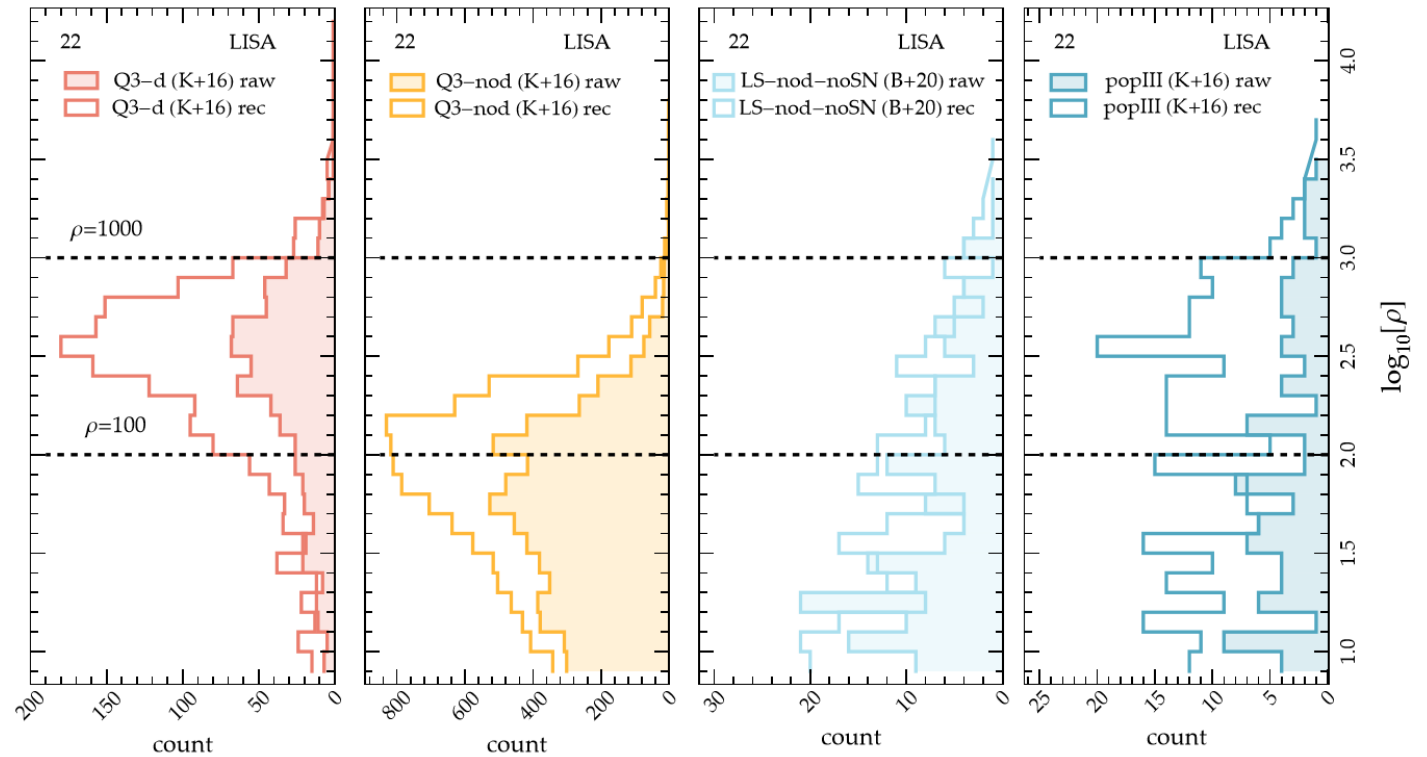
- Horizontal gradient \rightarrow mass-dependent
 \rightarrow optimal mass range

Detector	Max ρ_{22}	Mass Range [M_\odot]
LISA	10^3	$10^{6.5} - 10^7$
B-DECIGO	10^3	$10^{4.5} - 10^5$

Results: population inference

SNR distribution of the primary mode

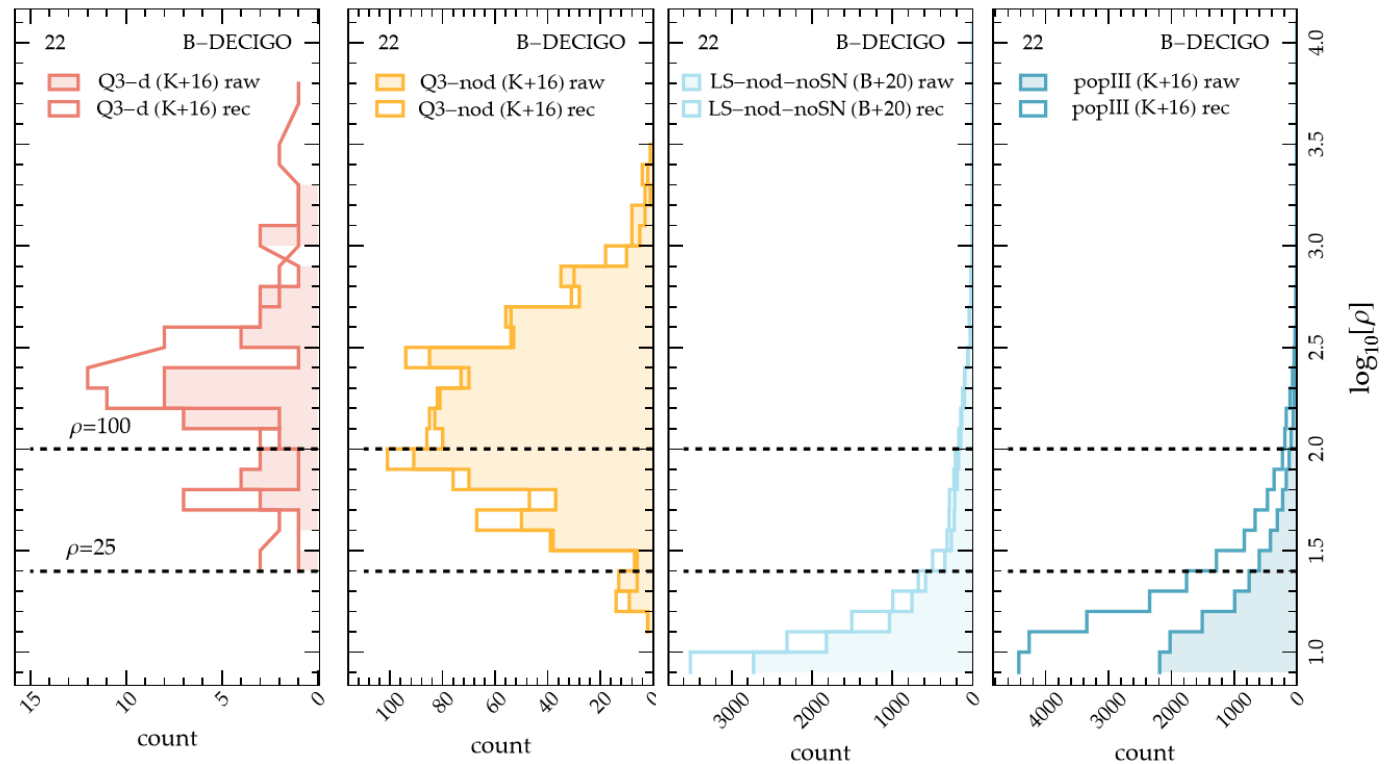
- Events detectable ($\rho \geq 8$) by LISA



Results: population inference

SNR distribution of the primary mode

- Events detectable ($\rho \geq 8$) by B-DECIGO



Results: population inference

Detectable events during a nominal mission (4 yr)

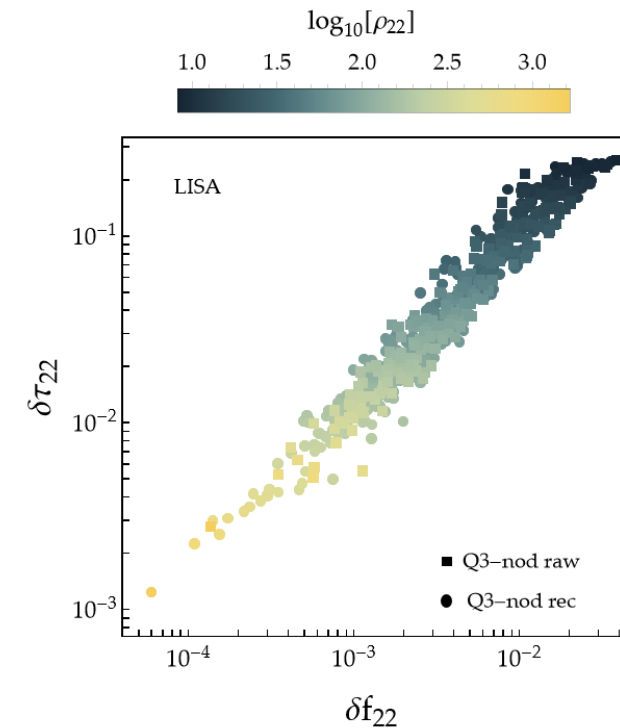
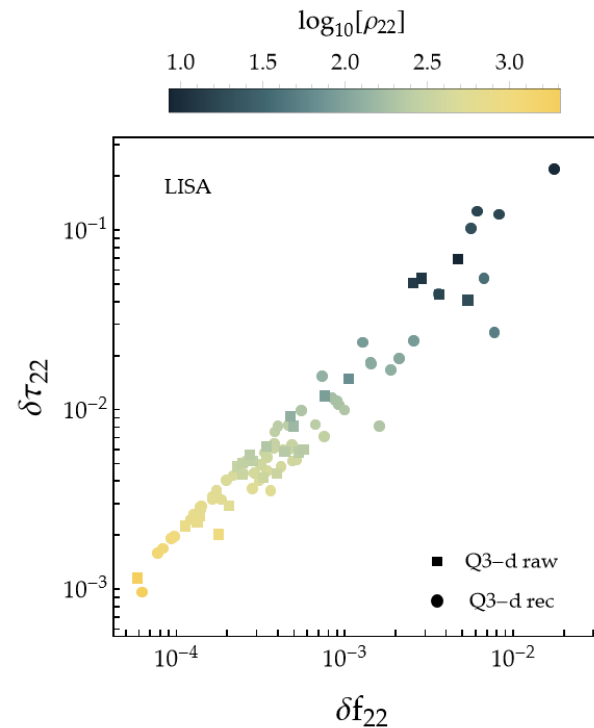
Catalog	LISA	B-DECIGO
Q3-d (K+16)	27(64)	2(2)
Q3-nod (K+16)	244(389)	37(39)
LS-nod-SN (B+20)	0(0)	58(81)
LS-nod-noSN (B+20)	7(9)	361(592)
popIII (K+16)	4(10)	388(827)

→ LISA more effective with **heavy** seed models, B-DECIGO with **light** seed models

Results: population inference

Distribution of QNM frequency and damping time relative uncertainties

- LISA
- Catalogs with ≥ 10 detectable events in 4 yr



Conclusions

- Parameter space exploration

→ optimal mass range LISA: $10^{6.5} - 10^7 M_{\odot}$, B-DECIGO: $10^{4.5} - 10^5 M_{\odot}$

- Population inference

→ LISA more effective with heavy seed catalogs, $\rho \sim 10^2 - 10^3$

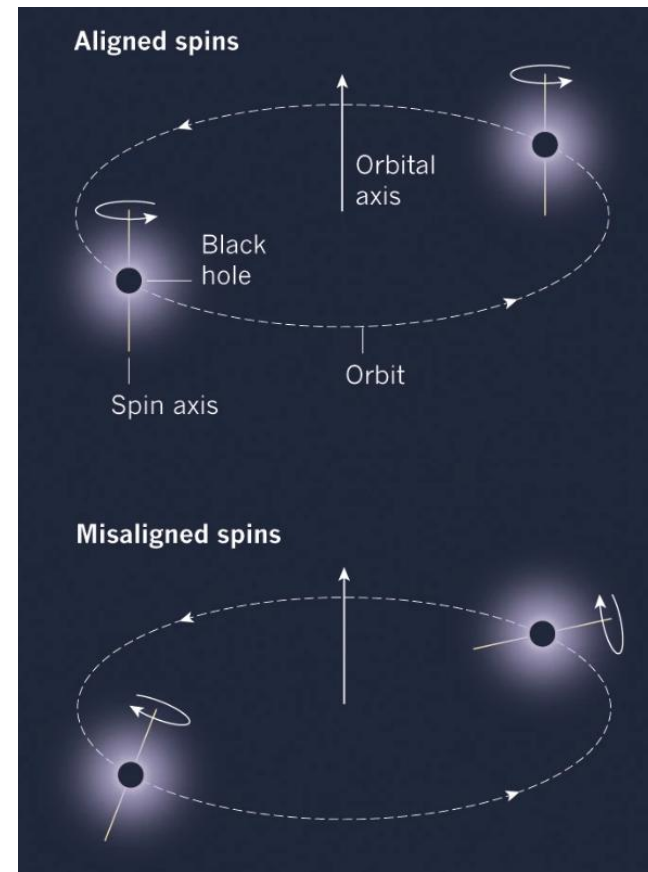
→ B-DECIGO more effective with light seed catalogs, $\rho \sim 10^1 - 10^2$

→ Minimum relative uncertainties on QNM parameters: $\delta\omega_{22} \sim 10^{-4}$, $\delta\tau_{22} \sim 10^{-3}$

- Possible further studies: new catalogs, unequal-spin BHs, higher mass ratios

What I am doing now

- 2nd year PhD student at University of Milano-Bicocca
- Broad research topic: GW data analysis for current and future detectors
- First project: parameter estimation of the **ringdown** of GW190521 with a ringdown model capturing **spin precession**



Credit: <https://www.nature.com/articles/548397a> (adapted)