Particle Dark Matter Direct Detection

Lecture 2

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U.S. DEPARTMENT OF ENERGY | Office of Science DE-SC0009937

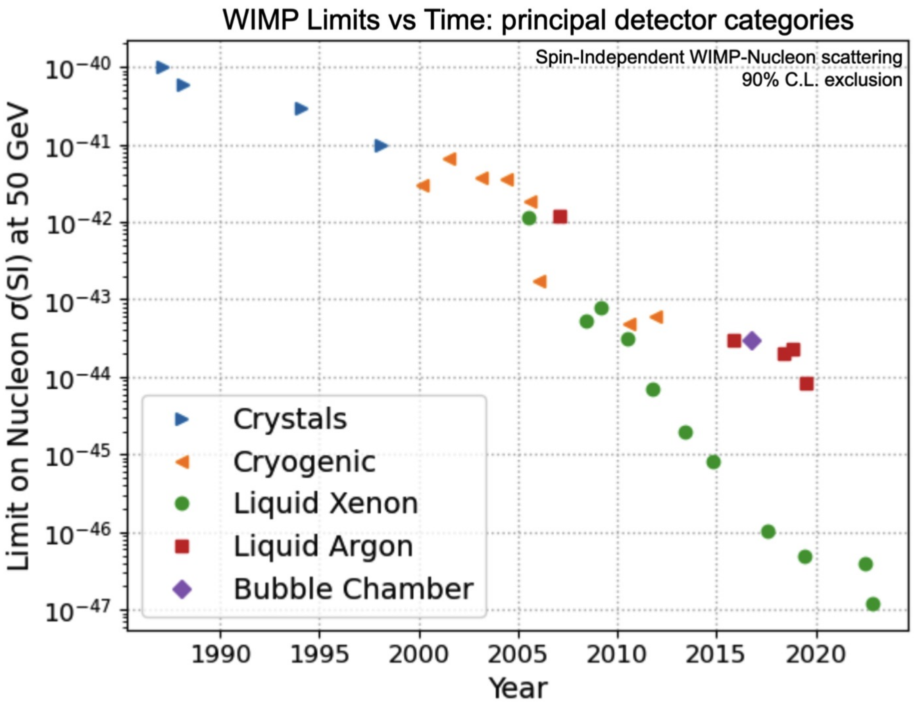
Theory Meets Experiments 2025, GGI, Florence, Nov 10-21 2025

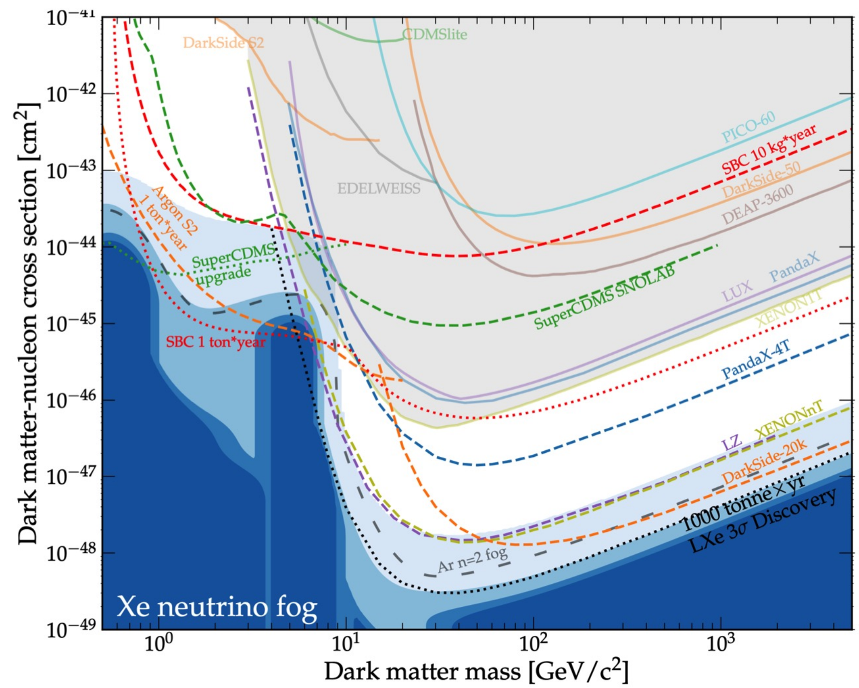
Direct Detection

- Lecture 1:
  – Brief review of the observational evidence for Dark Matter (DM)
  – What we know about DM and implications for DM candidates mass and interaction (PBH or particles? CDM, WDM, PIDM, DDDM, SIDM? Millicharge DM, kinetic mixing, Hidden (or Dark) Photons (HP or DP), Atomic DM, Mirror DM, WIMPs, FIMPs, SIMPs, ELDERs, Axions, ALPs, WISPs, FIPs...)
  – The Standard Halo Model (SHM) and its main parameters
- Lecture 2:
  – Introduction to DM Direct Detection (DD)
  – Non-directional DD of WIMPs
- Lecture 3:
  – Halo model implications, Halo-Independent Data Analysis, Directional DD
- Lecture 4:
  – DD of Light Dark Matter

Disclaimer: idiosyncratic choice of subjects and not complete lists of citations

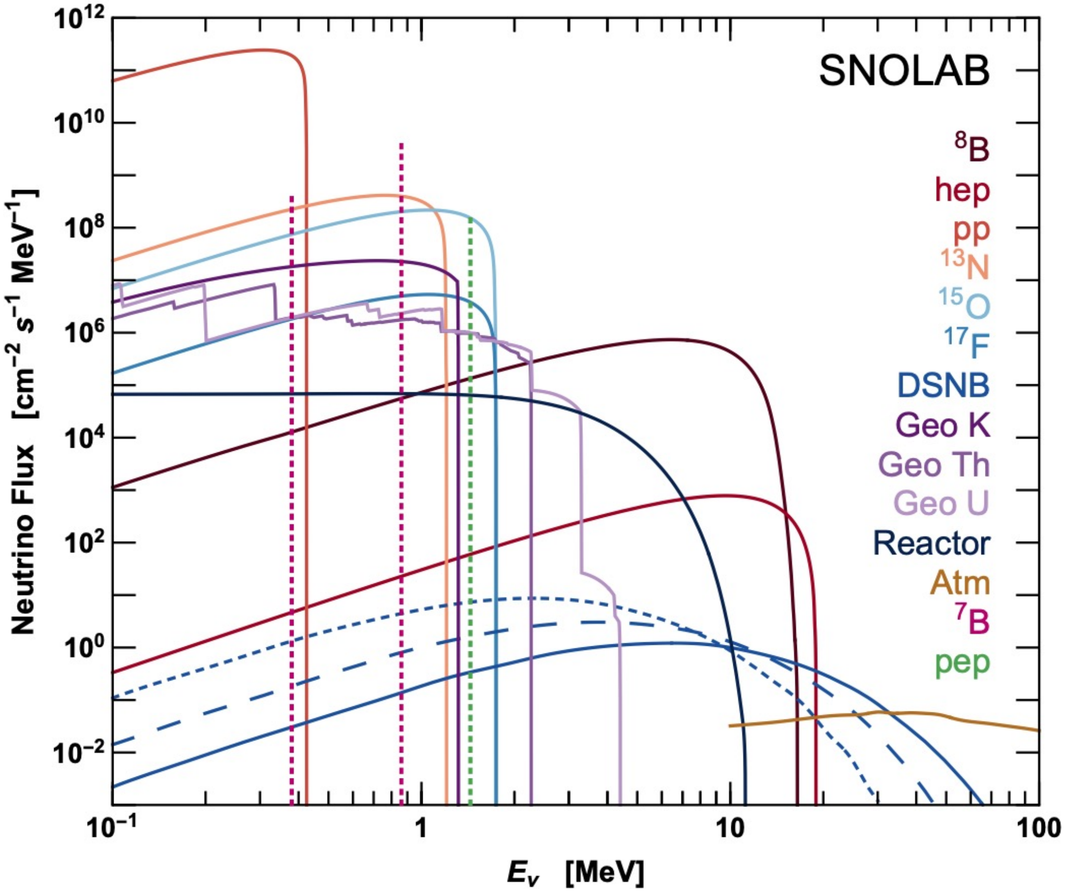
Leading WIMP DD experiments (Plot for SI int.) (Snowmass paper 2203.08084)





Present multi-tonnne liquid noble gas experiments: with Xe, XENONnt (5.9 t), LZ (7 t), PANDA-X-4T (3.7 t), and with Ar, DarkSide-20k (20 t), will observe solar neutrinos- In future only two k-tonne size experiments one with Xe, "XLZD Consortium", and one with Ar, "Global Ar DM Collab." (GADMC), will explore the "neutrino fog" of solar and atmospheric neutrinos.

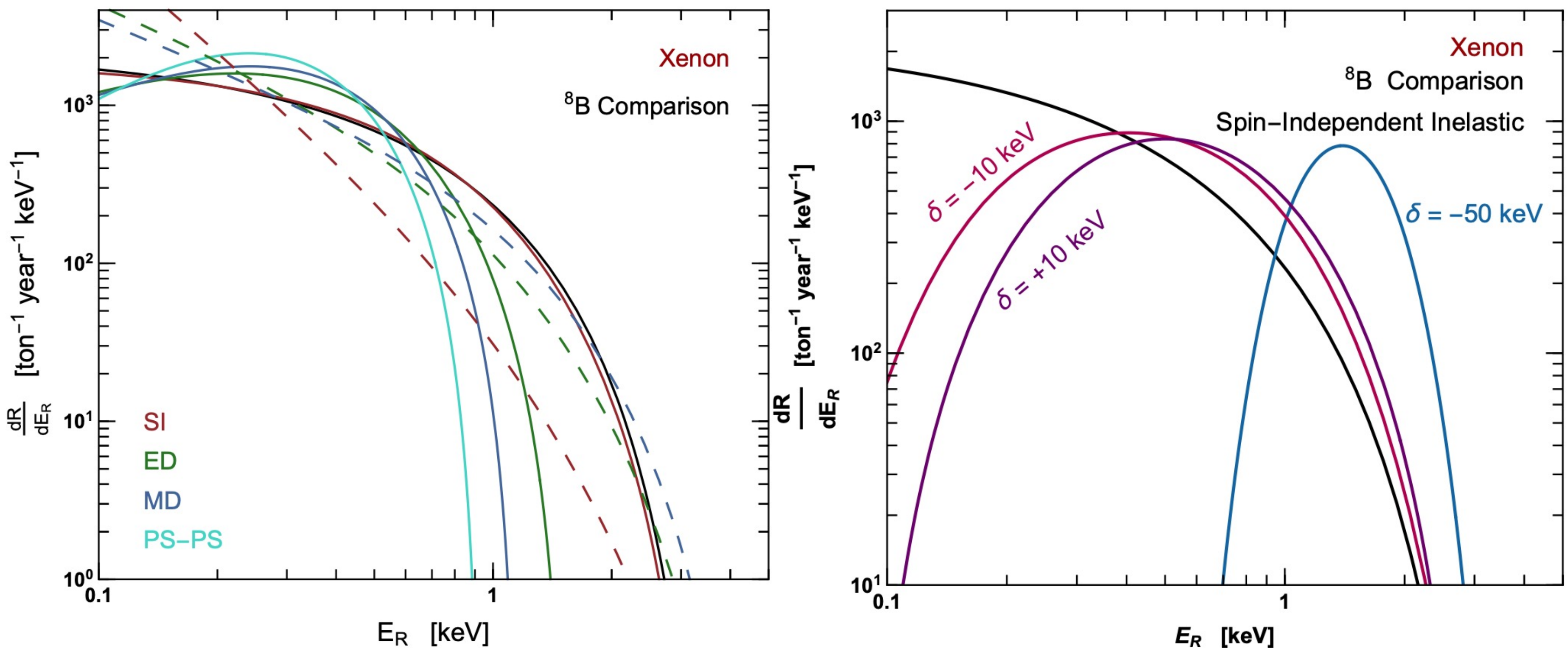
Neutrino spectra



^{8}B solar neutrinos will be the first to be detected.

Degeneracy with ν spectrum depends on interaction type

Gelmini, Takhistov and Witte 1804.01638



- Left: elastic scattering, different interactions- for m leading to most spectral degeneracy (all $m <$ 10 GeV)
- Right: SI inelastic scattering ($\delta = m' - m >$ 0 endothermic, $\delta < 0$ exothermic)

Discovery reach of future DD experiments The relevance of the neutrino fog depends on the WIMP interaction and mass

Gelmini, Takhistov and Witte 1804.01638

Differential cross section momentum exchange q dependence from LESS to MOST sensitive to the low energy ν background:
(HM: heavy mediator, LM:light mediator)

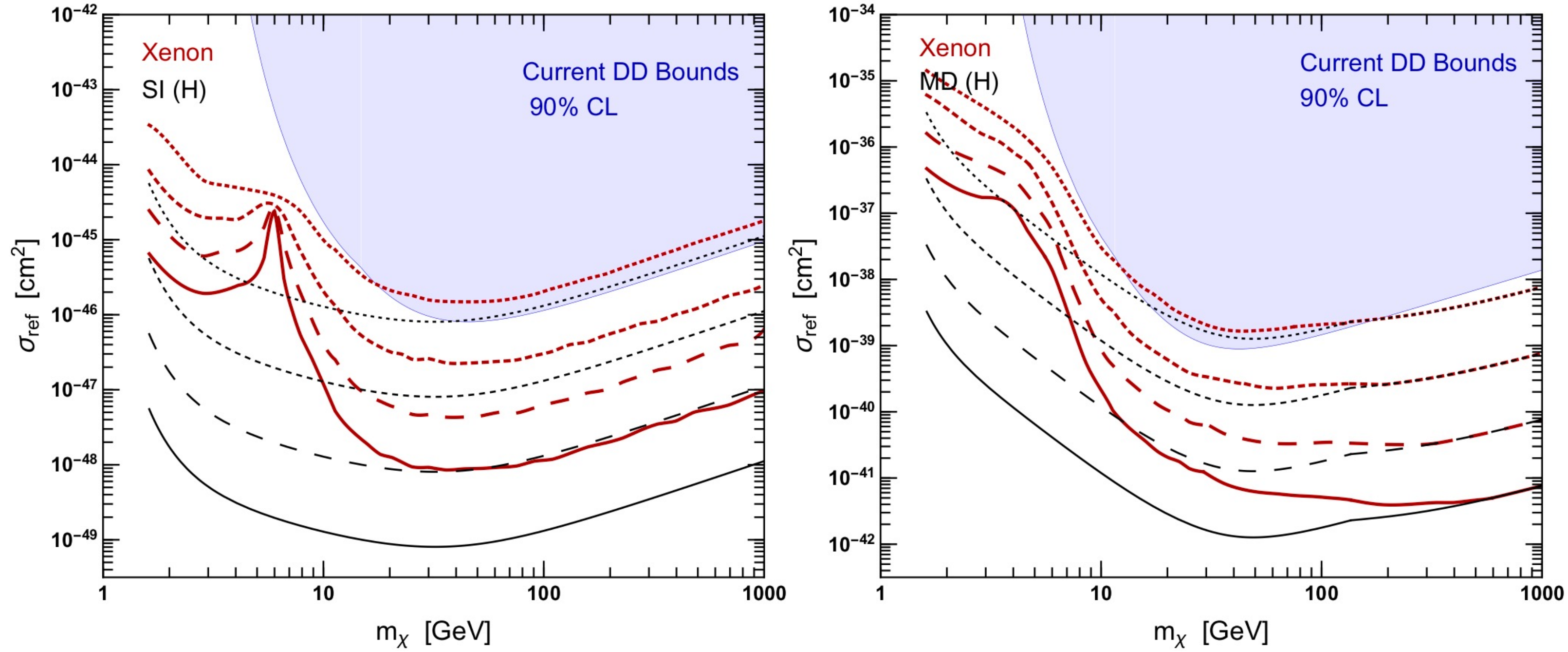
– q^b with $b = 2, 4, 6$ (HM: MD, ED, PS-S, S-PS, PS-PS, Ana)
spectrum large at large energies, so less affect by low energy background

– q^0 - no dependence (HM: SI, SD, mC, AV-V; LM: PS-PS, MD)

– $1/q^b$ with $b = 2, 4$ (LM: SI, SD, mC, AV-V, ED, PS-S, S-PS)
no spectral degeneracy, but enhanced at low energy

Discovery reach of future DD experiments The relevance of the neutrino fog depends on the WIMP interaction and mass

Gelmini, Takhistov and Witte 1804.01638

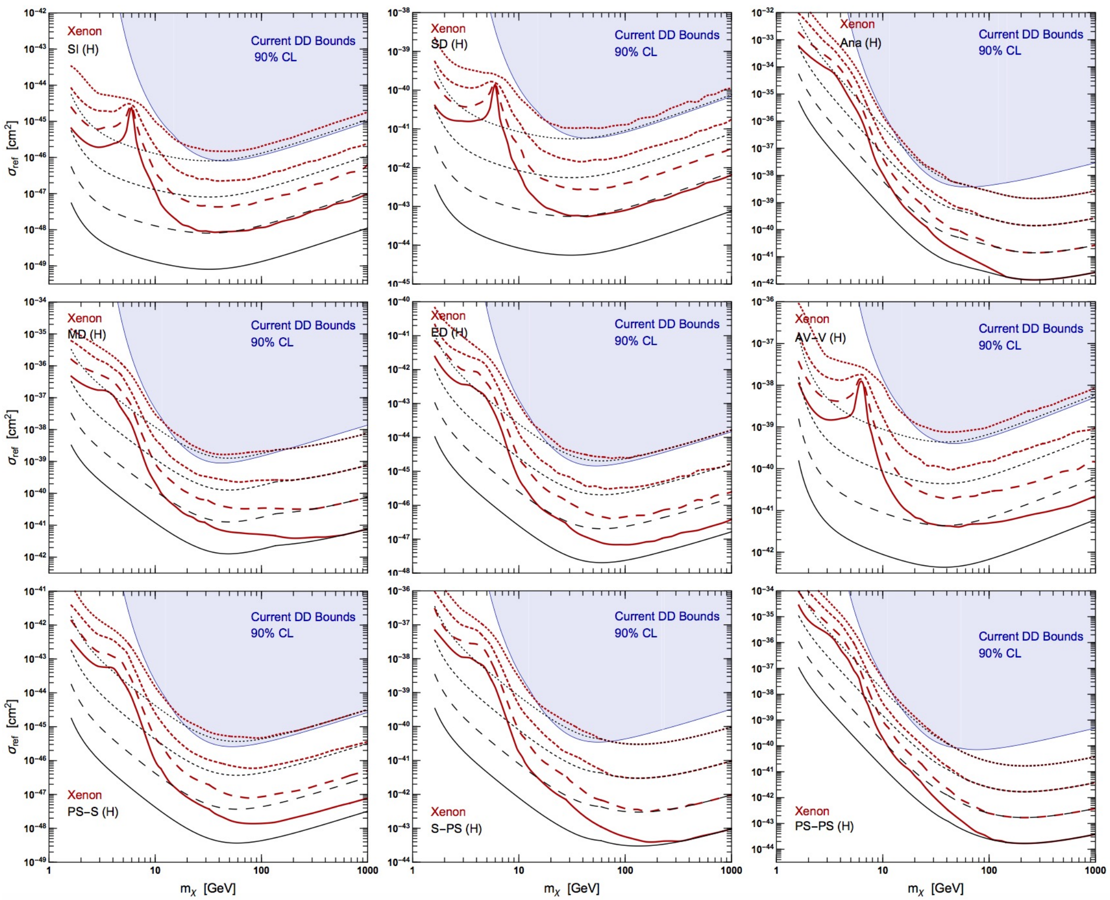


Lines for 0.1, 1, 10, 100 ton y- black (red) without (with) ν background.

The Xe 3σ discovery limit of heavy q^2 MD interacting DM is not affected by the neutrino floor (for exposures $\leq$ 100 tonne y) spectrum peaks at a higher E.

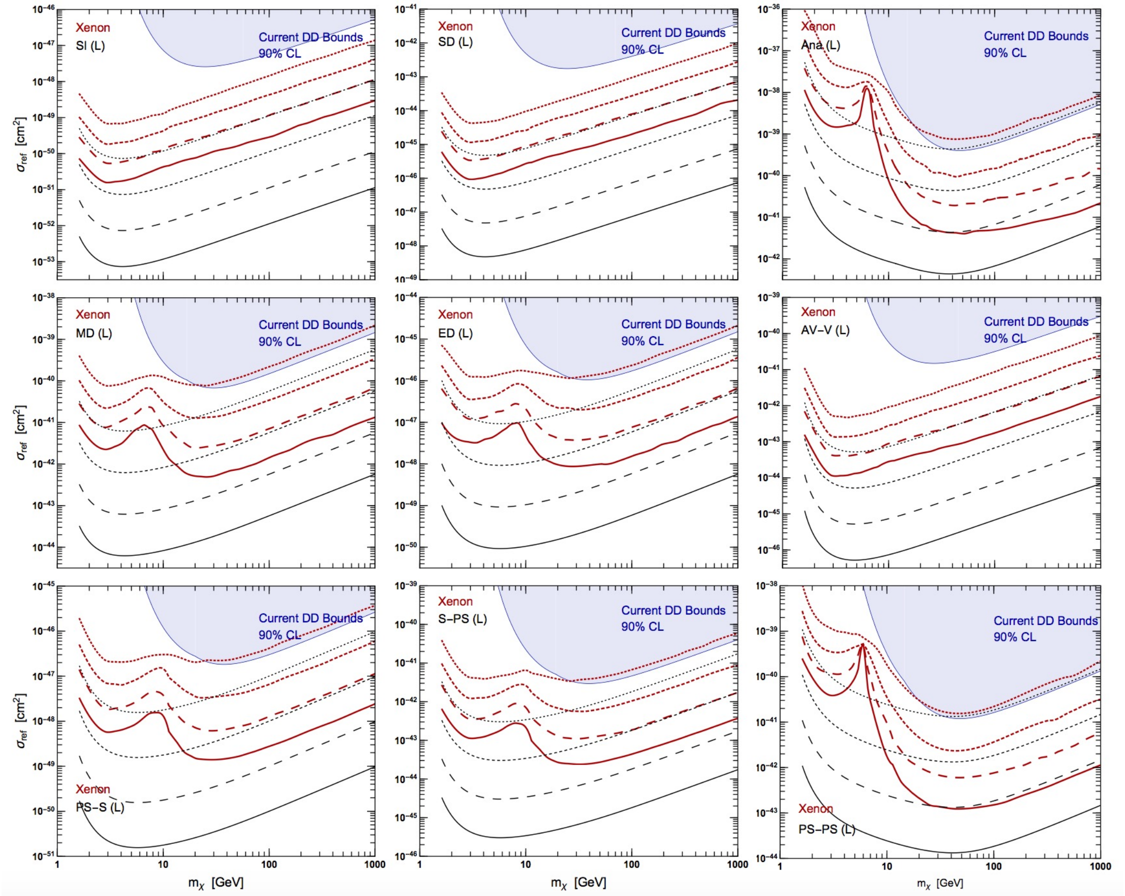
For Xe target, elastic scattering with different interactions -Heavy Mediator)

Gelmini, Takhistov and Witte 1804.01638



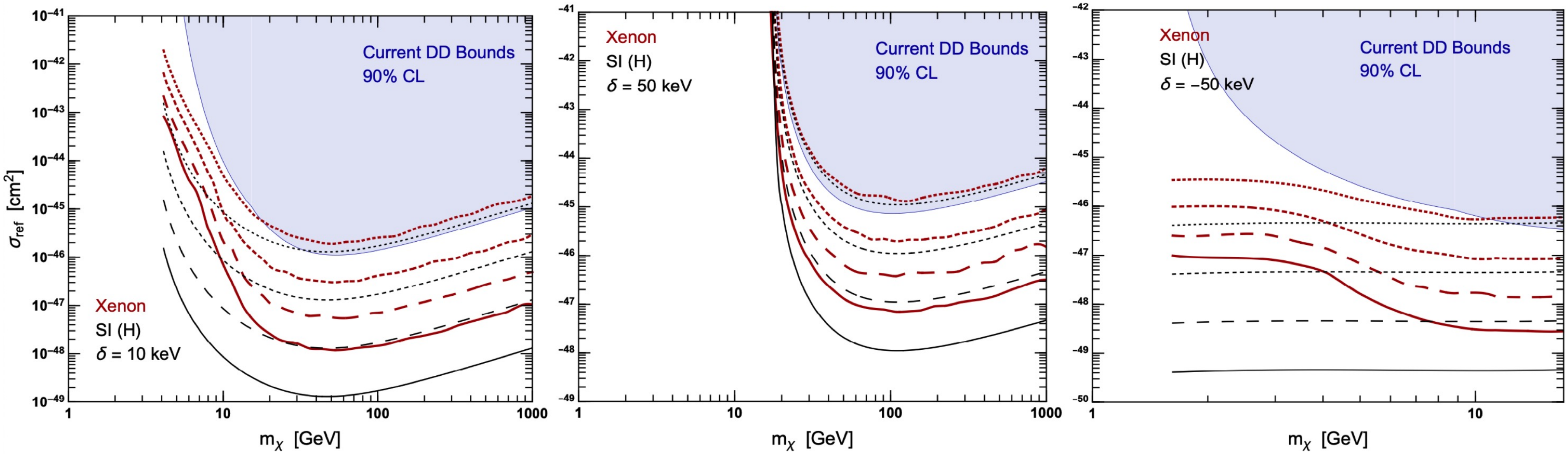
For Xe target, elastic scattering with different interactions -Light Mediator)

Gelmini, Takhistov and Witte 1804.01638



For Xe target, inelastic scattering SI interaction -Heavy Mediator)

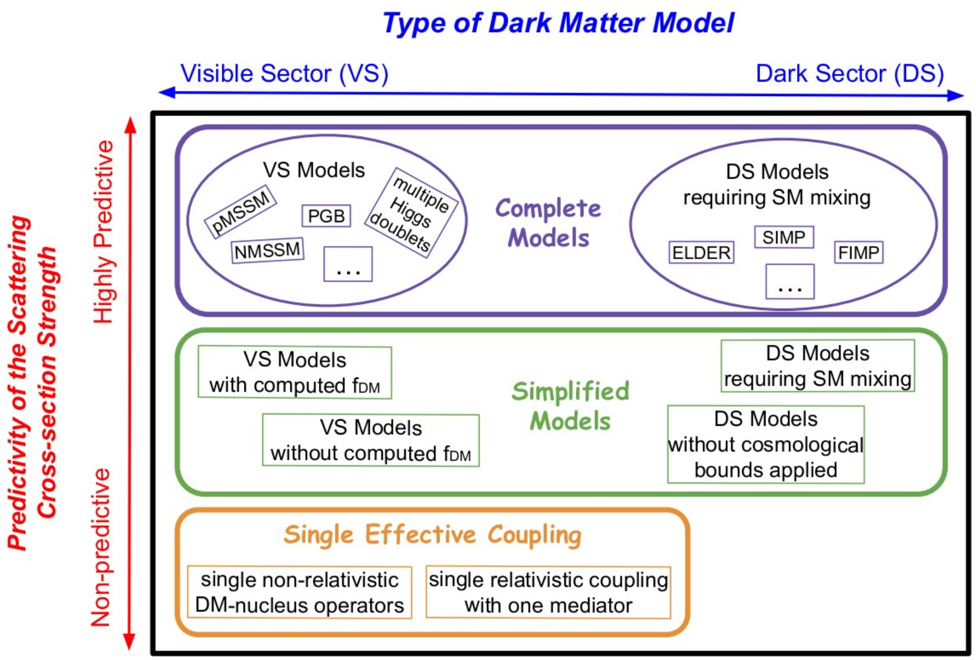
Gelmini, Takhistov and Witte 1804.01638

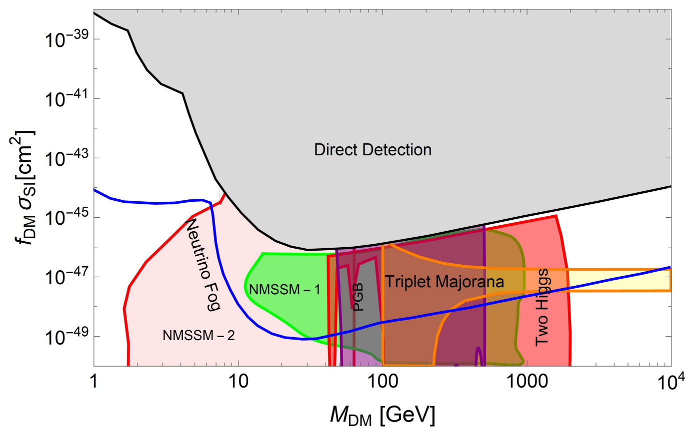


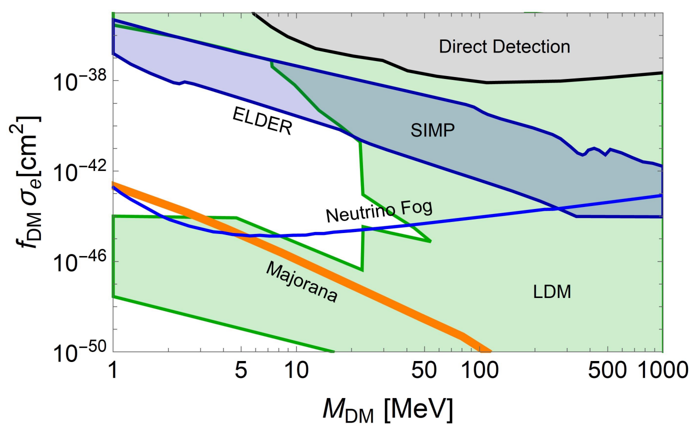
Complementarity of different targets is essential to determine the character of the interaction once a signal is detected!

Some particle models to test beyond the neutrino fog

Akerib et al Snowmass 2021, 2203.08084 [hep-ex]







Elements of the Event Rate in Direct DM detection

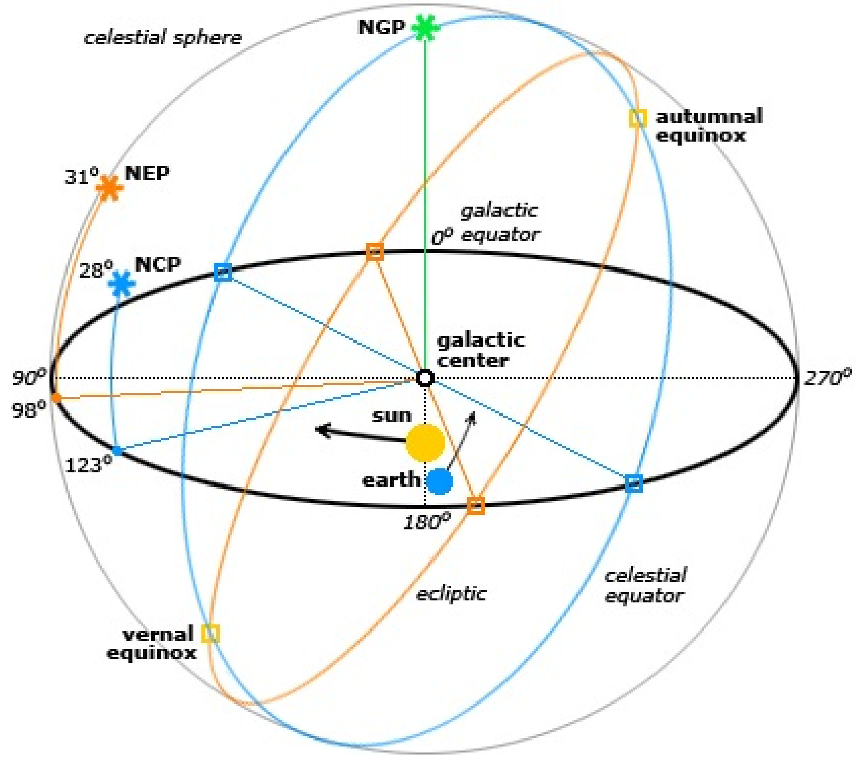
[Event Rate] = [Detector Response] x [Cross Section] x [Halo Model]

How many DM particles are passing through the detector and with which velocity distribution?

The usually assumed Standard Halo Model is a good first approximation although not expected to be correct. The SHM implies an annual modulation of the rate.

Annual Modulation of the Rate

$|\vec{v}_{\odot} + \vec{v}_{\oplus}|$ is maximum at the beginning of June (with the 2021 conventions on June 2). $\vec{v}_{\odot}$ and $\vec{v}_{\oplus}$ are at 60^o, so $\simeq v_{\oplus} \cos 60^o$ sums or subtructs from $v_{\odot}$.



Annual Modulation of the Rate in the SHM

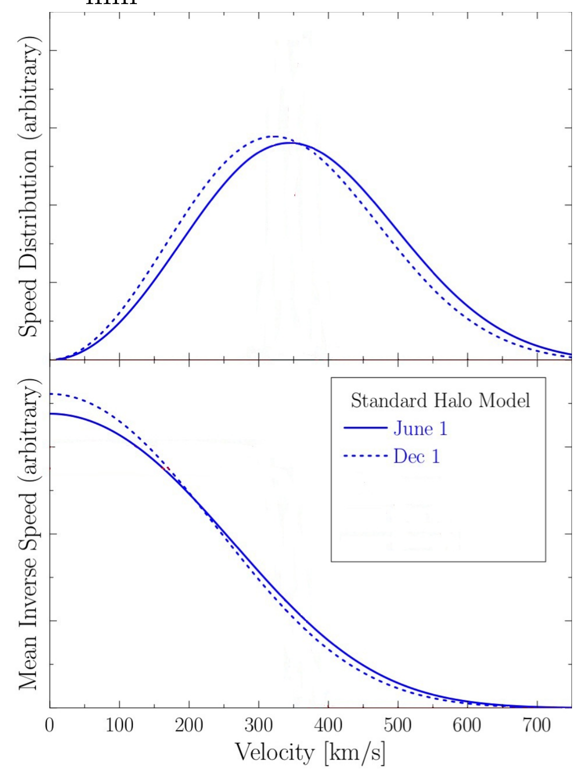
Schematic speed distribution $F(v,t)$ and integral $\eta(v)$ with arbitrary normalization, where

$$\eta(v_{\rm min},t) = \int_{v>v_{\rm min}} \frac{f(\vec{v},t)}{v} d^3v = \int_{v>v_{\rm min}} \frac{F(v,t)}{v} dv \qquad (^*)$$

Notice that the maximum of η changes from June 1 to Dec. 1 at $v_{\rm min} < 200$km/s. Annual modulation is due only to $\vec{v}_\oplus$, so $\eta(v_{\rm min},t) \simeq \eta(v_{\rm min}, v_\oplus = 0) + \vec{v}_\oplus \cdot \frac{\partial \eta}{\partial \vec{v}_\odot}\Big|_{v_\oplus = 0}$

Thus the annual modulation amplitude is linear in Earth's orbital speed $v_\oplus$

(fig. from Freese, Lisanti & Savage 1209.3339)



Annual Modulation of the Rate in the SHM

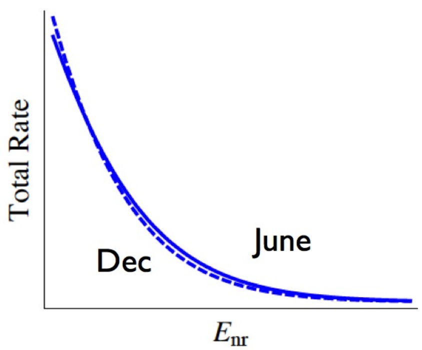
The rate can be well approximated by the 1st term of a harmonic expansion

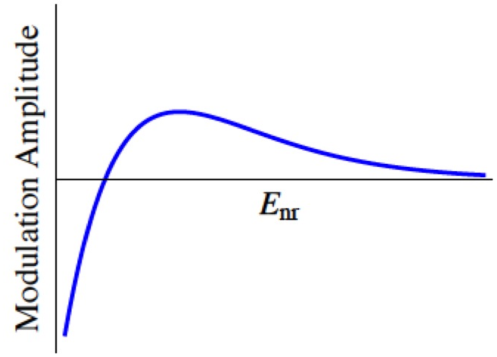
$$\frac{dR}{dE_r}(E_R, t) \simeq S_0(E_R) + S_m(E_R)\cos[\omega(t - t_0)]$$

t_0 is the phase, $\omega = 2\pi/$ year. Written in this way, the annual modulation amplitude

$$S_m(E_R) = \frac{1}{2}\left[\frac{dR(June1)}{dE_R} - \frac{dR(Dec1)}{dE_R}\right]$$

changes sign at low recoil energies. $S_m/S_0 \simeq$ 1- 10% for most $v_{\rm min}$ values ($\sim v_\oplus/v_\odot \simeq$ 10%)

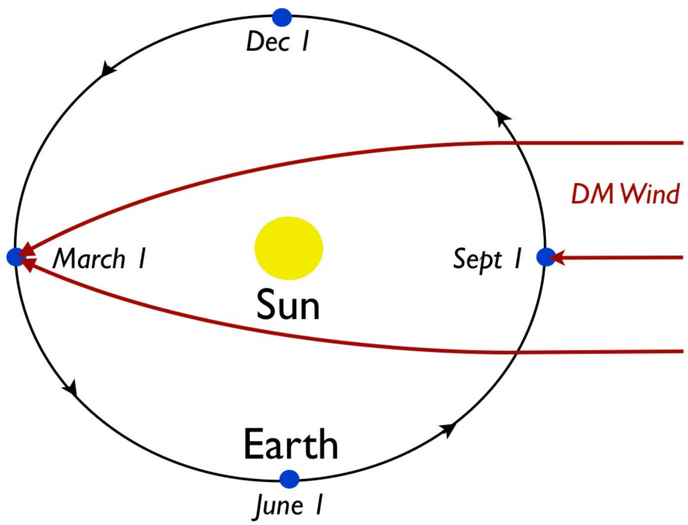


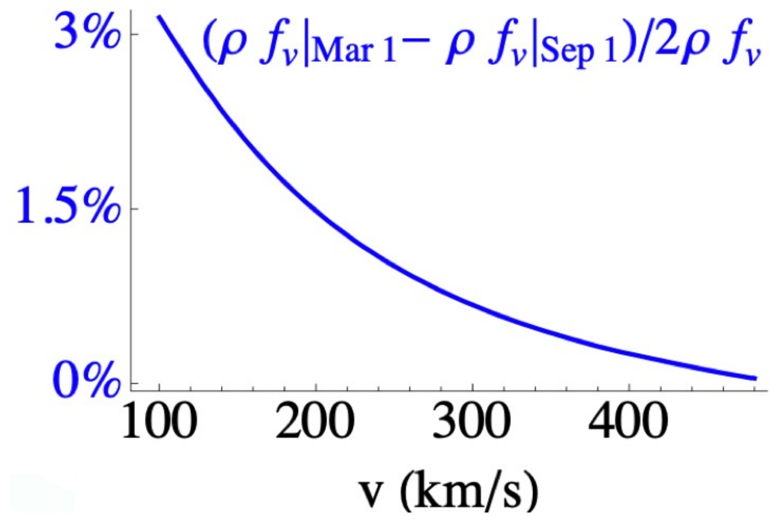


Recall $E_R = 2\mu v_{\rm min}^2/M$, so sign change is at E_R corresponding to $v_{\rm min} = 200$km/s

Gravitational Focussing by the Sun affects the Annual Modulation for low $v_{\rm min}$

Lee, Lisanti, Peter & Safdi 1308.1953 for the GF effect, see e.g. Alenazi & Gondolo in 2006, 0608390





Lee, Lisanti, Peter & Safdi found that for v_{min} <200 km/s the maximum of $\eta(v_{min}, t)$ shifts to 21 days later

Target dependence of the Annual Modulation

The annual modulation could be different in different experiments even as function of $v_{\rm min}$, if the speed dependence of the cross section does not factorize.

In some cross sections the v dependence does not factorize, e.g. Magnetic Dipole DM

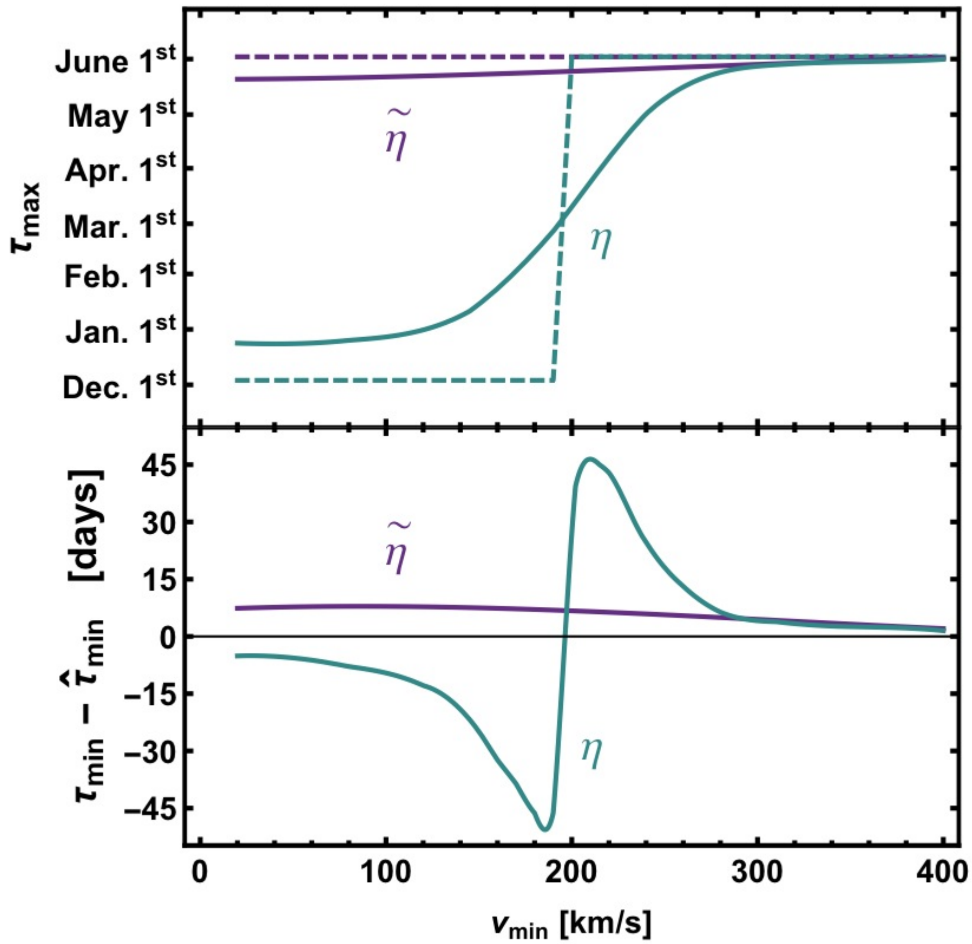
$$\frac{d\sigma_T}{dE_R} = \frac{\alpha d_m^2}{v^2}\left\{Z_T^2 \frac{M}{2\mu_T^2}\left[\frac{v^2}{v_{\rm min}^2} - \left(1 - \frac{\mu_T^2}{m^2}\right)\right] F_{{\rm SI},T}^2(E_R) + \frac{d_{mT}^2}{\mu_N^2}\frac{M}{m_p^2}\left(\frac{S_T+1}{3S_T}\right) F_{{\rm M},T}^2(E_R)\right\}$$

one term $\sim v^{-2}$ another $\sim v^0$ yield two different functions of $v_{\rm min}$, with different detector dependent coefficients

$$\eta(v_{\rm min}, t) \equiv \int_{v \geq v_{\rm min}} \frac{f(\vec{v}, t)}{v} d^3v, \qquad \tilde{\eta}(v_{\rm min}, t) \equiv \int_{v \geq v_{\rm min}} v f(\vec{v}, t) d^3v$$

In the rate, the combination of these is target dependent: $dR_T/dE_R = C_1^T \eta(v_{min}, t) + C_2^T \tilde{\eta}(v_{min}, t)$ with C_1^T, C_2^T target nucleus T dependent coefficients.Thus, the annual modulation is target material dependent!

Times of max and (min − 1/2 y from max) of η and $\tilde{\eta}$



Del Nobile, Gelmini, Witte 1504.06772

τ_{max}: time of η or $\tilde{\eta}$ maximum

$\hat{\tau}_{min}$: 1/2 year apart from τ_{max}

τ_{min}: time of η or $\tilde{\eta}$ minimum

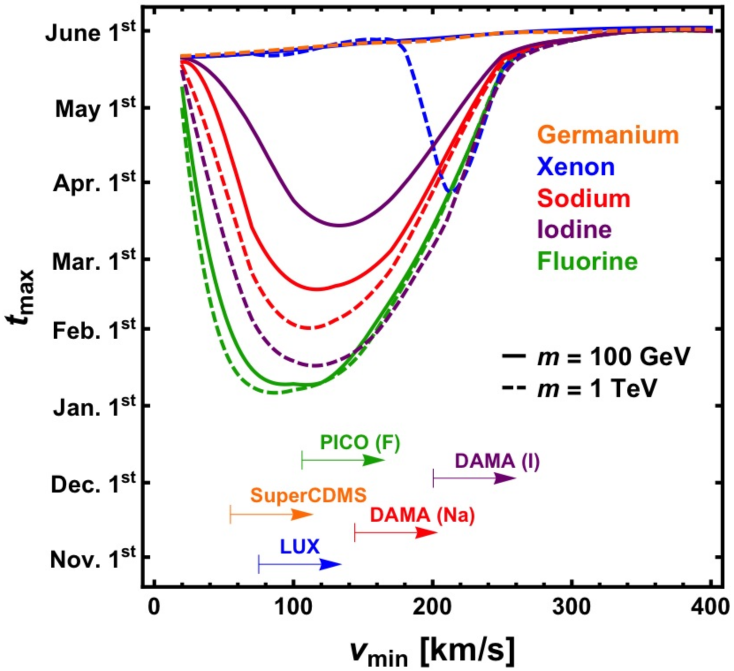
including (solid lines) and neglecting (dashed lines) GF

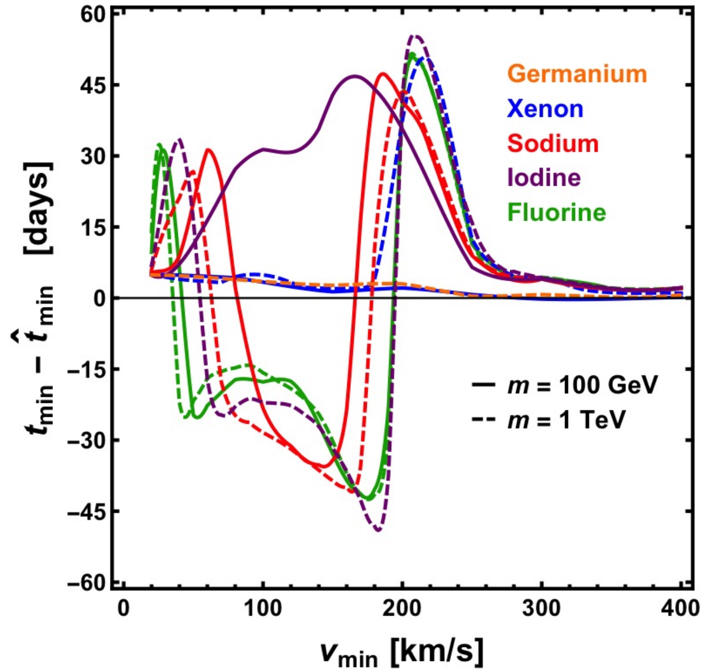
$$\eta(v_{\rm min}, t) \equiv \int_{v \geq v_{\rm min}} \frac{f(\vec{v}, t)}{v} d^3 v,$$

$$\tilde{\eta}(v_{\rm min}, t) \equiv \int_{v \geq v_{\rm min}} v f(\vec{v}, t) d^3 v$$

Times of max and (min − 1/2 y from max) of the rate

Del Nobile, Gelmini, Witte 1504.06772 and 1512.03961





Solid (dashed) lines for $m = 100$ GeV (1 TeV) for magnetic dipole DM scattering elastically

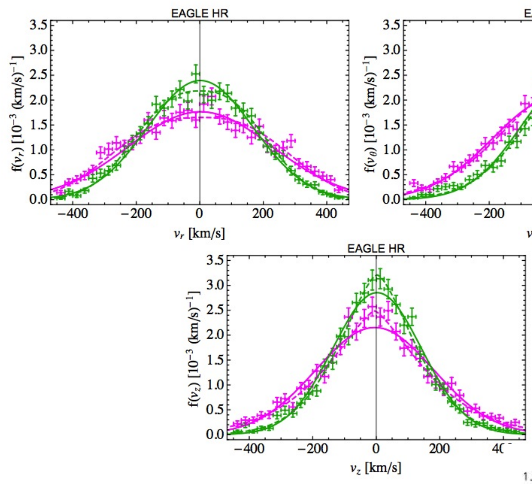
E.g. $t_{\rm max}$ in Xe and F could differ by 4 months

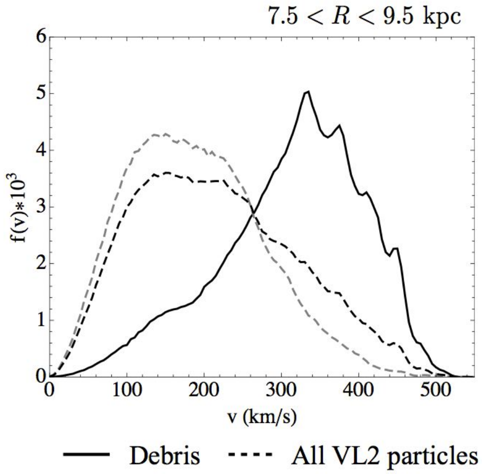
and modulation in Xe better described by a sinusoidal t-dependence than in F

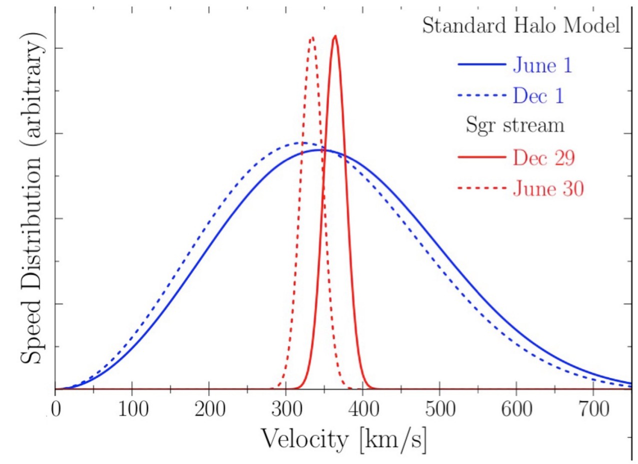
Deviations from the SHM

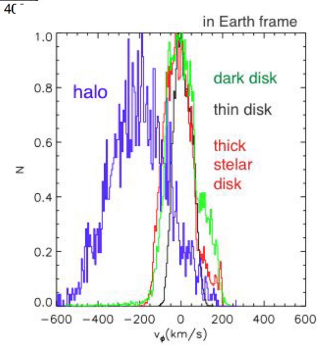
Triaxiality, debris flows, streams, dark disk ? ....these are ∼ 10 y old plots

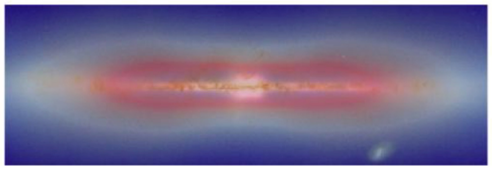
Distributions of radial, azimuthal, and vertical velocity components:









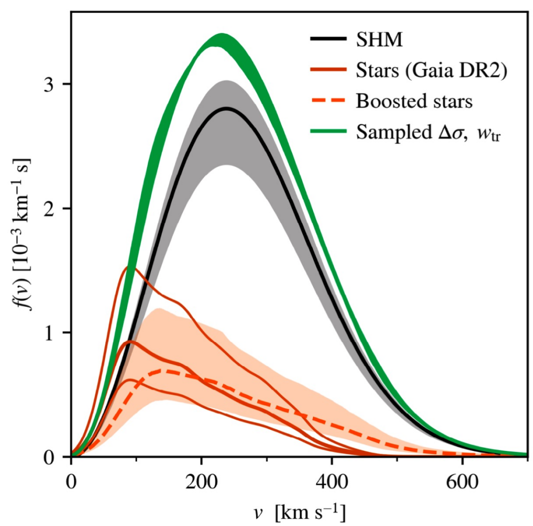


Simulations find SHM is a good approximation

Shpigel, Folsom, Lisanti, Necib, Vogelsberger,Hernquist 2510.21914

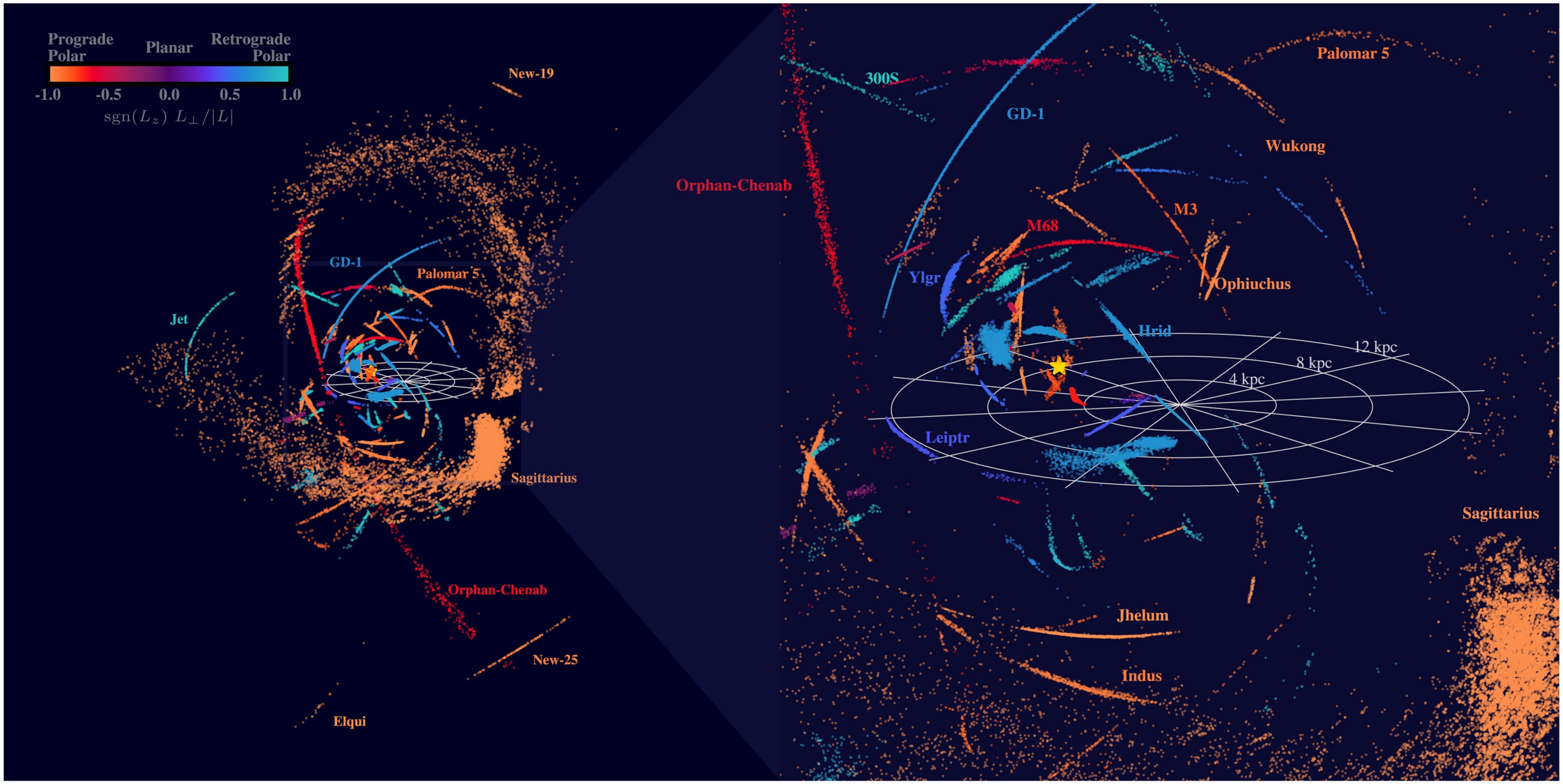
Using 98 Milky way analogues in IllustrisTNG50 simulations tuned to latest data

Gaia found a merger 8-10$10^9$ y ago: Gaia-Enceladus contributing 10 -30% of DM in the solar neighborhood (in orange) Full reconstructions (solid green), which lie 6^{+5}_{-3} km/s from a SHM-only model.

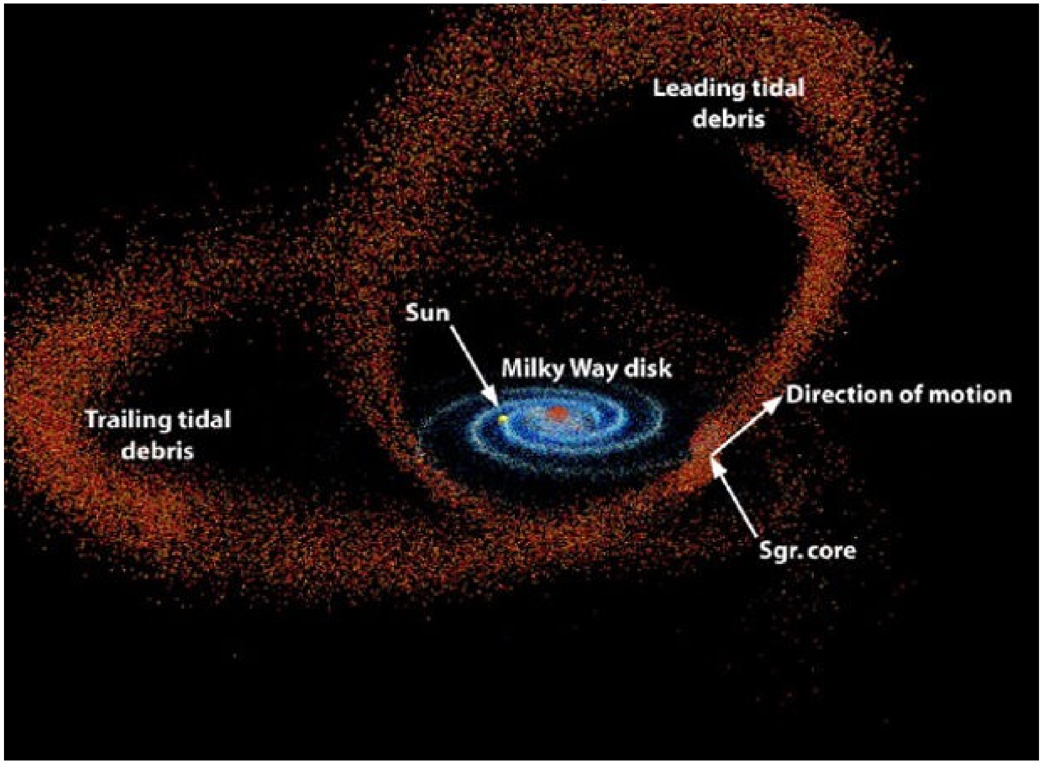


Caveats:Analysis may not resolve local velocity structures: (1) speed distribution is an azimuthal average of particles within an annulus around the galactic center, (2) the simulations have 288pc softening length. Additionally gravitational effect of the Sun is not included Folson et al 2505.07924

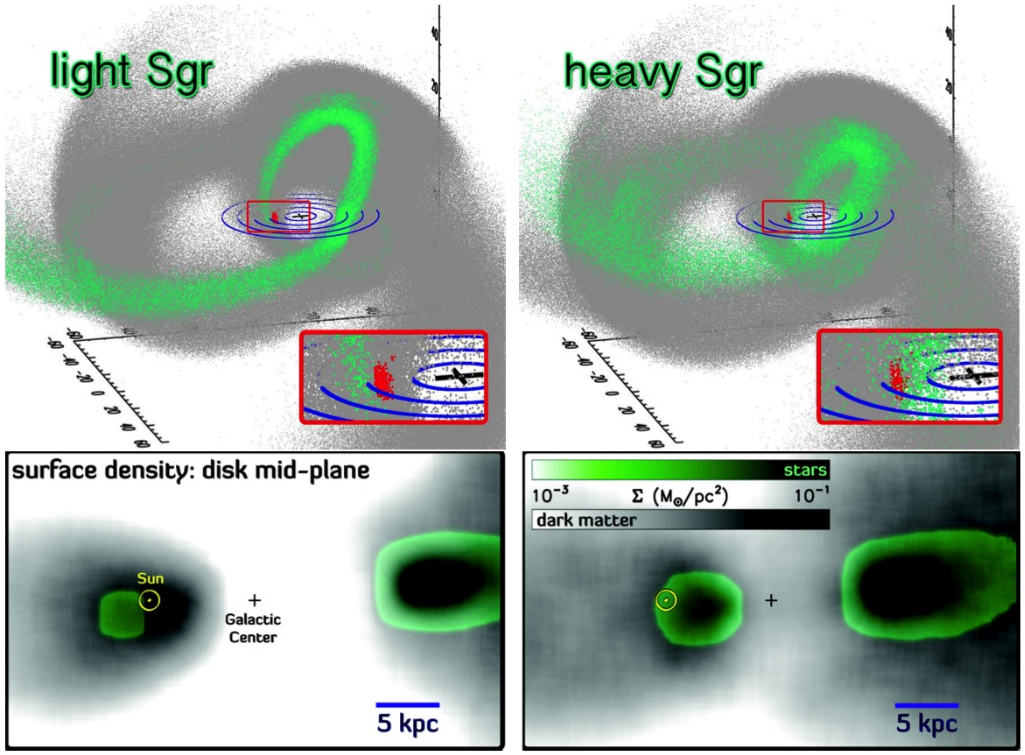
GAIA has observed many stellar streams WBonaca and Price-Whelan 2405.19410



Sagittarius stream is the most prominent

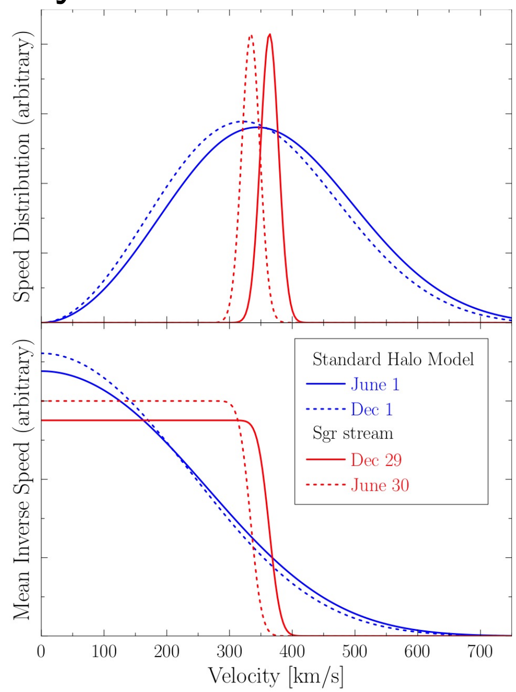


Sgr. leading trail DM may be passing through the Solar System



Large uncertainties in local DM stream density, $\rho_{Sgr} < 5\%\ \rho_{SHM}$ and velocity $v \simeq 250 - 400$ km/s w.r.t the Sun Purcell, Zentner, Wang 1203.6617

SHM + DM in the Sgr. leading trail Schematic speed distribution and integral $\eta(v)$ with arbitrary normalization Freese, Lisanti & Savage 1209.3339

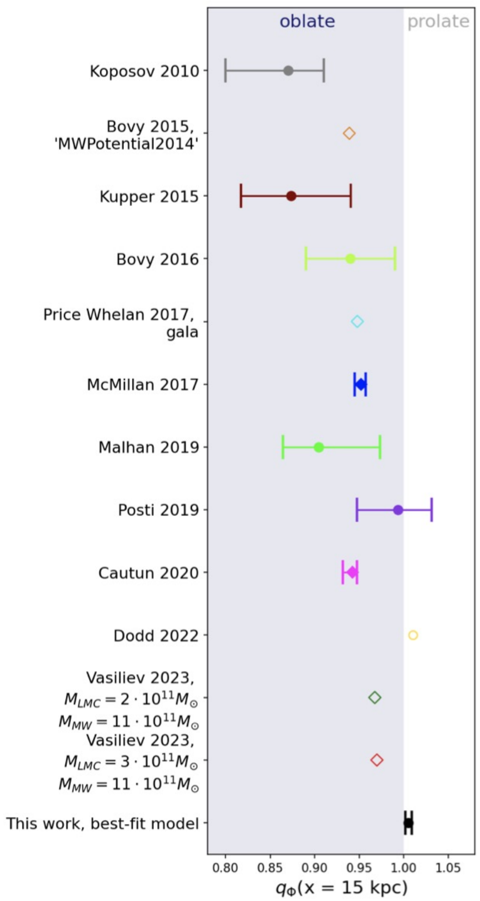


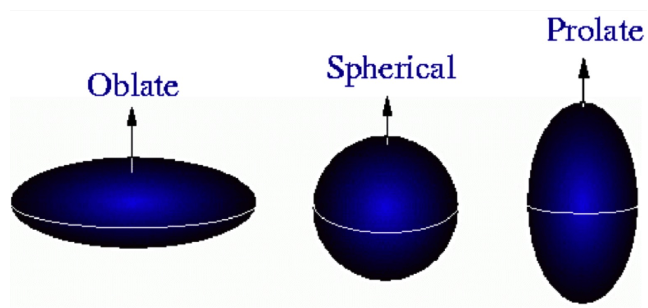
For $m_\chi < 20$ GeV, Sgr DM stream could enhance Direct DM detection rate by 20% to 45%, reduce the annual modulation amplitude by as much as 50% and change its phase by 20 days (but large uncertainties) Purcell, Zentner, Wang 1203.6617

Graciela Gelmini-UCLA

Very mild triaxiality in the inner 20kpc of the Milky Way dark halo

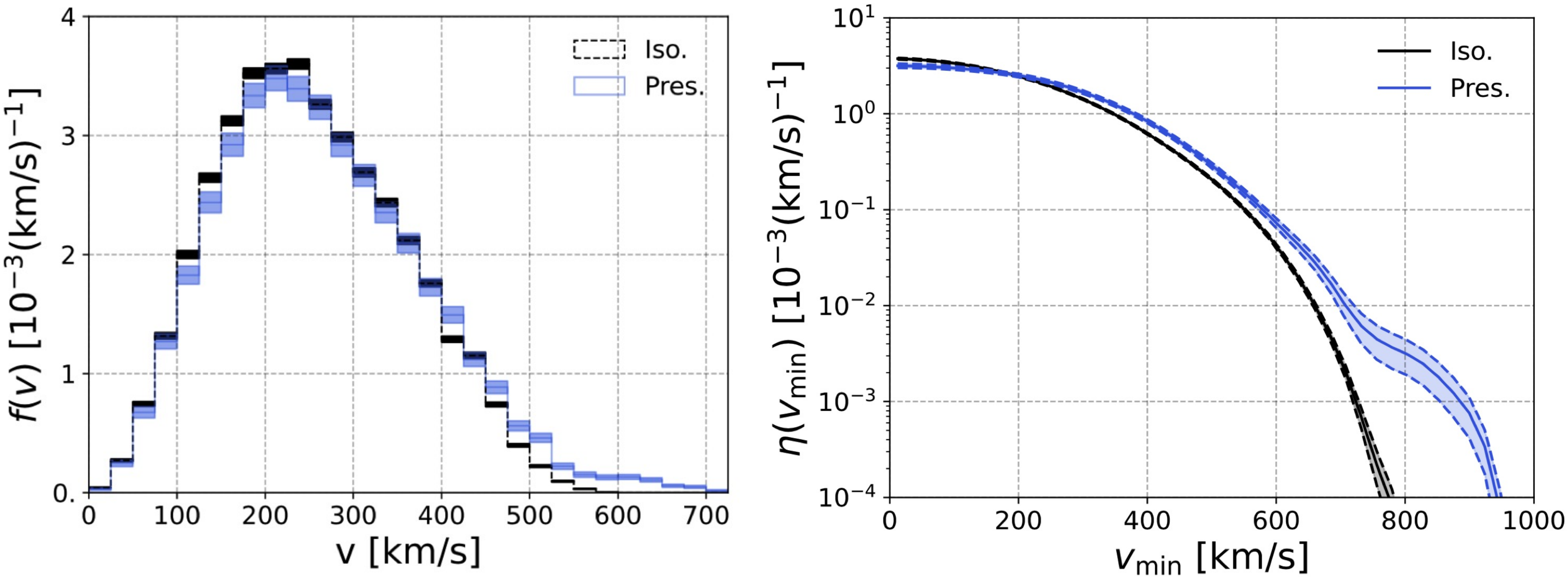
using stellar stream Gaia DR3 data Woudenberg and Helmi 2407.21790





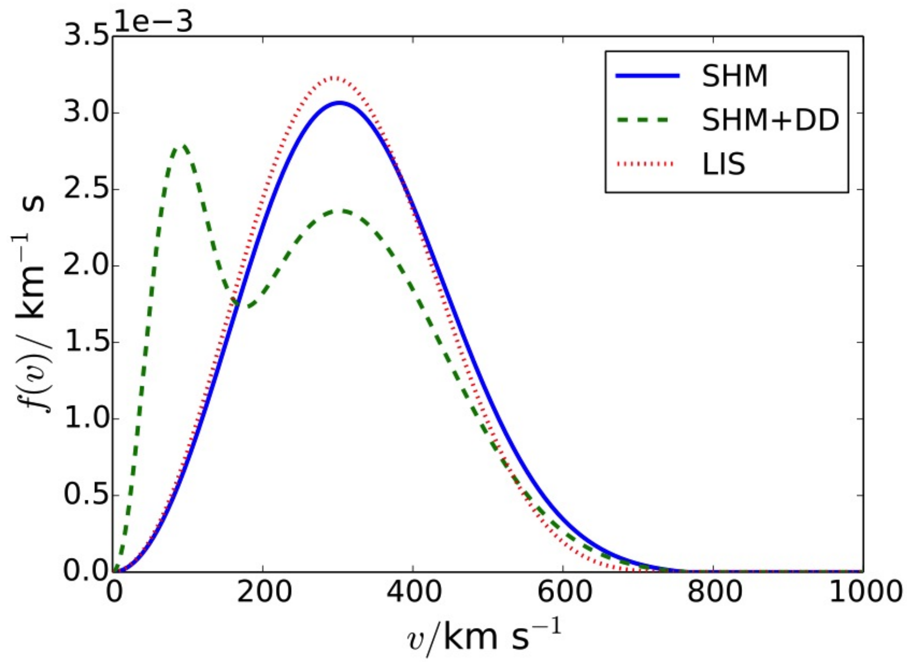
Effect of the Large Magellanic Cloud

LMC is coming into the MW (for the 1st time) Enhances high speed DM, by contributing its own and accelerating some originating in the MW. Reynoso-Cordova, Bozorgnia, Piro 2409.09119



A Dark Disk would enhanced population at very low speeds

Rare feature shown in simulations with simple CDM Read, Lake, Agertz, Debattista MNRA 389, 8/2008;Read, Mayer, Brooks, Governato, Lake 0902.0009, pervasive if part of the DM is dissipative, as in DDDM)Fan, Katz, Randall & Reece 1303.1521-1303.3271 (Fig from Peter, Gluscevic, Green, Kavanah & Lee 1310.7039)

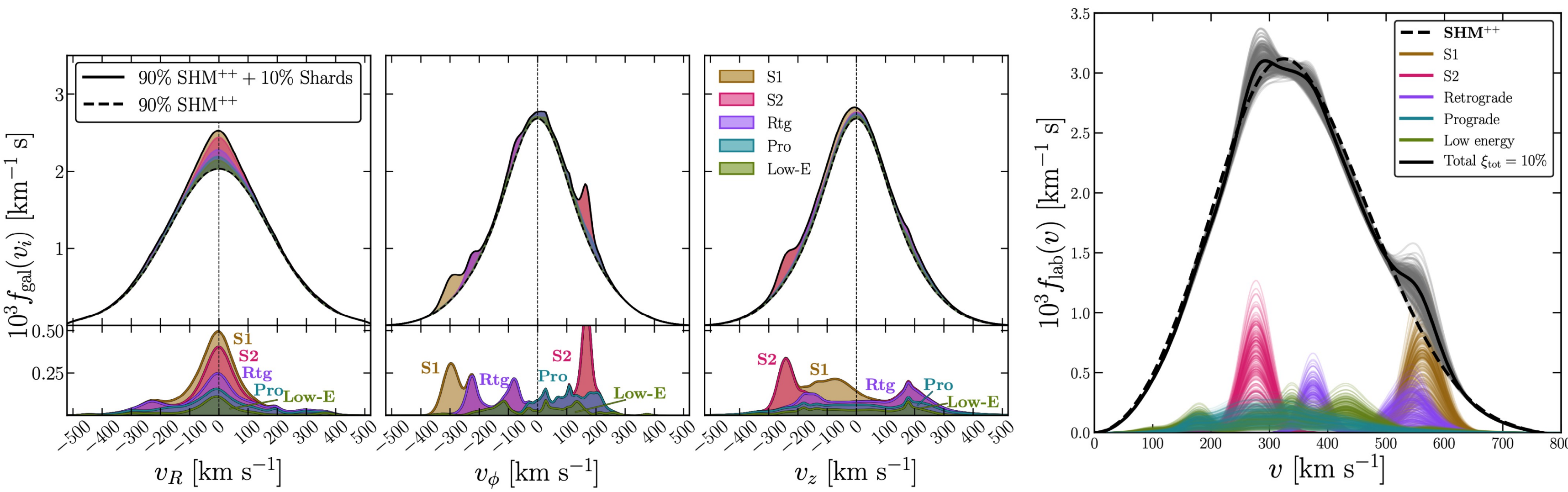


GAIA data: dissipative DM <1% of DM (for DD thickness <20 pc)

Schutz, Lin, Safdi, Wu 1711.03103

"Dark Shards" Here three components: smooth (SHM) + Gaia-Enceladus=SHM^{++}, and "shards" remaining from destroyed subtructure, modeled as many Gaussians; with uncertain density O'Hare, Evans, McCabe, Myeong, Belokurov 1909.04684, SHM^{++}: Evans, O'Hare, McCabe, 1810.11468,

$$f_{\mathrm{SHM}^{++}} = f_{\mathrm{SHM}} + \frac{1}{(2\pi)^{3/2}\sigma_r\sigma_\theta^2 N_{\mathrm{S,esc}}} \exp\left(-\frac{v_r^2}{2\sigma_r^2} - \frac{v_\theta^2}{2\sigma_\theta^2} - \frac{v_\phi^2}{2\sigma_\phi^2}\right) \times \Theta(v_{\mathrm{esc}} - |\vec{v}|)$$



Effects depend crucially on "Shards" density fraction, here assumed10%

“Halo-Independent” DD scattering data analysis

complementary to the usual “Halo Dependent”

Recall Event rate: events/(unit mass of detector)(keV of recoil energy)day

$$\frac{dR}{dE_R} = \sum_T \int_{v>v_{\min}} N_T \times \frac{d\sigma_T}{dE_R} \times n v f(\vec{v}, t) d^3 v$$

-E_R: nuclear recoil energy

- T: each target nuclide (elements and isotopes)

- $N_T = C_T/M_T$= Number of nuclides T in the detector = (mass fraction $\times$ Number of nuclides T per unit target mass);

- $v_{\min}$ min WIMP speed to impart E_R to the target T, $v_{\min}(E_R) = \frac{1}{\sqrt{2M_T E_R}} \left| \frac{M_T E_R}{\mu_T} + \delta \right|$

$\mu_T = m M_T/(m + M_T)$, reduced mass; $\delta = m' - m$ for DM inelastic scattering

-$\rho = nm$, $f(\vec{v}, t)$: local DM density and $\vec{v}$ distribution depend on halo model.

"Halo-Independent": Recall the event rate:

For a WIMP-nucleus contact differential cross section (for momentum transfer and velocity-independent interaction operators) e.g. for Spin Independent interactions

$$\frac{d\sigma_T}{dE_R} = \frac{\sigma_T(E_R)\ M_T}{2\mu_T^2 v^2} \qquad \sigma_T(E_R) \sim \sigma_{\rm ref}$$

$$\frac{dR}{dE_R} = \sum_T \frac{\sigma_T(E_R)}{2m\mu_T^2}\ \rho\eta(v_{\rm min-T}, t),\ \ \eta(v_{\rm min}, t) = \int_{v>v_{\rm min}} \frac{f(\vec{v}, t)}{v} d^3v = \int_{v_{\rm min}} \frac{F(v, t)}{v} dv$$

- ρ, $f(\vec{v}, t)$: local DM density, Earth's frame $\vec{v}$ distribution depend on halo model (Notice: given E_R, $v_{\rm min}$ depends on the target mass: $v_{\rm min-T}$)

"Halo-Dependent": Given $\rho\eta(v_{\rm min})$ plots in $(m, \sigma_{\rm ref})$ plane (usual)

"Halo-Independent": Given m, $d\sigma_T/dE_R$ plots in $(v_{\rm min}, \tilde{\eta}(v_{\rm min}))$ plane,

$$\tilde{\eta}(v_{\rm min}, t) = \rho\eta(v_{\rm min}, t)$$

contains all halo dependence in ANY experiment!

Fox, Liu, Weiner 1011.1915; Frandsen et al 1111.0292; Gondolo-Gelmini 1202.6359...

With several nuclides (elements or isotopes) in the detector, there is no unique relation between $v_{\min}$ and E_R: need to chose one of them as independent variable. E_R is not directly accessible to experiments, they observe only a proxy E'.

Observed event rate:

$$\frac{dR}{dE'} = \epsilon(E') \int_0^\infty dE_R \sum_T G_T(E_R, E') \frac{dR_T}{dE_R}$$

- E': detected energy (in keVee or number of PE), C_T: mass fraction in target nuclide T;
- $\epsilon(E')$: counting efficiency or cut acceptance; $G_T(E_R, E')$: energy response function

$$\frac{dR_T}{dE_R} = \frac{C_T}{M_T} \int_{v>v_{\min}} \frac{d\sigma_T}{dE_R} \times \frac{\rho}{m} v f(\vec{v}, t) d^3v$$

Early versions of the HI method used the recoil spectrum dR_T/dE_R, a unique relation E_R-$v_{\min}$ and SI interactions. Fox, Liu, Weiner 1011.1915; Frandsen et al 1111.0292

HI for ANY type of interaction, form factors, energy resolutions, efficiencies...

Gondolo-Gelmini 1202.6359; Del Nobile, Gelmini, Gondolo and Huh, 1306.5273

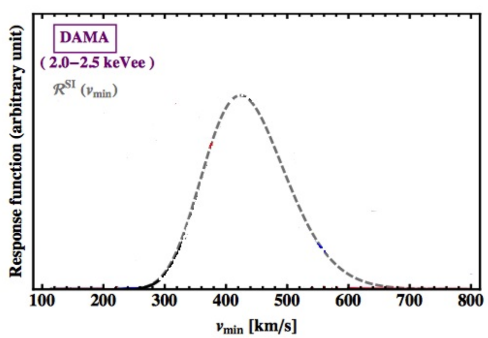
We choose $v_{\rm min}$ as independent variable, change the order of integration and write the predicted observable rate for any cross section as

$$\frac{dR}{dE'} = \int_0^\infty dv_{\rm min}\, \frac{d\mathcal{R}}{dE'}(v_{\rm min})\, \tilde{\eta}(v_{\rm min}, t)$$

or for integrated rates:

$$R_{[E'_1,E'_2]} = \int_0^\infty dv_{\rm min}\, \mathcal{R}_{[E'_1,E'_2]}(v_{\rm min})\, \tilde{\eta}(v_{\rm min}, t)$$

$\mathcal{R}_{[E'_1,E'_2]}$: **experiment and interaction dependent response function**
(non zero only for an interval in $v_{\rm min}$ given a measured energy interval $[E'_1, E'_2]$)
Every experiment is sensitive to a "window" in v



"Halo Independent" data analysis

1- **Find the predictions of Direct Detection data for the halo**, e.g. for the coefficients of the harmonic expansion of $\tilde{\eta}(v_{\rm min}, t)$ (mostly its time average).

2- **Compare data from different experiments by comparing their predictions for the halo**, e.g. for the time average of $\tilde{\eta}(v_{\rm min})$ of $\tilde{\eta}(v_{\rm min}, t)$:
– putative measurements translate into regions in the $(v_{\rm min}, \tilde{\eta}(v_{\rm min}))$ plane,
– upper limits into upper limits on $\tilde{\eta}(v_{\rm min})$

Main Problem: Likelihood methods are good for parameter estimation, **but here we want to estimate a function,** $\tilde{\eta}$ or the local WIMP speed distribution F which the predicted rates depend on

$$R_{[E'_1,E'_2]} = \int_0^\infty dv_{\rm min}\, \mathcal{R}_{[E'_1,E'_2]}(v_{\rm min})\, \tilde{\eta}(v_{\rm min}) = \int_0^\infty {\rm d}v\, \mathcal{H}_{[E'_1,E'_2]}(v)\, F(v,t)$$

2014-2015 Solved the problem only for unbinned data (Extended Likelihood)

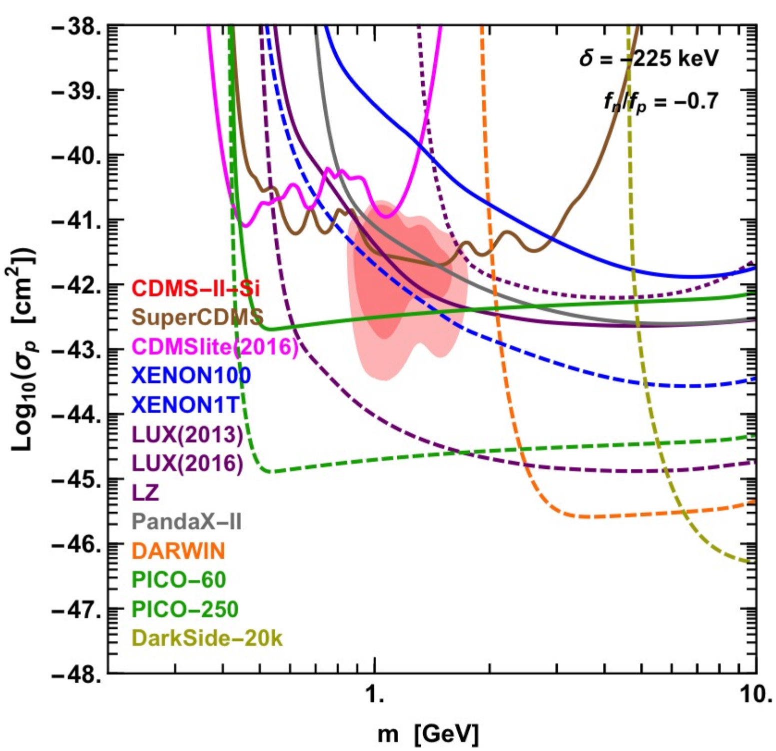
Halo-Independent analysis

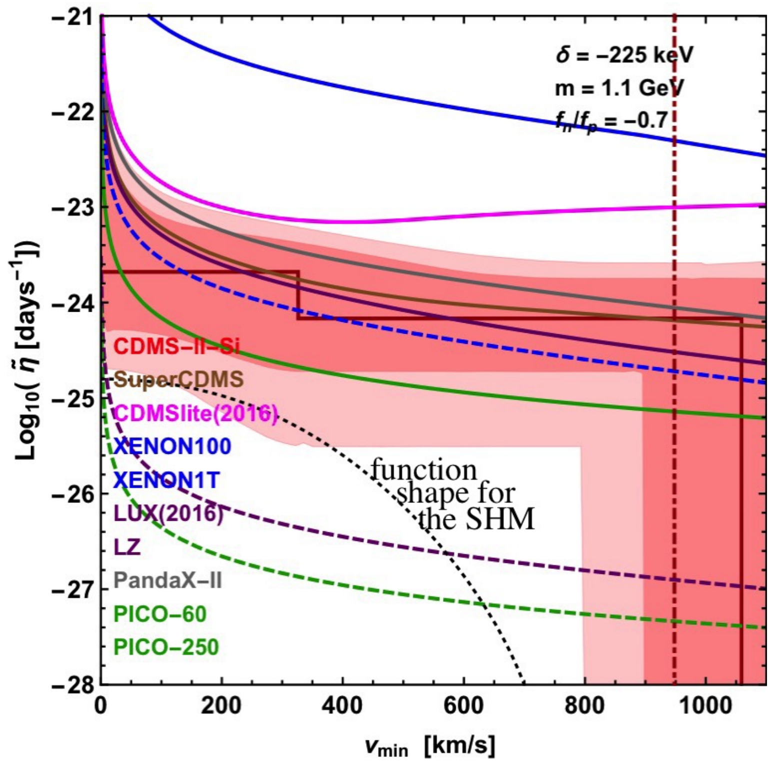
Regions for putative DM (time averaged) rate measurements:
With unbinned data (e.g. CDMS-II-Si), using at least one extended likelihood, we found (Fox, Kahn and McCullough 1403.6830; Gelmini, Georgescu, Gondolo and Huh 1507.03902; Gelmini, Huh and Witte 1607.02445)

- a unique piecewise constant best fit $\tilde{\eta}(v_{\min})$ with a number of downward steps $\leq$ number of data points, by extending to functionals the Karush-Kuhn-Tucker (KKT) maximization conditions (Fox, Kahn and McCullough 1403.6830), and a

- statistically meaningful two-sided point-wise band at a chosen CL. (Gelmini, Georgescu, Gondolo and Huh, 1507.03902)

Halo-Dependent and Independent analyses CDMS-II-Si data inelastic exothermic DM with SI IV coupling, $\delta = -225$ keV Witte, Gelmini 1703.06892





LEFT: assuming the SHM RIGHT: Halo independent, m=1.1GeV

We found piecewise constant best-fits, with a small number of downward steps

A deeper understanding of Halo-Independent methods for all Likelihoods Gelmini, Huh and Witte 1707.07019

Why a piecewise constant best fit $\tilde{\eta}(v_{\rm min})$ with the number of downward steps $\leq$ the number of data points???

Well known theorems in convex geometry (Caratheodory, Fenchel-Eggleston) provide the answer: for d (time average) predicted rates the DM speed distribution $F(v)$, normalized to 1, is given by

$$F(v) = \sum_{n=1}^{d} F_n \, \delta(v - v_n)$$

Now we have at most 2d parameter F_n, v_n to estimate using the Likelihood

and the integral $\tilde{\eta}(v_{\rm min}) = {\rm const.} \int_{v_{min}}^{\infty} {\rm d}v \, \frac{F(v)}{v}$ of a sum of at most d delta functions is piecewise constant with at most d downward steps (d= number of data points) Lots of work yet to do to develop this method....

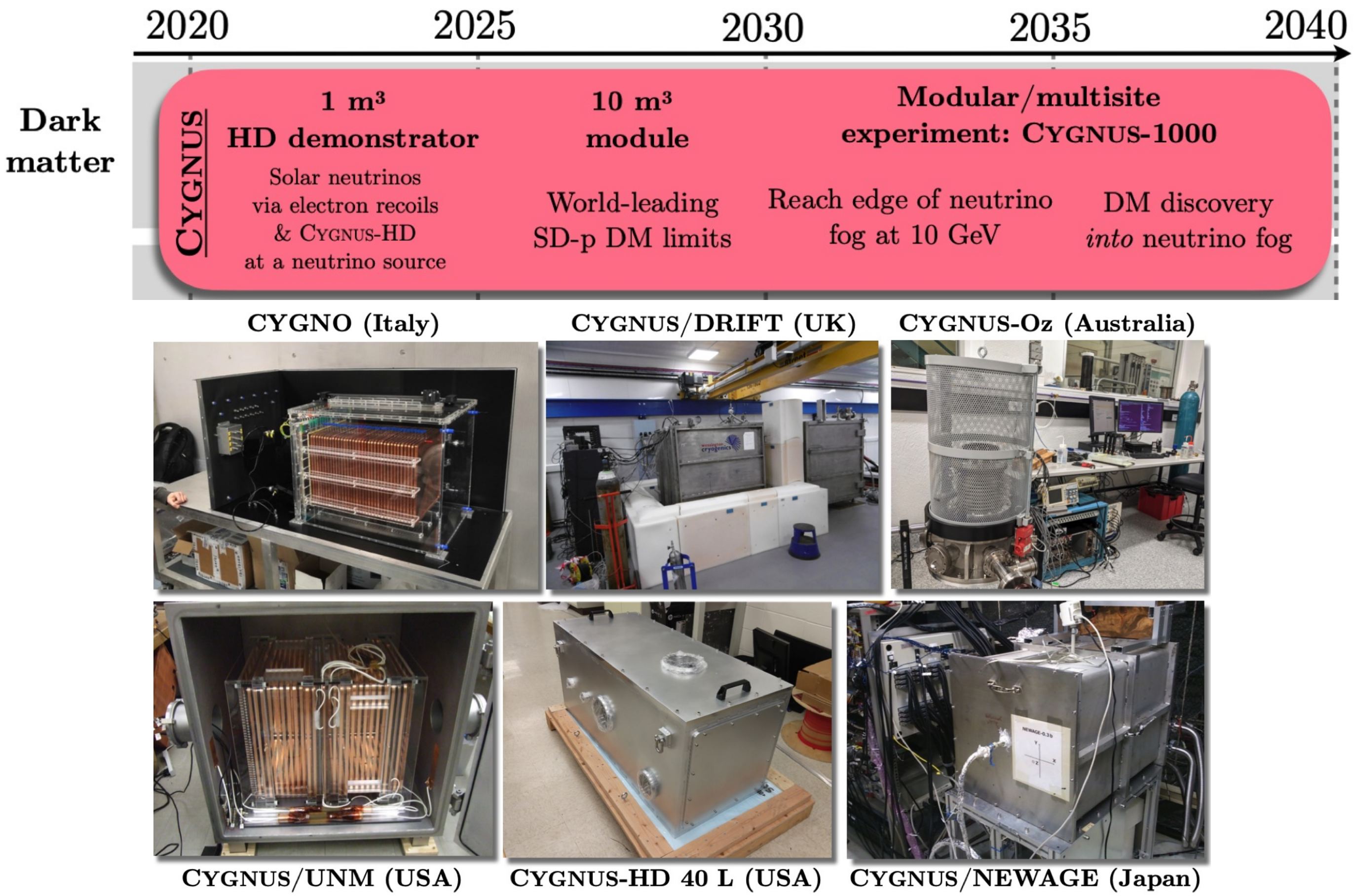
Present directional direct Dark Matter detectors

A recent review: O'Hare et al. Snowmass 2021 whitepaper 2203.05914

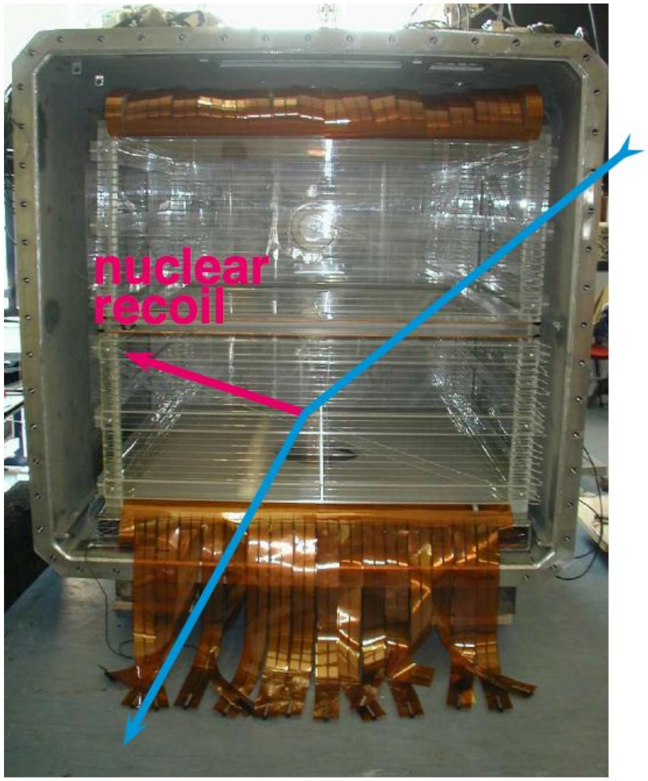
Directional detectors can measure both the energy and direction of the WIMP-induced recoils.

Name	Detector, [TPC readout]	Directionality	Status
NEWAGE	**Gas TPC**, GEM + μPIC, **NID**	3d	Running underground (Kamioka), scaling up to $1m^3$
DRIFT	**Gas TPC**, MWPC, **NID**	1.5d	Ran $1m^3$ underground (Boulby). MPGD R&D at Sheffield.
MIMAC	**Gas TPC**, Micromegas + Strips	3d	Ran underground (Modane), scaling up
DMTPC	**Gas TPC**, Optical readout	2d	Ran underground (WIPP), scaled up, stopped
D^3 / BEAST / CYGNUS HD	**Gas TPC**, 2xGEM + CMOS pixel, **NID**	3d	Prototypes evaluated, ran above-ground, scaling up
New Mexico readout R&D / CYGNUS HD	**Gas TPC**, Optical readout, **NID**	2d	Prototypes evaluated
CYGNO	**Gas TPC**, 3xGEM + CMOS optical + PMT	3d / 2d+1d	Prototypes evaluated, funded to scale up
CYGNUS-Oz	**Gas TPC**, Optical and electronic	?	Prototyping, then scale up
NEWSdm	Nuclear Emulsions	2d	Prototyping / going underground

Future: all unified into CYGNUS At present various prototypes



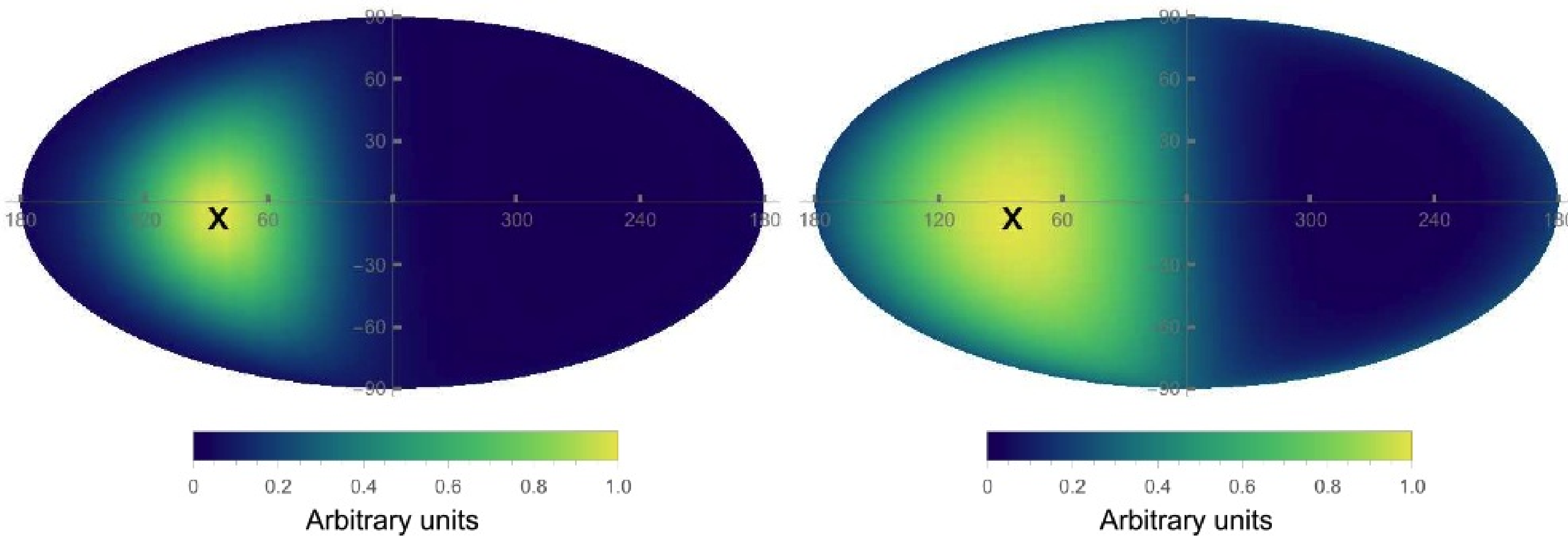
Directional DD detectors low density gas TPCs, e.g. DRIFT (CS_2) and DM-TPC (CF_4) Measure direction of recoil - tracks reconstructed through drift of e



DRIFT

Directional direct DM detectors, dipole feature

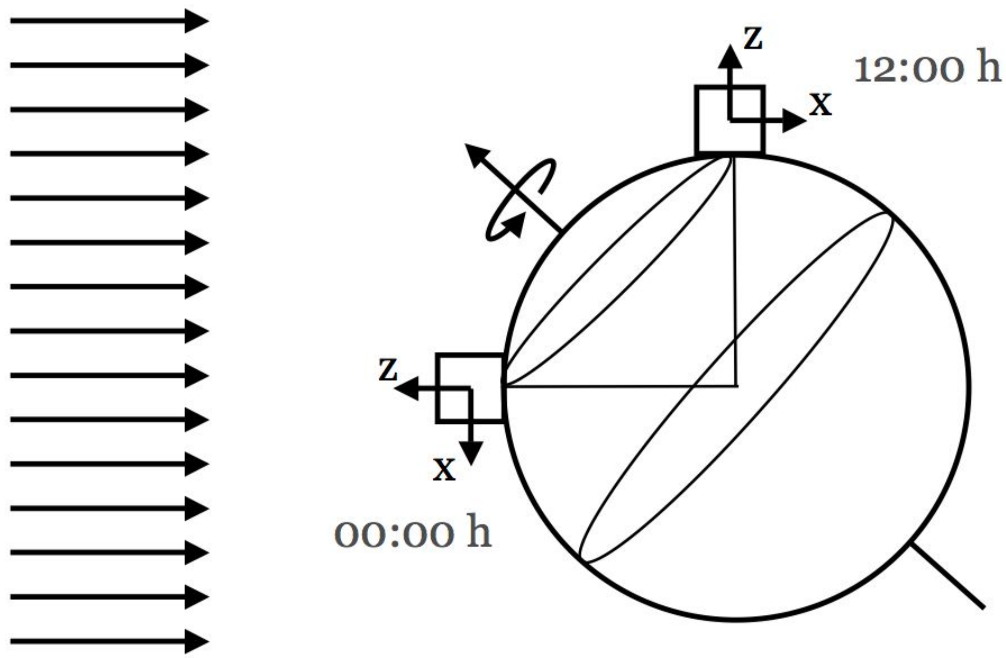
Left: Flux of 100 GeV WIMPs with $v > v_{\min}$ for E_R =25 keV F recoils arriving on Earth.
Right: Angular distribution of the energy differential recoil rate in F for WIMP m =100 GeV, E_R =25 keV. Maps are incoming direction of WIMP-induced recoils in Mollweide equal-area projections, in Galactic coordinates.



A few dozen events would be enough to detect the dipole feature. Unmistakeable DM signature: no known backgrounds can mimic this directional signature!

Directional DD, will see a daily modulation

Because of the Earth's rotation, the peak recoil direction in the lab frame varies over the course of a day: daily modulation



Directional recoil rate

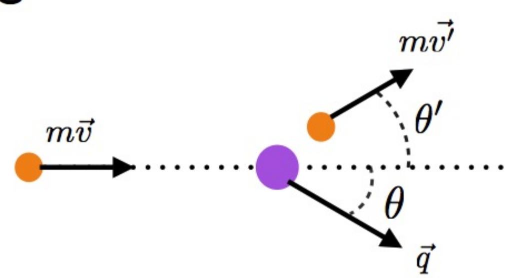
- The double differential event rate per unit detector mass as a function of both the recoil energy and direction is given by:

$$\frac{d^2R}{dEd\Omega} = \frac{\rho}{mM} \int_{v>v_m} d^3v \, \frac{d^2\sigma}{dEd\Omega} v \; f(\mathbf{v})$$

$d\Omega$: infinitesimal solid angle around the recoil direction.

- Azimuthal symmetry of the scattering around the WIMP arrival direction:

$$d\Omega = 2\pi \; d\cos\theta$$



Directional recoil rate

- Recall:

$$E = \frac{2\mu^2 v^2}{M} \cos^2\theta$$

which gives:

$$\cos\theta = \frac{v_m}{v}$$

- Impose this relation through a Dirac delta function:

$$\begin{aligned} \frac{d^2\sigma}{dE d\Omega} &= \frac{d\sigma}{dE} \frac{1}{2\pi} \, v \; \delta\left(v\cos\theta - v_m\right) \\ &= \frac{d\sigma}{dE} \frac{1}{2\pi} \, v \; \delta\left(\mathbf{v}.\hat{\mathbf{q}} - v_m\right) \end{aligned}$$

Directional recoil rate

- Remember the energy differential cross section for standard contact interactions:

$$\frac{d\sigma}{dE} = \frac{M}{2\mu^2 v^2}\ \sigma_0\ F^2(E)$$

- We have:

$$\frac{d^2R}{dE d\Omega} = \frac{\rho\ \sigma_0\ F^2(E)}{4\pi m \mu^2}\ \hat{f}(v_m, \hat{\mathbf{q}})$$

$\hat{f}(v_m, \hat{\mathbf{q}})$ is the three-dimensional *Radon transform* of the velocity distribution: **[Gondolo, hep-ph/0209110]**

$$\hat{f}(v_m, \hat{\mathbf{q}}) = \int d^3v\ \delta\left(\mathbf{v}.\hat{\mathbf{q}} - v_m\right) f(\mathbf{v})$$

Johan Radon, austrian mathematician, proposed Radon transform in 1917. Used in tomography where f is density and $\hat{f}$ scattering data output; etc

Directional recoil rate

- Using the Radon transform, one can evaluate the event rate analytically for most halo models.

- For the SHM with a truncated Maxwellian velocity distribution, the Radon transform in the lab frame is:

$$\hat{f}(v_m, \hat{\mathbf{q}}) \propto \left(\exp\left[-\frac{(v_m + \hat{\mathbf{q}}.\mathbf{v}_{\text{lab}})^2}{2\sigma_v^2} \right] - \exp\left[-\frac{v_{\text{esc}}^2}{2\sigma_v^2} \right] \right)$$

if $v_m + \hat{\mathbf{q}}.\mathbf{v}_{\text{lab}} < v_{\text{esc}}$, and zero otherwise.

$\mathbf{v}_{\text{lab}}$: velocity of the lab with respect to the Galaxy.

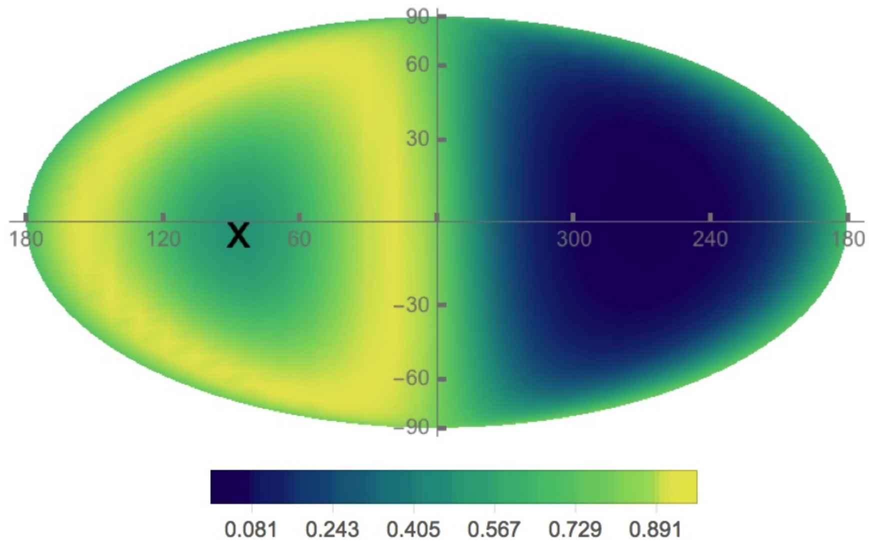
Directional recoil rate

$$\hat{f}(v_m, \hat{\mathbf{q}}) \propto \left(\exp\left[-\frac{(v_m + \hat{\mathbf{q}}.\mathbf{v}_{\rm lab})^2}{2\sigma_v^2} \right] - \exp\left[-\frac{v_{\rm esc}^2}{2\sigma_v^2} \right] \right)$$

- Two regimes of interest:
  - If $v_m > v_{\rm lab}$, argument of first exponential minimized when $\hat{\mathrm{q}}$ is in the opposite direction of $\mathbf{v}_{\rm lab}$. → *dipole feature*
  - If $v_m < v_{\rm lab}$, i.e. for low recoil energies and large WIMP masses, maximum of the Radon transform happens at an angle between $\hat{\mathrm{q}}$ and $\mathbf{v}_{\rm lab}$. → *ring-like feature* in the recoil map. [Bozorgnia, Gelmini, Gondolo, 1111.6361]

Directional recoil rate

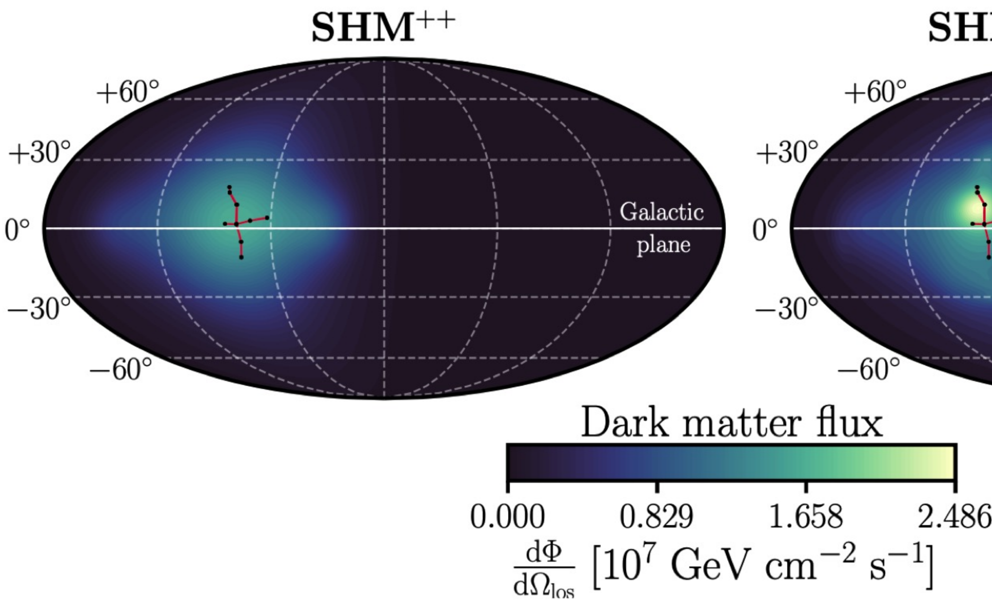
- Differential directional recoil rate in fluorine, assuming 5 keV recoil energy and 100 GeV WIMP.



Once this can be detected, we will be in the real of WIMP astronomy!

Directional recoil rate has sensitivity to streams and dark shards

O'Hare et al 1909.04684



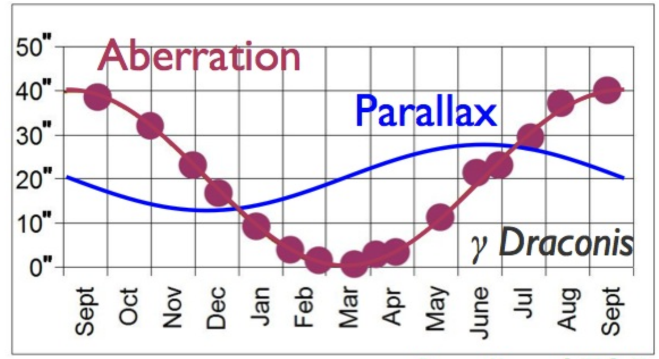
and particle models in some instances, e.g. for inelastic exothermic dark matter, the mean recoil direction as well as a secondary feature, a ring of maximum recoil rate around the mean recoil direction, could instead be opposite to the average DM arrival direction Bozorgnia, GG, Gondolo 1611.01750

In the distant future: WIMP astronomy

Fig. from Gondolo

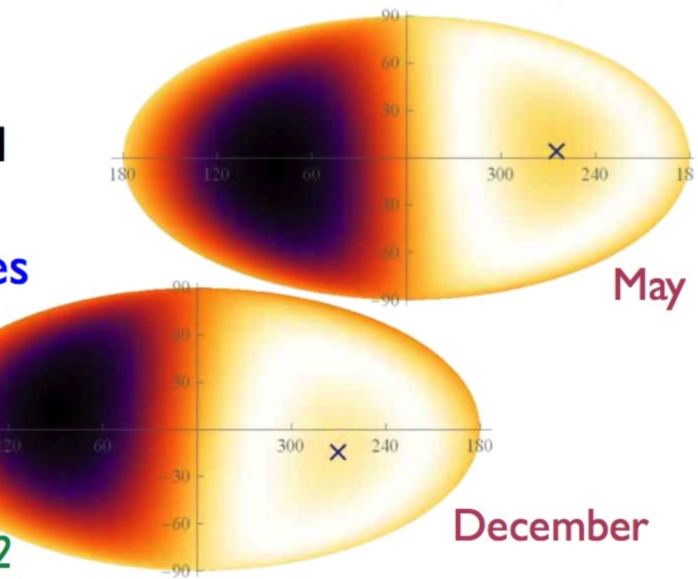
Aberration of WIMPs

Photon arrival direction
20 arcsec



Bradley 1725

WIMP arrival direction
10 degrees



Bozorgnia, Gelmini, Gondolo 2012