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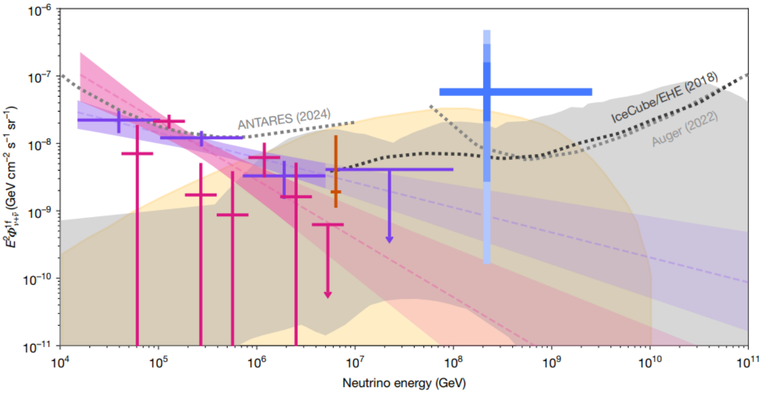
Exploring ultra-high energy neutrino experiments through the lens of the transport equation

Based on arXiv:2507.10665

with about 50% of audience

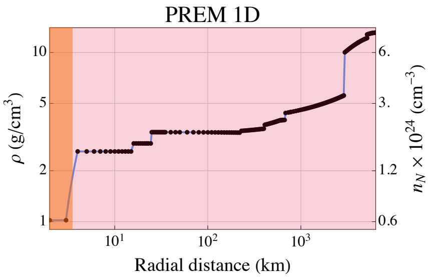
Outline of the talk

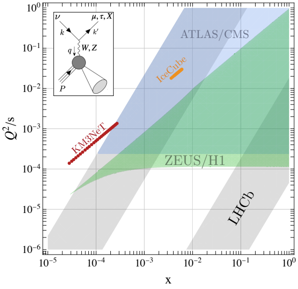
In February, KM3NeT announced highest-energy event ever KM3NeT Collaboration, '25



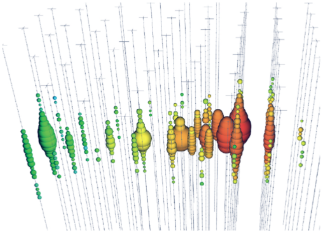
In this sense neutrino telescopes are huge "fixed" target experiments

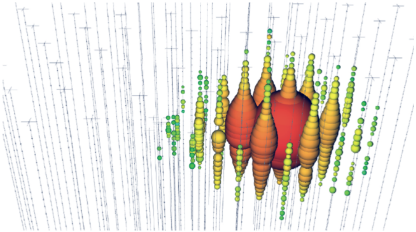
- Measure flux of neutrinos (or other particles!)
- Infer properties of neutrino, Earth density profile, ...
- Indirect detection of dark matter





- Neutral current interactions within detector → cascade events
- Charged current (far away) → Mostly muon tracks



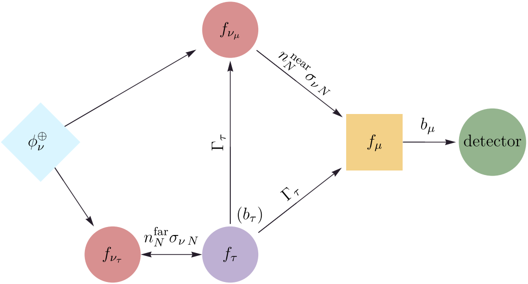


- Computation of rate of muon tracks from first principles

$$\langle N_\mu \rangle = \int_{\mathcal{V}} \int \frac{d^3 p_\mu}{(2\pi)^3} \mathbf{f}_\mu (t, \vec{x}, \vec{p}) \longrightarrow \text{Boltzmann Equation}$$

Beacom, et al., 02
Dutta, et al., 02
Dutta, et al., 05
Gonzalez-Garcia, et al., 09
Gaisser, et al., 16

- Only unknown →



- Solve neutrino-tau system once and for all, feed into equation for muons

Muon transport equation

Boltzmann Equation

$$\frac{\partial f_\mu}{\partial t} + \dot{\vec{x}} \cdot \frac{\partial f_\mu}{\partial \vec{x}} + \dot{\vec{p}} \cdot \frac{\partial f_\mu}{\partial \vec{p}} = K\left(t, \vec{x}, \vec{p}\right)$$

- Kernel encodes upscatterings $\supset \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\mu}$, decays $\supset \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 p_\mu}$
- QED processes are fast, and they contribute to $\dot{\vec{p}}$ (here it gets tricky)

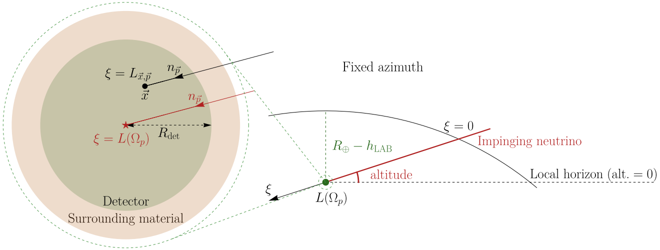
$$-\frac{\mathrm{d}E_\ell}{\mathrm{d}t} = -v\frac{\mathrm{d}E_\ell}{\mathrm{d}x} \approx -b_\ell v E_\ell$$

- Muon decay and ν_μ repopulation is slow, we neglect it

Under these approximations, system of PDEs

Find trajectories along which equation is ordinary

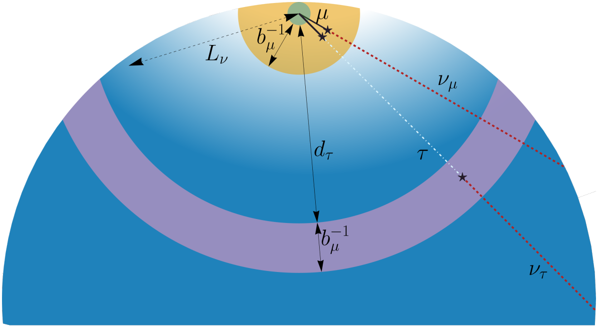
$$\begin{cases} \mathrm{d}t/\mathrm{d}\xi = 1/v \\ \mathrm{d}\vec{x}/\mathrm{d}\xi = \hat{n}_p \\ \mathrm{d}\vec{p}/\mathrm{d}\xi = -b_\mu(\xi)\vec{p} \end{cases} \implies \begin{cases} t(\xi) = t + (\xi - L_{x,p})/v \\ \vec{x}(\xi) = \vec{x} - (L_{x,p} - \xi)\hat{n}_p \\ \vec{p}(\xi) = p\,\hat{n}_p\,\exp(\int_\xi^{L_{x,p}} \mathrm{d}\xi' b(\xi')) \end{cases}$$



- $\frac{\mathrm{d}f_\mu}{\mathrm{d}\xi} = K\left(t(\xi), \vec{x}(\xi), \vec{p}(\xi)\right) \longrightarrow f_\mu\left(t, \vec{x}, \vec{p}\right) = \int_0^L \mathrm{d}\xi K(t(\xi), \vec{x}(\xi), \vec{p}(\xi))$
- Neutrino energy - conversion point relation:

$$E_\mu^{\text{meas}} \sim E_\mu^{\text{prod}} e^{-b_\mu \xi} \sim E_\nu e^{-b_\mu \xi}$$

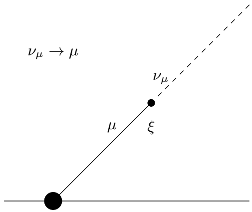
- Integral ranges over $R_{\text{eff}} \sim 1/b_\mu \gtrsim R_{\text{det}}$

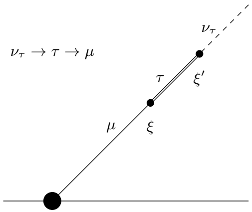


$$\left\langle \frac{\mathrm{d}N_\mu}{\mathrm{d}t\mathrm{d}E_\mu\mathrm{d}\Omega_\mu} \right\rangle = \frac{\text{number of muons}}{\text{time} \times \text{muon energy} \times \text{solid angle}} =$$

$$4\pi A_T E_\mu^2 \Bigg[\int^L \mathrm{d}\xi \int \mathrm{d}E_\nu \, (\mathrm{Att.}) \, n_N(\xi) \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\mu(\xi)} \phi_\nu^\oplus (E_\nu)$$

$$+ \int^L \mathrm{d}\xi \int^\xi \mathrm{d}\xi' \int dE_\nu \, \mathrm{d}^3 p_\tau \, (\mathrm{Att.}) \, n_N(\xi') \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\tau(\xi')} \frac{\mathrm{d}\Gamma}{\mathrm{d}^3 p_\mu(\xi)} D_\tau \left(\xi, \xi'\right) \phi_\nu^\oplus (E_\nu) \Bigg]$$





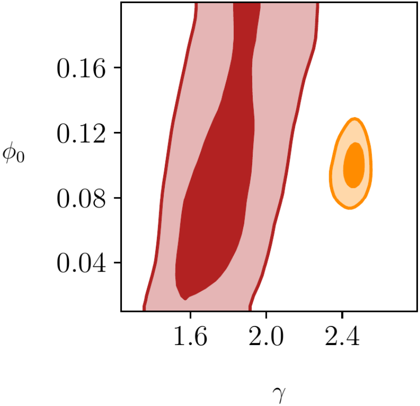
Case study: KM3-230213A

- Assessment of tension with IceCube
- Tension quantified in terms of

$$\Delta = \frac{\#_{\mathrm{IC}}}{\#_{\mathrm{KM3NeT}}}\Bigg|_{\text{KM3-230213A bin}}$$

1. Diffuse flux, both power law and energy localized
2. Point source, which may be transient

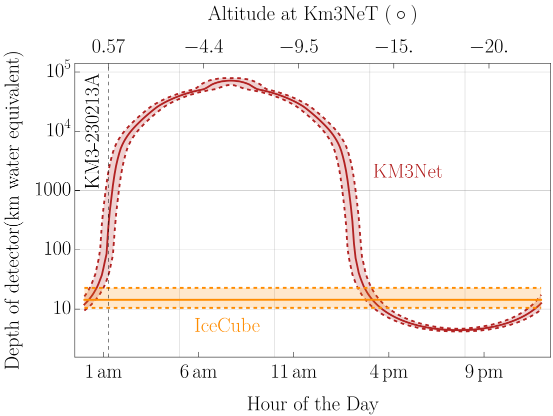
$$\phi_\nu^\oplus = \phi_0^* \left(\frac{E_\nu}{E_*}\right)^{-\gamma}$$ $\longrightarrow \Delta \sim 70 \longrightarrow$ $3.1\,\sigma$ tension compatible w/ Li, et al., 25



- Egg: IceCube through-going
- Veg. bacon: KM3NeT

Point source

$$\phi_\nu^\oplus(\Omega_\nu) \propto \delta^{(2)}\left(\Omega_\nu - \Omega_{\mathrm{PS}}(t)\right)$$

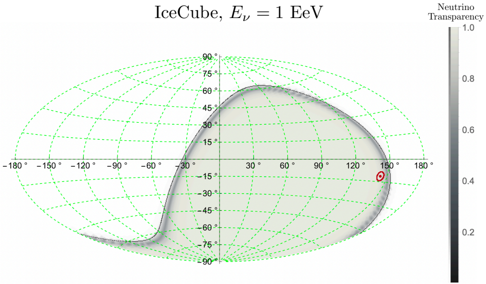


$$\Delta_{\mathsf{PS}} \approx \frac{[TA]_{\mathrm{IC}}}{[TA]_{\mathsf{KM3NeT}}} \cdot \frac{\langle \mathrm{Att.}\rangle_{\mathrm{IC}}}{\langle \mathrm{Att.}\rangle_{\mathsf{KM3NeT}}} \cdot \frac{R_{\mathrm{eff}}^{\mathrm{IC}}}{R_{\mathrm{eff}}^{\mathsf{KM3NeT}}} \approx 140$$

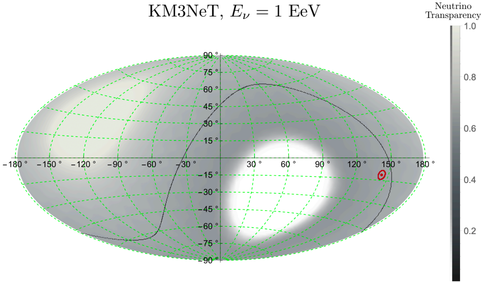
Larger tension than diffuse

unlike Li, et al., 25

Neutrino transparency



$\langle e^{-L/L_\nu} \rangle_{\mathrm{IC}} \approx 0.99$



$\langle e^{-L/L_\nu} \rangle_{\mathrm{KM}} \approx 0.48$

Take-home

Summary of tension with IC

Model	$\Delta_{\mathrm{IC/KM}}$	B factor	$\Delta\sigma$
Diffuse (power-law)	70	21	3.1
PS	140	≈ 35	3.8
PS (transient)	12	≈ 2.2	1.6
Diffuse (energy localized)	70	≈ 3.4	2.4

Advantages

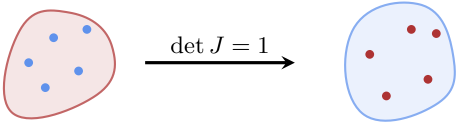
- Physical parameters (ϕ_ν, new-physics, ...) $\leftrightarrow$ Measured quantity (E_μ, E_{had})
- No need to re-run simulations for new-physics/unprobed regions of SM
- Fit directly from measured quantities, not relying on ν energy reconstruction

Outlook

- Extension to cascades and other event topologies
- Neglects diffusion, does not conserve number of particles (dissipative system)
- τ-induced muons: several scales, subtle
- Implementation of formalism in a public code

Thank you for the attention!
And thank you for a great year!

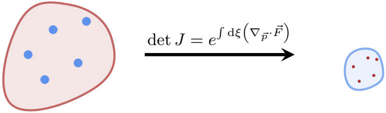
Hamiltonian systems



$\mathrm{d}^3p\,\mathrm{d}^3x\,f(t,\vec{x},\vec{p})$ $\qquad$ $\mathrm{d}^3p'\,\mathrm{d}^3x'\,f(t',\vec{x}',\vec{p}')$

$$\dot{f} + \dot{\vec{x}}\cdot\nabla f + \dot{\vec{p}}\cdot\nabla_{\vec{p}} f = 0 \rightarrow C[f]$$

Dissipative systems



$\mathrm{d}^3p\,\mathrm{d}^3x\,f(t,\vec{x},\vec{p})$ $\qquad$ $\mathrm{d}^3p'\,\mathrm{d}^3x'\,f(t',\vec{x}',\vec{p}')$

$$\dot{f} + \dot{\vec{x}}\cdot\nabla f + \dot{\vec{p}}\cdot\nabla_{\vec{p}} f + \nabla_{\vec{p}}\cdot\left(\vec{F} f\right) = 0 \rightarrow C[f]$$

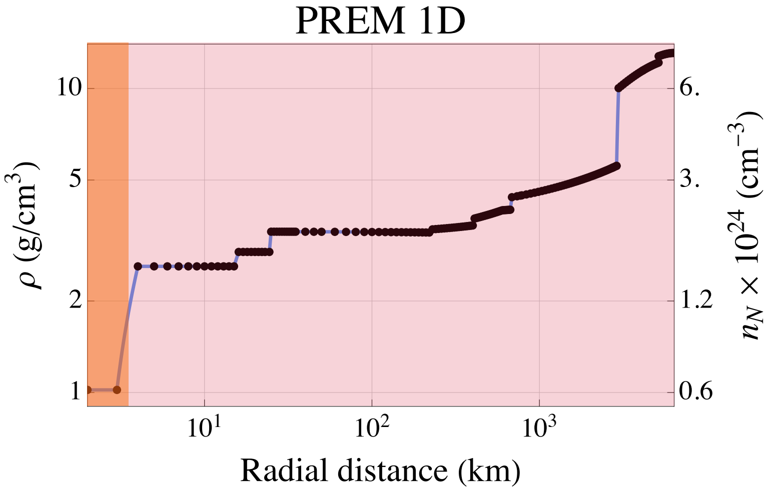
$$\dot{f} + \dot{\vec{x}} \cdot \nabla f + \dot{\vec{p}} \cdot \nabla_{\vec{p}} f + \underbrace{\vec{F} \cdot \nabla_{\vec{p}}}_{\dot{\vec{p}}=-b_\mu \vec{p}}(f) = C[f]$$

- Does not conserve number of particles even in the collisionless limit
  - Would need an extra source term $\sim 3b_\mu f_\mu$

Comments

- Neglecting diffusion underestimates energy loss $\rightarrow$ larger effective volume
- Loss of particles mostly from $R > 1/b_\mu$ $\rightarrow$ smaller effective volume

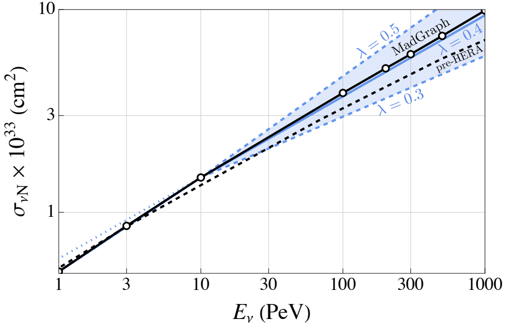
Interpretation of b_μ as a nuisance can take into account approximations


PREM 1D
ρ (g/cm^3)
10
5
2
1
6.
3.
1.2
0.6
$n_N \times 10^{24}$ (cm^{-3})
10^1
10^2
10^3
Radial distance (km)

- In the limit of small x ($E_\nu > 10\,\mathrm{PeV}$), neutrino DIS cross section scales as

$$\frac{\mathrm{d}^2\sigma_{\nu N}}{\mathrm{d}x\mathrm{d}y} \approx \frac{G_F^2 M_W^2}{\pi x^\lambda}\left[\frac{(1+(1-y)^2)\tilde{s}}{(1+xy\tilde{s})^2}\right]$$

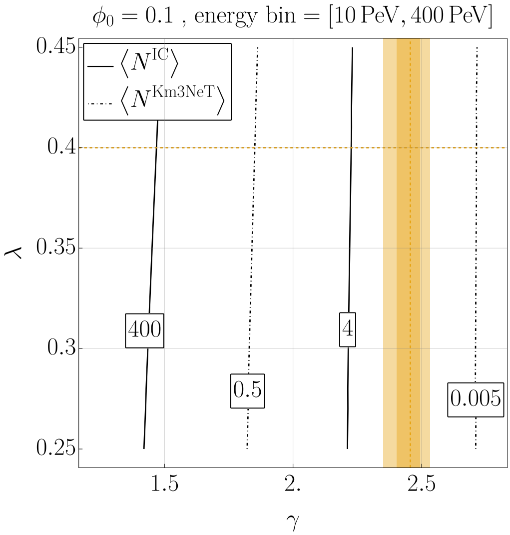
$$\sigma_{\nu\,N}(E_\nu) 1.48\times 10^{-33}\ \mathrm{cm}^2 \times \left(\frac{E_\nu}{10\ \mathrm{PeV}}\right)^\lambda$$



LHAPDF6

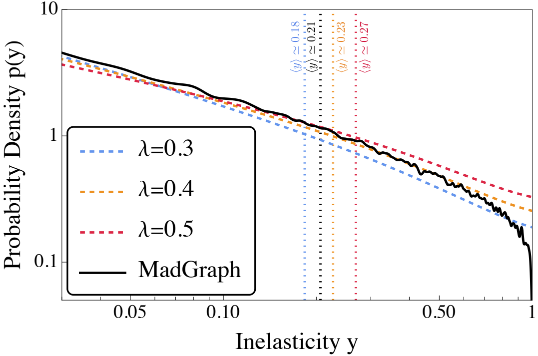
- Eventually it can scale at most as $\log^2(E_\nu)$ Froissart, 1961

- Expected number of events in UHE bins largely independent on λ

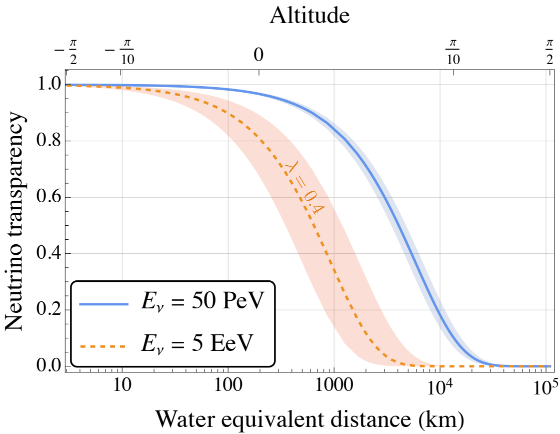


Inelasticity

$$\frac{1}{\sigma_{\nu N}}\frac{\mathrm{d}\sigma_{\nu N}}{\mathrm{d}^3 p_\mu} = \delta^{(2)}(\Omega_\nu - \Omega_p)\frac{(1-y)}{E_\mu^3}\mathcal{P}(y)$$



$$D(\xi, E_\nu) = \exp\left(- \int_0^\xi \frac{d\zeta}{L_\nu(\zeta, E_\nu)} \right) = \exp\left(- \frac{\xi_{\text{we}}}{L_\nu^{\text{water}}(E_\nu)} \right),$$



$$\begin{cases} \boxed{\dfrac{\partial f_{\nu_\mu}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\nu_\mu}}{\partial \vec{x}} + \dot{\vec{p}}_{\nu_\mu} \cdot \dfrac{\partial f_{\nu_\mu}}{\partial \vec{p}_{\nu_\mu}} = C_{\text{flavor-ind.}}(t, \vec{x}, \vec{p}_{\nu_\mu}) + K^{\tau}_{\text{decay}}(t, \vec{x}, \vec{p}_{\nu_\mu})}\,, \\ \dfrac{\partial f_{\nu_\tau}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\nu_\tau}}{\partial \vec{x}} + \dot{\vec{p}}_{\nu_\tau} \cdot \dfrac{\partial f_{\nu_\tau}}{\partial \vec{p}_{\nu_\tau}} = C_{\text{flavor-ind.}}(t, \vec{x}, \vec{p}_{\nu_\tau}) + K^{\tau}_{\text{decay}}(t, \vec{x}, \vec{p}_{\nu_\tau})\,, \\ \boxed{\dfrac{\partial f_{\tau}}{\partial t} + \dot{\vec{x}} \cdot \dfrac{\partial f_{\tau}}{\partial \vec{x}} + \dot{\vec{p}}_{\tau} \cdot \dfrac{\partial f_{\tau}}{\partial \vec{p}_{\tau}} = K^{\nu_\tau}_{\text{scattering}}(t, \vec{x}, \vec{p}_{\tau}) - \Gamma_\tau(\vec{p}_\tau) f_\tau} \end{cases}$$

- Neutrino attenuation Beacom, et al., 02

$$\boxed{\frac{\partial \phi_\nu}{\partial \xi} = -\frac{1}{\lambda(E_\nu)}\left[\phi_\nu(\xi, E_\nu) - \int_0^1 \frac{dy}{1-y}\mathcal{P}(y)\frac{\sigma_{\text{NC}}}{\sigma_{\text{tot}}}\phi_\nu(\xi, \frac{E_\nu}{1-y})\right]}$$

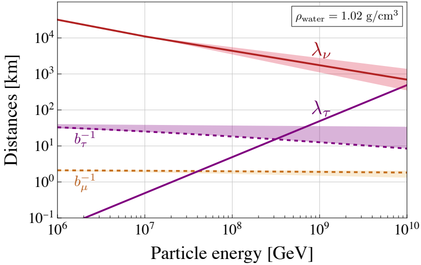
- Taus are more complex Dutta, et al., 05

$$\boxed{f_\tau \sim \int d\xi' \int dE_\nu \text{Att.}\phi_\nu^{\oplus}(E_\nu) n_N^{\text{far}} E_\tau^2 \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_\tau(\xi')} \exp\left[(m_\tau/E_\tau)/(d_\tau b_\tau)\left(1 - e^{-b_\tau \xi'}\right)\right]}$$

$$-\left\langle \frac{\mathrm{d}E_\ell}{\mathrm{d}x} \right\rangle = a_\ell(E_\ell) + b_\ell(E_\ell)E_\ell \approx b_\ell E_\ell$$ PDG

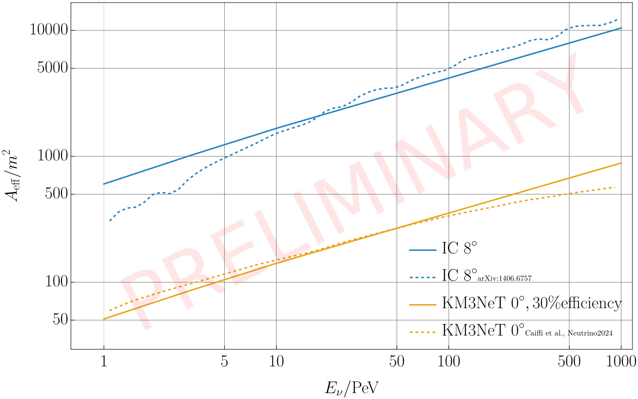
Very different for the three leptons

- $b_\mu^{-1} \sim$ few km $\rightarrow$ tracks, bangs
- $b_e^{-1} \sim \left(\frac{m_e}{m_\mu}\right)^2 b_\mu^{-1} \approx 0 \rightarrow$ no tracks, only bangs
- $b_\tau^{-1} \sim$ tens of km, with large uncertainties due to DIS
  - But decays in shorter lenghts (unless very high energy) $\rightarrow$ muon tracks, double bangs



Effective Area

$$A_{\text{eff}}(E_\nu,\Omega_\nu) = A_T \int_0^{L(\Omega_{\vec{p}})} \mathrm{d}\xi \int_{\Delta_{E_\mu}\otimes\Delta_{\Omega_{\vec{p}}}} \mathrm{d}E_\mu \mathrm{d}\Omega_p \underbrace{\epsilon\left(E_\mu,\Omega_{\vec{p}}\right)}_{???} \overbrace{\mathcal{T}(E_\mu,\Omega_{\vec{p}}|E_\nu,\Omega_\nu;\xi)}^{4\pi E_\mu^2(\text{Att.})\frac{d\sigma(p(\xi))}{d^3p(\xi)}n_N(\xi)}$$



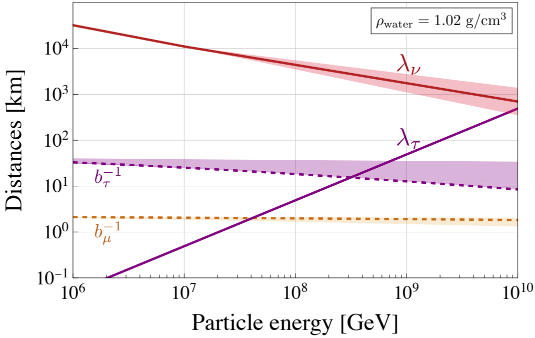
IC: Aartsen, et al., 14

KM3NeT: Caiffi, et al., 25

- Neglecting ξ' dependence of attenuation, as $b_\tau \to 0$ $(p_\tau(\xi) \approx p_\tau)$

$$f_\tau \propto \left(1 - e^{\xi/d_\tau}\right) \to \left(1 - e^{-L/d_\tau}\right)$$

- Naïvely, at $E_\tau \gtrsim 1\,\mathrm{EeV}$, yield from PS is
  - unsuppressed at KM3NeT
  - supressed as $L_{\mathrm{IC}}/d_\tau \lesssim 0.2$
- $\min\{b_\tau^{-1}, d_\tau\}$ sets the rules of the game, never drastically larger than L_{IC}



$$B_{1\to 2}(D) \equiv \frac{\int P(\hat{\theta}_1|D)d\hat{\theta}_1}{\int P(\hat{\theta}_2|D)d\hat{\theta}_1} = \frac{\int \mathcal{L}_D(\hat{\theta}_1)\pi_1(\hat{\theta}_1)d\hat{\theta}_1}{\int \mathcal{L}_D(\hat{\theta}_2)\pi_2(\hat{\theta}_2)d\hat{\theta}_2},$$

- $B_{1\to 2} \gg 1 \to$ model 1 favored by data wrt model 2

$$B_{D_1/D_2} = \frac{\int \mathcal{L}_{D_2}(\hat{\theta}|\mathrm{M})\pi(\hat{\theta})d\hat{\theta}}{\int \mathcal{L}_{D_2}(\hat{\theta}|\mathrm{M})\pi_{\mathrm{ref}_{\mathrm{D1}}}(\hat{\theta})d\hat{\theta}}$$

- $\pi_{\mathrm{ref}_{\mathrm{D1}}}(\hat{\theta})$ yields a good fit of D_1 for model M
- If $\pi(\hat{\theta})$ such that $B_{D_1/D_2} \gg 1 \to$ tension between datasets D_1 and D_2

Remark ($B \sim \chi^2$ with d d.o.f)

$$\Delta\sigma = \sqrt{2}\,\mathrm{erfc}^{-1}\left[Q\left(\frac{d}{2};2\ln B\right)\right],$$

where $Q(a,z) \equiv \Gamma(a,z)/\Gamma(a,0)$, and $\Gamma(a,z) = \int_z^\infty dt\, t^{a-1}e^{-t}$.

Energy-Localized Sources

$$\mathcal{L}_{\mathrm{IC},0}(\hat{N}) = e^{-\Delta\times\hat{N}} \quad \text{and} \quad \mathcal{L}_{\mathrm{KM3},1} = \hat{N}e^{-\hat{N}}\,.$$