# **String Theory** & **Modern QFT**

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*Florence 17/03/2026*

After carelessly sending the title to our host, Fulvio, & to Massimo (Porrati)

I had to define my ambition:

Do better than (the paying version of) Chat-gpt

# String Theory before GGI

′68 — Veneziano amplitude

′74 — spin 2 is inevitable → theory of gravity

[Scherk, Schwarz; Yoneya]

280 ctns,

12 < GS

′84 — Green-Schwarz anomaly cancelation → finite q-gravity

′85

′88

} Unification of interactions; $\mathrm{CFT}_2$ → $\alpha'$ corrections tamed

**Landscape** : *Our Universe ?*

*beyond perturbation theory ?*

'95

**String dualities** → UV theory is unique

[Witten; Hull, Townsend]

**D-branes**: string solitons, "trap" non-abelian YM

[Polchinski]

'96

**Microscopic** (susy)**Black Holes** → Bekenstein-Hawking

[Strominger, Vafa]

'97

**AdS/CFT :** String theory in a box is a QFT

[Maldacena]

[Gubser, Klebanov, Polyakov; Witten]

# the GGI years

1 **"Prove", decipher** & **exploit** AdS/CFT

*Much progress, more open questions*


*cf also Massimo's talk*

2 **Q**uantum **F**ield **T**heory **/** gen-Z

*Inspired by (in symbiosis with) string theory & cond-mat physics*

3 **Unification** of particle physics & cosmology, **tests ?**

*Some, but timid progress; need new ideas & some luck*

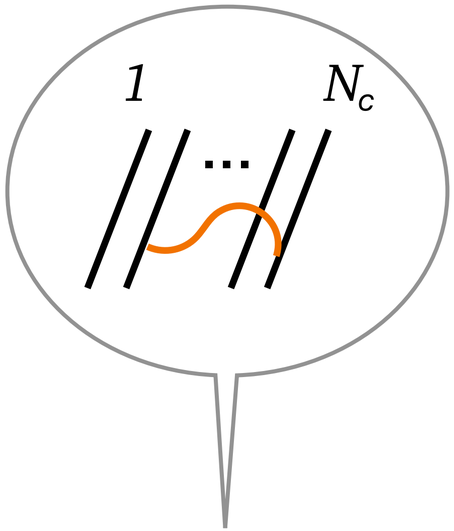
# 1. AdS/CFT

Key idea :

- **solitons** have *zero modes, i.e.* massless transversely localised excitations

- in D*p*-branes these come in **Yang-Mills** $U(N_c)$ *supermultiplets*:

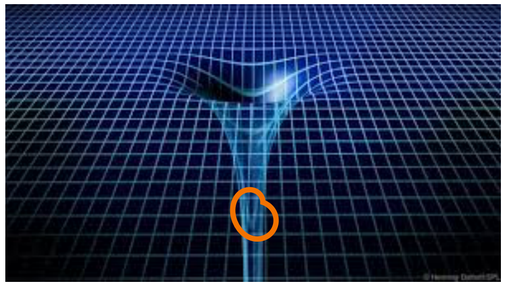
$$(A_\mu^{ab},\ \Phi_i^{ab},\ \text{fermions})$$




- for large tension $(N_c \gg 1)$ the soliton becomes an ***extremal brane;***

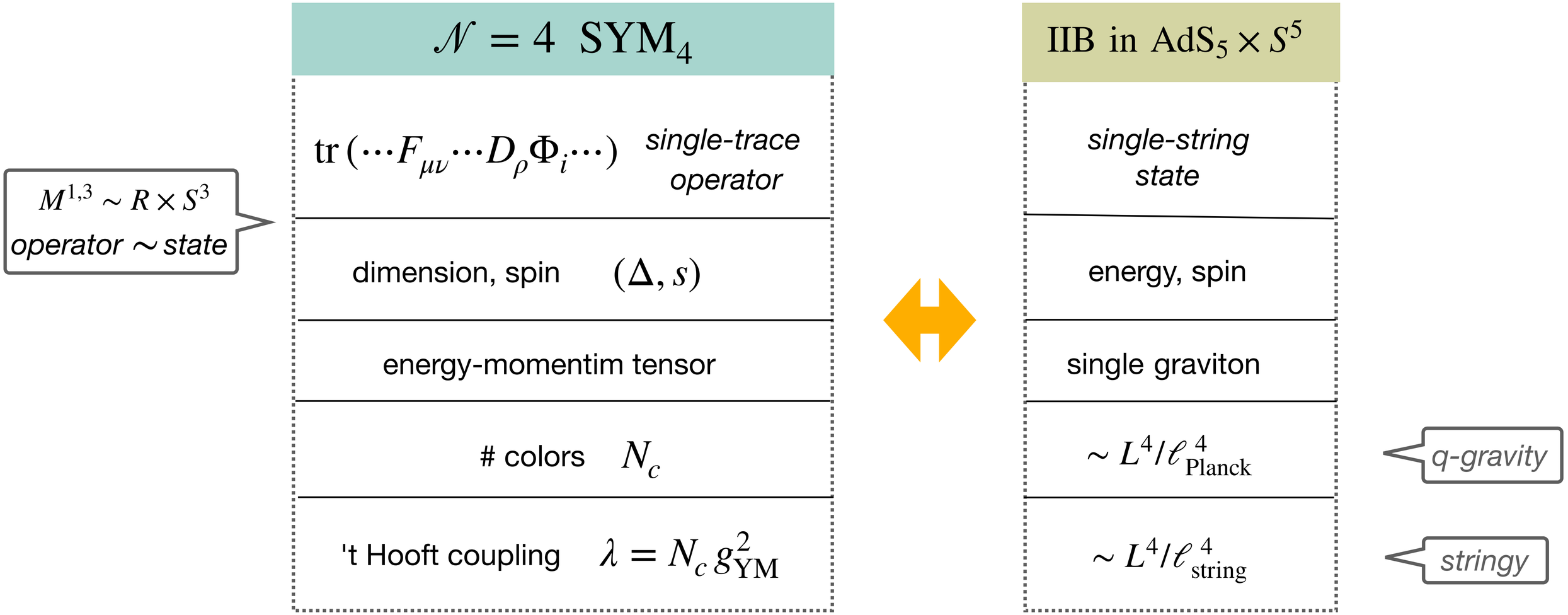
zero modes → **closed-strings** ∞-ly red-shifted near *horizon*

$\sim \mathrm{AdS}_{d+1} \times Y$



Unlike other dualities, AdS/CFT relates a **CFT** with **quantum gravity** "in a box".

The prototypical case is



An example of **top-down holography**

- *q*-string theory in the UV
- dual CFT exactly identified
- but **unrealistic** *(brane engineered)*

A very convincing *tests* of the conjecture: exact derivation of the **spectrum** in the **planar limit** $(N_c \to \infty)$, for all values of $\lambda$

This mathematical tour de force started with the reduction to *integrable spin chains* at weak $\lambda$, and ended with derivation of the **Quantum Spectral Curve**

[*see reviews by* Beisert; Gromov]

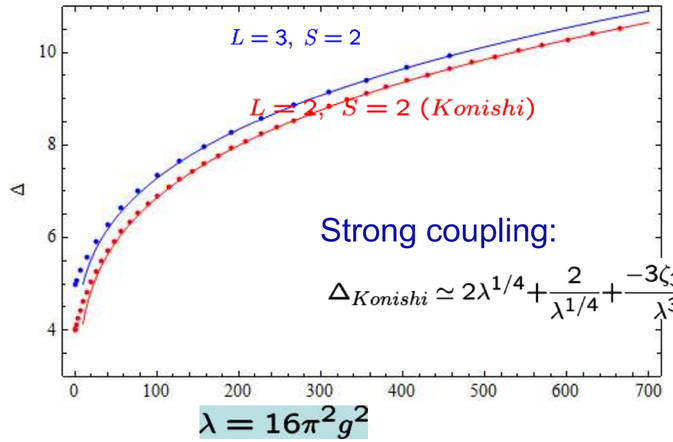
# Dimensions of twist-2,3,… operators $\mathrm{Tr}\,(\Phi\,\nabla^S\,\Phi)$

Gromov,VK,Leurent, Volin ‘13

- Numerics, weak and strong coupling from Quantum Spectral Curve;

Gromov,V.K.,Vieira ‘09
Frolov ‘10
Gromov,Valatka ‘12

Old TBA results:



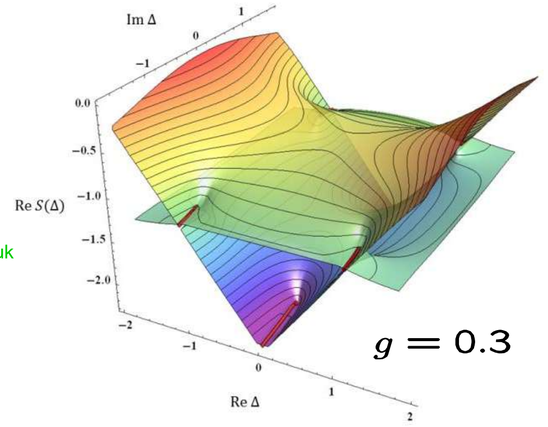

Strong coupling:

$$\Delta_{Konishi} \simeq 2\lambda^{1/4} + \frac{2}{\lambda^{1/4}} + \frac{-3\zeta_3+\frac{1}{2}}{\lambda^{3/4}} + \frac{\frac{15}{2}\zeta_5+6\zeta_3-\frac{1}{2}}{\lambda^{5/4}} + \dots$$

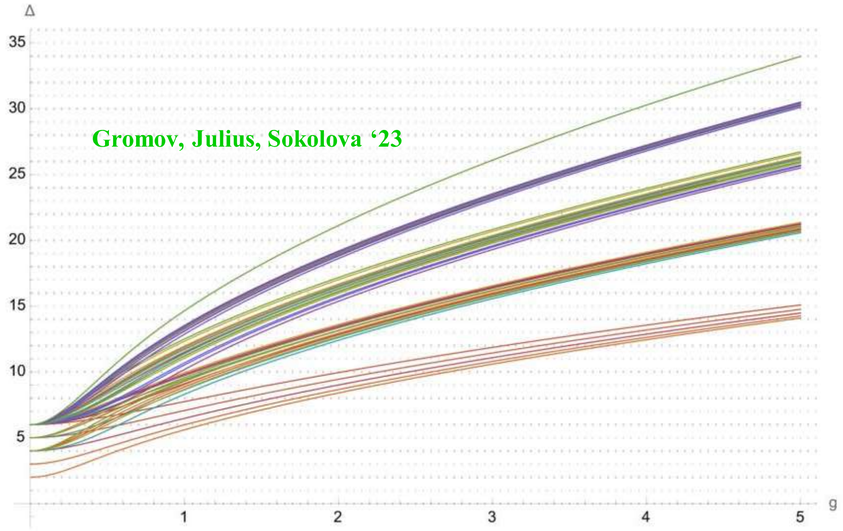
Gromov, Valatka, Sizov, Levkovich-Maslyuk
Gromov,Shenderovich,
Serban, Volin
Roiban, Tseytlin
Vallilo, Mazzucato
Gubser, Klebanov, Polyakov

Function of complex conformal spin $\Delta(S,g)$

Gromov, Levkovich-Maslyuk, Sizov ‘15




Recent results, “unlimited” precision (~30-50 digits)




*courtesy*

V. Kazakov

Weak couping (11 loops)

$$\gamma_{Konishi} = \sum_{j=1}^{\infty} g^{2j}\,\gamma_j$$

Gromov,VK,Leurent, Volin ‘13
Leurent, Serban, Volin ‘12
Volin, Marboe ‘18

$$\begin{aligned}
\gamma_{11} = {} & -242508705792 + 107663966208\zeta_3 + 70251466752\zeta_3^2 - 12468142080\zeta_3^3 \\
& +1463132160\zeta_3^4 - 71663616\zeta_3^5 + 180173002752\zeta_5 - 16655486976\zeta_3\zeta_5 \\
& -24628230144\zeta_3^2\zeta_5 - 2895575040\zeta_3^3\zeta_5 + 19278176256\zeta_5^2 - 9619845120\zeta_3\zeta_5^2 \\
& +2504494080\zeta_3^2\zeta_5^2 + \tfrac{882108048384}{175}\zeta_5^3 + 45602231040\zeta_7 + 14993482752\zeta_3\zeta_7 \\
& -12034759680\zeta_3^2\zeta_7 + 1406730240\zeta_3^3\zeta_7 + 30605033088\zeta_5\zeta_7 + 21217637376\zeta_3\zeta_5\zeta_7 \\
& -\tfrac{1309941061632}{275}\zeta_5^2\zeta_7 - 13215327552\zeta_7^2 - 4059901440\zeta_3\zeta_7^2 - 69762034944\zeta_9 \\
& +23284599552\zeta_3\zeta_9 - 3631889664\zeta_3^2\zeta_9 - 11032374528\zeta_5\zeta_9 - 6666706944\zeta_3\zeta_5\zeta_9 \\
& -23148129024\zeta_7\zeta_9 - 10024051968\zeta_9^2 - 54555179184\zeta_{11} + \tfrac{10048541184}{5}\zeta_3\zeta_{11} \\
& -726029568\zeta_3^2\zeta_{11} - 8975463552\zeta_5\zeta_{11} - 22529041920\zeta_7\zeta_{11} - \tfrac{1437993422496}{175}\zeta_{13} \\
& +\tfrac{1504385419392}{35}\zeta_3\zeta_{13} - 30324602880\zeta_5\zeta_{13} - \tfrac{151130039581392}{875}\zeta_{15} - 41375093760\zeta_3\zeta_{15} \\
& -\tfrac{196484147423712}{275}\zeta_{17} + 309361358592\zeta_{19} - 1729880064Z_{11}^{(2)} - \tfrac{1620393984}{5}\zeta_3 Z_{11}^{(2)} \\
& -131383296\zeta_5 Z_{11}^{(2)} + \tfrac{138107420928}{175}Z_{13}^{(2)} + \tfrac{3543865344}{35}\zeta_3 Z_{13}^{(2)} - \tfrac{5716780416}{7}Z_{13}^{(3)} \\
& -\tfrac{674832384}{7}\zeta_3 Z_{13}^{(3)} + \tfrac{48227088384}{175}Z_{15}^{(2)} + \tfrac{3581880576}{25}Z_{15}^{(3)} + 754974720 Z_{15}^{(4)} \\
& -\tfrac{854924544}{11}Z_{17}^{(2)} + \tfrac{4963244544}{55}Z_{17}^{(3)} + \tfrac{818159616}{275}Z_{17}^{(4)} + \tfrac{175363688448}{1925}Z_{17}^{(5)}\,. \qquad \text{(A.}
\end{aligned}$$

- QSC for ABJM, AdS3/CFT2

Cavaglia, Fioravanti,Gromov,Tateo ‘14
Cavaglia, Gromov, Stefanski,Torielli ‘21

This *goes beyond kinematics/symmetries*. But $\left\{\begin{array}{l}\text{it only tests stringy corrections, not finite } N_c \\ \text{it does not provide the OPE coefficients}\end{array}\right.$

- Considerable ongoing effort to "solve" planar $\mathcal{N}=4$ $\mathrm{SYM}_4$ by combining integrability with *supersymmetric localisation* & the *conformal bootstrap.* Also count **operators** with $\Delta \sim N_c^2$

*see later*

(susy) ***black hole*** *entropy*

- Other top-down dualities: $\mathrm{ABJM}_3$ ; $\mathrm{SymOrb}_2$ ; $\mathrm{SCFT}_6$ $\mathrm{dCFT}_{d,p}$

**Great mathematical physics,** uses & develops powerful tools.

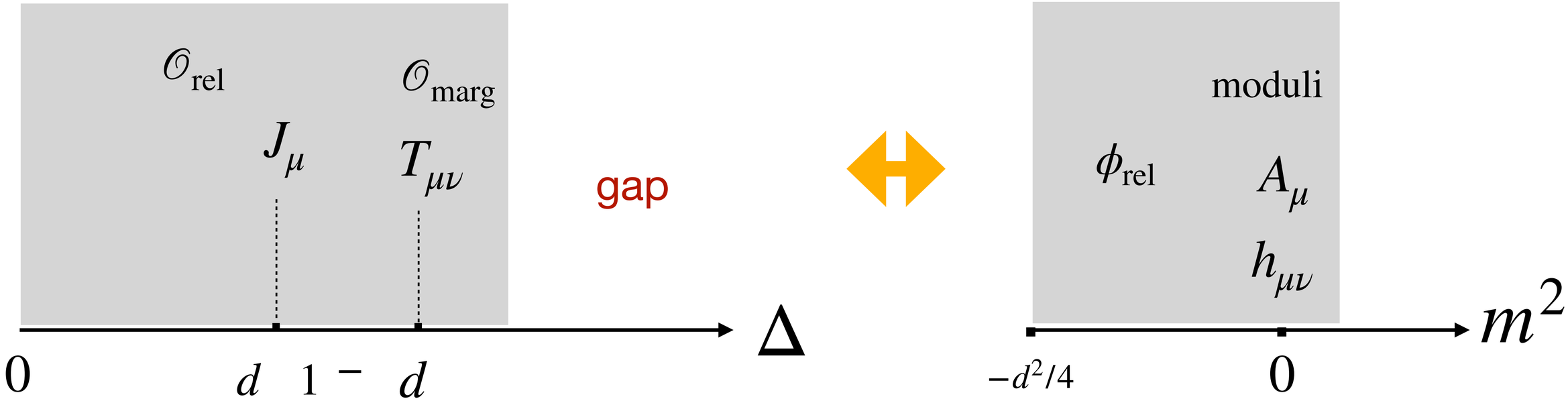
Results confirm AdS/CFT; but *few new conceptual lessons/surprises.*

At other end **bottom-up** holography

- putative CFT with given type of spectrum
- low-E effective gravitational theory
- flexible, *but could lie in* ***swampland***

**q-gr** : *no UV completion*
**condmat**: no ab initio theory

typical:



This has lead to new models of strong-coupling "phenomena":

**AdS/CMT:** *holographic superconductors, strange metals, quantum phase transitions*

**AdS/QCD:** new EFTs, intuition on confinement, chiral symmetry breaking

Not quantitatively precise 😡 , but allows study of **real-time** phenomena out-of-reach of numerical simulations (transport at finite $T, \mu$ ; far out-of-equilibrium ) 🙂

It has also greatly enhanced our understanding of the dynamics of **black-hole horizons**.

$T > 0$

I will mention two specific aspects, *for more see* Massimo Porrati's talk

## **Gravity/fluid** correspondence

Dynamics of s*trongly-coupled plasmas* mapped to that *of* ***planar horizons***.

Low-frequencies governed by hydrodynamic modes (related to conserved quantities).

An early result in (3+1)*d* : horizon $\sim$ **most-perfect** of all fluids

$$\frac{\textit{shear viscosity}}{\textit{entropy density}} \qquad \boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \sim 0.08}$$

[Policastro, Son, Starinets '01]

Remarkably close to the **quark-gluon** plasma from data at RHIC and the LHC

*see Urs Wiedemann's talk*

$$cf \quad \begin{cases} \gg 1 & \textit{perturbative QCD} \quad \text{[Arnold, Moore, Yaffe '03]} \\ \simeq 0.7 & \textit{liquid He} \\ \gtrsim 0.5 & \textit{cold atoms @ unitarity} \quad \text{[Schäfer '08]} \end{cases}$$

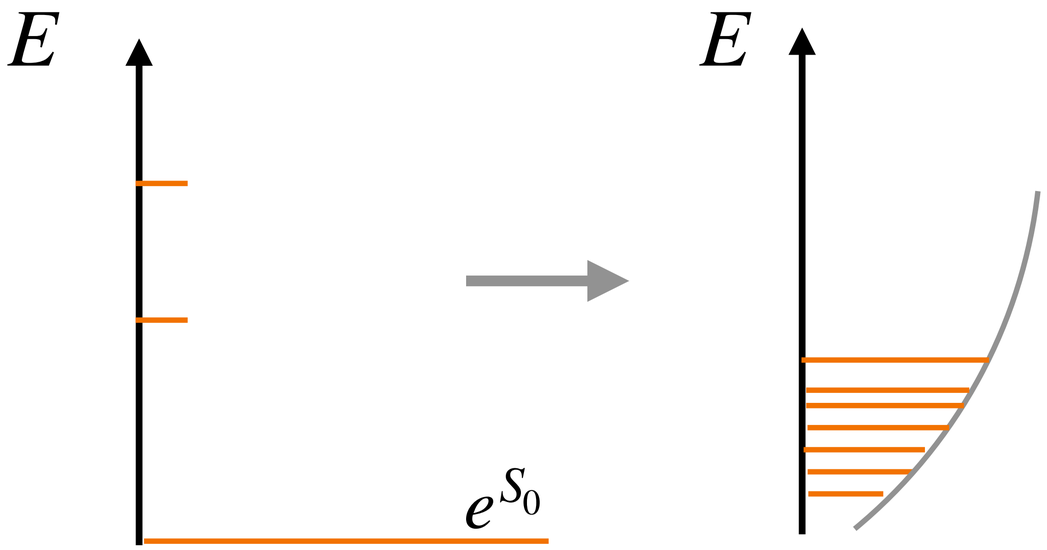
# Non-susy $AdS_2$ horizons

Prototype: extremal charged (*Reissner-Nordstrom*) black hole $M = |Q|$

$\mathrm{AdS}_2 \times S^2$ horizon with Bekenstein-Hawking entropy $S_0 = \pi r_h^2 \Longrightarrow e^{S_0}$ ground states.

& full 1d conformal symmetry

But strong quantum fluctuations of very low-*E* gravity mode lift this degeneracy so that




$$\rho(E) \sim e^{S_0} \sinh(2\pi\sqrt{E})$$

with $\delta E \sim e^{-S_0}$ ; given by random-matrix model

This low-*E* dynamics is captured by the Schwarzian mode, present in both

**J**ackiw-**T**eitelboim gravity

$$S = \frac{1}{16\pi G}\left[\int_{\mathscr{M}} d^2x\sqrt{-g}\,\phi\left(R+\frac{2}{\ell^2}\right)+2\int_{\partial\mathscr{M}} dx\sqrt{-h}\,\phi K\right]$$

$\mathrm{NAdS}_2$

?

**S**achdev-**Y**e-**K**itaev model

$$H_{\mathrm{SYK}} = \sum_{i<j<k<l} J_{ijkl}\,\psi_i\psi_j\psi_k\psi_l$$

$$\langle J_{i_1\cdots i_4}\rangle = 0\,,\quad \langle J^2_{i_1\cdots i_4}\rangle = \frac{6J^2}{N^3}$$

$\mathrm{NCFT}_2$

[Kitaev; Almheiri, Polchinski; Jensen; Maldacena, Stanford, Yang; . . . ; Iliesiu, Murthy, Turiaci]

Solvable model of { spin liquid with no quasi-particles

**chaotic** dynamics of BH horizon

JT gravity: reproduces the self-averaging $\overline{\rho(E)}$ of SYK in the $N \sim e^{S_0} \to \infty$ limit.

Sum over different topologies (gravitational wormholes) controled by double-scaled matrix model. Leads to a spectral form factor $\overline{Z(\beta + i\alpha)\, Z(\beta - i\alpha)}$ matching qualitative expectations & numercis of SYK. But for large $\alpha$ this is not self-averaging.

[Saad, Shenker, Stanford]

Quenched disorder is confusing:

How to reconcile with top-down exact holography ?

*Are wormholes ruled out in q-gravity?*

What does the gravitational path integral compute ?

*If not, what inherent uncertainty do they compute?*

# 2. **QFT** / *gen-Z*

String theory/holography & "QFT 2.0" have developed hand-in-hand during the past 30 years.

Quick inventory of some landmarks:

- **Supersymmetric QFTs**

  [Seiberg-Witten '94] solution of $\mathcal{N} = 2$ $\mathrm{SYM}_4$ ; strong/weak *dualities*;

  . . . . . ; *localisation*: functional $\longrightarrow$ matrix integral; . . . .

  [Pestun '07 ]

- **Amplitudes**

  Efficient on-shell techniques beyond Feynman diagrams: *Spinor helicity formalism*;

  *Recursion relations; Double copy; . . . ; Unitarity cuts*

  Bern, Dixon, Kosower
  *(2023 Galileo Medal)*

- **Conformal** & **S-matrix bootstrap** in $d > 2$

Perturbative string theory grew out of the bootstrap idea in spacetime [Veneziano] & the worldsheet [Belavin, Polyakov, Zamolodchikov].

causality + unitarity + high-E fall-off
+ crossing symmetry

Its modern revival was prompted by the *unexpected efficiency* of the numerical CFT bootstrap

[Rattazzi, Rychkov, Tonni, Vichi '08; El Showk, Paulos, Poland, Rychkov, Simmons-Duffin '12; . . . ]

& has grown into active area of research

*Snowmass white papers*

- The Numerical Conformal Bootstrap
- Analytic Conformal Bootstrap
- S-matrix Bootstrap
- Bootstrapping String Theory
- The Cosmological Bootstrap

*see Ricardo Rattazzi's talk*

The beautiful technical progress in these  directions

has been integrated in areas where high precision is needed:

**QCD at LHC, Gravity waves, critical phenomena**

Let me here mention two other, more "conceptual" developments

- **Defects, symmetries** & **anomalies**

Exisence of extended solitons (branes) of any dimension in string theory ushered a revival of Defect QFT/CFT. Non-local observables are important as:

- **Order parameters** for phase transitions [Wilson-Polyakov loops $\longrightarrow$ de-confinement ]

- **RG flows** [Kondo impurities]

- **Infrared-safe** observables, eg. ANEC operators $\mathscr{E}(\vec{x}_\perp) = \int_{-\infty}^{+\infty} dx^- \, T_{--}(x^-, x^+ = 0, \vec{x}_\perp)$

[. . . ; Hoffman, Maldacena '08; . . ]

*carolimeter observables in conformal colliders*

With help from holography, bootstrap & susy DCFT became a very active research area.

Focus here on one specific aspect: *quantum symmetries & anomalies*

Current understanding of QFT symmetries was developped in the **60s & 70s.**

Symmetries can be global or gauged. Gauge symmetries are redundancies $\Longrightarrow$ always intact,

whereas global symmetries can be broken $\left\{\begin{array}{l}\text{explicitly } \textit{(softly or hardly)} \\ \text{spontaneously} \Longrightarrow \text{Nambu-Goldstone bosons} \\ \text{by quantum anomalies} \quad \text{[Adler, Bell, Jackiw]}\end{array}\right.$

*e.g.* QCD

$$SU(N_c) \times U(1)_B \times SU(N_F)_{\rm v} \times SU(N_F)_{\rm a} \times U(1)_{\rm a}$$

*gauged* — *exact* — *approximate* — *spontaneously broken* — *anomalous*

Anomalies are *obstructions to gauging*, must be preserved by RG flow ( $\Longrightarrow$ missing $\eta$ meson)

['t Hooft]

Gauge anomalies cancel in Standard Model $\Longleftarrow$ quark-lepton families are complete.

[Bouchiat, Iliopoulos, Meyer]

Novelties in the **80s & 90s:**

- *p*-form symmetries $G^{(p)}$ acting on dynamical *p*-branes & gauged by

  *(p+1)*-form gauge fields, e.g. { *p*=0: particles; *p*=1: strings

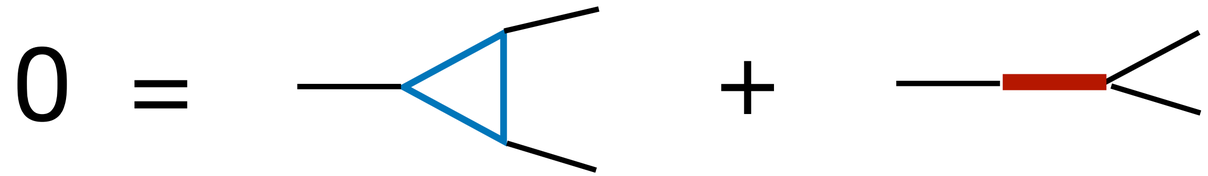
- $G^{(p)}$ need ***not commute*** with $G^{(q)}$ ⟶ ***classical anomalies*** which can be cancelled by quantum corrections [Green, Schwarz]

e.g. *axion shifts*

$$\mathscr{L} \sim m^2(A - da)^2 + aF \wedge F + \bar{\psi} D \gamma^5 \psi$$

$$\delta A = d\Lambda\,,\ \delta a = \Lambda$$

pure quantum symmetry




- Closely related anomaly inflow: quantum anomalies on soliton worldvolume cancelled by classical topological bulk term [Callan, Harvey]

Ideas generalized to *global, discrete & non-invertible* symmetries in **00s & 10s** by implementing their action via *(d-p-1)*-dim topological defects that link $p$-branes and $p$-extended operators. Group theory replaced by category theory.

[ CFT in *d=2* ; Frohlich, Fuchs, Runkel, Schweigert; Kontsevich, Douglas; . . . ; Gaiotto, Kapustin, Seiberg, Willett '14 ; . . . ]

one application: Pure $SU(N)$ YM with $\theta = 0,\ \pi$ has higher-group symmetry $\mathbb{Z}_2^{(0)} \times \mathbb{Z}_N^{(1)}$

[GKSW '17]

*CP or T*

*center*

mixed 't Hooft anomaly for $\theta = \pi \implies$ trivial confining vacuum ruled out

*unbroken* $\mathbb{Z}_N^{(1)}$

Example of symmetry-protected topological phase (?) : Generalized symmetries play important role in topological phases of matter, quantum tensor networks . . .

- **Bounds** from **horizons**

Inspired by BH thought experiments, proven for any (non-gravitational) QFT

"spreading" of operators

i) A diagnostic of chaos in many-body $q$-systems is the early-time growth of

$$-\langle [\mathcal{O}_1(t), \mathcal{O}_2(0)]^2 \rangle_T \sim \epsilon \exp(\lambda_L t)$$

A universal *bound* $\boxed{\lambda_L \leq \frac{2\pi}{\beta}}$ [Maldacena, Shenker, Stanford '15]

saturated by AdS black holes: black holes as "optimal scramblers"

ii) Bekenstein's heuristic entropy bound: $S \leq 2\pi RE$ in region of size $R$

converted into rigorous statement ( $\Longleftarrow$ +vity of relative entropy):

$$\boxed{S(\rho_A) - S(\rho_{0,A}) \leq \langle K_A \rangle - \langle K_{0,A} \rangle} = 2\pi \int x T_{00} \quad \text{for } A \text{ the Rindler wedge}$$

[Casini '08]

iii) **Q**uantum **N**ull **E**nergy **C**ondition

NEC violated by Casimir energy & Hawking radiation, but bounded from below:

$$\boxed{n^\mu n^\nu \langle T_{\mu\nu} \rangle \geq \frac{\hbar}{2\pi\, \delta A} n^\mu n^\nu \partial_\mu \partial_\nu S_{\text{vN}}} \quad \text{for } n^2 = 0$$

[Bousso, Fisher, Leichenauer, Wall '15;
Balakrishnan, Faulkner, Khandker, Wand '17]

# 3. Q-gravity & Unification

Original hope: sharp predictions on physics **B**eyond the **S**tandard **M**odel & **B**eyond **E**instein **G**ravity, stalled due to problem of *vacuum selection/stability* and absence of supersymmetry in LHC data.

Since no **E**ffective **F**ield **T**heory could be singled out because { **anthropic** ? / not smart enough ? } (unfortunate possibility)

try at least to constrain space of *EFTs consistent with UV quantum gravity*:

**Landscape** versus **Swampland** program

[Vafa '05; . . . . ]

Swampland conjectures supported by { general arguments
**string lamp post**

ex.1 **No global symmetries** and its refinement:

**Weak Gravity** conjecture : $\exists$ particle with $M \leq |Q|$

$\Uparrow$

BHs can discharge, no remnants

[Arkani-Hamed, Motl, Nicolis, Vafa '06; . . . ]

ex.2 **Distance** conjectures : limits on scalar-field excursions (*models of inflation/quintessence*)

ex.3 Bounds on **Wilson coefficients** from dispersion relations

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06; . . . ]

- No general proofs, but point to loopholes if conjectures were not obeyed.

- Sharper with extra assumptions: *higher susy, perturbative string theory, etc.*

Ex. Type II string theory or M theory on Calabi-Yau 3-fold :

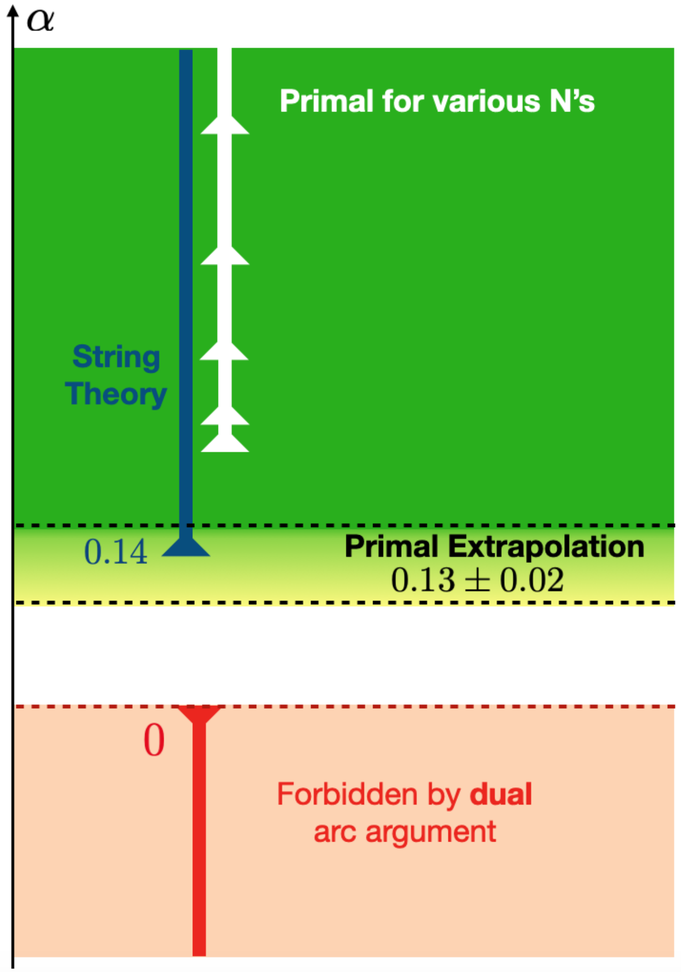
Classify all infinite-distance limits in classical moduli space, & prove

Thm: towers of light ( $\ll M_{\text{Planck}}$) modes condense as either

$$M_n \sim n\, M_{\text{KK}} \quad \text{or} \quad M_n \sim \sqrt{n}\, M_{\text{string}}$$

*decompactification* *tensionless strings*

[Lee, Lerche, Weigand '19]




## Where is String Theory?

Assume EFT = maximal supergravity in $d > 4$, constrain coefficient of $R^4$ term (or better the corresponding physical S-matrix element)

[Guerrieri, Penedones, Vieira '21]

IIB theory covers entire allowed range

$$\alpha^{\text{IIB}} = 2^{-6} E_{3/2}(\tau, \bar{\tau}) \gtrsim 0.1389$$

[Green, Gutperle, Vanhove]

***String universality** in UV ?*


*all this is nice, but one elephant still in the room:*

**No controlled vacuum with broken susy**

Difficulties identified in late 80s on persist:

- EFT: No stable $\Lambda = 0$ vacuum at weak coupling from non-perturbative effects [Dine, Seiberg]

- Worldsheet: Unlike gauge symmetries, susy continuously restaured only in $\infty$ - volume limits [Banks, Dixon; Antoniadis, CB, Lewellen, Tomaras ]

Flux [KKLT] compactifications showed a possible route, but still inconclusive

Are these just **technical** problems or **much deeper ?**

# what next ?

Theorists not  good in this -- if they really knew they would be writing a paper

but, paraphrasing Marc Mezard paraphrasing/quoting Giorgio Parisi *(on spin glasses)*

"If you bump into an interesting physics problem follow it to the end of the world"

ok if interesting physics & new  math tools keep crossing one's horizon

**Happy birthday** & many thanks

to this wonderful Institute

& those that made it work

# **Ithaka** *(by CP Cavafy)*

Translated by Edmund Keeley
and Philip Sherrard.

. . .

Keep Ithaka always in your mind.
Arriving there is what you're destined for.
But don't hurry the journey at all.
Better if it lasts for years,
so you're old by the time you reach the island,
wealthy with all you've gained on the way,
not expecting Ithaka to make you rich.

Ithaka gave you the marvelous journey.
Without her you wouldn't have set out.
She has nothing left to give you now.

hope not !

. . .