Confining Flux Tubes in a Gapless Phase

Part II

arxiv:2602.17758 w\ J. Aguilera Damia and G. Galati

Defects and Extended Excitations in Quantum Field Theory, Quantum Matter and Statistical Models”

Galileo Galilei Institute, Firenze, Italy

Giovanni Rizi (IHES) 19/05/2026

$4d\ \mathbb{CP}^1$NLSM $\quad S = \frac{2}{g^2}\int d^4x \frac{\partial_\mu \omega \partial^\mu \overline{\omega}}{(1+|\omega|^2)^2} \qquad \omega_n(z) = \frac{P(z)}{Q(z)} \qquad 2n$ moduli

Spectrum

Quadratic fluctuations: $\omega = \omega_0 + g\delta\omega \quad \omega_0 = \left(\frac{\lambda}{z}\right)^n$

Rotational symmetry $\longrightarrow H\xi_M(u) = \kappa^2 \xi_M(u) \qquad M \in \mathbb{Z}\,, u = |z|/\lambda$

$$H = A^\dagger A \qquad A = \frac{d}{du} - \frac{1}{u}(M+1/2) - \frac{2n}{u(u^{2n}+1)} \longrightarrow \kappa^2 \geq 0$$

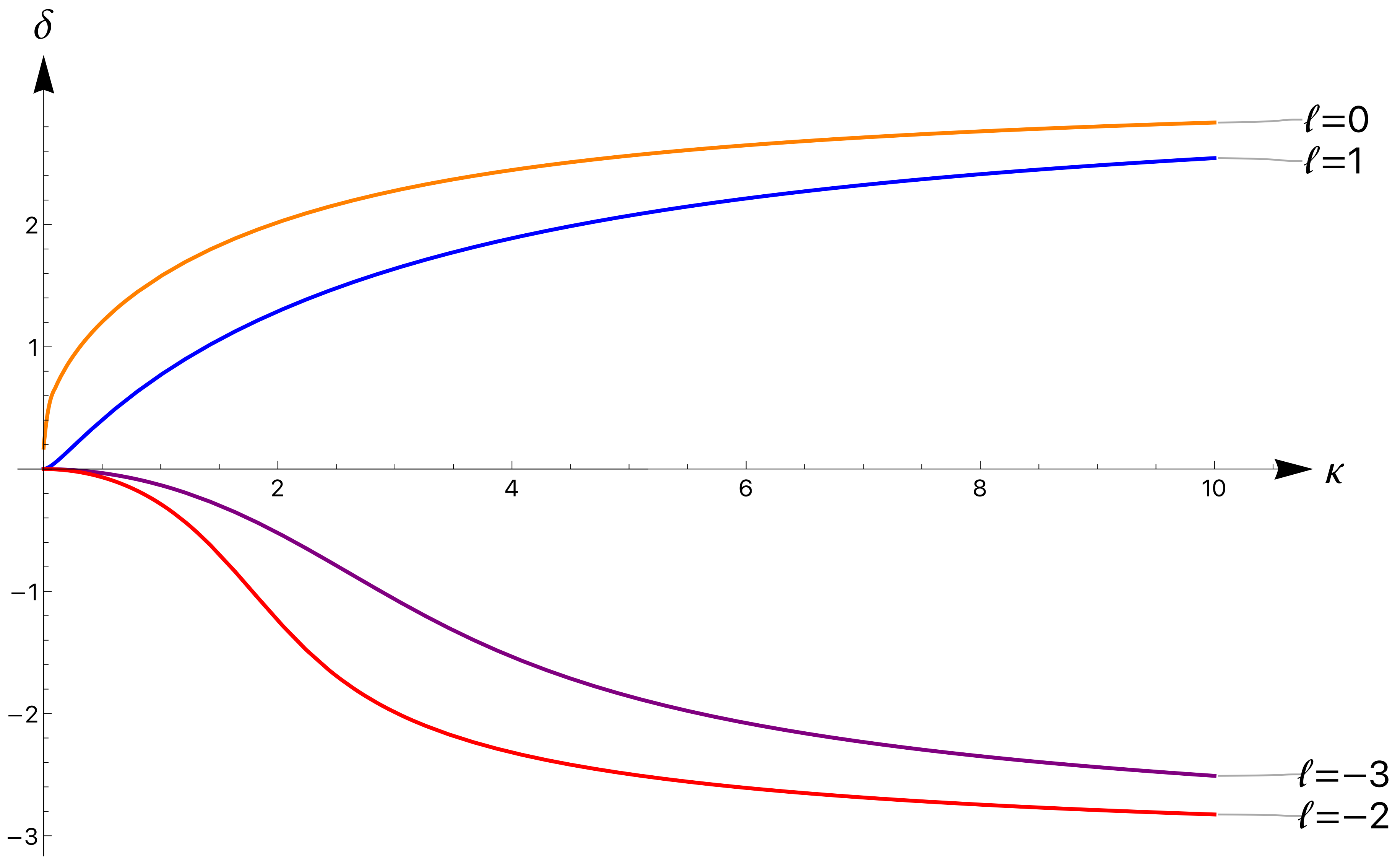
- $\kappa = 0$ Zero Modes $\delta\omega = z^M$
  - $M = -2,.., -2n$ Normalizable
  - $M = -1$ Non-Normalizable

$$\omega_n(z) \simeq \frac{\lambda}{z} + O(z^{-2})$$

λ frozen in infinite volume

[Intrilligator, Seiberg ’13]

- $\kappa^2 > 0$ $u \gg 1$ $\xi_M(u) \simeq A_{n,M}(\kappa)(J_{|M|}(\kappa u) + \tan(\delta_{n,M}(\kappa))Y_{|M|}(\kappa u))$



Phase shifts

Ground state energy Charge $n = 1$ string of length L $\omega_0 = \frac{\lambda}{z}$

Classical action

String tension correction

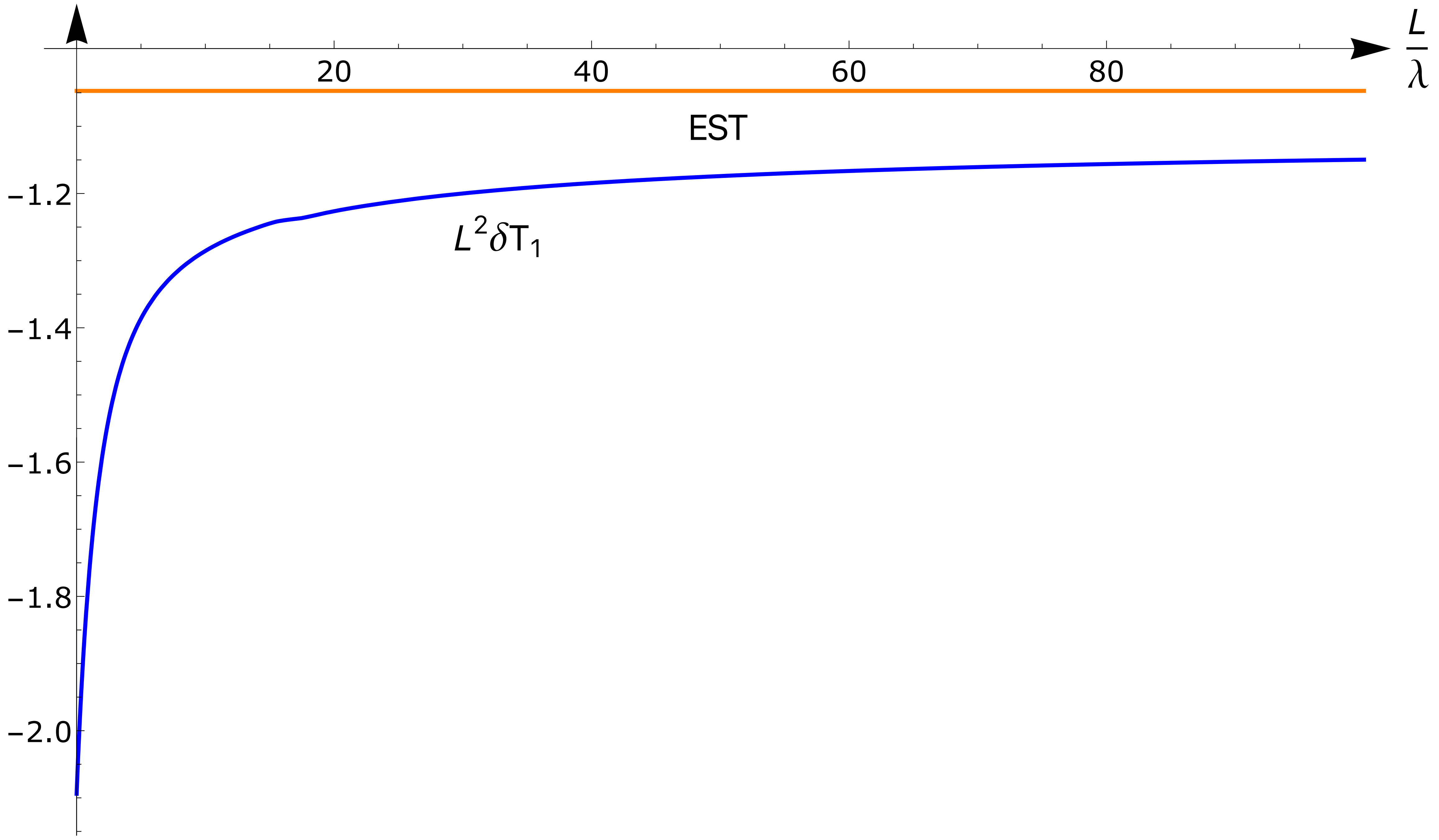
$$E_1(L) = \frac{L}{g^2}\left(4\pi + \delta T_1\right) + \delta E_1(L)$$

$$\delta T_1 = \frac{g^2}{\lambda^2}\alpha$$

Finite size corrections

$$\delta E_1(L) = -\frac{\pi}{3L}\left[1 + J(L/\lambda)\right]$$

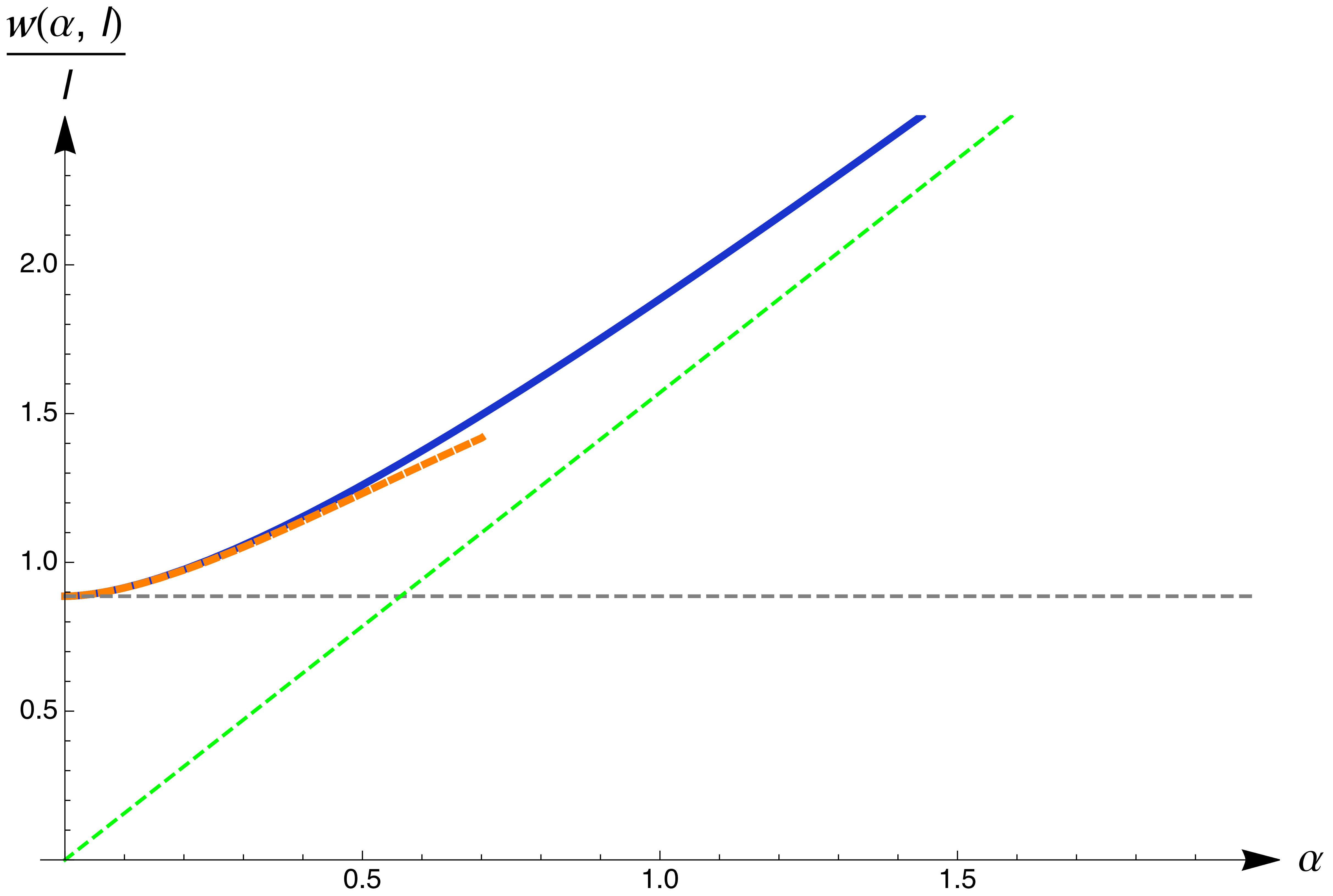
$$J(x) \sim \frac{1}{\log(x)}, \quad x \gg 1$$



String width

$$\mathscr{W} = \frac{\int d^2x_\perp |x_\perp| \langle E_\| \rangle}{\int d^2x_\perp \langle E_\| \rangle}$$

$$E_\| = J_{0\|} = \frac{\lambda^2}{((z-z_0)^2+\lambda^2)^2}$$



$$\mathscr{W}_{\text{EST}} = \frac{\int d^2x_\perp |x_\perp| e^{-\frac{x_\perp^2}{l^2}}}{\int d^2x_\perp e^{-\frac{x_\perp^2}{l^2}}} = \frac{\sqrt{\pi}}{2} l$$

$$\mathscr{W} \simeq \frac{\sqrt{\pi}}{2} l + O(\alpha^2) \qquad \alpha = \frac{\lambda}{l}$$

Final Remarks

UV completions:

[Córdova, Dumitrescu '24]

- $SU(N_c)$ Adjoint QCD with $N_f = 2$ Weyl fermions $\quad \dfrac{SU(2) \times \mathbb{Z}_{4N_c}}{\mathbb{Z}_2} \to U(1) \rtimes \mathbb{Z}_2$

  Large N_c: $g^{-1} \sim N_c \Lambda_{QCD} \to T \sim 1/g^2 \sim N_c^2 \Lambda_{QCD}^2$ for $\mathbb{CP}^1$ strings

  Inconsistent with expected scaling $T \sim \Lambda_{QCD}^2$

[Witten '83]

- $Spin(N_c)$ QCD with $N_f = 2$ Weyl fermions in vector irrep. Same SSB pattern

  Large N_c: $g_{SO}^{-2} \sim N_c \Lambda_{QCD}^2 \to T \sim N_c \Lambda_{QCD}^2$ for $\mathbb{CP}^1$ strings

  Consistent with expected scaling $T \sim N_c \Lambda_{QCD}^2$

Thank you !