The state/defect correspondence

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based on
work to appear soon
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(+ 2406.02662 + 2507.01104)
w/ D. M. Hofman

📍 Defects and Extended Excitations in Quantum Field Theory,
Quantum Matter and Statistical Models
GGI – 19.05.2026

The state/operator correspondence relates states of a CFT to local operators.

The state/operator correspondence relates states of a QFT to extended operators.*

*Terms and conditions apply

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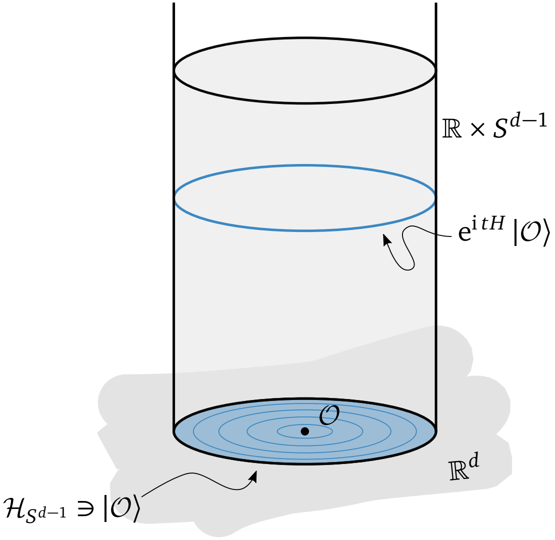
State-operator correspondence in CFT

local operators on $\mathbb{R}^d$ $\Longleftrightarrow$ states on S^{d-1}



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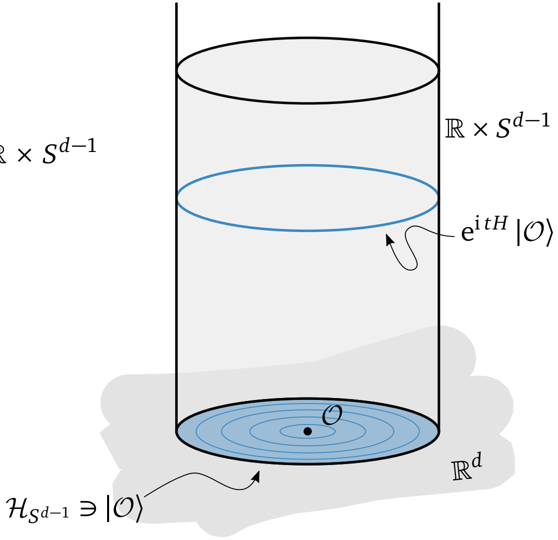

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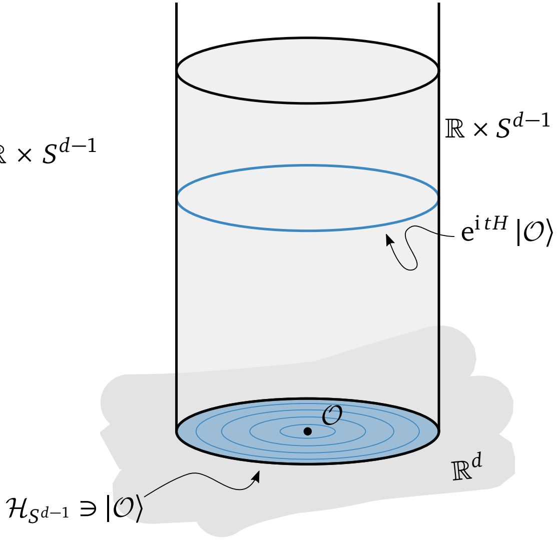

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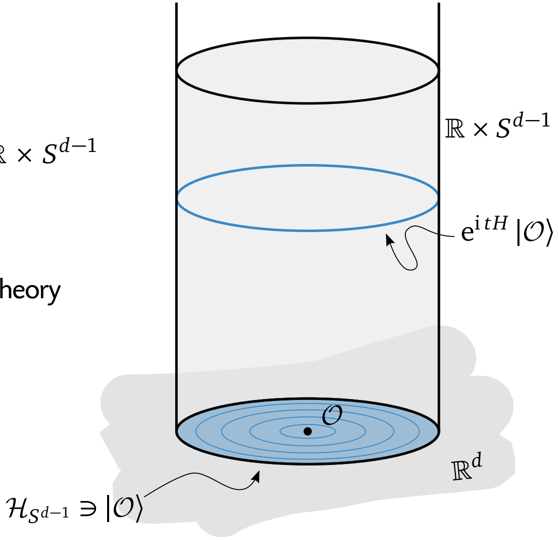
Important for

- Consistency of CFT [Moore, Seiberg '89] $\Longrightarrow$ string theory
- Bootstrap
- Holography
- Entanglement entropy [Calabrese, Cardy '09]
- Large charge [Hellerman et al. '15; Monin et al. '16]
- Fuzzy sphere [Zhu, Han, Huffman, Hofmann, He '23]
- ...



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(A) Doesn't work in non-conformal theories

operator $\Longrightarrow$ state *e.g.* $\mathcal{O}(0)\,|vac\rangle = |\mathcal{O}\rangle$ *or* 

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Workaround for (B): *In 4d CFTs with continuous one-form symmetries* [Hofman, **SV** '24]

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Today: *combine both*

QFTs with $\mathrm{U}(1) \times \mathrm{U}(1)$ *symmetry + anomaly*

⇣

infinitely many charges

⇣

Kac–Moody algebra

⇣

state/defect correspondence

Consider $\mathrm{U}(1)^{[p]} \times \mathrm{U}(1)^{[d-p-2]}$ symmetry + anomaly $\frac{\mathrm{i}}{2\pi}\int \mathcal{B} \wedge \mathrm{d}\mathcal{A}$

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$$S = \int a \wedge G(f) + \int \star \mathrm{P}\left(f^2, f\tilde{f}\right) + \text{higher derivatives}$$

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Allowed only when $d = (n+1)p + n$
Gives Chern-Weil symmetry
[Heidenreich et al. '20]

topological term
$G(f) \sim f \wedge f \wedge \cdots \wedge f$

"theta" term
(only when $d = 2p + 2$)

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It's non-linear QED [Heisenberg, Euler; '37] $\leftrightsquigarrow$ higher-form superfluid [Armas, Jain; '18]

P is the pressure $\rightsquigarrow$ determined by the equation of state/UV microscopics $\rightsquigarrow$ not fixed by symmetry

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Alternative to Goldstone theorem [Delacrétaz, Hofman, Mathys '19]

p-form $\mathrm{U}(1)$ symmetry

Current: $J_{[p+1]} = \mathrm{P}'\left(f^2\right) f_{[p]}$

Counts electrically charged defects

Conserved on-shell

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$$\begin{aligned} \mathrm{d}\star J_{[p]} &= 0 \\ \mathrm{d}\star \widetilde{J}_{[d-p-1]} &= -\mathcal{F}_{[p+2]} \end{aligned} \iff S_{\text{anom}} = \frac{\mathrm{i}}{2\pi}\int_{d+1} \mathcal{B}_{[d-p-1]} \wedge \mathrm{d}\mathcal{A}_{[p+1]}$$

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Very similar to *axial* anomaly in 2d *(more similarities later)*

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solutions guaranteed by [Witten '82]

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Kac–Moody-*like* algebra of conserved charges:

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$d = 2,\ p = 0$: Kac–Moody algebra of compact scalar CFT ⇝ mode-expand: $[Q_n, Q_m] = k\, n \delta_{n+m,0}$

$d = 4,\ p = 1$: Kac–Moody in "photonised" CFT_4 [Hofman, Iqbal '18]

recent application for soft amplitudes [Tizzano '26]

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generic $d,\ p = 0$: *non-conformal* Kac–Moody in superfluids [**SV** '25]

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here generic d, generic p, non-conformal

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Charges $\mathcal{Q}_\eta[\Sigma_{d-1}] \rightsquigarrow$ expand in a nice basis: $\mathcal{Q}_{n\sigma}$:

$\sqrt{\text{eigenvalue of Laplacian on } \Sigma}$

$$[\mathcal{Q}_{n\sigma}, \mathcal{Q}_{m\sigma'}] = i\sigma\sqrt{\lambda_n}\,\delta_{n,m}\delta_{\sigma,-\sigma'}$$

a parity label $\sigma = \pm$

Quantise on $\Sigma_{d-1} = S^p \times S^{d-p-1}$

can quantise generally; this is just a convenient choice

Hamiltonian takes Sugawara form: $$H \sim J^2 + \widetilde{J}^2 \sim Q_{\mathrm{e}}^2 + \sum_{n \neq 0} A_n^\dagger A_n$$

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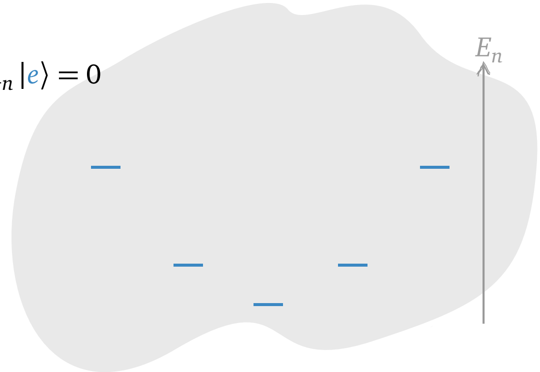
$d{=}2p{+}2$ *has also magnetic charge; ignore for now will patch later*

$d{=}2p{+}1$ *has more electric charges; more subtle*

States

Primary: $Q_{\mathrm{e}} \left|e\right\rangle = \left(\frac{g^2 V_p}{V_{d-p-1}}\right)^{1/2} e \left|e\right\rangle$ and $A_n \left|e\right\rangle = 0$

$e \in \mathbb{Z}$ from Dirac quantisation



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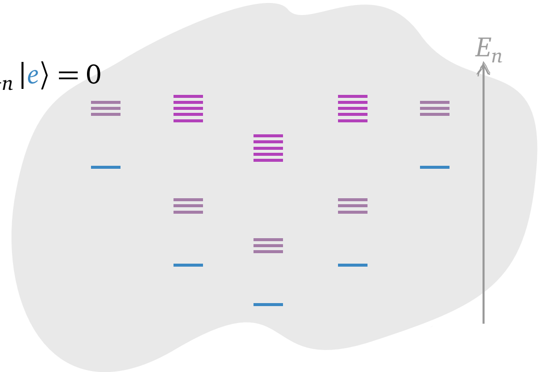

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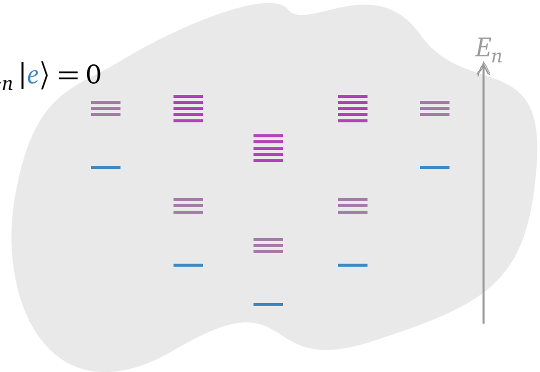

Hilbert space realises representation theory of Kac–Moody algebra [Fliss, **SV** '23]

States done. Operators:

Wilson defects: $W_e(\gamma_p) = \mathrm{e}^{\mathrm{i}\, e \int_{\gamma_p} a}$ *definite electric charge* $U_\alpha W_e = \mathrm{e}^{\mathrm{i}\, e\, \alpha}\, W_e \quad e \in \mathbb{Z}$

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can be viewed as Wilson defects of magnetic dual theory; ignore

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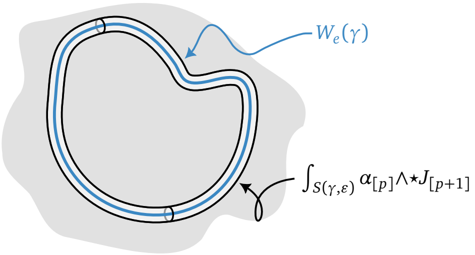
Equivalent description as "disorder" operators *e.g.*

$$\mathcal{D}_{\alpha,e}(\gamma_p) = \lim_{\varepsilon\to 0} \int_{S(\gamma,\varepsilon)} \alpha_{[p]} \wedge \star J_{[p+1]} \times W_e(\gamma) =$$



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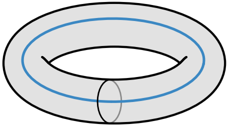
$$|\mathcal{D}\rangle := \int \mathrm{D}a\, \mathrm{e}^{-S[a]}\, \mathcal{D}(S^p \times \{0\}) =$$ 

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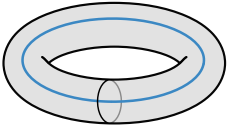

Charges act by surrounding

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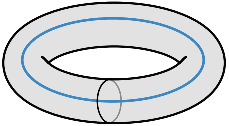
$$|\mathcal{D}\rangle := \int \mathrm{D}a\, \mathrm{e}^{-S[a]}\, \mathcal{D}(S^p \times \{0\}) =$$ 

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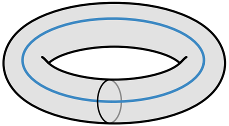

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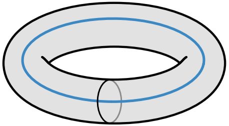

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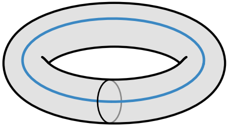

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Overall

states on $S^p \times S^{d-p-1}$
in one-to-one correspondence with
p-dimensional defects on S^p

Recap

QFTs with $\mathrm{U}(1)^{[p]} \times \mathrm{U}(1)^{[d-p-2]}$

symmetry with mixed anomaly

⇣

are higher-form superfluids

⇣

have infinitely many charges

⇣

satisfy Kac–Moody

⇣

state/defect correspondence

Now what?

Gapped theories

Add Chern-Simons term, gaps out theory

$\mathrm{U}(1)^{[p]} \to \mathbb{Z}_k^{[p]}$ but Kac–Moody survives!

state/defect correspondence? [WIP]

More general symmetries

Non-abelian Kac–Moody in higher d?

Non-invertible? [Hofman, **SV** '24]

Biform and higher-spin symmetries (gravity) [Hinterbichler et al '22; '24]

Tool for entanglement?

Other applications?

Thank you!