Field theories for (non-Abelian) anyon condensation transitions

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Motivation

Gapped phases with symmetry S ⟷ **Topological manipulations** with S

Example: bosonic group symmetry

$\mathsf{S} = (G, \omega), \qquad \omega \in H^{d+1}(G, U(1))$

anomaly-free $H \subset G, \qquad \alpha \in H^{d}(H, U(1))$

1. Gapped phase

$H =$ preserved subgroup

$\alpha =$ SPT phase

2. Topological manipulation

$H =$ gauged subgroup

$\alpha =$ discrete torsion

Motivation

One side is purely topological, while the other is closer to dynamics:

phase transitions between gapped phases

- Topological manipulations:

$$M_1,\ M_2$$

- Corresponding gapped phases:

$$\mathcal{P}(M_1), \qquad \mathcal{P}(M_2)$$

- What is the field theory describing a phase transition

$$\mathcal{P}(M_1) \longrightarrow \mathcal{P}(M_2)\,?$$

Gapped phases in (2+1)d

This talk: *phases in (2+1)d*

- Gapped phases are described by MTCs
- Topological manipulations are ”connected commutative separable algebras” or ”Condensable algebras”

$$\mathcal{A} = \bigoplus_a n_a X_a \ , \qquad X_a = \text{bosons}$$

- TQFTs: $X_a \longleftrightarrow$ topological lines generating anomaly-free 1-form symmetry.
- Lattice models: $X_a \longleftrightarrow$ particles that proliferate in the gapped phase.
- Field theory?
  related recent works [Ji, Lanzetta, Zhou, Wang ’26; Cheng, Seiberg ’26]

A first guess

MTC $\mathcal{T} \;\simeq\; G_k$ Chern–Simons

$$X_a \;\simeq\; W_{R_a} := \mathrm{Tr}_{R_a} \mathcal{P} \exp\left(i \oint A \right) \qquad R_a = \text{integrable irrep of } \mathfrak{g}_k$$

$$S = S_{\mathrm{CS}_{G_k}}(A) + \int \sum_a \left| (\partial - i A^\alpha T^\alpha_{R_a}) \Phi_a \right|^2 + V(\{\Phi_a\})$$

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Doesn't work:

$$X_a \simeq W_{R_a}$$

breaks once we couple with scalars.

Example: $\mathbb{Z}_4$ gauge theory

$$S = \frac{4i}{2\pi}\int a \wedge db \qquad G = U(1)^2, \qquad K = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$$

$$e \equiv e^{i\oint a}, \qquad m \equiv e^{i\oint b}$$

$$\mathcal{A}_{e^2} = 1 \oplus e^2 \rightsquigarrow \qquad \frac{4i}{2\pi}\int a\,db + \int |D_{2a}\phi_{e^2}|^2 + V(\phi_{e^2})$$

$m^2 > 0$:

$$\mathcal{T}$$

$m^2 < 0$:

$$\underbrace{\frac{4i}{2\pi}\int a\,db + \frac{i}{2\pi}\int c\,d(2a)}_{(c\mapsto c-2b)}$$

$$\frac{2i}{2\pi}\int c\,da = \mathcal{T}/\mathcal{A}_{e^2}$$

Similarly for $\mathcal{A}_{m^2} = 1 \oplus m^2$: $\qquad \frac{4i}{2\pi}\int a\,db + \int |D_{2b}\phi_{m^2}|^2 + V(\phi_{m^2})$

What about $\mathcal{A}_{e^2,m^2} = 1 \oplus e^2 \oplus m^2 \oplus e^2m^2$?

$$S = \frac{4i}{2\pi}\int a\, db + \int \left(|D_{2a}\phi_{e^2}|^2 + |D_{2b}\phi_{m^2}|^2 + V(\phi_{e^2}, \phi_{m^2})\right)$$

doesn't work: $\langle \phi_{e^2} \rangle, \langle \phi_{m^2} \rangle \neq 0 \quad \Longrightarrow \quad \mathbb{Z}_2 \times \mathbb{Z}_2$ gauge theory

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$$G = U(1)^4, \quad K = \begin{pmatrix} 2 & 2 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}, \quad e^2 \equiv e^{i\oint \widetilde{a}}, \ m^2 \equiv e^{i\oint(\widetilde{a}+c)}, \ e^2m^2 \equiv e^{i\oint c}$$

$$\frac{2i}{2\pi}\widetilde{a}\,d\widetilde{b} + \frac{i}{2\pi}c\,d\widetilde{a} + \frac{2i}{2\pi}x\,dc + \frac{i}{2\pi}\widetilde{a}\,d\widetilde{a} + |D_{\widetilde{a}}\phi_{e^2}|^2 + |D_{\widetilde{c}}\phi_{e^2m^2}|^2 + V$$

- $\langle\phi_{e^2}\rangle \neq 0: \quad \frac{2i}{2\pi}\int x\,dc \ \equiv \ (\mathbb{Z}_2)_0 \equiv \mathcal{T}/\mathcal{A}_{e^2}$
- $\langle\phi_{e^2m^2}\rangle \neq 0: \quad \frac{2i}{2\pi}\int \widetilde{a}\,db + \frac{i}{2\pi}\int \widetilde{a}\,d\widetilde{a} \ \equiv \ (\mathbb{Z}_2)_1 \equiv \mathcal{T}/\mathcal{A}_{e^2m^2}$

Lessons

- The dual description is not out of the blue: $\widetilde{a}$ and c both appear as ”discrete gauge fields”.
- General for Abelian: $\forall\ U(1)^r$ and $K \in M_{r.r}(\mathbb{Z})$, for any algebra $\mathcal{A}$ we can find a dual description $U(1)^{r'}$, K' that can be coupled to Φ to get the transition $\mathcal{T} \mapsto \mathcal{T}/\mathcal{A}$.
- In the dual description $X_a \in \mathcal{A}$ are Wilson lines for ”discrete gauge fields”.
- What is the analogue of a discrete gauge field in the case of non-Abelian anyons?

Closest analogue is $\mathrm{TV}_{\mathcal{C}}$, with $\mathcal{C}$ a fusion category with the objects ”confined” by $\mathcal{A}$.

Non-Abelian anyons

Consider $SU(2)_3 \times SU(2)_{-3}$.

$$(0,0)$$

$$\mathcal{A}_1 = (0,0) \oplus \left(\frac{3}{2},\frac{3}{2}\right)$$

$$\mathcal{A}_2 = (0,0) \oplus (1,1)$$

$$\mathcal{L} = (0,0) \oplus \left(\frac{1}{2},\frac{1}{2}\right) \oplus (1,1) \oplus \left(\frac{3}{2},\frac{3}{2}\right)$$

$$\mathcal{T}/\mathcal{A}_1 = \text{Fib} \otimes \overline{\text{Fib}}\,, \qquad \mathcal{T}/\mathcal{A}_2 = (\mathbb{Z}_2)_1\,, \qquad \mathcal{T}/\mathcal{L} = 1$$

$\mathcal{T}/\mathcal{L} = 1$

$$SU(2)_3 \times SU(2)_{-3} \; + \; \underbrace{\Phi \in (1/2, 1/2)}_{} \qquad V(\Phi) = \lambda\Big(\mathrm{Tr}(\Phi^\dagger \Phi) - v^2\Big)^2$$

2×2 matrix $\begin{pmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{pmatrix}$

Higgs phase: $\langle \Phi_{ij} \rangle \sim v\, \delta_{ij}$

↓

$SU(2)_{\mathrm{diag}}$

↓ confinement

trivial

$\mathcal{T}/\mathcal{L} = 1$

$$SU(2)_3 \times SU(2)_{-3} \; + \; \underbrace{\Phi \in (1/2, 1/2)}_{2\times 2 \text{ matrix} \begin{pmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{pmatrix}} \qquad V(\Phi) = \lambda\Big(\mathrm{Tr}(\Phi^\dagger \Phi) - v^2\Big)^2$$

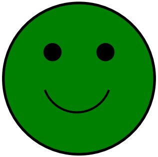
Higgs phase: $\langle \Phi_{ij} \rangle \sim v\,\delta_{ij}$

$\downarrow$

$SU(2)_{\text{diag}}$

$\downarrow$ confinement

trivial



$\mathcal{T}/\mathcal{A}_2 = (\mathbb{Z}_2)_1$

$$SU(2)_3 \times SU(2)_{-3} \; + \; \underbrace{\Phi \in (1,1)}_{3\times 3 \text{ real matrix}} \qquad V(\Phi) = \lambda\Big(\mathrm{Tr}(\Phi^{\mathsf{T}}\Phi) - v^2\Big)^2$$

Higgs phase: $\langle \Phi_{ab} \rangle \neq 0$

$$\rightsquigarrow$$

$$(\mathbb{Z}_2)_1 \times (\mathbb{Z}_2)_1$$

$\mathcal{T}/\mathcal{A}_2 = (\mathbb{Z}_2)_1$

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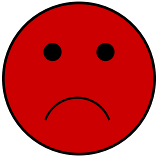
$$(\mathbb{Z}_2)_1 \times (\mathbb{Z}_2)_1$$



$\mathcal{T}/\mathcal{A}_2 = (\mathbb{Z}_2)_1$ and $\mathcal{T}/\mathcal{A}_1 = \text{Fib} \otimes \overline{\text{Fib}}$

No rep of $SU(2) \times SU(2)$ works ...?

$\mathcal{T}/\mathcal{A}_2 = (\mathbb{Z}_2)_1$ and $\mathcal{T}/\mathcal{A}_1 = \text{Fib} \otimes \overline{\text{Fib}}$

No rep of $SU(2) \times SU(2)$ works ...?

Duality: $\boxed{SU(2)_3 \simeq (G_2)_1 \otimes U(1)_{-2}}$

$$\longrightarrow SU(2)_3 \times SU(2)_{-3} \simeq (G_2)_1 \times (G_2)_{-1} \times U(1)^2_K \,, \quad K = \begin{pmatrix} 2 & 2 \\ 2 & 0 \end{pmatrix}$$

$$S = \frac{i}{4\pi} \int \text{Tr} \left(x\, dx - \frac{2}{3} x^3 \right) - \frac{i}{4\pi} \int \text{Tr} \left(y\, dy - \frac{2}{3} y^3 \right)$$
$$+ \frac{2i}{2\pi} \int a\, db + \frac{i}{2\pi} \int a\, da + \int |D_a \Phi|^2 + |D_{x,y} \Psi_{(\mathbf{7},\mathbf{7})}|^2 + V(\Phi, \Psi)$$

- $\langle \Psi \rangle \neq 0 \quad \Longrightarrow \quad U(1)^2_K \cong (\mathbb{Z}_2)_1 = \mathcal{T}/\mathcal{A}_2$
- $\langle \Phi \rangle \neq 0 \quad \Longrightarrow \quad (G_2)_1 \otimes (G_2)_{-1} \cong \text{Fib} \otimes \overline{\text{Fib}} = \mathcal{T}/\mathcal{A}_1$

Conclusions

- The identification of anyons with scalar fields depend on the choice of duality frame.
- The condensed bosons must be realized from ”TV-gauge fields”.
- While less systematic, many non-Abelian examples can be worked out explicitly (some guess-work required).
- Chiral TQFTs
- New 3d dualities?
- Use these field theories to learn about the phase transitions?