



Monodromy defects in Chern-Simons

Symmetries, defects, and holography

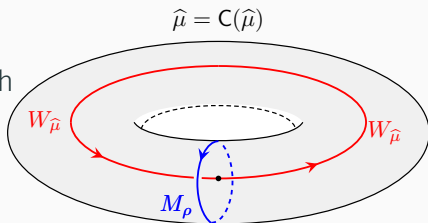
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[Perimeter Institute]

May 26, 2026

Defects and Extended Excitations, GGI

Based on work to appear soon with
Jaume Gomis & Suriyah Kannagi



Topological theory in 2+1 d

Gauge group G , level k

$$S_{\text{CS}}[A] = \frac{k}{4\pi} \int_{\mathcal{M}^{(3)}} \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

No local operators


Line operators : $W_R(\mathcal{K}) = \text{Tr}_R \mathcal{P} \exp \oint_{\mathcal{K}} A$

Topological lines \leftrightarrow 1-form symmetry $\mathcal{Z}(G)$

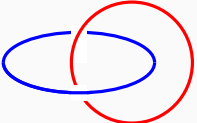
Link and knots

Interesting observables on $\mathcal{M}^{(3)} = S^3$

Knots \leftrightarrow VEV $\langle W_R(\mathcal{K}) \rangle_{S^3} = \left\langle \text{img} \right\rangle$



Links $\langle W_{R_1}(\mathcal{K}_1) W_{R_2}(\mathcal{K}_2) \rangle_{S^3} = \left\langle \text{img} \right\rangle$



3D TFT- 2d CFT correspondence

Computed by 2d CFT!

$$\boxed{\text{CS}(G)_k \longleftrightarrow G_k \quad \text{WZW-model}}$$

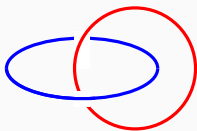
$$\text{(Un)knots} \quad \langle W_R(S^1) \rangle = \left\langle \bigcirc \right\rangle = \boxed{\frac{S_{R0}}{S_{00}}}$$

$$\text{Hopf} \quad \langle W_{R_1}(S^1) W_{R_2}(S^1) \rangle = \left\langle \bigcirc \bigcirc \right\rangle = \boxed{\frac{S_{R_1, R_2}}{S_{00}}}$$

$$S_{ij} = \text{Modular S-Matrix of } \widehat{\mathfrak{g}}_k^{(1)}$$

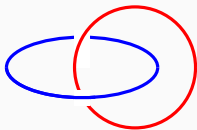
Symmetries of CS theory

1-form symmetry \longleftrightarrow Topological lines



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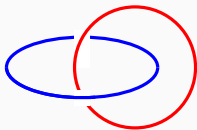
Today:

Charge conjugation

0-form symmetry \longleftrightarrow Topological surfaces!

Symmetries of CS theory

1-form symmetry \longleftrightarrow Topological lines



Today:

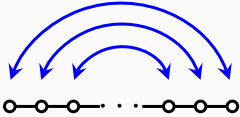
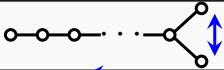
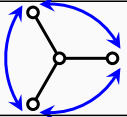
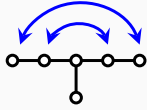
Charge conjugation

0-form symmetry \longleftrightarrow Topological surfaces!

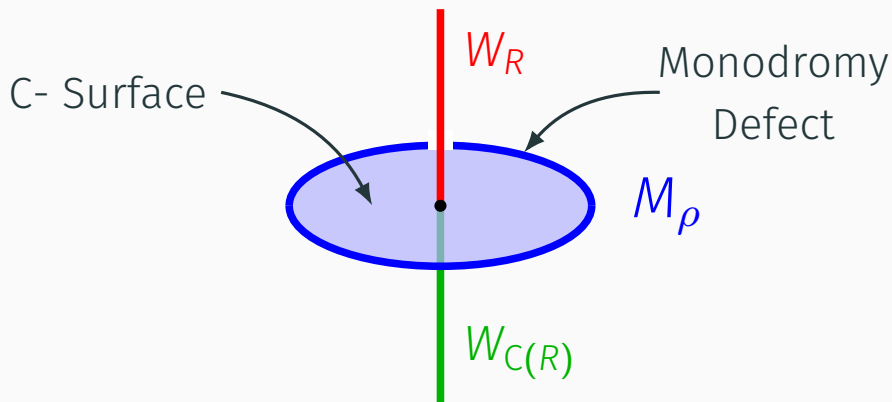
New observables in CS theory!

Charge conjugation symmetry

Charge conjugation $C \leftrightarrow \text{Out}(\mathfrak{g})$

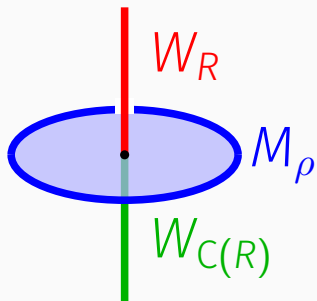
\mathfrak{g}	$\text{Out}(\mathfrak{g})$	Dynkin diagram
$\mathfrak{su}(N) \quad N > 2$	\mathbb{Z}_2	
$\mathfrak{so}(2N)$	\mathbb{Z}_2	
$\mathfrak{so}(8)$	S_3	
\mathfrak{e}_6	\mathbb{Z}_2	

Topological operators of charge conjugation



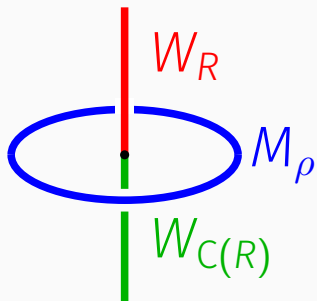
Monodromy defects! (Cod 2)

Monodromy defects:



- ★ Monodromy around M_ρ :
$$A_\mu(\phi + 2\pi i) = C \cdot A_\mu(\phi)$$
- ★ M_ρ line defects of CS
 - How to compute with them?
- ★ CS theory \leftrightarrow Top Strings
 - Fit in the duality?

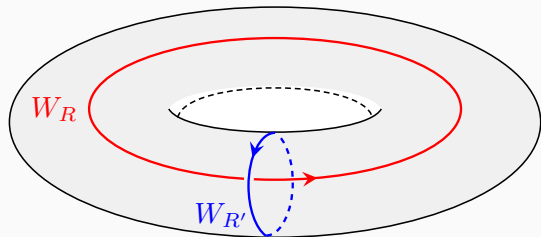
Monodromy defects:



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Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2
No monodromy defects:



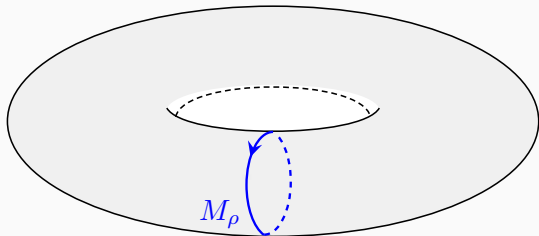
$$\mathcal{H}^{(1,1)} \leftrightarrow \widehat{\mathfrak{g}}_k^{(1)}$$

$$R, R' \subset \text{Int. rep } (\widehat{\mathfrak{g}}_k^{(1)})$$

$$\text{Modular-S } (\widehat{\mathfrak{g}}_k^{(1)})$$

Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2
Monodromy defect wrapping A-cycle



$$\mathcal{H}^{(C,1)} \leftrightarrow \widehat{\mathfrak{g}}_k^{(2)} \\ \simeq \text{Conf.blocks}(\mathfrak{g}_k^{(2)})$$

C-twisted boundary conditions on B cycle

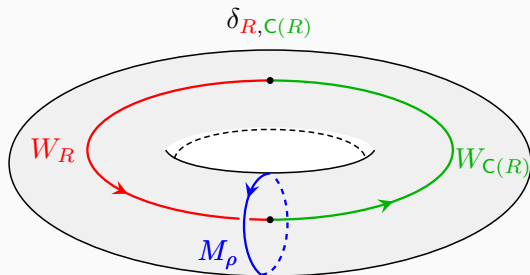
$$\mathcal{J}^\alpha(z e^{2\pi i}) = C(\mathcal{J}^\alpha(z))$$

Monodromy = New line defects \leftrightarrow Int. Rep($\widehat{\mathfrak{g}}^{(2)}$)

Monodromy defects from twisted 2d CFT

Quantization of CS on solid T^2

Monodromy defect wrapping A-cycle, Wilson on B



$$\begin{aligned} \mathcal{H}^{(C,1)} &\leftrightarrow \widehat{\mathfrak{g}}_k^{(2)} \\ &\simeq \text{Conf.blocks}(\widehat{\mathfrak{g}}_k^{(2)}) \\ &\neq 0 \text{ iff } R = C(R) \end{aligned}$$

C-twisted boundary conditions on B cycle

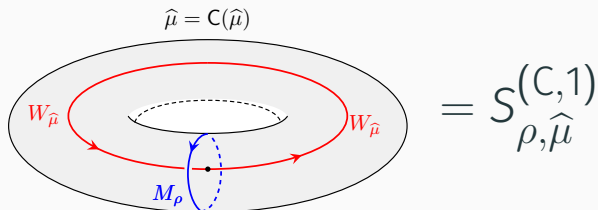
$$\mathcal{J}^\alpha(z e^{2\pi i}) = C(\mathcal{J}^\alpha(z))$$

Monodromy = New line defects \leftrightarrow Int. Rep($\widehat{\mathfrak{g}}^{(2)}$)

Twisted affine Lie algebras

$$\mathcal{H}^{(C,1)} \simeq \text{Conf.blocks}(\widehat{\mathfrak{g}}_k^{(2)})$$

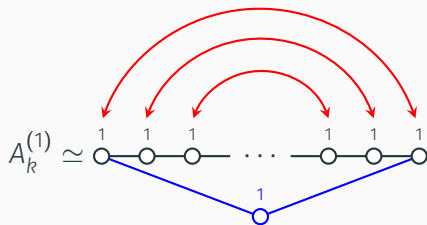
Twisted affine $\widehat{\mathfrak{g}}_k^{(2)}$ are classified [Fuchs, Schweigert][Gaberdiel, Gannon][Kac]



- $\widehat{\mu}$: C-invariant weight of $\widehat{\mathfrak{g}}^{(1)} \sim$ Wilson Lines that can link
- ρ : Int. Rep($\widehat{\mathfrak{g}}_k^{(2)}$) \sim Monodromy defects

$\widehat{\mathfrak{g}}^{(2)}$ obtained by folding

Twisted affine Lie algebras: SU(N) example



Fold by automorphism:

$$A_{2m-1}^{(1)} \mapsto D_{m+1}^{(2)} : \quad \begin{array}{c} 1 \quad 2 \quad 2 \quad \dots \quad 2 \quad 2 \quad 1 \\ \circ \rightrightarrows \circ - \circ \quad \dots \quad - \circ - \circ \leftleftarrows \circ \end{array}$$

$$A_{2m}^{(1)} \mapsto A_{2m}^{(2)} : \quad \begin{array}{c} 1 \quad 2 \quad 2 \quad \dots \quad 2 \quad 2 \quad 2 \\ \circ \rightrightarrows \circ - \circ \quad \dots \quad - \circ - \circ \rightrightarrows \circ \end{array}$$

Some more details

$\mathfrak{g}^{(1)}$	(1, 1)	(C, 1)	(1, C)	(C, C)
$A_{2n-1}^{(1)} (n \geq 2)$	$A_{2n-1}^{(1)}$	$A_{2n-1}^{(2)}$	$D_{n+1}^{(2)}$	$B_n^{(1)}$
$A_{2n}^{(1)} (n \geq 1)$	$A_{2n}^{(1)}$	$A_{2n}^{(2)}$	$A_{2n}^{(2)}$	$A_{2n}^{(2)}$
$D_n^{(1)} (n \geq 4)$	$D_n^{(1)}$	$D_n^{(2)}$	$A_{2n-3}^{(2)}$	$C_{n-1}^{(1)}$
$E_6^{(1)}$	$E_6^{(1)}$	$E_6^{(2)}$	$E_6^{(2)}$	$F_4^{(1)}$

Some more details

$\mathfrak{g}^{(1)}$	(1, 1)	(C, 1)	(1, C)	(C, C)
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$A_{2n}^{(1)} (n \geq 1)$	$A_{2n}^{(1)}$	$A_{2n}^{(2)}$	$A_{2n}^{(2)}$	$A_{2n}^{(2)}$
$D_n^{(1)} (n \geq 4)$	$D_n^{(1)}$	$D_n^{(2)}$	$A_{2n-3}^{(2)}$	$C_{n-1}^{(1)}$
$E_6^{(1)}$	$E_6^{(1)}$	$E_6^{(2)}$	$E_6^{(2)}$	$F_4^{(1)}$

With exact results comes hard questions

Now we know how to compute exactly with monodromy defects

What can we learn about holography?

$SU(N)_k$ CS on $S^3 \leftrightarrow$ Top. Strings on Resolved Conifold

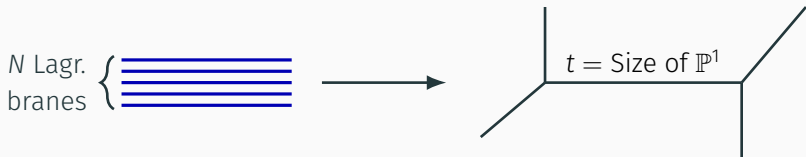
$$g_s = \frac{-2\pi i}{k + N}, \quad \text{'t Hooft: } t = Ng_s \text{ fixed}$$



Originally at Large N , even at finite!

Holographic duality

N Lagrangian branes wrapping $S^3 \rightarrow$ geometry



$$\log S_{0,0}^{SU(N)} = (\text{non-per}) + \sum_{m>0} \frac{e^{-mt}}{m[m]^2} = \log \mathcal{Z}_{\text{con}}$$

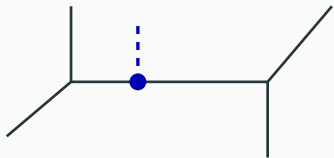
Partition function of the Resolved Conifold!

Count maps from $\Sigma_g \rightarrow \mathbb{C}Y_3$

Holographic duality

N Langrangian branes wrapping $S^3 \rightarrow$ geometry

We can add branes to the geometry:



$$N \rightarrow N + 1 \quad t \rightarrow t + g_s$$

$$x \sim n$$

$$\log S_{\Lambda^n, 0}^{SU(N)} = \sum_{m>0} \frac{e^{-m\hat{t}}}{m[m]^2} + \sum_{m>0} \frac{e^{-mx} + e^{-m(\hat{t}-x)}}{m[m]} = \log \mathcal{Z}_{\text{con}} + \mathcal{Z}_{\text{open}}$$

Adding orientifold planes

Projection by



[Hori, Vafa][Sinha,Vafa]

$\sigma_- : CY_3 \rightarrow CY_3$ target anti-hol inv. $\omega(z) = -1/\omega(\bar{z})$ on \mathbb{P}^1



$$\log S_{00}^{SO/Sp} = \frac{1}{2} \underbrace{\sum_m \frac{e^{-\hat{m}t}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ odd}} \frac{e^{-\hat{m}t/2}}{m[m]}}_{\text{not-oriented}}$$

$$SU CS \rightarrow SO/Sp CS \quad (\hat{t} = t \mp g_s)$$

maps $\Sigma \rightarrow CY_3$ and unoriented equivariant $\Sigma/\sigma_- \rightarrow CY_3$
(e.g $\mathbb{RP}^2 \rightarrow CY_3 \dots$)

Adding monodromy defects

What is the dual to monodromy defects? We find:

Projection by $\Omega \circ \sigma_+^{SO/Sp}$

$\sigma_+ : CY_3 \rightarrow CY_3$ target anti-hol inv. $\omega(z) = 1/\omega(\bar{z})$

Fixed locus $S^1 \subset S^3$ ($\mathbb{R}P^1 \subset \mathbb{P}^1$): support of M_0



$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2} \sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}$$

Monodromy defects add a **non-oriented sector!**

Tricky signs

	Theory	Background
$SU(2N + 1)_k$,	k odd	$\Omega\sigma_+^{\text{SO}} \oplus D$
$SU(2N + 1)_k$,	k even	$\Omega\sigma_+^{\text{SO}}$
$SU(2N)_k$,	k odd	$\Omega\sigma_+^{\text{Sp}}$
$SU(2N)_k$,	k even, $N \leq k$	$\Omega\sigma_+^{\text{Sp}}$
$SU(2N)_k$,	k even, $2N > k + 2$	$\Omega\sigma_+^{\text{SO}}$

Very non trivial dependence on k, N

New feature of monodromy defects

You can ask about it if interested

Monodromy defects of $SO(2N)$

$SO(2N)$ has “chirality flip”:



Start with $SU(2N)$: $t = 2Ng_s$

$2N$ Lagr.
branes



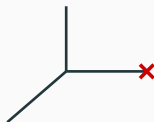
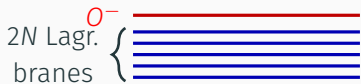
Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \frac{1}{2} \underbrace{\sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}, \quad \hat{t} = (2N - 2)g_s \checkmark$$

Monodromy defects of $SO(2N)$

$SO(2N)$ has “chirality flip”:

Insert O^- : $t \rightarrow (2N - 1)g_s$



$$\{1, \Omega\sigma_-\}$$

Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2} \sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}, \quad \hat{t} = (2N - 2)g_s\sqrt{}$$

Monodromy defects of $SO(2N)$

$SO(2N)$ has “chirality flip”:



Insert other O^- plane with fixed locus: $t \rightarrow (2N - 2)g_s$

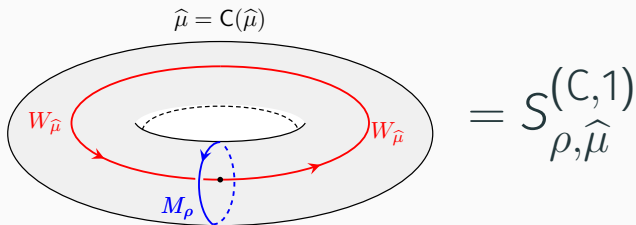


$$\{1, \Omega\sigma_-, \Omega\sigma_+, \underbrace{\sigma_-\sigma_+}_{\text{orbifold}}\} \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$$

Using $D_N^{(2)}$ we find:

$$\log S_{00}^{(C,1)} = \underbrace{\frac{1}{2} \sum_m \frac{e^{-m\hat{t}}}{m[m]^2}}_{\text{oriented}} \mp \underbrace{\sum_{m \text{ even}} \frac{e^{-m\hat{t}/2}}{m[m]}}_{\text{not-oriented}}, \quad \hat{t} = (2N - 2)g_s\sqrt{}$$

Summary



- Monodromy = ∂ Charge conjugation defects
- New line defects of CS:
 - Wilson lines $\leftrightarrow \text{Int}(\hat{\mathfrak{g}}_k^{(1)})$
 - Monodromy defects $\leftrightarrow \text{Int}(\hat{\mathfrak{g}}_k^{(2)})$
- Non-oriented sector in topological strings!
- Test of holography at finite N and k
- New Gromov-Witten invariants, new knots invariants?

Resolved conifold

Simplest CY_3 : $\mathcal{O}(-1) \oplus \mathcal{O}(-1) \rightarrow \mathbb{P}^1$:

$$\underbrace{\{|z_1|^2 + |z_4|^2 - |z_2|^2 - |z_3|^2 = t\}}_{\mathbb{P}^1 \text{ size } t}$$

Start from $T^*S^3 \simeq xy - uv = a \sim 1/N$

$a \rightarrow 0$ singular, resolve by $x = \lambda v, u = \lambda y$

$$x = z_1 z_3, \quad y = z_2 z_4, \quad u = z_1 z_2, \quad v = z_2 z_3, \quad \mathbb{P}^1 \ni \lambda = z_1/z_4$$

gives exactly the resolved conifold. S^3 is Lagrangian in T^*S^3
branes wrapping

Transition replaces S^3 by \mathbb{P}^1

Involutions

$$\sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{\bar{z}_4, -\bar{z}_3, -\bar{z}_2, \bar{z}_1\}$$

Fixed locus : $z_1 z_4 - z_2 z_3 = |z_1|^2 + |z_2|^2 = t \leftrightarrow S^3$

On the $\mathbb{P}^1 \ni z$ after transition $f(z) \mapsto -f(1/\bar{z})$

$$\sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4\}$$

Fixed locus : $z_i \in \mathbb{R}$: real quartic $z_1 z_4 - z_2 z_3 = t$ $f(z) = 1/f(\bar{z})$

$$\text{Fix}(\sigma_+) \cap \mathbb{P}^1 \simeq S^1$$

$$\text{Clearly : } \sigma_{\pm}^2 = 1 = \Omega^2$$

Orbifold : $\sigma_+ \sigma_- : \{z_1, z_2, z_3, z_4\} \mapsto \{z_4, -z_3, -z_2, z_1\}$ holomorphic

Action of charge conjugation

$$\mathfrak{su}(N) : \quad A_\mu(ze^{2\pi i}) = CA_\mu(z) = A_\mu^*(z) = -A_\mu^T \\ W_R \rightarrow W_{\bar{R}}$$

$$\mathfrak{so}(2N) : \quad A_\mu(ze^{2\pi i}) = CA_\mu(z) = MA_\mu(z)M^T \\ M \in O(2N)/SO(2N), \quad W_{S^+} \rightarrow W_{S^-}$$

Non trivial result: $M \leftrightarrow$ odd partition of $2N$

Inv algebra : $= SO(2N - 1) \rightarrow SO(N_1) \times SO(N_2)$

$$N_1 + N_2 = 2N - 1$$

Resulting $\widehat{\mathfrak{g}}_k^2$ are isomorphic