Mixed-State Topological Order: Classification & Excitations

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R. Sohal, A. Prem, PRX Quantum 6, 010313 (2025)

See also: Ellison, Cheng PRX Quantum (2025)

Turning a Bug into a Feature

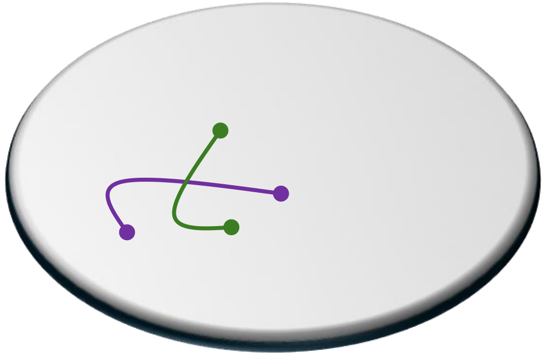
Decoherence is a *non-unitary* process: can drive new dynamics and stabilize novel many-body phases which may furnish resources for quantum information tasks.

Possibility of *intrinsically mixed-state order* – patterns of many-body entanglement that are forbidden in ground states of local Hamiltonians.

Ground states of local gapped Hamiltonians in 2+1D are widely believed to be classified:

Unitary modular tensor category (UMTC) captures universal data.

Alternative perspective: spontaneous breaking of generalized 1-form symmetries.



Key Assumption: Unitary quantum dynamics

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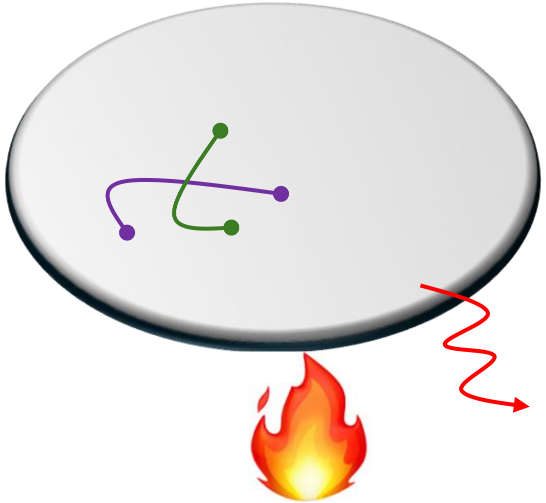
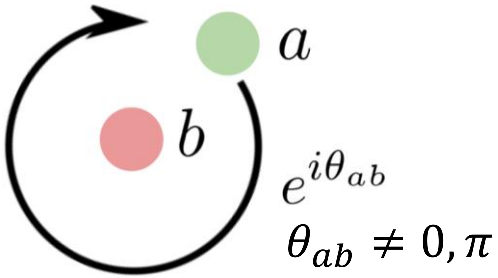
Can we obtain new topological phases of matter intrinsic to open quantum systems?

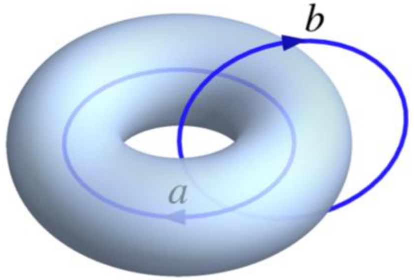
How do we classify them?



Topological Order in d = 2

Gapped phases of matter supporting fractionalized “anyon” excitations and locally indistinguishable ground-states on manifolds with non-trivial genus.





Classified by Unitary Modular Tensor Categories (UMTCs), which encode algebraic data describing braiding and fusion of anyons.

- Modular: every anyon braids non-trivially with at least one other anyon.

Serve as error correcting codes: robust to local errors. Natural to study in mixed-states.

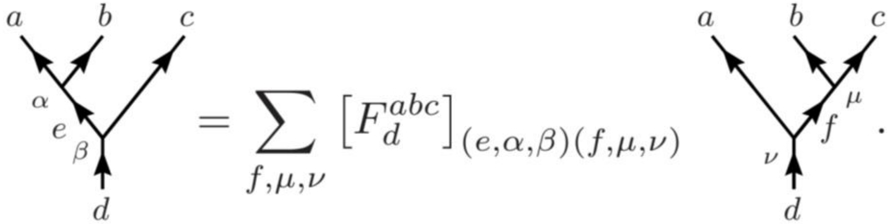
General Anyon Theories in d = 2

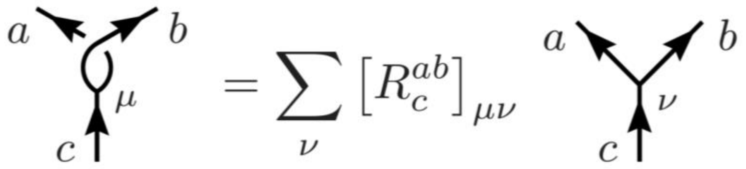
Universal data for 2+1D TO: UMTC $\mathcal{C}$

Anyons/Simple objects: $\{a, b, \ldots, N\}$

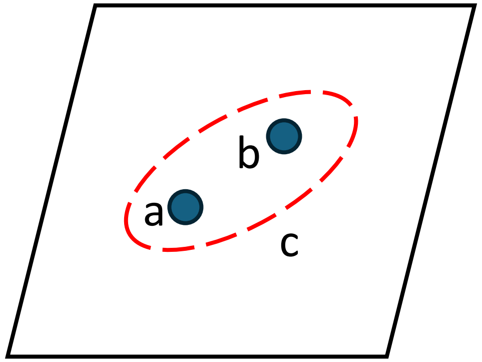
Fusion rules: $a \otimes b \simeq \oplus_c N_{ab}^{c}\, c$

Physical Constraints: $1 \otimes a \simeq a, a \otimes a^{*} \simeq 1 \oplus \ldots$

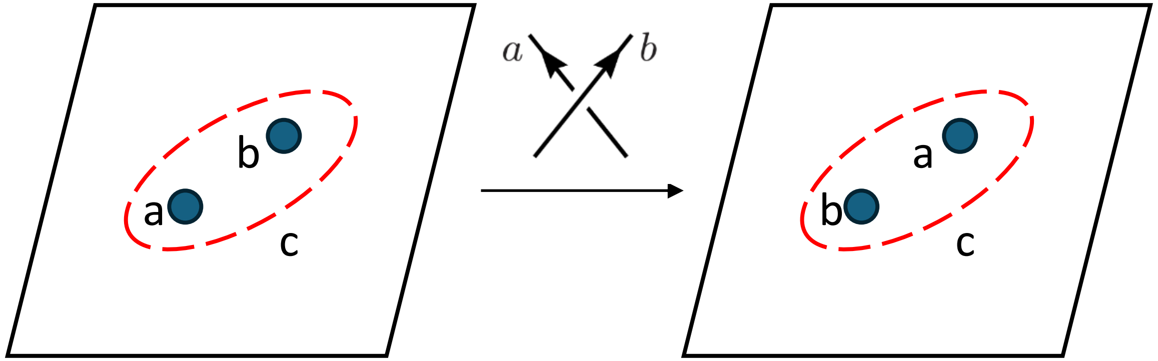




Braiding tracked via R-symbols
R-symbols satisfy hexagon equation



Multiple ways of creating same wavefunction
F-symbols satisfy pentagon equation

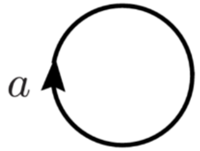


General Anyon Theories in d = 2

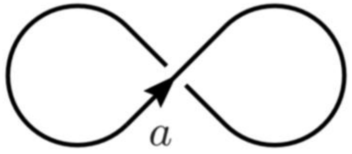
Universal data for 2+1D TO: UMTC $\mathcal{C}$

Gauge invariant data:

$d_a =$



$$\theta_a = \sum_{c,\mu} \frac{d_c}{d_a} \left[R^{aa}_c\right]_{\mu\mu} = \frac{1}{d_a}$$



$$\mathcal{D} = \sqrt{\sum_a d_a^2};$$

$$\mathcal{B}_\theta(a,b) = R_c^{ab} R_c^{ba} = \frac{\theta_c}{\theta_a \theta_b} \mathbb{I}$$

Local gapped Hamiltonians in 2+1D believed to support only UMTCs.
(BFCs can occur at surfaces of 3+1D systems)

UMTCs $\mathcal{C}$ satisfy **braiding non-degeneracy**: for every a $\in \mathcal{C}$ $(a \neq 1), \exists b \in \mathcal{C}: B_\theta(a,b) \neq 1$

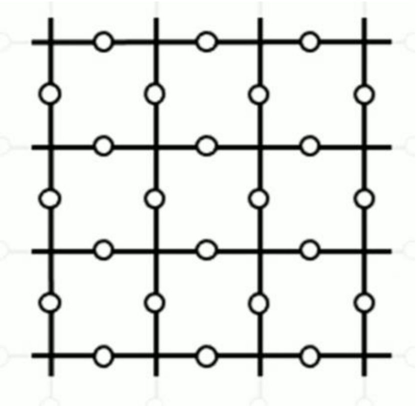
Topological Stabilizer Codes (TSCs)

Hilbert space and Hamiltonian:

$X_j Z_j = \omega\, Z_j X_j$

$X_j^d = 1, Z_j^d = 1$

$\omega = e^{2\pi i/d}$



○ $\mathcal{H}_j = \mathbb{C}^d$

$$H_{\mathcal{C}} = \sum_{i,\alpha}\left(\frac{1-\theta_i^\alpha}{2} + h.c.\right)$$

Stabilizers: $\left[\theta_i^\alpha, \theta_j^\beta\right] = 0\ \forall i, j, \alpha, \beta$

Stabilizer group: $\mathcal{S} = \langle\{\theta_i^\alpha\}\rangle$

Code space/Ground space: $\mathcal{H}_C = \{|\psi\rangle \mid S\,|\psi\rangle = |\psi\rangle\ \forall S \in \mathcal{S}\}$

Can realize *all* Abelian topological phases with gapped boundaries via Pauli stabilizer models.

Abelian topological order $\mathcal{C}$: fully specified by (i) anyon types, (ii) fusion rules, (iii) braiding statistics

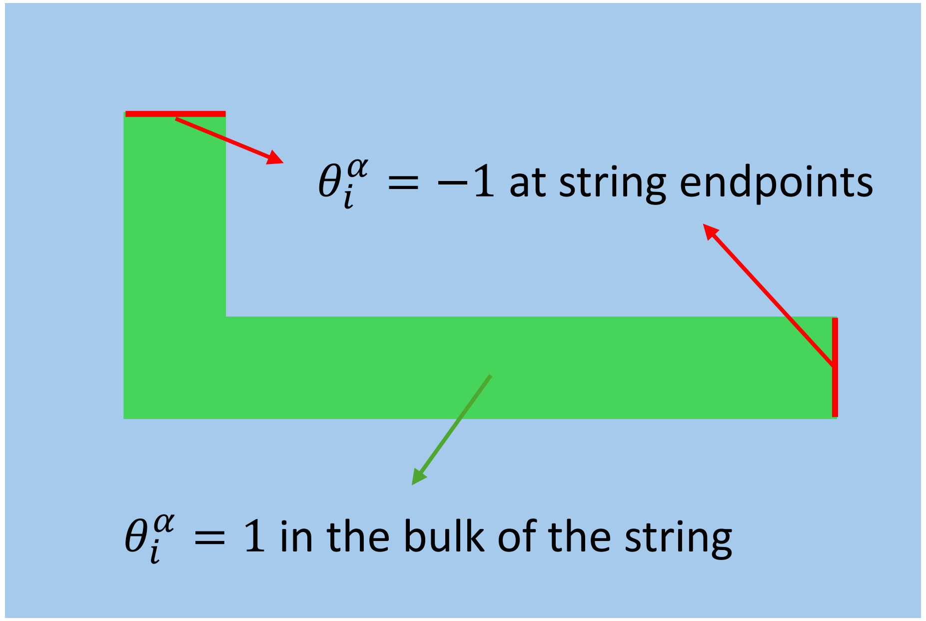
(i) $\{a, b, c, \ldots\}$ (ii) $a \times b = c$ (iii) $\mathcal{B}_\theta(a, b) \in U(1),$ $\theta_a \in U(1)$

Modularity: for every a $\in \mathcal{C}$ $(a \neq 1), \exists b \in \mathcal{C}$: $B_\theta(a, b) \neq 1$

Ellison et al, PRX Quantum (2023)

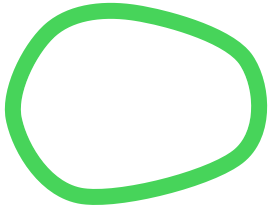
Topological Stabilizer Codes (TSCs)

<u>Anyons:</u> Created by “string-like” operators



$\theta_i^\alpha = -1$ at string endpoints

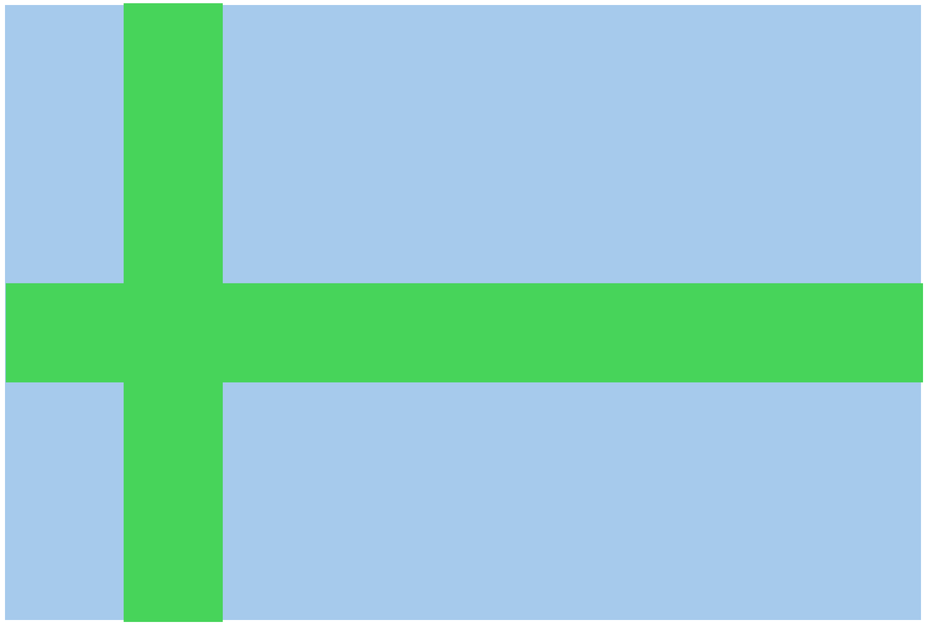
$\theta_i^\alpha = 1$ in the bulk of the string



$$|\psi\rangle = |\psi\rangle$$

Closed anyon loops ~ 1-form symmetries

<u>Logical operators:</u> Non-contractible Wilson-loops



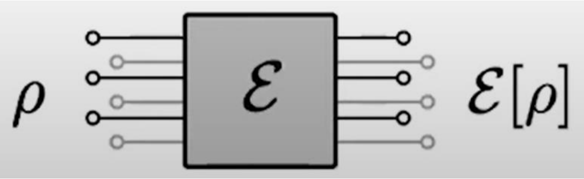
$$[W_{x,y}^a, \theta_i^\alpha] = 0$$

$$W_x^a W_y^b = e^{i\Theta_{ab}} W_y^b W_x^a$$

Encode braiding/mixed anomalies

Locally Decohering Pure-State TO

General strategy: decohere familiar ground states via *local* noise channels.



Goal: characterisation of decohered states in terms of *strong* & *weak* symmetries.

Symmetries in Open Quantum Systems

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|$$

Weak symmetry: Ensemble of asymmetric states, but ρ symmetric *on average.*

$$U\rho U^\dagger = \rho, \qquad U\,|\psi_i\rangle = e^{i\gamma_i}|\psi_i\rangle$$

Strong symmetry: *Each state* in the ensemble is symmetric.

$$U\rho = e^{i\gamma}\rho, \qquad U\,|\psi_i\rangle = e^{i\gamma}|\psi_i\rangle$$

Strong & Weak Symmetries

Fundamentally new physics can arise.

Buca, Prosen (2012); Albert, Jiang (2014)

Decohered TSCs

Consider ρ in the code-space of a TSC ⟷ ground-space of some topological order $\mathcal{C}$

$$S\rho = \rho S = \rho \ \forall S \in \mathcal{S}$$

$$\mathcal{S} = \langle \{\theta_i\} \rangle$$

$$\xrightarrow{\rho_{\mathcal{E}} = \mathcal{E}_a[\rho]}$$

Each anyon in $\mathcal{C}$ furnishes a strong 1-form symmetry for the initial state.

Topological Stabilizer Code

$$\mathcal{H} = \mathcal{H}_C \oplus \mathcal{H}_C^{\perp}$$

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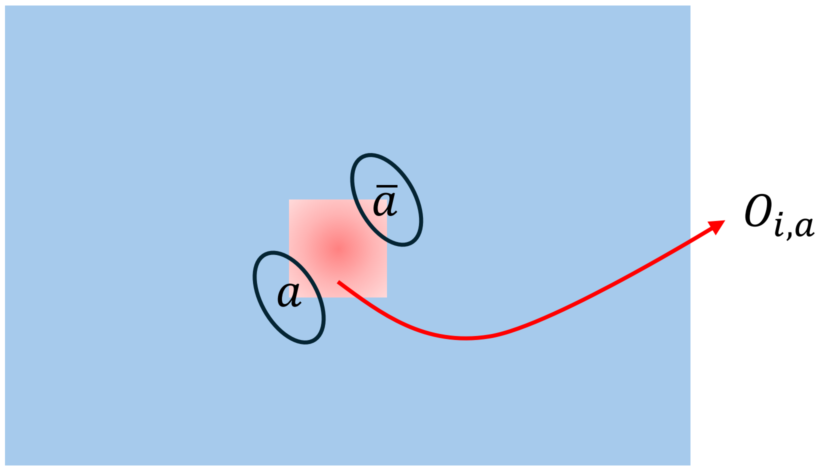
$$\mathcal{E}_a[\rho] = \prod_i \mathcal{E}_{i,a}[\rho]\,, \qquad \mathcal{E}_{i,a} = \sum_m^{k-1} \frac{1}{k}\, O_{i,a}^m\, \rho \left(O_{i,a}^m\right)^{\dagger}$$



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$$\xrightarrow{\rho_{\mathcal{E}} = \mathcal{E}_a[\rho]}$$

Strong 1-form:

$$S\rho_{\mathcal{E}} = \rho_{\mathcal{E}}, \qquad S \in \mathcal{S}\colon [S, O_{i,a}] = 0$$

Weak 1-form:

$$S\rho_{\mathcal{E}}S^\dagger = \rho_{\mathcal{E}}, \quad S \in \mathcal{S}\colon [S, O_{i,a}] \neq 0$$

Topological Stabilizer Code

$$\mathcal{H} = \mathcal{H}_C \oplus \mathcal{H}_C^\perp$$

Topological Subsystem Code

$$\mathcal{H} = (\mathcal{H}_{\mathcal{G}} \otimes \mathcal{H}_{\mathcal{L}}) \oplus \mathcal{H}_C^\perp$$

Factorization of code-space into a *noiseless subsystem* (remaining strong symmetries) & a gauge subsystem induced by maximal anyon decoherence.

Topological Order in the Decohered State

Decohered state: $S\rho_{\mathcal{E}} = \rho_{\mathcal{E}}S^{\dagger} = \rho_{\mathcal{E}}\ \forall S : \left[S, O_{i,a}\right] = 0$

These correspond to anyons in $\mathcal{C}$ that braid trivially with set of decohered anyons $\hat{\mathcal{A}}$.

Equivalently, $\rho_{\mathcal{E}}$ is *strongly* symmetric under 1-form symmetries with no mixed anomaly w/ 1-form generated by Wilson loops of decohered anyons; remainder are broken to *weak* 1-form symmetries.

Some anyons become "transparent" i.e., braid trivially with all other remaining anyons.

TO in decohered state:

$$\mathcal{A} \equiv \{b \in \mathcal{C} | \mathcal{B}_{\theta}(a, b) = 1\ \forall a \in \hat{\mathcal{A}}\}$$

(Unitary braided fusion category in general, not modular)

Maximal Decoherence ~ "Gauging Out"

<u>Gauging out:</u>

Append string operators for set of anyons $\hat{\mathcal{A}}$ to $\mathcal{S}$ to form "gauge group" $\mathcal{G} = \langle \mathcal{S}, \mathcal{F} \rangle$

$\mathcal{F} \equiv$ short-string operators for anyons $a \in \hat{\mathcal{A}}$

Stabilizers of $\rho_{\mathcal{E}}$: $\mathcal{S}_{\mathcal{E}} = \mathcal{Z}(\mathcal{G})$

Resulting anyons: $\mathcal{A} \equiv \{b \in \mathcal{C} | \mathcal{B}_\theta(a, b) = 1 \ \forall a \in \hat{\mathcal{A}}\}$

Can generate *chiral or premodular* anyon theories from non-chiral UMTCs

<u>Decoherence = physical mechanism for gauging out</u>

Anyon Condensation vs Gauging Out

$$H \to H + \lambda \sum_i \mathcal{O}_{a,i}$$

Anyon condensation of *a*

- Only bosons condense $\theta(a) = 1$
- Some anyons become confined
- Some anyons become identified
- Coherently proliferated anyons

$$|\psi\rangle \to |\psi^a\rangle \propto \sum_a |a\rangle, \qquad \mathcal{O}_a|\psi^a\rangle = |\psi^a\rangle$$

$$\rho \to \mathcal{E}_a^p[\rho] = \prod_i \mathcal{E}_{a,i}^p[\rho]$$

Gauging out of *a*

- Arbitrary $\theta(a)$ can be gauged out
- Some anyons become confined
- Anyons are *not* identified
- Incoherently proliferated anyons

$$\rho = |\psi\rangle\langle\psi| \to \rho^a \propto \sum_a |a\rangle\langle a|, \qquad \mathcal{O}_a\rho^a\mathcal{O}_a = \rho^a$$

Beyond Abelian Anyon Theories

General UMTC $\mathcal{C}$: Gauge out proper subset of anyons $\hat{\mathcal{A}} \in \mathcal{C}$

$$\boxed{\text{imTO } \mathcal{A} \equiv C_{\mathcal{C}}(\hat{\mathcal{A}}) = \{x \in \mathcal{C} | \mathcal{B}_\theta(x, y) = 1 \forall y \in \hat{\mathcal{A}}\}}$$

- <u>Transparent anyons:</u> If $y \in \hat{\mathcal{A}}$ is transparent in $\hat{\mathcal{A}}$, then y is transparent in $\mathcal{A}$.
- $\mathcal{A}$ is a UMTC $\Leftrightarrow$ $\hat{\mathcal{A}}$ is a UMTC ($\mathcal{C} = \mathcal{A} \otimes \hat{\mathcal{A}}$ in this case).

Classification of imTO: *Non-modular* braided fusion categories.

(imTO: TO not believed to occur in ground states of gapped local Hamiltonians in d=2.)

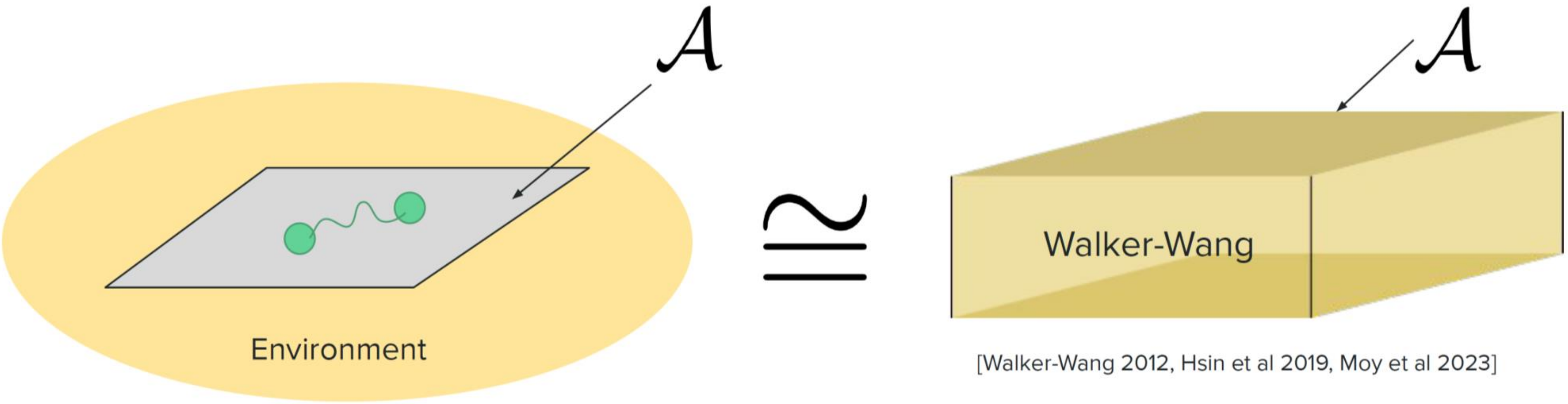
Mueger (2002)

Classification

Intrinsically mixed-state topological order $\cong$ mixed strong & weak symmetries.

As pure states, can only exist on surfaces in $d=3$.

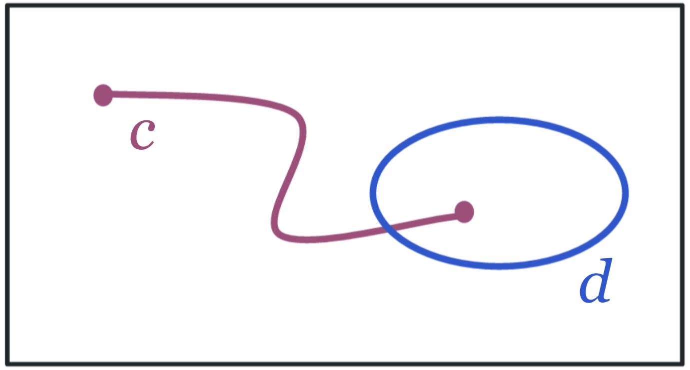
Classification: Non-modular braided fusion category $\mathcal{A}$



[Walker-Wang 2012, Hsin et al 2019, Moy et al 2023]

See also: Hsin, Kobayashi, Prem (2025); Kobayashi, Prem, Yu (forthcoming) for imTO in 3+1D

Locally Detectable Excitations



$$\mathcal{C} \xrightarrow{\text{Decoherence of } \mathbf{a}} \mathcal{A}$$

Excitations:

$$\rho_c = W^c_{x_1,x_2}\rho(W^c_{x_1,x_2})^\dagger$$

$$c \in \mathcal{C}$$
$$d \in \mathcal{A}$$

Observables:

$$\mathrm{Tr}[W^d_\Gamma \rho_c] = \mathcal{B}_\theta(c,d)\mathrm{Tr}[W^d_\Gamma \rho] = \mathcal{B}_\theta(c,d)$$

- Anyon $c \in \mathcal{C}$ is an “excitation” if detectable via braiding with some $d \in \mathcal{A}$
- If $\mathcal{A}$ is a UMTC, it labels all detectable excitations. Not true if $\mathcal{A}$ non-modular.
- Wilson lines corresponding to transparent anyons do not generate detectable excitations.

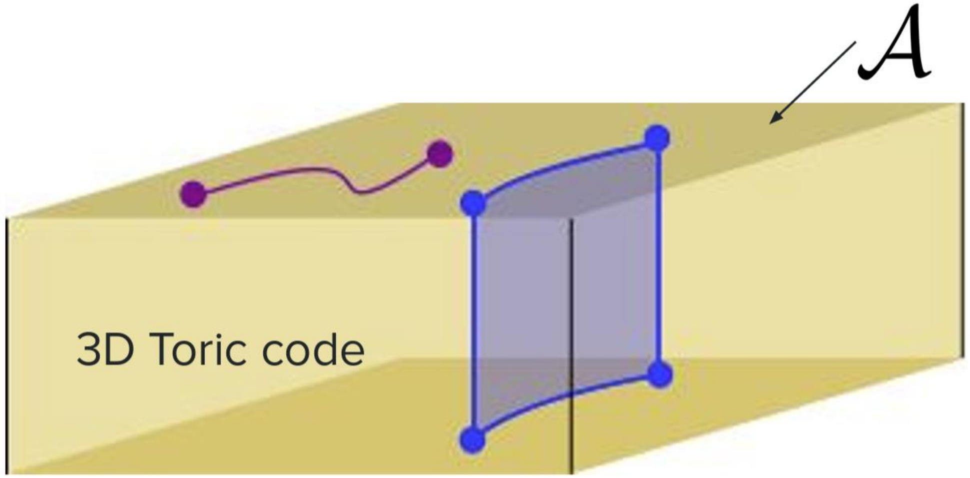
Locally Detectable Excitations

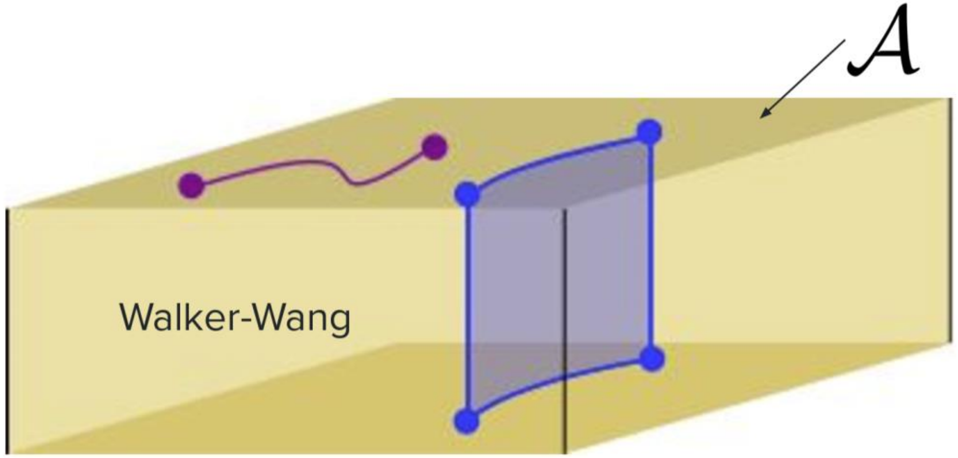
$$\mathcal{C}_{\mathbb{Z}_2} = \{1, e, m, f\} \longrightarrow \mathcal{A}_{\mathbb{Z}_2^{(0)}} = \{1, e\}$$

$$\mathcal{E}_{i,e}[\rho] = \frac{\rho + Z_i \rho Z_i}{2}$$

$$\rho_{\mathcal{E},e} \equiv Z_i \rho_{\mathcal{E}} Z_i = \rho_{\mathcal{E}}$$

$$\rho_{\mathcal{E},m} \equiv X_i \rho_{\mathcal{E}} X_i \qquad \mathrm{Tr}[Z_i \rho_{\mathcal{E},m}] = -1$$





Excitations characterised by opaque anyons on surface **and** bulk surface-like excitations terminating on surface.

Summary

Partial classification of intrinsically mixed-state topological order via non-modular strong symmetries.

Locally Detectable Excitations: “Quantum” excitations form a UMTC but “classical” excitations also remain well-defined.

Pure-state Topological Order

Intrinsically Mixed Topological Order

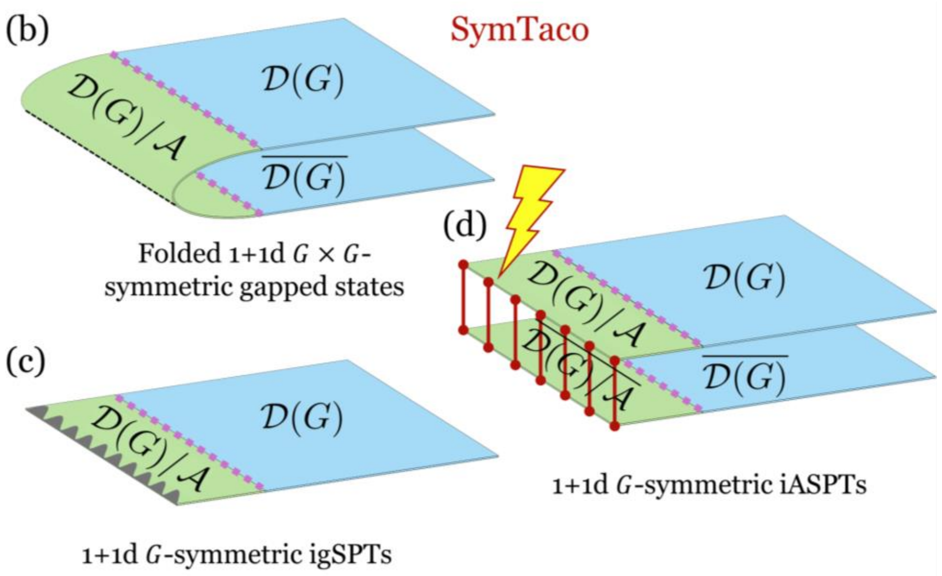
$$|\psi\rangle \xrightarrow{\text{Decoherence}} \rho$$

UMTC

Non-modular BFC

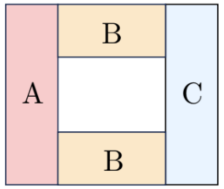
Future Directions/Open Questions

Mixed State SPTs ~ Gapless Topological Phases



Qi, Sohal, Chen, Stephen, Prem (2025)

Characterization of mixed-state topological orders

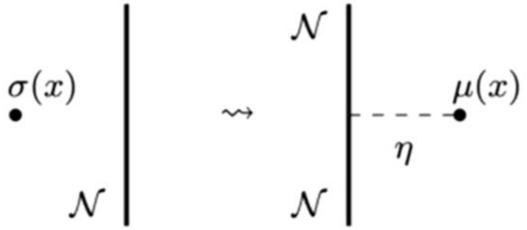


Ex: topological entanglement negativity

Fate of conformal field theories under noise?

New universality classes under measurement & decoherence?

Non-invertible symmetries in mixed-states



Non-invertible symmetries

Schafer-Nameki, Tiwari, Warman, Zhang (2025)

Duality Webs from Gauging Strong and Weak Symmetries

Lam, Prem, Qi, Sohal (forthcoming)