Pseudo-supersymmetry: a tale of alternate realities

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Outline

- Goal : Construct different supergravity actions from one 'complex' action by taking different real slices.

- Motivation :
  1. The domain-wall vs. cosmology correspondence (Townsend, Skenderis) suggests that this can be done. Explicit realisation of this correspondence in a supergravity setting.
  2. 'Variant' supergravities in 10 and 11 dimensions have been considered by looking at time-like T-duality, e.g. the so-called *-theories. (Hull, Bergshoeff, Van Proeyen, Vaula). Can we construct these explicitly?

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There is a correspondence between domain-walls and cosmologies (Townsend, Skenderis).

- Domain wall metric

$$ds^2 = dz^2 + \mathrm{e}^{2\beta\varphi}\left[-\frac{\mathrm{d}\tau^2}{1+k\tau^2} + \tau^2(\mathrm{d}\psi^2 + \sinh^2\psi \mathrm{d}\Omega^2_{d-2})\right].$$

  where $k = 0, \pm 1$, $\varphi = \varphi(z)$.

- FLRW cosmology

$$ds^2 = -dt^2 + \mathrm{e}^{2\beta\phi}\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\Omega^2_{d-2})\right].$$

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Related via analytical continuation : $(t, r, \theta) = -\mathrm{i}(z, \tau, \psi)$ and $\phi(t) = \varphi(\mathrm{i}t)$.

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Introduction and motivation

Domain-walls vs. cosmologies

- Considering gravity coupled to scalars:

$$\mathcal{L} = \sqrt{-g}\Big[R - \frac{1}{2}(\partial\sigma)^2 - \eta V(\sigma)\Big]\,, \quad \eta = \pm 1\,.$$

DW for $(\eta = 1, k = \pm 1 \text{ or } 0) \rightarrow$ cosmology for $(\eta = -1, k = \mp 1 \text{ or } 0)$.

- For the DW (fake supersymmetry)

$$V = 2\left(|W'|^2 - \alpha^2|W|^2\right) \quad \text{and} \quad (D_\mu - \alpha\beta W\Gamma_\mu)\epsilon = 0$$

- For the cosmology (fake pseudo-supersymmetry)

$$V = -2\left(|W'|^2 - \alpha^2|W|^2\right) \quad \text{and} \quad (D_\mu - \mathrm{i}\alpha\beta W\Gamma_\mu)\epsilon = 0$$

- $\Gamma^\mu D_\mu\epsilon = M\epsilon$:
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- From a supergravity point of view this correspondence looks rather strange:
  - Supersymmetric domain walls can be generically found, supersymmetric cosmologies not.
  - $V \to -V$, $W \to \mathrm{i}W$?
  - In real supergravity you do care about reality of fermions $\leftrightarrow$ fake supergravity.
- Is there a way of realizing this in a supergravity context, i.e. see the Killing spinor conditions as arising from $\delta_\epsilon \psi_\mu = 0$?
- Strategy :
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 & IIA & \longrightarrow & IIB & \\
T_t & \downarrow & & \downarrow & T_t \\
 & IIB^* & \longrightarrow & IIA^* & \\
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Also leads to theories in other signatures.

- RR-fields become ghosts

$$\text{e.g.} \quad L_{IIA^*} = \sqrt{-g}\left\{ e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}H\cdot H\right] + \frac{1}{2}\sum_{n=0,1,2} F^{(2n)}\cdot F^{(2n)} \right\}$$

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- Has a unique real form.
- Imposing different reality conditions on the complex algebra $\longrightarrow$ different parametrizations of this real form $\longrightarrow$ Hull's theories (Bergshoeff, Van Proeyen)
- dualities then relate the various parametrizations
- All this was on the level of the algebra
- $\Rightarrow$ We'd like to do a similar thing on the level of the action? (Vaula, Nishino, Gates)

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- Consider the standard type IIA action in signature $(t,s)=(1,9)$:

$$S_{IIA} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\Big[R-4\big(\partial\phi\big)^2+\tfrac{1}{2}H\cdot H+-2\partial^\mu\phi\chi_\mu^{(1)}$$
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$$+\sum_{n=0}^{2}\tfrac{1}{2}G^{(2n)}\cdot G^{(2n)}+G^{(2n)}\cdot\Psi^{(2n)}\Big\}$$

- $\bar{\lambda}=\bar{\lambda}^\dagger\Gamma_0=\lambda^T\mathcal{C}$ = reality condition.
- If $\bar{\lambda}=\lambda^T\mathcal{C}$, supersymmetry does not really depend on the reality of the fields.
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The strategy

Reality conditions on the fields

- Impose suitable reality conditions on the fermions:

$$\chi^* = R\chi\,.$$

- Compatibility with Lorentz invariance implies

$$R = \alpha\, B \quad \text{or} \quad R = \alpha\, B\,\Gamma_{11} \quad \text{with} \;\; B = \mathcal{C}\Gamma_0\,.$$

- This is a good reality condition as in both cases * is an involution : $\chi^{**} = \chi$.
- There are then two possibilities to impose reality conditions on the fermions:

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- Next step : determine all the α-factors. This is done by imposing reality of the action and by checking consistency with the supersymmetry transformation laws.

$$\begin{aligned} \delta_\epsilon b = \bar{\epsilon}\Gamma f \quad &\Rightarrow \quad (\delta_\epsilon b)^* = (\bar{\epsilon}\Gamma f)^* \\ \delta_\epsilon f = b\epsilon \quad &\Rightarrow \quad (\delta_\epsilon f)^* = (b\epsilon)^*\,. \end{aligned}$$

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- Next step : determine all the α-factors. This is done by imposing reality of the action and by checking consistency with the supersymmetry transformation laws.

$$\begin{aligned} \delta_\epsilon b = \bar{\epsilon}\Gamma f \qquad &\Rightarrow \qquad (\delta_\epsilon b)^* = (\bar{\epsilon}\Gamma f)^* \\ \delta_\epsilon f = b\epsilon \qquad &\Rightarrow \qquad (\delta_\epsilon f)^* = (b\epsilon)^*\,. \end{aligned}$$

- This leads to a set of relations between the α-factors. For type IIA in $(1,9)$ it turns out that both sets of reality conditions on the fermions give a consistent choice of α-factors.

The strategy

IIA and IIA*

- Two different reality conditions → two different theories.

IIA	IIA*
$\epsilon^* = -\mathcal{C}\Gamma_0\epsilon$	$\epsilon^* = -\mathcal{C}\Gamma_0\Gamma_{11}\epsilon$
$\psi_\mu^* = -\mathcal{C}\Gamma_0\psi_\mu$	$\psi_\mu^* = -\mathcal{C}\Gamma_0\Gamma_{11}\psi_\mu$
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${e_\mu}^{a*} = {e_\mu}^a$	${e_\mu}^{a*} = {e_\mu}^a$
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  1. Replace $\chi^T\mathcal{C}$ by $-\alpha_\chi^{-1}\chi^\dagger\Gamma_0$ (IIA) or by $\alpha_\chi^{-1}\chi^\dagger\Gamma_0\Gamma_{11}$ (IIA*).
  2. In IIA case, this gives a good action. In IIA* case, express everything in real fields by redefining $C^{(m)} = i\tilde{C}^{(m)}$ and $\lambda = i\tilde{\lambda}$.
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More generally

Type II theories in different signatures

- So far, we've found real slices of the complex action, leading to IIA and IIA* theories in $(1,9)$ signature, but using more general reality conditions, one can find IIA theories in other signatures.
- Results for type IIA

t mod 4	0	1		2
type	SM	MW	*MW	M
α_B	-	+	+	-
$\alpha_{-1} = \alpha_3$	+	+	-	-
α_1	-	+	-	+

- Similar analysis for type IIB

t mod 4	1		3
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- Variant supergravities can be constructed by taking real slices of one complex action.
- In some signatures (e.g.$(1, 9)$), two distinct possibilities occur.
- Relation with extended supersymmetry.
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- Consider a truncation of mIIA:

$$S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} \left(\partial\phi \right)^2 - \tfrac{1}{2} e^{5\phi/2} m^2 \right) ,$$

Note that m is a real mass parameter! The potential can be expressed in terms of a real superpotential W

$$V = 8 (\frac{\delta W}{\delta \phi})^2 - \frac{9}{2} W^2 = \frac{1}{2} e^{5\phi/2} m^2 , \quad W = \frac{1}{4} e^{5\phi/4} m .$$

The supersymmetry transformations are then

$$\delta \psi_\mu = \left(D_\mu - \frac{1}{8} W \Gamma_\mu \right) \epsilon ,$$

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The domain-wall cosmology correspondence

An example in mIIA and mIIA*

- We can now construct the *-version mIIA*, by imposing different reality conditions.
  1. ϕ, e_μ^a real.
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  3. $\rightarrow$ m is now purely imaginary!, redefine : $\tilde{m} = -\mathrm{i}m$.
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This is precisely the setup as proposed in the DW-cosm correspondence:

- mIIA has a supersymmetric domain wall solution ($D8$ brane)

$$\mathrm{d}s^2 = H^{1/8}[-\mathrm{d}t^2 + (\mathrm{d}x^\mu)^2] + H^{9/8}\mathrm{d}z^2 \qquad (H = 1 + mz)$$

  The Killing spinor obeys:

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