Rotation of Linear Polarization Plane from Cosmological Pseudoscalar Fields

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based on a work with:

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Overview

- Pseudoscalar – photon coupling.
- Main effects on CMB polarization.
- Modified Einstein – Boltzmann equations for a time dependent linear polarization rotation angle.
- Fixed DM (or DE) model:
  - full linear polarization angular power spectra;
  - comparison with constant rotation angle approximation.

Work based on:
Pseudoscalar – photon coupling

Pseudoscalar fields are invoked to solve the strong CP-problem of QCD [R. Peccei and H. Quinn PRL 38 (1977)]

\[ \mathcal{L}_{QCD} = \mathcal{L}_{PERT} + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \frac{g^2}{32\pi^2} \phi \frac{G^a_{\mu\nu}}{f_a} \tilde{G}^{\mu\nu}_a \]

They are also good candidates for cold dark matter (misalignment axion production).

Pseudoscalar particles interact with ordinary matter: photons, nucleons, [electrons].

The coupling with photons play a key role for most of the searches:

\[ \mathcal{L}_{\phi \gamma} = g_\phi \mathbf{E} \cdot \mathbf{B} \phi = -\frac{g_\phi}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \phi \]

where:

\[ F^{\mu\nu} \equiv \nabla^\mu A^\nu - \nabla^\nu A^\mu \quad \text{and} \quad \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \]
Most of this searches make use of the **Primakoff effect**, by which pseudoscalars convert into photons in presence of an external electromagnetic field.

- Dichroism in laser experiments
- Solar axions (e.g. CAST)
- Birefringence in laser experiments
- Light shining through walls experiments
Current Constraints

[Battesti et al., arXiv:0705.0615]
We want to evaluate the effect on CMB polarization of a coupling of this kind between pseudoscalar field and photon, improving the estimate obtained by D. Harari and P. Sikivie in 1992 [Phys. Lett. B 289 67] for linear polarization:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) - \frac{g_\phi}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \]
Pseudoscalar – photon coupling

- Assume a spatially flat Roberson-Walker universe:

\[ ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\eta) \left[ -d\eta^2 + d\mathbf{x}^2 \right] \]

- Neglect the spatial variations of the pseudoscalar field:

\[ \phi = \phi(\eta) \]


For a plane wave propagating along z-axis, the equation for Fourier transform of the vector potential (in the Coulomb Gauge \( \nabla \cdot \mathbf{A} = 0 \) ):

\[ \tilde{A}_+''(\eta, k) + \left[ k^2 + g\phi k \frac{d\phi}{d\eta} \right] \tilde{A}_+(\eta, k) = 0 \]

\[ \tilde{A}_-'''(\eta, k) + \left[ k^2 - g\phi k \frac{d\phi}{d\eta} \right] \tilde{A}_-(\eta, k) = 0 \]
Adiabatic solution

It is possible to search a solution in this form:

\[ \tilde{A}_s = \frac{1}{\sqrt{2\omega_s}} e^{\pm i \int \omega_s d\eta} \quad \text{where:} \quad \omega_s(\eta) = k \sqrt{1 \pm \frac{g\phi}{k} \phi'} \equiv k \sqrt{1 \pm \Delta(\eta)} \]

It is a good approximation of the solution when:
\[ \frac{3\omega_s'^2}{4\omega_s^4} \ll 1 \quad \text{and} \quad \frac{\omega_s''}{2\omega_s^3} \ll 1. \]

If also \( \Delta(\eta) \ll 1 \):

\[ \tilde{A}_\pm \approx \frac{1}{\sqrt{2k (1 \pm \Delta/4)}} \exp \left[ \pm ik \left( \eta \pm \frac{1}{2} \int \Delta(\eta) d\eta \right) \right] \]
\[ = \frac{1}{\sqrt{2k (1 \pm g\phi \phi' k/4)}} \exp \left[ \pm i (k\eta \pm g\phi \phi/2) \right]. \]
Adiabatic solution

The two main effects on the propagation of the wave are:

- a $k$-independent shift between the two polarized waves, which corresponds to rotation of the plane of linear polarization of an angle:

$$\theta(\eta) = \frac{g\phi}{2} [\phi(\eta) - \phi(\eta_{\text{rec}})]$$

- production of a certain degree of circular polarization (dependent on $k$):

$$\tilde{\Pi}_V(\eta) \equiv \frac{V}{T} = \frac{\left|\tilde{A}_+^\prime\right|^2 - \left|\tilde{A}_-^\prime\right|^2}{\left|\tilde{A}_+^\prime\right|^2 + \left|\tilde{A}_-^\prime\right|^2} \simeq \frac{\Delta(\eta)}{2} = \frac{g\phi'(\eta)}{2k}$$
CMB Polarization

• Linear polarization of CMB was **predicted** soon after CMB discovery in 1968 by Martin Rees [Rees, ApJ 153 1968] (Thomson scattering of anisotropic radiation at last scattering give rise to linear polarization).

• The first **detection** of CMB polarization was made by the Degree Angular Scale Interferometer (DASI, Kovac et al., Nature 420, 2002).

• First **full-sky polarization map** released from WMAP in 2006.

Plot of signal for TT, TE, EE, BB for the best fit model. [Page et al., 2006]
**E and B linear polarization**

Potential sources of B polarization:

- *Cosmological gravitational waves* (tensor perturbation of the metric)
- *Gravitational lensing* of E-mode polarization
- *Faraday Rotation* of E-mode polarization (magnetic fields)
- Coupling of CMB photons with a *pseudoscalar field* (e.g. axion). …

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[Zaldarriaga, astro-ph/0305272]
One of the main effects of coupling between photons and pseudoscalar fields is **cosmological birefringence**:

$$\theta(\eta) = \frac{g \phi}{2} \left[ \phi(\eta) - \phi(\eta_{\text{rec}}) \right]$$

Including the time dependent rotation angle contribution in the Boltzmann equation for polarization [Liu et al., PRL 97, 161303 (2006)]:

$$\Delta'_{Q\pm iU}(k, \eta) + i k \mu \Delta_{Q\pm iU}(k, \eta) = -n_e \sigma_T a(\eta) \left[ \Delta_{Q\pm iU}(k, \eta) \right. \right.$$

$$+ \sum_m \sqrt{\frac{6\pi}{5}} Y^m_2 S_{\text{P}}(m)(k, \eta) \left. \right]$$

$$\mp i 2\theta'(\eta) \Delta_{Q\pm iU}(k, \eta) .$$
Polarization Boltzmann equation

Following the line of sight strategy for scalar perturbations, we have an additional term in polarization sources:

\[ \Delta_T(k, \eta) = \int_0^{\eta_0} d\eta \, g(\eta) \, S_T(k, \eta) \, J_\ell(k\eta_0 - k\eta), \]
\[ \Delta_E(k, \eta) = \int_0^{\eta_0} d\eta \, g(\eta) \, S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \cos[2\theta(\eta)], \]
\[ \Delta_B(k, \eta) = \int_0^{\eta_0} d\eta \, g(\eta) \, S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \sin[2\theta(\eta)]. \]

If \( \theta \) is constant in time the new terms exit from the time integrals and:

\[ \Delta_E = \Delta_E(\theta = 0) \cos(2\bar{\theta}), \]
\[ \Delta_B = \Delta_E(\theta = 0) \sin(2\bar{\theta}). \]
Constant rotation angle

In the constant rotation angle approximation new polarization power spectra are given by [A. Lue, L. Wang, M. Kamionkowski PRL 83, 1506 (1999)]:

\[
\begin{align*}
C_{\ell}^{EE,\text{obs}} &= C_{\ell}^{EE} \cos^2(2\theta), \\
C_{\ell}^{BB,\text{obs}} &= C_{\ell}^{EE} \sin^2(2\theta), \\
C_{\ell}^{EB,\text{obs}} &= \frac{1}{2} C_{\ell}^{EE} \sin(4\theta), \\
C_{\ell}^{TE,\text{obs}} &= C_{\ell}^{TE} \cos(2\theta), \\
C_{\ell}^{TB,\text{obs}} &= C_{\ell}^{TE} \sin(2\theta).
\end{align*}
\]

Where \( C_{\ell}^{XY} \) are the primordial power spectra produced by scalar fluctuations in absence of parity violation, while \( C_{\ell}^{XY,\text{obs}} \) are what we would observe in the presence of an for an isotropic, k-independent rotation \( \theta \) of the plane of linear polarization.
Constraints on the rotation angle

- analyzing a subset of WMAP3 and BOOMERANG data
  [B. Feng, et al., PRL 96 221302 (2006)]
  \[-13.7 \text{ deg} < \bar{\theta} < 1.9 \text{ deg} \ (2\sigma)\]

- analyzing WMAP three years polarization data
  [P.Cabella, et al., PRD 76 123014 (2007)]
  \[-8.5 \text{ deg} < \bar{\theta} < 3.5 \text{ deg} \ (2\sigma)\]

- analyzing WMAP five years polarization data
  [E. Komatsu, et al., arXiv:0803.0547]
  \[-5.9 \text{ deg} < \bar{\theta} < 2.4 \text{ deg} \ (2\sigma)\]

- analyzing QUaD experiment second and third season observations
  [QUaD Collaboration, arXiv:0811.0618]
  \[-1.2 \text{ deg} < \bar{\theta} < 3.9 \text{ deg} \ (2\sigma)\]
Cosine-type potential

Assuming that dark matter is given by massive pseudoscalar particles (e.g. axions), we consider the potential:

\[ V(\phi) = m^2 \frac{f_a^2}{N^2} \left( 1 - \cos \frac{\phi N}{f_a} \right) \simeq \frac{1}{2} m^2 \phi^2 \]

the evolution of \( \phi \) is given by the equation:

\[ \ddot{\phi} + 3H \dot{\phi} + m^2(T)\phi = 0 \]

If \( m \ll 3H \)  the solution simply is: \( \phi \simeq \phi_i \)

If \( m > 3H \)  the field begins to oscillate and the solution, in a matter dominated universe ( \( \dot{a}/a = 2/3t \) ), is:

\[ \phi(t) \simeq \frac{\phi_0}{mt} \sin(mt) \]
Cosine-type potential

\[ \theta(\eta) = \sqrt{\frac{3}{\pi}} \frac{g\phi M_{pl}}{2m\eta_0} \left\{ \left( \frac{\eta_0}{\eta} \right)^3 \sin \left[ m \frac{\eta_0}{3} \left( \frac{\eta}{\eta_0} \right)^3 \right] - \left( \frac{\eta_0}{\eta_{rec}} \right)^3 \sin \left[ m \frac{\eta_0}{3} \left( \frac{\eta_{rec}}{\eta_0} \right)^3 \right] \right\} \]

\[ \theta_0 \sim 0.506 \text{ rad} \]

\[ m = 10^{-22} \text{ eV}, \quad g\phi = 10^{-20} \text{ eV}^{-1} \]
Cosine-type potential

\[ \theta = \theta(\eta) \]

\[ r = 0.1 \]

\[ \theta = \theta(\eta) \]

\[ \theta_0 \sim 0.506 \text{ rad} \]

BB

\[ \theta = \theta_0 \]

TE

\[ \theta = \theta_0 \]

\[ \theta = \theta(\eta) \]
Cosine-type potential \[\textit{note vertical axis}\]

WMAP collaboration

[arXiv:0803.0593]

Cosine-type potential with:

\[m = 10^{-22} \text{ eV}, \quad g_\phi = 10^{-20} \text{ eV}^{-1}\]
Parity odd correlators

In absence of parity-violating interactions, the ensemble of fluctuations is statistically parity symmetric and therefore the parity odd correlators have to vanish. In this case photons interact with pseudoscalars:

\[ \mathcal{L}_{\phi\gamma} = g_{\phi} \mathbf{E} \cdot \mathbf{B}_\phi = -\frac{g_{\phi}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \phi \]

therefore also parity-odd correlators should be considered:

\[
\begin{align*}
C_{l}^{TT} & \xrightarrow{P} C_{l}^{TT} \\
C_{l}^{TE} & \xrightarrow{P} C_{l}^{TE} \\
C_{l}^{EE} & \xrightarrow{P} C_{l}^{EE} \\
C_{l}^{TB} & \xrightarrow{P} -C_{l}^{TB} \\
C_{l}^{TV} & \xrightarrow{P} -C_{l}^{TV} \\
C_{l}^{EB} & \xrightarrow{P} -C_{l}^{EB} \\
C_{l}^{EV} & \xrightarrow{P} -C_{l}^{EV} \\
C_{l}^{BB} & \xrightarrow{P} C_{l}^{BB} \\
C_{l}^{BV} & \xrightarrow{P} C_{l}^{BV} \\
C_{l}^{VV} & \xrightarrow{P} C_{l}^{VV}
\end{align*}
\]
Cosine-type potential

\[ \theta = \theta(\eta) \]

\[ \theta_0 \sim 0.506 \text{ rad} \]

\[ \theta = \theta_0 \]
Cosine-type potential \( [\text{note vertical axis}] \)

### WMAP collaboration

\[ m = 10^{-22} \text{ eV}, \quad g_\phi = 10^{-20} \text{ eV}^{-1} \]
Photon coupling with pseudoscalar fields

Oscillating behaviour:

axion-like particles

\[ \mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \]
\[ -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{2} m^2 \phi^2 \]
\[ -\frac{g_\phi}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \]
In 1995 Frieman et al. [PRL 75, 2077] proposed a quintessence model based on a pseudoscalar field.

Cosmology with Ultralight Pseudo Nambu-Goldstone Bosons

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We explore the cosmological implications of an ultralight pseudo Nambu-Goldstone boson. With
global spontaneous symmetry breaking scale \( f \approx 10^{18} \) GeV and explicit breaking scale comparable to
Mikheyev-Smirnov-Wolfenstein neutrino masses, \( M \approx 10^{-3} \) eV, such a field, which acquires a mass
\( m_\phi \sim M^2/f \sim H_0 \), would currently dominate the energy density of the Universe. The field acts as
an effective cosmological constant before relaxing into a condensate of nonrelativistic bosons. Such
a model can reconcile dynamical estimates of the density parameter, \( \Omega_m \sim 0.2 \), with a spatially flat
universe, yielding \( H_0 t_0 = 1 \) consistent with limits from gravitational lens statistics.
Ultralight pseudo Nambu-Goldstone bosons

In 1995 Frieman et al. [PRL 75, 2077] proposed a quintessence model based on a pseudoscalar field.

This model is still in agreement with observations and can be probed by future experiment reaching stage 4 of DETF methodology (Planck CMB measurements, future SNIa surveys, baryon acoustic oscillations, and weak gravitational lensing). This analysis can be improved considering also birefringence of CMB polarization:

\[
\mathcal{L}_\phi = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - M^4 \left( 1 + \cos \frac{\phi}{f} \right)
\]

\[
\mathcal{L}_{\phi \gamma} = -\frac{1}{4f} \phi F^{\mu \nu} \tilde{F}_{\mu \nu}
\]

where: \( M \sim 10^{-3} \text{ eV} \) and \( f \lesssim \frac{M_{\text{pl}}}{\sqrt{8\pi}} \)
Ultralight pseudo Nambu-Goldstone bosons

Fixed \( M = 8.8 \times 10^{-4} \text{ eV} \), \( f = 0.3 \frac{M_{\text{pl}}}{\sqrt{8\pi}} \), \( \Theta_i \equiv \frac{\phi}{f} = 0.25 \), \( \dot{\Theta}_i = 0 \)

the pseudoscalar field mimes the behaviour of the cosmological constant:

\[
\Omega_{\text{RAD}} \quad \Omega_{\text{MAT}}
\]

\[
\Omega_\phi
\]

\[
\omega_\phi
\]

\[
\log a
\]

\[
\Theta(a) = \Theta_{\text{TOT}}
\]

\[
\Theta(a)
\]

\[
\log a
\]
Ultralight pseudo Nambu-Goldstone bosons

\[ \theta = \theta(\eta) \]

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\( \theta_0 \sim 0.54 \text{ rad} \)

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Ultralight pseudo Nambu-Goldstone bosons [note vertical axis]

\[ M = 8.8 \times 10^{-4} \text{ eV}, \quad f = 0.3 \frac{M_{\text{Pl}}}{\sqrt{8\pi}}, \]

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\[ \theta = \theta(\eta) \]

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Conclusions & Developments

We discuss the effects of coupling between pseudoscalar fields and photons on Cosmic Microwave Background Polarization:

• how the public code CAMB can be modified in order to take into account the rotation of the linear polarization plane by a cosmological pseudoscalar field acting as dark matter from last scattering surface to nowadays.

• Polarization power spectra strongly depend on the kinematics of the pseudoscalar field.

CMB birefringence constraints are complementary to experiments and astroparticle observations.