

# Rotation of Linear Polarization Plane from Cosmological Pseudoscalar Fields

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*based on a work with:*

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# Overview

- Pseudoscalar – photon coupling.
- Main effects on CMB polarization.
- Modified Einstein – Boltzmann equations for a time dependent linear polarization rotation angle.
- Fixed **DM** (or **DE**) model:
  - full linear polarization angular power spectra;
  - comparison with constant rotation angle approximation.

## Work based on:

- F. Finelli and MG, “*Rotation of Linear Polarization Plane and Circular Polarization from Cosmological Pseudoscalar Fields*”, arXiv:0802.4210 [astro-ph], *accepted in Phys. Rev. D.*
- F. Finelli and MG, “*CMB Cosmological Birefringence and Ultralight Pseudo Nambu-Goldstone Bosons*”, *in preparation.*

# Pseudoscalar – photon coupling

Pseudoscalar fields are invoked to solve the **strong CP-problem of QCD** [R. Peccei and H.Quinn PRL **38** (1977)]

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{PERT}} + \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi + \frac{g^2}{32\pi^2} \frac{\phi}{f_a} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$

They are also good candidates for cold dark matter (misalignment axion production).

Pseudoscalar particles **interact with ordinary matter:** photons, nucleons, [electrons].

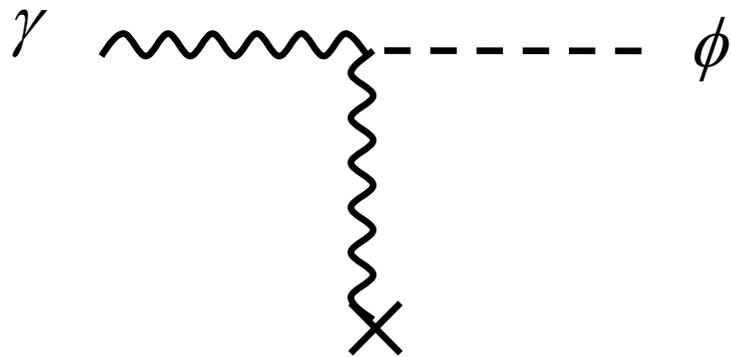
The **coupling with photons** play a key role for most of the searches:

$$\mathcal{L}_{\phi\gamma} = g_{\phi} \mathbf{E} \cdot \mathbf{B} \phi = -\frac{g_{\phi}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \phi$$

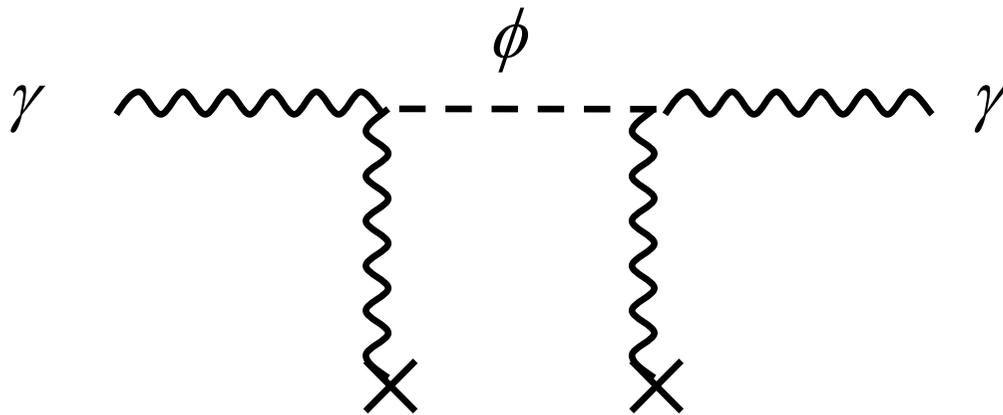
where:  $F^{\mu\nu} \equiv \nabla^{\mu} A^{\nu} - \nabla^{\nu} A^{\mu}$  and  $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

# Pseudoscalar – photon coupling

Most of these searches make use of the *Primakoff effect*, by which pseudoscalars convert into photons in presence of an external electromagnetic field.

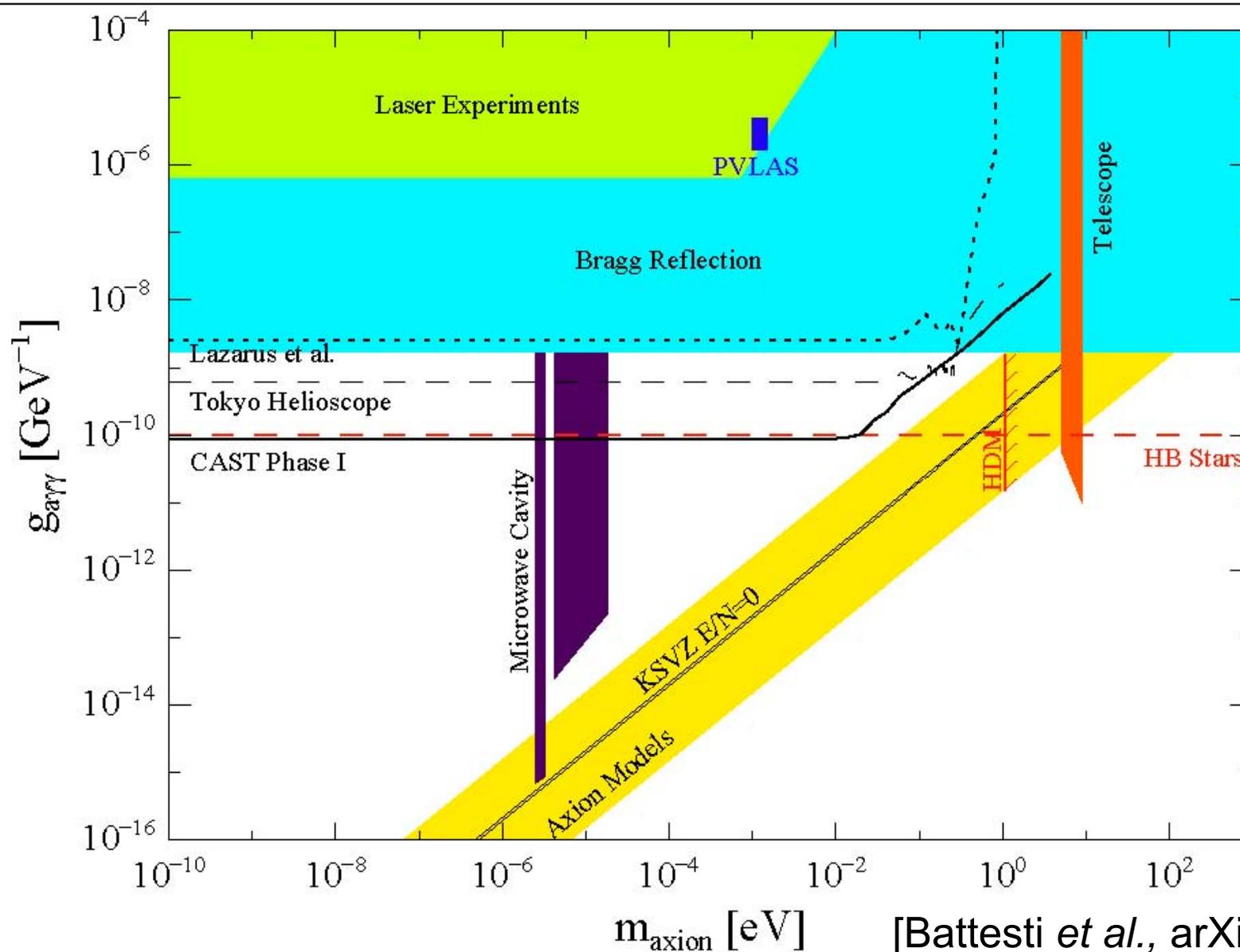


- Dichroism in laser experiments
- Solar axions (e.g. CAST)



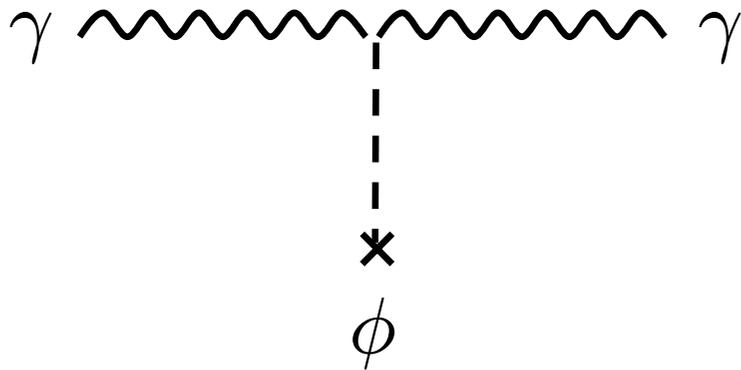
- Birefringence in laser experiments
- Light shining through walls experiments

# Current Constraints



[Battesti *et al.*, arXiv:0705.0615]

# Cosmological background



Photon propagation in a **time dependent background** of pseudoscalar particles acting as **DM** (e.g. axion-like particles) or **DE** (e.g. ultralight pseudo Nambu-Goldstone bosons)

We want to evaluate the effect on CMB polarization of a coupling of this kind between **pseudoscalar field** and **photon**, improving the estimate obtained by D. Harari and P. Sikivie in 1992 [Phys. Lett. B **289** 67] for linear polarization:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\nabla_{\mu}\phi\nabla^{\mu}\phi - V(\phi) - \frac{g\phi}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

# Pseudoscalar – photon coupling

- Assume a spatially flat Robertson-Walker universe:

$$ds^2 = -dt^2 + a^2(t)dx^2 = a^2(\eta) [-d\eta^2 + dx^2]$$

- Neglect the spatial variations of the pseudoscalar field:

$$\phi = \phi(\eta)$$

$\phi$  is homogeneous throughout our universe (inflation occurs after the PQ-symmetry breaking): PQ scale is much higher than  $10^{11-12}$  GeV, case motivated by anthropic considerations [Linde, Phys. Lett. B **201** (1988), M. Tegmark, A. Aguirre, M. Rees, F. Wilczek Phys. Rev. D **73** (2006), M.P. Hertzberg, M. Tegmark, F. Wilczek Phys. Rev. D **78** (2008) ]

For a plane wave propagating along z-axis, the equation for Fourier transform of the vector potential (in the Coulomb Gauge  $\nabla \cdot \mathbf{A} = 0$  ) :

$$\tilde{A}_+''(\eta, k) + \left[ k^2 + \underline{g_\phi k \frac{d\phi}{d\eta}} \right] \tilde{A}_+(\eta, k) = 0$$

$$\tilde{A}_-''(\eta, k) + \left[ k^2 - \underline{g_\phi k \frac{d\phi}{d\eta}} \right] \tilde{A}_-(\eta, k) = 0$$

# Adiabatic solution

It is possible to search a solution in this form:

$$\tilde{A}_s = \frac{1}{\sqrt{2\omega_s}} e^{\pm i \int \omega_s d\eta} \quad \text{where:} \quad \omega_s(\eta) = k \sqrt{1 \pm \frac{g_\phi}{k} \phi'} \equiv k \sqrt{1 \pm \Delta(\eta)}$$

It is a good *approximation* of the solution when:  $\frac{3\omega_s'^2}{4\omega_s^4} \ll 1$  and  $\frac{\omega_s''}{2\omega_s^3} \ll 1$ .

If also  $\Delta(\eta) \ll 1$  :

$$\begin{aligned} \tilde{A}_\pm &\simeq \frac{1}{\sqrt{2k(1 \pm \Delta/4)}} \exp \left[ \pm ik \left( \eta \pm \frac{1}{2} \int \Delta(\eta) d\eta \right) \right] \\ &= \frac{1}{\sqrt{2k(1 \pm \underline{g_\phi \phi' k/4})}} \exp \left[ \pm i \left( \underline{k\eta \pm g_\phi \phi/2} \right) \right]. \end{aligned}$$

# Adiabatic solution

The two main effects on the propagation of the wave are:

- a k-independent shift between the two polarized waves, which corresponds to **rotation of the plane of linear polarization** of an angle:

$$\theta(\eta) = \frac{g\phi}{2} [\phi(\eta) - \phi(\eta_{\text{rec}})]$$

- production of a certain **degree of circular polarization** (dependent on k):

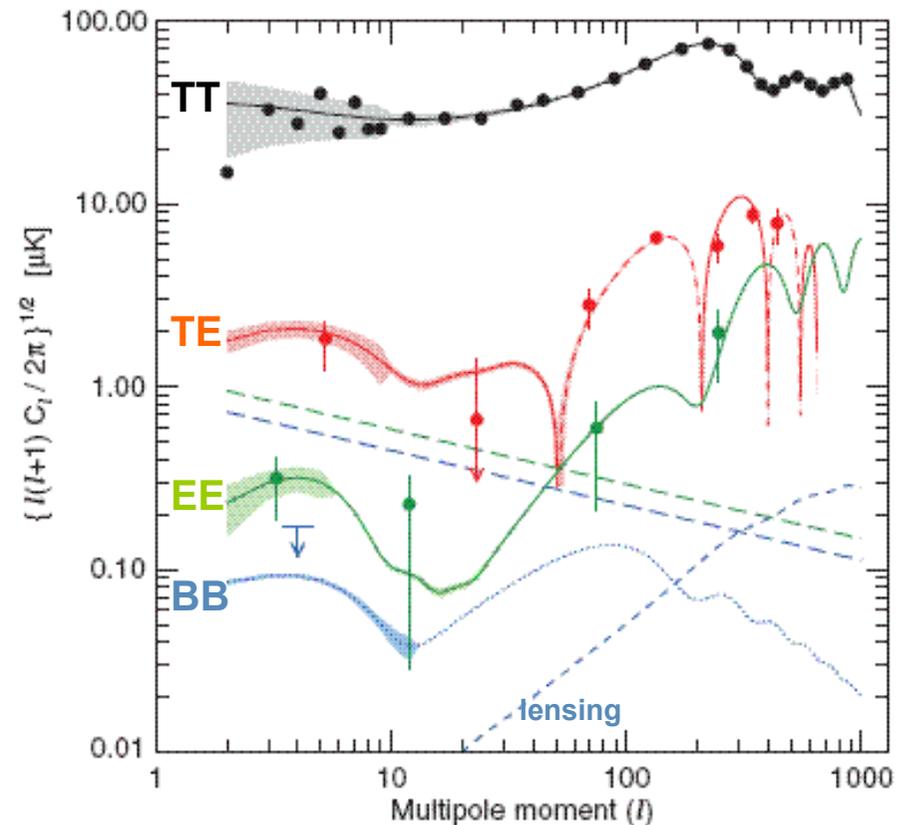
$$\tilde{\Pi}_V(\eta) \equiv \frac{V}{T} = \frac{|\tilde{A}'_+|^2 - |\tilde{A}'_-|^2}{|\tilde{A}'_+|^2 + |\tilde{A}'_-|^2} \simeq \frac{\Delta(\eta)}{2} = \frac{g\phi'(\eta)}{2k}$$

# CMB Polarization

- Linear polarization of CMB was **predicted** soon after CMB discovery in 1968 by Martin Rees [Rees, ApJ 153 1968] (Thomson scattering of anisotropic radiation at last scattering give rise to linear polarization).

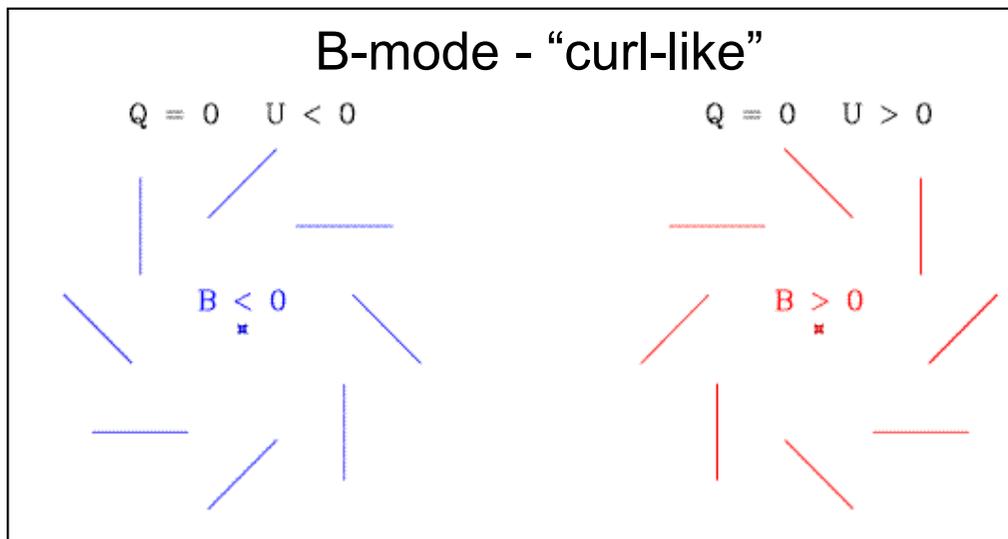
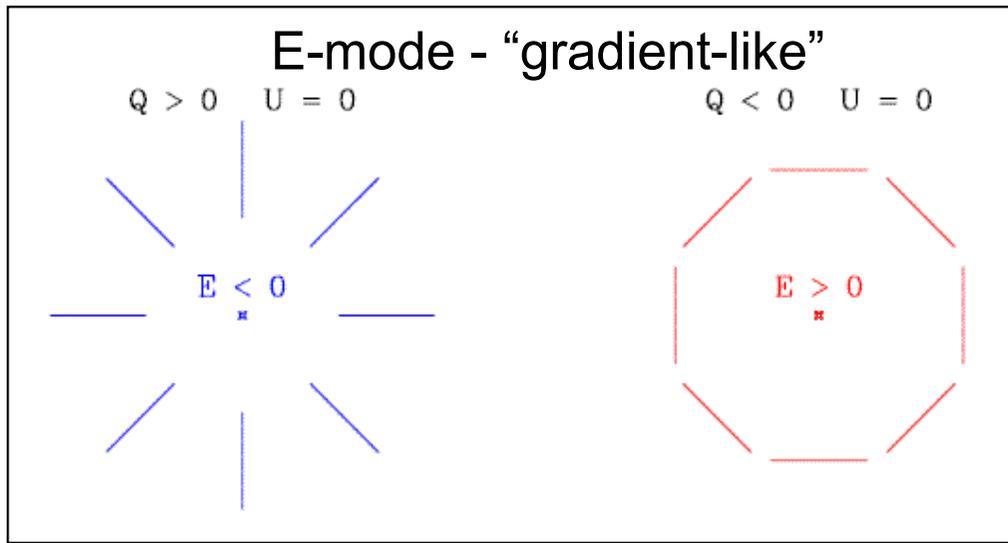
- The first **detection** of CMB polarization was made by the Degree Angular Scale Interferometer (DASI, Kovac *et al.*, Nature 420, 2002).

- First **full-sky polarization map** released from WMAP in 2006.



Plot of signal for TT, TE, EE, BB for the best fit model.  
[Page *et al.*, 2006]

# *E* and *B* linear polarization



## Potential sources of *B* polarization:

- *Cosmological gravitational waves* (tensor perturbation of the metric)
- *Gravitational lensing* of E-mode polarization
- *Faraday Rotation* of E-mode polarization (magnetic fields)
- Coupling of CMB photons with a *pseudoscalar field* (e.g. axion). ...

[Zaldarriaga, astro-ph/0305272]

# Polarization Boltzmann equation

One of the main effects of coupling between photons and pseudoscalar fields is **cosmological birefringence**:

$$\theta(\eta) = \frac{g_\phi}{2} [\phi(\eta) - \phi(\eta_{\text{rec}})]$$

Including the time dependent rotation angle contribution in the Boltzmann equation for polarization [Liu *et al.*, PRL 97, 161303 (2006)] :

$$\Delta'_{Q\pm iU}(k, \eta) + ik\mu\Delta_{Q\pm iU}(k, \eta) = -n_e\sigma_T a(\eta) \left[ \Delta_{Q\pm iU}(k, \eta) + \sum_m \sqrt{\frac{6\pi}{5}} {}_{\pm 2}Y_2^m S_P^{(m)}(k, \eta) \right] \mp i2\theta'(\eta)\Delta_{Q\pm iU}(k, \eta).$$

# Polarization Boltzmann equation

Following the line of sight strategy for scalar perturbations, we have an additional term in polarization sources:

$$\Delta_T(k, \eta) = \int_0^{\eta_0} d\eta g(\eta) S_T(k, \eta) j_\ell(k\eta_0 - k\eta),$$

$$\Delta_E(k, \eta) = \int_0^{\eta_0} d\eta g(\eta) S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \cos[2\theta(\eta)],$$

$$\Delta_B(k, \eta) = \int_0^{\eta_0} d\eta g(\eta) S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \sin[2\theta(\eta)].$$

If  $\theta$  is constant in time the new terms exit from the time integrals and:

$$\Delta_E = \Delta_E(\theta = 0) \cos(2\bar{\theta}),$$

$$\Delta_B = \Delta_E(\theta = 0) \sin(2\bar{\theta}).$$

# Constant rotation angle

In the constant rotation angle approximation new polarization power spectra are given by [A. Lue, L. Wang, M. Kamionkowski PRL **83**, 1506 (1999)]:

$$\begin{aligned}C_{\ell}^{EE,obs} &= C_{\ell}^{EE} \cos^2(2\bar{\theta}), \\C_{\ell}^{BB,obs} &= C_{\ell}^{EE} \sin^2(2\bar{\theta}), \\C_{\ell}^{EB,obs} &= \frac{1}{2} C_{\ell}^{EE} \sin(4\bar{\theta}), \\C_{\ell}^{TE,obs} &= C_{\ell}^{TE} \cos(2\bar{\theta}), \\C_{\ell}^{TB,obs} &= C_{\ell}^{TE} \sin(2\bar{\theta}).\end{aligned}$$

Where  $C_{\ell}^{XY}$  are the primordial power spectra produced by scalar fluctuations in absence of parity violation, while  $C_{\ell}^{XY,obs}$  are what we would observe in the presence of an for an isotropic, k-independent **rotation**  $\theta$  of the plane of liner polarization.

# Constraints on the rotation angle

- analyzing a subset of **WMAP3** and **BOOMERANG** data  
[B. Feng, *et al.*, PRL **96** 221302 (2006)]

$$-13.7 \text{ deg} < \bar{\theta} < 1.9 \text{ deg} \quad (2\sigma)$$

- analyzing **WMAP three years polarization data**  
[P.Cabella, *et al.*, PRD **76** 123014 (2007)]

$$-8.5 \text{ deg} < \bar{\theta} < 3.5 \text{ deg} \quad (2\sigma)$$

- analyzing **WMAP five years polarization data**  
[E. Komatsu, *et al.*, arXiv:0803.0547]

$$-5.9 \text{ deg} < \bar{\theta} < 2.4 \text{ deg} \quad (2\sigma)$$

- analyzing **QUaD** experiment second and third season observations  
[QUaD Collaboration, arXiv:0811.0618 ]

$$-1.2 \text{ deg} < \bar{\theta} < 3.9 \text{ deg} \quad (2\sigma)$$

# Cosine-type potential

Assuming that dark matter is given by **massive pseudoscalar particles** (e.g. axions), we consider the potential:

$$V(\phi) = m^2 \frac{f_a^2}{N^2} \left( 1 - \cos \frac{\phi N}{f_a} \right) \simeq \frac{1}{2} m^2 \phi^2$$

the evolution of  $\phi$  is given by the equation:

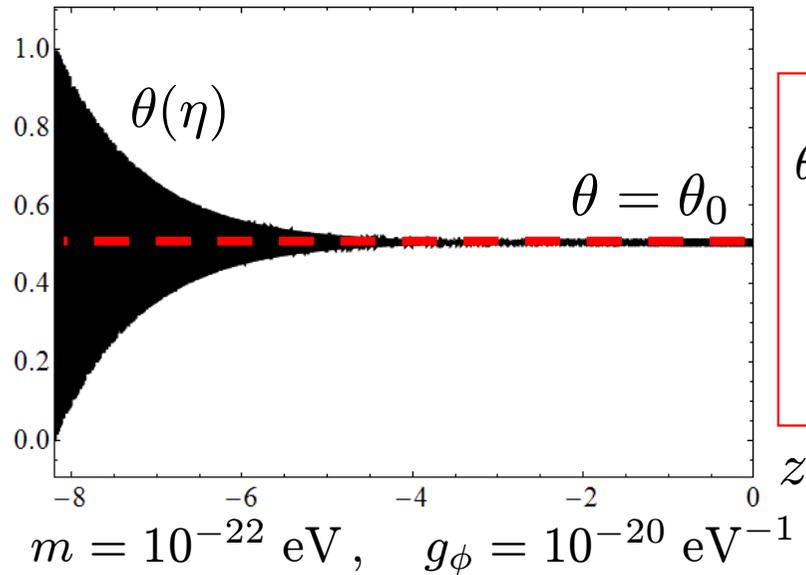
$$\ddot{\phi} + 3H\dot{\phi} + m^2(T)\phi = 0$$

If  $m \ll 3H$  the solution simply is:  $\phi \simeq \phi_i$

If  $m > 3H$  the field begins to oscillate and the solution, in a matter dominated universe ( $\dot{a}/a = 2/3t$ ), is:

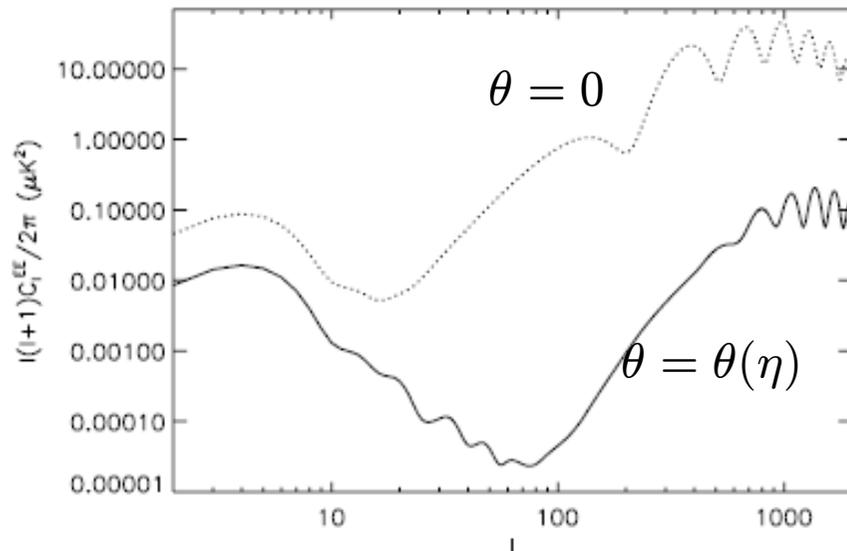
$$\phi(t) \stackrel{m_\phi t \gg 1}{\simeq} \frac{\phi_0}{mt} \sin(mt)$$

# Cosine-type potential

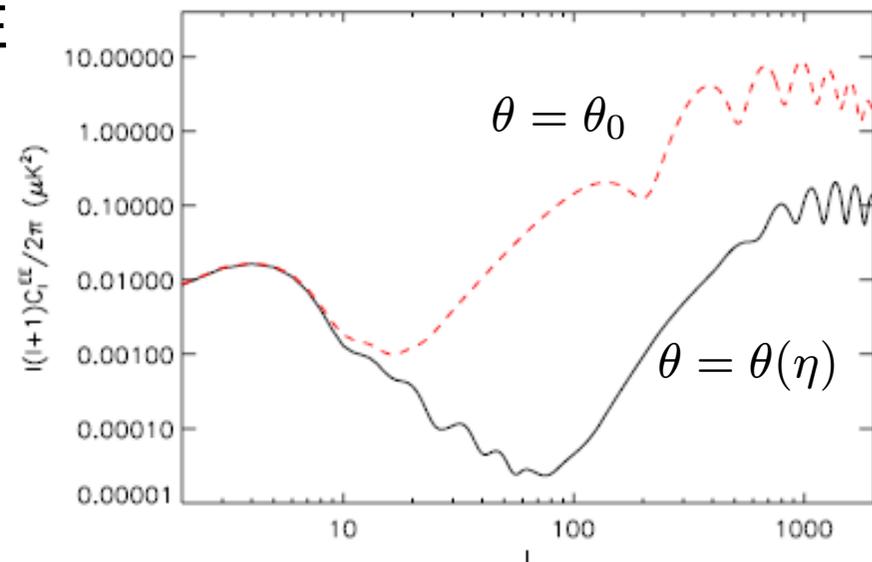


$$\theta(\eta) = \sqrt{\frac{3}{\pi}} \frac{g_\phi M_{\text{pl}}}{2m\eta_0} \left\{ \left( \frac{\eta_0}{\eta} \right)^3 \sin \left[ m \frac{\eta_0}{3} \left( \frac{\eta}{\eta_0} \right)^3 \right] - \left( \frac{\eta_0}{\eta_{\text{rec}}} \right)^3 \sin \left[ m \frac{\eta_0}{3} \left( \frac{\eta_{\text{rec}}}{\eta_0} \right)^3 \right] \right\}$$

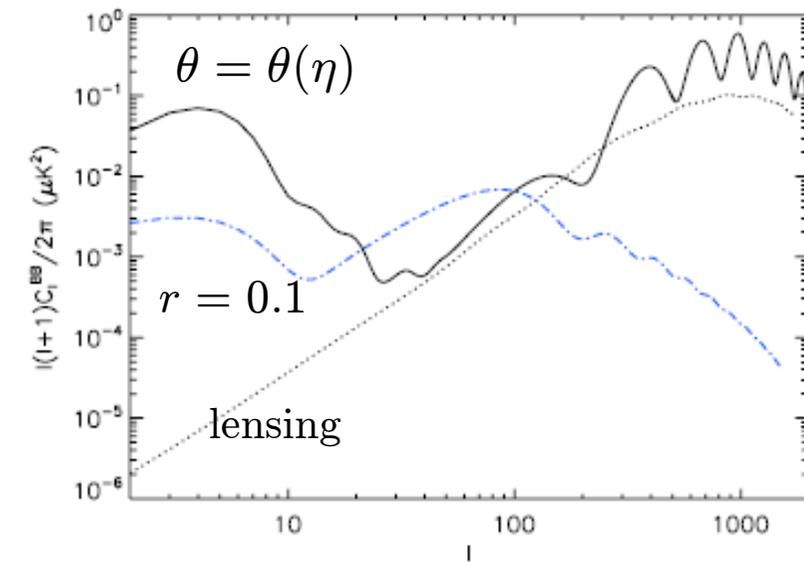
$$\theta_0 \sim 0.506 \text{ rad}$$



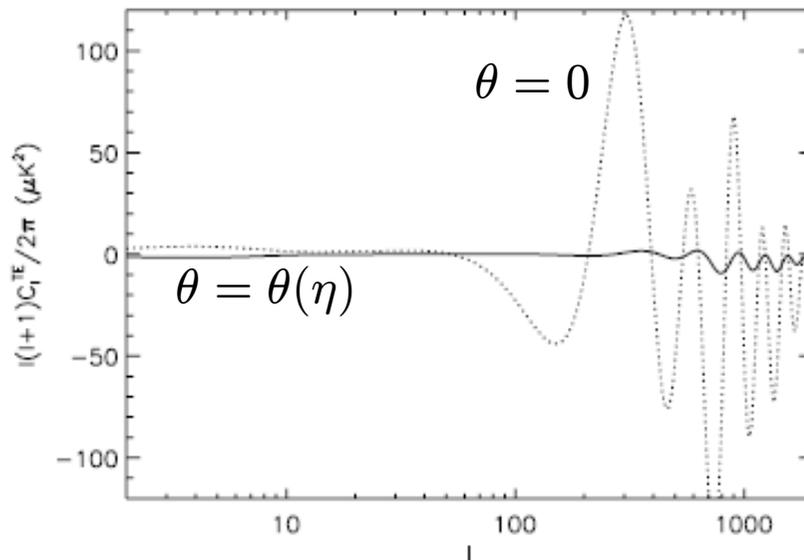
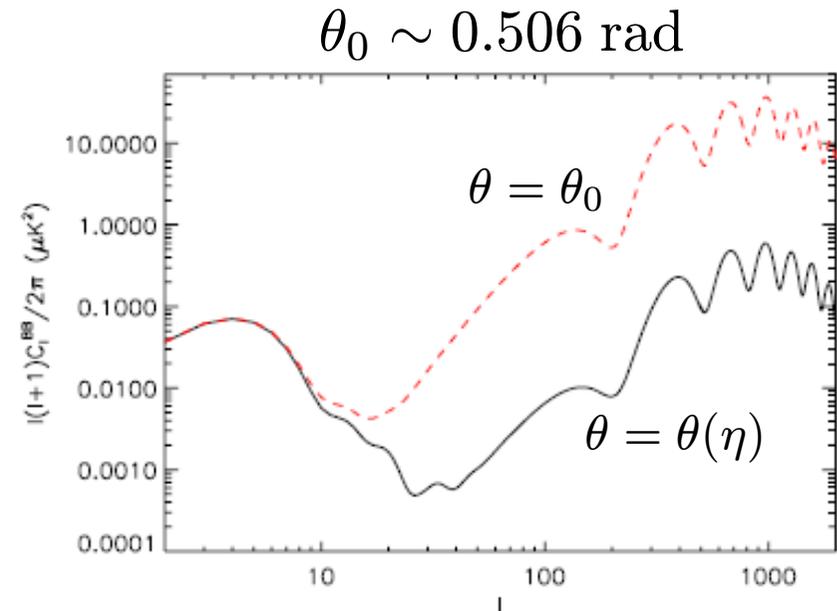
EE



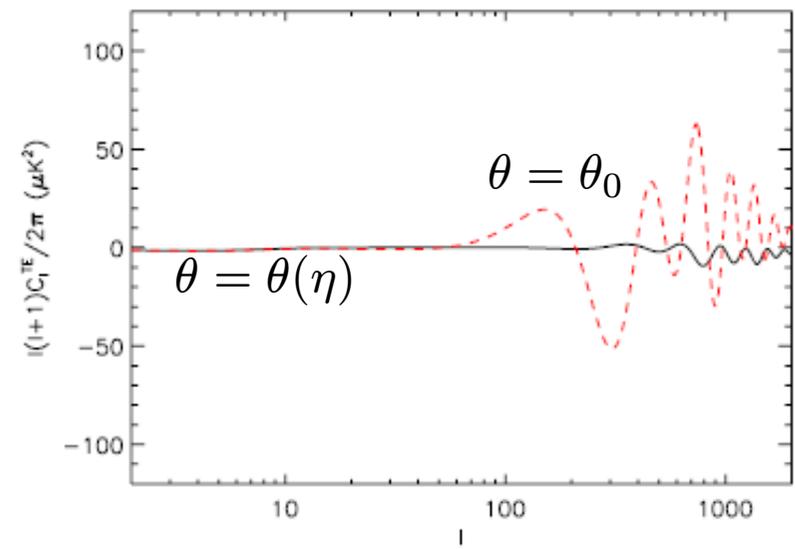
# Cosine-type potential



BB

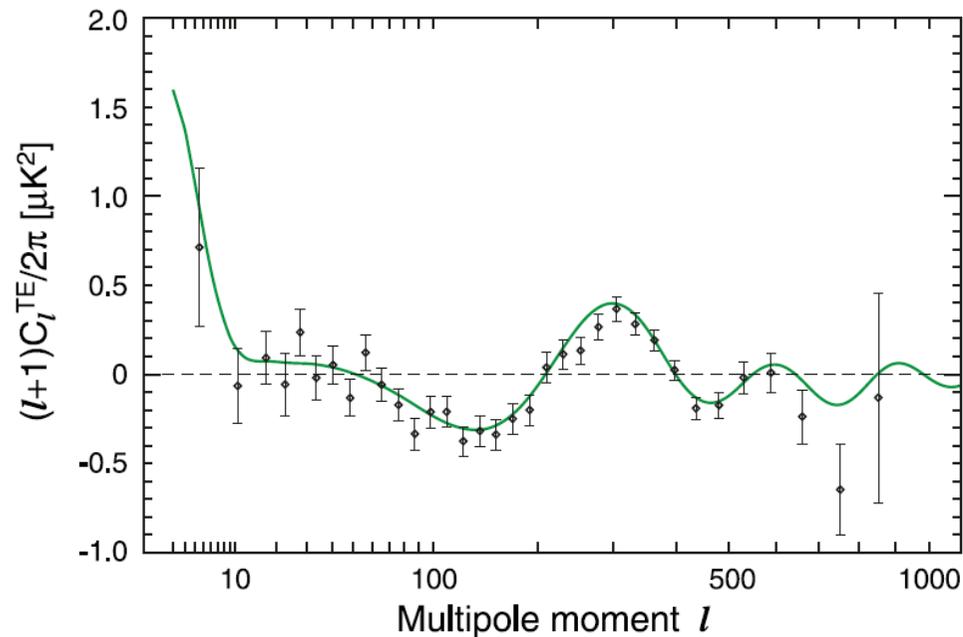


TE

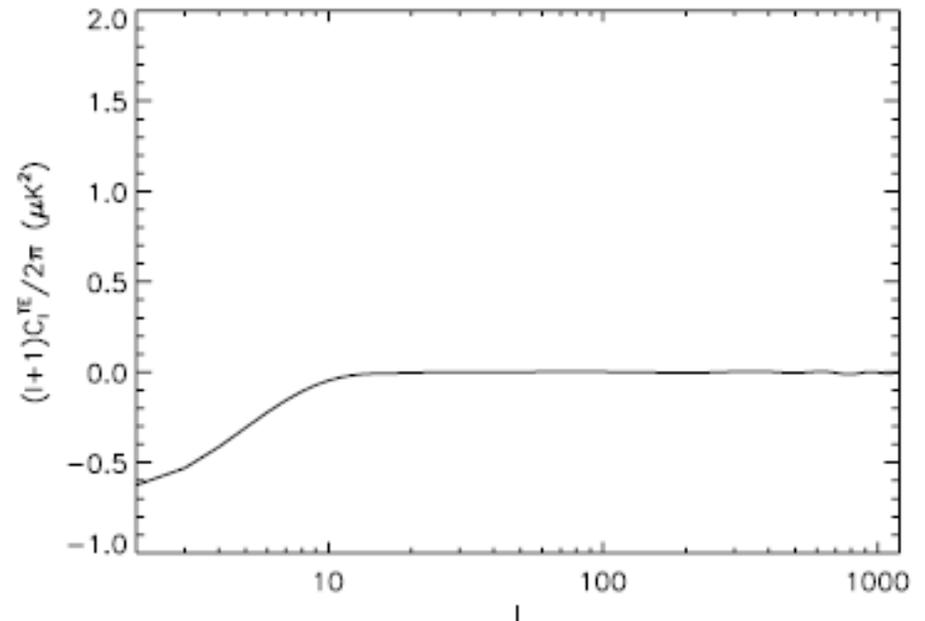


# WMAP 2008 - TE

## Cosine-type potential [note vertical axis]



WMAP collaboration  
[arXiv:0803.0593]



Cosine-type potential with:

$$m = 10^{-22} \text{ eV}, \quad g_\phi = 10^{-20} \text{ eV}^{-1}$$

# Parity odd correlators

In absence of parity-violating interactions, the ensemble of fluctuations is statistically parity symmetric and therefore the parity odd correlators have to vanish.

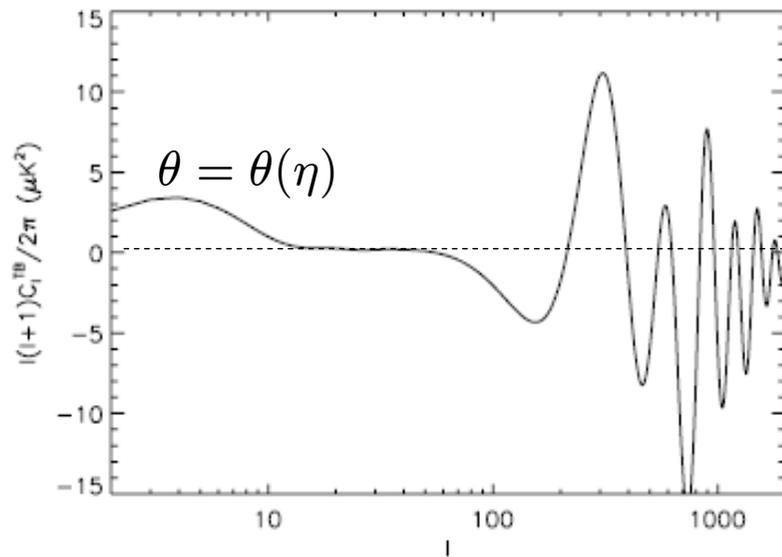
In this case photons interact with pseudoscalars:

$$\mathcal{L}_{\phi\gamma} = g_\phi \mathbf{E} \cdot \mathbf{B} \phi = -\frac{g_\phi}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \phi$$

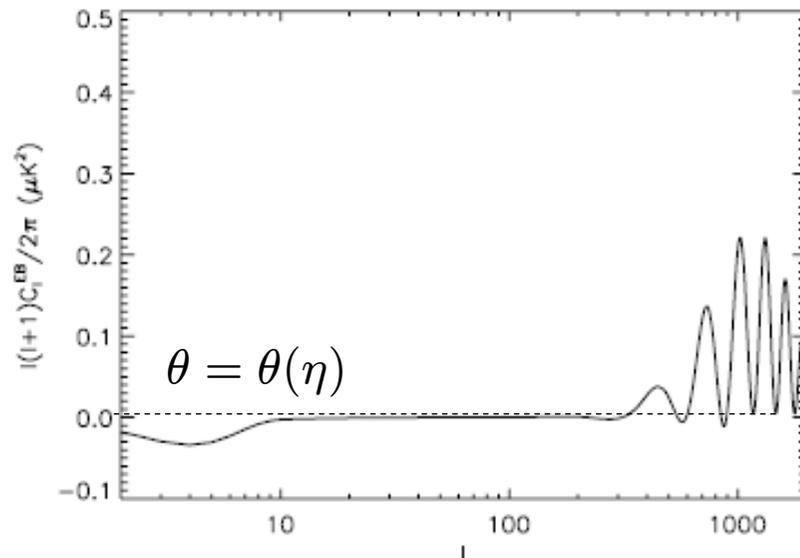
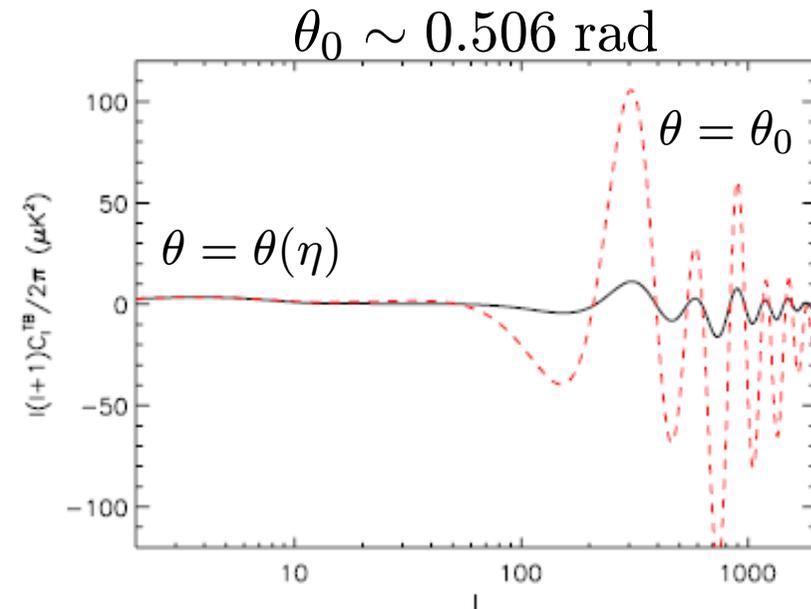
therefore also parity-odd correlators should be considered:

$$\begin{array}{ccccccc}
 C_l^{TT} \xrightarrow{P} C_l^{TT} & & & & & & \\
 C_l^{TE} \xrightarrow{P} C_l^{TE} & C_l^{EE} \xrightarrow{P} C_l^{EE} & & & & & \\
 C_l^{TB} \xrightarrow{P} -C_l^{TB} & C_l^{EB} \xrightarrow{P} -C_l^{EB} & C_l^{BB} \xrightarrow{P} C_l^{BB} & & & & \\
 [C_l^{TV} \xrightarrow{P} -C_l^{TV} & C_l^{EV} \xrightarrow{P} -C_l^{EV} & C_l^{BV} \xrightarrow{P} C_l^{BV} & C_l^{VV} \xrightarrow{P} C_l^{VV}] & & & 
 \end{array}$$

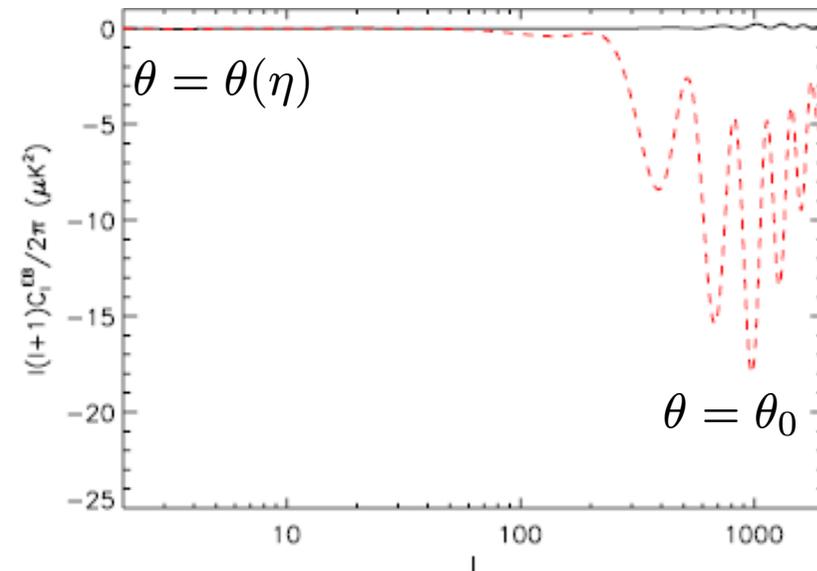
# Cosine-type potential



TB

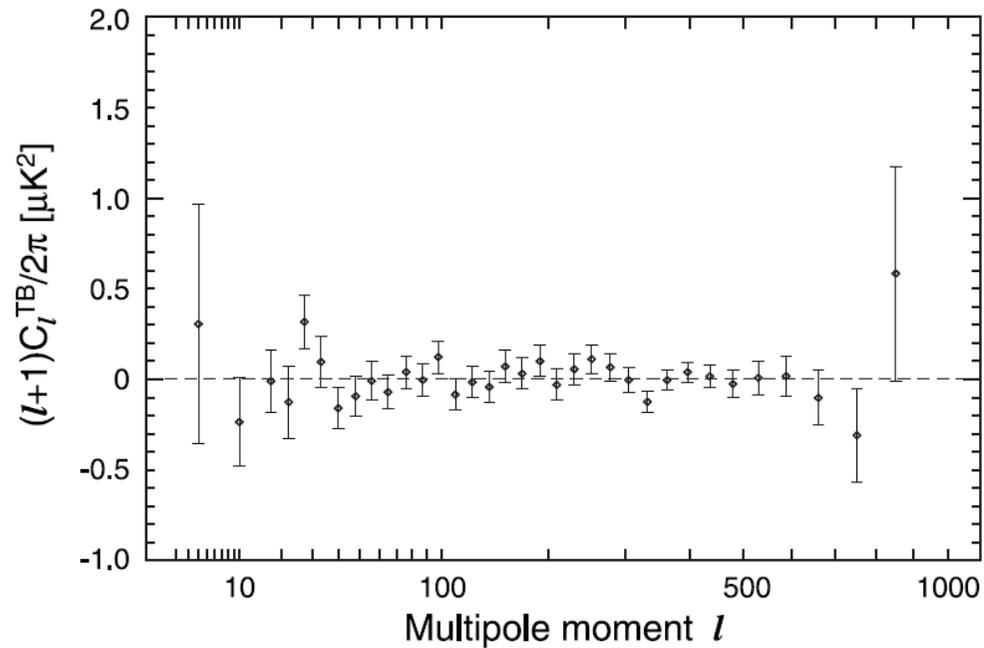


EB

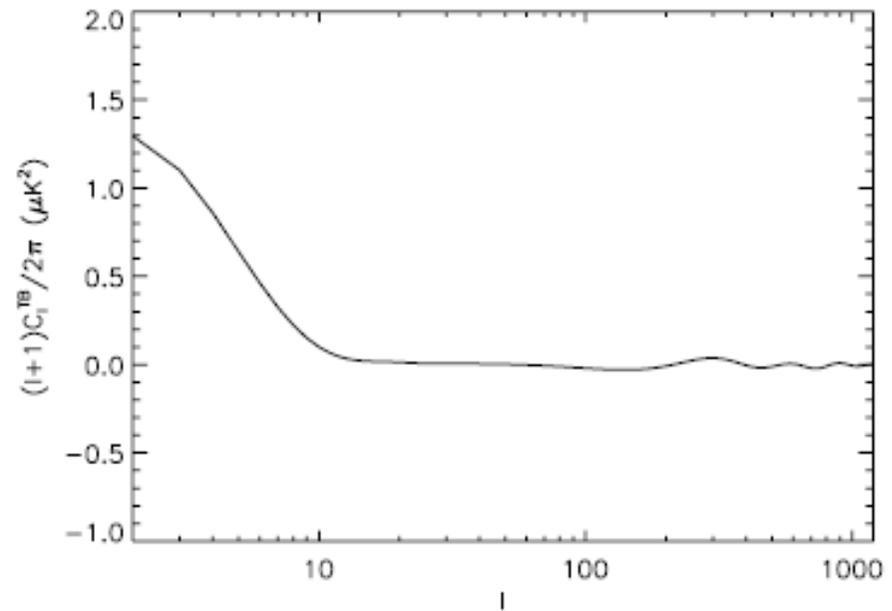


# WMAP 2008 - TB

## Cosine-type potential *[note vertical axis]*



WMAP collaboration  
[arXiv:0803.0593]



Cosine-type potential with:

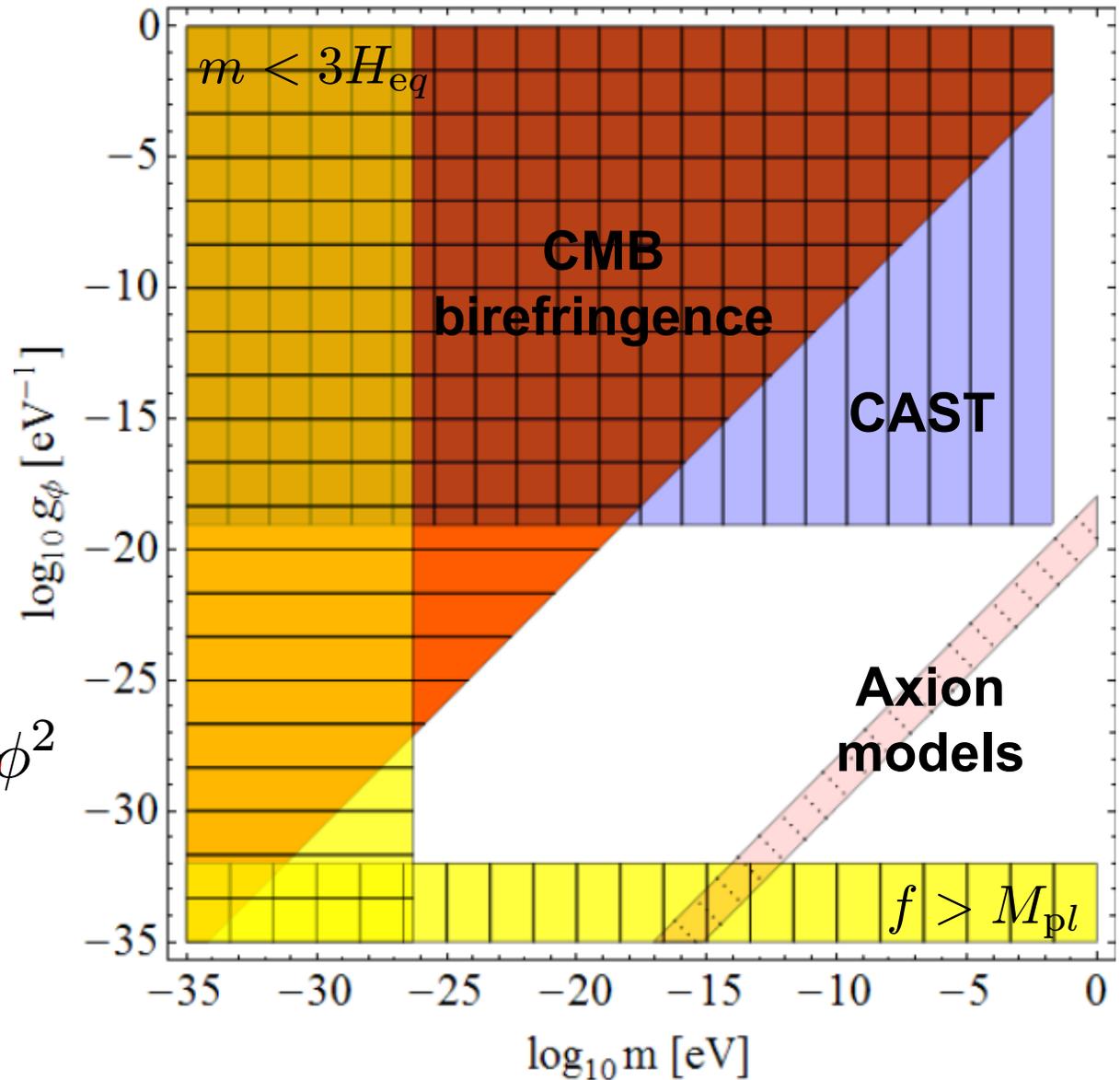
$$m = 10^{-22} \text{ eV}, \quad g_\phi = 10^{-20} \text{ eV}^{-1}$$

# Photon coupling with pseudoscalar fields

Oscillating  
behaviour:

*axion-like particles*

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \frac{g_{\phi}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$



# Ultralight pseudo Nambu-Goldstone bosons

In 1995 Frieman *et al.* [PRL **75**, 2077] proposed a quintessence model based on a pseudoscalar field.

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## Cosmology with Ultralight Pseudo Nambu-Goldstone Bosons

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(Received 17 May 1995)

We explore the cosmological implications of an ultralight pseudo Nambu-Goldstone boson. With global spontaneous symmetry breaking scale  $f \approx 10^{18}$  GeV and explicit breaking scale comparable to Mikheyev-Smirnov-Wolfenstein neutrino masses,  $M \sim 10^{-3}$  eV, such a field, which acquires a mass  $m_\phi \sim M^2/f \sim H_0$ , would currently dominate the energy density of the Universe. The field acts as an effective cosmological constant before relaxing into a condensate of nonrelativistic bosons. Such a model can reconcile dynamical estimates of the density parameter,  $\Omega_m \sim 0.2$ , with a spatially flat universe, yielding  $H_0 t_0 \approx 1$  consistent with limits from gravitational lens statistics.

# Ultralight pseudo Nambu-Goldstone bosons

In 1995 Frieman *et al.* [PRL **75**, 2077] proposed a quintessence model based on a pseudoscalar field.

This model is still in agreement with observations and can be probed by future experiment reaching stage 4 of DETF methodology (Planck CMB measurements, future SNIa surveys, baryon acoustic oscillations, and weak gravitational lensing). This analysis can be improved considering also birefringence of CMB polarization:

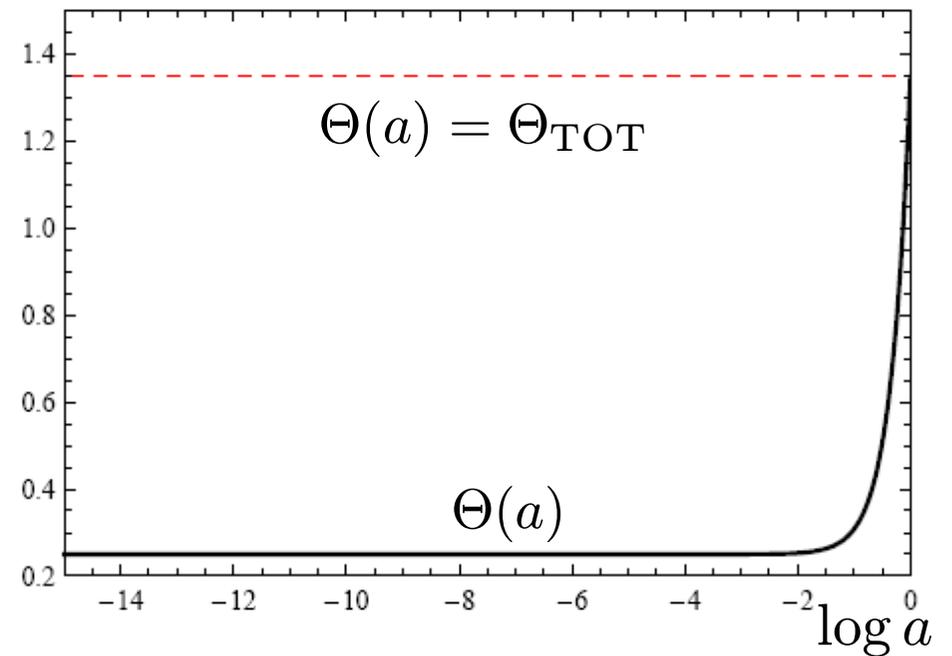
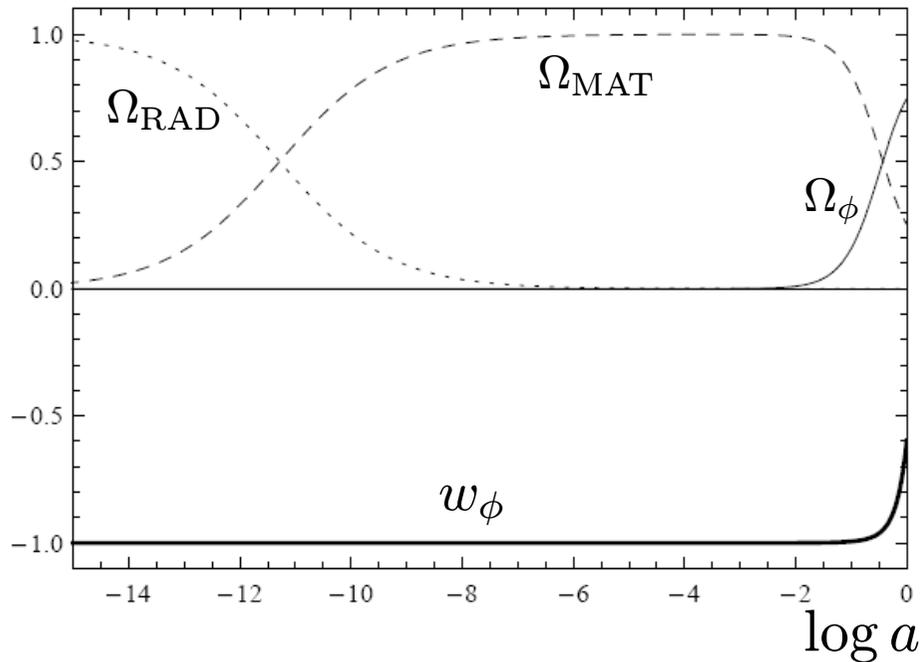
$$\begin{aligned}\mathcal{L}_\phi &= -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - M^4\left(1 + \cos\frac{\phi}{f}\right) \\ \mathcal{L}_{\phi\gamma} &= -\frac{1}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}\end{aligned}$$

where:  $M \sim 10^{-3}$  eV and  $f \lesssim M_{\text{pl}}/\sqrt{8\pi}$

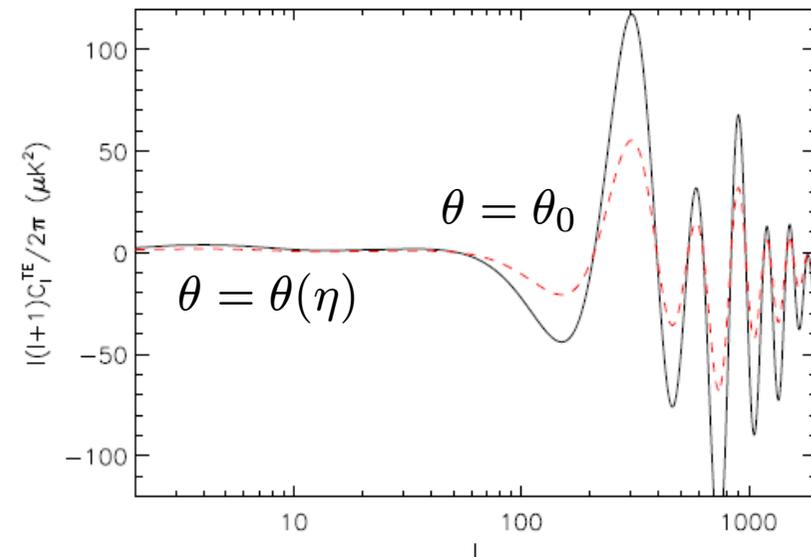
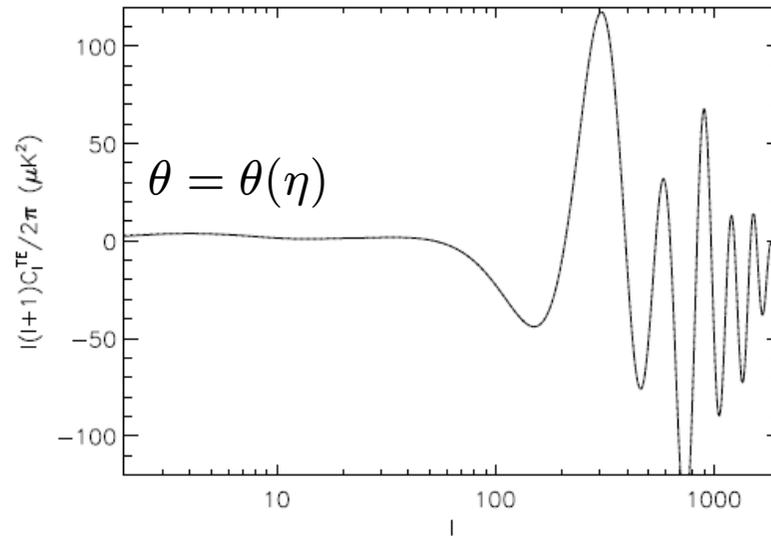
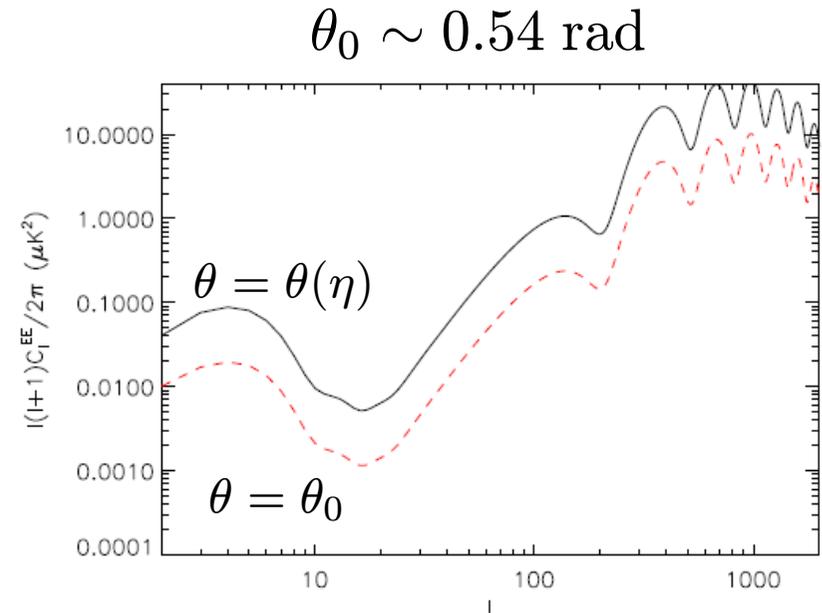
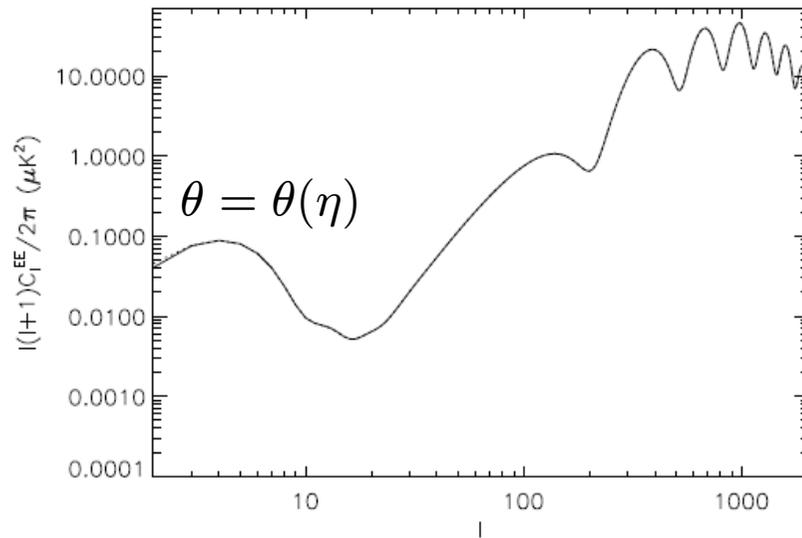
# Ultralight pseudo Nambu-Goldstone bosons

Fixed  $M = 8.8 \times 10^{-4} \text{ eV}$ ,  $f = 0.3 \frac{M_{\text{pl}}}{\sqrt{8\pi}}$ ,  $\Theta_i \equiv \frac{\phi}{f} = 0.25$ ,  $\dot{\Theta}_i = 0$

the pseudoscalar field mimes the behaviour of the cosmological constant:

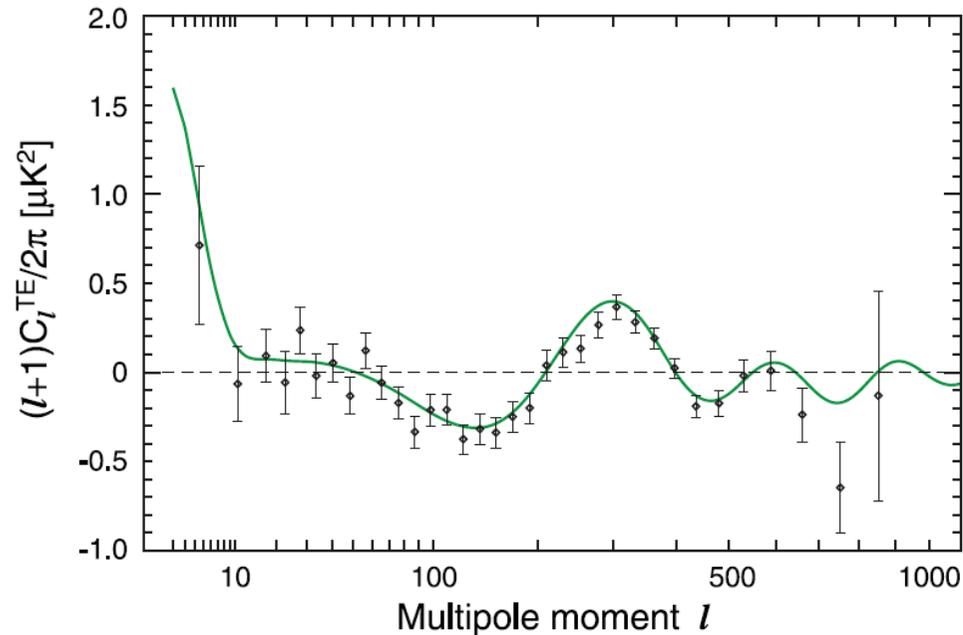


# Ultralight pseudo Nambu-Goldstone bosons

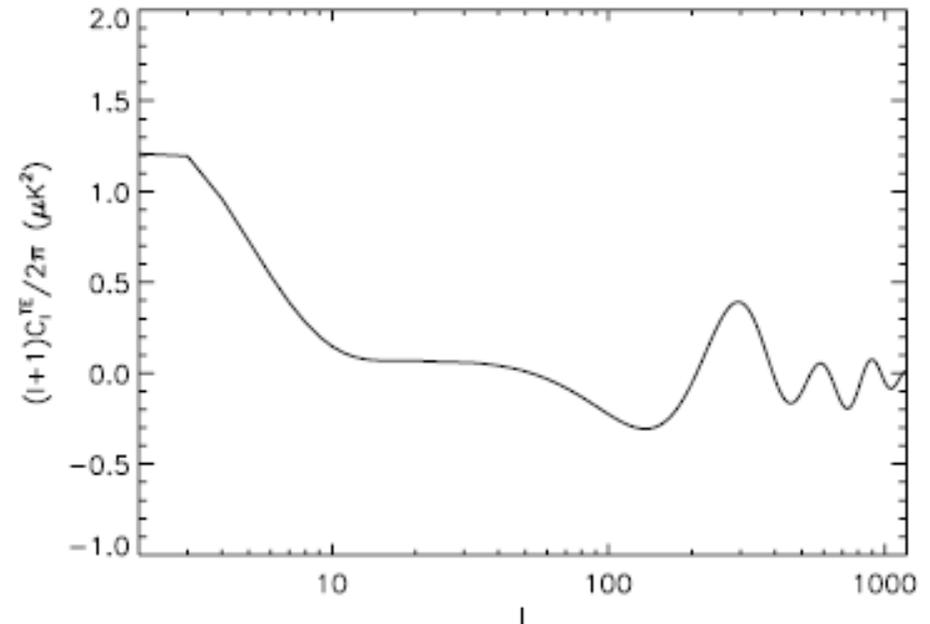


# WMAP 2008 - TE

Ultralight pseudo Nambu-Goldstone bosons [note vertical axis]



WMAP collaboration  
[arXiv:0803.0593]

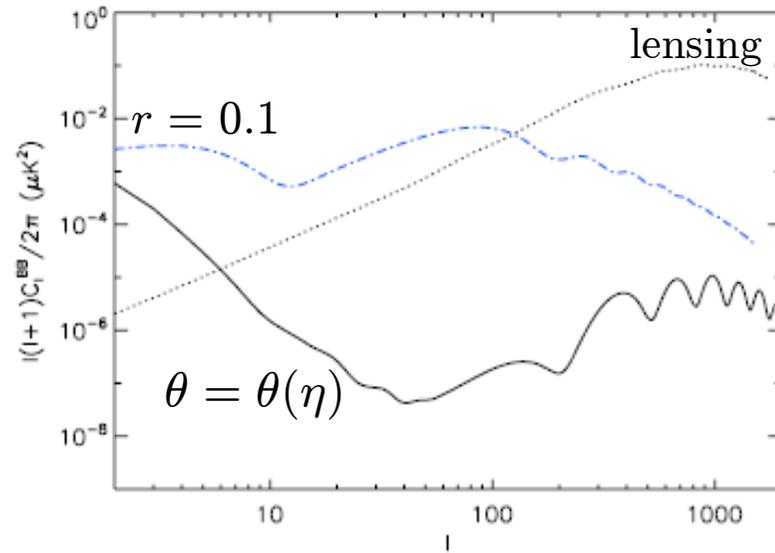


Fixed:

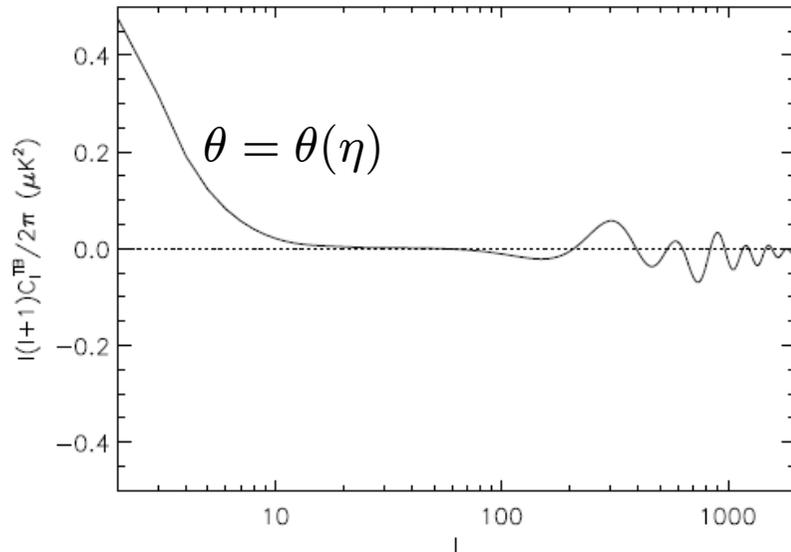
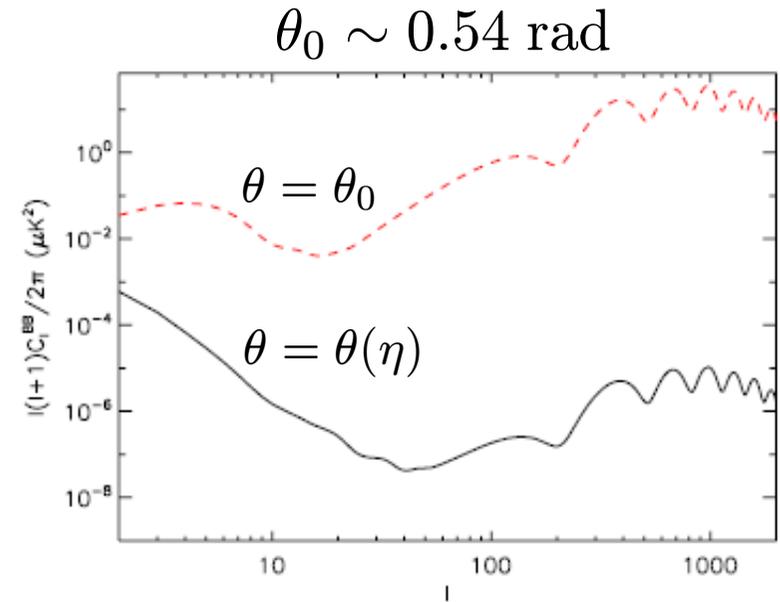
$$M = 8.8 \times 10^{-4} \text{ eV}, \quad f = 0.3 \frac{M_{\text{pl}}}{\sqrt{8\pi}},$$

$$\Theta_i \equiv \frac{\phi}{f} = 0.25, \quad \dot{\Theta}_i = 0$$

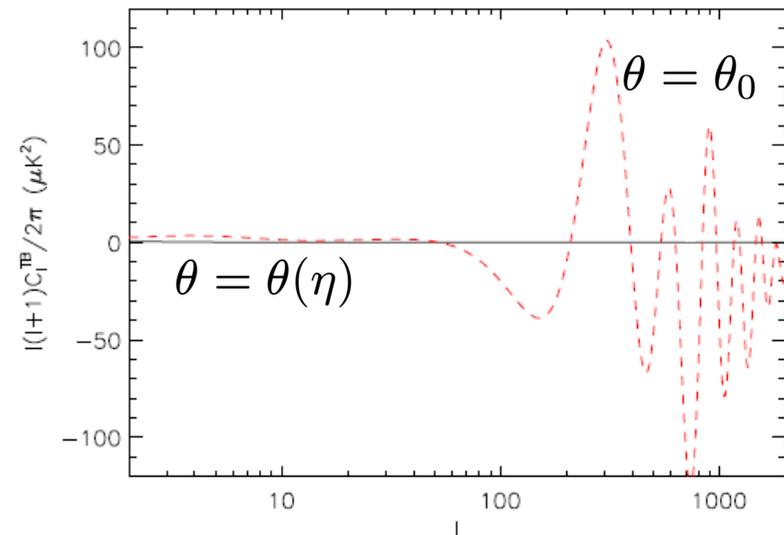
# Ultralight pseudo Nambu-Goldstone bosons



BB

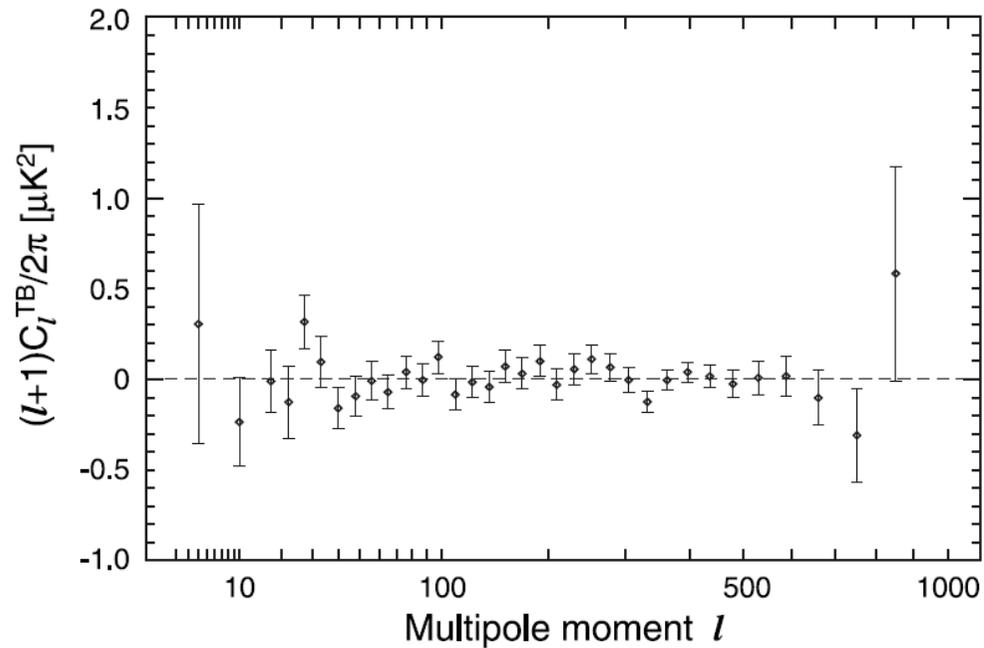


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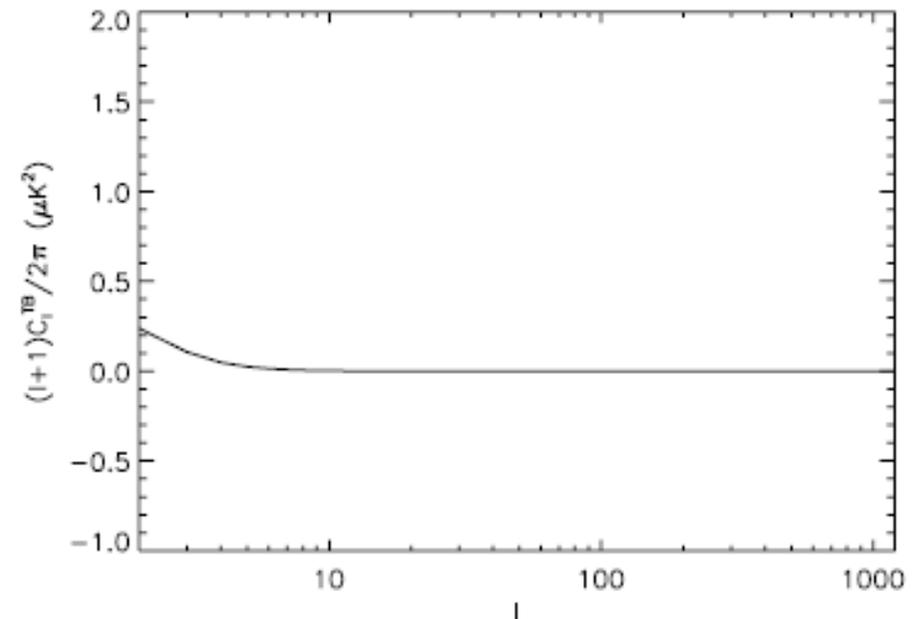


# WMAP 2008 - TB

Ultralight pseudo Nambu-Goldstone bosons [note vertical axis]



WMAP collaboration  
[arXiv:0803.0593]



Fixed:

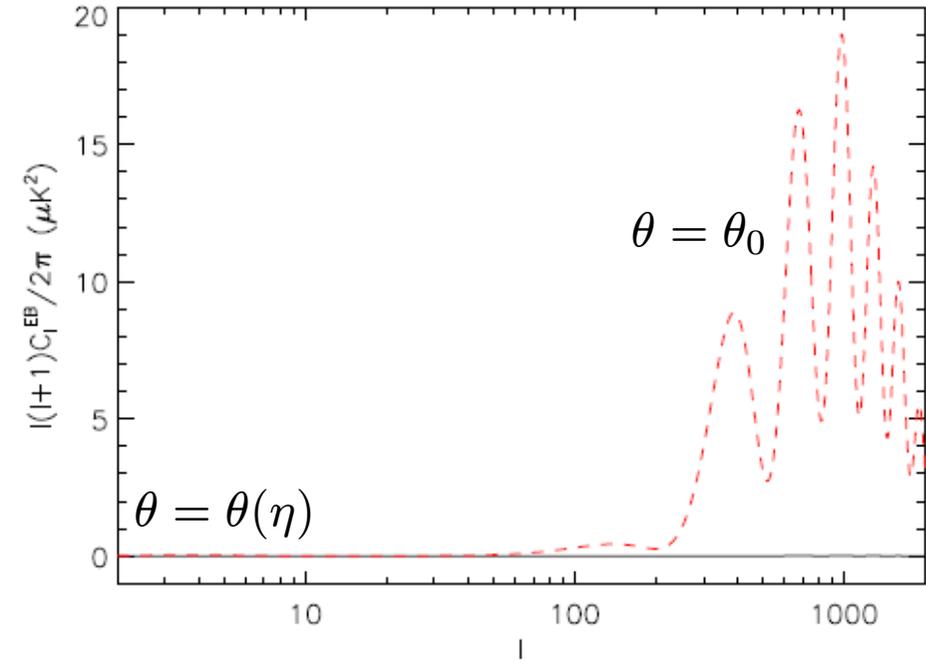
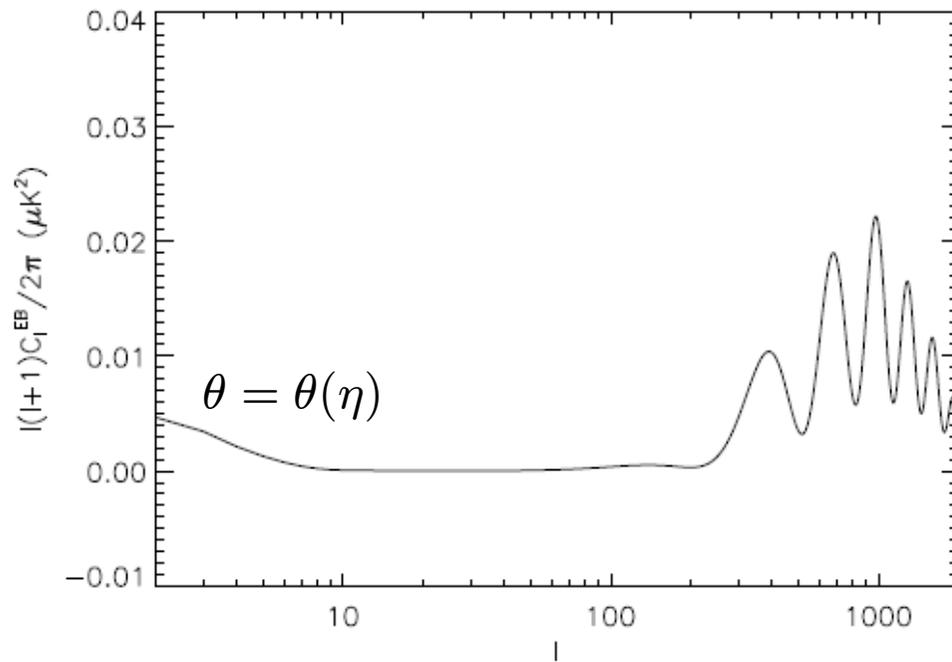
$$M = 8.8 \times 10^{-4} \text{ eV}, \quad f = 0.3 \frac{M_{\text{pl}}}{\sqrt{8\pi}},$$

$$\Theta_i \equiv \frac{\phi}{f} = 0.25, \quad \dot{\Theta}_i = 0$$

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EB

$\theta_0 \sim 0.54$  rad



# Conclusions & Developments

We discuss the effects of coupling between pseudoscalar fields and photons on Cosmic Microwave Background Polarization:

- how the public code CAMB can be modified in order to take into account the rotation of the linear polarization plane by a cosmological pseudoscalar field acting as dark matter from last scattering surface to nowadays.
- Polarization power spectra strongly depend on the kinematics of the pseudoscalar field.

CMB birefringence constraints are complementary to experiments and astroparticle observations

