Stability of form inflation

Tomi Koivisto (ITP Heidelberg)

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Why forms?

- **To test the robustness of scalar**

  is it the only natural possibility?

  scalars have not been detected yet
- **Forms exist in fundamental theories**

  string theory

  nonsymmetric gravity
- **Possibility to generate anisotropy**

  present anomalies in CMB

  Planck could detect small anisotropy

Outline

- The question: can single-field inflation be generalised to forms?
- Furthermore: are the resulting models stable (shear, perturbations, ghosts)?
- We will:
  - 0) Introduce the action
  - 1) Discuss vector & 2form: **anisotropic inflation**
  - 2) Discuss 3form & 4form: **new isotropic inflation**
  - 3) Summarise and look out

Stability in flat space

- Parity and Lorentz-invariant,quadratic

$$\mathcal{L}_f = -\frac{1}{4}a(\partial A)^2 - \frac{1}{2}b(\partial \cdot A)^2 - \frac{1}{4}m^2 A^2, \quad (1)$$

- Ghost or nonlocality unless a(a+b)=0

van Nieuwenhuizen, Nucl. Phys. B69, 478 (1973)

- If a=-b: Maxwell recovered
- If a=0: Dual theory

Stability in curved space

- General curvature couplings:

$$\mathcal{L}_c = -\frac{1}{2}\sqrt{-g}\left(\xi R A^2 - c R_{\mu\nu} A^{\mu\alpha} A_\alpha{}^\nu - R_{\mu\nu\alpha\beta} A^{\mu\nu} A^{\alpha\beta}\right)$$

- FRW stability: c=d
- Schwarzchild solutions : d=0
- …we're left with a coupling to R

Janssen and Prokopec, CQG 23, 467 (2006)

The models

- Thus we consider the case

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2(n+1)!} F^2 - \frac{1}{2}(m^2 + \xi R) A^2 \right]$$

- Notations: $F = (n+1)\mathcal{A}\partial A$

$$\begin{aligned} F_{M_1 \dots M_{n+1}}(A) &\equiv F_{M_{n+1}} \\ &\equiv (n+1)\partial_{[M_1} A_{M_2 \dots M_{n+1}]} \end{aligned}$$

- EOM: $\nabla \cdot F = n! \left(2V' + \xi R \right) A$

Stückelberg form: $A = B + \frac{1}{M}\mathcal{A}\partial\Sigma$

- We get the Lagrangian

$$\mathcal{L} = -\frac{1}{2(n+1)!}F^2(B) - \frac{1}{2}M^2\left(B + \frac{1}{m}\mathcal{A}\partial\Sigma\right)^2$$

- Gauge invariance restored, for an (n–1) form Δ:

$$\begin{aligned} \Sigma &\rightarrow \Sigma + \Delta, \\ B &\rightarrow B - \frac{1}{M}\mathcal{A}\partial\Delta \end{aligned}$$

- We can choose a gauge where

$$\mathcal{L} = -\frac{1}{2(n+1)!}F^2(B) - \frac{1}{2}M^2B^2 - \frac{1}{2}sgn(M^2)\,(\mathcal{A}\partial\Sigma)^2 .$$

- Thus: eff. mass negative -> **a (n–1)-ghost**!

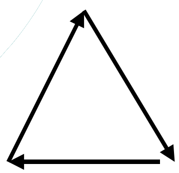
Vector field cosmology

Ford: Phys.Rev.D40:967 (1989)

FRW symmetry problematic:

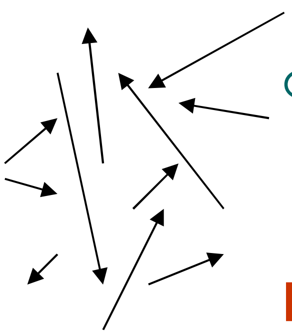
- A spatial vector not compatible
- Time-like field trivial

Proposed solutions:



- Introduce a “triad” of three spatial (stability?)

Armendariz-Picon: JCAP 0407:007 (2004)



- Introduce a large number of random fields (tractability?)

Golovnev, Mukhanov & Vanchurin: JCAP 0806 (2008)

But generation of anisotropy was among our original moti

Vector field: Background

- **In Bianchi I universe, a vector must be aligned along a spatial axis!**
- **So, consider axisymmetry with shear σ:**

$$
\begin{aligned}
ds^2 &= -dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2+dz^2\right)\right] \\
&= -dt^2 + a^2(t)dx^2 + b^2(t)\left(dy^2+dz^2\right)
\end{aligned}
$$

- **The EOM for the comoving field X=A/a is**

$$
\ddot{X} + 3H\dot{X} + \left[2V' + (1+6\xi)(\dot{H}+2H^2) - 2H\dot{\sigma} - 2\ddot{\sigma} - (4-6\xi)\dot{\sigma}^2\right]X = 0
$$

- **For slow roll one needs conformal coupling**

Golovnev, Mukhanov & Vanchurin JCAP 0806 (2008)

- **More general couplings:**

TK & D. Mota: JCAP 0806 (2008)
TK & D.Mota: JCAP 0808 (2008)

Vector field: perturbations

- **Consider A=(α0, α,i+αi) in Minkowski**

$$S=\int d^4x\left\{\frac{1}{2}\left[|\alpha_i'|^2-\left(k^2+M^2\right)|\alpha_i|^2\right]+\frac{1}{2}\left[k^2|\alpha'|^2-k^2(\alpha'^*\alpha_0+cc)-M^2k^2|\alpha|^2+\left(k^2+M^2\right)|\alpha_0|^2\right]\right\}$$

- **Solve α0 and plug back:**

$$S=\int d^4x\frac{k^2M^2}{2}\left[\frac{|\alpha'|^2}{k^2+M^2}-|\alpha|^2\right]$$

- **If M^2<0, α becomes a ghost**

  and now indeed M^2 = -R/6+m^2~-H^2
- **The ghost is confirmed by full computation**

  Himmetoglu, Contaldi & Peloso arXiv:0812.1231

- **We already learned it with Stückelberg!**

Other vector models: remarks

- Several cases exist in the literature studying inflation with "vector impurity"

  e.g.: Kanno, Kimura, Soda, Yokoyama JCAP 0808:034,2008

- Our arguments apply to these models as such though the vector isn't dominating

- The fixed-norm case

$$L=-\alpha_1(\nabla A)^2-\alpha_2 F^2-\alpha_3(\nabla\cdot A)^2+\lambda(A^2-m^2)$$

  Ackerman, Carroll and Wise Phys.Rev.D75:083502,2007

  has a similar instability of the longitudinal vector mode

Two-form: background

$$ds^2 = -dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2+dz^2\right)\right]$$
$$= -dt^2 + a^2(t)dx^2 + b^2(t)\left(dy^2+dz^2\right)$$

- Symmetry allows only A=XdyΛdz/b. Then

$$\ddot{X} + 3H\dot{X} + 2\left[2V'(A^2) + (1+6\xi)\dot{H} + (1+12\xi)H^2 - \dot{\sigma}H + \ddot{\sigma} - 2(1-3\xi)\dot{\sigma}^2\right]X = 0.$$

- Thus, slow roll requires conformal/2 coupling

- Now effective mass contributions remain due to, in addition to shear, −ϵH^2

- At the level of action:

  M^2 = -R/12+m^2 ~ -H^2/2,

  Stückelberg says we expect a vector ghost

2-form: perturbations

- **Go to Minkowski and decompose with transverse potentials**

$$\begin{aligned} A_{0i} &= \partial_i E + E_i \\ A_{ij} &= \epsilon_{ijk}(\partial^k B + B^k) \end{aligned}$$

$$\begin{aligned} S &= \int d^4x \left[\frac{1}{2}B_i' B^{i\prime} + \frac{1}{2}\partial_i B' \partial^i B' - \frac{1}{2}\Delta B \Delta B + \frac{1}{2}\partial_i E_j \partial^i E^j - B^{i\prime}\epsilon_{ijk}\partial^k E^j\right] \\ &+ \int d^4x \left[-\frac{1}{2}M^2 B_i B^i + \frac{1}{2}M^2 E_i E^i - \frac{1}{2}M^2 \partial_i B \partial^i B + \frac{1}{2}M^2 \partial_i E \partial^i E\right] \end{aligned}$$

- **with further decomposition we can write the constraints as**

$$B^i = \sum_{a=1,2} \mathrm{i} B^a e_a^i$$

$$\begin{aligned} E &= 0 \\ (M^2 + k^2)E_a &= \pm \mathcal{M}_a{}^b k B_b \end{aligned} \qquad \mathcal{M}_a{}^b = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- **Plugging back into action yields the result…**

2form: perturbations

- **The action for perturbations is:**

$$S = \int d^4x \left\{ \frac{k^2}{2} \left[|B'|^2 - \left(k^2 + M^2\right) |B|^2 \right] + \frac{M^2}{2} \left[\frac{|B_i'|^2}{k^2 + M^2} - |B_i|^2 \right] \right\}$$

- **There is a well behaved scalar part**
- **There is also a vector ghost when M^2<0**

- **We conclude that massive 1- or 2-forms cannot support inflation**

Three-form

- **Symmetry allows only FRW and**

$$A = e^{3\alpha(t)} X(t) dx \wedge dy \wedge dz$$

- **The EOM becomes**

$$\ddot{X} + 3H\dot{X} + 3\left[4V'(A^2) + 24\xi H^2 + (1 + 12\xi)\dot{H}\right] X = 0$$

- **Coupling nothing but introduces large mass**

  -> set ξ = 0

- **Promote the mass term into V(x^2)**

  This time S. only requires V>0 for stability

Three-form $A = e^{3\alpha(t)} X(t) dx \wedge dy \wedge dz$

- X is not equivalent to scalar, but

$$\rho_X = \frac{1}{2}\left(\dot{X} + 3HX\right)^2 + V(X),$$
$$p_X = -\frac{1}{2}\left(\dot{X} + 3HX\right)^2 + V'(X)X - V.$$

- **Always** **ρ = + kinetic + potential**
- **If V is constant** **p = – kinetic – potential**
- **If V is mass term** **p = – kinetic + potential**

$$\frac{6\ddot{a}}{a} = \left(\dot{X} + 3HX\right)^2 + 2V - 3V'(X)X > 0$$

- It seems a minimally coupled 3-form inflates easily
- Phantom inflation occurs whenever V’(A^2)<0

Four-form

- The only possibility: A=X(t)dt∧dx∧dy∧dz
  the kinetic term is trivial. Call A^2 = φ
- Algebraic EOM: $V'(\varphi) = \xi R/2$
- Plugging back gives an f(R) theory:
  $$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(\frac{1}{\kappa^2} - \xi\varphi(R)\right)R - V(\varphi(R))\right]$$
- If V is quadratic, this just the R^2 inflation

Starobinsky, Phys. Lett. B91, 99 (1980)

Dual

$$(*A)_{M_{n+1}\dots M_d} = \frac{1}{n!}\epsilon_{M_d} A^{M_n}$$

- Maps A into the orthogonal subspace of (d-n)-forms

- The field strenght transforms as

  $$¿\, F = (-1)^{(d-n)d} \nabla \cdot (*A)$$

  yielding the only stable kinetic term for 2form in flat space

- The resulting theory is not equivalent since ksi

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2(n+1)!} (\nabla \cdot (*A))^2 - \frac{1}{2}(m^2 + \xi R)(*A)^2 \right]$$

- EOM: $\nabla\nabla \cdot (*A) = 2(n+1)! \left(V' - \frac{1}{2}\xi R \right) (*A)$

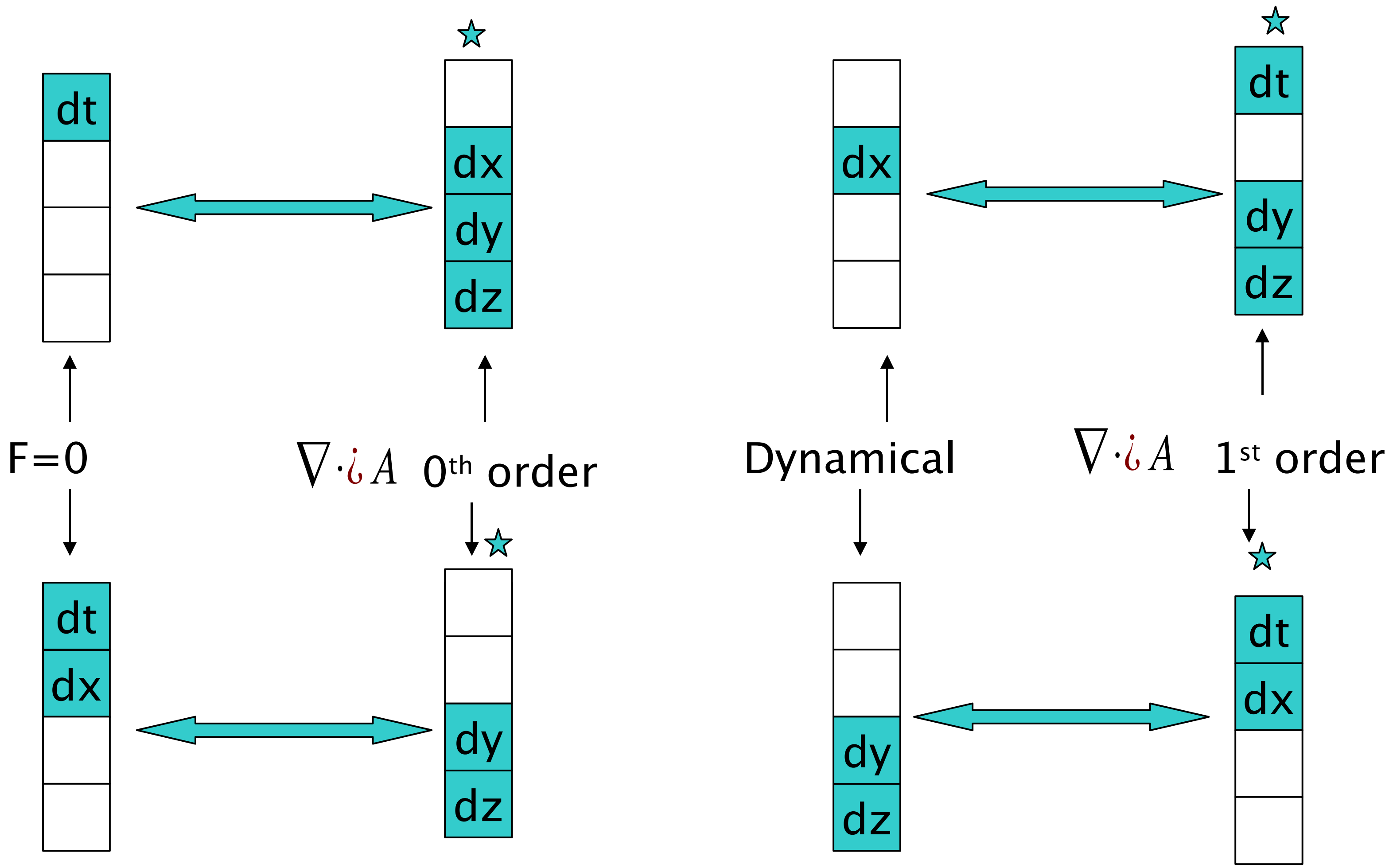
Reformulation

- So one may write the dual as a (d-n-1)-index generalisation of a massive scalar

$$L=-\frac{(n+1)!}{2}m^2\varphi^2-V\left(A^2([\nabla\varphi]^2)\right)-\frac{1}{2}\xi RA^2([\nabla\varphi]^2)$$

A		**Φ**	**n=d-1**
Mass term	↔	Canonical kinetic	Quintessence
Nonquadratic potential	↔	Noncanonical kinetic	K-essence
Dual kinetic	↔	**Mass term**	Chaotic inf.
Nonquadratic dual kinetic	↔	**General potential**	General scalar potential

Forms in axisymmetric B(I)



form	#dof	shear	coupling	comments
0	1	0	0	A scalar field
*0=4	1	0	0	1st order EOM
1	4	~X^2	-1/6	Scalar ghost appears
*1=3	4	0	0	1st order EOM
2	6	~X^2	-1/12	Vector ghost appears
*2=2	6	Not 0	0	1st order EOM
3	4	0	0	Isotropic inflation
*3=1	4	0	0	Equivalent to scalar
4	1	0	not zero	Metric f(R) gravity
*4=0	1	0	not zero	Equivalent

Outlook

- **To find stable models supporting anisotropy**
  - Go to nonquadratic theories
  - Consider scalar inflaton + forms
- **To see if the new isotropic inflations are viable**
  - Check stability of perturbations
  - Compute the fluctuation spectrum
- **Other applications**
  - Origin of 4 large dimensions
  - Dark energy