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Comparison of atom and photon interferometers using 5D optics

Christian J. Bordé

Académie des Sciences

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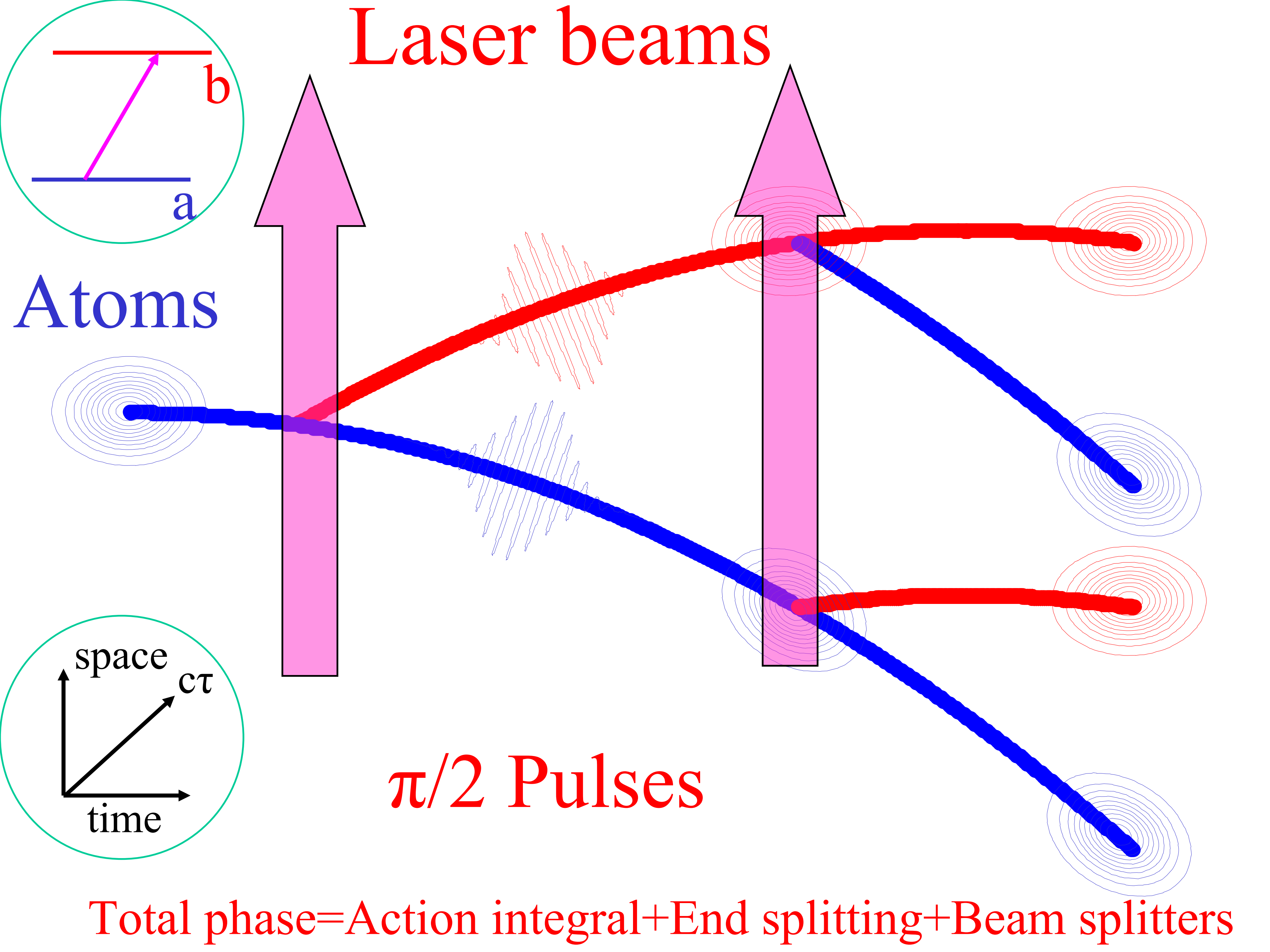
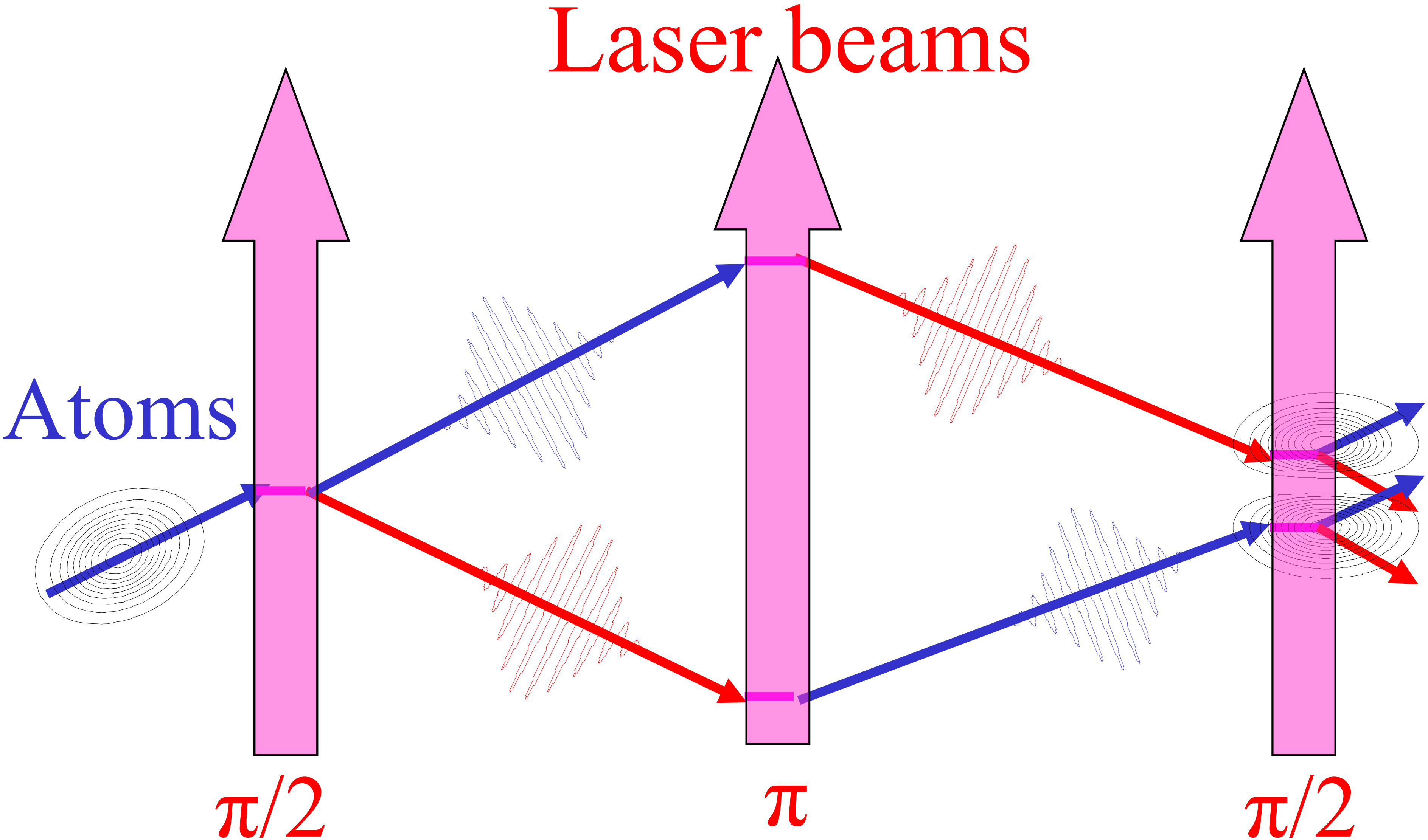


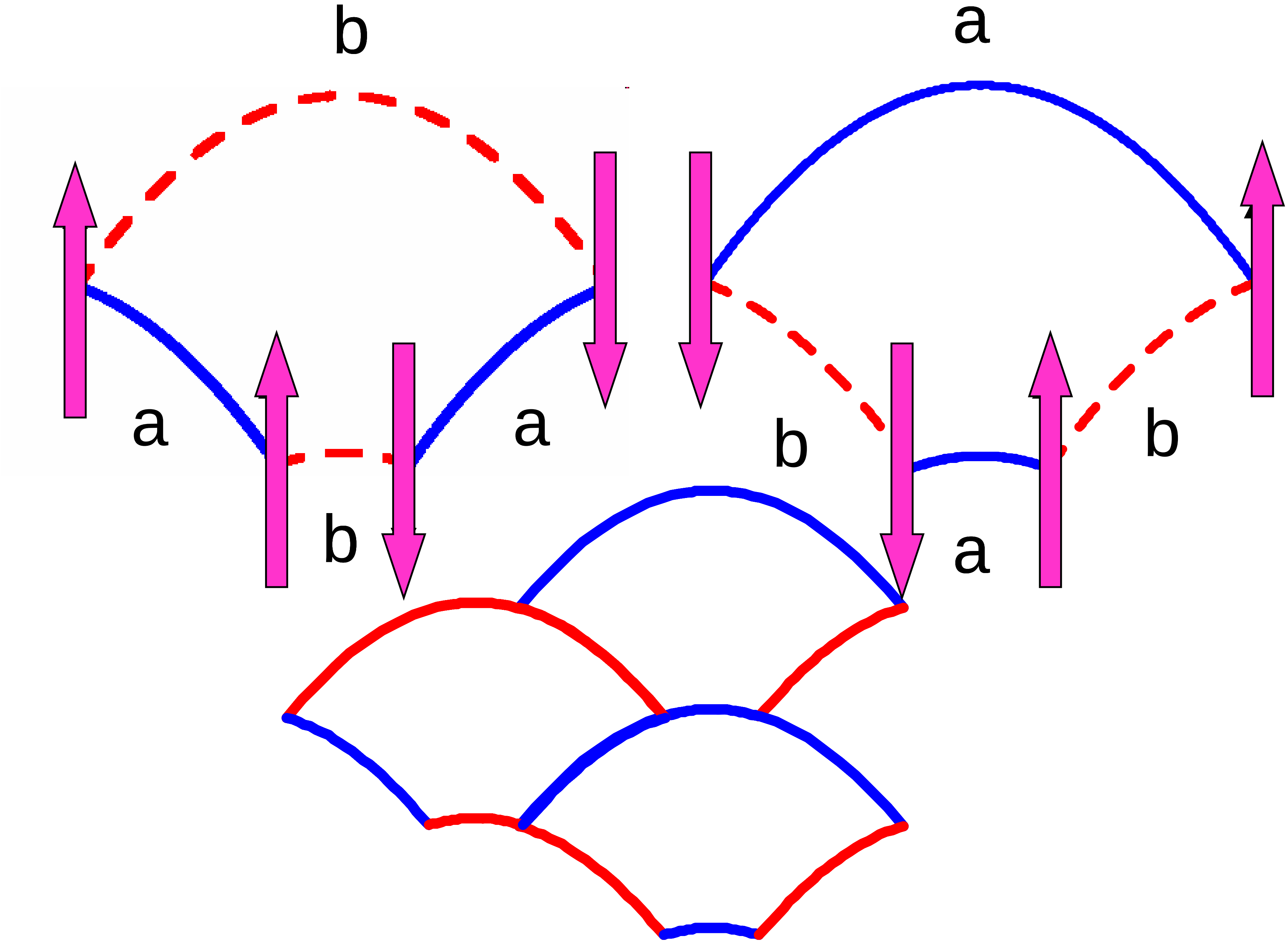
Laser beams
b
a
Atoms
space
cτ
time
π/2 Pulses
Total phase=Action integral+End splitting+Beam splitters



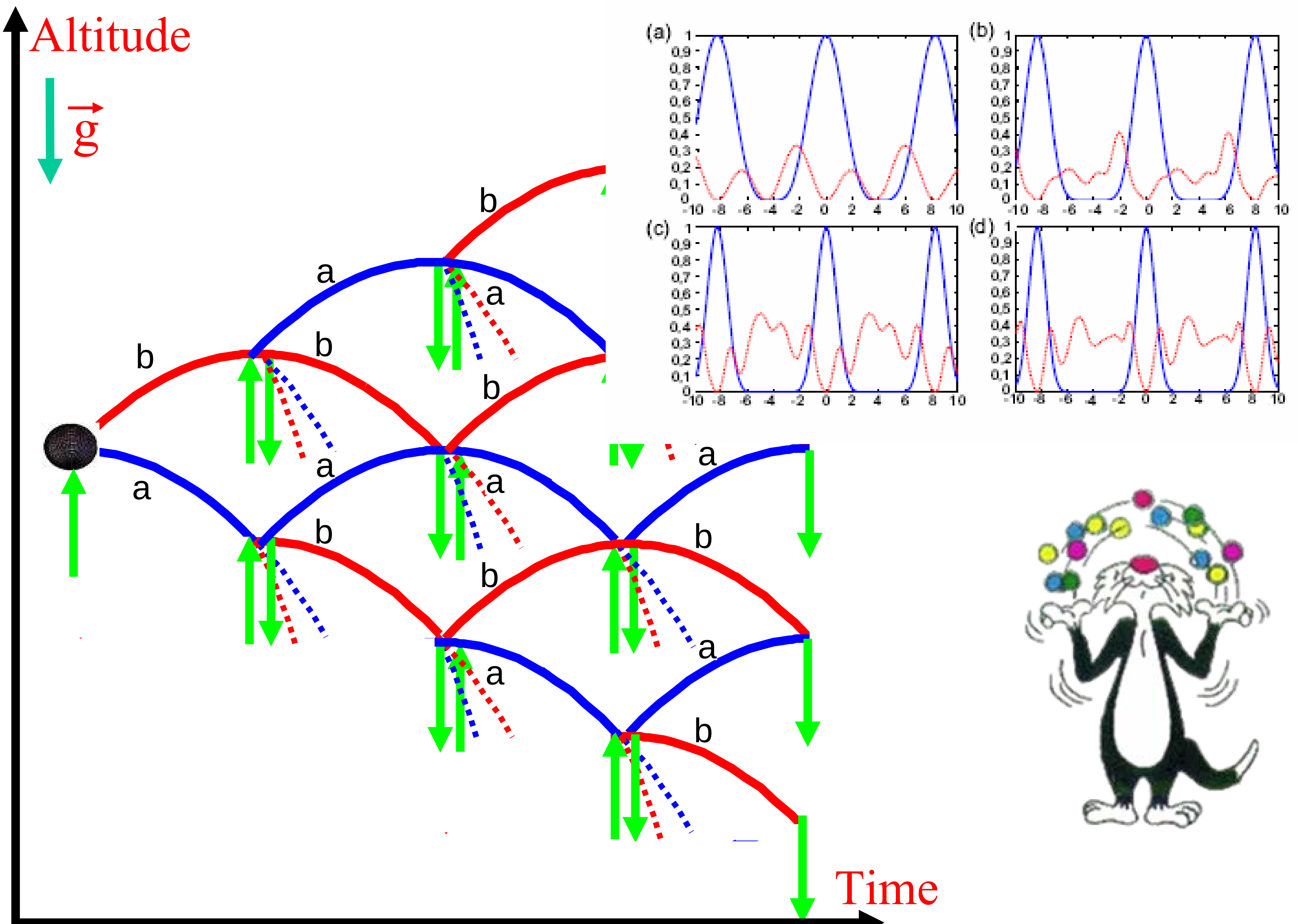
BORDÉ-CHU INTERFEROMETER

Total phase=Action integral+End splitting+Beam splitters

BORDÉ-RAMSEY INTERFEROMETERS



Multiple wave interferometer



NEW OPTICAL ATOMIC INTERFEROMETERS FOR PRECISE MEASUREMENTS OF RECOIL SHIFTS. APPLICATION TO ATOMIC HYDROGEN

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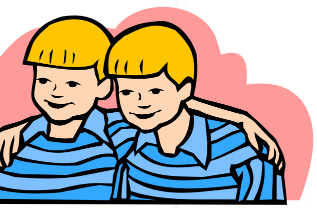
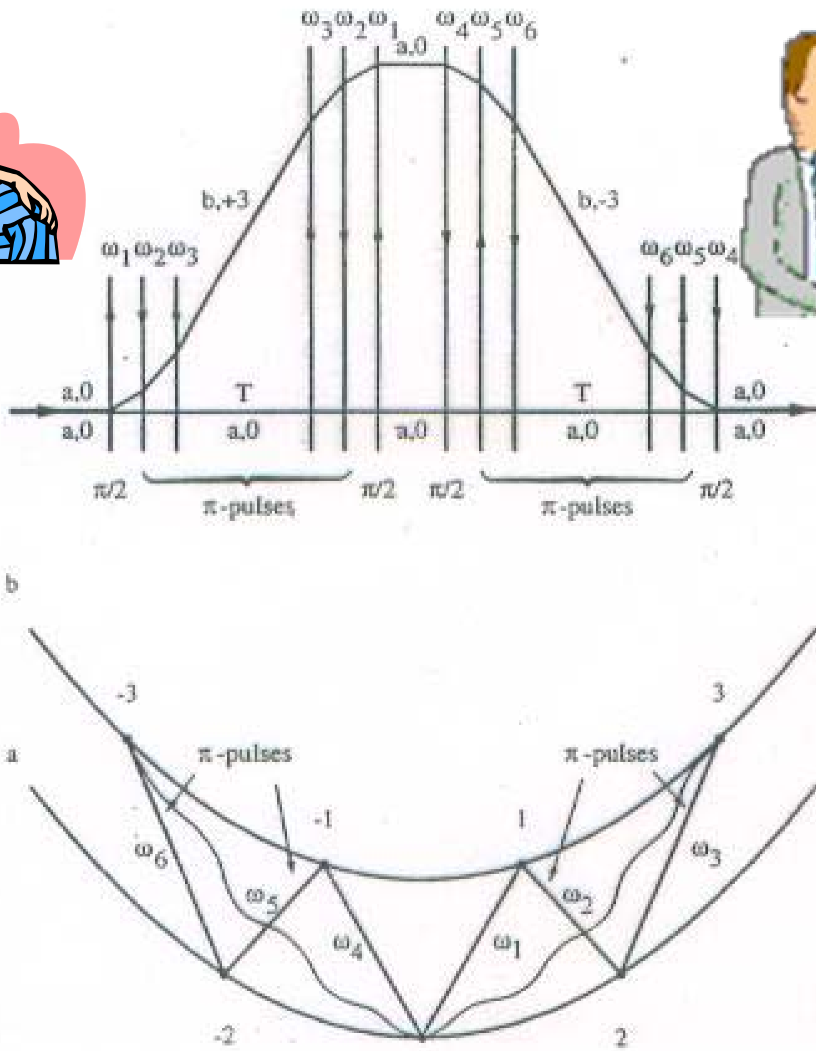




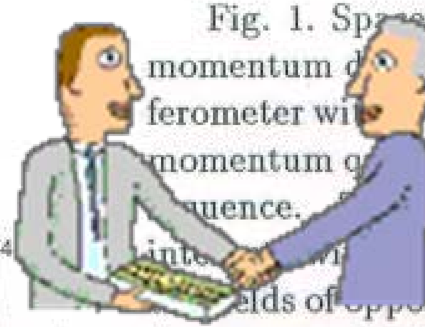
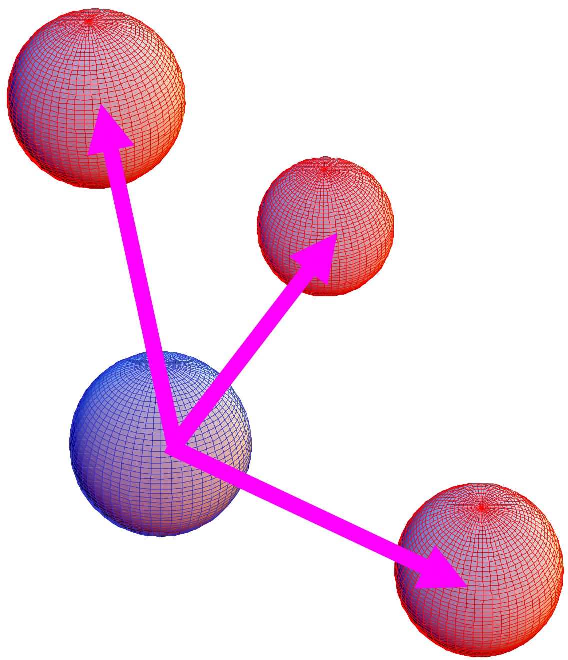


Fig. 1. Spa-time and energy-momentum d ms for an interferometer wi $_1| = 3$ exchanged momentum q a per interaction uence. wo-level system inte wi ective multipho- elds of op ite directions either perpendicular or collinear to the atomic motion. The space-time diagram displays the deflection along the optical axis versus the proper time in the "atomic frame" at the velocity $\vec{p}_0/M$. A coherent superposition of the two states $|a,0>$ and $|b,m_1>$ is created (wiggly line) and travels freely during the time T leading to a phase shift $\varphi = (\omega_1 - \omega_2 + \omega_3 + \omega_4 - \omega_5 + \omega_6 - 2\omega_0 - 18\delta)T$. A second interferometer with opposite recoil shift is obtained by exchanging the roles of states a and b.



$$\Box \equiv g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$$

$$\Box\varphi = 0$$

$$p^{\mu}p_{\mu} = M^{2}c^{2} \rightarrow \Box\varphi + \frac{M^{2}c^{2}}{\hbar^{2}}\varphi = 0$$

FROM 3 TO 4 SPATIAL DIMENSIONS

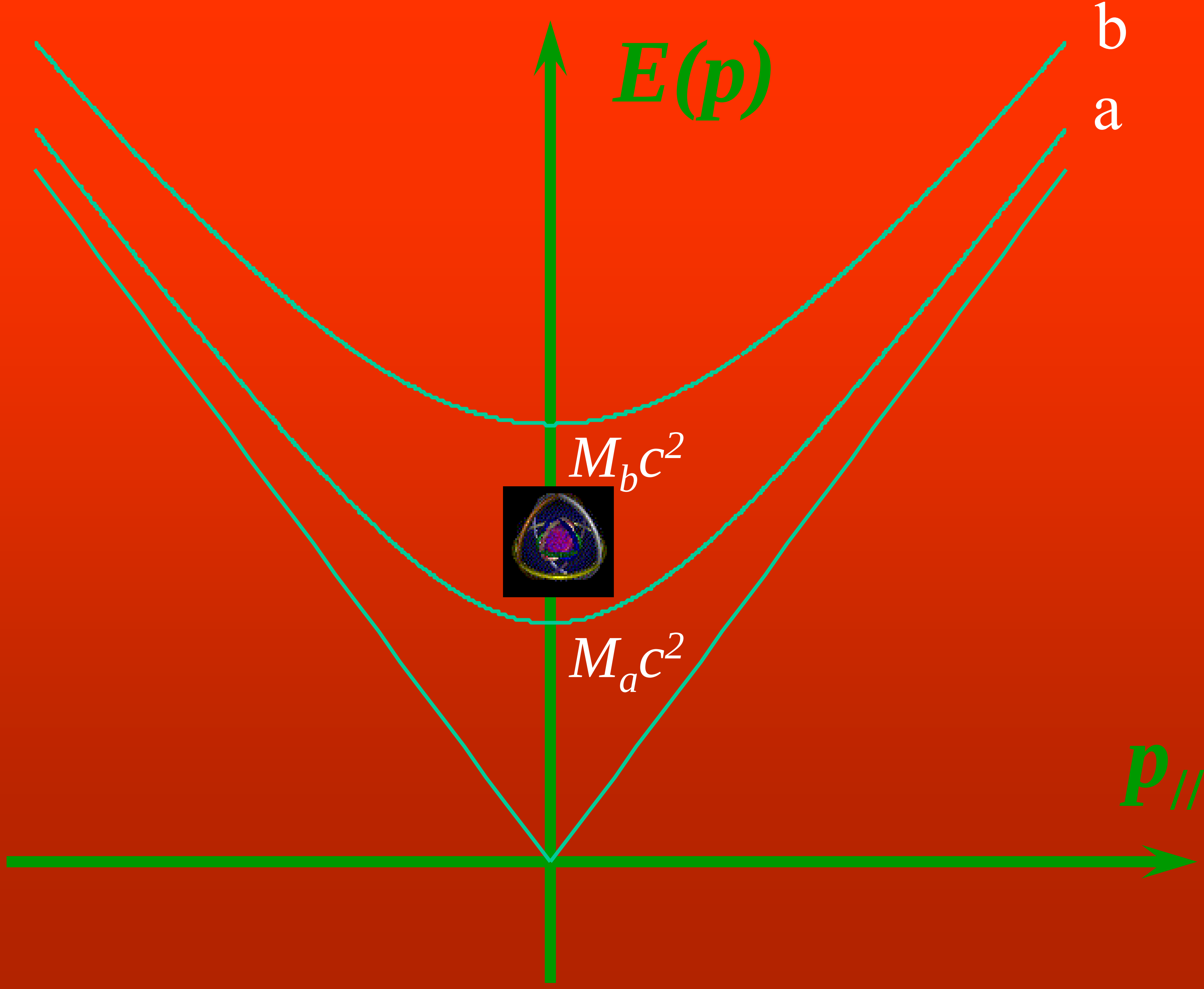
$$\varphi(x, c\tau) = \exp\left[i \frac{Mc^2}{\hbar} (\tau - \tau_0) \right] \varphi(x, Mc)$$

$$i\hbar \frac{\partial \varphi(x, c\tau)}{\partial \tau} = -Mc^2 \varphi(x, \tau)$$

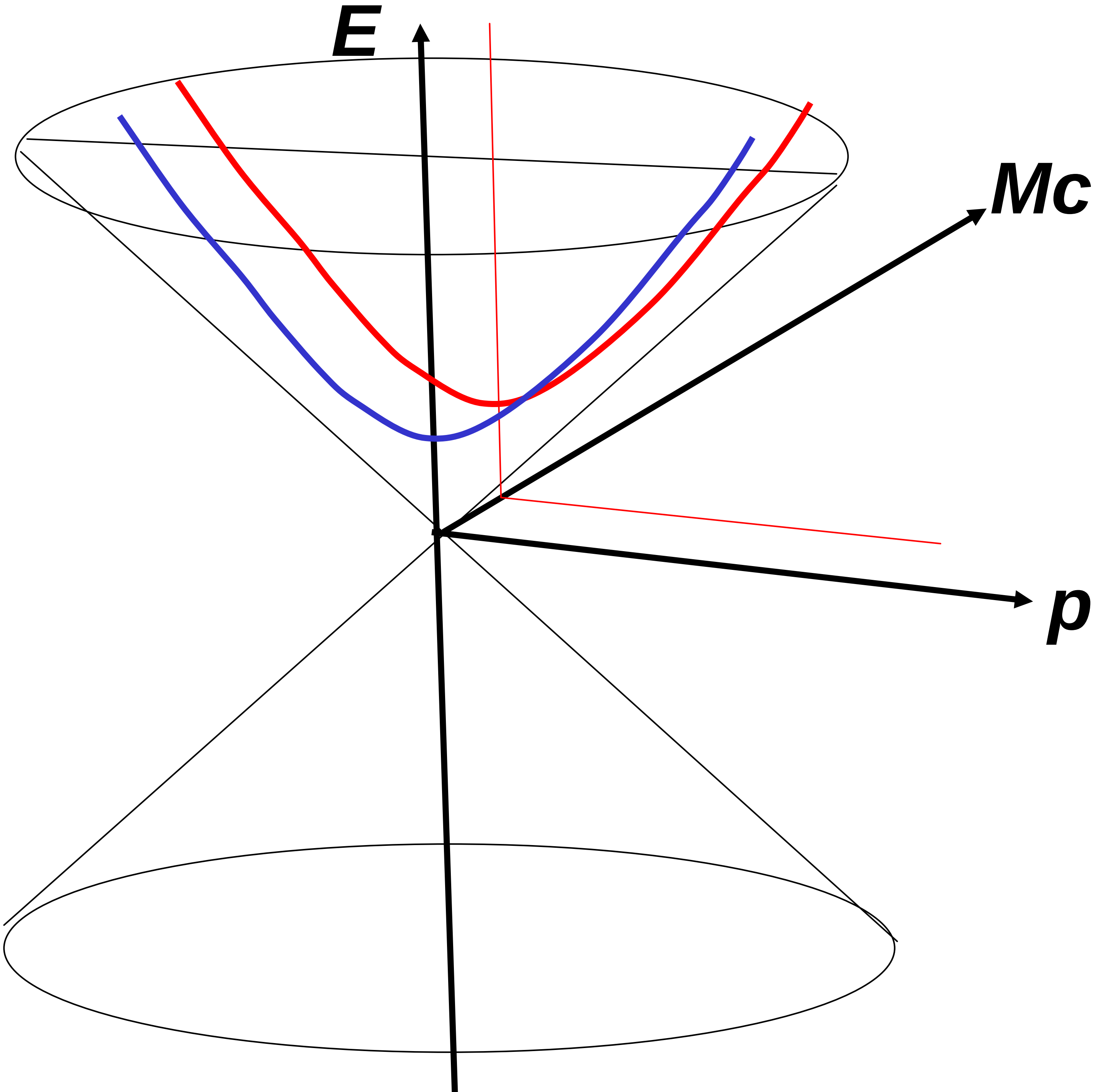
$$\frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \Delta \varphi + \frac{M^2 c^2}{\hbar^2} \varphi = 0$$

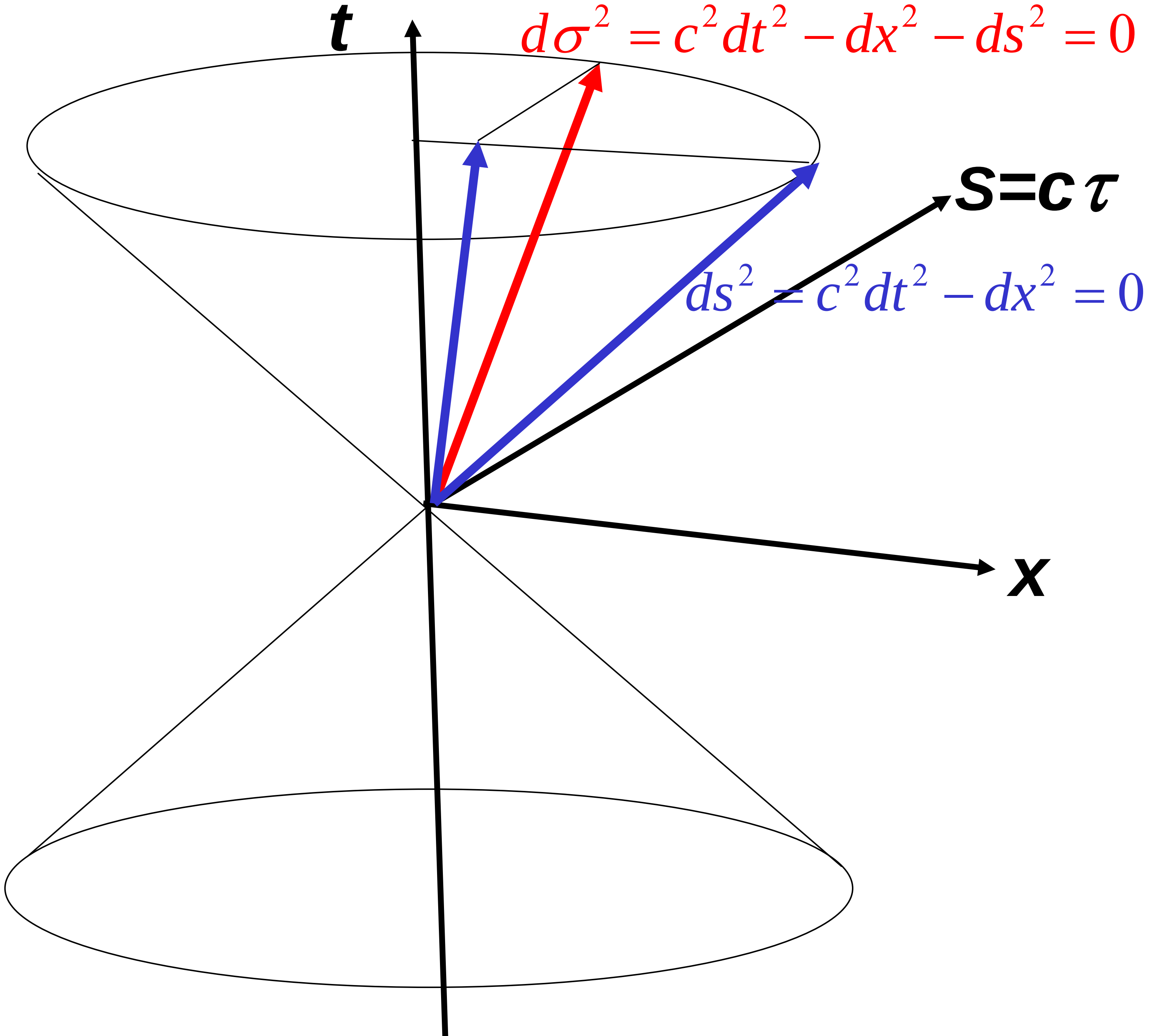
$$\hat{\Box} \varphi \equiv \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \Delta \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial \tau^2} = 0$$

$$\varphi(x, c\tau) = \int \frac{d(Mc)}{\sqrt{2\pi\hbar}} \exp\left[i \frac{Mc^2}{\hbar} (\tau - \tau_0) \right] \varphi(x, Mc)$$



E(p)
b
a
$M_b c^2$
$M_a c^2$
$p_{//}$


E
Mc
p


t
$d\sigma^2 = c^2dt^2 - dx^2 - ds^2 = 0$
S=cτ
$ds^2 = c^2dt^2 - dx^2 = 0$
x

OPTICAL PATH & FERMAT'S PRINCIPLE IN (4+1)D

$$\hat{\Box}\varphi \equiv \Box\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial\tau^2} = 0$$

eikonal equation in 5D ($\hat{\mu},\hat{\nu} = 0,1,2,3,4$):

$$g^{\hat{\mu}\hat{\nu}}\partial_{\hat{\mu}}\phi\,\partial_{\hat{\nu}}\phi = 0$$

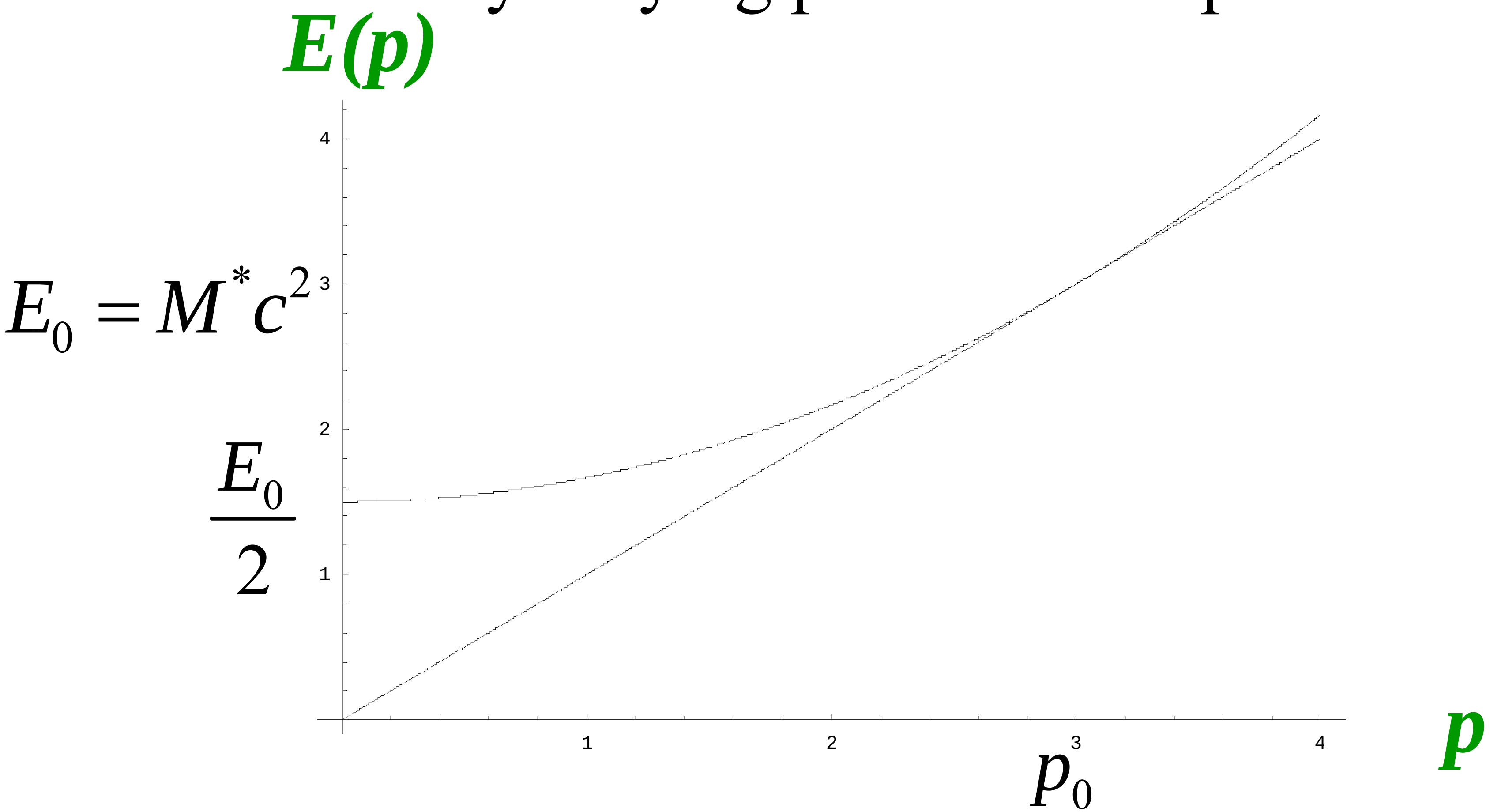
$$\phi = -\frac{E}{h}\left((t - t_0) - \int \frac{dl^{(4)}}{c\sqrt{g_{00}}} + \int \frac{g_{j0}}{c g_{00}} dx^j\right)$$

$$dl^{(4)} = \sqrt{-f_{ij}dx^i dx^j + c^2 d\tau^2}$$

$$\lambda^{(4)} = \frac{h}{E} c\sqrt{g_{00}}$$

$$f_{ij} = g_{ij} - \frac{g_{0i}g_{0j}}{g_{00}}$$

BASICS OF ATOM /PHOTON OPTICS
Parabolic approximation of slowly varying phase and amplitude



BASICS OF ATOM /PHOTON OPTICS

Schroedinger-like equation for the atom /photon field:

$$i\hbar\frac{\partial\varphi}{\partial t}=\frac{M^{*}c^{2}}{2}\varphi-\frac{1}{2M^{*}}\left[p^{j}p_{j}+p^{4}p_{4}\right]\varphi+\frac{1}{2M^{*}}p_{\mu}h^{\mu\nu}p_{\nu}\varphi$$

$$p_{j}=i\hbar\partial_{j};\quad p_{4}=i\hbar\partial_{c\tau};\quad p_{0}=M^{*}c\quad(\hbar\omega/c\text{ for photons})$$

- gravitation field: $h^{00}=-2\vec{g}.\vec{q}/c^{2}-\vec{q}.\overset{\Rightarrow}{\gamma}.\vec{q}/c^{2}$

- rotation field: $\vec{h}=-\overset{\Rightarrow}{\alpha}.\vec{q}/c$

- gravitational wave: $\overset{\Rightarrow}{h}=\overset{\Rightarrow}{\beta}-\overset{\Rightarrow}{\delta}$

$$H_{ext}=\vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q}+\vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M^{*}-M^{*}\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2-M^{*}\vec{g}.\vec{q}+\vec{f}.\vec{p}$$

ABCDξϕ LAW OF ATOM/PHOTON OPTICS

$$wavepacket(q,t) = \exp\left[ip_c(t)\left(q - q_c(t)\right)/\hbar\right] F\left(q - q_c(t), X(t), Y(t)\right)$$

$$p = (p_x, p_y, p_z, Mc); \quad q = (x, y, z, c\tau)$$

$$q_c(t) = Aq_c(t_0) + Bp_c(t_0)/M^* + \xi(t, t_0)$$

$$p_c(t)/M^* = Cq_c(t_0) + Dp_c(t_0)/M^* + \phi(t, t_0)$$

$$X(t) = AX(t_0) + BY(t_0)$$

$$Y(t) = CX(t_0) + DY(t_0)$$

Ehrenfest theorem

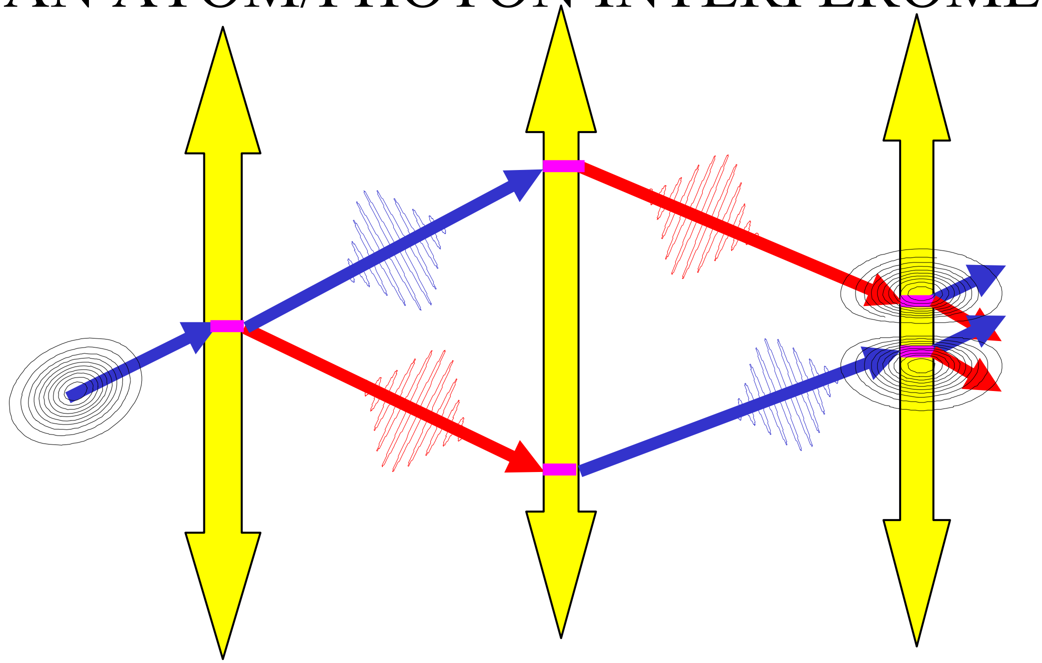
+

Hamilton equations

$$H_{ext} = \vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q} + \vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M^{*} - M^{*}\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2 - M^{*}\vec{g}.\vec{q} + \vec{f}.\vec{p}$$

$$\begin{pmatrix} A(t,t_0) & B(t,t_0) \\ C(t,t_0) & D(t,t_0) \end{pmatrix} = \mathcal{T}\exp\left[\int_{t_0}^{t}\begin{pmatrix} \alpha(t') & \beta(t') \\ \gamma(t') & \alpha(t') \end{pmatrix}dt'\right]$$

GENERAL FORMULA FOR THE PHASE SHIFT OF AN ATOM/PHOTON INTERFEROMETER

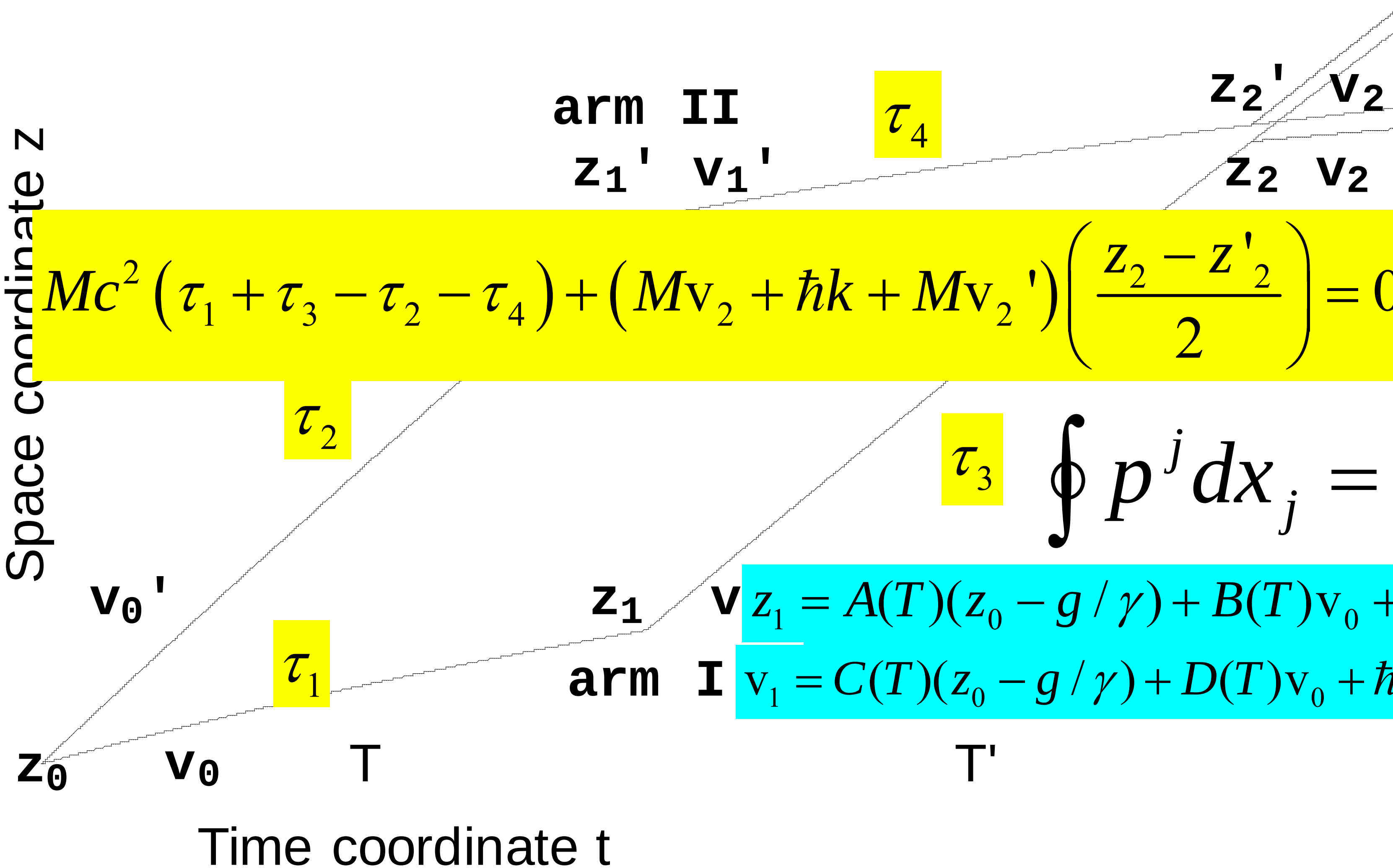


$$\delta\varphi = \sum_{j=1}^{N} (\delta\tilde{k}_j^{(5)} . q_j^{(5)} + \delta\varphi_j)$$

$$\tilde{k}^{(5)} = \left[(k_x, k_y, k_z, \frac{\omega^{(0)}}{c}), \frac{\omega}{c} \right]; \quad \tilde{q}^{(5)} = \left[(x, y, z, c\tau), ct \right]$$

$$\delta\tilde{k}^{(5)} = \tilde{k}_{\beta j}^{(5)} - \tilde{k}_{\alpha j}^{(5)}; \quad q_j^{(5)} = \left(q_{\beta j}^{(5)} + q_{\alpha j}^{(5)} \right) / 2$$

Atomic Gravimeter



$$Mc^2\left(\tau_1+\tau_3-\tau_2-\tau_4\right)+\left(M\mathrm{v}_2+\hbar k+M\mathrm{v}_2'\right)\left(\frac{z_2-z'_2}{2}\right)=0$$

$$\oint p^j dx_j = 0$$

$$z_1 = A(T)(z_0-g/\gamma)+B(T)\mathrm{v}_0+g/\gamma$$

$$\mathrm{v}_1 = C(T)(z_0-g/\gamma)+D(T)\mathrm{v}_0+\hbar k/M$$

$$\delta\varphi = -k(z_2-z_1-z'_1+z_0)+k(z_2-z'_2)/2$$

Exact phase shift for the atom gravimeter

$$\delta\varphi = -k(z_2 - z_1 - z'_1 + z_0) + k(z_2 - z'_2)/2$$
$$= \frac{k}{\sqrt{\gamma}}\left\{\left[\sinh\left(\sqrt{\gamma}\,(T+T')\right) - 2\sinh\left(\sqrt{\gamma}T\right)\right]\left(v_0 + \frac{\hbar k}{2M}\right)\right.$$
$$\left. + \sqrt{\gamma}\left[1 + \cosh\left(\sqrt{\gamma}\,(T+T')\right) - 2\cosh\left(\sqrt{\gamma}T\right)\right]\left(z_0 - \frac{g}{\gamma}\right)\right\}$$

which can be written to first-order in γ, with T=T':

$$\delta\varphi = kgT^2 + k\gamma T^2\left[\frac{7}{12}gT^2 - \left(v_0 + \frac{\hbar k}{2M}\right)T - z_0\right]$$

Reference: Ch. J. B., Theoretical tools for atom optics and interferometry, C.R. Acad. Sci. Paris, 2, Série IV, p. 509-530, 2001

ARBITRARY 3D TIME-DEPENDENT GRAVITO-INERTIAL FIELDS

Hamiltonian: $H = \vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q} + \vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M^* - M^*\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2$

Hamilton's equns: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = T \exp \int dt \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix}$

Example: Phase shift induced by a gravitational wave

Einstein coord.: $\overset{\Rightarrow}{\beta} = \overset{\Rightarrow}{1} + \overset{\Rightarrow}{h} \cos(\xi t + \phi), \overset{\Rightarrow}{\gamma} = 0, \text{ with } \overset{\Rightarrow}{h} = \{h^{ij}\}$

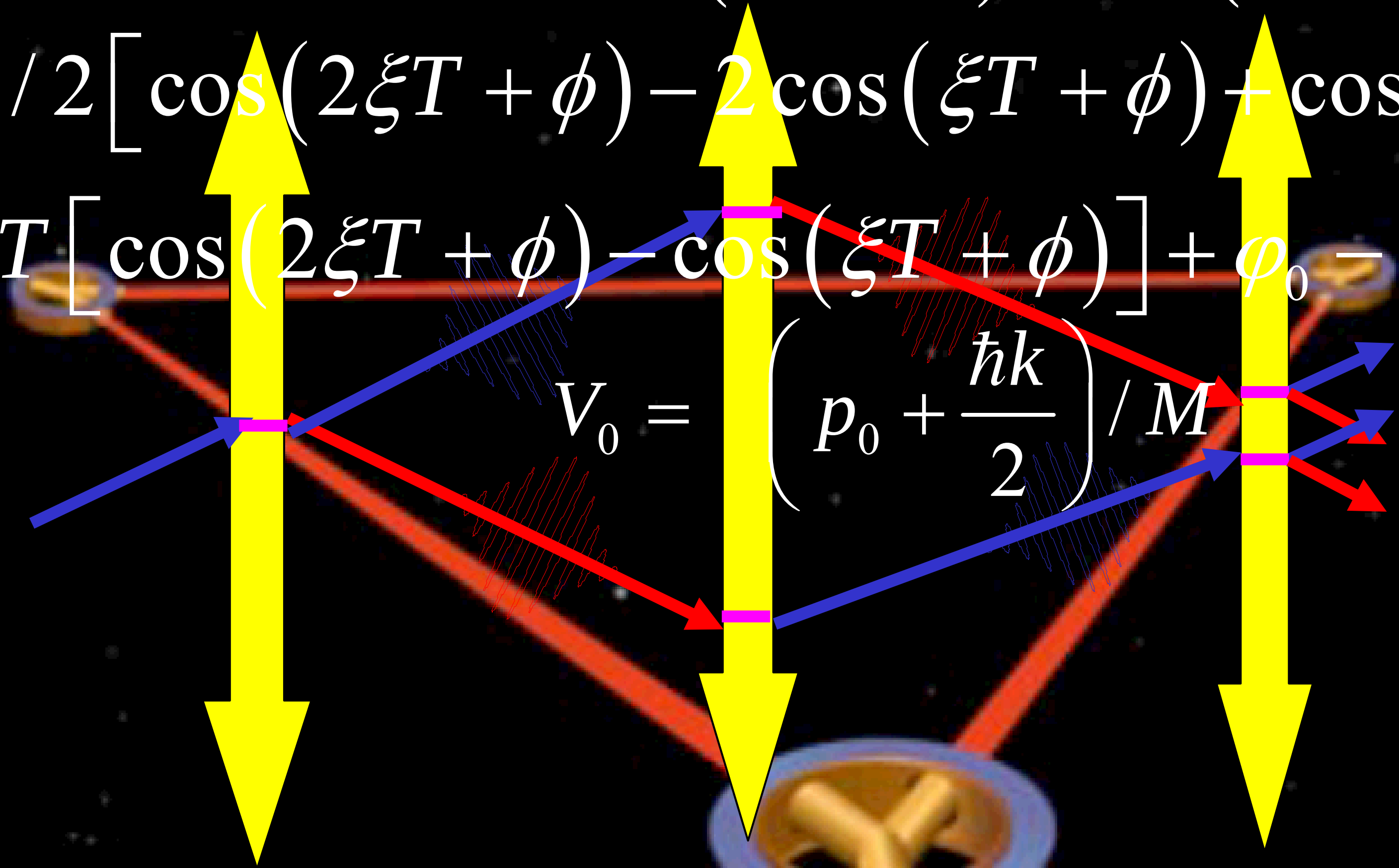
Fermi coord.: $\overset{\Rightarrow}{\beta} = \overset{\Rightarrow}{1}, \overset{\Rightarrow}{\gamma} = (\xi^2/2)\overset{\Rightarrow}{h} \cos(\xi t + \phi)$

Einstein coord.: $\begin{cases} A = 1 \\ B = t + \dfrac{h}{\xi}\left[\sin(\xi t + \phi) - \sin\phi\right] \end{cases}$

Fermi coord.: $\begin{cases} A = 1 - \dfrac{h}{2}\left[\cos(\xi t + \phi) - \cos\phi\right] - \dfrac{h\xi t}{2}\sin\phi \\ B = t + \dfrac{h}{\xi}\left[\sin(\xi t + \phi) - \sin\phi\right] - \dfrac{ht}{2}\left[\cos(\xi t + \phi) + \cos\phi\right] \end{cases}$

Atomic phase shift induced by a gravitational wave

$$\delta\varphi = -khV_0\xi T^2 \sin\left(\xi T+\phi\right)\mathrm{sinc}^2\left(\xi T/2\right)$$
$$-khq_0/2\left[\cos\left(2\xi T+\phi\right)-2\cos\left(\xi T+\phi\right)+\cos\phi\right]$$
$$-khV_0T\left[\cos\left(2\xi T+\phi\right)-\cos\left(\xi T+\phi\right)\right]+\varphi_0-2\varphi_1+\varphi_2$$



$$V_0=\left(p_0+\frac{\hbar k}{2}\right)/M$$

Ch.J. Bordé, Gen. Rel. Grav. 36 (March 2004)

Ch.J. Bordé, J. Sharma, Ph. Tourrenc and Th. Damour, *Theoretical approaches to laser spectroscopy in the presence of gravitational fields*, J. Physique Lettres 44 (1983) L983-990

J. Physique — LETTRES **44** (1983) L-983 - L-990 15 DÉCEMBRE **1983,** PAGE L-983

Classification
Physics Abstracts
32.70J — 04.80

Theoretical approaches to laser spectroscopy in the presence of gravitational fields

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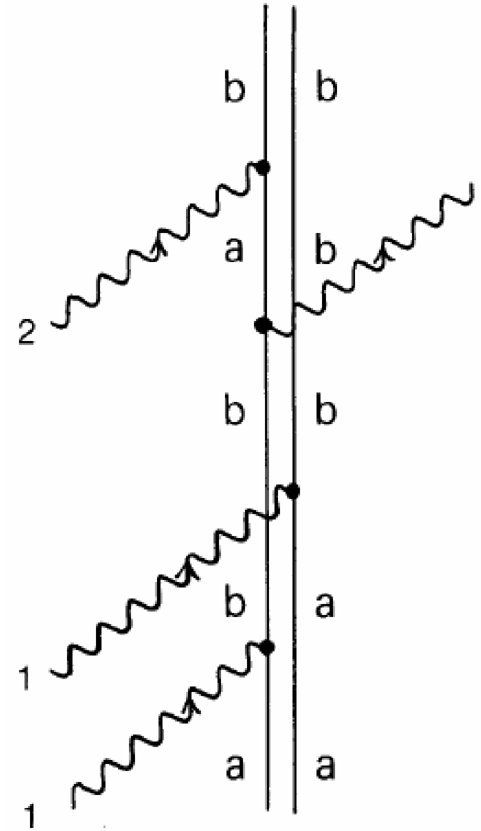

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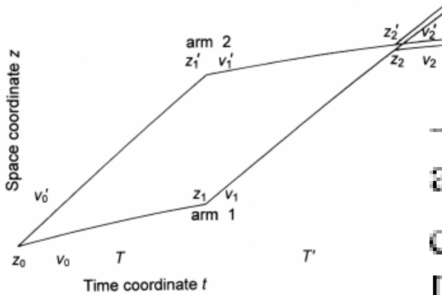


Figure 18. Space-time diagram of the atomic gravimeter.

what could be obtained with light rays, since the times T can be of the order of $1s$ and gT^2 is of the order of 10 m compared with an optical wavelength. The next term is a significant correction due to the gravitational field gradient γ. One can also measure directly these field gradients, with two gravimeters using two clouds of cold atoms and sharing the same vertical laser beam splitters. It is then no longer necessary to have a very sophisticated inertial platform for the reference mirror and it is possible to measure directly the differential acceleration between these two clouds. This is the principle, illustrated in Figure 19, of the gradiometers developed first in Stanford then in Yale by Kasevich and co-workers [27, 59]. The current sensitivity is 4×10^{-9} $s^{-2}/\sqrt{Hz}$ and the uncertainty is 1×10^{-9} $s^{-2}/\sqrt{Hz}$ for an extrapolated 10 m separation between accelerometers. For the future, this principle may be considered for gravitational wave detection in space [60].

the test mass energy Mc^2 has to be replaced by the photon energy $h\nu$ in the case of light waves. This expression can be derived by a number of approaches, the best of which is to use the rotation operator in the Schrödinger equation and the derived propagator and $ABCD$ matrices, as outlined above, which gives the Sagnac shift thanks to an exact formula. The same approach applies to the trapezoid geometry used in the previous experiment as well as to the parallelogram geometry, as suggested in [24], analogue of the Mach-Zehnder optical interferometer, which has been used in more recent experiments and which has the advantage of being insensitive to laser detuning. For the illustration, the formula calculated with the $ABCD\xi$ formalism is

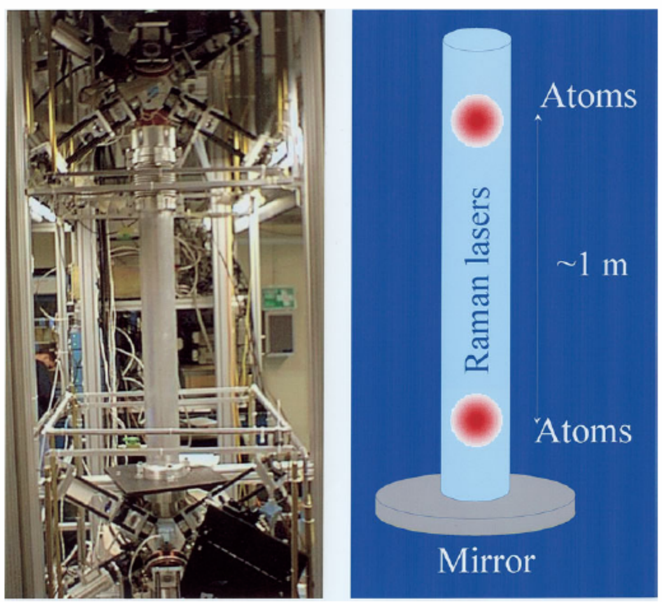
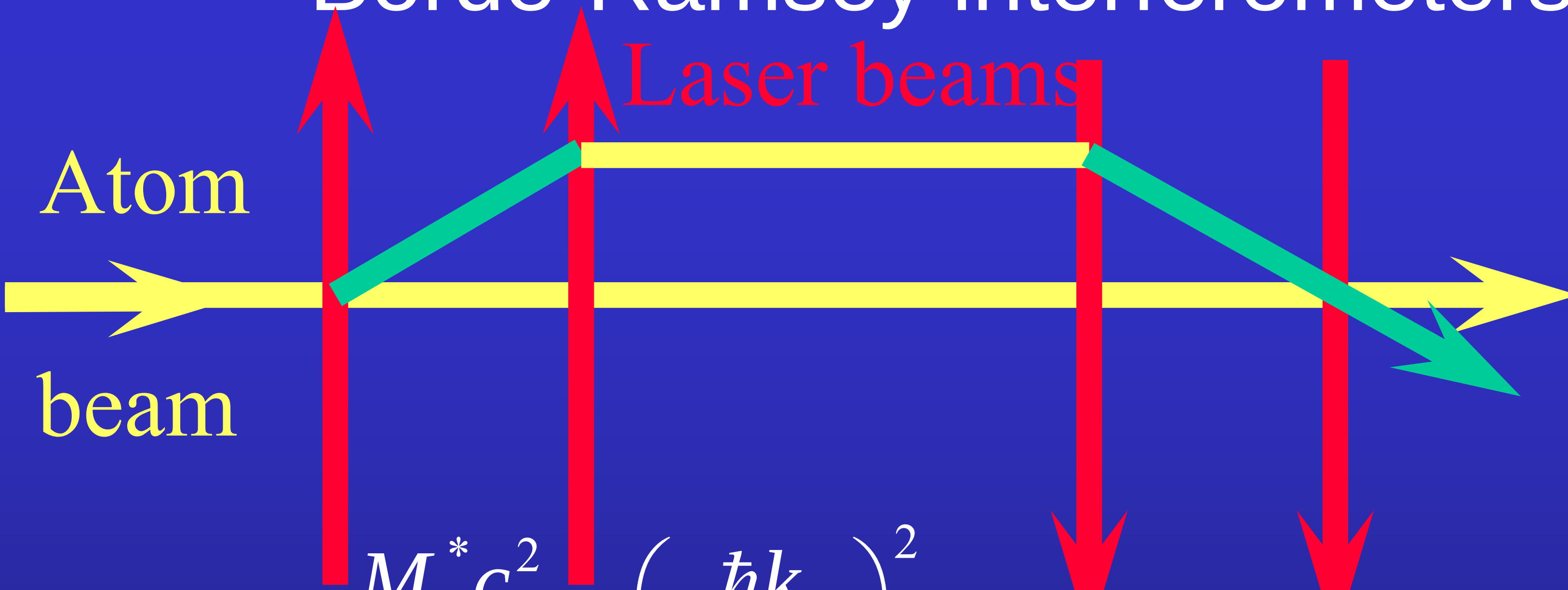


Figure 19. Atom wave gradiometer of Yale University. The two clouds of atoms share the same Raman beams, which generate two atom interferometers separated vertically by 1 m.

Metrologia, 2002, 39, 435-463

Bordé-Ramsey interferometers



$$\delta\varphi = -\frac{M^{*}c^{2}}{\hbar}T\left(\frac{\hbar k}{M^{*}c}\right)^{2} h\cos\left(\xi T+\phi\right)\mathrm{sinc}\left(\xi T\right)$$