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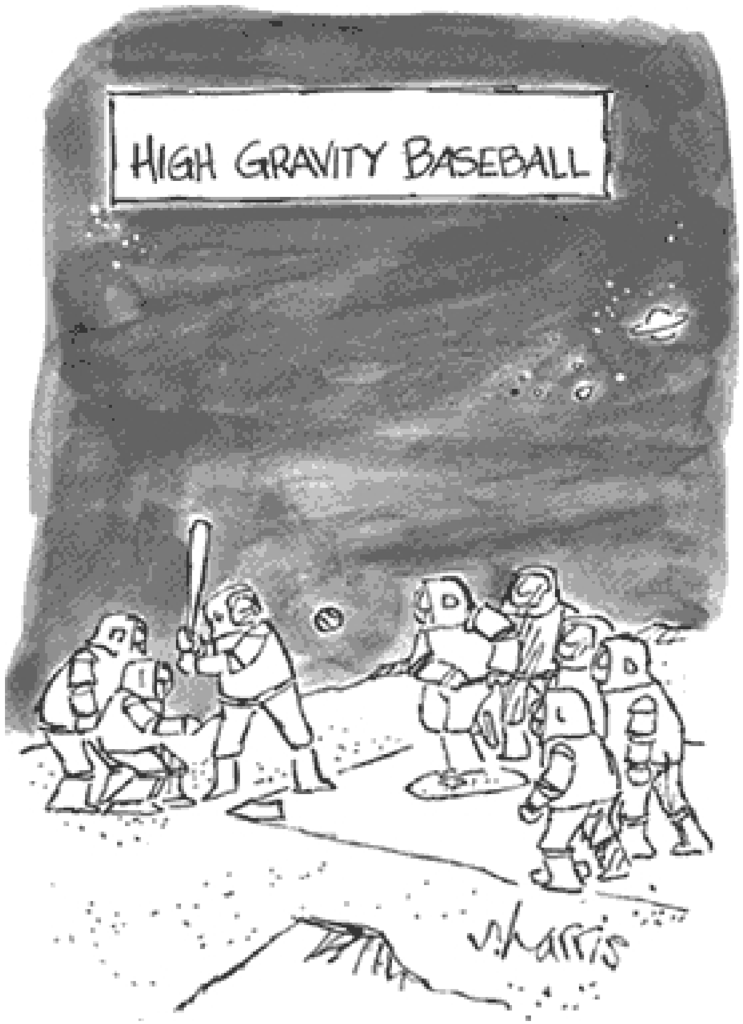
Sagnac effect in a proper reference frame

Endre Kajari, International Workshop on "Gravitational Waves Detection with Atom Interferometry", Florenze, 23th February 2009

Drop Tower in Bremen

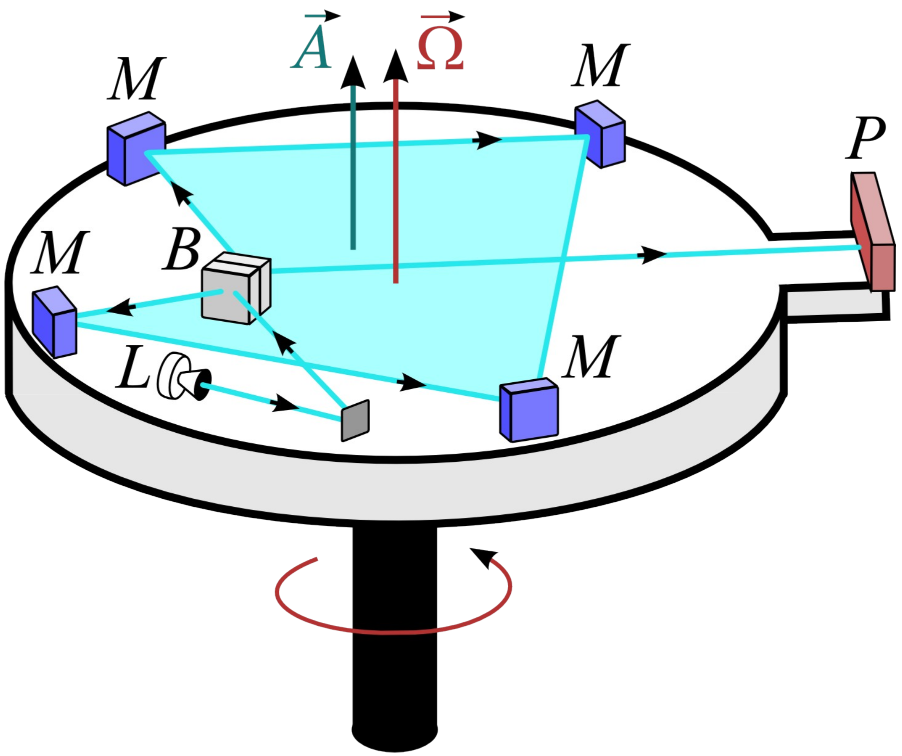
Outline of the talk:

- Sagnac's original experiment
- Sagnac effect in general relativity
- Definition of a proper reference frame (PRF)
- Sagnac time delay in a PRF and the double eight-loop interferometer (DELI)
- Comment on gravitational wave detection



S. Harris

Sagnac's Original Experiment



$$\Delta t = \frac{4}{c^2}\vec{A}\cdot\vec{\Omega}$$

Sagnac's Conclusion:

"The observed interference effect is clearly the optical whirling effect due to the movement of the system in relation to the ether and directly manifests the existence of the ether, supporting necessarily the light waves of Huygens and of Fresnel."

C. R. Acad. Sci. **157**, 708 and 1410 (1913), translated by R. Hazelett

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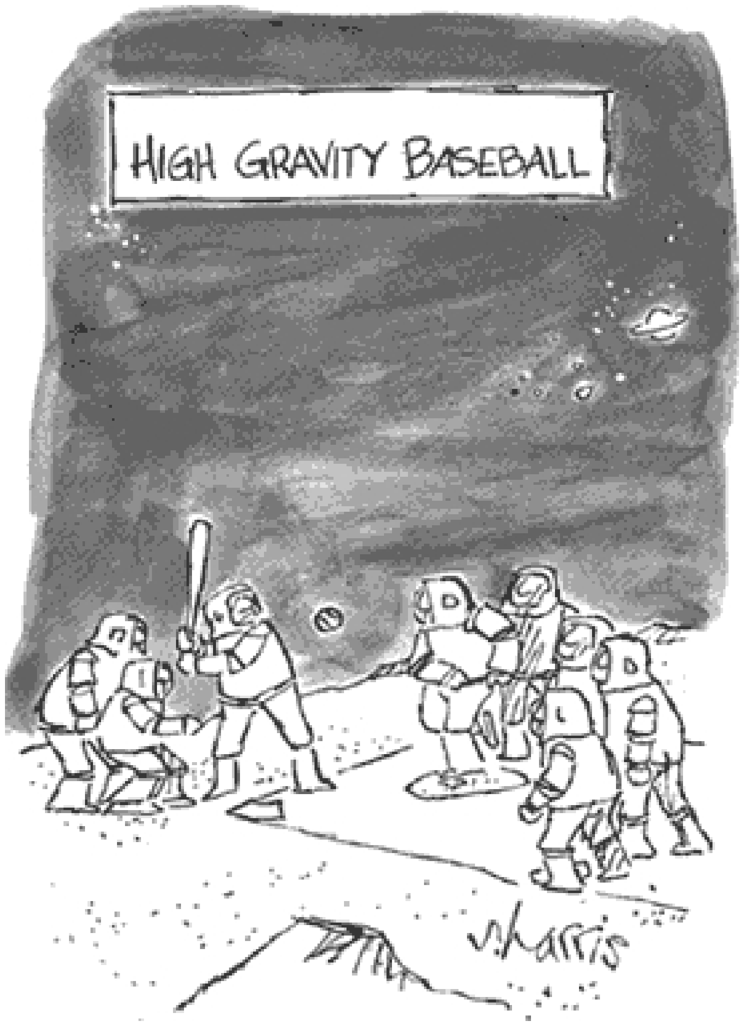
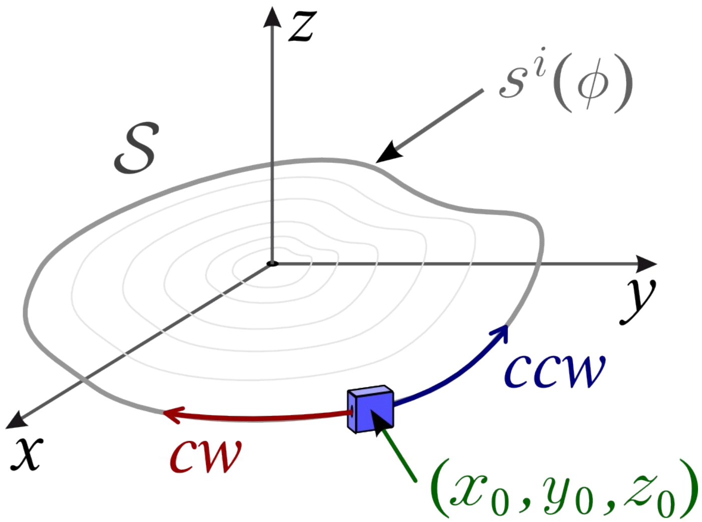


S. Harris

Sagnac Time Delay in General Relativity

for a time independent metric



Sagnac Time Delay in General Relativity

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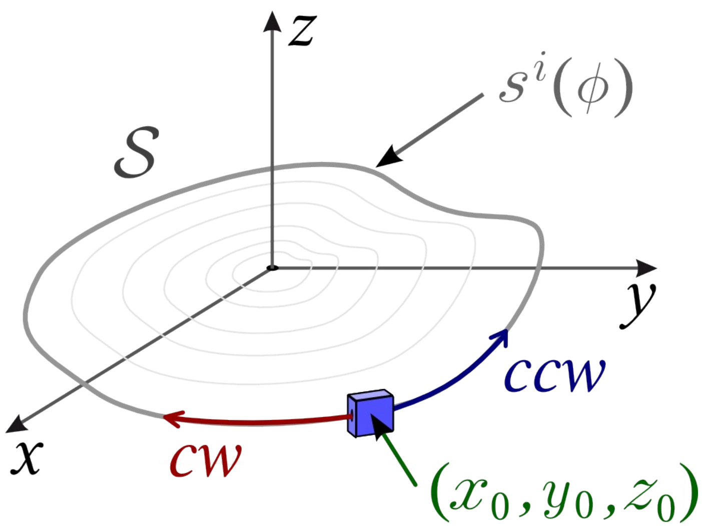

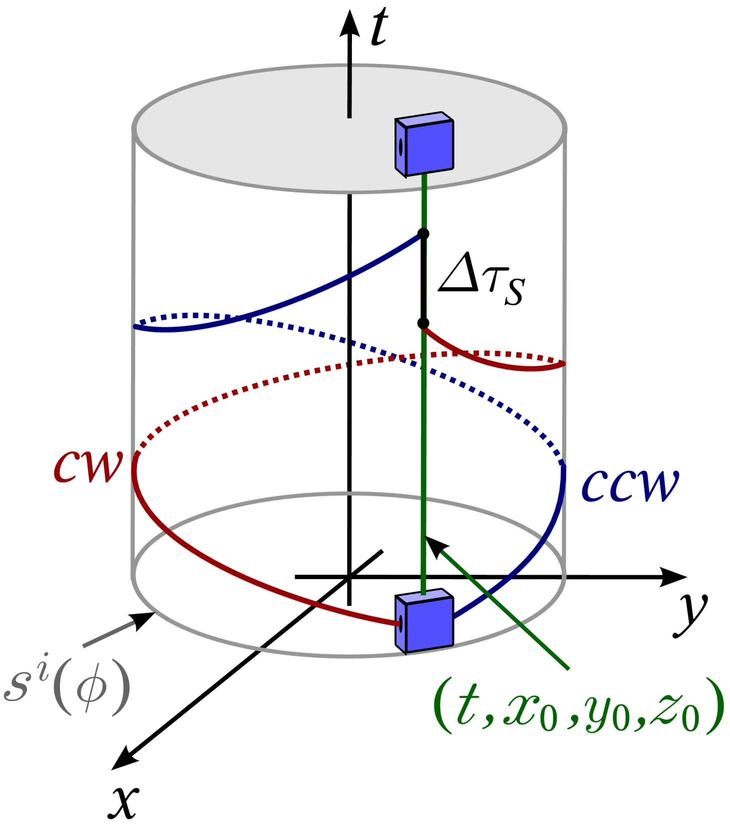

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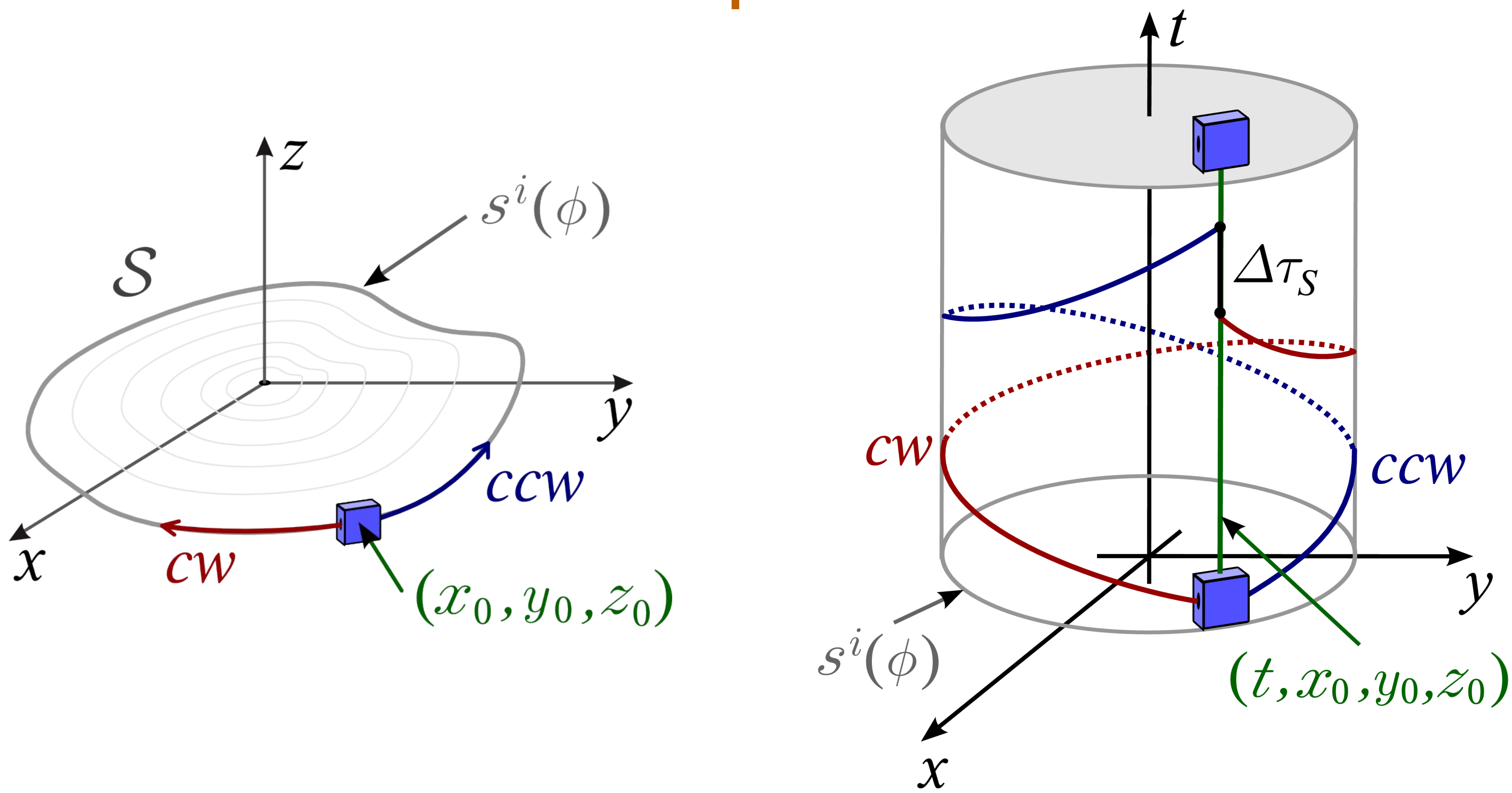
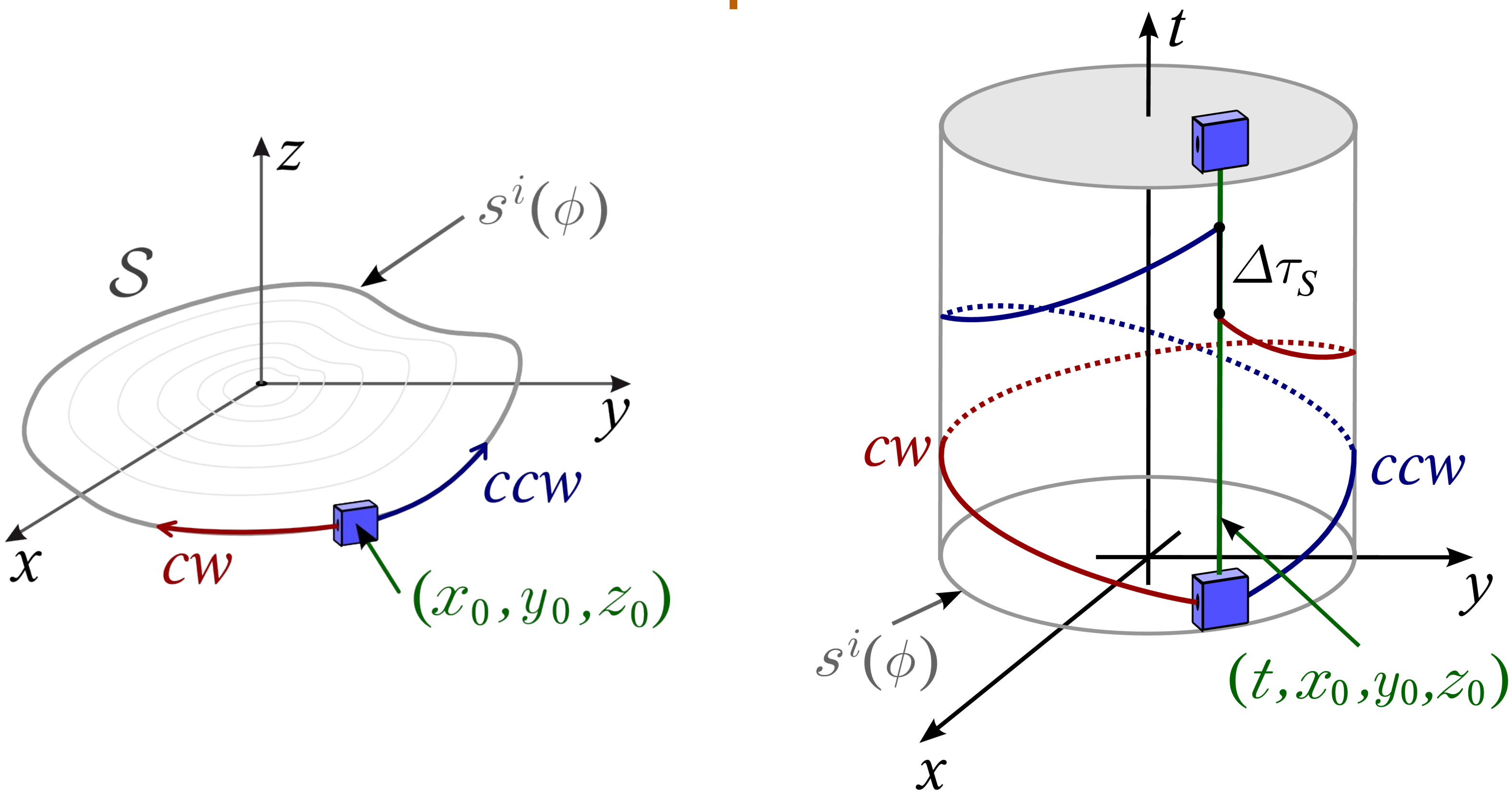


$$g_{\mu\nu}\big|_{\mathcal{S}} \frac{dx^\mu}{d\phi}\frac{dx^\nu}{d\phi} = g_{00}\big|_{\mathcal{S}}\left(\frac{dt}{d\phi}\right)^2 + 2\, g_{0i}\big|_{\mathcal{S}}\frac{ds^i}{d\phi}\frac{dt}{d\phi} + g_{ik}\big|_{\mathcal{S}}\frac{ds^i}{d\phi}\frac{ds^k}{d\phi} = 0$$

Sagnac Time Delay in General Relativity

for a time independent metric



$$\Delta\tau_S = -\frac{2}{c}\sqrt{g_{00}(q^r)}\oint_S \frac{g_{0i}}{g_{00}}\,ds^i$$

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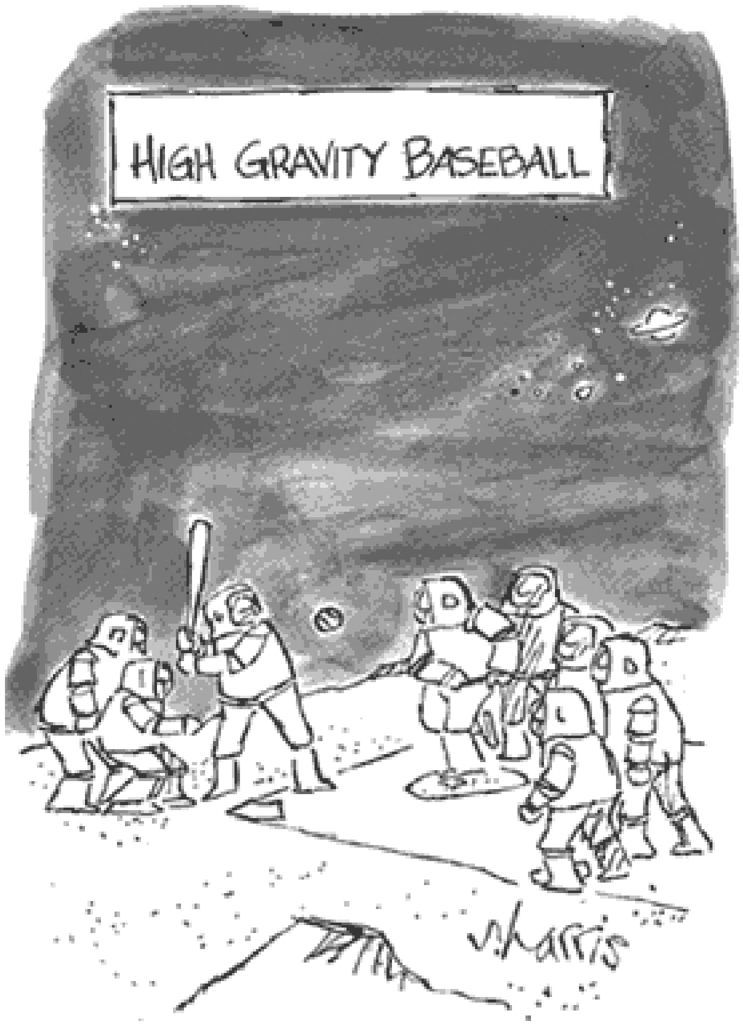

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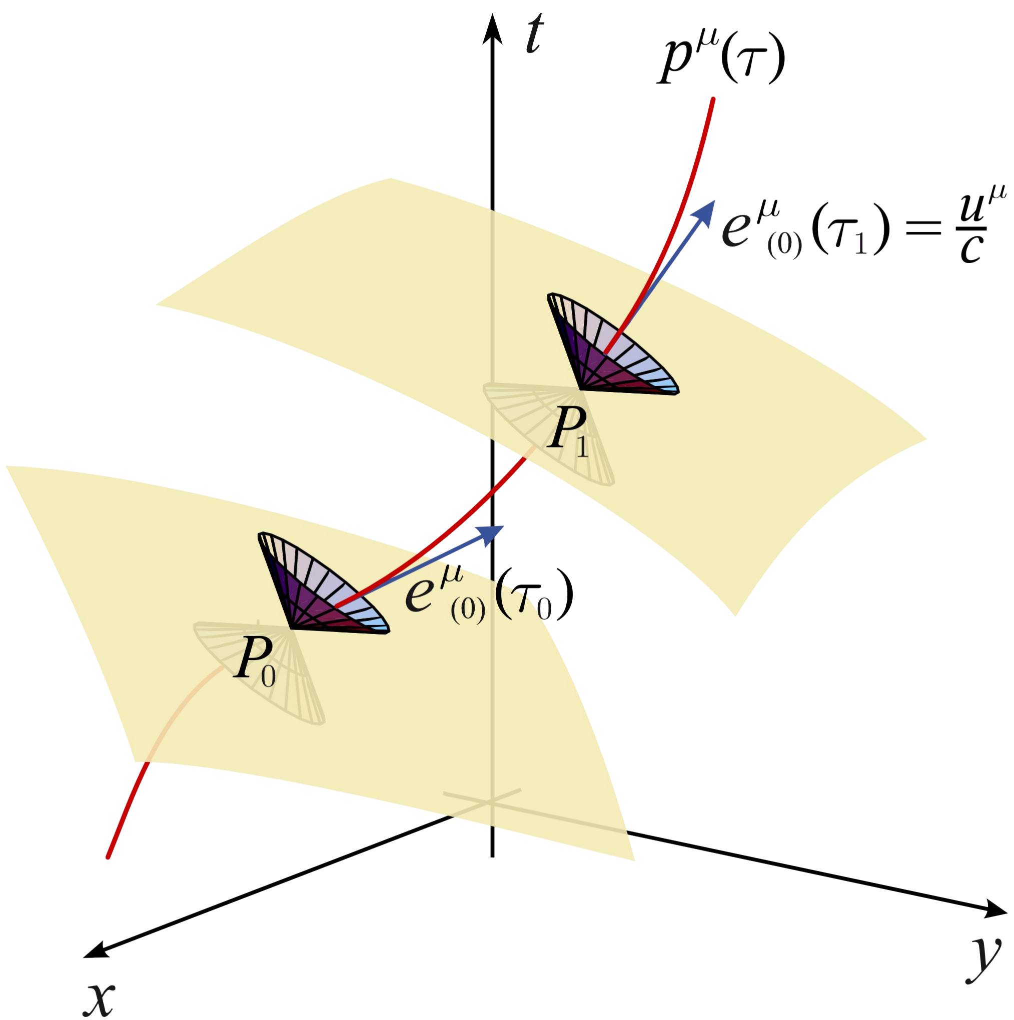
Proper Reference Frame (PRF)



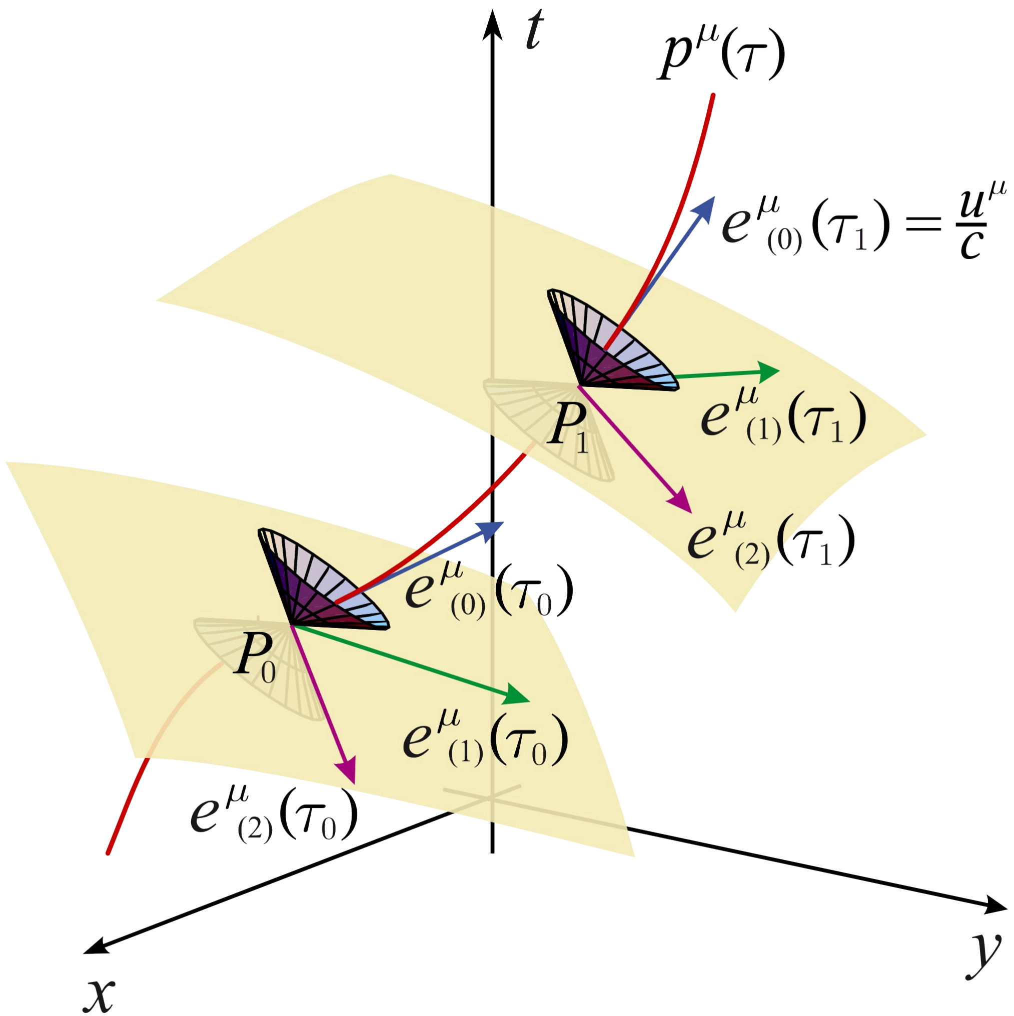
four-velocity:

$$u^\mu(\tau) = \frac{dp^\mu}{d\tau}$$

four-acceleration:

$$a^\mu(\tau) = u^\mu{}_{;\nu}\, u^\nu$$

Proper Reference Frame (PRF)



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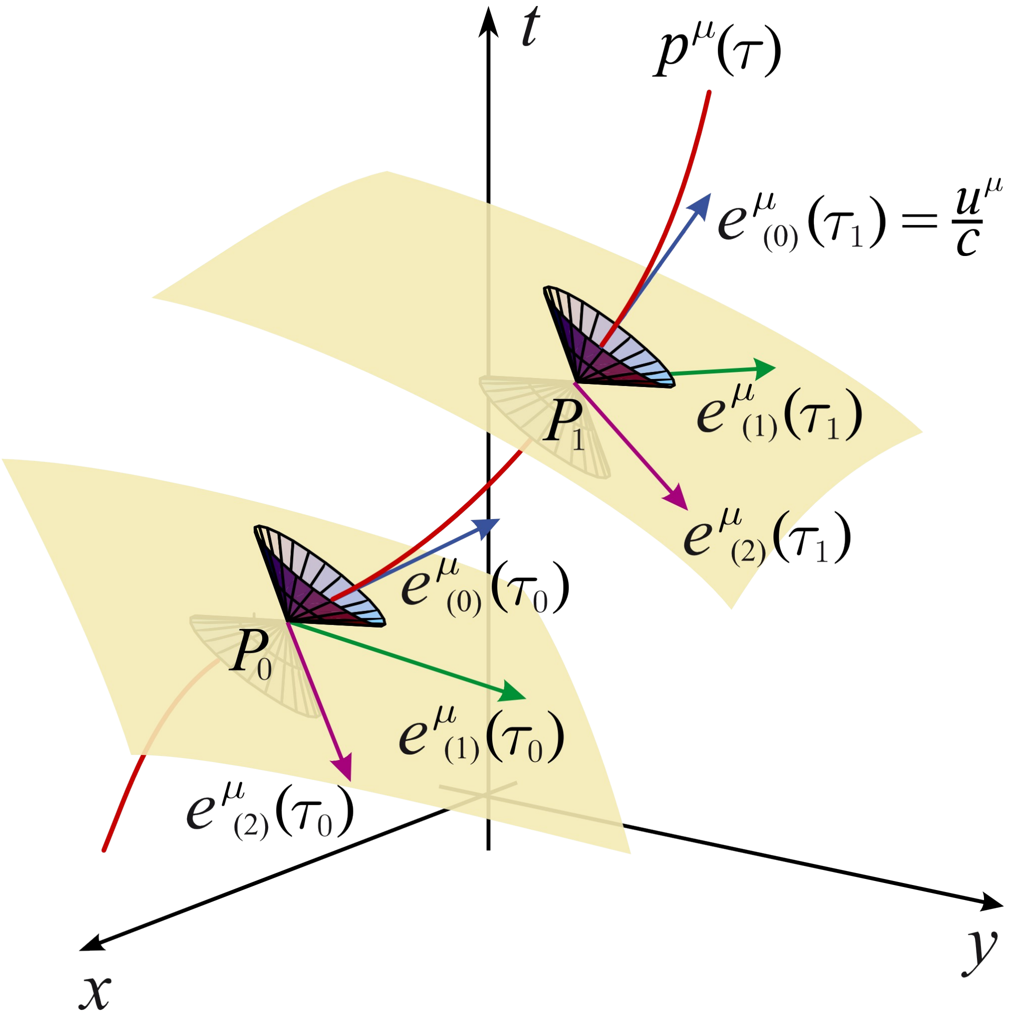
orthonormal tetrads:

$$e^{\mu}_{(\alpha)}(\tau)\, e^{\nu}_{(\beta)}(\tau)\, g_{\mu\nu}(p^{\sigma}(\tau)) = \eta_{(\alpha\beta)}$$

Proper Reference Frame (PRF)



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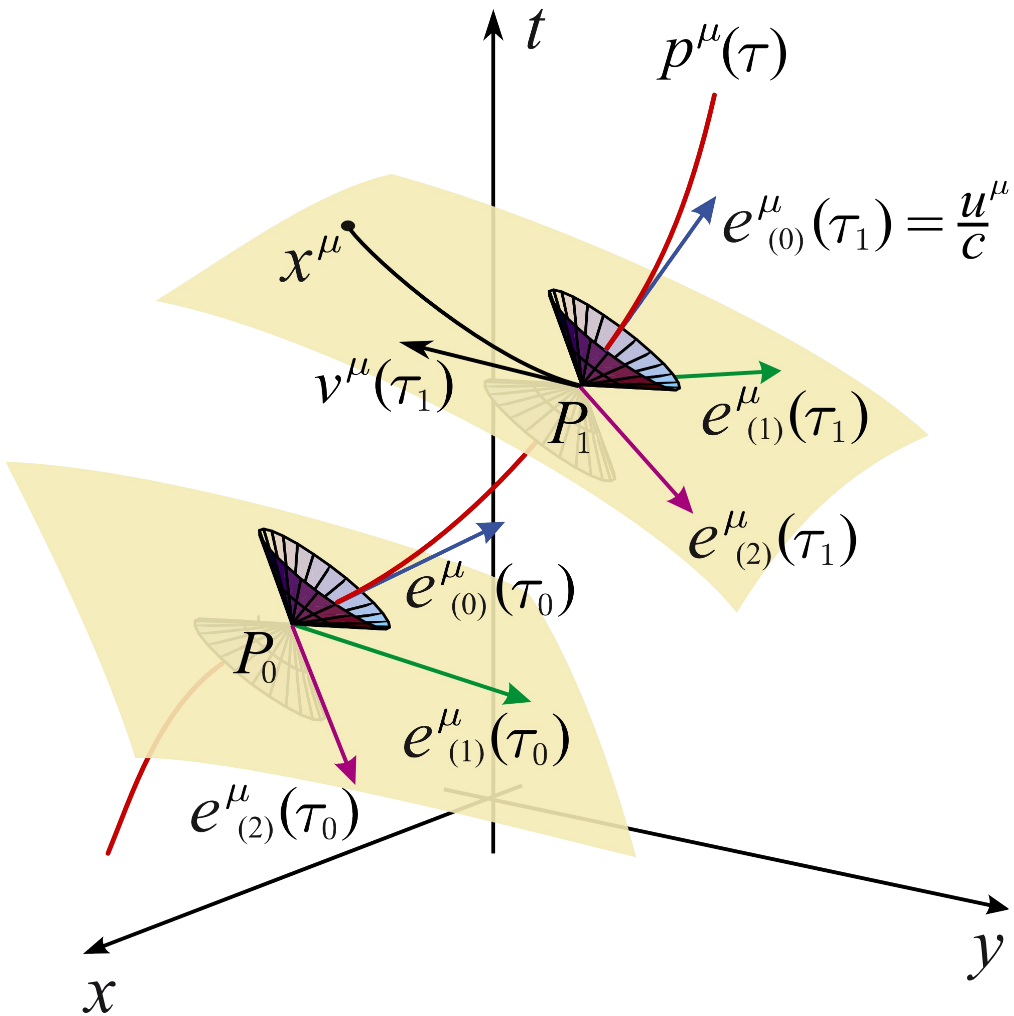
proper transport:

$$e^\mu{}_{(\alpha);\nu}\, u^\nu = -\Omega^\mu{}_\nu\, e^\nu{}_{(\alpha)}$$

transport matrix:

$$\Omega^{\mu\nu} = -\frac{1}{c^2}\left(a^\mu u^\nu - a^\nu u^\mu\right) + \frac{1}{c}\, u_\rho\, \omega_\sigma\, \varepsilon^{\rho\sigma\mu\nu}$$

Proper Reference Frame (PRF)



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Metric Expansion in PRF

$$g_{(00)}(x^{(\sigma)}) = 1 - \frac{2}{c^2} a_{(i_1)} x^{(i_1)} + R_{(0)(i_1)(i_2)(0)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)}$$

$$+ \frac{1}{c^2}\left(\frac{1}{c^2} a_{(i_1)} a_{(i_2)} + \omega_{(i_1)}\omega_{(i_2)} - \omega^{(l)}\omega_{(l)}\,\eta_{(i_1 i_2)}\right) x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)\,,$$

$$g_{(0k)}(x^{(\sigma)}) = \frac{1}{c}\,\varepsilon_{(0kli_1)}\,\omega^{(l)} x^{(i_1)} + \frac{2}{3} R_{(0)(i_1)(i_2)(k)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)\,,$$

$$g_{(jk)}(x^{(\sigma)}) = -\delta_{(jk)} + \frac{1}{3} R_{(j)(i_1)(i_2)(k)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)$$

Metric Expansion in PRF

$$
\begin{aligned}
g_{(00)}(x^{(\sigma)}) &= 1 - \frac{2}{c^2} a_{(i_1)} x^{(i_1)} + R_{(0)(i_1)(i_2)(0)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)} \\
&\quad + \frac{1}{c^2}\left(\frac{1}{c^2} a_{(i_1)} a_{(i_2)} + \omega_{(i_1)} \omega_{(i_2)} - \omega^{(l)} \omega_{(l)}\, \eta_{(i_1 i_2)}\right) x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)\,, \\
g_{(0k)}(x^{(\sigma)}) &= \frac{1}{c} \varepsilon_{(0kli_1)}\, \omega^{(l)} x^{(i_1)} + \frac{2}{3} R_{(0)(i_1)(i_2)(k)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)\,, \\
g_{(jk)}(x^{(\sigma)}) &= -\delta_{(jk)} + \frac{1}{3} R_{(j)(i_1)(i_2)(k)}(p^{(\sigma)})\, x^{(i_1)} x^{(i_2)} + \mathcal{O}(x^3)
\end{aligned}
$$

- red-shift mainly due to acceleration term
- spacetime curvature occurs in the second order
- local inertial frame for freely falling and non-rotating observer

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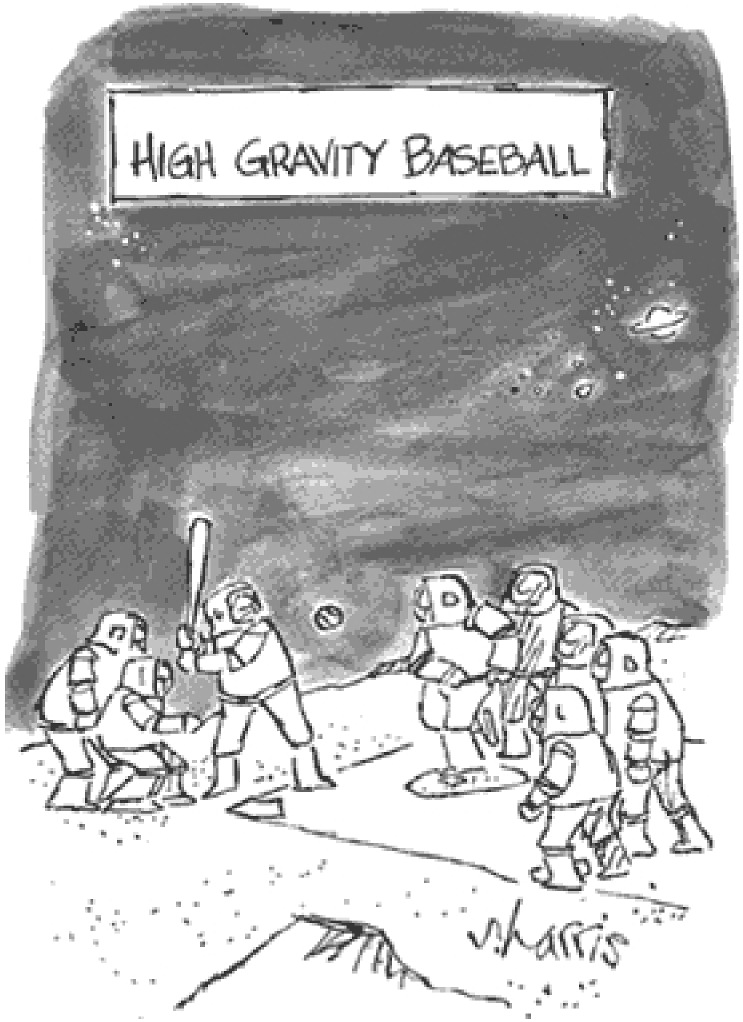

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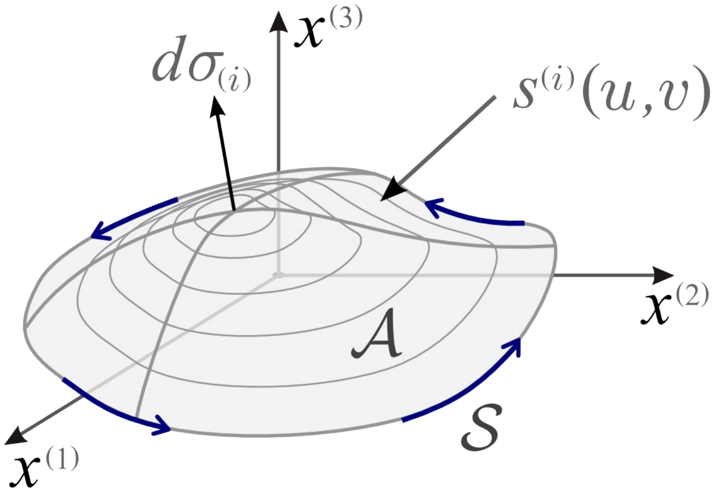
Sagnac Time Delay in a PRF

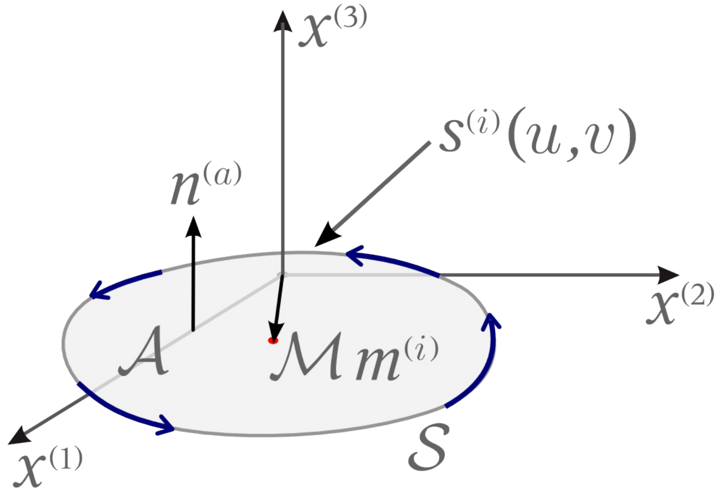


general surface

$$A_{(a)} = \iint_{\mathcal{A}} d\sigma_{(a)}$$

$$A_{(a)}^{(i_1)} = \iint_{\mathcal{A}} s^{(i_1)} d\sigma_{(a)}$$

Sagnac Time Delay in a PRF

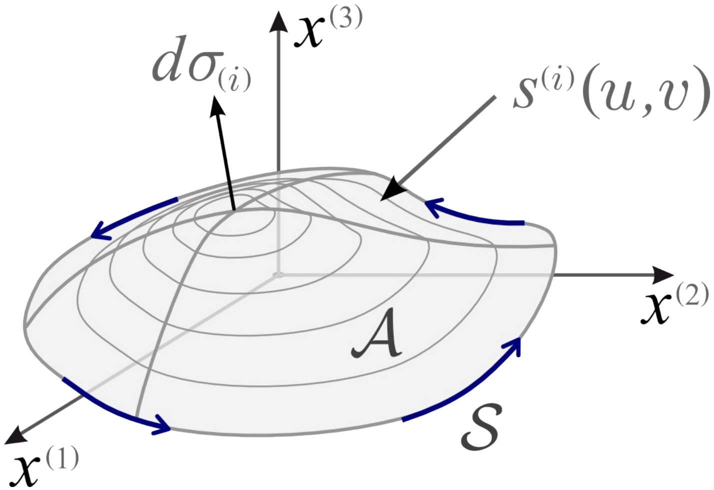


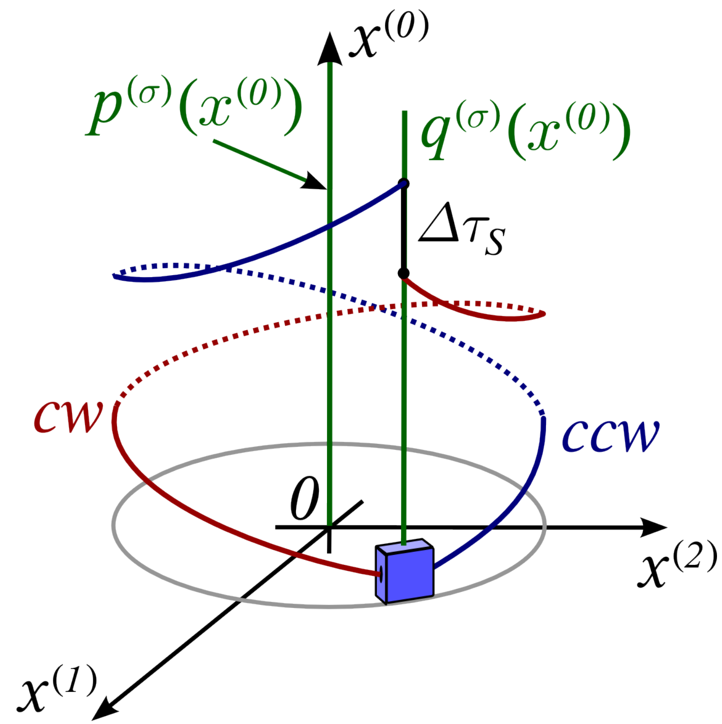
planar surface

$$A_{(a)} = \mathcal{A}\, n_{(a)}$$

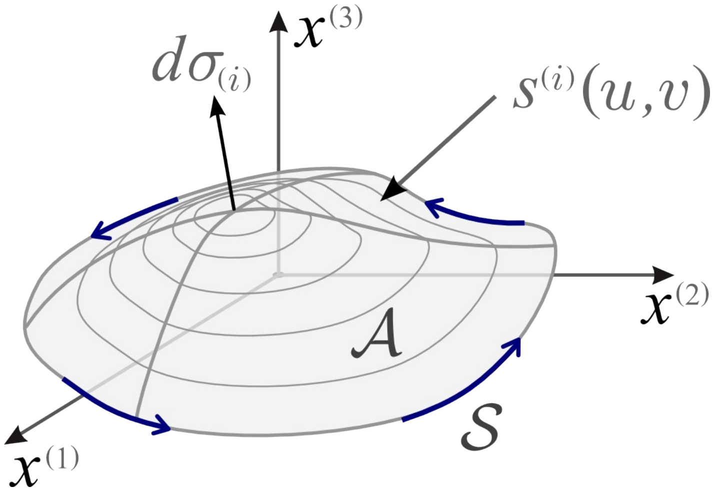
$$A_{(a)}^{(i_1)} = \mathcal{M}\, m^{(i_1)}\, n_{(a)}$$

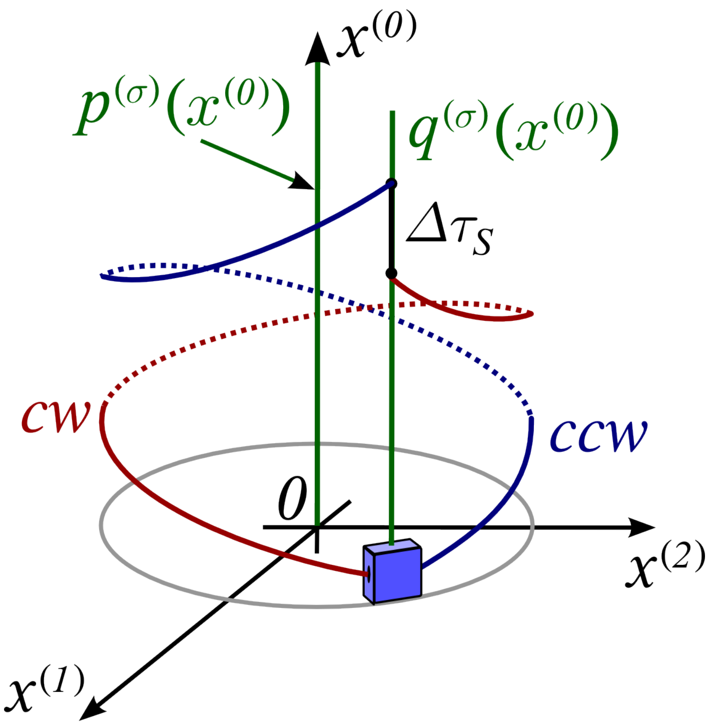
Sagnac Time Delay in a PRF





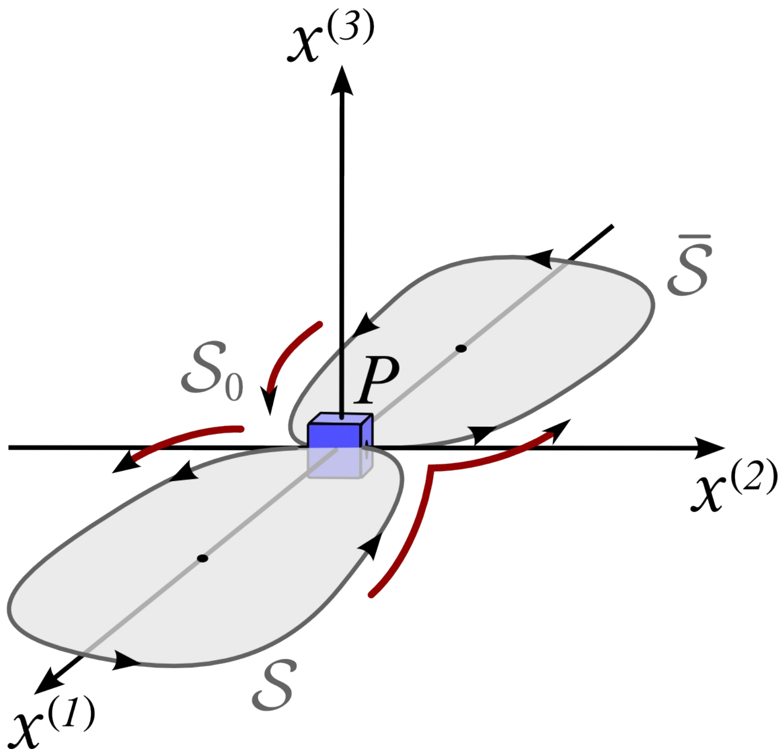
Sagnac Time Delay in a PRF





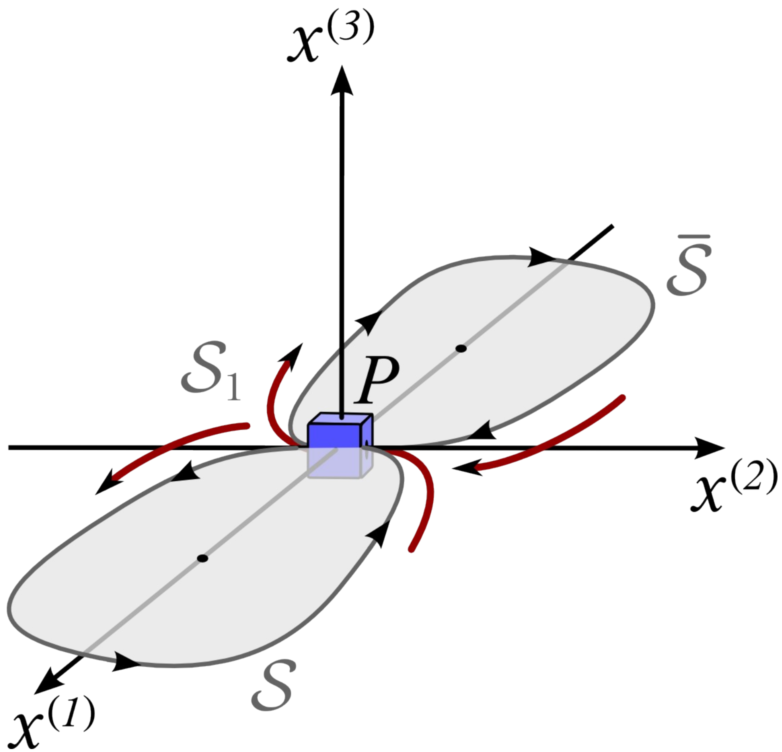
$$\Delta\tau_S = \frac{4}{c^2}\sqrt{g_{(00)}(q^{(r)})}\left[-\omega^{(a)}A_{(a)} + \frac{2c}{3}\,\varepsilon^{(0ajk)}\,R_{(0)\{(i_1)(j)\}(k)}(p^{(r)})\,A_{(a)}^{(i_1)} + \frac{1}{c^2}\left(\omega^{(l)}a_{(l)}\,\delta_{(i_1)}^{(a)} - 3\,\omega^{(a)}a_{(i_1)}\right)A_{(a)}^{(i_1)} + \mathcal{O}\left(A_{(a)}^{(i_1)(i_2)}\right)\right]$$

DELI: Measurement mode 1



$$\Delta\tau_{Sp}(\mathcal{S}_0) = -\frac{4}{c^2}\,\omega^{(a)} n_{(a)} \cdot 2\mathcal{A} + \mathcal{O}\left(A_{(a)}^{(i_1)(i_2)}\right)$$

DELI: Measurement mode 2



$$\Delta\tau_{Sp}(\mathcal{S}_1) = n_{(a)}\left[\frac{8}{3c}\varepsilon^{(0ajk)}R_{(0)\{(i_1)(j)\}(k)}(p^{(r)}) + \frac{4}{c^4}\left(\omega^{(l)}a_{(l)}\delta^{(a)}_{(i_1)} - 3\omega^{(a)}a_{(i_1)}\right)\right]m^{(i_1)}\cdot 2\mathcal{M}$$

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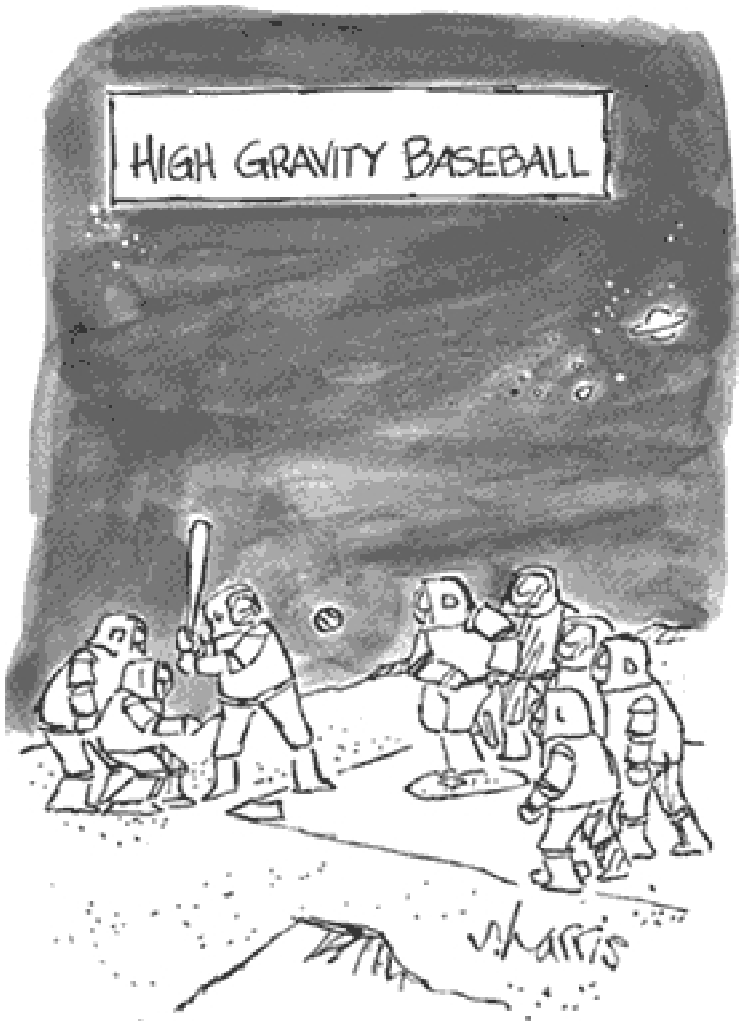


S. Harris

Comment on gravitational wave detection

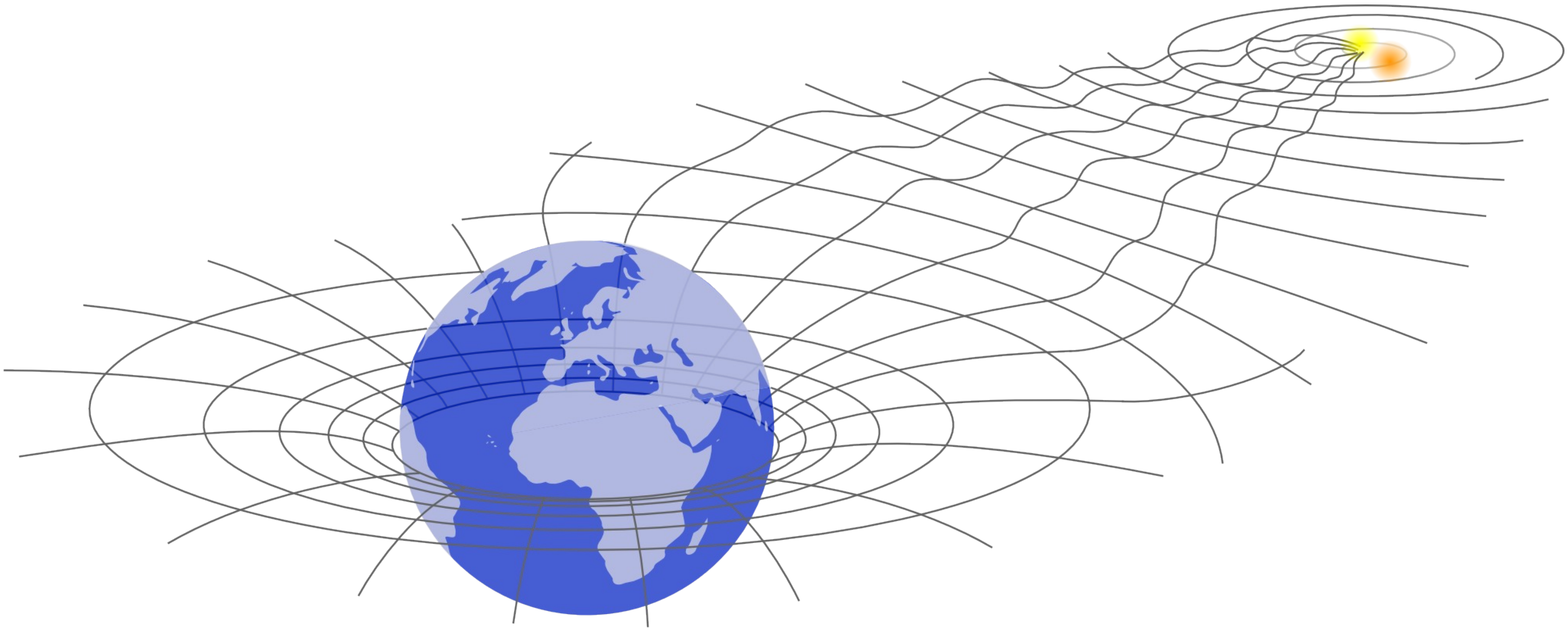
general background-metric

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + h_{\mu\nu}$$

flat background-metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

non-linear regime



The team

(Institute of Quantum Physics, Ulm University)

Michael Buser

Cornelia Feiler

Wolfgang P. Schleich

E. Kajari, M. Buser, C. Feiler and W. P. Schleich,
Proceedings of the International School of Physics "Enrico Fermi"
Course "Atom Optics and Space Physics"

Thank you very much for your attention!

"If tachyons do exist, and if they do go faster than the speed of light, then I'm determined to find something that goes faster than tachyons."

S. Harris