The sensitivity of atom interferometers to gravitational waves

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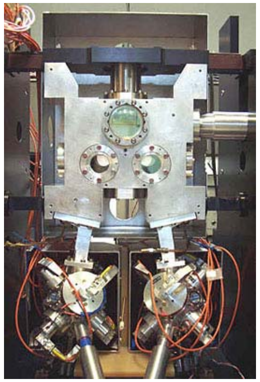
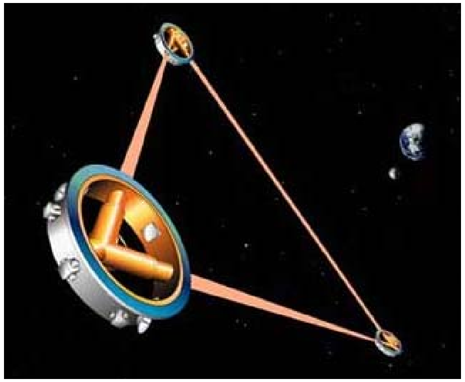
- **Actual laser interferometers**: first detection soon? Very few events expected (<1 detection/year).
- **Amelioration** of terrestrial antennas (2013) → ~1det./day to 1det./week.
- **Exploration** of a new frequency range (low frequency): LISA (ESA/NASA 2018).

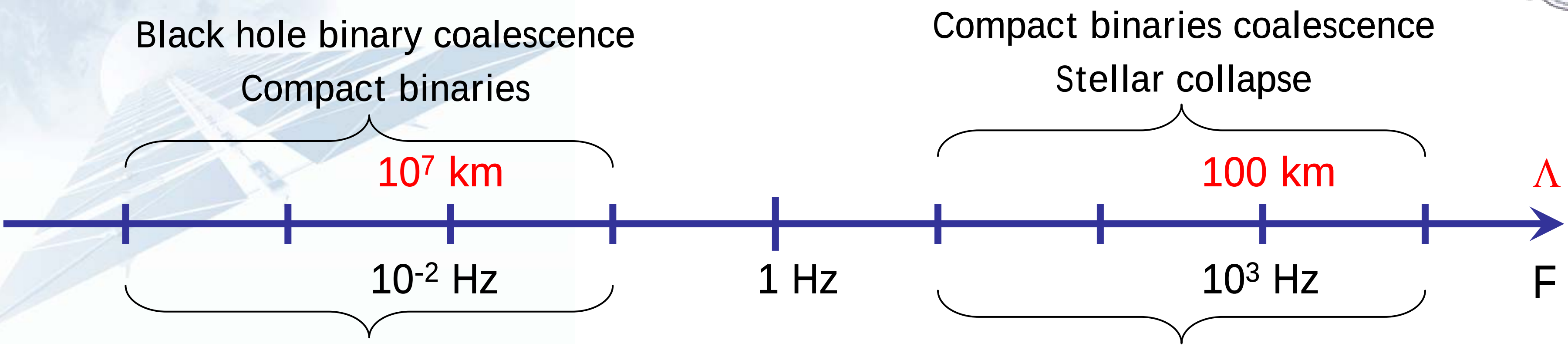
- **New type of detectors**: atom interferometers.
- **Applications**: inertial sensors, gyrometer and absolute gravimeter (see the review by Miffre et al. *Phys. Scr.* 74, 2006)





1. Interest
2. The phase difference
   1. Operational coordinates
   2. Active and passive change of coordinates
3. MWI vs. LWI
4. Sensitivity curves
5. Another configuration
6. Conclusion

MWI interest



Space based interferometer

LISA : **L_{tot} ~ 5.10^6 km**

Ground based interferometer with Fabry-Perot cavities

VIRGO (3 km) + Fabry-Pérot (Finesse = 50) : **L_{tot} ~ 150 km**

- The interferometer frequency domain depends only on the flight time T of the particle in the interferometer arm

$F \sim 1 / T \sim V / L_{tot}$

- For the same frequency domain, reducing the particle velocity → reduce the arm length

Particle = atoms

- Reducing the dimension of the interferometer helps to fight the different noises, and especially thermal noise

- Weak-field approximation $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} \ , \ H_{\mu\nu} \ll 1$

- Calculation of the phase difference within the **eikonal** and the **weak-field** approximation (Linet & Tourrenc 1976).

$$[\phi]_A^B = [\phi_o]_A^B + [\delta\phi]_A^B$$

$$[\phi_o]_A^B = k_\mu x_B^\mu - k_\mu x_A^\mu$$

$$[\delta\phi]_A^B = \frac{\hbar c^2}{2} \int_{t_A}^{t_B} H_{\mu\nu} k^\mu k^\nu \frac{\mathrm{d}t}{E}$$

- In the Einstein frame : $\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + h_{rs}\mathrm{d}x^r\mathrm{d}x^s \ ; \ r,s = 1,2$

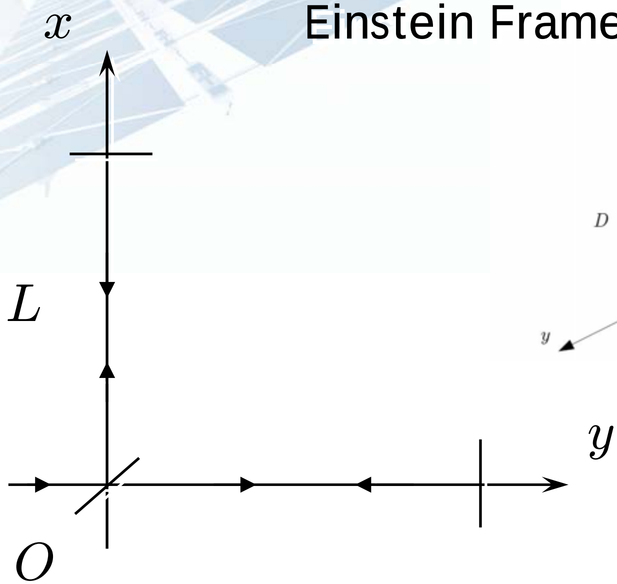
$\mathrm{Z} \simeq z \simeq 0$

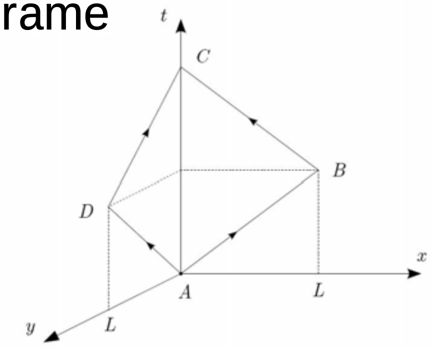
$\mathrm{X}^{\hat{\imath}} \leq \zeta \ll \Lambda$

$$\begin{aligned} x^a &= f^a + \frac{1}{4}\dot{h}_{jk} X^{\hat{\jmath}} X^{\hat{k}} + \mathcal{O}(\zeta^4) \\ x^r &= f^r + X^{\hat{r}} - \frac{1}{2}\bar{h}^r_s X^{\hat{s}} + \mathcal{O}(\zeta^4) \end{aligned}$$

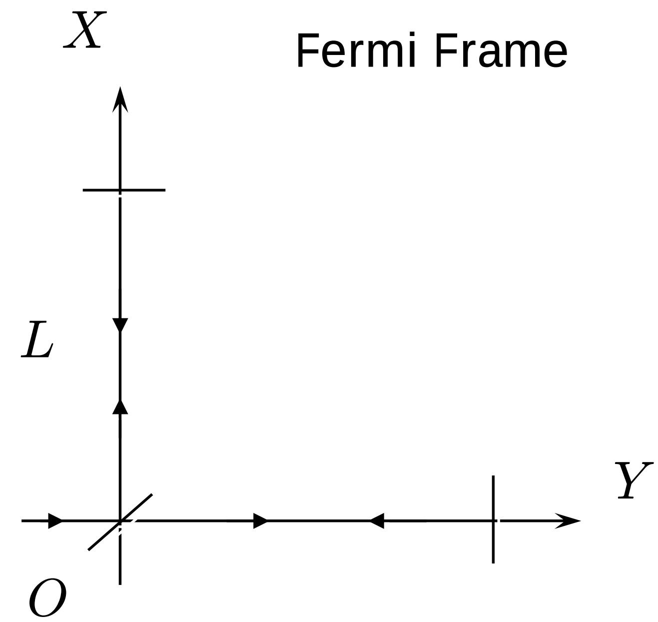
- In the Fermi frame : $\mathrm{d}s^2 = \eta_{\hat{\alpha}\hat{\beta}}\mathrm{d}X^{\hat{\alpha}}\mathrm{d}X^{\hat{\beta}} + \frac{1}{2}\ddot{h}_{rs}X^{\hat{r}}X^{\hat{s}}\mathrm{d}T^2 \ ; \ r,s = 1,2$

Einstein Frame





Fermi Frame



$$\psi = \Omega T/2$$

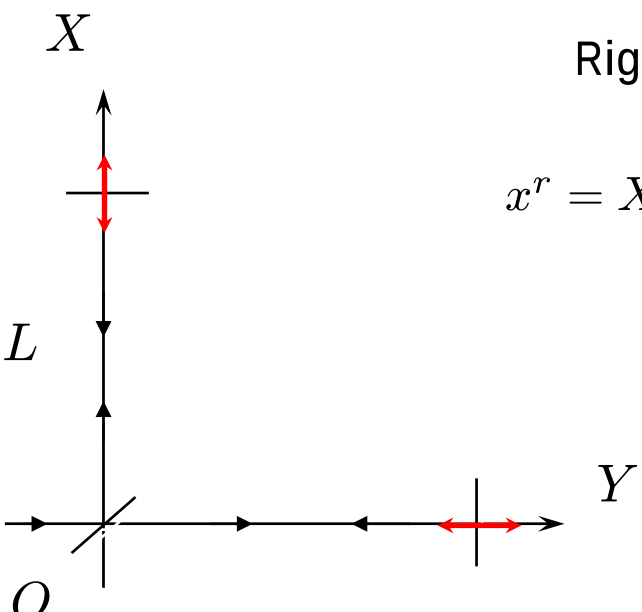
$$\Delta\phi = -4\pi \frac{L}{\lambda} \frac{\sin 2\psi}{2\psi} \tilde{h}_+$$

$$\Delta\phi = 4\pi \frac{L}{\lambda} \left[1 - \frac{\sin 2\psi}{2\psi} \right] \tilde{h}_+$$

WHY ? -> TWO DIFFERENT EXPERIMENTS

Operational coordinates

- By defining our atom interferometer in a non covariant way (ie. its definition depends on the coordinate system we use), we assume that we can experimentally realize this coordinate system with a certain protocol -> we give a physical meaning to the coordinate system -> **operational coordinates**
- **Free experiment** -> the different part of the interferometer do not move in the Einstein frame
- **“Rigid” experiment** -> the different part of the interferometer do not move in a Fermi frame



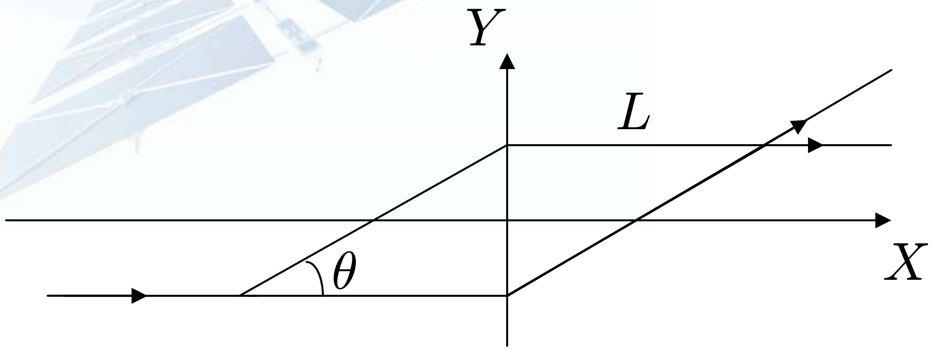
Free Michelson in the Fermi Frame

Rigid Michelson in the Fermi Frame

$$x^r = X^{\hat{r}} - \frac{1}{2}\bar{h}^r_s X^{\hat{s}} + \mathcal{O}(\zeta^4) \longrightarrow \Delta\phi_o$$

$$\Delta\phi = 4\pi \frac{L}{\lambda}\left[\cancel{1} - \frac{\sin 2\psi}{2\psi}\right]\tilde{h}_+$$

Delva et al. 06



$$\Omega = \frac{2\pi c}{\Lambda}\ ,\ \Lambda \gg L = v_0 T$$

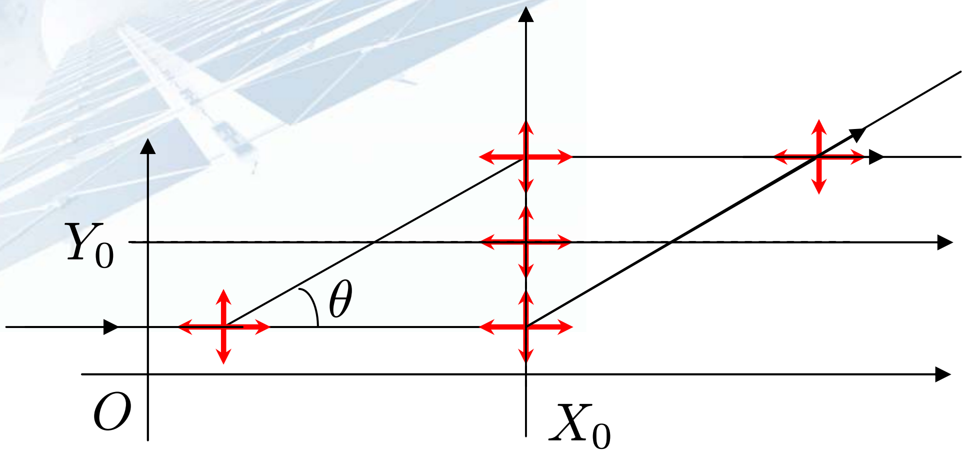
- We assume that the center of mass of the interferometer (= origin of the frame) is located at the center of symmetry of the atom trajectory

$$\Delta\phi(\Omega) = 4\pi \frac{L}{\lambda} F_0(\Omega) \tan\theta \left(h_\times(\Omega) - \frac{\tan\theta}{2} h_+(\Omega) \right)$$

$$\begin{cases} F_0(\Omega) = i \sin\Psi \left(\cos\Psi - \dfrac{\sin\Psi}{\Psi} \right) \\ \Psi = \dfrac{\Omega T}{2} = \dfrac{\Omega L}{2 v_0} \end{cases}$$

$\propto \Psi^3$ at low frequency

D'Ambrosio et al. 07



$$\Delta\phi = \phi_o + \delta\phi + \cancel{\delta\phi} + \cancel{\phi_o}$$

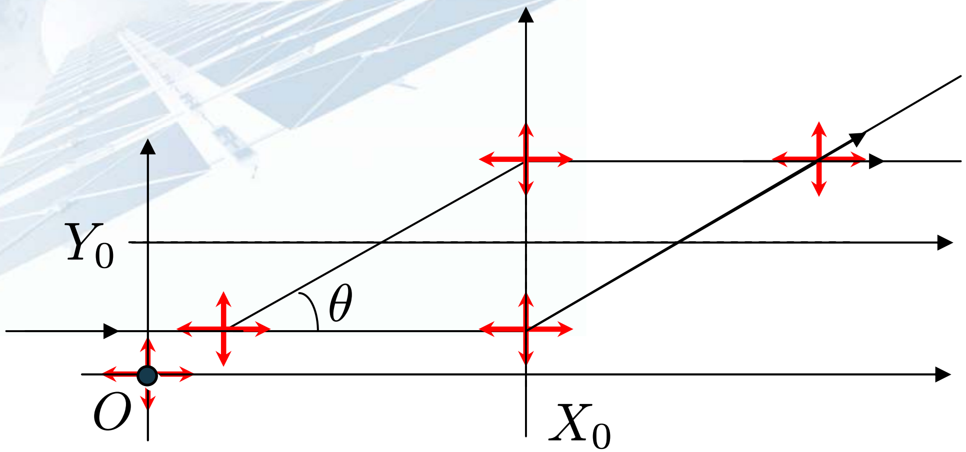
- The center of mass follows a geodesic (doesn't move in the Einstein frame)

$$\Delta X^r = \frac{1}{2}\bar{h}^r_s X^s_0$$

- Same result as before
- As should be, the phase difference does not depend on the origin of the frame -> **passive change of coordinates**

D'Ambrosio et al. 07

Change of the center of mass of the *apparatus (1/2)*

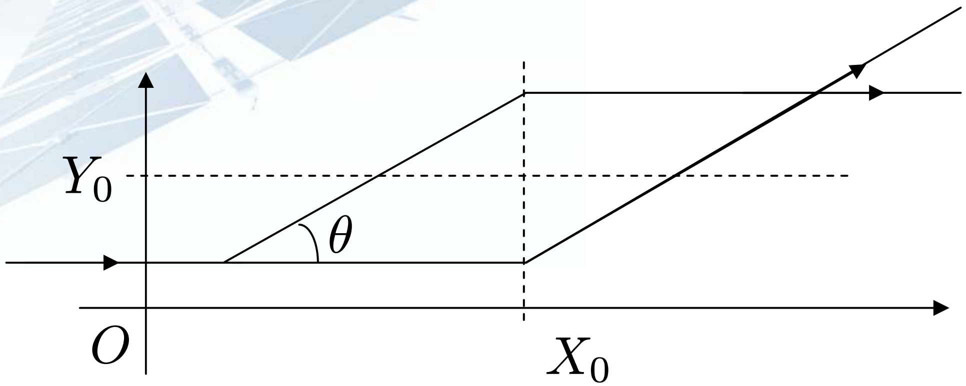


$$\Delta\phi = \phi_o + \delta\phi + \delta\phi$$

- The center of mass follows a geodesic (doesn't move in the Einstein frame)

$$\Delta X^r = \frac{1}{2}\bar{h}^r_s X^s_0$$

- There is a supplementary term
- It can be seen also as an **active change of coordinates**: we define a *DIFFERENT* experiment



$$\Delta\phi(\Omega) = 4\pi \frac{L}{\lambda} \tan\theta \left[\left(F_0(\Omega) + F_X(\Omega, X_0)\right) \tilde{h}_\times - \frac{\tan\theta}{2} \left(F_0(\Omega) + F_Y(\Omega, Y_0)\right) \tilde{h}_+ \right]$$

$$\begin{cases} F_0(\Omega) = i \sin\Psi \left(\cos\Psi - \dfrac{\sin\Psi}{\Psi} \right) \\ \Psi = \dfrac{\Omega T}{2} = \dfrac{\Omega L}{2 v_0} \end{cases}$$

- at low frequency $\Psi \ll 1$:

$$F_X \simeq \frac{X_0}{L}\Psi^2 \qquad F_Y \simeq -\frac{Y_0}{L}\Psi^2$$

$$F_0 \simeq -\frac{i}{3}\Psi^3$$

$$\Delta\phi(\Omega) = 4\pi\frac{L}{\lambda}F_0(\Omega)\tan\theta\left(h_\times(\Omega) - \frac{\tan\theta}{2}h_+(\Omega)\right)$$

$$\begin{cases} F_0(\Omega) = i\sin\Psi\left(\cos\Psi - \frac{\sin\Psi}{\Psi}\right) \\ \Psi = \frac{\Omega T}{2} = \frac{\Omega L}{2v_0} \end{cases}$$

- The maximum phase difference is obtained for T~1/Ω. Then, if $\tan\theta \simeq 1$

$$\widetilde{\Delta\phi} \sim 4\pi|h|\cdot\frac{L_{\mathrm{mw}}}{\lambda_{\mathrm{mw}}}$$

- For a light wave interferometer in a Michelson configuration, the maximum phase difference is obtained for L~c/Ω

$$\widetilde{\Delta\phi} \sim 4\pi|h|\cdot\frac{L_{\mathrm{lw}}}{\lambda_{\mathrm{lw}}}$$

- The shot noise ultimately limit the sensitivity

$$\widetilde{\Delta\phi} \sim \frac{1}{2\sqrt{\dot{N}t}}$$

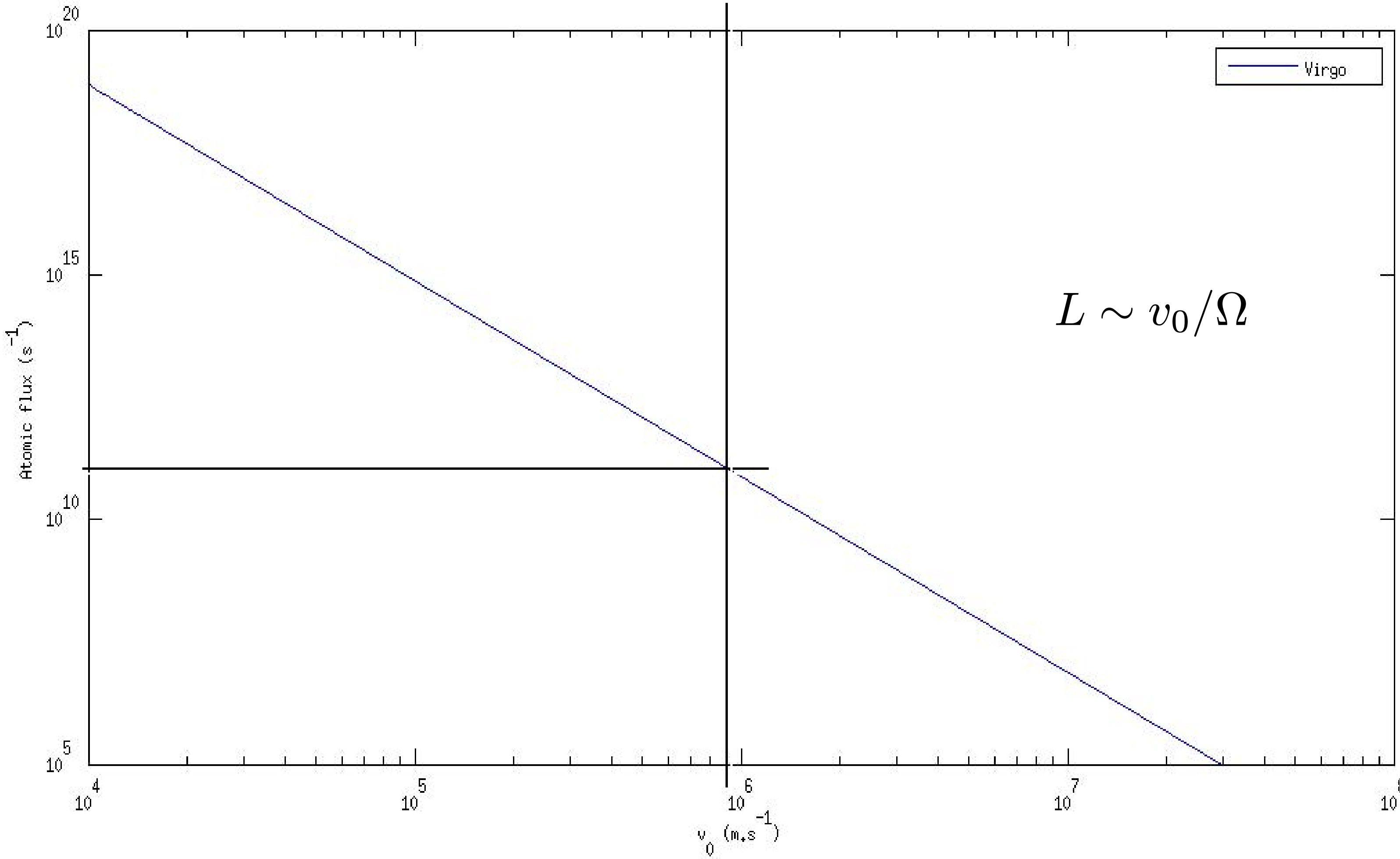
(*Gustavson* et al.) → $\dot{N}_{\mathrm{mw}} \sim 10^{11}\ \mathrm{s}^{-1}$

Virgo → $\dot{N}_{\mathrm{lw}} \sim 10^{23}\ \mathrm{s}^{-1}$

LISA → $\dot{N}_{\mathrm{lw}} \sim 10^{8}\ \mathrm{s}^{-1}$

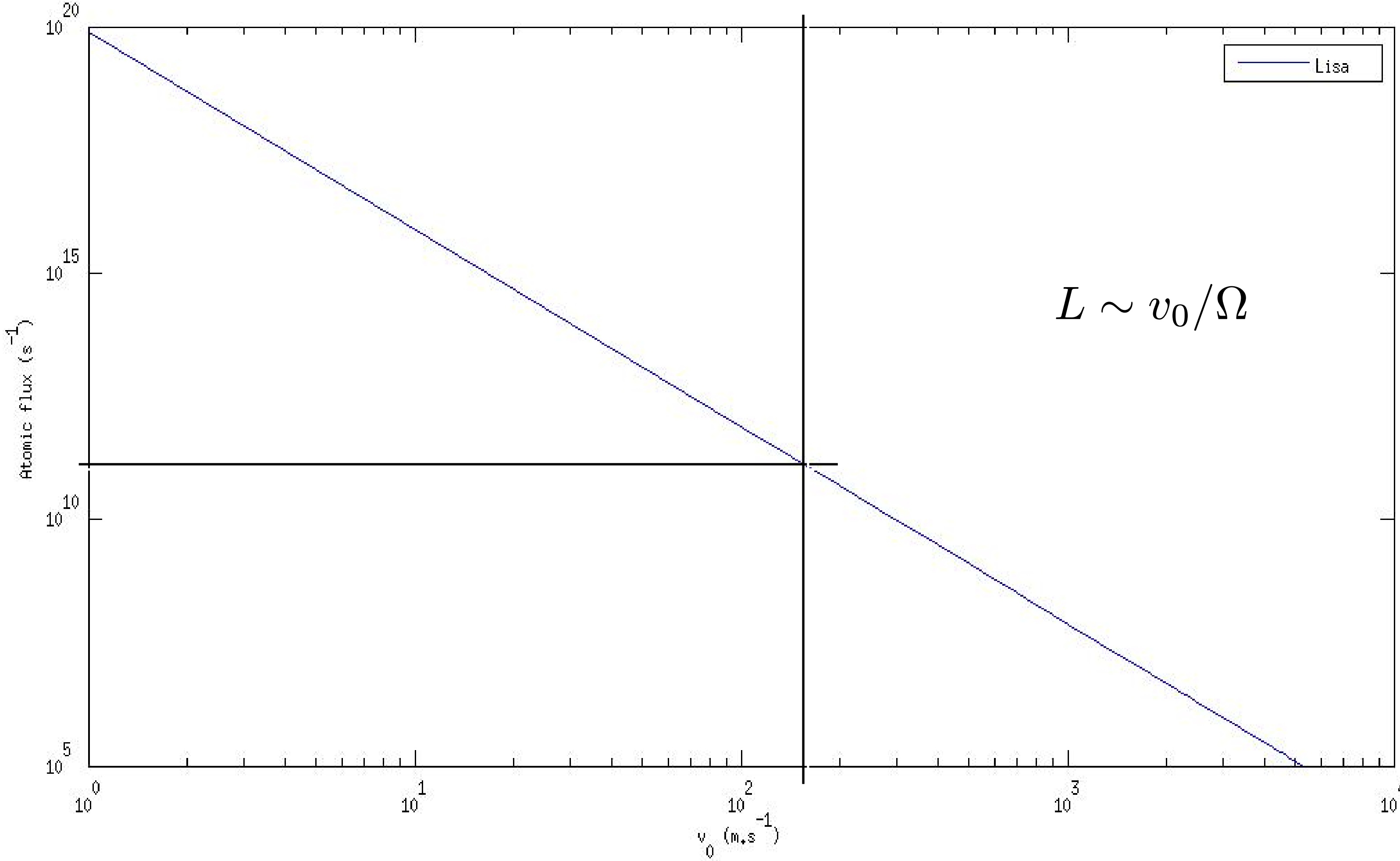
Relativistic velocities needed to reach VIRGO sensitivities

(Matter wave acceleration, deviation of atoms, measurement frequency)

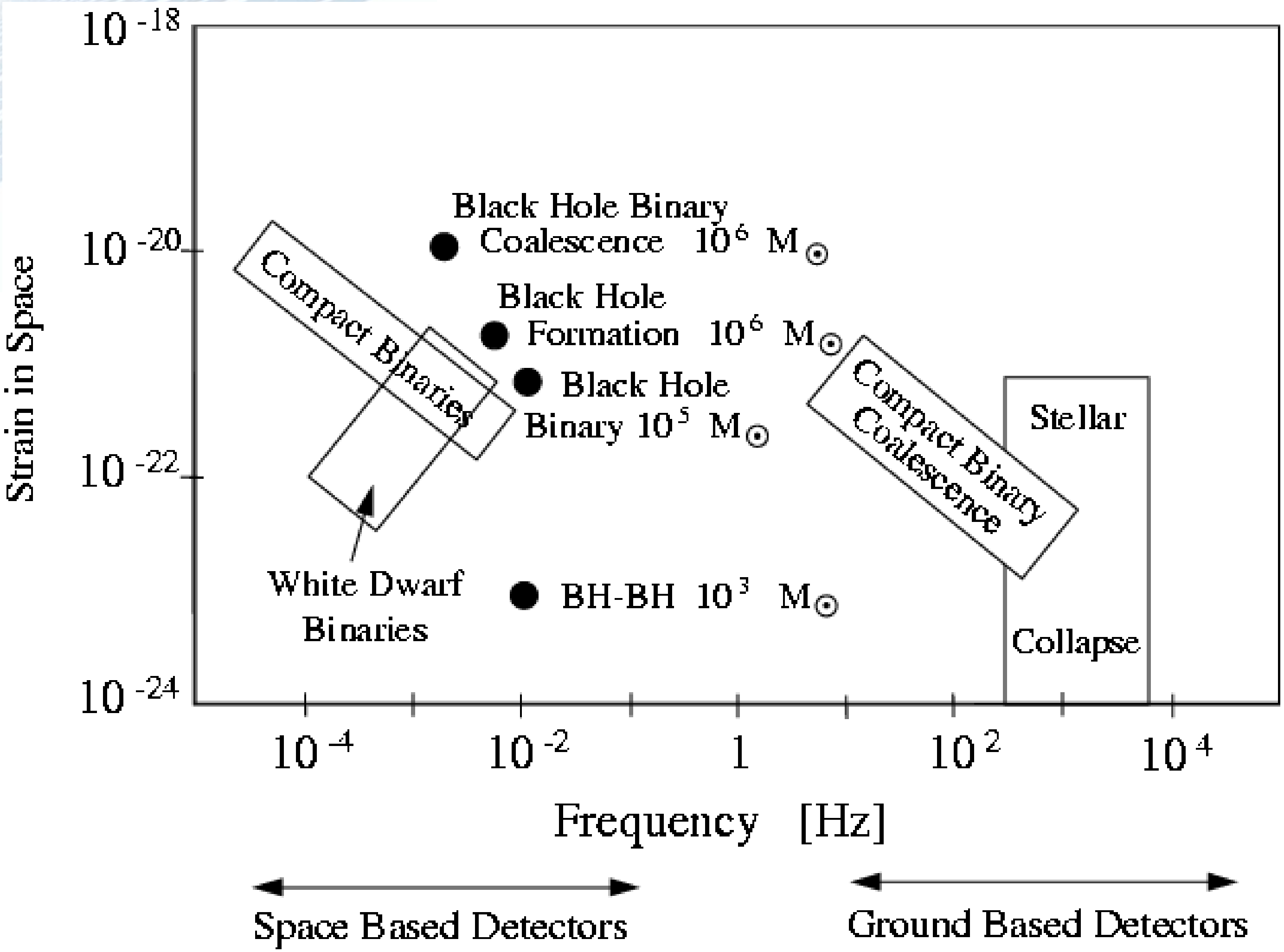


Kilometric interferometer to reach the sensitivity of LISA with thermal atoms

(Matter wave cavity ?)

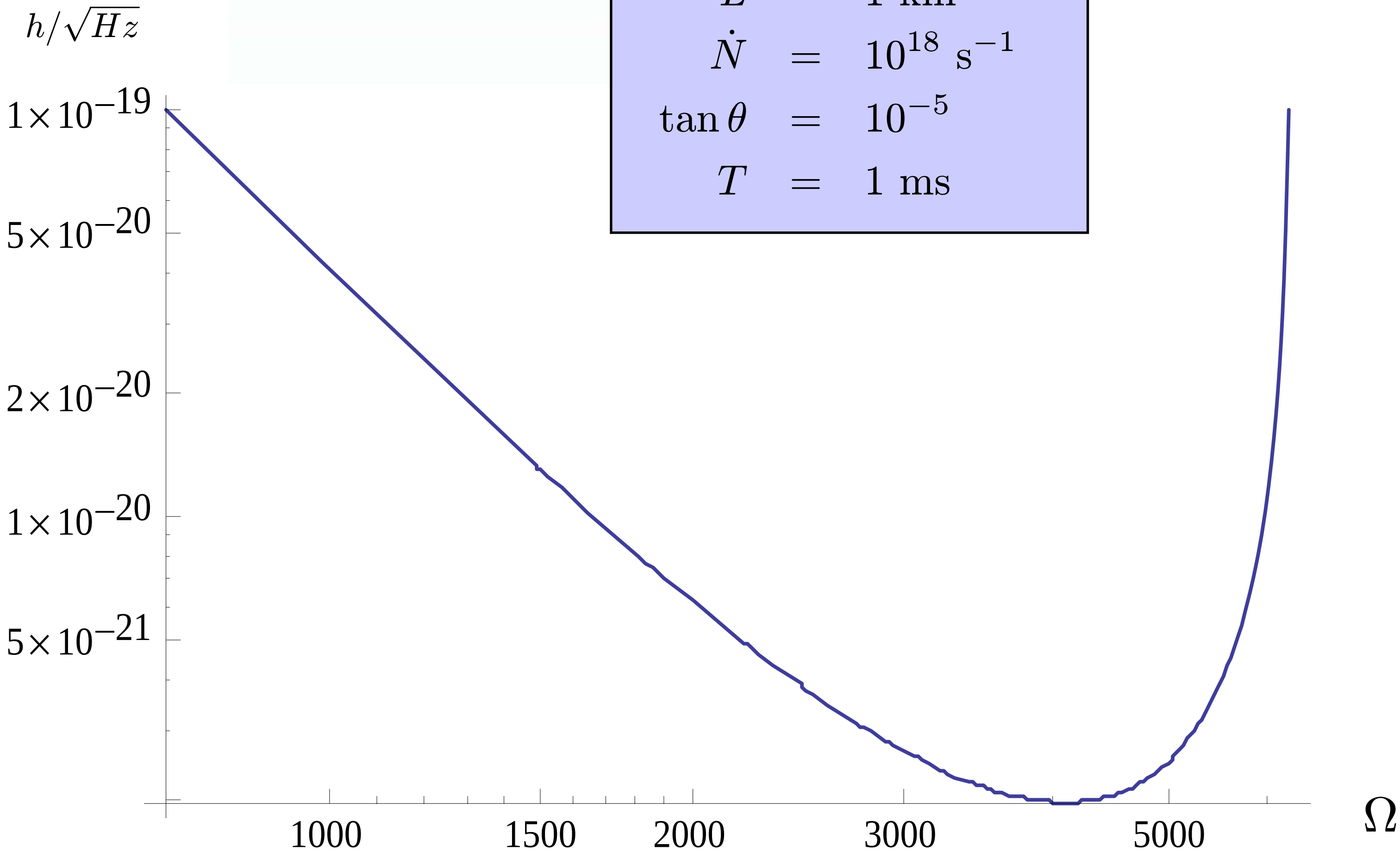


$$L \sim v_0/\Omega$$


10 -18
10 -20
10 -22
10 -24
Strain in Space
Black Hole Binary
Coalescence 10⁶ M⊙
Black Hole
Formation 10⁶ M⊙
Black Hole
Binary 10⁵ M⊙
BH-BH 10³ M⊙
Compact Binaries
White Dwarf
Binaries
Compact Binary
Coalescence
Stellar
Collapse
10 -4
10 -2
1
10 2
10 4
Frequency [Hz]
Space Based Detectors
Ground Based Detectors

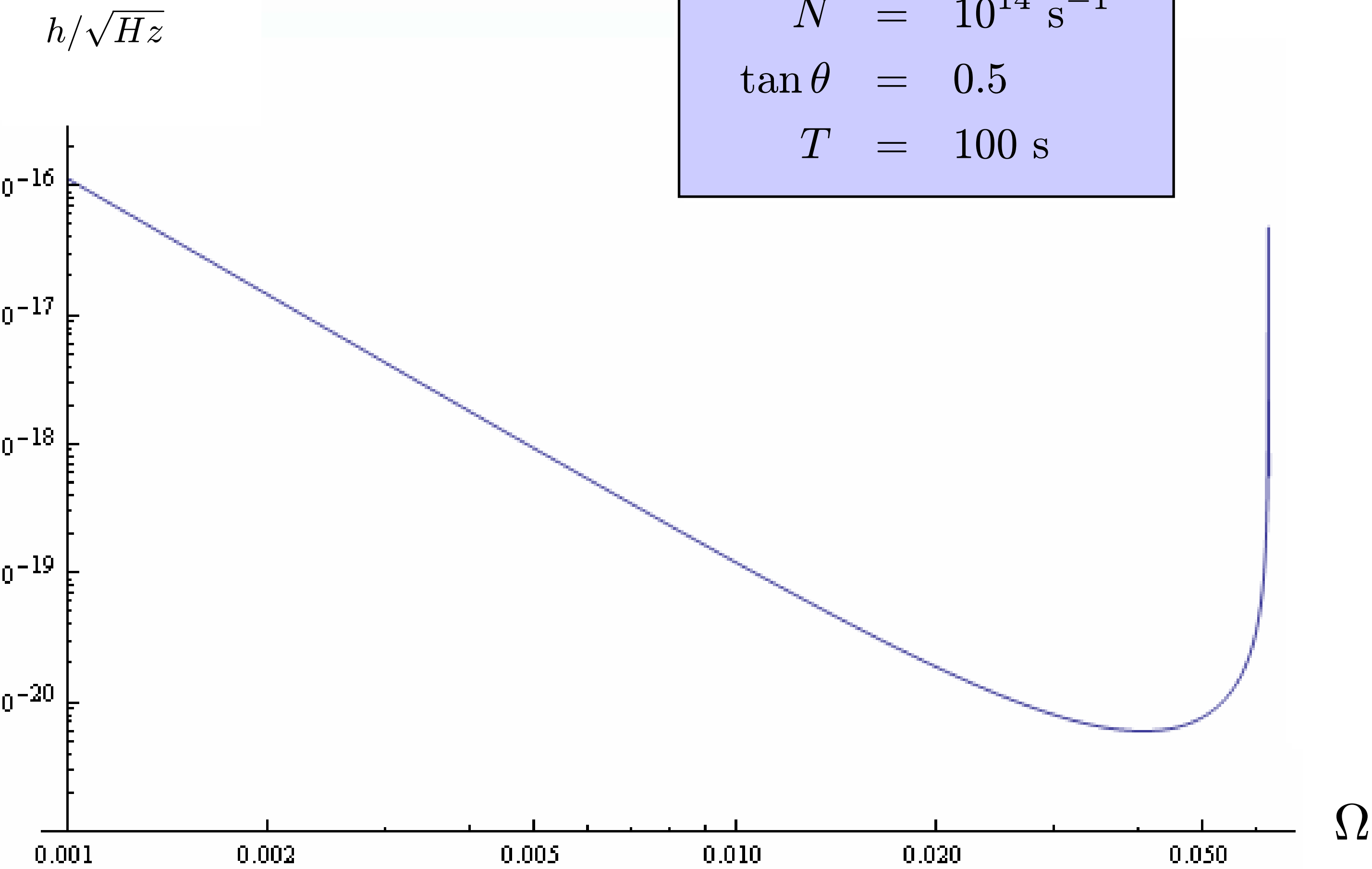
Terrestrial configuration

$$
\begin{aligned}
v &= 10^6\ \mathrm{m.s^{-1}} \\
L &= 1\ \mathrm{km} \\
\dot{N} &= 10^{18}\ \mathrm{s^{-1}} \\
\tan\theta &= 10^{-5} \\
T &= 1\ \mathrm{ms}
\end{aligned}
$$

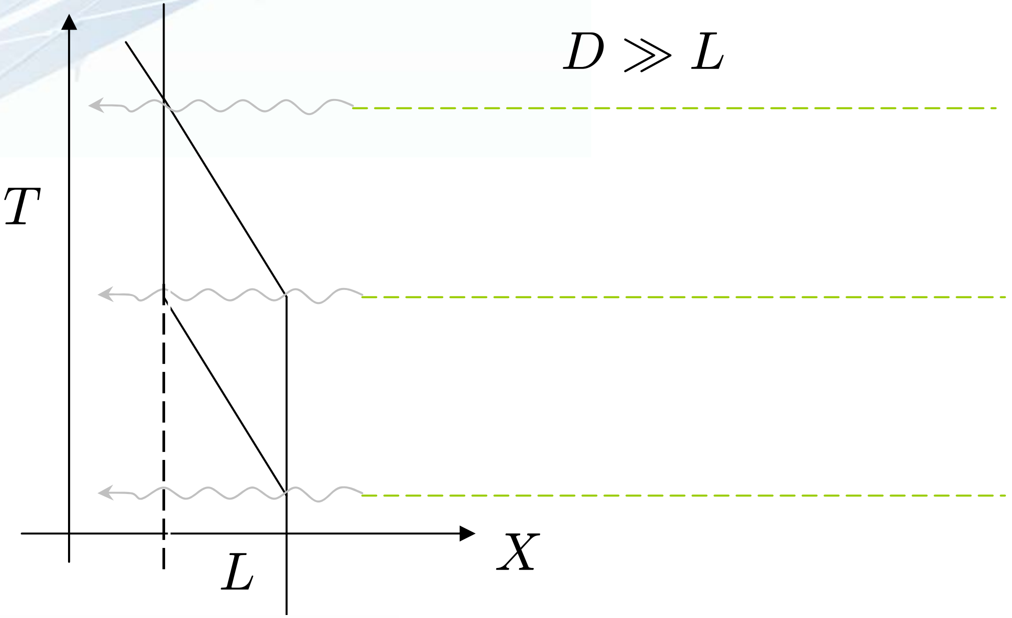


$$
\begin{aligned}
v &= 10 \text{ m.s}^{-1} \\
L &= 1 \text{ km} \\
\dot{N} &= 10^{14} \text{ s}^{-1} \\
\tan\theta &= 0.5 \\
T &= 100 \text{ s}
\end{aligned}
$$

Spatial configuration

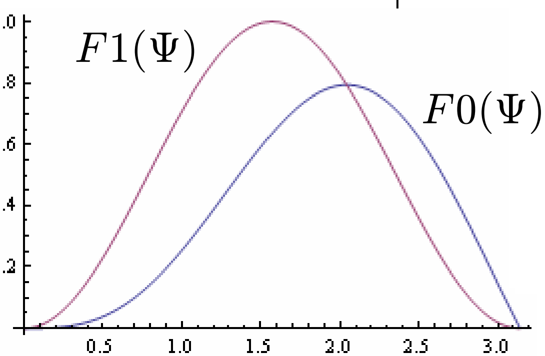


- Dimopoulos et al. (2007) proposed a different configuration for the detector that takes advantage of the distance between the center of mass of the interferometer (lasers) and the center of symmetry of the atoms trajectory



$$\Delta\phi(\Omega) = 4\pi h \frac{D}{\lambda_r} F_1(\Omega)$$

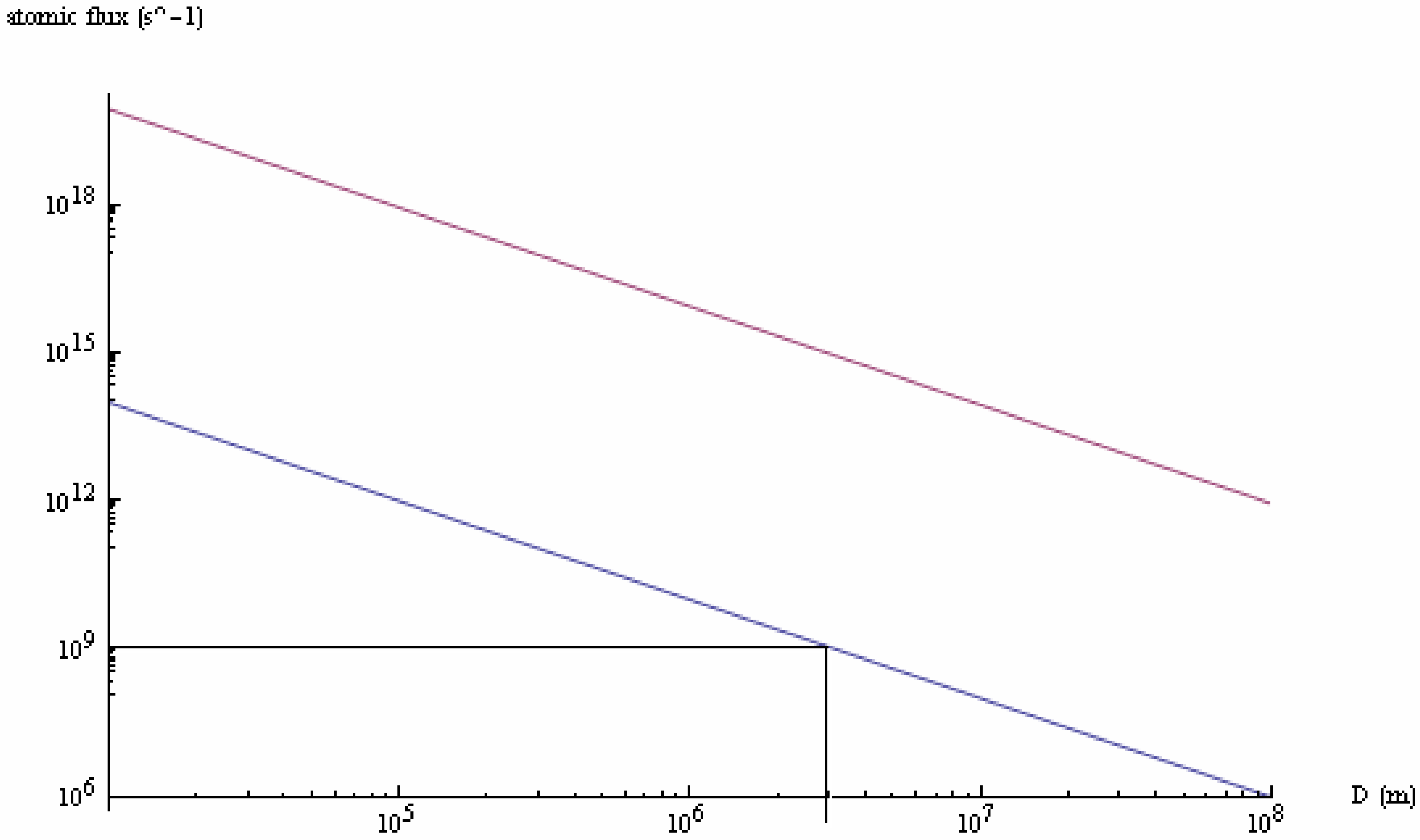
$$\begin{cases} F_1(\Omega) = i \sin^2 \Psi \\ \Psi = \dfrac{\Omega T}{2} \end{cases}$$



- The atom wavelength is fixed by the impulsion of the laser

$$\lambda_r = \frac{2\pi\hbar}{mv_r} = \frac{2\pi}{k_{eff}}$$

- The distance in the amplitude is the distance between the atom interferometer and the laser


atomic flux (s^-1)
10^18
10^15
10^12
10^9
10^6
10^5
10^6
10^7
10^8
D (m)

- Atom interferometers have not reach their best sensitivities.
- Important difficulties remain to reach good sensitivities in order to detect gravitational waves: matter wave cavities, efficient splitting, collisions, flux.
- Matter wave interferometers could compete with space based interferometers such as LISA (low frequency range), but not with earth based ones (high frequency range).
- Importance of **operational coordinates**, difference between **passive** and **active** change of coordinates
- Sensitivity comparison (with same flux)

Atom interferometer

$$h_{\min} \sim \frac{\lambda}{L} \tan\theta \qquad \lambda \sim \mathrm{pm}$$

Atom interferometer with far away lasers

$$h_{\min} \sim \frac{\lambda_r}{D} \qquad \lambda_r \sim \mathrm{nm}$$

LISA

$$h_{\min} \sim \frac{\lambda}{D} \qquad \lambda \sim \mu\mathrm{m}$$

THANK YOU

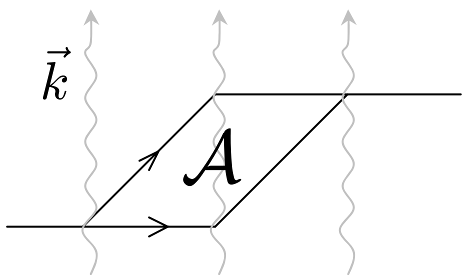
- Métrique : $$\mathrm{d}s^2 = (\eta_{\mu\nu} + K_{\mu\nu})\,\mathrm{d}x^\mu \mathrm{d}x^\nu \ , \ K_{\mu\nu} \ll 1$$

- Différence de phase dans un interféromètre :

$$\Delta\phi \sim \frac{c^2}{\hbar} \int K_{\mu\nu} p^\mu p^\nu \frac{\mathrm{d}t}{E}$$

(Formule de Linet-Tourrenc)

Accélération a	Rotation Ω	Onde Gravitationnelle h^{TT}
$K_{00} \sim \frac{aL}{c^2}$	$K_{0i} \sim \frac{\Omega L}{c}$	$K_{ij} \sim h^{\mathrm{TT}}$
$\Delta\phi \sim \frac{ma}{\hbar} \cdot \frac{\mathcal{A}}{v}$	$\Delta\phi \sim \frac{m\Omega}{\hbar} \cdot \mathcal{A}$	$\Delta\phi \sim \frac{mh^{\mathrm{TT}}}{\hbar} \cdot \frac{\mathcal{A}}{T}$
$\Delta\varphi \sim kaT^2$	$\Delta\varphi \sim k\Omega vT^2$	$\Delta\varphi \sim kh^{\mathrm{TT}}vT$



$$\mathcal{A} = \frac{\hbar k v T^2}{m}$$