The Galileo Galilei Institute for Theoretical Physics

Rovibrational Quantum Interferometers and Gravitational Waves

D. Lorek, A. Wicht, C. Lämmerzahl, H. Dittus

Motivation

Molecular Interferometry

- basic features of Atom Interferometry
- coherent manipulation of internal molecular quantum states
- molecules can distinguish between different directions
- molecular states are sensibel to non–isotropic effects
- $\longrightarrow$ Gravitational Wave Detectors

Contents

Quantum Interferometers

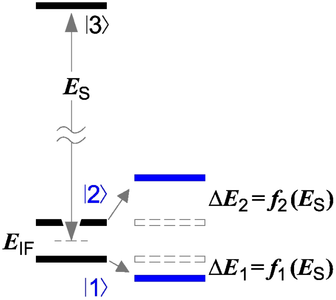
Atom and Molecular Interferometry

- applications: fine structure constant, gravitational acceleration, gravity gradients, inertial sensors, test of GR, ...
- frequency measurement $\leftrightarrow$ second
- phase sensitive frequency measurement
  $\rightarrow$ sensitivity increases $\sim T$ (not $\sim \sqrt{T}$)
- truly differential phase (frequency) measurement - cancellation of common mode phase evolution (common frequency)
- Stanford atom interferometer: relative shift of 10^{-19}

Quantum Interferometers

Atom Interferometric "Lever Arm"

- energy difference between paths: E_{IF}
- assumption: effect causes a shift, that scales with optical frequency E_s: $\Delta E = \Delta E_2 - \Delta E_1 = h \cdot E_s$
- sensitivity $\Delta E / E > \epsilon_{\mathrm{ref}} \sim 10^{-15}$

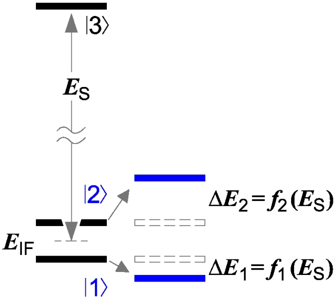


Quantum Interferometers

Atom Interferometric "Lever Arm"

- energy difference between paths: E_{IF}
- assumption: effect causes a shift, that scales with optical frequency E_s: $\Delta E = \Delta E_2 - \Delta E_1 = h \cdot E_s$
- sensitivity $\Delta E / E > \epsilon_{\text{ref}} \sim 10^{-15}$

- laser spectroscopy: $h \cdot E_s / E_s > \epsilon_{\text{ref}}$
- atom interferometry: $h \cdot E_s / E_{IF} > \epsilon_{\text{ref}}$
- minimal detectable h: $h_{\text{min}} = \epsilon_{\text{ref}} \frac{E_{IF}}{E_s}$



Molecular Quantum Interferometry

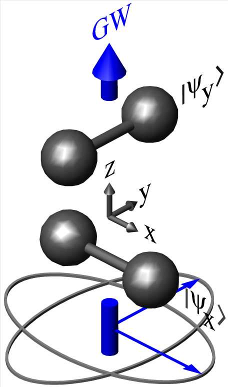
Rovibrational Quantum Interferometers

- coherent manipulation of different individual rotational–vibrational molecular quantum states
- molecules are not spherically symmetric
- $\longrightarrow$ molecules can distinguish between different directions in space w. r. t. their internuclear axis
- $\longrightarrow$ molecular spectra depend on the orientation of the molecule, if a non–isotropic situation is considered

Molecular Quantum Interferometry

Gravitational Wave Detection

- prepare molecules in a coherent superposition of two mutually orthogonal orientations in the x–y–plane ↔ two paths of a quantum interferometer
- linearly polarized GW (z–direction)
- the GW modifies the internuclear distance periodically
- free quantum evolution: non–isotropic perturbation removes the orientational degeneracy → states will acquire a quantum phase difference



Gravitational Waves and Charged Point Masses

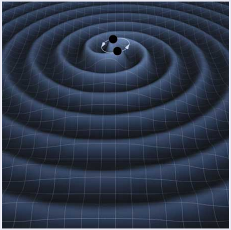
Gravitational Waves

- linearized gravity: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

  $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)\,, \quad |h_{\mu\nu}| \ll 1$

- TT gauge :

$$\Box h_{ij} = 0\,, \qquad h_{\mu 0} = 0\,,$$
$$\delta^{kl} h_{ik,l} = 0\,, \qquad \delta^{ij} h_{ij} = 0$$



Gravitational Waves and Charged Point Masses

Quantum Physics

- Klein–Gordon equation minimally coupled to gravity and to the Maxwell field:

$$g^{\mu\nu} D_\mu D_\nu \psi - \frac{m^2 c^2}{\hbar^2} \psi = 0$$

  - covariant derivative: $D_\mu T^\nu = \partial_\mu T^\nu + \left\{ {\nu \atop \mu\sigma} \right\} T^\sigma - \frac{ie}{\hbar c} A_\mu T^\nu$
  - Christoffel symbol: $\left\{ {\nu \atop \mu\sigma} \right\} := \frac{1}{2} g^{\nu\rho} \left(\partial_\mu g_{\sigma\rho} + \partial_\sigma g_{\mu\rho} - \partial_\rho g_{\mu\sigma} \right)$
  - Maxwell potential: A_μ

Gravitational Waves and Charged Point Masses

Quantum Physics

- Klein–Gordon equation minimally coupled to gravity and to the Maxwell field:

$$g^{\mu\nu} D_\mu D_\nu \psi - \frac{m^2 c^2}{\hbar^2} \psi = 0$$

- insert: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Ansatz [1]: $\psi = \exp\left(\frac{i}{\hbar}[c^2 S_0 + S_1 + c^{-2} S_2 + \ldots]\right)$
- compare equal powers of c^2:

[1] C. Kiefer, T. P. Singh, Phys. Rev. D **44**, 1067–1076 (1991)

Gravitational Waves and Charged Point Masses

Schrödinger Equation: $i\hbar\partial_t\tilde{\phi} = \mathcal{H}\tilde{\phi}$

- Hamiltonian [1]:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\left(\delta^{ij} - h^{ij}\right)\partial_i\partial_j - eA_0 + \frac{ie\hbar}{m}\frac{A_i}{c}\left(\delta^{ij} - h^{ij}\right)\partial_j$$

[1] S. Boughn, T. Rothman, Class. Quantum Grav. **23**, 5839–5852 (2006)

Gravitational Waves and Charged Point Masses

Schrödinger Equation: $i\hbar\partial_t\tilde{\phi} = \mathcal{H}\tilde{\phi}$

- Hamiltonian:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\left(\delta^{ij} - h^{ij}\right)\partial_i\partial_j - eA_0 + \frac{ie\hbar}{m}\frac{A_i}{c}\left(\delta^{ij} - h^{ij}\right)\partial_j$$

- for non–relativistic systems $A_i \ll A_0 \rightarrow$
  Interaction Hamiltonian:

$$\mathcal{H}_{\mathrm{I}} = \frac{\hbar^2}{2m}h^{ij}\partial_i\partial_j$$

Gravitational Waves and Charged Point Masses

Electric Potential A_μ

- inhomogenous Maxwell equations coupled to gravity:

$$4\pi j^\mu = D_\nu \left(g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \right)$$

  - Field-Strength Tensor: $F_{\rho\sigma} = \partial_\rho A_\sigma - \partial_\sigma A_\rho$
- insert: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- point charge: $j_0 = q\delta^3(r)$ and $j_i = 0$

Gravitational Waves and Charged Point Masses

Electric Potential A_μ

- inhomogenous Maxwell equations coupled to gravity:

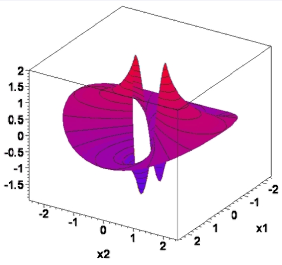
$$4\pi j^\mu = D_\nu \left(g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \right)$$

- insert: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- point charge: $j_0 = q\delta^3(r)$ and $j_i = 0$
- periodic gravitational waves: $h_{\mu\nu} = h^0_{\mu\nu} \cdot \exp(i[\vec{k}\vec{x} - \omega t])$
- Influence of the GW is adiabatic (low frequency) and quasi–constant (long wavelength) $\longrightarrow$ potentials are static
- Ansatz: $A_0 = q/r + qA_0^{(1)}, \quad A_i = qA_i^{(1)}$

Gravitational Waves and Charged Point Masses

Potential A_0 of a Point Charge q in the Field of a GW

$$A_0 = \frac{q}{r}\left(1 - \frac{x^i h^0_{ij} x^j}{2r^2} e^{i(\vec{k}\vec{x}-\omega t)}\right)$$



HD^+ Molecular Hamiltonian

- molecular Hamiltonian contains:

$$\mathcal{H} = T_e + V_{en1} + V_{en2} + V_{nn} + T_{n1} + T_{n2} \quad + \delta\mathcal{H}$$

  - T: kinetic energy
  - V: potential energy
  - e, $n1$, $n2$: electron, first nucleus, second nucleus

$\vdots$

Perturbation to Molecular Hamiltonian

electronic kinetic energy:

$$(h \cdot R_\infty)\,\delta\tilde{T}_e(R)\cos(2\phi)(1-\cos(2\theta))$$

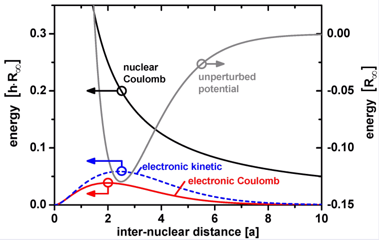
electronic Coulomb energy:

$$(h \cdot R_\infty)\,\delta\tilde{V}_{en}(R)\cos(2\phi)(1-\cos(2\theta))$$

nuclear Coulomb energy:

$$-(h \cdot R_\infty)\,(2R)^{-1}\cos(2\phi)(1-\cos(2\theta))$$

Radial dependence of perturbations



Perturbation to Molecular Hamiltonian

electronic kinetic energy:

$$(h \cdot R_\infty)\,\delta \tilde{T}_e(R)\cos(2\phi)(1-\cos(2\theta))$$

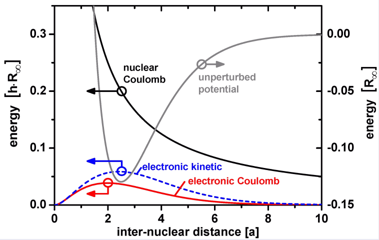
electronic Coulomb energy:

$$(h \cdot R_\infty)\,\delta \tilde{V}_{en}(R)\cos(2\phi)(1-\cos(2\theta))$$

nuclear Coulomb energy:

$$-(h \cdot R_\infty)\,(2R)^{-1}\cos(2\phi)(1-\cos(2\theta))$$

Radial dependence of perturbations



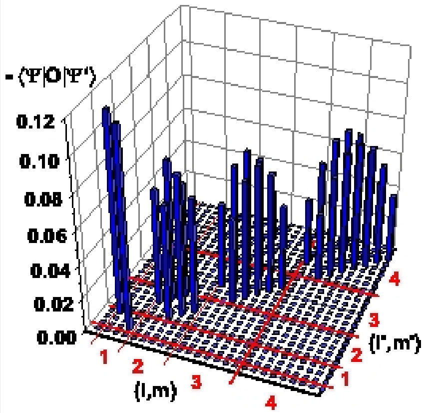
Perturbation Energy: $\sim 0.1 \cdot h \cdot R_\infty$

Gravitational Waves and the HD^+ Molecule

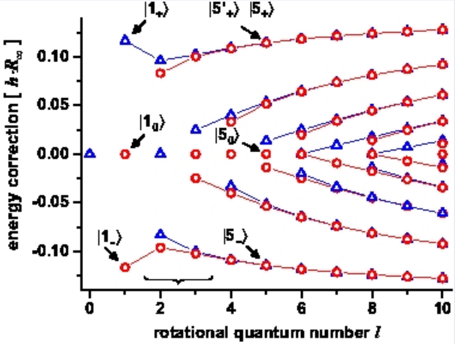
Total Perturbation Operator

- GW can not drive rotational, vibrational or electronic transitions
- GW only couples states with $|\Delta m| = 2$ as expected from quadrupole nature of GW

Perturbation Matrix



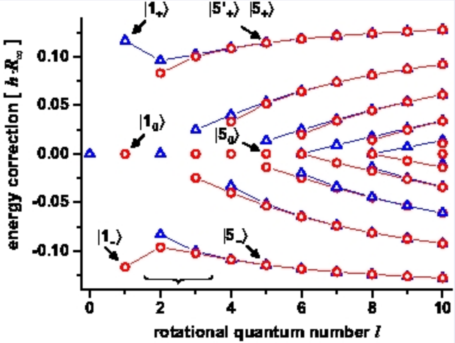
Eigenvalues



eigenvalues for the total perturbation operator for the vibrational ground state $v = 0$

Gravitational Waves and the HD^+ Molecule

Eigenvalues

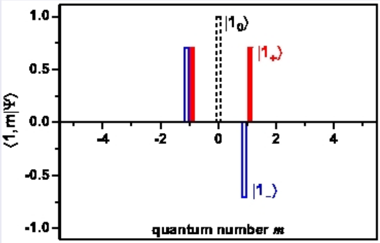


Differential Energy Shift: 60 μHz for $h = 10^{-19}$

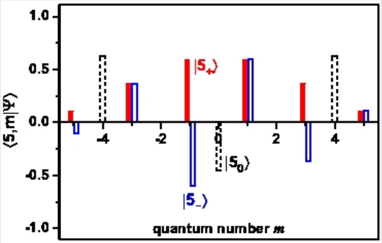
Gravitational Waves and the HD^+ Molecule

State Spectra

state spectrum for $l = 1$ eigenstates

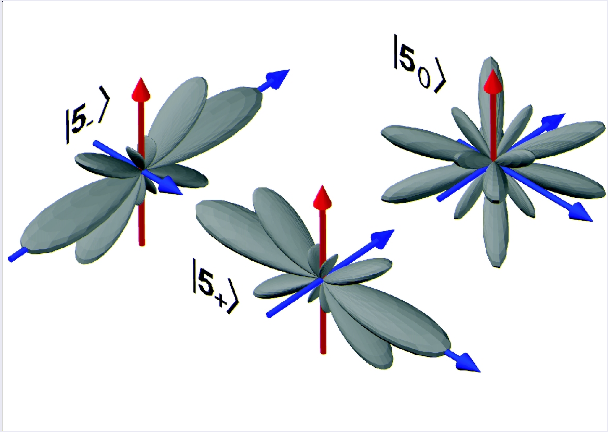


state spectrum for some $l = 5$ eigenstates



Gravitational Wave Detection

Spherical Part of the Probability Distribution $|\langle\vec{R}|\psi\rangle|^2$



Comparison with "Classical" Detectors

Advantages

- most accurate measurement methods available
- physics of the probe and the interaction between the probe and the environment is well understood
- exactly identical, well defined probes exist
- no storage time limit
- spectral-, polarization- and directional-sensitivity can be chosen and modified within milliseconds

Challenges

- ultra-cold molecules ($\ll$mK) to be prepared in a specific state
- multi-chromatic narrow linewidth laser-fields required
- control/suppress environmental perturb. (em. fields, vibrations)

Conclusion

Quantum Interferometry

- Basic ideas of AI $\longrightarrow$ Rovibrational states of molecules
- Molecular states are sensibel to non–isotropic effects
- Quantum sensor for fundamental physics

Gravitational Waves and HD^+

Prospects

Conclusion

Quantum Interferometry

Gravitational Waves and HD^+

- Hamiltonian of a charged particle in a GW field
- Electric potential of a point charge in a GW field
- $\rightarrow$ Perturbation operator of HD^+ and energy shifts
- Adequate construction

Prospects

Conclusion

Quantum Interferometry

Gravitational Waves and HD^+

Prospects

- $h = 10^{-19} \rightarrow$ 60 μHz (current AI: $\approx$ 100 μHz)
- Further tests of fundamental physics
- AI

Conclusion

Quantum Interferometry

Gravitational Waves and HD^+

Prospects

A. Wicht, C. Lämmerzahl, D. L., H. Dittus, Phys. Rev. A **78**, 013610

Thanks for your attention!