Atom vs. laser interferometers for gravitational-wave detection

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Based on work in collaboration with
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Outline

1. Gravitational-wave (GW) detection:
   opening a new window to the universe
   - Laser interferometers
   - Atom interferometers
2. Response of atom interferometers to GWs
   - Linear GW geometry (rigid vs. comoving coordinates)
   - Phase shift due to GWs
   - Sources of discrepancy
3. Discussion of the results
4. Future prospects

Gravitational-wave detection: a new window to the universe

- New window for astrophysics and cosmology.

  coherent sources, low frequencies
  and almost undisturbed propagation

  - Tests of GR in *strong* gravitational fields, *event horizons*
  - Early universe: pre-BBN *phase transitions*, direct probe of *inflation*

- Detection requirements:

  $h \sim \delta L/L \sim 10^{-22} \longrightarrow \delta L \sim 10^{-19}\,\mathrm{m}$ for $L \sim 1\,\mathrm{km}$

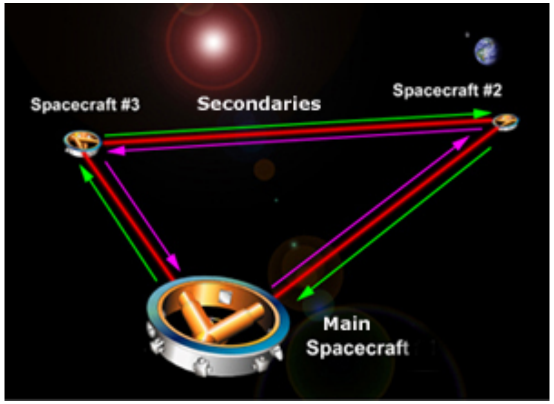
Laser interferometers

- LIGO (VIRGO, GEO, TAMA)

$$h \sim 10^{-22}$$
$$\omega \sim 10^2 - 10^3 \,\mathrm{Hz}$$
$$L \sim 4\,\mathrm{km}$$

- LISA

$$h \sim 10^{-20}$$
$$\omega \sim 10^{-3} - 10^{-1} \,\mathrm{Hz}$$
$$L \sim 10^5\,\mathrm{km}$$



Atom interferometers (AIs)

- Extremely precise devices based on atomic physics (e.g. atomic clocks)
- Atomic interferometers currently used to measure non-inertial and gravitational properties:
  - *Sagnac effect* (gyroscopes)
  - *gravitational redshift* on Earth (gravimeters/gradiometers)
- $mc^2/h\nu \gg 1$ (although $v/c \ll 1$)
  BUT *lower* flux & *difficult* manipulation
- GW detection?

Response of AIs to GWs

- Earlier studies $\longrightarrow$ response *qualitatively* similar to laser interferometers

  Linet, Tourrenc; Stodolsky; Cai, Papini; Bordé

- Recently (*5 yrs.* ago) $\longrightarrow$ qualitative *differences* leading to higher sensitivity

  Chiao, Speliotopoulos; Foffa, Gasparini, Papucci, Sturani

- Detailed analysis to identify *source of discrepancy*.

- Consider rigid arms vs. freely suspended mirrors.

Linear GW geometry

- Linear GW in *TT gauge:* $h_{i3} = 0 \quad h^i_i = 0$

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij}(t-z))dX^i dX^j$$

- *Geodesic* equation for *non-relativistic* particle:

$$\frac{d^2 X^i}{dt^2} = -\dot{h}^i_j \frac{dX^j}{dt} + O(h^2_{ij}, v^2 h_{ij})$$

Geodesic (comoving) coordinates.

- Non-relativistic Lagrangian:

$$L(X^i, \dot{X}^i) = \frac{m}{2}\left(\dot{X}^i \dot{X}_i + h_{ij}(t)\dot{X}^i \dot{X}^j - 2\right)$$

$|x^i| \ll \lambda_{\rm GW}$

- Rigid coordinates: $t \to t\,, \quad x^i \to x^i = X^i + \frac{1}{2}h^i_j X^j$
- Metric: $$\begin{aligned} ds^2 &= -dt^2 - \dot{h}_{ij}(t-z)x^i dt dx^j + \delta_{ij}dx^i dx^j \\ &\quad + \dot{h}_{ij}(t-z)x^i dz dx^j + O(h_{ij}^2) \end{aligned}$$
- *Geodesic* equation for *non-relativistic* particle:
$$\frac{d^2x^i}{dt^2} = \frac{1}{2}\ddot{h}^i_j x^j + O(h^2_{ij}, v^2 h_{ij}) \qquad z = 0$$
- Non-relativistic Lagrangian:
$$L(x^i, \dot{x}^i) = \frac{m}{2}\Big(\dot{x}^i\dot{x}_i - \dot{h}_{ij}(t)x^j\dot{x}^i - 2\Big)$$
- Equivalent results (classically and quantum mechanically).

Phase shift due to GWs

- WKB approx. $\longrightarrow$ solve H-J equation (*perturbatively*):

$$0 = \frac{\partial S}{\partial t} + H\left(\frac{\partial S}{\partial y^i}, y^i, h_{ij}(t)\right)$$

$$v^i = v\, n^i = \frac{1}{m}\frac{\partial S_0}{\partial y^i}$$

$$\frac{dS_1}{dt} = \frac{\partial S_1}{\partial t} + v^i \frac{\partial S_1}{\partial y^i} = -H^{(1)}\left(\frac{h}{\lambda} n_i, y^i, h_{ij}(t)\right)$$

$$\lambda = 2\pi\hbar/mv$$

- Alternatively, evaluate the classical action along (*perturbed*) classical *solutions*:

$$\begin{aligned} S_E(y^i,t) - S_E(y^i, t-T) &= \int_{t-T}^{t} dt' L\left[x_0(t') + x_1(t'); h_{ij}(t')\right] \\ &= \int_{t-T}^{t} dt' \left(L^{(0)}\left[x_0(t') + x_1(t')\right] + L^{(1)}\left[x_0(t')\right]\right) \end{aligned}$$

Sources of discrepancy

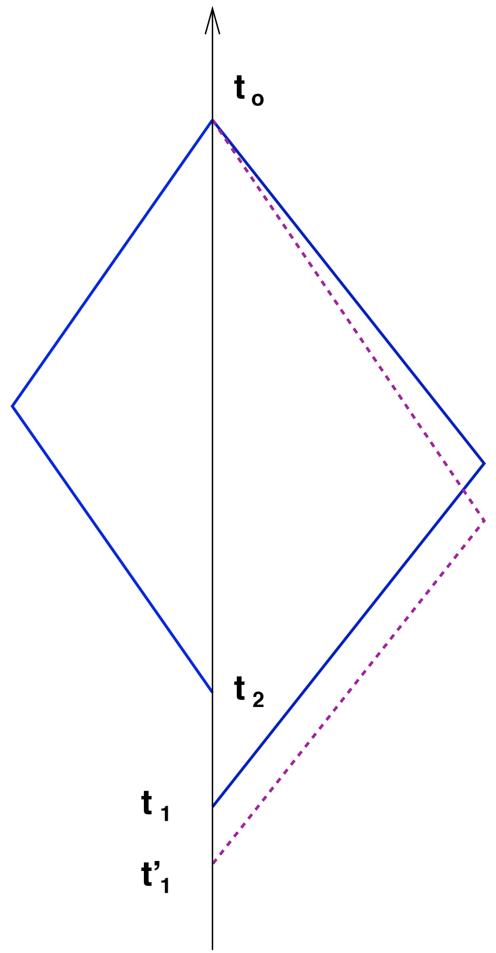
$$\delta S_0[x_0(t') + x_1(t')] = (p_B \Delta x_B - H(x_B, p_B)\Delta t_B) - (p_A \Delta x_A - H(x_A, p_A)\Delta t_A)$$

- *Chiao & Speliotopoulos*
  $\Delta t_{A,B} = 0$ rather than $\Delta x_{A,B} = 0$.

- *Foffa et al.* took $\Delta t_{A,B} \neq 0$,
  but *missed* the *phase difference*
  at the source: $E(\Delta t_1 - \Delta t_2)$.

  Later on they *missed*
  $\vec{p} \cdot (\Delta \vec{x}_2 - \Delta \vec{x}_1)$ when $\Delta t_i = 0$.

- Extra term $\propto n^i n^j \bar{h}_{ij} L^2 \omega / (\lambda v)$.



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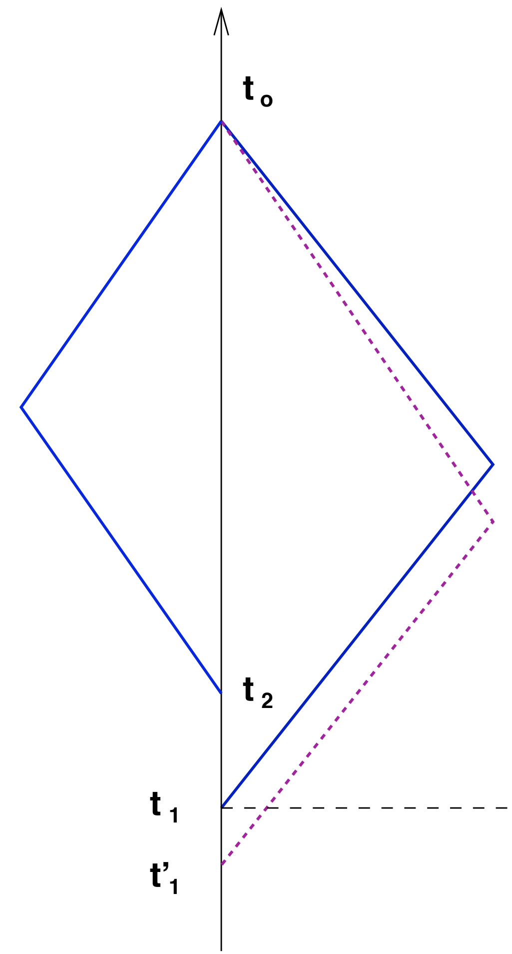

Discussion of the results

- *Michelson-type* interferometer: $h_{ij}(t) = \bar{h}_{ij} \sin(\omega t + \varphi)$

$$\Delta\phi(t) = 4\pi n^i n^j \bar{h}_{ij} \left[\frac{L}{\lambda} - \frac{v}{\lambda\omega} \sin\left(\frac{\omega L}{v}\right) \right] \sin\left(\omega t + \varphi - \frac{\omega L}{v}\right)$$

rigid arms

- High frequency (*rigid* arms): $\leftarrow$ $v/c \ll 1$

$$\Delta\phi \propto 4\pi n^i n^j \bar{h}_{ij} (L/\lambda) \qquad \omega L/v \gg 1$$

Limited by *shot noise* (need for large flux).

- Low frequency (*freely suspended* mirrors):

$$\Delta\phi \propto -4\pi n^i n^j \bar{h}_{ij} (L/\lambda) \qquad \omega L/v \ll 1$$

Other sources of noise (e.g. *seismic, suspension thermal, gravity gradient*).

Future prospects

- Interesting proposal: Dimopoulos, Graham, Hogan, Kasevich, Rajendran

  Two *atom fountain* interferometers sharing *common lasers* (analogous to a *gradiometer*) separated by a *large* vertical *distance* (several kilometers).

- Effect of GW on laser propagation.

- Similar to LIGO with atom interferometers replacing suspended mirrors.

- Less sensitive to vibration (suspension) noise.

- Could be competitive with laser interferometers?