Conformal Defects in/and String Theory

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based on work with:

J. de Boer, R. Dijkgraaf and H. Ooguri (hep-th/0111210),
M. Gaberdiel (hep-th/0411067),
W. Abou Salem and J. Fröhlich (unpublished),
and in progress.

- In the informal spirit of this workshop, I want to discuss some ideas that are speculative, but (I hope!) interesting. My own published work on this is not new. I will simply try to put it in a different context, discuss some more recent results by other authors, and end with some questionmarks.

- A way to introduce these ideas is as follows: the classical (super)gravity equations admit a set of solution-generating 'symmetries' . These include U dualities, which are relics of spontaneously-broken gauge symmetries, such as large reparametrizations of tori . They include, however, also transformations which change the physical properties of the solution. The simplest example are scale transformations, e.g. for 11D supergravity:

$$g_{\mu\nu} \to \lambda^2 g_{\mu\nu} \ , \quad A_{\mu\nu\rho} \to \lambda^3 A_{\mu\nu\rho} \ , \quad \mathcal{L} \to \lambda^2 \mathcal{L} \ .$$

Flux quantization breaks (in general) this symmetry to a Z^+ subgroup, which survives (in many cases) all other 'quantum corrections'.

- A more intriguing example are the Ehlers-Geroch transformations of pure Einstein gravity, and its extensions to effective supergravities. Assuming e.g. the existence of a Killing vector field ξ^μ, one may define on-shell the complex scalar $z = \omega + i|\xi|^2$, where

$$\xi_\mu \xi^\mu = |\xi|^2 \ , \quad \epsilon^{\mu\nu\rho\sigma}\xi_\nu \nabla_\rho \xi_\sigma = D^\mu_{(h)}\omega \ , \quad \text{and} \quad h_{\mu\nu} = |\xi|^2 (g_{\mu\nu} - \xi_\mu \xi_\nu / |\xi|^2) \ .$$

The remaining equations are then invariant under SL(2,R) transformations of z . This symmetry of the equations becomes infinite - dimensional if there are two Killing vectors. It can be extended easily to effective string-theory actions.

Johnson+Myers, Sen, Bakas, ...

- The question that one may ask is whether such transformations can be defined for the exact string equations of motion. I will now argue that there exist candidate generators, at least in the case of open string theory, the conformal defects. This is one of their possible uses in string theory – the other will come in the end.

• Solutions of open-string theory can be thought of as boundary states that annihilate the (non-anomalous) super-Virasoro algebra:

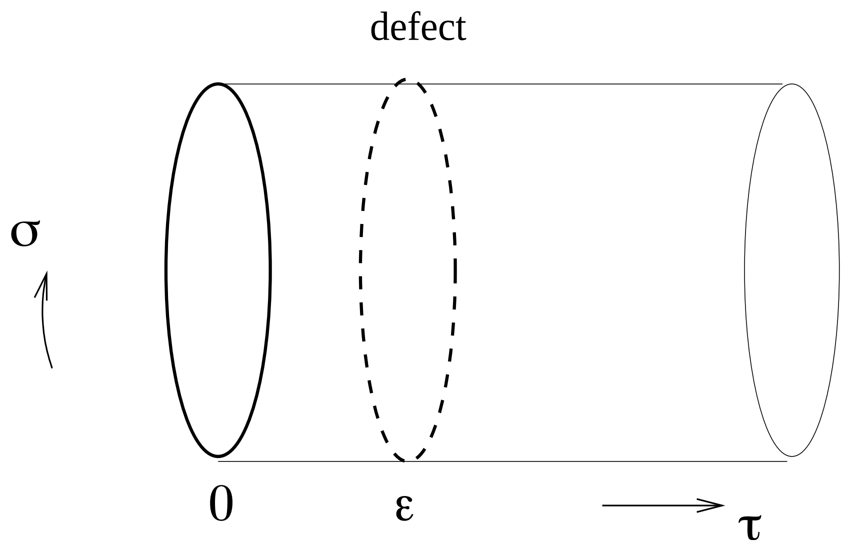
$$(L_N - \bar{L}_{-N})\,|\mathcal{B}\gg \;=\; (G_r - i\bar{G}_{-r})\,|\mathcal{B}\gg \;=\; 0 \qquad \text{for all } N, r \ .$$

In the open channel, this is the condition that no energy and supercharge flow to/from the worldsheet boundary. This must be supplemented by the condition of tadpole cancellation for certain RR fields that correspond to top-forms in the non-compact spacetime. The latter requires in general the introduction of an additional crosscap state $|\mathcal{C}\gg$. Let us for now ignore this complication, and focus on (space non-filling) D-branes, which do not couple to RR top forms.

• Let $\mathcal{O}$ be now an operator acting (formally) on the space of boundary states. For this to be a symmetry of the equations, we must demand:

$$[L_N - \bar{L}_{-N}\ , \mathcal{O}] \;=\; [G_r - i\bar{G}_{-r}\ , \mathcal{O}] \;=\; 0 \ .$$

This is precisely the condition satisfied by conformal defects.



An arbitrary operator $\mathcal{O}$ is not a priori an admissible transformation. We must further require that it be of the following form:

$$\mathcal{O} \;=\; \mathrm{tr}\, P \exp\left(- \int_0^{2\pi} d\sigma\, H_{\mathrm{I}}(\phi, s) \right) \;,$$

corresponding to an impurity which interacts locally with the (worldsheet) bulk fields ϕ. Here the trace is over the (finite-dimensional) space of states s of the defect. Any given impurity Hamiltonian will flow, in principle, in the IR to a conformally- invariant defect. Thus such conformal operators are common.

Conformal defects were first considered in the condensed-matter literature, as impurities of quantum wires *(Fisher + Kane '92)* , or as lines of 'weak links' in the Ising model *(Affleck + Oshikawa '96)*. They are special cases of conformal interfaces, which obey the generalized condition:

$$(L_N^{(1)} - \bar{L}_{-N}^{(1)}) \times \mathcal{O} \;=\; \mathcal{O} \times (L_N^{(2)} - \bar{L}_{-N}^{(2)}) \ .$$

Their algebraic properties have been analyzed by many authors (see later).

In string theory they first appeared as the holographic duals of AdS2 branes *(CB, de Boer, Dijkgraaf + Ooguri '01; Karch + Randall '01; CB '02).*

• General conformal defects are partly-transmitting and partly-reflecting. It has been conjectured by *Quella, Runkel + Watts '06* (and it has been checked in many examples) that

$$\mathcal{T} \leq \frac{2\text{min}(c_1, c_2)}{c_1 + c_2} \quad , \qquad \mathcal{R} \geq \frac{|c_1 - c_2|}{c_1 + c_2} \ .$$

Such defects cannot appear on the worldsheet of physical strings, since they only preserve a subset of the Virasoro charges (the one that leaves their time-like worldlines unchanged).

• The special defects that are purely transmitting are called topological. They obey the more stringent conditions *(Petkova + Zuber '00)*:

$$[L_N, \mathcal{O}_{top}] \ = \ [\bar{L}_{-N}, \mathcal{O}_{top}] \ = \ 0 \quad \text{ for all } \ N \ .$$

An example are chiral conformal defects, that can be pictured as moving on the lightcone. On a Euclidean worldsheet, topological defect lines can be deformed continuously in any arbitrary way, as long as they do not encounter boundaries and/or bulk-operator insertions.

Topological defects <u>can</u> appear on the worldsheet of physical strings. But is this story interesting? (postpone this to the end of the talk).

- Non-interacting defects give the (trivial) operators $n\mathbf{1}$. These correspond to the scale transformations of the dual supergravity solutions. Furthermore, to each $\mathcal{O}_{top}$ one can associate the orientation-reversed defect $\mathcal{O}'_{top}$. It can be also shown, by purely algebraic means, that topological defects obey a fusion algebra *(Petkova + Zuber '00)*:

$$\mathcal{O}^a_{top} \times \mathcal{O}^b_{top} = n^{ab}{}_c \, \mathcal{O}^c_{top} \ .$$

Automorphisms of the operator algebra of the CFT (such as $\phi \to \phi + a$, or $\phi \to -\phi$ for a free scalar field) correspond to invertible defects, which satisfy $\mathcal{O}_{top} \times \mathcal{O}'_{top} = \mathbf{1}$. They form a group that generates real perturbative-symmetry transformations (such as translations or T-dualities of D-branes in some flat dimensions).

Fuchs, Fröhlich, Runkel + Schweigert '05

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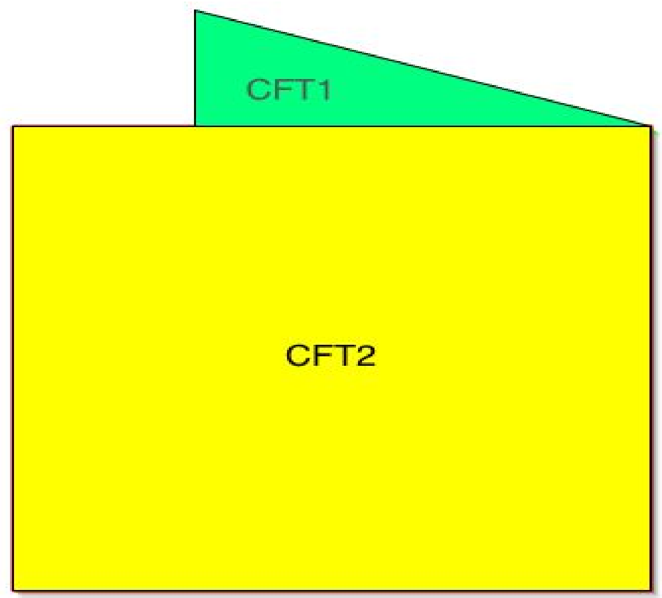
The proposal is that more general conformal defects can be used as solution-generating transformations of open-string theory.

The challenge is to construct explicit examples, and to understand more generally whether/when such transformations are well-defined.

A very useful technical device for constructing conformal interfaces is the folding trick *(Affleck + Oshikawa '96)*. This maps interfaces onto boundary states of the product theory CFT1 $\otimes$ CFT2 (with left- and right-movers in CFT2 interchanged). The operation obviously respects the locality and the unitarity of the underlying theories.

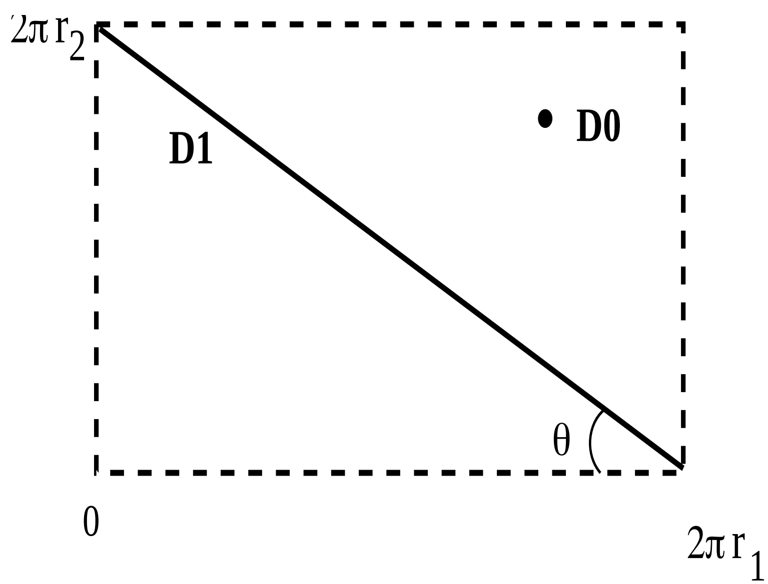


The trivial defect corresponds to a permutation brane, for which:

Left (right) movers of CFT1 $\leftrightarrow$ Right (left) movers of CFT2.

In geometric language this is the diagonally-embedded, middle-dim. D- brane. For a free scalar field, for instance, this brane imposes $\partial_\pm \phi^1 = \partial_\mp \phi^2$. The D-brane is deformed continuously when the volume (or other continuous moduli) are changed in one of the two CFTs. The deformed brane $\mathcal{O}(r_1, r_2)$ is not topological, since $\mathcal{R} = |r_1^2 - r_2^2|/(r_1^2 + r_2^2) \neq 0$.

CB, de Boer, Dijkgraaf, Ooguri



Perturbations of the trivial diagonal brane (with CFT1 = CFT2) are described in the sigma-model approach by the following, renormalizable by power-counting, interactions:

$$\mathcal{O} \;=\; \mathrm{tr}\,\mathsf{P}\exp\left(i\int_0^{2\pi} d\sigma \left[A_M(X)\,\partial_\sigma X^M + \frac{1}{2\pi\alpha'}\,Y_M(X)\,\partial_\tau X^M\right]\right) \;.$$

Here the X^M are the worldsheet fields defined in a target-space $\mathcal{M}$, and A and Y are gauge and coordinate fields of the D-brane. These can be matrix-valued so as to account for Chan-Paton factors. In general, the tachyon field may be also turned on the diagonal brane.

♠ The conformal-invariance conditions of a defect line can thus be derived perturbatively from a (matrix-valued) Dirac-Born-Infeld action, describing the dynamics of a diagonal D-brane in the doubled target space $\mathcal{M}\times\mathcal{M}$.

The action of the defect on a D-brane submanifold, $\mathcal{D} \subset \mathcal{M}$, amounts semi-classically to pulling back the fields A_M and Y_M from $\mathcal{M}$ to $\mathcal{D}$. In the process, half of the fields ($A_\perp$ and $Y_\parallel$) are set as expected to zero.

This adds an entry to the (extensive) dictionary between worldsheet and target-space features, and it gives a geometric interpretation to the solution-generating transformations induced by defects. ♠♠

Note that since defects respect unitarity and locality, there is no need to check the Cardy, or other consistency conditions for the transformed branes. This is not true for the crosscap state, which obeys:

$$(L_N - (-)^N \bar{L}_{-N}) \, |\mathcal{C} \gg \; = \; 0$$

(with similar conditions for the fermionic generators). Although topological defects respect this condition, they violate in general the requirement that in the closed channel of the Klein bottle multiplicities must be ± 1 (so as to project out part of the closed-string spectrum).

This additional requirement is obeyed by defects that act as Z_2 automorphisms of the operator algebra, and (possibly) by interfaces that correspond to allowed deformations of the orientifold theory. Other defects have no consistent action on the crosscap state.

- Besides the free-field examples discussed so far, many conformal defects and interfaces have been constructed in a variety of models in the last few years *(Quella, Fredenhagen, Recknagel, Schomerus, Graham, Watts and others).*

In the rest of this talk I will focus on conformal defects in WZW models, and then discuss the construction of defects that induce the decay of unstable D-branes in bosonic string theory.

WZW defects

D-branes of WZW models are by now very well (though still not completely) understood, both from the algebraic and from the geometric viewpoint.

Cardy; Bianchi, Pradisi, Sagnotti + Stanev

Alekseev, Schomerus+Recknagel; CB, Douglas + Schweigert

Maldacena, Moore + Seiberg; Gaberdiel + Gannon; Gawedzki $\cdots$

The RG flows between symmetric D-branes describe the (partial) screening of magnetic impurities by the electron gas in a metal – the Kondo problem *(Affleck + Ludwig '91)*. In geometric language, it describes the blowing-up of a magnetic brane (dual to the dielectric Myers effect).

Let us now see how these flows can be obtained from the action of topological defects. A similar universality of flows has been first proposed in minimal models *(Graham + Watts '03)*.

♠ The existence of conformal defects in WZW models can be inferred from a semiclassical argument, similar to the one used by Witten '84 to infer conformal invariance in the bulk. One starts with the classical currents

$$J(x^+) = -i\kappa\,(\partial_+ g)g^{-1} \quad \text{and} \quad \bar{J}(x^-) = i\kappa\, g^{-1}\partial_- g\ ,$$

where $x^\pm = \tau \pm \sigma$, the $g(x^+, x^-)$ take values in a Lie group G, and $\kappa = \psi^2 k/2$ with ψ the length of long roots and k the level of the current algebra. The currents generate the left and right symmetry transformations

$$g \to u(x^+)^{-1}\, g\, \bar{u}(x^-)\ ,$$

under which they themselves transform in the same way as the components of a 2D gauge field:

$$J \to u^{-1} J u + i\kappa\, u^{-1}\partial_+ u \qquad \text{and} \qquad \bar{J} \to \bar{u}^{-1}\bar{J}\bar{u} + i\kappa\, \bar{u}^{-1}\partial_- \bar{u}\ .$$

Thus the following 'Wilson loops' [with t^a the Lie algebra generators in the representation R] will be invariant under <u>all</u> the symmetry transformations:

$$\mathcal{O}_{\text{chir}}(\lambda; R) = \text{Tr}_R\, \text{P} \exp\left(i\lambda \oint_C dx^+\, J^a t^a\right) \;,$$

if we choose $\boxed{\lambda = \lambda^* \equiv -1/\kappa}$. Note that $(A_+, A_-) = \lambda(J^a t^a, 0)$ is a flat connection for any value of λ, so that $\mathcal{O}_{\text{chir}}(\lambda; R)$ is always topological at the classical level [the contour can be deformed by the non-abelian Stokes theorem] . But this does not survive renormalisation, which introduces (through dimensional transmutation) a length scale. For $\lambda = \lambda^*$ on the other hand:

$$\{J^a_n\,,\, \mathcal{O}_{\text{chir}}(\lambda^*; R)\} = \{\bar{J}^a_n\,,\, \mathcal{O}_{\text{chir}}(\lambda^*; R)\} = 0\;.$$

If these relations survive quantization, then the above special Wilson loops will describe topological defect lines. [In the classical theory they measure the monodromies of a solution.]

A similar semiclassical argument helps us identify also a class of conformal (but not topological) defects, by considering the more general 'Wilson lines'

$$\mathcal{O}(\lambda, \bar{\lambda}; R) = \mathrm{Tr}_R \, \mathrm{P} \exp \left(i \int_0^{2\pi} d\sigma \, (\lambda \, J^a \; - \; \bar{\lambda} \, \bar{J}^a) \, t^a \right) \; .$$

For $\boxed{\lambda = \bar{\lambda} = \lambda^*/2}$ these are invariant under (vector-like) transformations, i.e. transformations with $u(x) = \bar{u}(-x)$. It follows that

$$\left\{ J_n^a + \bar{J}_{-n}^a \, , \, \mathcal{O}\left(\frac{\lambda^*}{2}, \frac{\lambda^*}{2}; R \right) \right\} \; = \; 0 \quad .$$

If these relations survive quantization, they would imply that $\mathcal{O}(\lambda^*/2, \lambda^*/2; R)$ are (G-symmetric) conformal defects. As we shall see, they correspond to unstable fixed points of the RG flow.

In order to construct the quantum defects *(CB + Gaberdiel '04)* we start with the formal expression

$$\mathcal{O}_{\mathsf{chir}}(\lambda; R) = \sum_{N=0}^{\infty} (i\lambda)^N \, \mathcal{O}^{(N)}(R) \ ,$$

where

$$\mathcal{O}^{(N)}(R) = \mathsf{Tr}_R \left(t^{a_1} \cdots t^{a_N}\right) \left(\prod_{i=1}^{N} \int_0^{2\pi} d\sigma_i\right) \theta_{\sigma_1 > \cdots > \sigma_N} \, J^{a_1}(\sigma_1) \cdots J^{a_N}(\sigma_N) \ .$$

Classically the order of the currents is irrelevant, but in the quantum theory there is an ambiguity due to the short-distance singularities of the OPE.

• To guide the choice, we insist that the following two symmetries be preserved: (i) the path can start at any point σ_0 on the circle, and (ii) the result is invariant if the loop orientation is reversed, and R is traded for its conjugate representation. These symmetries can be preserved by the following (non-unique) regularization prescription:

$$\mathcal{O}^{(N)}_{\text{reg}}(R) = \mathrm{Tr}_R\,(t^{a_1} \quad \cdots t^{a_N}) \left(\prod_{i=1}^{N} \int_0^{2\pi} d\sigma_i\right)\,\theta_{\sigma_1 > \cdots > \sigma_N} \times$$
$$\times \frac{1}{2N}\left(J^{a_1}_{\text{reg}}(\sigma_1) \cdots J^{a_N}_{\text{reg}}(\sigma_N)\ + \text{cyclic}\ + \text{reversal}\right)\ ,$$

where

$$J^a_{\text{reg}}(\sigma) = \sum_{n \in \mathbf{Z}} J^a_n\, e^{-in\sigma - |n| s/2} \quad .$$

Note (i) that since the bare currents at non-coincident points commute, the choice of ordering is part of the regularisation prescription and (ii) that the prescription guarantees that $\mathcal{O}^{(N)}_{\text{reg}}(R)$ commutes with the generator $L_0 - \bar{L}_0$. Thus, even without being topological, it can be transported to the boundary of the half-cylinder freely.

Plugging the mode expansion (with $\tilde{J}^a_n \equiv J^a_n\, e^{-|n|s/2}$) and performing explicitly the integrals leads to the following expressions for the first few values of N :

$$\mathcal{O}^{(2)}_{\rm reg}(R) = 2\pi^2\ {\rm Tr}_R(t^a t^b)\ J^a_0 J^b_0\ ,$$

$$\mathcal{O}^{(3)}_{\rm reg}(R) = \frac{2\pi^2}{3}\ {\rm Tr}_R(t^a t^b t^c)\left[\frac{\pi}{3}\ J^a_0 J^b_0 J^c_0 + \sum_{n\neq 0}\frac{i}{n}\tilde{J}^a_{-n}\tilde{J}^b_n J^c_0\ + {\rm cyclic} + {\rm reversal}\right]\ ,$$

$$\begin{aligned}\mathcal{O}^{(4)}_{\rm reg}(R) \ &=\ \frac{\pi^2}{2}\ {\rm Tr}_R(t^a t^b t^c t^d)\left[\frac{\pi^2}{6}\ J^a_0 J^b_0 J^c_0 J^d_0 + \sum_{n\neq 0}\frac{i\pi}{n}\tilde{J}^a_{-n}\tilde{J}^b_n J^c_0 J^d_0\right.\\ &\quad + \sum_{n\neq 0}\frac{1}{n^2}\left(\tilde{J}^a_{-n}\tilde{J}^b_n J^c_0 J^d_0 - \tilde{J}^a_{-n} J^b_0 \tilde{J}^c_n J^d_0\right) + \sum_{\substack{m,l,n\neq 0\\ m+n+l=0}}\frac{1}{ml}\,\tilde{J}^a_m \tilde{J}^b_n \tilde{J}^c_l J^d_0\\ &\quad \left. -\frac{1}{2}\sum_{m,n\neq 0}\frac{1}{mn}\tilde{J}^a_{-n}\tilde{J}^b_n \tilde{J}^c_{-m}\tilde{J}^d_m\ +\ {\rm cyclic} + {\rm reversal}\right].\end{aligned}$$

After normal ordering, *i.e.* moving all positive modes to the right of negative modes, we get :

$$\mathcal{O}^{(2)}_{\text{reg}}(R) = 2\pi^2 I_R\, J^a_0 J^a_0\,,$$

$$\begin{aligned}\mathcal{O}^{(3)}_{\text{reg}}(R) \;&=\; \frac{2\pi^3}{3}\, I^{(3)}_R\, d^{abc}\, J^a_0 J^b_0 J^c_0 \;+\; 4\pi^2 I_R\, f^{abc} \sum_{n>0} \frac{1}{n}\, J^a_{-n} J^b_0 J^c_n \;- \\ &- 4\pi^2 i\; I_R\, h^\vee\, \psi^2 \left[\, \sum_{n>0} \frac{1}{n} J^a_{-n} J^a_n \;-\; \frac{1}{2}\, J^a_0 J^a_0 \left(\sum_{n>0} \frac{e^{-ns}}{n} \right) \;+\; \frac{\kappa}{6}\,\text{dim}(g)\; \left(\sum_{n>0} e^{-ns} \right) \right],\end{aligned}$$

$$\begin{aligned}\mathcal{O}^{(4)}_{\text{reg}}(R) \;&=\; :\mathcal{O}^{(4)}_{\text{reg}}(R): -\, 2\pi^2\, I_R\, h^\vee\, \psi^2\, \kappa \left[\, \sum_{n>0} \frac{1}{n}\, J^a_{-n} J^a_n -\; J^a_0 J^a_0\; \left(\sum_{n>0} \frac{e^{-ns}}{n} \right) + \right. \\ &\left. + \frac{\kappa}{4}\,\text{dim}(G)\; \left(\sum_{n>0} e^{-ns} \right) \right] \;+\; \text{subleading}\,.\end{aligned}$$

where $I_R = C(R) \times \text{dim}(R)/\text{dim}(G)$, $\text{Tr}_R\,(t^a t^b t^c) = \frac{i}{2}\, f^{abc}\, I_R + \frac{1}{2}\, d^{abc}\, I^{(3)}_R$, and h^V is the dual Coxeter number.

• We can absorb all divergences at this order with the help of the two local counterterms (a mass and coupling-constant renromalization):

$$\int_0^{2\pi} d\sigma(\Delta m + i\Delta\lambda\, J^a t^a)\ .$$

These are the only relevant operators consistent with the global G symmetry of the problem. The explicit form of these renormalizations is:

$$\Delta m = \pi\, C(R)\, h^\vee\, \psi^2 \left(\frac{1}{3}\kappa\lambda^3 + \frac{1}{4}\kappa^2\lambda^4 + \text{subleading}\right) \times \frac{1}{s}\ ,$$

$$\lambda_{\text{eff}} = \lambda + \frac{1}{2}(\lambda^2 + \kappa\lambda^3)\,\xi + \frac{1}{4}\lambda^3\xi^2 + O(\lambda^4)\ ,$$

where $\xi = h^\vee\psi^2 \log s$. Note that λ_{eff} is independent of the representation R. Note also that s is the ratio of the (only) two length scales in the problem: the short-distance cutoff and the circumference L of the cylinder.

The β-function of the chiral defect thus reads:

$$\beta(\lambda_{\text{eff}}) = -\frac{d\lambda_{\text{eff}}}{d\log s} = -\frac{1}{2}h^{\vee}\psi^2\left(\lambda_{\text{eff}}^2 + \kappa\lambda_{\text{eff}}^3 + O(\lambda_{\text{eff}}^4)\right) .$$

It is asymptotically free, and has an infrared fixed point at the critical value

$$\lambda^* = -\frac{1}{\kappa} + O\left(\frac{1}{\kappa^2}\right) .$$

This is perturbatively-small for large κ. If one brings this defect to a D0-brane boundary, one recovers the RG flow of the Kondo problem (derived in a non-conventional way).

The basic point, however, is that this is a universal RG flow, applicable to any UV fixed point.

More generally, the fusion of these topological defects, among themselves and with the Cardy boundary states, is the same as the fusion of primary fields. For instance, in the case $G = SU(2)$:

$$j \otimes j' = |j - j'| \oplus \cdots \oplus \mathrm{max}(j + j', k - j - j') \ .$$

This can be verified explicitly from the above, and follows also from more formal arguments of *Petkova + Zuber*, or by lifting to a TFT in 3D *(Fröhlich, Fuchs, Runkel + Schweigert)* .

At level $k = 1$, the $j = 0$ and $j = 1/2$ Cardy branes are the D0 and D1 branes on the $r = 1$ circle. The $j = 1/2$ defect is in this case a symmetry transformation that maps one D-brane to the other.

<u>NB1</u>: The topological defects constructed here are central elements in the envelopping current algebra. Their existence <u>to all higher orders</u> (an open mathematical problem) has been proved recently by *Alekseev + Monnier '07*.

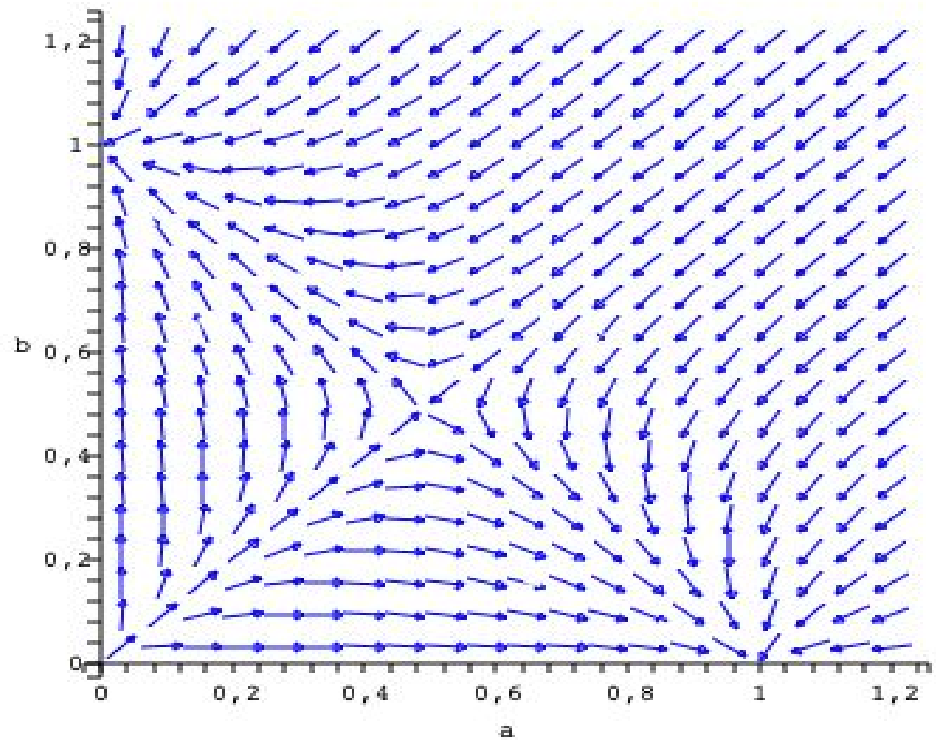
NB2: A nice alternative argument showing that $\lambda^{-1} = k+2$ is a fixed point is due to *Affleck*: the impurity Hamiltonian can in this case be absorbed through the redefinition $J_n^a \to J_n^a + t^a$, which preserves the current algebra.

NB3: According to the semiclassical argument, WZW models also have conformal but not topological defects. The leading-order renormalizations in the general case can be computed in the same way as above:

$$\begin{aligned} \lambda_{\text{eff}} &= \lambda + \frac{1}{2}\xi\,\left(\lambda^2 + \kappa(\lambda^3 + \lambda\,\bar\lambda^2)\right) + \cdots \\ \bar\lambda_{\text{eff}} &= \bar\lambda + \frac{1}{2}\xi\,\left(\bar\lambda^2 + \kappa(\bar\lambda^3 + \bar\lambda\,\lambda^2)\right) + \cdots\ . \end{aligned}$$

and lead to the following flow diagram:

The conformal defect respecting the diagonal-current symmetry can be seen to be an unstable fixed point :



NB: Its action on the WZW branes is not known.

• As another non-trivial example, consider the decay of an unstable Dp-brane to a D(p-1)-brane in the bosonic string. According to Sen this is described by a tachyon-lump background on the Dp-brane. Alternatively, he described it by the following sequence of marginal bulk and boundary transformations [only one coordinate plays here a role]:

• Change the radius $r \to 1$

• SU(2)-rotate the Neumann to the Dirichlet brane

• Change back the radius $1 \to r$

This suggests the following composition of conformal interfaces and defects:

$$\mathcal{O}_{\mathsf{lump}} = \mathcal{O}(r,1) \times \mathcal{O}_{\mathsf{chir}}(j = 1/2) \times \mathcal{O}(1,r) \ .$$

The product is here formal, since the fusion of general conformal defects is singular. It should be understood in a similar way as the OPE of local operators. Defining these singular products is a very interesting open problem [see also Kapustin + Witten '06].

In the simplest case of $\mathcal{O}(r_1, r_2) \times \mathcal{O}(r_2, r_1)$ the leading term in the product can be extracted from the 'striped' torus amplitude:

CB, de Boer, Dijkgraaf, Ooguri

$$\mathcal{N}^2 \prod_{n=1}^{\infty} [1 - (q_1^{2n} + q_2^{2n})\cos^2 2\theta - 2q_1^n q_2^n \sin^2 2\theta + q_1^{2n} q_2^{2n}]^{-1}$$

where $\tan\theta = r_1/r_2$, $q_1 = e^{-2\pi d/T}$ and $q_2 = e^{-2\pi(L-d)/T}$. In the limit $d \to 0$ (when the two interfaces collapse) the result reads:

$$\mathcal{N}^2 \prod_{n=1}^{\infty} \sin^2 2\theta (1 - q_2^n)^{-2}$$

which is the expected (closed-string) torus amplitude, up to a (vanishingly small) normalization constant. ζ-function regularization gives:

$$\mathcal{O}(r_1, r_2) \times \mathcal{O}(r_2, r_1) \ \simeq \ (\sin^2 2\theta)^{\zeta(0)} \, \mathbf{1} + \ \cdots$$

Summary

• Conformal defects and interfaces have the potential to unify all symmetry tranformations of the field equations for open strings.

• They also have a plethora of condensed-matter-physics applications.

• Their fusion rules and their realization in open-string field theory deserve, further study.