Modifications of gravity

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Tensor-Vector-Scalar theory

(Sanders 1997, Bekenstein 2004)

Ingredients

- Tensor field (metric) $\tilde{g}_{ab}$
- Unit-timelike Vector field A_a
- Scalar field ϕ

Physical metric

$$g_{ab} = e^{-2\phi}(\tilde{g}_{ab} + A_a A_b) - e^{2\phi} A_a A_b$$

- toy-theory (phenomenological)
- gives MOND in the non-relativistic limit
- good platform for studying alternatives to ΛCDM

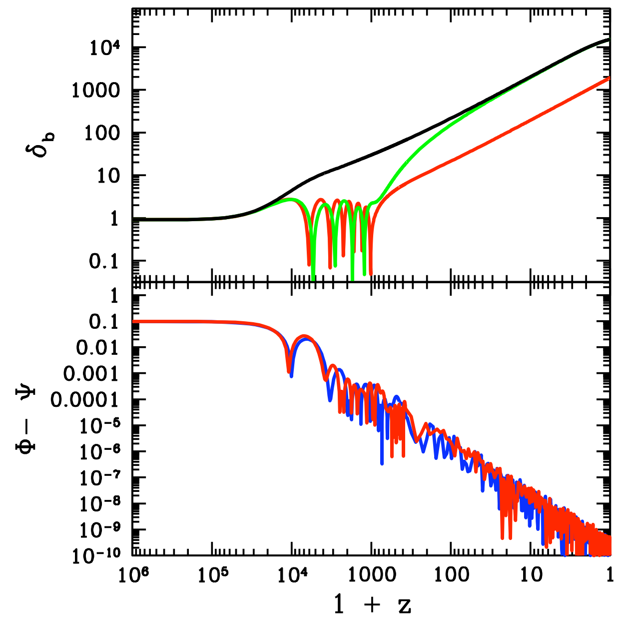
See topical review in CQG (C.S. on arXiv next week)

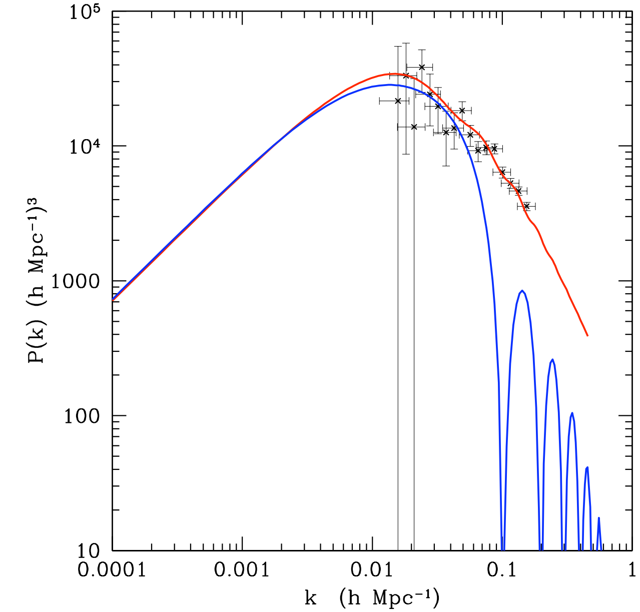
Growth of structure in CDM and TeVeS

ΛCDM

growth in CDM sources baryons

$\Phi - \Psi$ small



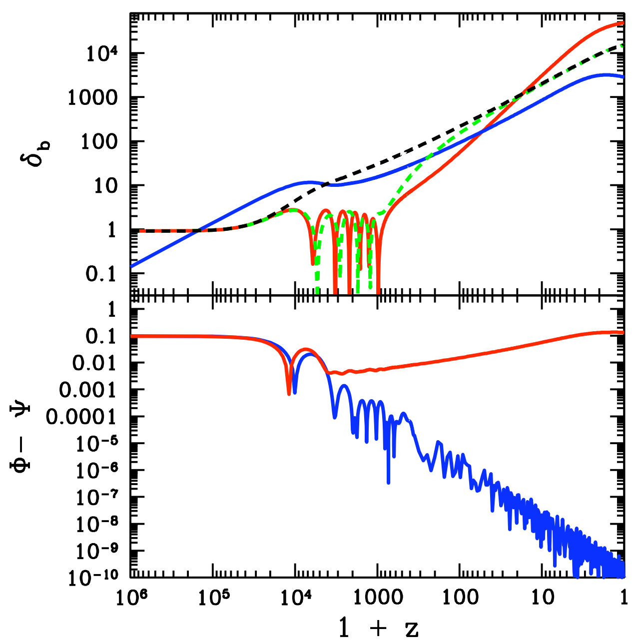


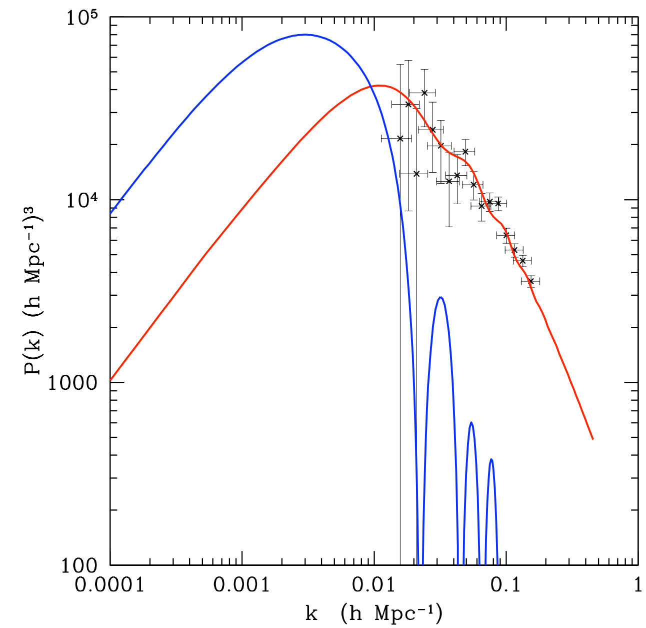
TeVeS

$$A_i = \vec{\nabla}_i \alpha$$

growth in vector sources baryons

$\Phi - \Psi$ large





(C.S., D. Mota, P. Ferreira, C.Boehm, 2005) (S.Dodelson & M.Liguori, 2006)

Lessons from TeVeS

- gravity may depend on additional fields
- these fields may mimic dark matter
- however, some other effect different from dark matter may appear

example :

Both ΛCDM and TeVeS give same $P(k)$

But $\Phi - \Psi$ is very different

Can be distinguished from combinations of other observables

How special is General Relativity?

- A principle theory (SEP, diffeo-invariance)
- Lovelock-Grigore theorem : In any dimension, the only local diffeomorphism invariant action leading to 2nd order field equations and which depends only on a metric is a linear combination of the E-H action with a cosmological constant up to a total derivative.

Any other theory must : (at least one applies)

- be non-local (Sousa-Woodard, Dvali-Gabadaze-Porrati, etc)
- Have absolute elements (stratified theories)
- Depend on other fields (JFBD, TeVeS, etc)
- have higher than 2nd order field equations (Weyl gravity)

What is/not gravity?

- "f(R) and scalar-tensor theories are not modified gravity because they are equivalent to GR + scalar"
- "Scalar-tensor and TeVeS theories are not modified gravity because they depend on extra fields"
- TeVeS is not modified gravity because it can be written in a single-metric frame without coupling to matter.

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\, K^{abcd}(g, B_c)\nabla_a B_b \nabla_c B_d + S_m[g]$$

- "If $\Phi - \Psi \neq 0$ then we have modified gravity"

these are certainly all incorrect statements

Cosmological tests of gravity

How do we treat complicated theories of gravity?

$$f_{ab}\left[g_{cd}, Riem, Ric, R, \phi^A, \ldots\right] = 8\pi G h_{ab}\left[T_{cd}, g_{cd}, Ric, R, \phi^A, \ldots\right]$$

metric (→ g_{cd}) curvature (→ $Riem$) extra fields (→ ϕ^A) matter (→ T_{cd})

Trick : add G_{ab} to both sides

add and subtract $8\pi G T_{ab}$ on RHS

Collect terms $G_{ab} = 8\pi G T_{ab}^{(known)} + U_{ab}$

where $U_{ab} = 8\pi G\left[h_{ab} - T_{ab}\right] + G_{ab} - f_{ab}$

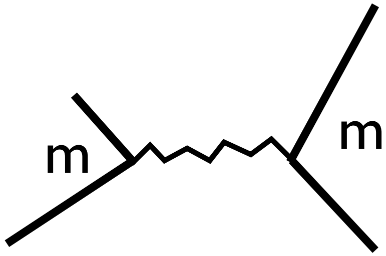
Bianchi identity : $\nabla_a U^a{}_b = 0$ ➤ Field equations for ϕ^A

C.S. (arXiv:0806.1238)

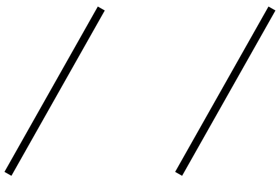
What is gravity

"The natural phenomenon of attraction between physical objects with mass or energy."

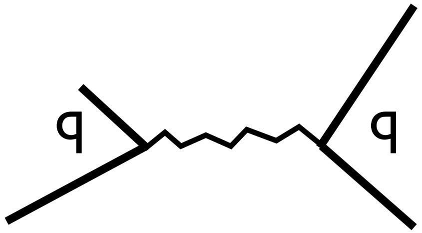
Gravity: generated by mass, affects mass



Fluid: contributes only to total energy density



Other force: generated by/affects other charges



To distinguish gravity from fluids or other forces we must specify the field content

Can we distinguish modified gravity from matter at the FRW level?

FRW $ds^2 = -dt^2 + a^2 d\ell_K^2$

$$G^0{}_0: \quad 3H^2 + \frac{3K}{a^2} = 8\pi G \sum_i \rho_i + X$$

$$G^i{}_i: \quad -2\frac{\ddot{a}}{a} - H^2 = 8\pi G \sum_i P_i + Y$$

Bianchi identity gives $\dot{X} + 3H(X+Y) = 0$

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the answer is therefore : NO

Linear perturbation level

C.S. (arXiv:0806.1238)

Metric has 4 scalar dof : Ψ , ζ , Φ, ν

Given a vector field ξ^a s.t. $\xi_\mu = a(-\xi, \vec{\nabla}_i \psi)$

reduced to 2 by gauge transformations

Distinguishing gravity from fluids : field content

- Linearized equations must be gauge form-invariant.

$$\delta G^\mu{}_\nu = \sum_i \mathcal{O}_i \Delta_i \to \sum_i \mathcal{O}_i \Delta_i + [FRW eq.]\xi$$

  - Holds iff background equations satisfied.
  - Fixes all gauge non-invariant terms

- Bianchi identity holds (local energy conservation)

Parameterizing field equations

$$2(\vec{\nabla}^2 + 3K)(\Phi - \vec{\nabla}^2\nu) - 6\frac{\dot{a}}{a}(\dot{\Phi} - \frac{1}{3}\vec{\nabla}^2\zeta) - 6\frac{\dot{a}^2}{a^2}\Psi = 8\pi G a^2 \rho\delta$$

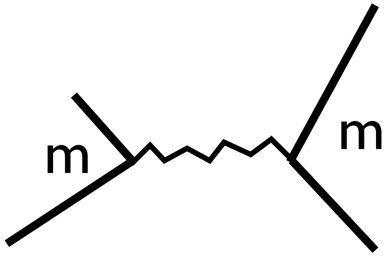
gauge ↓ transform

$$2(\vec{\nabla}^2 + 3K)(\Phi' - \vec{\nabla}^2\nu') - 6\frac{\dot{a}}{a}(\dot{\Phi}' - \frac{1}{3}\vec{\nabla}^2\zeta') - 6\frac{\dot{a}^2}{a^2}\Psi' = 8\pi G a^2 \rho\delta' + [FRW eq.]\xi$$

- Parameterizations in a fixed gauge are inconsistent
- Parameterizations using gauge invariant combinations are consistent but may be too arbitrary (from Stewart-Walker lemma)
- All parameterizations must take into account the field content.

Distinguishing gravity from fluids at the linear level

- Must specify field content
- Specify the parameterization
- Determine the force between 2 well separated masses in vacuum



This requires writing an action leading to the parameterized equations

The extended ΛCDM model

C.S. (arXiv:0806.1238)

Background : ΛCDM
No additional fields
No higher than 2 time derivatives

→ No Gauge Non-Invariant terms allowed

$\delta U^a{}_b$ contains Φ_{GI} Ψ_{GI} and derivatives

$$\Psi_{GI} = \Psi - \ddot{\nu} - \dot{\zeta} - \frac{\dot{a}}{a}(\dot{\nu} + \zeta)$$ contains 2nd derivatives

$$\Phi_{GI} = \Phi + \frac{1}{3}\vec{\nabla}^2\nu + \frac{\dot{a}}{a}(\dot{\nu} + \zeta)$$ contains 1st derivatives

Constructing the $U-$tensor

$$G_{ab} = 8\pi G T_{ab}^{(known)} + U_{ab}$$

Constraints : 1st derivatives

$$\delta U^0{}_0 = \mathcal{A}\Phi_{GI}$$

$$\delta U^0{}_i = \mathcal{B}\Phi_{GI}$$

Propagation : 2nd derivatives

$$\delta U^i{}_i = \mathcal{C}_1\Phi_{GI} + \mathcal{C}_2\dot{\Phi}_{GI} + \mathcal{C}_3\Psi_{GI}$$

$$\delta U^i{}_j - \frac{1}{3}\delta U^k{}_k\delta^i{}_j = \mathcal{D}_1\Phi_{GI} + \mathcal{D}_2\dot{\Phi}_{GI} + \mathcal{D}_3\Psi_{GI}$$

Bianchi identity gives

$$\mathcal{C}_3 = \mathcal{D}_3 = 0 \qquad \mathcal{A} = -\frac{\dot{a}}{a}\mathcal{C}_2 \qquad \mathcal{B} = \frac{1}{3}\mathcal{C}_2 + \frac{2}{3}\left(\vec{\nabla}^2 + 3K\right)\mathcal{D}_2$$

$$\dot{\mathcal{A}} + \frac{\dot{a}}{a}\mathcal{A} - \vec{\nabla}^2\mathcal{B} + \frac{\dot{a}}{a}\mathcal{C}_1 = 0 \qquad \dot{\mathcal{B}} + 2\frac{\dot{a}}{a}\mathcal{B} - \frac{1}{3}\mathcal{C}_1 - \frac{2}{3}\left(\vec{\nabla}^2 + 3K\right)\mathcal{D}_1 = 0$$

Corollary I: If $\mathcal{D}_1 = \mathcal{D}_2 = 0$

Then $U^a{}_b = 0$ no shear : no modification

Corollary II: If $\mathcal{A} = \mathcal{B} = 0$

Then $U^a{}_b = 0$ no constraints : no modification

Corollary III: If $\mathcal{D}_2 = \mathcal{B} = 0$ i.e. $\Phi_{GI} - \Phi_{GI} = \mathcal{D}_1\Phi_{GI}$

Then $U^a{}_b = 0$ no modification

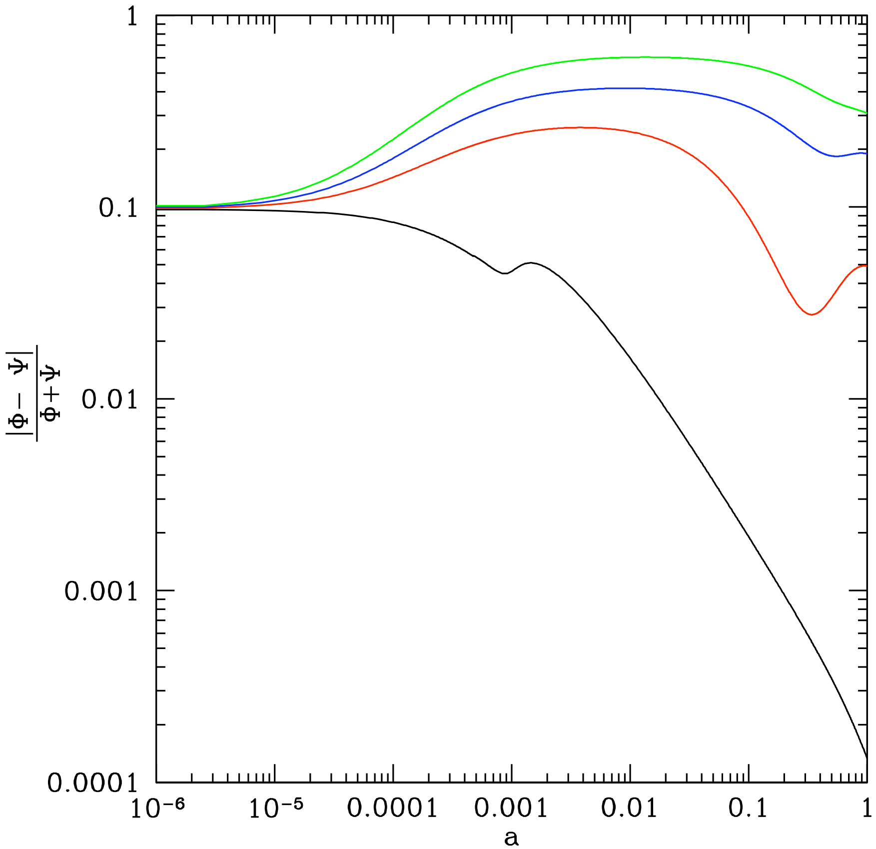
A simple model : $\Phi_{GI} - \Psi_{GI} = \mathcal{D}_2 \dot{\Phi}_{GI}$

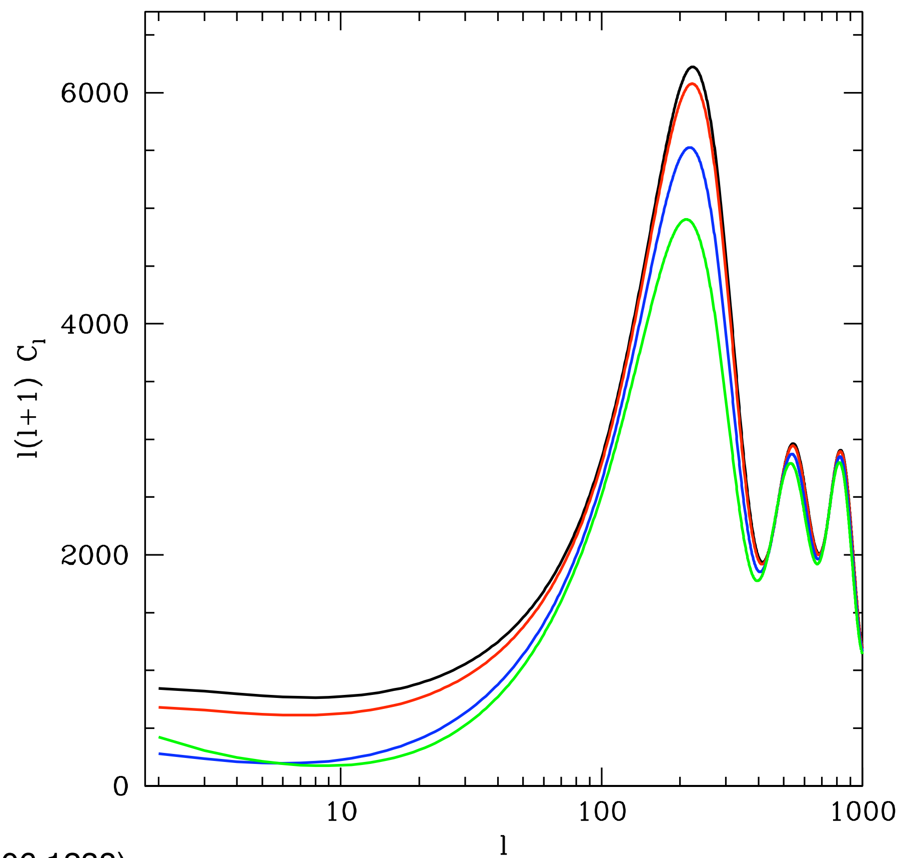
$$\mathcal{B} = \mathcal{C}_1 = \mathcal{D}_1 = 0$$

$$\mathcal{A} = \frac{\beta H_0^2}{a} \qquad \mathcal{C}_2 = -\frac{\beta H_0^2}{\dot{a}} \qquad \mathcal{D}_2 = \frac{\beta H_0^2}{2\dot{a}} \frac{1}{\vec{\nabla}^2}$$

new parameter β

non-locality





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Further consistency requirements

(in progress)

- Action for parameterized perturbed cosmological equations
- Quantize on de Sitter
- Eliminate ghosts : Should impose further constraints on the allowed terms
- Initial conditions : e.g. Inflation
- Modified gravity at the non-linear level (non-linear completion)

The end

- Detecting $\Phi - \Psi \neq 0$ not enough
- Parametrizing only in terms of the potentials can ignore important physics. Gravity may depend on additional fields.
- Constraints depend on the field content
- Field content important for distinguishing gravity from fluids.