Supersymmetric Wilson-loops in N=4 and their correlators

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Based on
A. Bassetto, L. Griguolo, F. Pucci and D.S. airXiv:0804.3973 [hep-th], JHEP 0806:083 (2008); D. Young, arXiv:0804.4098, JHEP 0805:077 (2008)
and on
A. Bassetto, L. Griguolo, F. Pucci, D.S., S-. Thambyahpillai and D. Young in preparation

Outline

- Supersymmetric Wilson loops:
  - Globally supersymmetric Wilson loops
  - Some examples and their solvability (the "circular" Wilson loop)
  - The emerging of a topological twist
- Supersymmetric Wilson-loops on S^3
  - Wilson loop on S^2 and the corresponding topological twist
  - The conjectured relation with two-dimensional YM
- Perturbative test of the relation with YM_2
- Correlators of Wilson-loops on S^2 :
  - Extending the relation with YM_2 to Correlators
  - Strong Coupling test of the relation
  - Weak coupling test of the relation

Motivations

- In the context of AdS/CFT Wilson loops are closely related to the fundamental string: in $AdS_5 \times S^5$ they are described by open strings ending along a curve on the boundary ⟹ at large λ Wilson-loops are described by classical strings

- When the Wilson-loop is endowed with a certain amount of SUSY, it could be used to check strong coupling results: suprisingly there are examples of loops that seems exactly computable as non trivial function of g^2 *and N: Correspondence beyond the leading order*

- Recently Wilson-loops appeared in a related but different (and maybe more intriguing) context: "N=4 scattering amplitudes". (see previous talk)

Supersymmetric loops: General Features

Apart from the usual coupling with the gauge connection A_μ, an appropriate Wilson loop operator for N=4 SYM generically contains couplings with the scalar fields Φ_I. A simple way to understand the origin of these couplings is to consider the reduction from 10 to 4 dim.

$$\mathcal{W}_{D=10}(C) = \frac{1}{N}\mathrm{Tr}\left[\mathrm{Pexp}\left(i\oint \mathbf{A}_a(X)dX^a\right)\right] \quad a = 0, 1, \ldots, 9$$

Dimensional reduction

$$\mathbf{A}_a = (A_\mu, \Phi_I)$$
$$X^a = (x^\mu, y^I)$$

$$\mathcal{W}_{D=4}(C) = \frac{1}{N}\mathrm{Tr}\left[\mathrm{Pexp}\left(\oint i(A_\mu(x)\dot{x}^\mu(s) + \Phi_I(x)\dot{y}^I(s))ds\right)\right]$$

$x^\mu(s)$ is the actual contour in 4 dimensions, while $y^I(s)$ can be thought of as the extra six coordinates of the ten dimensional space.

Remark: (a) Relations with string theory (AdS/CFT) needs these couplings; (b) Fermion couplings are possible, but we shall not examine this case here; (c) In euclidean the couplings $y^I(s)$ --> i $y^I(s)$. The Wilson loop is no longer a phase.

Locally Supersymmetric loops

By means of the transformations $\delta A_\mu = \psi\Gamma_\mu\varepsilon$ $\delta\Phi_I = \psi\Gamma_I\varepsilon$, we can show that a Wilson loop is locally supersymmetric;

$$\delta_\epsilon W_{D=4}(C) = 0 \quad \text{if} \qquad i\Gamma_\mu \dot{x}^\mu(s)\epsilon + \Gamma_I \dot{y}^I(s)\epsilon = 0$$

which admits non trivial solutions if and only if $\dot{x}^2(s) - \dot{y}^2(s) = 0$

This condition also ensures:

(a) Finiteness at coincident points

(b) Consistency with the AdS/CFT dual picture

The above condition implies the existence of an ε for any fixed s. But ε may change with s. Thus no (global) supersymmetry is generically preserved.

Then, how do we construct supersymmetric Wilson-loop?

- By selecting particular contours
- By choosing the scalar couplings in a suitable way

Let's briefly reviews some known examples:

the infinite straight-line:

The simplest way to realize a super-symmetric Wilson-loop is to consider the infinite straight-line: x(s)=r s+b and y(s)= r s+b ($\frac{1}{2}$ BPS).

It is perturbative trivial: $W_{\mathrm{straight-line}} = 1!$. In fact

$$\langle (iA_1 \cdot r + \Phi_1 \cdot R)(iA_2 \cdot r + \Phi_2 \cdot R)\rangle \propto \frac{R^2 - r^2}{(s_1 - s_2)^2} = 0!$$

SUSY also prevents non-perturbative quantum corrections

Tuning the scalars couplings:

Zarembo has endowed any contour by a certain amount of supersymmetry by choosing the following scalar couplings:

$$y^I(s) = M^I_\mu \dot{x}^\mu(s)$$

M^I_μ is a 3x6 constant matrix, satisfying $\sum_{I=1}^6 M^I_\mu M^I_\nu = \delta_{\mu\nu}$. The number of the preserved supersymmetries depends on the dimension of the subspace where the contour lies

D=1: straight line: 1/2 BPS

D=2: contour in R^2: 1/4 BPS

D=3: contour in R^3: 1/8 BPS

D=4: generic contour in R^4: 1/16 BPS

These loops are again trivial $\mathcal{W}[C]=1!$ both at the perturbative level

$$\langle (iA^a_{1\mu}+M^I_\mu \Phi^a_{1I})(iA^b_{2\mu}+M^I_\mu \Phi^b_{2I})\rangle_0 = \frac{g^2\delta^{ab}(\delta_{\mu\nu}-M^I_\mu M^I_\nu)}{4\pi^2(x_1-x_2)^2}=0.$$

[Zarembo (two-loop computation) and Guralnik and Kulik provided a complete field theoretical proof from loop equations.] This result is consistent also at strong coupling with the ADS/CFT prediction.

Less trivial example: the circular Maldacena-Wilson-loop

A more interesting example of supersymmetric loop was considered by **Erickson, Semenoff , Zarembo hep-th/0003055 and Drukker, Gross: hepth/0010274**

$$\mathcal{W}_M(C)=\frac{1}{N}\mathrm{Tr}\left[\mathrm{Pexp}\left(\oint_C (iA_\mu dx^\mu+\theta_I\Phi^I|\dot{x}|)\right)\right]$$

where C is a circle and we have chosen $\dot{y}^I(s)=|\dot{x}|\,\Theta^I,\ \Theta_I=\mathrm{const.}$ and $\Theta^I\Theta^I=1$

It is ½ BPS, but with respect to the superconformal algebra. In fact

$$\delta_\epsilon W_{D=4}(C)=0 \quad \text{if} \qquad i\Gamma_\mu\dot{x}^\mu(s)\epsilon+\Gamma_I\theta^I|\dot{x}(s)|\epsilon=0,$$

for the conformal killing spinors $\epsilon=\epsilon_0+x^\mu\Gamma_\mu\epsilon_1$ with $\varepsilon_1=-i/R\,\Gamma_{12}\,\theta_I\,\Gamma^I\,\varepsilon_0.$ The other example were annihilited by Poincarè supercharges.

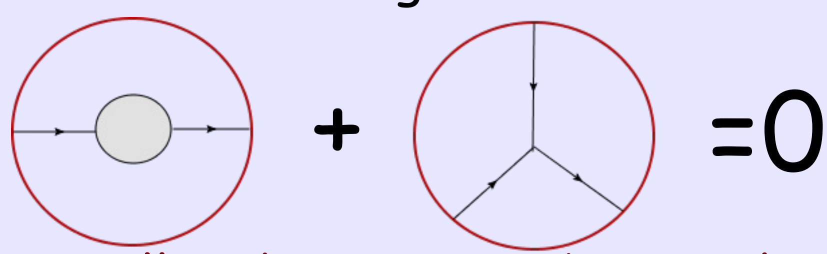
The effective propagator propagator for this loop is constant:

$$\langle (iA^a(t_1) + \theta \cdot \Phi^a(t_1))(iA^b(t_2) + \theta \cdot \Phi^b(t_2))\rangle_0 = \frac{g^2\delta^{ab}}{4\pi^2}\frac{|\dot{x}_1||\dot{x}_2| - (\dot{x}_1 \cdot \dot{x}_2)}{(x_1 - x_2)^2} = \frac{g^2\delta^{ab}}{8\pi^2}$$

By means of simple combinatorial argument all the perturbative diagrams which does not contain interactions are, then, resummed by the matrix model

$$\mathcal{W}_{circle} = \frac{1}{Z}\int dM \frac{1}{N}\mathrm{Tr}\left(e^M\right)\exp\left(-\frac{2}{g^2_{SYM}}\mathrm{Tr}(M^2)\right) =$$
$$= \frac{1}{N}L^1_{N-1}(-g^2_{SYM}/4)\exp(g^2_{SYM}/8)$$

What about interactions? At the order g^4 the different contributions cancels!



If this cancellation extends to all order in perturbation theory, the matrix model is the exact result.

Strong coupling check: At large N $\mathcal{W}_{circle} = \frac{2}{\sqrt{\lambda}}I_1(\sqrt{\lambda}) \overset{\lambda\to\infty}{\simeq} \frac{e^{\sqrt{\lambda}}}{(\pi/2)^{1/2}(\lambda)^{3/4}}$

From ADS/CFT: the minimal surface can be found and the minimal "regularized Area" is -2π ⇒ W behaves as $e^{\sqrt{\lambda}}$. The factor is recovered by taking into account the zero-modes of the fluctuations around the minimal surface.

OK!

Topological Twist

An elegant way to understand the properties of some of these Wilson-loop is to view them as observables in a "topologically twisted theory"

For example, the Zarembo loops with the matrix M^I_μ given by the identity matrix can be viewed as observables in a twisting of N=4, where the SO(4) Lorentz group is enterely embedded in the R-charge group SO(6). After this twisting, we have two scalar supercharges and the loop are invariant under one of these two charges. (Twist 1 in the classification of Vafa-Witten.)

More interesting is the case of the circle. Pestun, formulating N=4 on S^4 and adding a Q-exact term with respect to one of the preserved SUSY (of the Wilson loop), has shown that the path-integral, for this observable, in the zero instanton sector, localizes on configurations with constant fields (bosonic): one scalar and one auxiliary field The matrix model is fully recovered!
He also argues that there are no instantonic correction. (There are previous computations, which appears to contradict this second conclusion [Bianchi, Green, Kovacs]).

Super-symmetric loops on S^3

Another very interesting class of supersymmetric Wilson loops was constructed by Drukker, Giombi, Ricci, Trancanelli [DGRT] (0704.2237, 0707.2699, 07113226). They Consider circuit on the unit sphere S^3 embedded in R^4 and they define the scalar couplings:

$$\dot{y}^I(s) = M^I_i \sigma^i_R \quad \text{with} \quad \sigma^i_R = \sigma^i_{\mu\nu} x^\mu dx^\nu$$

σ^I_R are the right invariant forms on S^3 and $\sigma^I_{\mu\nu}$ are the 't Hooft symbol. M is a 3X6 constant matrix.

$$\delta_\epsilon W_{S^3}(C) = 0 \quad \text{if} \quad i\Gamma_\mu \dot{x}^\mu \epsilon + \Gamma_I M^I_i \sigma^i_{\mu\nu} x^\mu \dot{x}^\nu \epsilon = 0,$$

For a generic contour on S^3 and ε conformal killing spinor $[\epsilon = \epsilon_0 + x^\mu \Gamma_\mu \epsilon_1]$ ⇒

$$\begin{cases} \Gamma_{\mu\nu}\epsilon_1 - iM^I_i \sigma^i_{\mu\nu} \Gamma^I \epsilon_0 = 0 \\ \Gamma_{\mu\nu}\epsilon_0 - iM^I_i \sigma^i_{\mu\nu} \Gamma_I \epsilon_1 = 0 \end{cases}$$

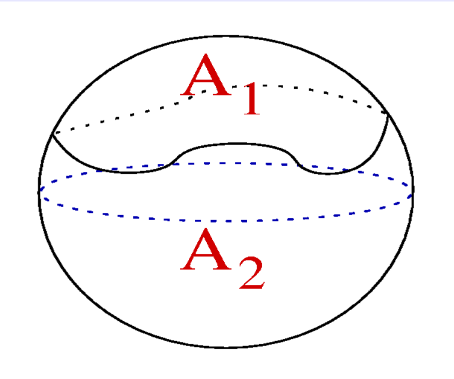
There exists always non trivial solutions:

Generic Loop on S^3: 1/16 BPS; **Generic Loop on S^2:** 1/8 BPS;
Equator: 1/2 BPS; **Latitude:** 1/4 BPS; **Wedge:** 1/4 BPS

We shall focus our attention on the loops on S^2

Loops on S^2

What happens when we try to compute these Wilson loops on S^2 at the lowest order?



The result is largely independent of the shape of the loop. It is determinated by a peculiar combinations of the areas:

$$\mathcal{W}_{S^2} = 1 + g^2_{SYM} N \frac{A_1 A_2}{A^2} + O(g^2_{SYM})$$

The peculiar dependece on the areas is typical of 2D-YM on S^2 and the above result actually reproduces the leading perturbative behavior for a Wilson loop in 2D-YM on S^2 in the "zero-instanton sector" once identifying

$$g^2_{YM_2} = -g^2_{SYM}/A$$

This observation leads DGRT to conjecture that these Wilson loops on S^2 are exactly described by the zero-instanton sector of two-dimensional QCD on S^2, namely by the matrix model:

$$\mathcal{W}(A_1, A_2) = \frac{1}{Z} \int dM \frac{1}{N} \mathrm{Tr}\left(e^M\right) \exp\left(-\frac{A^2}{2g^2_{SYM} A_1 A_2} \mathrm{Tr}(M^2)\right).$$

AdS/CFT correspondence supports this conjecture. For the loops for which the corresponding minimal surface can be computed, the result is consistent with the large N and strong coupling expansion of the above matrix model inspired by 2D-QCD. [For example the computation can be analytically performed for the “wedge”.]

Topological Twist

Also in this case one can argue a relation with a topologically twisted version N=4. The coupling with the scalars in the Wilson loops breaks the original R-symmetry group SU(4) to $SU(2)_A \times SU(2)_B$.

The first $SU(2)_A$ rotates the scalars coupled to the loop, while $SU(2)_B$ the remaining ones. Then the new Lorentz group is $SU(2)_R$ times the diagonal part of $SU(2)_L$ and $SU(2)_A$.

Two supercharges become scalars and closes on an R-symmetry transforma_ tions. (Witten Vafa tipo (ii): difference Poincarè charges versus Superconformal charges.)

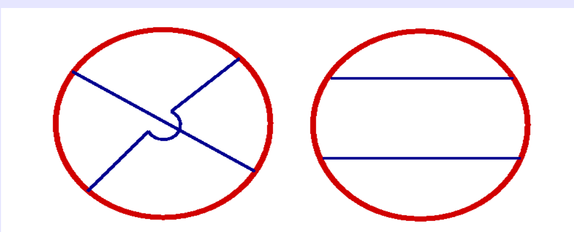
Perturbative Test of the Conjecture

General considerations:

The effective propagator in Feynman gauge and 2ω dimensions is

$$\Delta^{ab}(t_1,t_2)=\delta^{ab}\frac{\Gamma(\omega-1)}{4\pi^\omega}\frac{(\dot{x}_1\cdot\dot{x}_2)[(x_1\cdot x_2)-1]-(x_1\cdot\dot{x}_2)(x_2\cdot\dot{x}_1)}{((x_1-x_2)^2)^{\omega-1}}$$

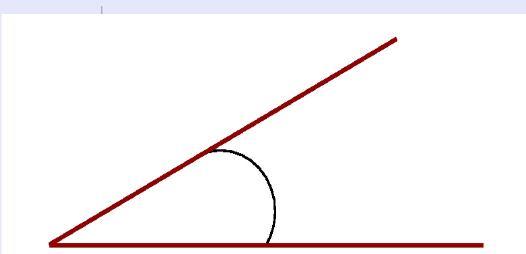
For a generic contour, it does **not** seem to exhibit any special property: e.g. it is not constant.



For example, double exchange diagrams become quite cumbersome and their evaluation is intricate and rarely can be performed analytically.

However the behavior at coincident points is more regular than in the standard case (Maldacena-Wilson loop). The singular behavior is smoothed out even in the presence of a cusp

$x_2 = x_0 + t_2 n_2 + O(t_2^2)$



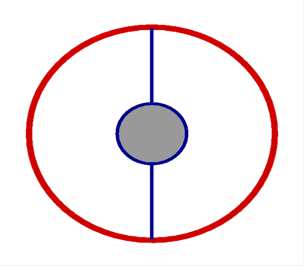
$x_1 = x_0 + t_1 n_1 + O(t_1^2)$

$$\Delta^{ab}(t_1,t_2)\sim\delta^{ab}\frac{\Gamma(\omega-1)}{4\pi^\omega}\frac{2(1-(n_1\cdot n_2)^2)t_1 t_2}{(t_1^2+t_2^2-2t_1t_2(n_1\cdot n_2))^{\omega-1}}.$$

There is no trace of the divergences related to the "celebrated" cusp anomaly.

Interactions: a contour independent analysis

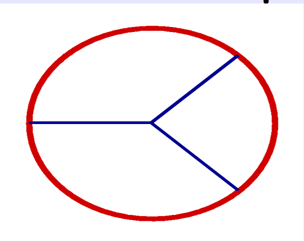
[1.] The bubble diagrams is divergent



$$S_2 = -\frac{g^4(N^2-1)\Gamma^2(\omega-1)}{2^7\pi^{2\omega}(2-\omega)(2\omega-3)}\oint d\tau_1\, d\tau_2 \frac{(\dot{x}_1\cdot\dot{x}_2)[(x_1\cdot x_2)-1]-(x_1\cdot\dot{x}_2)(x_2\cdot\dot{x}_1)}{\left[(x^{(1)}-x^{(2)})^2\right]^{2\omega-3}}$$

<u>Ultraviolet divergence coming from momenta integration</u>

[2.] The "spider" diagrams are also divergent (scalar+gauge contribution):



$$S_3 = \frac{g^4(N^2-1)}{4}\oint dt_1 dt_2 dt_3 \;\; \epsilon(t_1,t_2,t_3)\times$$
$$\times\,[(\dot{x}_1\cdot\dot{x}_3)[(x_1\cdot x_3)-1]-(x_1\cdot\dot{x}_3)(x_3\cdot\dot{x}_1)]\,\dot{x}_2^{\mu}\frac{\partial\mathcal{I}_1(x_3-x_1,x_2-x_1)}{\partial x_3^{\mu}}$$

where $\epsilon(t_{1'},t_{2'},t_3)$ is totally antisymmetric and $\epsilon(t_{1'},t_{2'},t_3)=1$ for $t_1 > t_2 > t_3$ and

$$\mathcal{I}_1(x_3-x_1,x_2-x_1)\equiv\int\frac{d^{2\omega}p_1 d^{2\omega}p_2}{(2\pi)^{4\omega}}\frac{e^{ip_1(x_3-x_1)+ip_2(x_2-x_1)}}{p_1^2p_2^2(p_1+p_2)^2}$$

<u>Divergences in the spider diagram arises in contour integration at coincident points:</u>

e.g. at $x_2=x_3$ since $\mathcal{I}_1(x_3-x_1,x_3-x_1)=\frac{\Gamma^2(\omega-1)}{(2\omega-3)(2-\omega)}\frac{1}{64\pi^{2\omega}\left[(x_1-x_3)^2\right]^{2\omega-3}}.$

One can combine these two results in an expression which is finite at D=4 for any circuit:

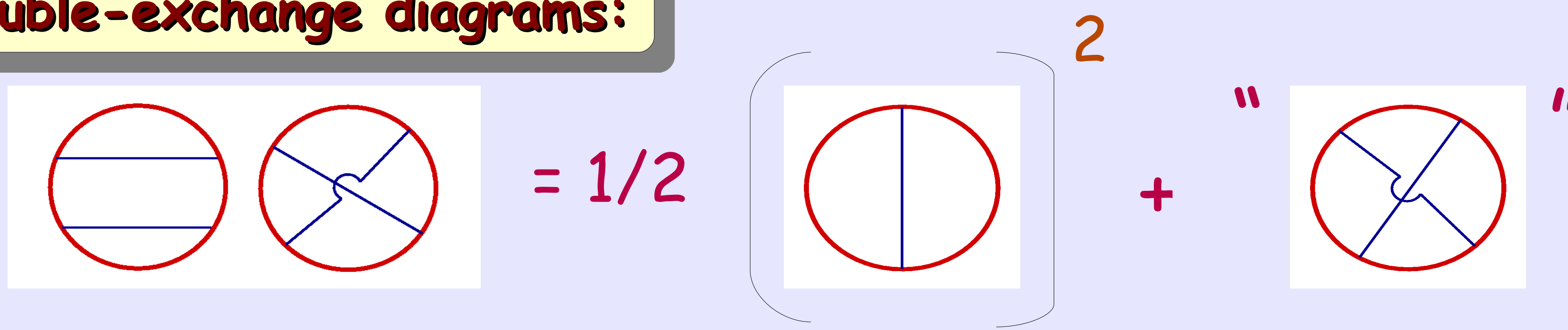
The sum of the two class of diagrams can be then put in the following compact form.

$$\frac{g^4(N^2-1)}{128\pi^4}\oint dt_1 dt_2 dt_3 \epsilon(t_1,t_2,t_3)\frac{(\dot{x}_1\cdot\dot{x}_3)[(x_1\cdot x_3)-1]-(x_1\cdot\dot{x}_3)(x_3\cdot\dot{x}_1)}{(x_3-x_1)^2}\times$$
$$\times\frac{(x_3-x_2)\cdot\dot{x}_2}{(x_3-x_2)^2}\log\left(\frac{(x_2-x_1)^2}{(x_3-x_1)^2}\right)$$

(a) The original power-like singularities has been replace by logarithmic ones, which are integrables.

(b) The interactions does not vanish for a generic loop

Double-exchange diagrams:

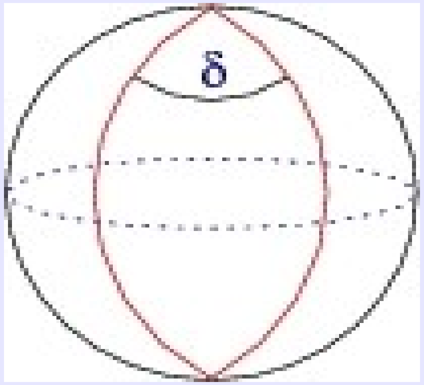


Abelian Maximally Non Abelian

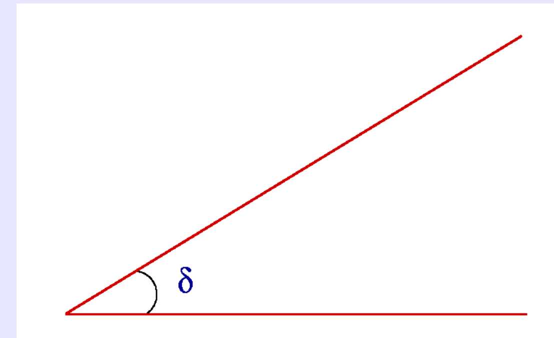
At the end of the day, nothing special seems to happen in general. In order to perform the integrals at least numerically let us choose a particular contour

The wedge

We shall consider a wedge on the sphere:



Stereographic projection



“wedge”

“infinite angle”

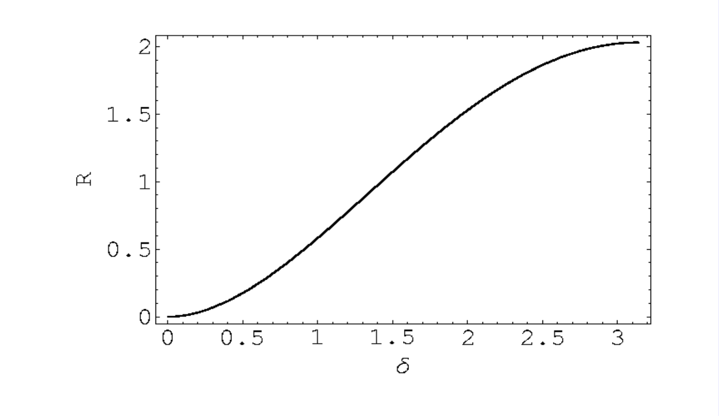
If the conjecture is right, the sum of all the g^4 contributions to the Wilson-loop should be equal to

$$-\frac{g^4(N^2-1)}{384\pi^4}\delta^2(2\pi-\delta)^2$$

We have certain numeber of diagrams to compute and most of them cannot be computed in closed form. We have to sum numerically all the different contributions:

Results:

The numerical analysis produces the following plot for the sum of the maximally non-abelian part and of the interaction diagrams:



Astonishingly the plotted function fitted very well by:

$$-\frac{g^4(N^2-1)}{384\pi^4}\delta^2(2\pi-\delta)^2$$

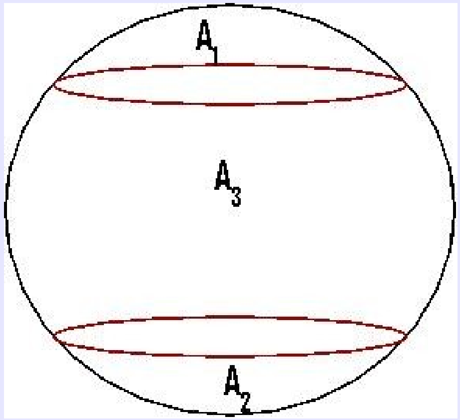
The exact result required by the conjecture!

Extending the conjecture to correlators

If we accept the idea that these Wilson loops are exactly computable because they can be interpreted as an "observable" in a topological sector of N=4 SYM, it is reasonable to expect that the correlators of these Wilson loops are also "observables" in the same sector. Recall in fact that all loops on S^2 share 4 super-conformal charges independently of their shape and position.

Then the trivial extension of the conjecture is to assume that the correlators are described by the zero instanton sector of QCD_2 as well. Then this correlator for two non-intersecting loops is given by the following two-matrix model. ($g^2_{YM_2} = -g^2_{SYM}/A$)

$$W(A_1, A_2) = \int dV_1 dV_2 \exp\left(-\frac{A(A_1+A_3)}{2g^2 A_1 A_3} Tr(V_1^2) - \frac{A(A_2+A_3)}{2g^2 A_2 A_3} Tr(V_2^2) + \frac{A}{g^2 A_3} Tr(V_1 V_2) \right) Tr(e^{V_1}) Tr(e^{V_2})$$



$$\exp\left(-\frac{1}{2g^2 A_1} Tr(V_1^2) \right)$$ Disc $\times$ $$Tr(e^{V_1})$$ Observable $\times$ $$\exp\left(\frac{1}{2g^2 A_3} Tr((V_1 - V_2)^2) \right)$$ Propagator $\times$

$\times$ $$Tr(e^{V_2})$$ Observable $\times$ $$\exp\left(-\frac{1}{2g^2 A_2} Tr(V_1^2) \right)$$ Disc

First Check: Drukker and Gross has shown that the correlator of two coincident circles in N=4 is described by the matrix model

$$W=\int dV\exp\left(-\frac{1}{g^2}Tr(V^2)\right)Tr(e^V)^2$$

This model is immediately recovered when A_3-> 0.

Strong coupling test of the conjecture:

The second test that can be easily done is to compare the proposed matrix model at strong coupling and large N with the AdS/CFT prediction for the correlator

The result for the two-matrix model at large N can be written in a simple and compact Form for SU(N):

$$W(A_1,A_2)-W(A_1)W(A_2)=\frac{1}{N^2}\sum_{k=2}^{\infty}k\left(\sqrt{\frac{A_1A_2}{\tilde{A}_1\tilde{A}_2}}\right)^k I_k\left(2\sqrt{\frac{\lambda A_1\tilde{A}_1}{A^2}}\right)I_k\left(2\sqrt{\frac{\lambda A_2\tilde{A}_2}{A^2}}\right)$$

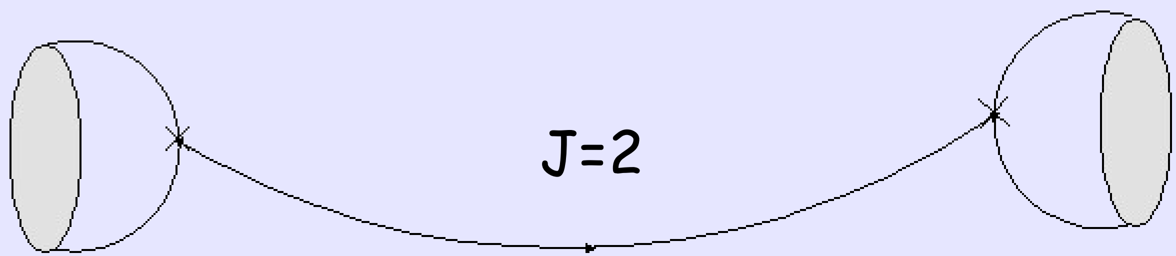
where $\tilde{A}_1=A-A_1$ and $\tilde{A}_2=A-A_2$.

At strong coupling the series is dominated by the first term and we can easily determine its behavior

For two opposite circles of latitude θ_0 and $\pi - \theta_0$, we have the following result

$$\frac{W_{conn.}(A_1, A_2)}{W(A_1)W(A_2)} \approx \frac{\lambda^2}{16N^2} \tan^4(\theta_0)\cos^2(\theta_0)$$

At the level of supergravity, this behavior at large distance (small θ_0 must be reproduced by considering the exchange of the lightest gravitational mode between the two worldsheets, which describe the Wilson loops



The lightest mode corresponds to a scalar propagation with J=2 and performing the computation we get

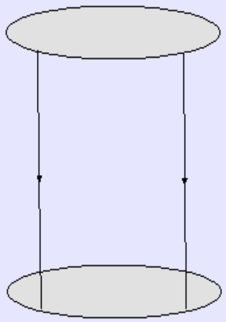
$$\frac{W_{conn.}(A_1, A_2)}{W(A_1)W(A_2)} \approx \frac{\lambda^2}{16N^2} \theta_0^4$$

OK! we find agreement. Adding the mode which corresponds to the next spherical Harmonics on S^5 reproduces also the first subleading term.

Perturbative test of the conjecture

(Preliminary results)

We start by considering the correlators of two latitude on the two-sphere. At the order λ^2 we find perfect agreement with two-dimensional QCD



$$= \frac{g^4}{2}\sin^4\left(\frac{\theta_1}{2}\right)\cos^4\left(\frac{\theta_2}{2}\right)$$

Areal test of the conjecture requires a computation to the order λ^3. However a complete and explicit computations is very hard.

Can we provide a simpler but not trivial test of the conjecture?

Yes, consider this correlator in the limit when one of the circle is shrinked to zero. In this limit, we probe the operator product expansion of the Wilson loop operator and thus we reduce to consider local objects.

Let's see what the OPE implies generically for this correlator?

Generically, we expect an OPE of the form:

$$\frac{W_1}{\langle W_1 \rangle} = 1 + \sum_k c_k r^{\Delta_k} O^{(k)}$$

Here $O^{(k)}$ is a basis a local gauge invariant operator, Δ_k denotes the dimension of the operator $O^{(k)}$, r the radius of the circle and C_k the Wilson coefficients.

$$\frac{\langle W_1 W_2 \rangle}{\langle W_1 \rangle \langle W_2 \rangle} = 1 + \sum_k c_k r^{\Delta_k} \frac{\langle O^{(k)} W_2 \rangle}{\langle W_2 \rangle}$$

If $O^{(k)}$ is the operator with the smallest classical dimension , which acquires an anomalous dimension at $\lambda \Delta_1^k$ one-loop, the above expansion predicts that

$$\frac{\langle W_1 W_2 \rangle}{\langle W_1 \rangle \langle W_2 \rangle} \simeq \lambda^3 c_k \Delta_1^k \left[\frac{\langle O^{(k)} W_2 \rangle}{\langle W_2 \rangle} \right]_{\lambda_2} r^{\Delta_0^k} \log r$$

for small r. This kind of behavior is incompatible with the matrix model ! Therefore the presence or the absence of these logs in the small r expansion of the correlator provide a consistency check on the validity of the conjecture.

It is instructive to apply this criterium first to the correlator of two standard circular Maldacena-Wilson loops (Arytunov, Plefka, Staudacher).

- We know that a circular Maldacena-Wilson loop is described by a matrix model
- What about the correlator?

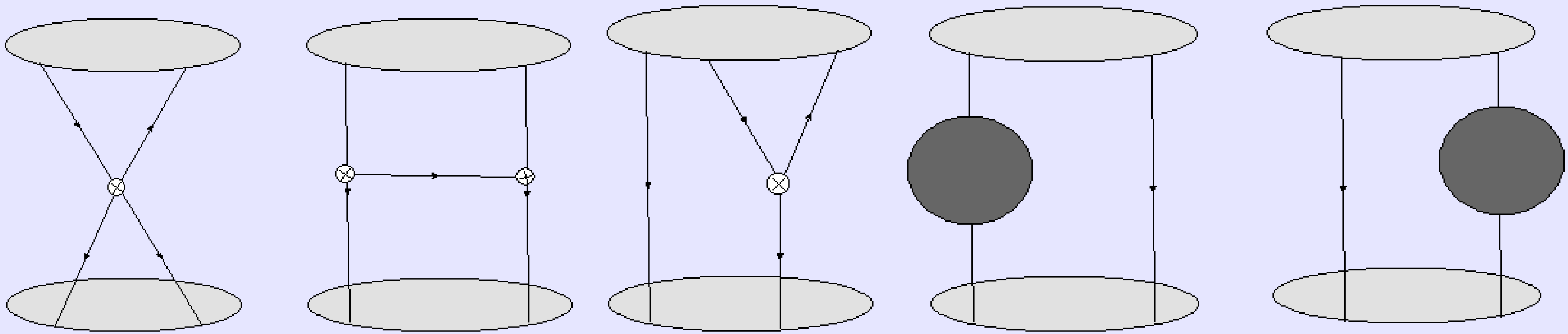
In the OPE of the Maldacena-Wilson loop appears, for example, at dimension 2 the Konishi Tr($\Phi\Phi$) operator and at dimension 3 the operator J= Tr(Φ F) . Both these operators acquires an anomalous dimension at one-loop. Then, by exploiting the previous discussion, we expect the following small r expansion for the correlator

$$\frac{\langle W_1 W_2 \rangle}{\langle W_1 \rangle \langle W_2 \rangle} = \frac{\lambda^3}{64\pi^2} \log(R_1) \frac{R_1^2 R_2^2}{(R_2^2 + h^2)^2} + \frac{\lambda^3}{32\pi^2} \log(R_1) \frac{R_1^3 R_2^3}{(R_2^2 + h^2)^3} + ...$$

Konishi J

Here no matrix model description!.

At the level of the correlator of the two Maldacena-Wilson loops these logs are generated by the diagrams



Extracting the log from these diagrams requires some care:

- Cancellation of the UV divergences
- Integration over the position of the vertices.

At the end, one recovers the previous result.

Back to the loops on S^2 :

Naively, one may think that the OPE exactly works as in the Maldacena-Wilson Loop. This is not true!

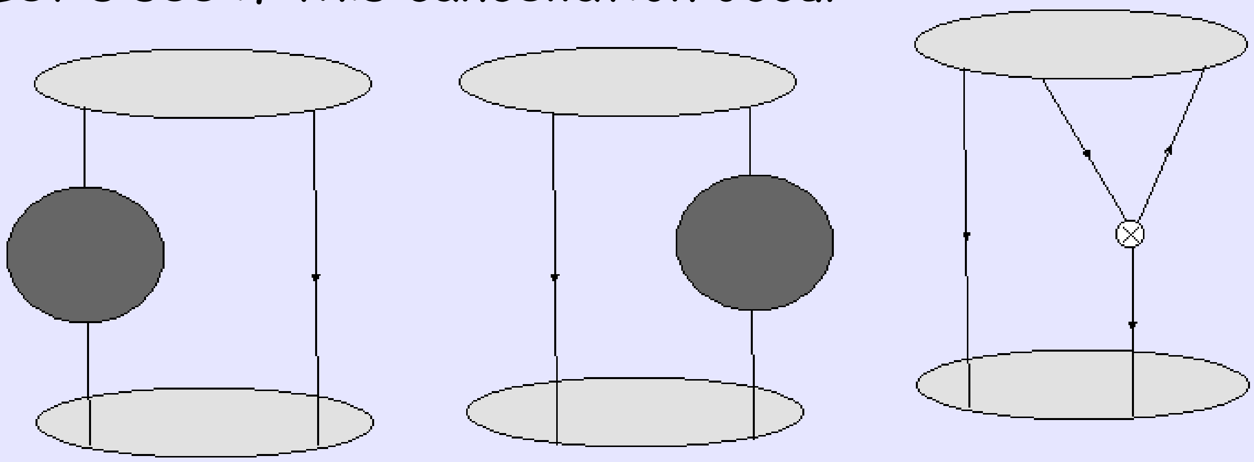
Because of the particular scalar couplings, in the OPE one of the scalar fields, let's say $\phi 3$, is not weighted by the radius r of the circle as naively expected, but by the ratio r^2/R (R sphere radius)

Since by conformal invariance the correlator can depend only on r/R, this means that $\phi 3$ effectively behaves as a field with dimension 2.

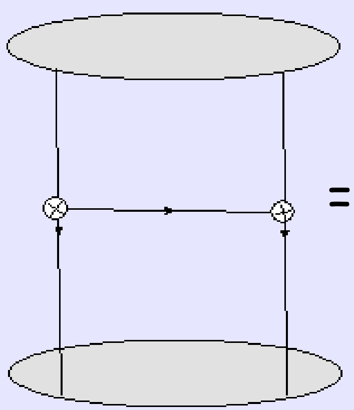
Consequences:

- The first potential logarithmic term appears at the order r^4
- Potentially there many operator that can contribute to this order, so cancellations are possibles. (This possibility was absent in the Maldacena-Wilson Loop, where there was only the Konishi operator at the leading order).

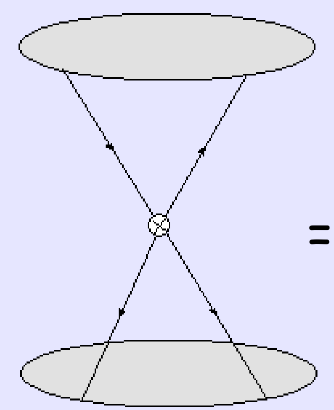
Let's see if this cancellation occur:



$$= -\frac{r^4}{256\pi^2}\log(r)\cos^4\left(\frac{\theta_0}{2}\right)$$



$$= -\frac{r^4}{384\pi^2}\log(r)\cos^4\left(\frac{\theta_0}{2}\right)$$



$$= \frac{5r^4}{768\pi^2}\log(r)\cos^4\left(\frac{\theta_0}{2}\right)$$

The sum is exactly zero! Conjecture OK!

At the level of the OPE What this cancellation means? A new (position dependent) protected operator constructed with the field strength of the twisted connection appearing in the Wilson-loop.

Outlooks

We have supported the conjectured equivalence between N=4 supersymmetric Wilson loops on the 2-sphere and 2D-YM Wilson-loops in the zero-instanton sector by ,means of a two-loop computation, but there are things to be done and clarified:

✔ One has to try to understand this relation through a localization extending the Pestun's result: Dimensional reduction to 2D as in topological strings

✔ Are there non-perturbative corrections[instanton contributions] to these Wilson-loops? [Reduced supersymmetry with respect to the circular case.]