On S-matrix for the twistorial A-model of large N Yang-Mills theory

1) Twistorial A-model on CP^2 with certain B-field and Wilson loop background

2) Same beta function as large N Yang-Mills

3) Vertex operators in the d-cohomology that correspond to plane waves i.e. representation of translations $\rightarrow$ S-matrix in a certain sector

Prologue

$N=4$ SUSY YM

Maldacena (1998)

Holography $AdS_5 \times S_5$ (off shell)

deformed to $N=1 \rightarrow N=0$?

Witten (2003)

Topological B-model

on twistor space (on shell)

strongly rigid ! $\rightarrow N=0$?

Neitzke - Vafa
conjecture (2004)

B - model S - dual to
A - model on the <u>same</u>
(super) twistor space

$$\Big\downarrow \quad \text{[illegible]}$$

$\mathcal{N}=0$ pure YM ?

R. Dijkgraaf, S. Gukov,
A. Neitzke, C. Vafa (2004)

Two problems: S-matrix problem

1) $\bar{\partial}$ cohomology on twistor space (B-model)

versus

$\mathcal{L}$ cohomology on Lagrangian submanifold (A-model)

2) Yang-Mills β function

The Yang-Mills string as the A-model on twistor space

MB hep-th/0811.2547 ; /0808.4662

1) β function in large N Yang-Mills
and in the A-model on
twistor space
semiclassical computation
involving gauge fields with
codimension-two singularities
in non-commutative Eguchi-Kawai
reduced gauge theory

2)
Holographic strings

versus

Topological strings
and gauge theories β function

$$Z_{YM} = \int e^{-\frac{N}{2g^2}\sum_{\mu\neq\nu}\int \mathrm{Tr}\, F_{\mu\nu}^2(A)} DA$$

$$g^2 = g_{YM}^2 N \qquad N \to \infty$$

$Z_{\text{CS+instantons}}$ =

$$= \int \exp\Big[-\frac{1}{g_s}\int \mathrm{Tr}\big(A\,dA + \frac{2}{3}A^3\big) +$$

$$- \sum_i e^{-\text{area}(\gamma_i)}\, \mathrm{Tr}\, P e^{i\int_{\gamma_i} A}\Big]\, DA$$

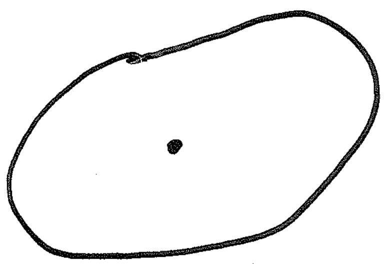
$$g_s = \frac{2\pi K}{N}$$

't Hooft electric/magnetic duality

electric charges characters of Z_N

magnetic charges

$\pi_2(SU(N)/Z_N) = \pi_1(Z_N) = Z_N$



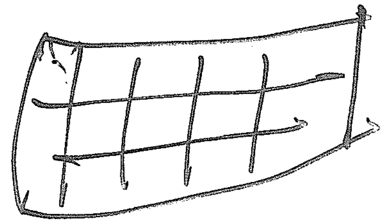
Z_N vortex $d=2$

$$P e^{i\int A} = e^{\frac{2\pi i k}{N}}$$

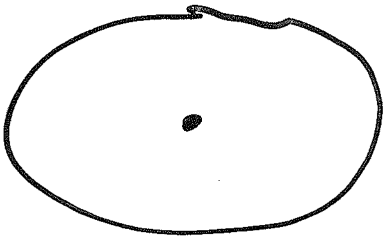
A has a pole $F_A \sim \delta^{(2)}$

vortex line $d=3$

vortex sheet $d=4$



$P e^{i\int A}$



gauge fields with any holonomy in $d=2$

gauge fields with codimension two singularities in $d=4$

→ "Surface operators"

in the geometric Langlands conjecture (Gukov Witten 2006)

gauge fields with Codimension two singularities as dynamical configurations (MB 1998)

$$S_{YM} = \int \mathrm{Tr}\, F^2\, d^4x \sim \int \delta^{(2)}\, \delta^{(2)}\, d^4x \sim$$

$$\sim \text{Area} \left(\frac{\Lambda}{2\pi}\right)^2$$

$$Z_{EK} = \int e^{-\left(\frac{2a}{\Lambda}\right)^d \frac{N}{2g^2} \sum_{\mu \neq \nu} \mathrm{Tr}\left([C_\mu, C_\nu] - 1 B_{\mu\nu}\right)^2} \, \mathcal{D}C$$

$$\left(\frac{2a}{\Lambda}\right)^d = \frac{V_d}{N_d}$$

Partial EK reduction

$d=4 \rightarrow d=2$

$$\frac{V_2}{N_2} = \left(\frac{2a}{\Lambda}\right)^2$$

cancels the quadratic divergence due to $\int \delta^{(2)} \cdot \delta^{(2)} \, d^4x$

$$Z = \int e^{-\frac{N}{2g^2}\sum_{\alpha\neq\beta}\int \mathrm{Tr}\, F_{\alpha\beta}^2} DA$$

$$= \int e^{-\frac{N8\pi^2}{g^2}Q - \frac{N}{4g^2}\sum_{\alpha\neq\beta}\int \mathrm{Tr}(F_{\alpha\beta}^-)^2} DA$$

$$1 = \int \delta(F_{\alpha\beta}^- - g\mu_{\alpha\beta}^-)\, D\mu_{\alpha\beta}^- \quad \left(\begin{matrix}\text{Nicolai}\\ \text{map}\end{matrix}\right)$$

$$F_{\alpha\beta}^- = F_{\alpha\beta} - \tilde{F}_{\alpha\beta}$$

$$\tilde{F}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}F_{\mu\nu}$$

$$Z = \int e^{-\frac{N8\pi^2}{g^2}Q - \frac{N}{4g^2}\sum_{\alpha\neq\beta}\int \mathrm{Tr}(g\mu_{\alpha\beta}^-)^2}$$

$$\times\ \delta(F_{\alpha\beta}^- - g\mu_{\alpha\beta}^-)\, DA\, D\mu^-$$

$$= \int e^{-\frac{N8\pi^2}{g^2}Q - \frac{N}{4g^2}\sum_{\alpha\neq\beta}\int \mathrm{Tr}(g\mu_{\alpha\beta}^-)^2}$$

$$\times$$

$$\times\ \mathrm{Det}^{-\frac{1}{2}}(-\Delta_A\delta_{\alpha\beta} + D_\alpha D_\beta + i\,\mathrm{ad}\,F_{\alpha\beta}^-)$$

$$\times\ D\mu_{\alpha\beta}^-$$

$$\mu_{EK} = \frac{N 8\pi^2}{N_2 g^2} Q + \frac{N}{g^2} \frac{2\pi}{N_2 B} \int \mathrm{Tr}\left(\bar{F}_{01}^{-2} + \bar{F}_{02}^{-2} + F_{03}^{-2}\right) d^2x$$

$$+ \log \mathrm{Det}^{-\frac{1}{2}}\left(-\Delta_A \delta_{\alpha\beta} + D_\alpha D_\beta + i\,\mathrm{ad}\, F^-_{\alpha\beta}\right)$$

$$[D_z, D_{\bar z}] - [D_u, D_{\bar u}] = \sum_p \mu_p^0\, \delta^{(2)}(x - x_p) - B\,1$$

$$[D_{\bar z}, D_u] = \sum_p m_p\, \delta^{(2)}(x - x_p)$$

$$[D_z, D_{\bar u}] = \sum_p \bar{m}_p\, \delta^{(2)}(x - x_p)$$

$$\left[\frac{N}{2g^2} - \left(-\frac{1}{3} + 2\right) \frac{N}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu}\right)\right] \times$$

$$\times \sum_{\alpha \neq \beta} \int \mathrm{Tr}\left(2\left(\frac{1}{2} F^-_{\alpha\beta}\right)^2\right)$$

$$\rightarrow -\left(2 - \frac{1}{3}\right) \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu}\right)$$

$$= -\frac{5}{3} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu}\right)$$

$$[D_z\, D_{\bar z}] - [D_u\, D_{\bar u}] = \sum_p g\, \alpha_p\, g^{-1}\, \delta^{(2)}(x - x_p) + -B1$$

$$[D_{\bar z}\, D_u] = 0$$

$$[D_z\, D_{\bar u}] = 0$$

$$\lambda_p \quad (k) - \frac{2\pi(N-k)}{N} \qquad ((N-k))\ \frac{2\pi k}{N}$$

$$\mathrm{Tr}\, \lambda^2 = (N-k)\left(\frac{2\pi k}{N}\right)^2 + k\left(\frac{2\pi(k-N)}{N}\right)^2 =$$

$$= (2\pi)^2\, \frac{k(N-k)}{N}$$

number of zero modes $= \frac{1}{2}\left(N^2 - \sum_i m_i^2\right) =$

$$= \frac{1}{2}\left(N^2 - k^2 - (N-k)^2\right) = k(N-k)$$

Action $\quad \frac{8\pi^2}{2g^2} k(N-k) + \frac{8\pi^2}{2g^2} k(N-k)$

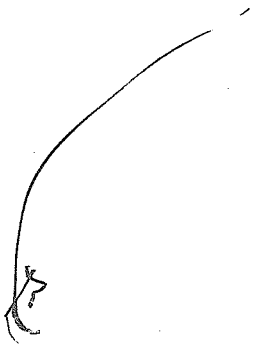
$$\longrightarrow \frac{16\pi^2}{2g^2} k(N-k)$$

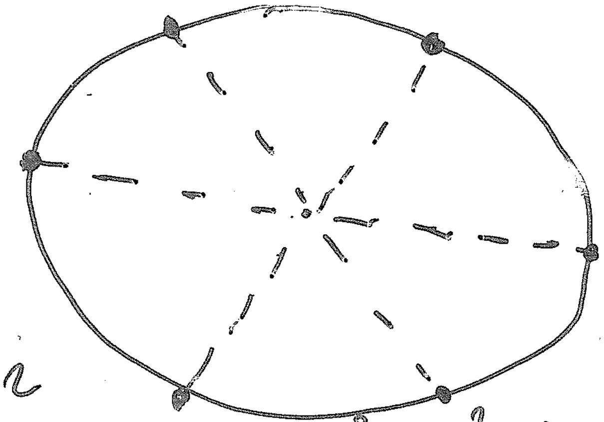
$$- \frac{5}{3}\, \frac{1}{16\pi^2}\, k(N-k)\, 16\pi^2 \log\left(\frac{\Lambda}{\mu}\right)$$

$$- 2 \quad k(N-k) \log\left(\frac{\Lambda}{\mu}\right)$$

$$16a^2 k(N-k)\left[\frac{1}{2g^2} - \left(\frac{5}{3}+2\right)\frac{1}{16a^2}\log\left(\frac{1}{\mu}\right)\right]$$

$$\beta_0 = \frac{11}{3}\frac{1}{(wa)^2}$$





(E) $\mathbb{R}^2 \times \mathbb{R}^2$ $\qquad$ $\mathbb{R}^2 \times \mathbb{R}^2$ (M)

$$\begin{array}{ccc} \mathbb{R}^2\times\mathbb{R}^2 & & \mathbb{R}^2\times\mathbb{R}^2 \\ \downarrow & & \downarrow \\ S^2\times S^2 & \to & S^2\times S^2/\mathbb{Z}_2 \end{array} \qquad (YM)$$

$$\to \mathbb{RP}^2 \qquad (CS)$$

$$\Psi(A+D) =$$

$$= P \exp i \int (A_z + D_u)\, dz + (A_{\bar z} + D_{\bar u})\, d\bar z$$

$$A_z\, dz + A_{\bar z}\, d\bar z + D_u\, du + D_{\bar u}\, d\bar u$$

$dz = du$
$d\bar z = d\bar u$
diagonal embedding

$dz = d\bar u$
$d\bar z = du$
Lagrangian embedding

$$P e^{i \int (A_z + D_{\bar u})\, dz + (A_{\bar z} + D_u)\, d\bar z}$$

$$\left\langle \frac{\delta \Gamma}{\delta \mu(x)} \Psi(x,x;A+D) \right\rangle =$$

$$= \frac{1}{2} \int \bar\partial^{-1}(x-y) \langle \Psi(x,y;A+D) \rangle \times$$

$$\times \langle \Psi(y,x;A+D) \rangle$$

$$\frac{\partial g_\omega}{\partial \log \Lambda} = -\beta_0 g_\omega^3 \quad ; \quad \beta_0 = \frac{11}{3}\frac{1}{(4\pi)^2}$$

$$\frac{\partial g_c}{\partial \log \Lambda} = \frac{-\beta_0 g_c^3 + \frac{\beta_J}{4} g_c^3 \frac{\partial \log Z}{\partial \log \Lambda}}{1 - \beta_J g_c^2}$$

$$\beta_J = \frac{4}{(4\pi)^2}$$

$$\frac{\partial \log Z}{\partial \log \Lambda} = \frac{\frac{1}{(4\pi)^2}\frac{10}{3} g_\omega^2}{1 + c\, g_\omega^2} \sim \frac{1}{(4\pi)^2}\frac{10}{3} g_c^2 + \cdots$$

$$\frac{\partial g_c}{\partial \log \Lambda} = -\beta_0 g_c^3 + \left(\frac{\beta_J}{4}\frac{1}{(4\pi)^2}\frac{10}{3} - \beta_0 \beta_J\right) g_c^5 + \cdots$$

$$= -\frac{1}{(4\pi)^2}\frac{4}{3} g_c^3 + \frac{1}{(4\pi)^4}\left(\frac{10}{3} - \frac{44}{3}\right) g_c^5 + \cdots$$

$$= -\frac{1}{(4\pi)^2}\frac{4}{3} g_c^3 - \frac{1}{(4\pi)^4}\frac{34}{3} g_c^5 + \cdots$$

1

A-model on twistor space

twistor space of a Riemannian 4-manifold is a bundle whose base is the manifold and whose fiber is the manifold of all its almost complex structures

thus locally

$TW(M_4) = M_4 \times CP^1$

A-model is a σ-model on $d=2$ whose target is twistor space

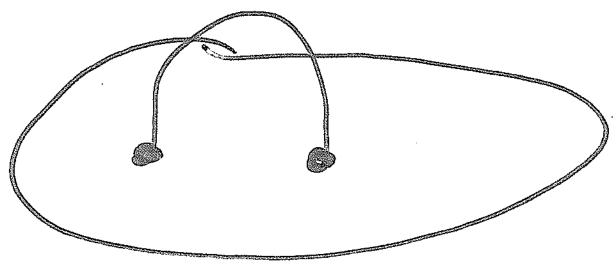
$M_4 = S^4, CP^2, T^4$

A-model on twistor space of CP^2

A-model is a topological σ-model

A-model is a theory of closed and open strings

the open string sector is defined on a Lagrangian submanifold of twistor space



A-model is topological → semiclassical approximation on the world-sheet is exact

→ effective action on closed form

→ CS + instantons

Local coordinates on twistor space

$$(w, \bar{w}, u, \bar{u})$$

$$(\lambda, \bar{\lambda}, u_1, \bar{u}_1, u_2, \bar{u}_2)$$

$$u_1 = w - \lambda \bar{u}$$
$$u_2 = u + \lambda \bar{w}$$

It is of the utmost importance that the 3 ASD or SD equations in 4-dimensional space-time can be written as <u>1</u> flatness condition on twistor space

$$D_{u_1} = D_{\bar{w}} - \lambda D_u$$

$$D_{u_2} = D_{\bar{u}} + \lambda D_w$$

$$[D_{u_1} D_{u_2}] = [D_{\bar{w}}, D_{\bar{u}}] - \lambda [D_u D_{\bar{u}}] +$$
$$+ \lambda [D_{\bar{w}} D_w] - \lambda^2 [D_u D_w] = 0$$

$$[D_w D_{\bar{w}}] + [D_u D_{\bar{u}}] = 0$$

$$[D_u D_w] = 0 \qquad (SD)$$

$$[D_{\bar{u}} D_{\bar{w}}] = 0$$

On the Lagrangian submanifold

$$D_{u_1} = D_{\bar{w}} - \lambda D_u$$

$$D_{u_2} = D_w + \frac{1}{\lambda} D_{\bar{u}}$$

$$[D_{u_1} D_{u_2}] = \frac{1}{\lambda}[D_{\bar{w}} D_{\bar{u}}] +$$

$$- \left([D_w D_{\bar{w}}] + [D_u D_{\bar{u}}]\right) +$$

$$+ \lambda [D_w D_u] = 0$$

$$\frac{1}{g_s}[D_{u_1} D_{u_2}] + \frac{1}{g_s} BI = 0$$

$$\frac{1}{g_s}[D_{u_1}, D_{u_2}] = -\frac{1}{g_s} B \mathbb{1} +$$

$$+ \delta^{(2)}(w - w_p)\, P e^{i\int_{\gamma_p} A_\lambda d\lambda}\, e^{-\mathrm{area}(\gamma_p)}$$

CS functional integral without instantons

$$Z = \int e^{\frac{1}{g_s}\mathrm{Tr}\int \tilde{A}\, \partial_\lambda \tilde{A}} \times$$

$$\times\ \delta\left(\frac{1}{g_s}[D_{u_1} D_{u_2}]\right) D\tilde{A}$$

$$\tilde{A} = \left(A_w + \frac{1}{\lambda} D_{\bar u}\right) dw +$$

$$+ \left(A_{\bar w} - \lambda D_u\right) d\bar w$$

In presence of instantons the δ-functional is modified; Within one-loop accuracy

$$\delta\left(\frac{1}{g_s}[D_{u_1} D_{u_2}]\right) \longrightarrow$$

$$\to \delta\left(\frac{1}{g_s}[D_{u_1} D_{u_2}] + \frac{1}{g_s} B I + e^{-d(p)} \delta^{(2)}(w - w_p)\, P e^{i \oint_{\gamma_p} A_\lambda d\lambda}\right)$$

If $\quad g_s e^{-\text{area}(\gamma_p)} \, P e^{i \oint_{\gamma_p} A_\lambda d\lambda} =$

$$= g \lambda_p g^{-1}$$

the argument of the δ-functional coincides with the Z_2 vortices equations of the YM theory!

The β function follows

Loop equation of the CS + + instantons

$$\left\langle \left\{ \frac{1}{g_S} [D_{u_1} D_{u_2}](w) + - \delta^{(2)}(w - w_p) e^{-\text{area}(\gamma_p)} \Psi(\gamma_p; A) \right\} \times \right.$$

$$\left. \times \Psi(\gamma_p; A) \right\rangle =$$

$$= \int \delta^{(2)}(w - w_p) \langle \Psi(\gamma_p; A) \rangle \times$$

$$\times \langle \Psi(\gamma_p; A) \rangle$$

But since $\langle \mathrm{Tr}\, \Psi(\gamma_p; A) \rangle = 0$ for the background that corresponds to a vortex the classical and quantum equation of motion coincide !!!

Thus at large N the beta function at one-loop is exact on the CS + instantons i.e. in the string theory

S-matrix

$$\delta_A \wedge (D_{\bar u}\delta z + D_u \delta \bar z) = 0$$

$$\delta \wedge (D_{\bar u}\delta z + D_u \delta \bar z) = 0$$

$$[D_{\bar u} A_z] - [D_u A_{\bar z}] = 0$$

$$\Psi \equiv D_{\bar u}\delta z + D_u \delta \bar z$$

$$\delta\Psi = 0$$

$$D_u = \partial_u + i A_u \rightarrow$$

$\rightarrow$ gauge equivalent to

$$i \quad e^{i\partial_u \bar u + i \partial_{\bar u} u} A_u \, e^{-i\partial_u \bar u - i\partial_{\bar u} u}$$

$$V = e^{i p_\alpha x_\alpha} + e^{-i p_\alpha x_\alpha}$$

$$\langle T^2 \ldots \int_{\gamma_1} V_1 \int_{\gamma_2} V_2 \ldots \rangle$$