SUPERFIELDS
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"New Perspectives in String Theory"

Flow Equations for Black Hole Attractors

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A. Ceresole, G.D., hep-th/0702088
G. L. Cardoso, A. Ceresole, G.D., J. Oberreuter and J. Perz, hep-th/0706.3373
I. Bena, G.D., S. Giusto, C. Ruef and N. Warner, hep-th/0902.4526
G.D., A. Gnecchi, w.i.p.

- Original idea (N=2):
  - A. Ceresole, G.D., hep-th/0702088
- New classes:
  - J. Perz, P. Smyth, T. van Riet, B. Vercnocke, hep-th/0901.4539
  - S. Ferrara, A. Gnecchi, A. Marrani, hep-th/0806.3196
  - D. Gaiotto, W. Li, M. Padi, hep-th/0710.1638
  - D. Astefanesi, H. Nastase, H. Yavartanoo, S. Yun, hep-th/0711.0036
- Higher dimensions:
  - G. L. Cardoso, A. Ceresole, G.D., J. Oberreuter, J. Perz, hep-th/0706.3373
  - A. Ceresole, S. Ferrara, A. Marrani, hep-th/0707.0964
  - K. Goldstein, S. Katmadas, hep-th/0812.4183
  - I. Bena, G.D., S. Giusto, C. Ruef and N. Warner, hep-th/0902.4526
- Higher susy (N>2):
  - L. Andrianopoli, R. D'Auria, E. Orazi, M. Trigiante, hep-th/0706.0712

Outline

- Lightning review of the Black Hole Attractor Mechanism
  - ***4d, N=2*** Black Hole Attractors (with SUSY)
  - Non-Supersymmetric Attractors
- **Attractor Flow Equations for Non-BPS Extremal BH's**
  - Hidden SUSY?
  - Can we split the non-BPS attractor?
- Summary and Outlook

Messages

i) Attractors exist also for extremal non-BPS BH's

ii) They are (almost) always associated to 1st order flow equations
[hence full solutions can be constructed]

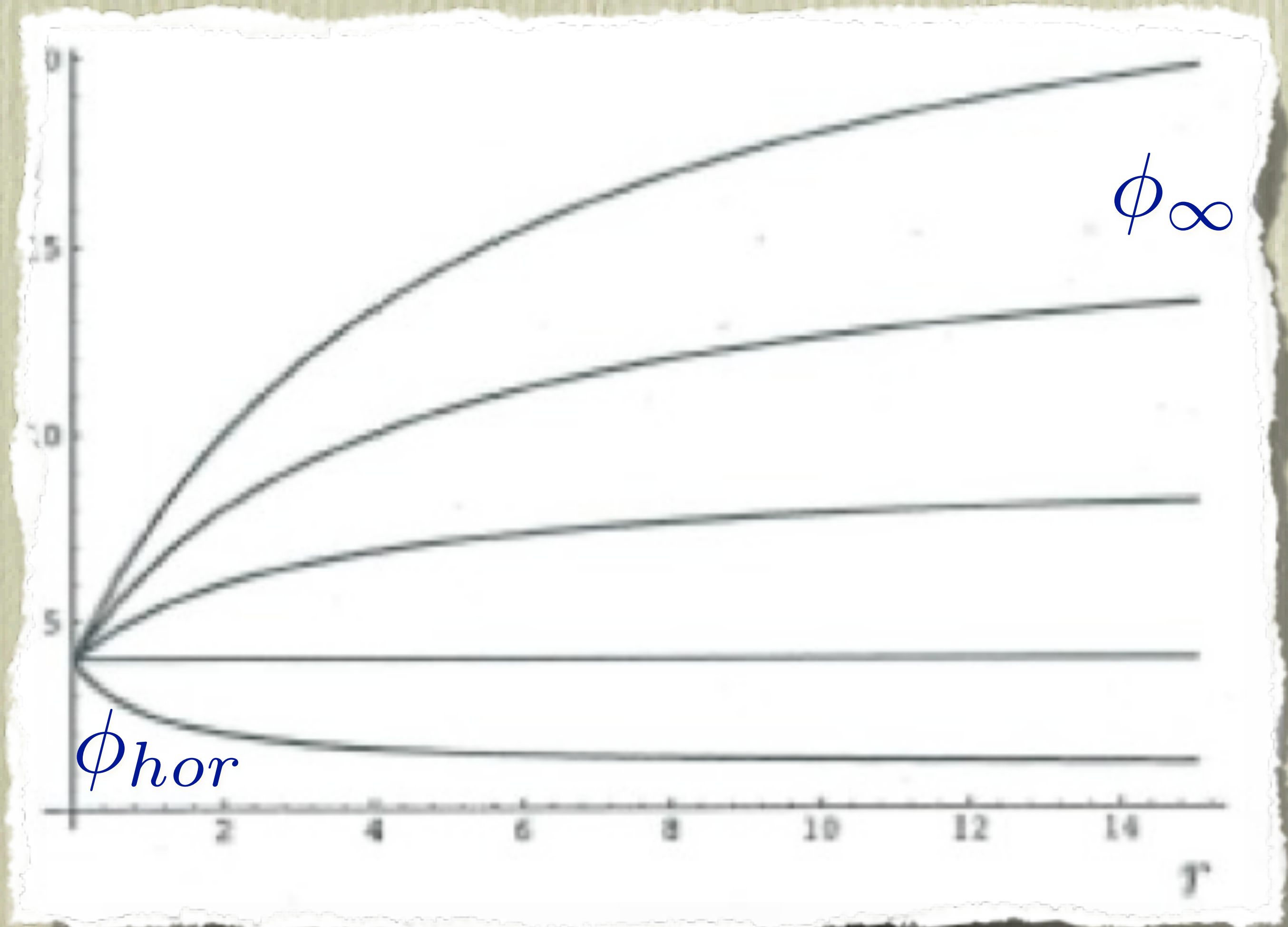
iii) The solutions can be expressed in terms of harmonic functions

iv) The flow equations do **not** (always) follow from hidden supersymmetry

The Attractor Mechanism

- Supersymmetric Black Holes with E&M charges arise as solitonic solutions of a 1d problem
- When scalars (ϕ^i) are present, the BH loses memory of ϕ^i_∞ at the horizon

$$\phi_{hor} = \phi(p,q)$$



- Supersymmetric Black Holes with E&M charges arise as solitonic solutions of a 1d problem
- When scalars (ϕ^i) are present, the BH loses memory of ϕ^i_∞ at the horizon

$$\phi_{hor} = \phi(p,q)$$

- This implies

$$S = \frac{A}{4} = \pi V_{BH}\left(\phi_{hor}(p,q), p, q\right)$$

- **SUSY** $\underset{\not\Longleftarrow}{\Longrightarrow}$ **Extremality**

$$M \geq |\mathcal{Z}|$$

- Extremal: *Minimal possible M for given Q* BPS bound

- Is this surprising?

- "No hair theorem" is not enough to explain it (valid only for static configs, no c.c., abelian groups, no scalars, 4 dimensions...)

- On the other hand **Attractors** are necessary for a sensible microscopic explanation of S
(And it is a stronger requirement!)

- Note: *Multiple basins of attractions are possible*

- For simplicity we (first) focus on static, spherically symmetric, asymptotically flat Black Holes
- Symmetries imply that $g_{\mu\nu}$ and ϕ^i depend only on r $\phi^i = \phi^i(r)$

$$ds^2 = -\mathrm{e}^{2U(r)}dt^2 + \mathrm{e}^{-2U(r)}\left(c^4\frac{dr^2}{\sinh^4(cr)} + \frac{c^2}{\sinh^2(cr)}d\Omega^2_{S^2}\right)$$

- Start from 4d N=2 supergravity with vector fields

$$\mathcal{L}_{4d} = -\frac{R}{2} + g_{i\bar{\jmath}}\partial_\mu\phi^i\partial_\nu\bar{\phi}^{\bar{\jmath}} + \mathcal{I}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\,\mu\nu} + \mathcal{R}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}\widetilde{F}^{\Lambda\mu\nu}$$

and assume $\int_{S^2} F^\Lambda = 4\pi p^\Lambda \quad \int_{S^2} G_\Lambda = 4\pi q_\Lambda$

- Integrating over $\mathbb{R}_t \times S^2$ (and performing a Legendre transform) reduces to

$$\mathcal{L} = (U'(r))^2 + g_{i\bar{\jmath}}\phi'^i\bar{\phi}'^{\bar{\jmath}} + \mathrm{e}^{2U} V_{BH}(\phi, q, p) - c^2$$

with a constraint

$$H = (U'(r))^2 + g_{i\bar{\jmath}}\phi'^i\bar{\phi}'^{\bar{\jmath}} - \mathrm{e}^{2U} V_{BH}(\phi, q, p) - c^2$$

Only if H=0 the eoms in 1d are consistent with the 4d ones

- For N=2 supergravity the potential reads

$$V_{BH}(\phi, q, p) = |\mathcal{Z}|^2 + 4g^{i\bar{\jmath}}\partial_i|\mathcal{Z}|\partial_{\bar{\jmath}}|\mathcal{Z}|$$

- For c = 0

FERRARA-GIBBONS-KALLOSH

$$S = \int dr \left[\left(U' \pm \mathrm{e}^{U} |\mathcal{Z}|\right)^2 + \left| z^{i\prime} \pm 2\mathrm{e}^{U} g^{i\bar{\jmath}} \partial_{\bar{\jmath}} |\mathcal{Z}| \right|^2 \mp 2 \frac{d}{dr} \left(\mathrm{e}^{U} |\mathcal{Z}|\right) \right]$$

Flow equations

ADM mass

- The flow stops at $\partial_i |\mathcal{Z}| = 0 \Rightarrow \partial_i V_{BH} = 0$

$AdS_2 \times S^2$

where $ds^2 = -\frac{r^2}{|\mathcal{Z}|_*^2} dt^2 + \frac{|\mathcal{Z}|_*^2}{r^2} \left(dr^2 + r^2 \Omega_{S^2}^2\right)$

$$S = \frac{A}{4} = \pi |\mathcal{Z}|_*^2 \left(\phi_*(p,q), p, q\right)$$

Non-BPS Extremal Black Holes

Non-BPS Extremal Black Holes

- **Non-BPS** extremal BH's share the attractor mechanism iff

Goldstein-Izuka-Jena-Trivedi
Ferrara-Gibbons-Kallosh

  - ▶ $\partial_i V_{BH} = 0$ at the horizon
  - ▶ The matrix $\partial_i \partial_j V_{BH}|_*$ is positive definite

- Obtained in generic theories with N<3 (at the two derivative level) also valid for AdS and higher dim. BH's.

- Difficult analysis (perturbative and 2nd order diff equations)

- Even more difficult to obtain the full solutions

- Interesting remark: non-BPS BH's have a **c-function**

GOLDSTEIN-JENA-MANDAL-TRIVEDI

$$A(r) \sim \mathrm{e}^{2U(r)}$$

which is *monotonically decreasing* from spatial infinity to the horizon

- Spontaneous relation with BPS domain walls
(c-function is again the warp factor)

- For domain-walls one can extend the BPS flow equations to the non-BPS case (Fake supergravity)

FREEDMAN-NUNEZ-SCHNABL-SKENDERIS
SKENDERIS-TOWNSEND
CELI-CERESOLE-G.D.-VAN PROEYEN-ZAGERMANN

- Can we do the same for Black Holes?

- Strong analogy with 1d-instantons in QM
- Consider a scalar field with lagrangian $\mathcal{L} = (\phi')^2 + V(\phi)$
- Instanton solutions with zero energy $H = (\phi')^2 - V(\phi) = 0$
- We can rewrite the 1d action as

$$S_{inst} = \int dt \left(\phi' \pm \sqrt{V(\phi)}\right)^2 \mp 2 \int dt \phi' \sqrt{V(\phi)}$$

- The last term is always a total derivative
- Instantons are then solutions to $\phi' = \mp\sqrt{V(\phi)}$
- EOM's are identically satisfied

- For BH Quantum Mechanics define $\vec{\phi} = (U, \phi^i)$

- The Lagrangian and constraint read

$$\boxed{\begin{aligned} \mathcal{L} &= (\vec{\phi}')^2 + V(\vec{\phi}) \\ H &= (\vec{\phi}')^2 - V(\vec{\phi}) = 0 \end{aligned}}$$

- We can rewrite the 1d action as

$$\frac{S_{1d}}{\mathcal{V}} = \int dr \left(\vec{\phi}' \pm \vec{n}\sqrt{V(\vec{\phi})} \right)^2 \mp 2 \int dr \vec{\phi}' \cdot \vec{n}\sqrt{V(\vec{\phi})}$$

- The last term is a total derivative iff $\vec{n} = \frac{1}{\sqrt{V}} \vec{\nabla} f(U, \phi^i)$

- BUT the vector has to be unit norm and this means that

$$e^{2U} V_{BH}(\phi, q, p) = (\partial_U f)^2 + g^{ab} \partial_a f \partial_b f$$

CERESOLE-G.D.

- Defining $f(U, \phi^i) = \mathrm{e}^U W(\phi^i)$, extremal black holes are described by

This is a PDE with b.c. the critical point of the superpotential

iff $$V_{BH}(\phi, q, p) = W^2 + 4g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$$

- BPS BH's are a special case with W = |Z|
- But we have found **more general solutions!**
- There are **classes** of **potentials** which allow for multiple *"superpotentials"*

Two Examples in N=2 Supergravity

The geometry of the moduli space follows from a prepotential F(X(z)), which defines the Kaehler potential

$$K = -\log\left[i(\bar{X}^\Lambda \partial_\Lambda F - X^\Lambda \bar{\partial}_\Lambda \bar{F})\right]$$

The **central charge** is

$$\mathcal{Z} = \mathrm{e}^{K/2}\left(p^\Lambda \partial_\Lambda F - q_\Lambda X^\Lambda\right)$$

and the Black Hole potential follows

$$V_{BH}(\phi, q, p) = |\mathcal{Z}|^2 + 4 g^{i\bar{\jmath}} \partial_i |\mathcal{Z}| \partial_{\bar{\jmath}} |\mathcal{Z}|$$

Example I: One modulus with prepotential

$$F(X^\Lambda) = -iX^0(\phi)X^1(\phi)$$

The potential is

$$V_{BH} = \frac{(p^1)^2 - iq_1(z-\bar{z})p^1 + {q_0}^2 + ip^0q_0(z-\bar{z}) + \left((p^0)^2 + (q_1)^2\right) z\bar{z}}{z+\bar{z}}$$

This can be written as $V_{BH}(\phi, q, p) = W^2 + 4g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$

for $W_{BPS} = |\mathcal{Z}| = \mathrm{e}^{K/2}\left|q_0 + ip^1 + (q_1 + ip^0)z\right|$

or for $W_{nonBPS} = \mathrm{e}^{K/2}\left|-q_0 + ip^1 + (q_1 - ip^0)z\right|$

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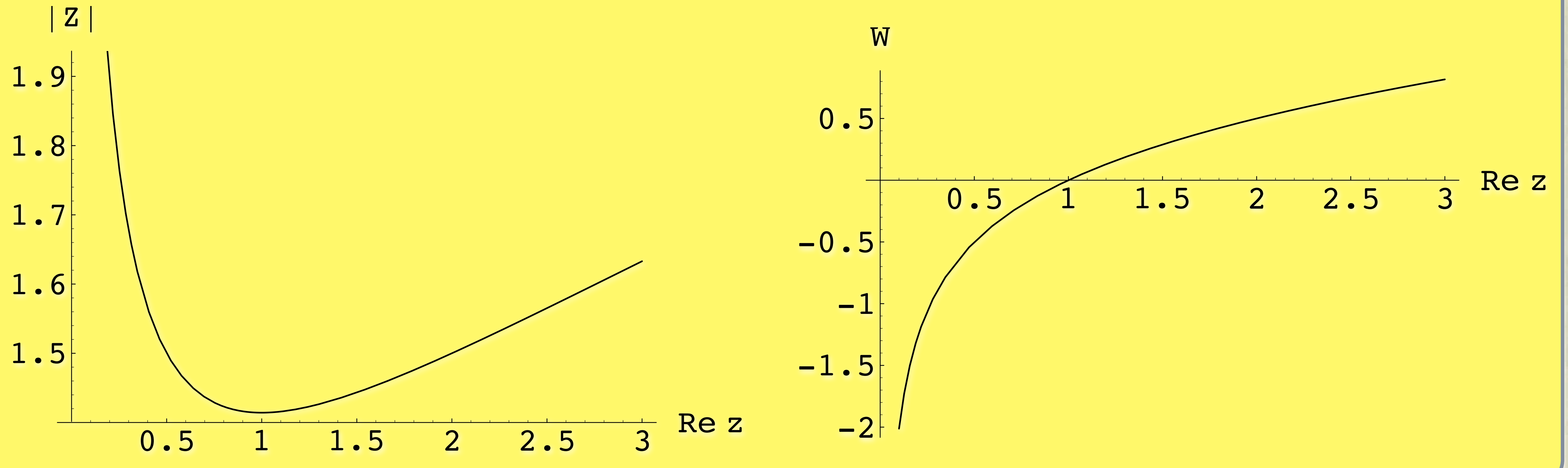

Plots of the sections of W and Z at Im z=0, for $q_0=1=p^0$.
Choosing $q_0=1$ $p^0=-1$ W and Z are exchanged

Note that at the minimum of W one has Z=0

Example II: T^2 x T^2 x T^2 with identified complex structure

$$F(X^\Lambda) = \frac{C_{IJK}X^I X^J X^K}{X^0} = \frac{(X^1)^3}{X^0}$$

Consider only two charges: q_0, p^1

The central charge is $\mathcal{Z} = \dfrac{q_0 - 3p^1 z^2}{\sqrt{-i(z-\bar{z})^3}}$

Susy BH's $\partial_z|\mathcal{Z}| = 0$ follow at $z^* = -i\sqrt{\dfrac{q_0}{p^1}}$

but they are inside the Kaehler cone only for

$$q_0 p^1 > 0$$

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The BPS solution is:

$$z = -i\sqrt{\frac{H_0}{H^1}}, \quad H_0 = h_0 + q_0 r, \quad H^1 = h^1 + p^1 r$$

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$$F(X^\Lambda) = \frac{C_{IJK}X^I X^J X^K}{X^0} = \frac{(X^1)^3}{X^0}$$

The same potential can be obtained from
$$W = \frac{q_0 - 3p^1 z\bar{z}}{\sqrt{-i(z-\bar{z})^3}}$$

Non-BPS BH's $\partial_z W = 0$ follow at $z^* = -i\sqrt{-\frac{q_0}{p^1}}$

and they are inside the Kaehler cone for $q_1 p^0 < 0$

Here at the minimum of W the central charge is not vanishing

Example II: T^2 x T^2 x T^2 with identified complex structure

$$F(X^\Lambda) = \frac{C_{IJK}X^I X^J X^K}{X^0} = \frac{(X^1)^3}{X^0}$$

The same potential can be obtained from

$$W = \frac{q_0 - 3p^1 z\bar{z}}{\sqrt{-i(z-\bar{z})^3}}$$

Non-BPS BH's $\partial_z W = 0$ follow at $z^* = -i\sqrt{-\frac{q_0}{p^1}}$

The non-BPS solution is given by:

$$z = \frac{b - i\,\mathrm{e}^{-2U}}{(H^1)^2}, \quad \mathrm{e}^{-4U} = c^2 - (H^1)^3 H_0,$$

$$H_0 = h_0 + q_0 r,$$
$$H^1 = h^1 + p^1 r$$

Uplifted (and Extended) Solutions

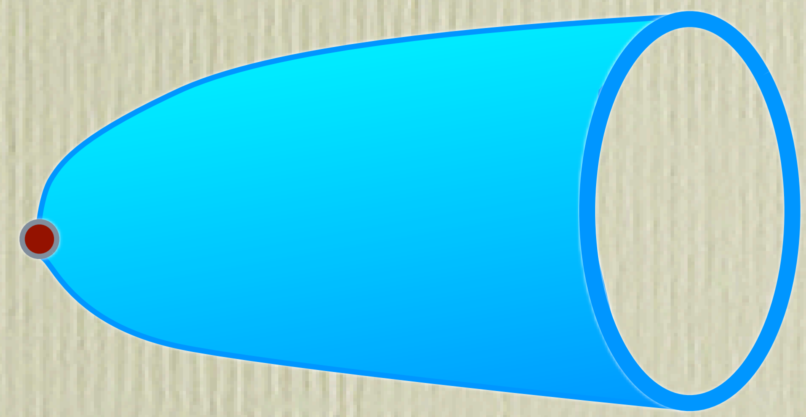
5d Attractors

- *Is all this a consequence of "supersymmetry without supersymmetry"?*
- We constructed single centre **rotating** 5d solutions that are lifts of 4d BH's

Cardoso-Ceresole-G.D.-Oberreuter-Perz

- 5d stationary Taub–Nut metrics

Close to BH d=5



Away from BH d=4

5d electric charges q_A= 4d *electric* charges (D2)

5d NUT charge p^0 = 4d *magnetic* charge (D6)

5d rotation q_0 = 4d *electric* charge (D0)

- By rewriting the action in a BPS^2 form, we discovered two types of solutions (for D6-D2 and D6-D2-D0):
  - **BPS** (everything given in terms of harmonic functions)

    Type I
  - **Non BPS.**
    Same solutions, but opposite sign for gauge potentials and different rotation parameter.

    Type II
- Dual D6-D0 solutions obtained by *Gimon, Larsen, Simon*.
- Observation by *Goldstein–Katmadas:* A new class of solutions can be obtained from BPS ones by a change of orientation.

- M-theory perspective. BPS solutions of M-theory with M2/M5-charges

Hyper-Kähler

$$
\begin{aligned}
ds^2 &= -(Z_1 Z_2 Z_3)^{-2/3}(dt+k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_4^2 \\
&\quad + \left(\tfrac{Z_2 Z_3}{Z_1^2}\right)^{1/3}(dx_1^2+dx_2^2) + \left(\tfrac{Z_1 Z_3}{Z_2^2}\right)^{1/3}(dx_3^2+dx_4^2) + \left(\tfrac{Z_1 Z_2}{Z_3^2}\right)^{1/3}(dx_5^2+dx_6^2) \\
C^{(3)} &= \left(a_1 - \tfrac{dt+k}{Z_1}\right)\wedge dx_1 \wedge dx_2 + \left(a_2 - \tfrac{dt+k}{Z_2}\right)\wedge dx_3 \wedge dx_4 + \left(a_3 - \tfrac{dt+k}{Z_3}\right)\wedge dx_5 \wedge dx_6 \, ,
\end{aligned}
$$

- Supersymmetric solutions are given by solutions of

$$\Theta_I = + *_4 \Theta_I$$

$$d *_4 dZ_I = \frac{C_{IJK}}{2} \Theta_J \wedge \Theta_K$$

$$dk + *_4 dk = Z_I \Theta_I \, .$$

$$\Theta_I = da_I$$

Bena-G.D.-Giusto-Ruef-Warner

- Non BPS solutions come from a change of orientation

$$\Theta_I = - *_4 \Theta_I$$

$$d *_4 dZ_I = \frac{C_{IJK}}{2} \Theta_J \wedge \Theta_K$$

$$dk - *_4 dk = Z_I \Theta_I \, .$$

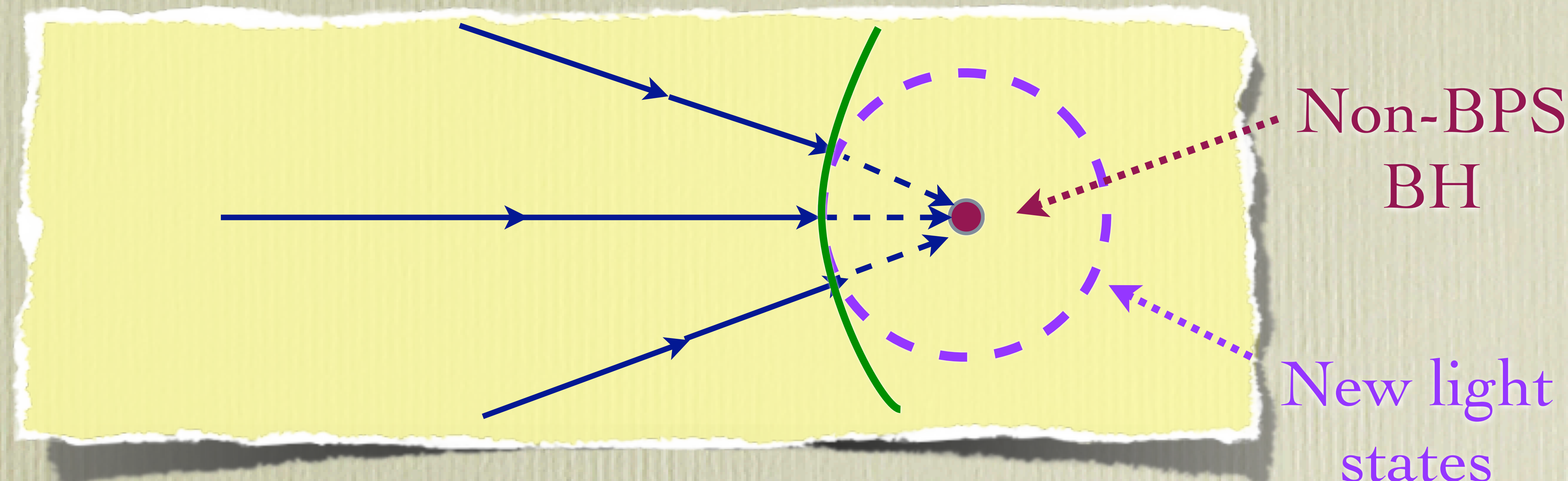
- When the 4-dimensional base is *flat* the orientation change is just a change of coordinates
- In Taub-NUT space, we have genuinely new solutions!!
- Also: new non-BPS Black Rings in 5d and the most general non-BPS extremal rotating BH in 4d

Split Attractors
(The Non-BPS Branch)

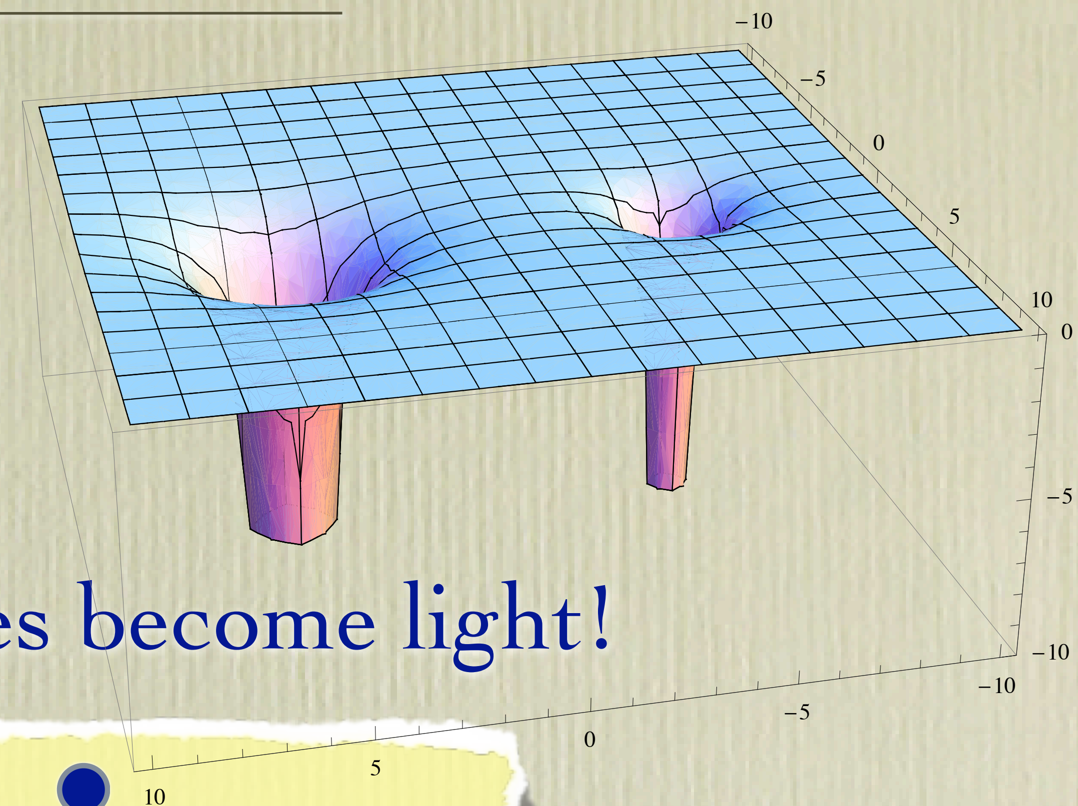
Example I: $\partial W = 0 \Leftrightarrow \mathcal{Z} = 0$

Can we trust supergravity?

Moore: at $Z = 0$ new stringy states become light!



Denef: a line of Marginal Stability leads to split attractors



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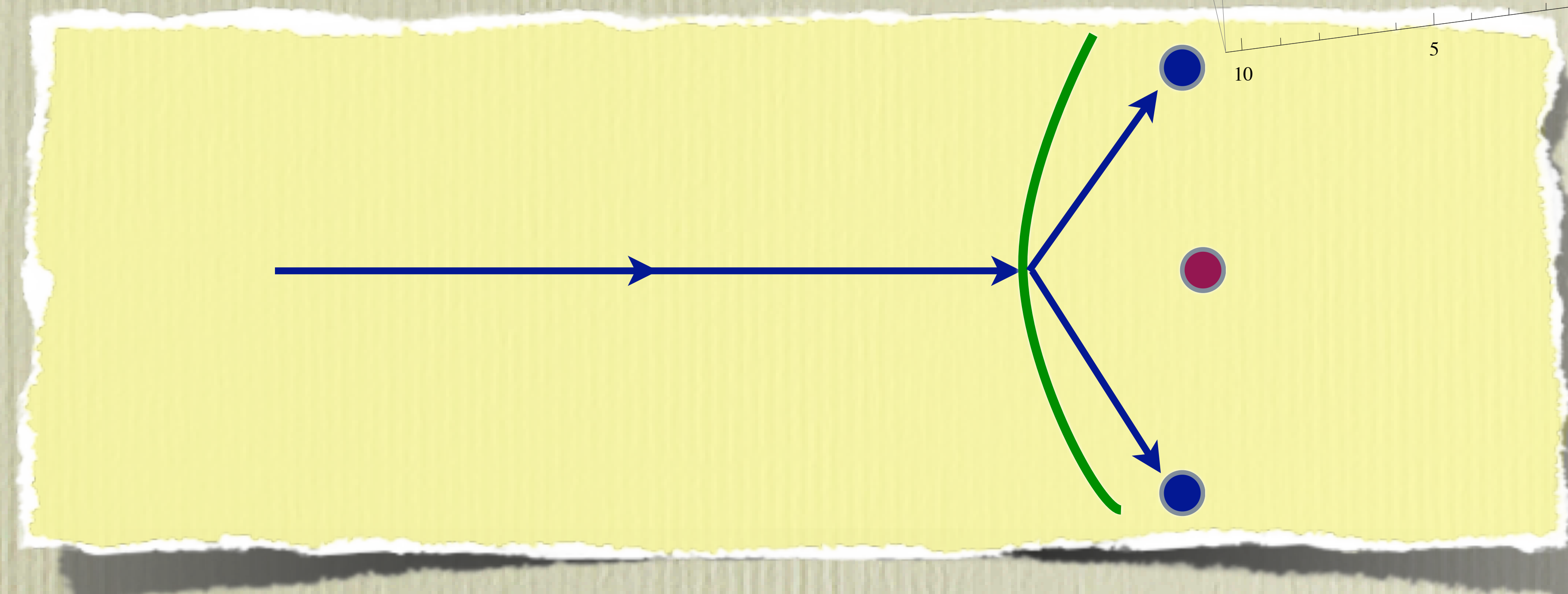


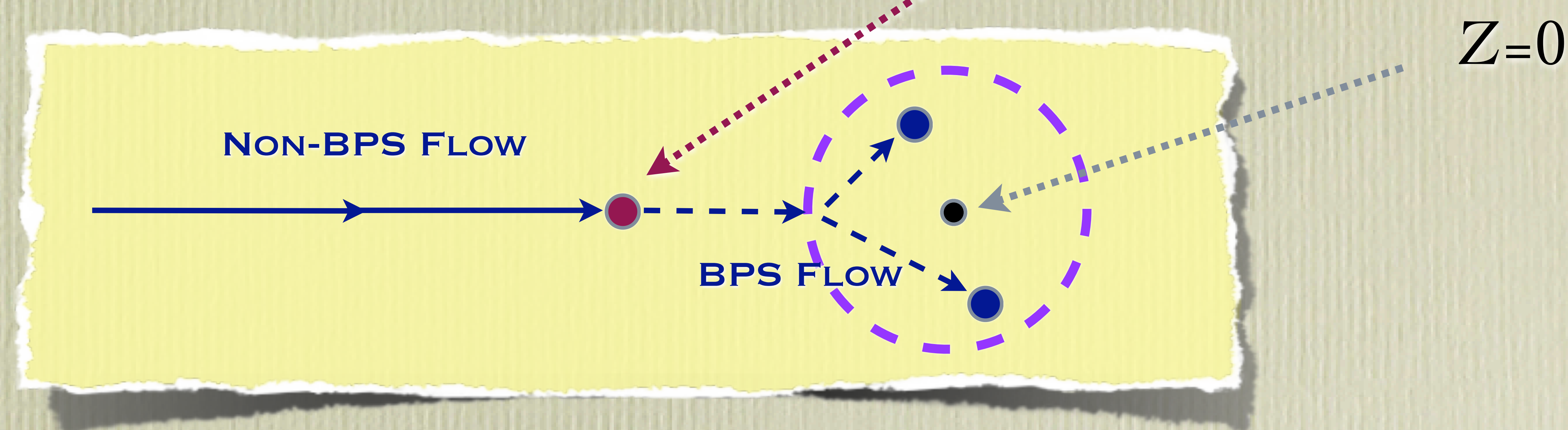
Non-BPS BH

Denef: a line of Marginal Stability leads to split attractors

MATTER GOES TO MULTIPLE BPS BH'S

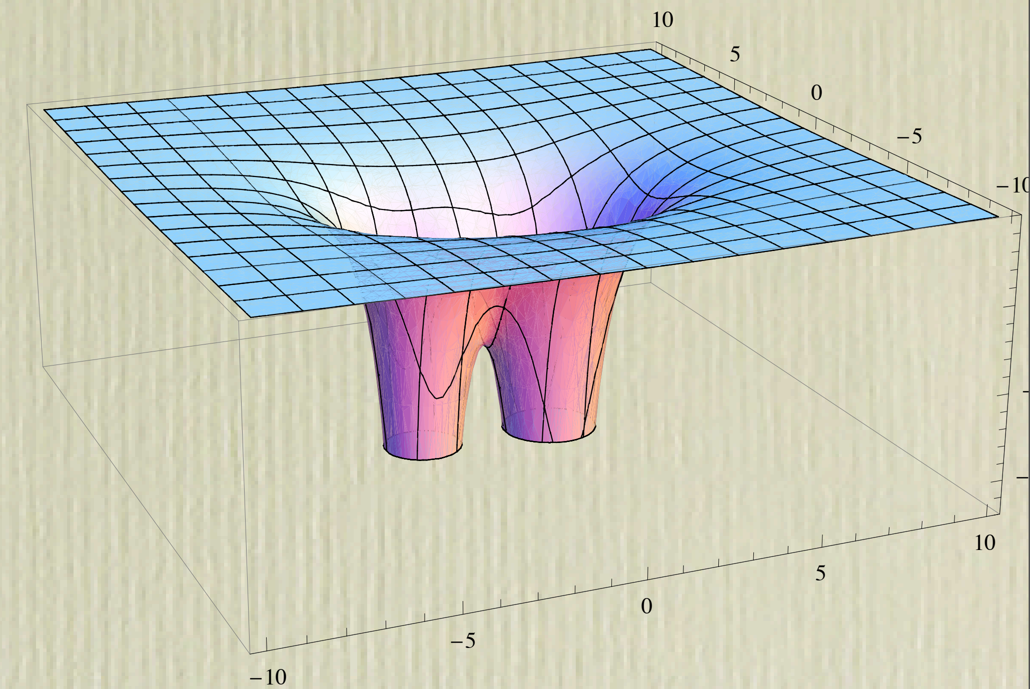
What about solutions like **example II**?

In this case $\partial W = 0 \not\Leftrightarrow \mathcal{Z} = 0$



- Same charges lead to **non-BPS single centre** or to a **BPS with multiple centres**
  - (*Non-BPS decay* and also *non-BPS microstates*!)

- Construct the instanton between non-BPS and multi-BPS
- Generalization of **Brill Instanton:**
- ***(Mixed) Non-BPS split attractor solutions (under construction)***



- Cut the asymptotics
- The Euclidean action of this solution is

$$I = -\frac{1}{2}\left(S_{BH}(\mathcal{Z}_1) + \ldots + S_{BH}(\mathcal{Z}_n) - S_{BH}(\mathcal{Z})\right)$$

- The decay amplitude goes with e^{-I}

Summary

i) *Attractors exist also for extremal non-BPS BH's*

ii) *They are (almost) always associated to 1st order flow equations*
[hence full solutions can be constructed]

iii) *The solutions can be expressed in terms of harmonic functions*

iv) *The flow equations do **not** (always) follow from hidden supersymmetry*

Outlook

- *General proof still missing (Lots of classes, all symmetric spaces...)*
- *Alternative description in terms of Harmonic functions*
- *Extension to **multiple centres** solutions*
- *Extension to gauged supergravity (asymptotically AdS)*
- ....