GGI: Arcetri, April 8, 2009 Firenze

« Form-dualities, BgKM superalgebras, E_{11} »

(« … and more », hep-th/0904.**)**

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Previous collaborators

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And also T. Damour, H. Nicolai

Outline

I Unification, finiteness, integrability?

II From U to V dualities: from scalar fields to p-form potentials D=3,4…

III $M^0=U/K$ $U=U^+$ fits inside $M^{prop}=(D-2)$-truncated-$V^+ < V^+$

IV Spin-Isospin correlation: E_{11}, E_{12}?, E_{oo}?

V GL(D,R) covariantized presentation.

Simple Cartan matrices for V.

New Low dimensions D=2 …

I. Motivations : nice theories

* 'Quantized' 11d Supergravity with its extended objects M2, M5. Its m=0 fields: G metric+$A^{(3)}$ gauge three-form+spin $\psi^{(1)}$ valued one-form
* Compactified on a small spacelike circle this is IIA superstring theory at small string coupling (on 10d Minkowski space), $(R_{11}/l_{Pl})^3 = g_{IIA}{}^2$
* Compactified on a zero volume two-torus this reduces to IIB superstring theory on 10d Minkowski space again.

HyperKaehler magic square of N=2 SUGRAS

$d=5$	$(D_4,0)$	$(A_1^3, SL(2))$ $(D_4, SU(2))$	$(A_1^3, SU(2,1))$
$d=4$	$(D_4,0)$	$(A_1^3, SL(2))$ $(D_4, SU(2))$	$(A_1^3, SU(2,1))$
$d=3$	$(D_4,0)$	$(A_1^3, SL(2))$ $(D_4, SU(2))$	$(A_1^3, SU(2,1))$

Real magic del Pezzo's square

$d=6$	$\frac{SO(9,1)}{SO(9)}\Big\vert^{45}_{37}$	$\frac{SO(5,1)\times SO(3)}{SO(5)\times SO(3)}\Big\vert^{18}_{13}$	$\frac{SL(2,\mathbb{C})\times U(1)}{SU(2)\times U(1)}\Big\vert^{8}_{4}$
$d=5$	$\frac{E_6}{F_4}\Big\vert^{78}_{52}$	$\frac{SU^*(6)}{Usp(6)}\Big\vert^{35}_{21}$	$\frac{SL(3,\mathbb{C})}{SU(3)}\Big\vert^{17}_{8}$
$d=4$	$\frac{E_7}{E_6\times U(1)}\Big\vert^{133}_{69}$	$\frac{SO^*(12)}{U(6)}\Big\vert^{66}_{36}$	$\frac{SU(3,3)}{S(U(3)\times U(3))}\Big\vert^{35}_{17}$
$d=3$	$\frac{E_8}{E_7\times SU(2)}\Big\vert^{248}_{136}$	$\frac{E_7}{SO(12)\times SU(2)}\Big\vert^{133}_{79}$	$\frac{E_6}{SU(6)\times SU(2)}\Big\vert^{78}_{38}$

String models have recently been constructed for these SUGRAS via IIA asymmetric orbifolds

Y. Dolivet, B. Julia and C. Kounnas
arXiv:0712.2867(hep-th)

M. Bianchi and S. Ferrara
arXiv:0712.2976.(hep-th)

S. Bellucci, S. Ferrara, A. Marrani and A. Yeranyan, arXiv:0802.0141.(hep-th)
Phenomenology? BH Entropy computations

I. Motivations :
Finiteness of 4d N=8 SUGRA?

* Bern et al. argued that N=8 and N=4 may have same (non-)divergences for any D.
  Absence of three loop divergence for N=8 SUGRA in 4d.

*Berkovits' superfield argument in the pure spinor superstring formalism accounts for some non-renormalization theorems up to 6 loops for the R^4 terms and its derivatives!

M. Green et al., found that the discrete SL(2,Z) gauge symmetry of IIB superstring theory forbids some counterterms even in high dimensions.

The real Lie group of classical dualities becomes the modular group SL(2,Z) preserving the lattice of quantum charges.

For the IIB effective action expansion $R+R^4+R^7$... see also Damour, Nicolai+ coll.

ADE (real) U-dualities

* Dd to 3d reduced gravity has A_{D-3} U-duality symmetry: stationary pure 4d gravity leads to magic Ehlers A1=SL(2,R)=E'1
* In type I string model one finds D_k symmetries. In M-theory reduced to (11-n) dimensions one gets E_n split symmetry .
* Starting from 3d symmetric space models E_n(n)/K E_n one gets upon decompactification the Symmetric magic triangle.

CJLP hep-th/9909099

II From U-dualities

Chronology

70's D,N=4,16 G=SL(2,R) ZSFC

D,N=4,32 $G=E_7(7)$ JC

80's D,N=D>2,32 $G=E_{11-D}(11-D)$ JC

D,N=D>2,N<32 Magic triangle$\times A_{D-1}$ J

D,N=2,32 G=E8(1):=E9 J;BM;NJ…

D,N=1,32/0 E10:=E8^^ / D24^^ J;N…

II To E_r not quite dualities r>9

Chronology

2000 D,N=1,32	G=WE10	HD
2001 D,N=11,32	G=E_{11} ? A_{10} reps	W
2002 D,N=1,32	E_{10}/KE_{10} ?	NHD

1997 Cargèse Lectures

D,N=D>2,N=32 UxGL=E_{11-D}xR*xA_{D-1} J

II To V-dualities

D,N=D>2,N=32 UxGL=E_{11-D}xR*xA_{D-1} J;W

1997-99 D>2,N=32 V>U=E_{11-D} PLJC

D>2,N<32 V>U PLJC

New features of V:

- (Super)algebras but for IIB
- All p-forms p=0 or not, propagating OR NOT
- Universal self-duality equation

	$N=7$	$N=6$	$N=5$	$N=4$
$d=11$	$+$			
$d=10$	$R\ \vert^{1}_{0}$			
$d=9$	$\frac{SL(2)}{SO(2)}\times R\ \vert^{4}_{1}$			
$d=8$	$\frac{SL(3)\times SL(2)}{SO(3)\times SO(2)}\ \vert^{11}_{4}$			
$d=7$	$\frac{SL(5)}{SO(5)}\ \vert^{24}_{10}$			
$d=6$	$\frac{SO(5,5)}{SO(5)\times SO(5)}\ \vert^{45}_{20}$	$\frac{SO(5,1)\times SO(3)}{SO(5)\times SO(3)}\ \vert^{18}_{13}$		
$d=5$	$\frac{E_6}{Usp(8)}\ \vert^{78}_{36}$	$\frac{SU^{\star}(6)}{Usp(6)}\ \vert^{35}_{21}$		
$d=4$	$\frac{E_7}{SU(8)}\ \vert^{133}_{63}$	$\frac{SO^{\star}(12)}{U(6)}\ \vert^{66}_{36}$	$\frac{SU(5,1)}{U(5)}\ \vert^{35}_{25}$	$\frac{SU(4)\times SU(1,1)}{SU(4)\times SO(2)}\ \vert^{18}_{16}$
$d=3$	$\frac{E_8}{SO(16)}\ \vert^{248}_{120}$	$\frac{E_7}{S0(12)\times S0(3)}\ \vert^{133}_{69}$	$\frac{E_6}{SO(10)\times SO(2)}\ \vert^{78}_{46}$	$\frac{SO(8,2)}{SO(8)\times SO(2)}\ \vert^{45}_{29}$

U magic triangle of pure 4d SUGRAS

CJ, C, J 1980

D	$n=8$	$n=7$	$n=6$	$n=5$	$n=4$	$n=3$	$n=2$	$n=1$
11	$+$							
10	R, A_1	$+$						
9	RxA_1	R						
8	A_1xA_2	RxA_1, A_2	A_1					
7	E_4	$R \times A_2$	$R \times A_1$	R	$+$			
6	E_5	$A_1 \times A_3$	$R \times A_1^2$	$R^2,\ A_1^2$	R			
5	E_6	A_5	A_2^2	$R \times A_1^2$	$R \times A_1$	A_1		
4	E_7	D_6	A_5	$A_1 \times A_3$	$R \times A_2$	RxA_1, A_2	R	$+$
3	E_8	E_7	E_6	E_5	E_4	A_1xA_2	RxA_1	$R,\ A_1$

Table : Disintegration (Oxidations) for split E_n Cosets

C J L P 1999

H J P 2002

CP_2 blown up in (11-D) points in general position for n=8 or more generally with A(8-n) singularity

III Universal V-self-duality equation

CJLP noted enlargment of U- to V-
duality: a spin mixing (super)algebra
p-form coefficients x supergenerators of
degree –p such that total degree is 0
E is valued in V

$$E^{-1}dE=S\{*(E^{-1}dE)\}$$

$S\{Q\}=Q^{\sim}$ and $S\{Q^{\sim}\}=+/-\ Q$

D=11 Nonabelian p-forms

- A a 3-form, B (dual) 6-form, D=11

  F=dA, dF=0, d*F+F.F=0

  G=dB+A.F   dG +F.F=0

- $E=e^{AQ3}\ e^{BQ^{\sim}6}$

  $\{Q_3,Q_3\} = Q^{\sim}{}_6$ commuting with Q_3

- $E^{-1}dE=S\{*(E^{-1}dE)\}$

Self-dual field strength

Superalgebras can(not) be named

*** They are κ **deformations** of:

[SL(1|11-D)$^+$ |x grSym.3-tensor] x

[its dual (coadjoint) rep.]

But these algebras do not belong to a nice classification! No name after deformation… except in 11d: $\{Q_3, Q_3\} = \kappa\, \tilde{Q}_6$

this is the nilpotent part Osp(1|2)$^{++}$ of

Osp(1|2) : $E = e^{A Q_3}\, e^{\tilde{A}\, \tilde{Q}_6}$

*** Miracle? they are: **degree truncations (of Borels) of Borcherds superalgebras.**

Borcherds (super-)algebras

Among infinite dimensional algebras one meets first affine Kac-Moody algebras, next hyperbolic ones. But Borcherds algebras generalize symmetrizable Kac-Moody algebras (BgKM).

Generalized Kac-Moody: Cartan algebra + quadratic form +positive part from simple roots+ negative part.

Borel = Cartan torus + Positive root spaces

Cartan-Kac-Moody-Borcherds matrices

$$(i) \quad a_{ij} \leq 0 \quad \text{if } i \neq j \qquad (1)$$

$$(ii) \quad \frac{2a_{ij}}{a_{ii}} \in Z \quad \text{if } a_{ii} > 0 \qquad (2)$$

Chevalley-Serre-Borcherds relations

$$(1) \quad [e_{\alpha_i}, f_{\alpha_j}] = \delta_{ij} h_{\alpha_i} \qquad (1)$$

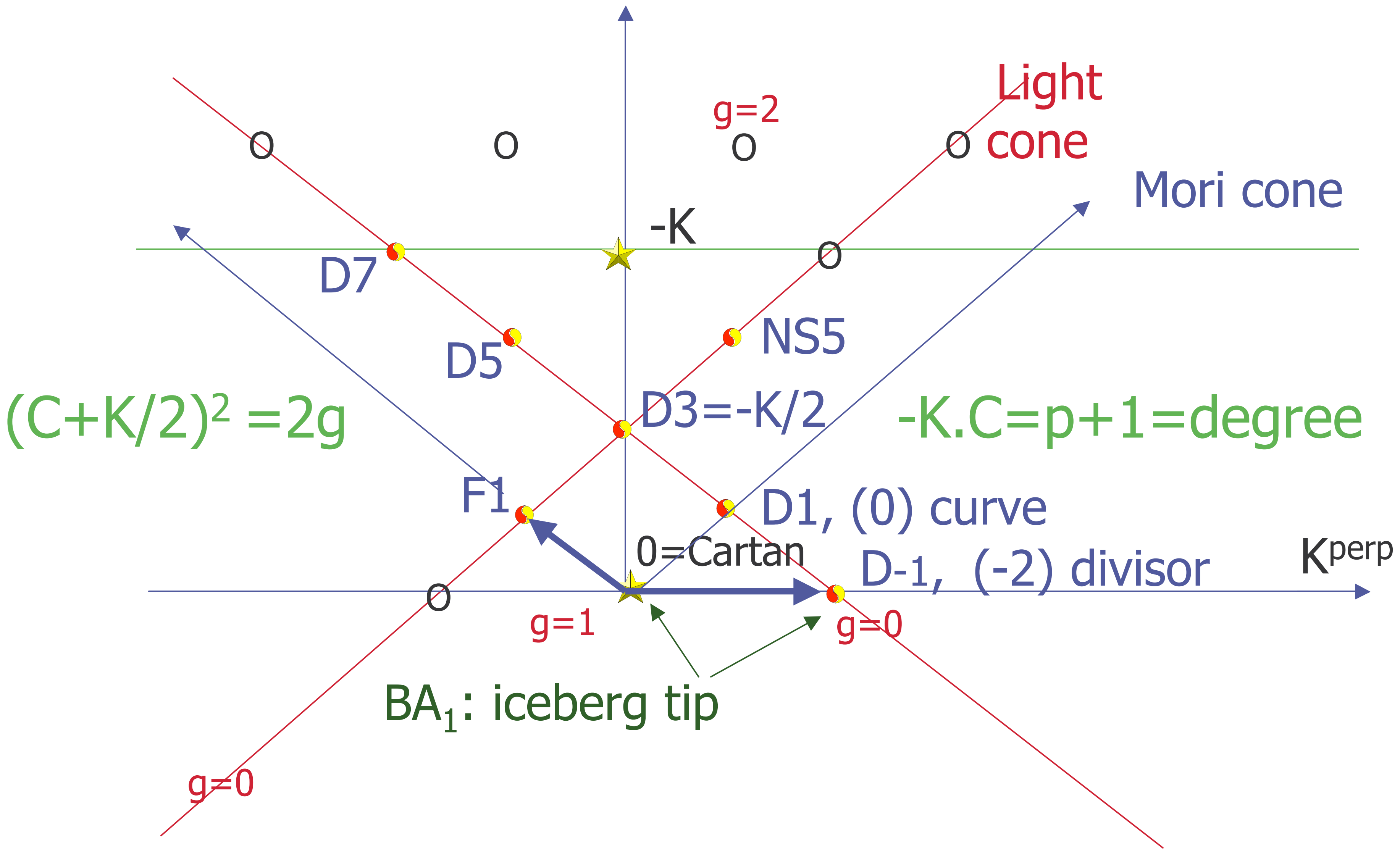
$$(2) \quad [h_{\alpha_j}, e_{\alpha_i}] = a_{ij} e_{\alpha_i}, [h_{\alpha_j}, f_{\alpha_i}] = -a_{ij} f_{\alpha_i} \qquad (2)$$

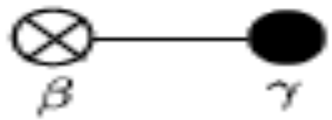
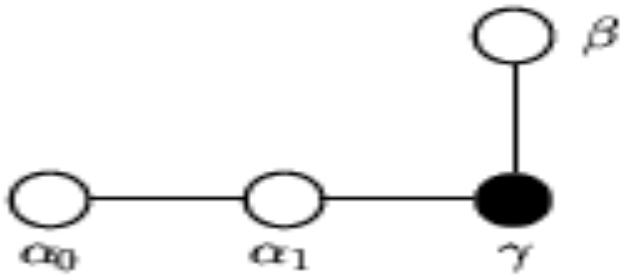
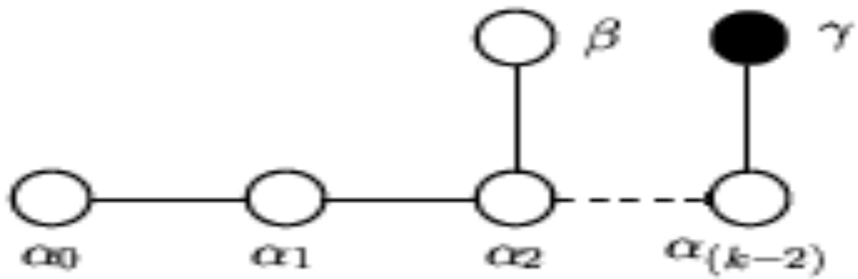
$$(3) \quad [h_{\alpha_i}, h_{\alpha_j}] = 0 \qquad (3)$$

$$(4) \quad ad(e_{\alpha_i})^{1-2\frac{a_{ij}}{a_{ii}}} e_{\alpha_j} = 0 = (ad(f_{\alpha_i}))^{1-2\frac{a_{ij}}{a_{ii}}} f_{\alpha_j} \text{ if } a_{ii} > 0 \qquad (4)$$

$$(5) \quad [e_{\alpha_i}, e_{\alpha_j}] = 0 = [f_{\alpha_i}, f_{\alpha_j}] \text{ if } a_{ij} = 0 \qquad (5)$$

IIB Iceberg: from BA_1 to BSA_2^{trc}



k	Simple superroot	Dynkin diagram
0	$\beta = H$	
1	$\beta = H - E_{11}$ $\gamma = E_{11}$	
2	$\alpha_0 = E_{11} - E_{10}$ $\beta = H - E_{11} - E_{10}$ $\gamma = E_{10}$	
3	$\alpha_0 = E_{11} - E_{10}$ $\alpha_1 = E_{10} - E_9$ $\beta = H - E_{11} - E_{10} - E_9$ $\gamma = E_9$	
4 to 8	$\alpha_i = E_{11-i} - E_{10-i},$ $0 \leq i \leq (k-2)$ $\beta = H - E_{11} - E_{10} - E_9$ $\gamma = E_{12-k}$	

A simple Borcherds algebra

Signature of Cartan matrix (1,1)

This is the signature of the homology of CP1xCP1

l_i are a basis of null vectors

Root lattice= Z + Z, simple roots: α_0, α_1

$l_i . l_j = 1 - \delta_{ij}$, $K = -2l_1 - 2l_2$, $K^{perp} = C(l_1 + l_2)$

$\alpha_0 = l_1 - l_2$, $\alpha_1 = l_2$

$[K]^{perp.}$ = Z = Root lattice of SL(2, (R or) C)

IIB Borcherds algebra

Cartan-Borcherds matrix :

$$\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix} .$$

Slansky's example 1993 of Borcherds algebra

(2) O--------⊗ (0)

= bosonic = IIB

IIA Superalgebra

(0_F) O--------⊗ (0)

Two relations between three theories (H-L,P,J 2002):

-V dualities (C,J,L,P 1998-99) of effective actions

-Borcherds Lie algebras (their truncated Borel)

-Algebraic surfaces (possibly singular del Pezzo surfaces)

e.g. IIB / Slanski algebra / CP1xCP1

Get V from U , from deformation, from Del Pezzo, from E_{11}

Benefits:

Systematic within group theory

Extends to lower dimensions: D=2 here

E_{11} was kinematic V=dynamical symmetry

Caveats:

E_{11} is too big V is too small

Needs stringy version

Thank you
You are welcome to join this project

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