E_7 description of N=2 backgrounds

Mariana Graña
CEA / Saclay
France

In collaboration with

Jan Louis, Aaron Sim, Daniel Waldram

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Motivation

Type II sugra on $M_{10} = M_4 \times_w M_6$

(N=2 eff. theory on M_4)

g + B $|_{M6}$ $\xrightarrow[\text{GCG}]{T \oplus T^*}$ Φ^+, Φ^- pure spinors $\xrightarrow[?]{+\,\mathrm{C}}$?

$(e^{B+iJ}, e^B\Omega_3)$

N=2 moduli spaces $\longrightarrow$ $\delta\Phi^+, \delta\Phi^-$ Special Kähler manifolds (Hitchin functionals) $\longrightarrow$ $\delta ?$ Special Kähler Quaternionic (?)

N=2 potentials $\longrightarrow$ $S_{AB} = \begin{pmatrix} \langle \Phi^+, d\Phi^- \rangle & \langle \Phi^+, F \rangle \\ \langle \Phi^+, F \rangle & \langle \Phi^+, d\bar{\Phi}^- \rangle \end{pmatrix}$ $\longrightarrow$ $S_{AB} = ?$

N=1 vacua $\longrightarrow$

$$d\Phi^+ = 0$$

$$d\Phi^- = F + i * F$$

$\longrightarrow$ $d\,? = 0$?

F: obstruction for integrability

Can we geometrize C?

Outline

-1. The setup

0. g + B $\xrightarrow[\text{GCG}]{T\oplus T^*}$ Φ^+, Φ^-

1. g + B + C $\xrightarrow[?]{}$

2. Pure spinors $\longrightarrow$?

3. N=2 moduli spaces $\longrightarrow$ $\delta ?$

4. N=2 potentials $\longrightarrow$ $S_{AB} = ?$

5. N=1 vacua $\longrightarrow$ $d\,? = 0$

-1. Pre-introduction : the setup

- Type II sugra on M$_{10}$ = M$_4$ x$_w$ M$_6$ $$ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + ds^2_6(y)$$
- Require effective theory on M$_4$ to have N=2 susy

Spinors $\epsilon^1 = \theta^1_+ \otimes \eta^1_+ + c.c.$

$$\epsilon^2 = \theta^2_+ \otimes \underbrace{\eta^2_+}_{} + c.c.$$

SU(3) x SU(3)
structure on T⊕T*

Off-shell SUSY:∃η^{1,2} → Topological condition

SU(3)x SU(3) structure on T⊕T*

On-shell SUSY:∃η / → Differential condition

$\delta_\epsilon\psi_m = 0$, $\delta_\epsilon\lambda = 0$ integrability of structure

∃ nowhere vanishing η : SU(3) structure

TM patched using g ∈ SU(3)

Spin(6) ≃ SO(6) ≃ SU(4) , η in 4 ⟶ 3 + 1

Stabilizer group G = {g ∈ Spin(6): g · η = η }= SU(3)

SO(6) ⟶ SU(3)

2-form 15 ⟶ 8 + 3 + $\bar{3}$ + 1 ← J

3-form 20 ⟶ 6 +$\bar{6}$+ 3 +$\bar{3}$+ 1+1 ← ρ

J : Sp(6,R) structure ↘

ρ : SL(3,C) structure ↗

∩

SU(3)

Generalized complex structures SU(3,3) on T⊕T*

Equivalent to nowhere vanishing ρ and J: $J\wedge\rho = 0$ → determine metric

0. g + B $\xrightarrow[\text{GCG}]{T\oplus T^*}$ Φ^+, Φ^-

Generalized complex geometry

Hitchin 02
Gualtieri 04

- Differential geometry on $T \oplus T^*$ sections are $x + \zeta = X$

Natural metric I on $T \oplus T^*$: $(x + \zeta , y + \eta) = i_x \eta + i_y \zeta$ $\quad I = \begin{pmatrix} 0 & 1_d \\ 1_d & 0 \end{pmatrix}$

On $T \oplus T^*$ define Generalized Almost Complex Structure (GACS) $\mathcal{J}$: $T\oplus T^* \to T\oplus T^*$

$$\boxed{\begin{array}{l} \mathcal{J}^2 = -1_{2d} \\ \mathcal{J}^t\, I\, \mathcal{J} = I \end{array}}$$

$\Pi_\pm$ projector onto holo/antiholo V

Integrability: $\Pi_-[\Pi_+ X, \Pi_+ Y]_C = 0$ $\quad [x+\zeta, y+\eta]_C = [x,y] + \mathcal{L}_x\eta - \mathcal{L}_y\zeta - \frac{1}{2}d(\iota_x\eta - \iota_y\zeta)$

Spinors Φ of O(6,6) : = p-forms

Weyl: positive chirality $S^+ \sim \Lambda^{\text{even}}$

negative chirality $S^- \sim \Lambda^{\text{odd}}$

Spinor bilinear: Mukai pairing

$$\Psi^\dagger\Phi \sim \langle\Psi, \Phi\rangle = \sum_p (-1)^{[p/2]}\Psi_p \wedge \Phi_{6-p}$$

Clifford action: $X^A\Gamma_A \underset{\substack{\uparrow \\ \Phi^+}}{\Phi} = \underbrace{x^m\, \iota_m\, \Phi + \zeta_m\, dx^m \wedge \Phi}_{\Phi^-}$

Pure spinor: annihilator space is maximal (6-dimensional)

$(v+\zeta) \in T \oplus T^*$ s.t. $(v+\zeta) \bullet \Phi = 0$

1-1 correspondance between pure spinors and generalized almost complex structures $\mathcal{J}$

$d\,\Phi = 0 \longrightarrow \mathcal{J}_\phi$ integrable   generalized Calabi-Yau manifold

$\exists$ pure spinor $\Phi \longrightarrow$ SU(3,3) structure on T⊕T*:

$\exists$ 2 compatible (3 common ann.)
pure spinors $\Phi^{1,2} \longrightarrow$ SU(3) x SU(3) structure on T⊕T*:

$$\langle \Phi_+, \Gamma^A \Phi_- \rangle = 0$$

Φ^1, Φ^2 compatible define metric and B-field:

$$\underset{\uparrow}{\mathcal{G}} = -I\mathcal{J}_1\mathcal{J}_2 = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix}$$

generalized metric

Consider O(6) x O(6) spinors

$$\Phi_\pm = e^B \eta^1_+ \otimes \eta^{2\,\dagger}_\pm = e^B \Phi^0_\pm$$ sum of forms

$$(\Phi^0_\pm)_{m1\ldots mp} = Tr(\Phi^0_\pm \gamma_{m1\ldots mp})$$
$$= \eta^{2\,\dagger}_\pm \gamma_{m1\ldots mp} \eta^1_+$$

Ex:

- $\eta^1 = \eta^2$ (SU(3) structure)
  - $\Phi_- = e^B \Omega_3$ → complex structure
  - $\Phi_+ = e^{B+iJ}$ → symplectic structure
- $\eta^2 = z \cdot \eta^1$ (static SU(2))
  - $\Phi_- = e^B z \wedge e^{ij}$ → 1d complex, 2d symplectic
  - $\Phi_+ = e^B \Omega_2 \wedge e^{z \wedge \bar{z}}$ → 2d complex, 1d symplectic

} generalized complex structures

Pure spinors $\Phi_\pm$ carry all information about metric and B-field on manifold

algebraic structures and B-field on manifold

1. g + B $\xrightarrow[\text{GCG}]{} T \oplus T^* \xrightarrow[?]{}$

g + B $\longrightarrow T \oplus T^*$ string momentum and winding

O(6,6) T-duality group

GCG

Hitchin 02
Gualtieri 04

g + B + C $\longrightarrow T \oplus T^* \oplus S^+ \oplus \Lambda^5 T^* \oplus \Lambda^5 T$

(Type IIA)

string momentum and winding
D-brane charges
5-brane charge + KK monopole

$E_{7(7)}$ U-duality group

EGG

Hull 07
Pires Pacheco ,Waldram 08

- Exceptional generalised tangent space ✓
- Exceptional generalised metric ✓
- Exceptional Courant bracket ✓

2. g + B $\longrightarrow$ Φ^+, Φ^- pure spinors $\xrightarrow[\text{EGG}]{+\,\text{C}}$?

E7 $\longrightarrow$ SL(2,R) x O(6,6)

S-duality $S = \overset{\substack{B_{\mu\nu}\\ \downarrow}}{a} + i\, e^{-2\varphi}$

56 $\longrightarrow$ (**2**,**12**) + (**1**,**32**)

$\lambda^{\mathcal{A}} \longrightarrow [\ \lambda^{A}_{i}\ ,\ \lambda^{\pm}\ ]$

$$\lambda = [0, \mathrm{Re}(\Phi^+)]$$

133 $\longrightarrow$ (**3**,**1**) + (**1**,**66**) + (**2**,**32**)

$\mu^{\mathcal{AB}} \longrightarrow [\,\mu^{i}{}_{j}\ ,\ \mu^{A}{}_{B}\ ,\ \mu^{i\pm}\,]$

To embed ϕ^- need a triplet of $SU(2)_R$

SL(2,R) doublet

$$\mu_+ = \mu_1 + i\,\mu_2 = [0, 0, u^i\bar{\Phi}^-]$$

$$\mu_- = \mu_1 - i\,\mu_2 = [0, 0, \bar{u}^i\Phi^-]$$

$\vdots$ μ_3

133 ⟶ **(3,1) + (1,66) + (2,32)**

$$\mu^{\mathcal{AB}} \longrightarrow [\,\mu^{i}{}_{j}\;,\quad \mu^{A}{}_{B}\;,\quad \mu^{i-}\,]$$

SL(2,R) doublet adds 2 dof ~ S ?

$$\mu_+ = \mu_1 + i\,\mu_2 = [0, 0, u^i\bar{\Phi}^-]$$

$$\mu_- = \mu_1 - i\,\mu_2 = [0, 0, \bar{u}^i\Phi^-]$$

$$\mu_3 = [u^i\bar{u}_j + c.c., |u|^2\mathcal{J}^A{}_B, 0]$$

$$\Phi^- \to c\Phi^-$$

$$u^i \to c^{-1}u^i$$

$$[\mu_a, \mu_b] = i\epsilon_{abc}\mu_c$$ SU(2) algebra

$$\mathcal{J}_{AB} = -i\frac{\langle\bar{\Phi}^-, \Gamma_{AB}\Phi^-\rangle}{\langle\bar{\Phi}^-, \Phi^-\rangle}$$

$$\frac{1}{12}\mathcal{J}_{AB}\Gamma^{AB}\Phi^- = i\Phi^-$$

Where is C$^-$??

In the last component of μ_3 ?

Sort of, but no so fast ...

O(6,6) : **B-field** $\Phi^\pm = e^B \eta^1_+ \eta^{2\dagger}_\pm = e^B \Phi^\pm_0$ $B = \begin{pmatrix} 0 & 0 \\ B_{mn} & 0 \end{pmatrix}$

E7 : **C-field**

$$\lambda = e^{\mathcal{C}} \lambda_0$$

$$\mu = e^{\mathcal{C}} \mu_0$$

↑ adjoint action

$$\mathcal{C} \in \mathbf{133}$$

$$\mathcal{C} = [\; 0 \;, \; 0 \;, v^i C^- \,]$$

rotates u^i

rotates ϕ

arbitrary v^i

$$\mu_+ = e^{\mathcal{C}} \, [0, 0, u^i \bar{\Phi}^-] = [\cdots, \cdots, u^i \bar{\Phi}^-]$$

$$\mu_3 = e^{\mathcal{C}} \, [u^i \bar{u}_j + c.c., |u|^2 \mathcal{J}^A{}_B, 0] = [\cdots, \cdots, \mathrm{Re} u^i \, \mathrm{C}^-]$$

$$\lambda = e^{\mathcal{C}} [0, \mathrm{Re}\Phi^+] = [v^i \langle C, \Gamma^A \lambda \rangle, \mathrm{Re}\Phi^+]$$

E7 ⟶ SL(2,R) x O(6,6)

56 ⟶ (**2**,**12**) + (**1**,**32**)

$\lambda^{\mathcal{A}} \longrightarrow [\lambda^A_i, \lambda^+]$

133 ⟶ (**3**,**1**) + (**1**,**66**) + (**2**,**32**)

$\mu^{\mathcal{AB}} \longrightarrow [\mu^i{}_j, \mu^A{}_B, \mu^{i-}]$

3. N=2 moduli spaces

$\delta\Phi^+$ Special Kähler manifolds (Hitchin functionals) $\longrightarrow$ $\delta\lambda$

$\delta\Phi^-$ $\delta ?$

$\delta\mu$

$\mathrm{Re}\Phi^+$ defines SU(3,3) ⊂ O(6,6) $\longrightarrow$ λ defines E₆ ⊂ E₇

$$\delta\mathrm{Re}\Phi^+ : \frac{O(6,6)}{SU(3,3)} \times \mathbf{R}^+$$

32 dim

special Kähler

$$K = \sqrt{H}$$

Hitchin functional
quartic invt of O(6,6)

$$= \sqrt{\langle \mathrm{Re}\Phi^+, \Gamma_{\mathrm{AB}}\mathrm{Re}\Phi^+\rangle\langle \mathrm{Re}\Phi^+, \Gamma^{\mathrm{AB}}\mathrm{Re}\Phi^+\rangle}$$

$$= i\langle \Phi^+, \bar{\Phi}^+\rangle$$

$$\delta\lambda \ : \frac{E_7}{E_6} \times \mathbf{R}^+$$

56 dim

special Kähler

Ceccotti 89
de Wit, Van Proeyen 93

$$K = \sqrt{q}|_\lambda$$

quartic invt of E₇

$$= \sqrt{\langle \mathrm{Re}\Phi^+, \Gamma_{\mathrm{AB}}\mathrm{Re}\Phi^+\rangle\langle \mathrm{Re}\Phi^+, \Gamma_{\mathrm{AB}}\mathrm{Re}\Phi^+\rangle}$$

$$= i\langle \Phi^+, \bar{\Phi}^+\rangle$$

$$\lambda = e^C[0, \mathrm{Re}\Phi^+]$$

3. N=2 moduli spaces

$\delta\Phi^-$ Special Kähler manifolds (Hitchin functionals) $\xrightarrow[\text{c-map}]{+\,C}$ $\delta\mu$ quaternionic

Wolf 65
Alekseevski 75

$$\delta\Phi^- \quad \frac{O(6,6)}{SU(3,3)\times U(1)} \xrightarrow[\text{c-map}]{} \frac{E_7}{SO(12)\times SU(2)}$$

Ceccotti, Ferrara, Girardello 89
de Wit, Van Proeyen 93

$SU(2) \longrightarrow$ stabilizer μ

local special Kähler

quaternionic

$\downarrow$ + compensator

orbit of μ_a

68 dim

$$\left\{ \frac{E_7}{SO(12)} \times \mathbf{R}^+ \right.$$

hyperKähler

$$\mu_+ = e^C \; [0, 0, u^i \bar{\Phi}^-]$$

$$\mu_3 = e^C \; [u^i \bar{u}_j + c.c., |u|^2 \mathcal{J}^A{}_B, 0]$$

triplet of complex structures

SU(2)$_R$ rotates among them

Kobak, Swann 00

$$\chi = \sqrt{tr(\mu_+\mu_-)} = \sqrt{|u|^2\langle\Phi^-, \bar{\Phi}^-\rangle}$$

$$= e^{-\varphi} e^{\frac{1}{2}K^-} \quad \Rightarrow$$

agree if

Rocek, Vafa, Vandoren 06

$$|u|^2 = e^{-2\varphi}$$

will see consistency

4. N=2 potentials

$$S_{AB} = \begin{pmatrix} \langle \Phi^+, d\Phi^- \rangle & \langle \Phi^+, F \rangle \\ \langle \Phi^+, F \rangle & \langle \Phi^+, d\bar{\Phi}^- \rangle \end{pmatrix} = e^{K^+} \sigma^a_{AB} P_a \longrightarrow S_{AB} = ?$$

M.G, Louis, Waldram 05

$$P_+ = e^{\frac{1}{2}K^- + \varphi} \langle \boldsymbol{\Phi}^+, \boldsymbol{d\Phi}^- \rangle$$

$$P_3 = e^{2\varphi} \langle \Phi^+, F \rangle \qquad \longrightarrow \qquad P_a = S(\lambda\, , D\cdot\, \mu_a)$$

Agreement if

$$|u|^2 = e^{-2\varphi}$$

✓Same requirement than for χ

d **6** of O(6) or 12 of O(6,6) → (2,12) ∈ 56 of E7

S : symplectic product in E7 ⊂ Sp(56, R)

$$d \longrightarrow D = [v^i d, 0]$$

$$S(\lambda, \lambda') = \epsilon_{ij} \lambda^{iA} \lambda'^{jB} \eta_{AB} + \langle \lambda^+, \lambda'^+ \rangle$$

$$\lambda = e^{\mathcal{C}} [0, \mathrm{Re}\Phi^+]$$

$$\mu_+ = e^{\mathcal{C}}\ [0, 0, u^i \bar{\Phi}^-]$$

$$\mu_3 = e^{\mathcal{C}}\ [u^i \bar{u}_j + c.c., |u|^2 \mathcal{J}^A{}_B, 0]$$

$$\mathcal{C} = [0, 0, v^i C^-]$$

E7 → SL(2,R) x O(6,6)

56 → (**2**,**12**) + (**1**,**32**)

$$\lambda^{\mathcal{A}} \longrightarrow [\lambda^A_i, \lambda^+]$$

133 → (**3**,**1**) + (**1**,**66**) + (**2**,**32**)

$$\mu^{\mathcal{AB}} \longrightarrow [\mu^i{}_j, \mu^A{}_B, \mu^{i-}]$$

5. N=1 vacua

$$d\Phi^+ = 0$$

$$d\Phi^- = (|a|^2 + |b|^2) F + i(|a|^2 + |b|^2) * F$$

$$d\Phi^- = F + i * F$$

M.G, Minasian, Petrini, Tomasiello 05

$$\longrightarrow$$

$$D\lambda_C = 0$$

$$d\, ? = 0\ ?$$

$$D\mu'_3 = 0$$

$$D\mu'_+|_{(0,1)} = 0$$

$$\Phi^+ = e^{-\varphi} a\bar{b}\, \eta^1_+ \eta^{2\dagger}_+$$

$$\Phi^- = e^{-\varphi} ab\, \eta^1_+ \eta^{2\dagger}_-$$

$$\begin{pmatrix} \epsilon^1 \\ \epsilon^2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \epsilon$$

U(1)$_R$ ⊂ SU(2)$_R$ $\longrightarrow$ $(r_+, r_-, r^3) = (\bar{a}\bar{b}, ab, |a|^2 - |b|^2)$

D-term : $r^a P_a$ $\qquad r^a \mu_a = \mu'_3$

Superpot. : $\omega^a P_a$ $\qquad \omega^a \mu_a = \mu'_+$

Vanishing of variation of superpotential and D-term equiv. to N=1 vacua eqs.

Koerber, Martucci 07
Bilal, Cassani 07

$$\lambda = e^{\mathcal{C}}[0, \mathrm{Re}\Phi^+] \quad \lambda_C = e^{\mathcal{C}}[0, \Phi^+]$$

$$\mu_+ = e^{\mathcal{C}}\ [0, 0, u^i \bar{\Phi}^-]$$

$$\mu_3 = e^{\mathcal{C}}\ [u^i \bar{u}_j + c.c., |u|^2 \mathcal{J}^A{}_B, 0]$$

Conclusions

O(6,6) ➔ E_7 : Geometrized C-field

- Relevant structures containing all dof: λ, μ_a
- Special Kähler and quaternionic moduli spaces: orbits of λ, μ_a

  - Kähler and hyperKähler potentials : invariants of E_7 specialized to orbits λ, μ_a
- Gravitino mass matrix (or Killing prepotentials) $P_a = S(\lambda\,, D\cdot\,\mu_a)$
- N=1 vacua equations : λ, μ'_3 integrable structures

Generalized complex geometry is a nice tool for a systematic description of flux backgrounds.
But may be strings see SO(d,d) structure of T⊕T* and we will learn more

Generalized complex geometry is tailor made for a systematic description of flux backgrounds.
Strings see SO(d,d) structure of T⊕T*

Exceptional generalized geometry is a nice tool
for a systematic description of flux backgrounds.
But may be strings see E_7 structure of ETS
and we will learn more