Topics in String Phenomenology

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Abstract

Notes taken by Cristina Timirgaziu of lectures by Angel Uranga in June 2009 at the Galileo Galilei Institute School "New Perspectives in String Theory". Topics include intersecting D-branes models, magnetized D-branes and an introduction to F-theory phenomenology.

1 Introduction

String phenomenology deals with building string models of particle physics. The goal is to find a generic scenario or even predictions at TeV scale. Topics in string phenomenology include heterotic strings model building both on smooth (Calabi-Yau) and singular (orbifold) manifolds and model building in Type II strings. The later has several branches: intersecting D-branes (type IIA strings), magnetized D-branes (type IIB), as well as F-theory models.

Common problems in string phenomenology include the issue of supersymmetry breaking, moduli stabilization, flux compactifications, non perturbative effects and applications to cosmology, in particular string inflation.

These lectures are concerned with model building using D-branes.

2 Intersecting D-branes

Model building using intersecting D-branes is a very active field in string phenomenology and many reviews are already present in the littereature, including [1]- [5]. For a review of recent progress see [6].

2.1 Basics of intersecting D-branes

In the weak coupling limit D-branes can be well described in the probe approximation as hyperplanes where open strings can end. A number of N overlapping D-branes generates a $U(N)$ gauge theory with 16 supercharges, which corresponds to $\mathcal{N} = 4$ supersymmetry in four dimensions. A Dp-brane, extending in the spacial directions $x^0 ... \ x^p$, breaks half the supercharges and the surving supersymmetry is given by $Q = \epsilon_L Q_L + \epsilon_R Q_R$, where Q_L and Q_R are left and right moving spacetime supercharges and $\epsilon_L = \Gamma^0\Gamma^1 ....\Gamma^p \epsilon_R$.[1]

The worldvolume dynamics of a Dp-brane is described by the Born-Infeld and Wess Zumino actions

$$S = -T_p \int d^{p+1}\xi \ e^{-\phi} \sqrt{-\det(G + 2\pi\alpha' F)} + \int C_{p+1},$$

where T_p is the tension of the brane, ϕ is the dilaton, G - the induced metric on the D-brane, F - the field strength of the world volume gauge field and C_{p+1} is the $p+1$ form that couples to the D-brane.

Dp-branes give rise to non-abelian gauge interactions and also to four dimensional chiral fermions provided the $\mathcal{N} = 4$ supersymmetry is broken to $\mathcal{N} = 1$ at the most. In order to obtain 4d chirality the six dimensional internal parity must be broken, since the 16 supercharges in ten dimensions split as follows under the breaking of the $SO(10)$ symmetry

$$SO(10) \to SO(6) \times SO(4); \quad 16 \to (4, 2_L) + (\bar{4}, 2_R).$$

[1] Γ^i denote de Dirac matrices.

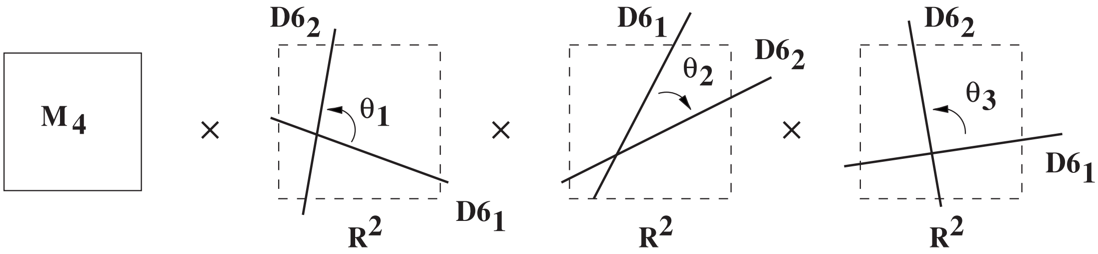


Figure 1: Intersecting D6-branes.

Consider two D-branes intersecting as in fig. 1, where a preferred orientation has been defined. The preferred orientation breaks the six dimensional parity. For phenomenological purposes we need to consider two stacks of N_1 and N_2 type IIA D6-branes overlapping over a 4d subspace and intersecting at angles θ_1, θ_2 and θ_3 in three 2-planes. The spectrum of open strings in this configuration contains several sectors

- 1-1 strings generate a $U(N_1)$ SYM theory in 7 dimensions with 16 supercharges
- 2-2 strings similarly lead to a $U(N_2)$ SYM theory in 7 dimensions with 16 supercharges
- 1-2 strings generate massless chiral fermions in the $(N_1, \bar{N}_2)$ representation in 4 dimensions[2] (since these states are located at the intersection of the two stacks), as well as other states, which could potentially be light scalars
- 2-1 strings generate the antiparticles of the states in sector 1-2.

The light scalars in the sector 1-2 exhibit the following masses[3]

$$\begin{aligned} \alpha' m^2 &= \frac{1}{2}(\theta_1 + \theta_2 + \theta_3) \\ \alpha' m^2 &= \frac{1}{2}(-\theta_1 + \theta_2 + \theta_3) \\ \alpha' m^2 &= \frac{1}{2}(\theta_1 - \theta_2 + \theta_3) \\ \alpha' m^2 &= \frac{1}{2}(\theta_1 + \theta_2 - \theta_3) \end{aligned} \tag{1}$$

[2]These chiral fermions leave in 4d because, due to the mixed boundary conditions (Neuman-Dirichlet) of the open strings stretched between the two stacks of D-branes, the zero modes of a Ramond fermion, corresponding to the 6d transverse space are not present.

[3]The oscillator modes in the expansion of the open strings will be shifted by $\pm\theta_i$ as in $b^{\alpha}_{-1/2+\theta_i}$ and this change will show in the mass of the states through contribution to the zero point energy.

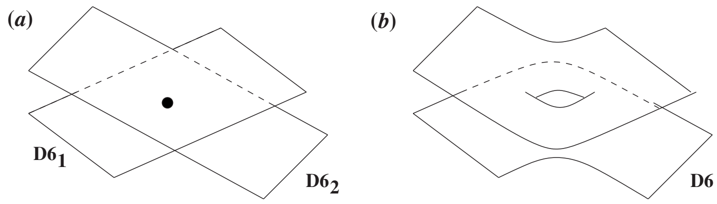


Figure 2: Recombination of two branes.

Generically the scalar spectrum laid out in (1) contains no massless states, in which case there is no supersymmetry preserved in the theory. The scalars in the sector 1-2 can also be tachyons indicating the instability of the brane configuration. In this case recombination of the branes will lead to a bound BPS state displaying the phenomenon of wall crossing (see figure 2). The initial configuration is described by a scalar potential, parametrized by a scalar ϕ with charges 1 and -1 under the gauge group generated by the two D-branes, $U(1) \times U(1)$

$$V_D = (|\phi|^2 + \xi)^2,$$

where the Fayet-Illiopoulos term is related to the intersection angles $\xi = \theta_1 + \theta_2 + \theta_3$. Three situations can present

- the massive case, $\xi > 0$: the minimum of V_D is at $<\phi>= 0$ and the $U(1) \times U(1)$ gauge group is unbroken
- the tachyonic case, $\xi < 0$: the scalar ϕ gets a VEV which breaks $U(1) \times U(1)$ to one $U(1)$ (one brane)
- the massless case, $\xi = 0$: supersymmetric case, $V_D = 0$, stable $U(1)^2$ gauge group.

Let's see under which conditions some supersymmetry can be preserved by configurations of intersecting D-branes. Remember that the supersymmetry transformations preserved by a D-brane are of the form $\epsilon_L Q_L + \epsilon_R Q_R$, where for two stacks of branes the spinor coefficients satisfy

$$\epsilon_L^1 = \Gamma^0 ... \Gamma^3 \; \Gamma_{\pi_1} \; \epsilon_R^1$$
$$\epsilon_L^2 = \Gamma^0 ... \Gamma^3 \; \Gamma_{\pi_2} \; \epsilon_R^2$$

Here π_1 and π_2 denote the compact directions of branes D6_1 and D6_2 respectively. Generically no supersymmetry transformations survive both conditions, but for special choices of the angles θ_i there may exist solutions. If R is the transformation that rotates the D6_1 branes into the D6_2 branes, $\Gamma_{\pi_1} = R \; \Gamma_{\pi_2} \; R^{-1}$, then R must be an element of the $SU(3)$ subgroup of the $SO(6)$ rotations group. If we assume the diagonal form of R

$$R = \text{diag}(e^{\theta_1}, e^{-\theta_1}, e^{\theta_2}, e^{-\theta_2}, e^{\theta_3}, e^{-\theta_3}),$$

the determinant of any sub three matrix should be zero, leading to one massless scalar among the states in (1) and, hence to $\mathcal{N}=1$ supersymmetry. The condition on the angles reads

$$\theta_1 \pm \theta_2 \pm \theta_3 = 0. \qquad (2)$$

A special case presents when one of the angles, say θ_3, is zero. The condition (2) becomes $\theta_1 = \pm\theta_2$, the rotation R is an element of $SU(2)$ and the configuration preserves $\mathcal{N}=2$ supersymmetry. In this case the spectrum is not chiral, but this distribution of D-branes can serve to generate the Higgs states of the MSSM in the hypermultiplet of $\mathcal{N}=2$. Spatial separation of the branes in the parallel directions allows to generate a mass for the Higgs.

2.2 Toroidal compactifications

Consider Type IIA theory compactified on a product of three 2-tori, $M^4 \times T^2 \times T^2 \times T^2$, and several stacks of N_a D6_a-branes wrapped on three-cycles Π_a factorized as the product of one-cycles with wrapping numbers (n_a^i, m_a^i), where i labels the i-th 2-tous and a labels the stack.

The homology class of the 3-cycles decomposes in a basis

$$\Pi_a = \prod_{i=1}^{3}(n_a^1[a_i] + m_a^1[b_i]),$$

with $[a_i]$ and $[b_i]$ being the fundamental 1-cycles of the torus T_i^2.

The chiral spectrum is given by

- aa strings give rise to a four dimensional $U(N_a)$ SYM
- ab strings generated four dimensional chiral fermions in the bi-fundamental representation $(N_a, \bar{N}_b)$ with multiplicity given by the number of intersections between stacks a and b, $I_{ab} = [\Pi_a]\cdot[\Pi_b] = \prod_i (n_a^i m_b^i - n_b^i m_a^i)$.

An important consistency condition of intersecting brane models is the RR tadpole cancellation, which arises from the Gauss law for RR-fields. The RR fields carry D-brane charges and in a compact space the total RR charge must vanish(flux lines cannot escape). The RR tadpole cancellation can be phrased as consistency of the equations of motion of the RR-fields. the D6-branes introduced previously are charged with respect to a 7-form C_7. The equation of motion for C_7 is derived from the spacetime action

$$\begin{aligned} S_{C_7} &= \int_{10d} H_8 \wedge *H_8 + \sum_a N_a \int_{M_4\times\Pi_a} C_7 \\ &= \int_{10d} C_7 \wedge dH_2 + \sum_a N_a \int_{10d} C_7 \wedge \delta(\Pi_a), \end{aligned}$$

where $H_8 = dC_7$ is the field strength of C_7, $H_2 = *_{10d}\, H_8$ its Hodge dual and $\delta(\Pi_a)$ a bump 3-form localized on Π_a. The equation of motion of C_7 reads then

$$dH_2 = \sum_a N_a\ \delta(\Pi_a),$$

which taken in homology yelds

$$0 = \sum_a N_a\ [\Pi_a].$$

Cancellation of the RR tadpoles implies cancellation of four-dimensional chiral anomalies in the effective field theory. The cubic $SU(N_a)^3$ anomalies are given by

$$A_a = \#\Box_a - \#\bar{\Box}_a = \sum_b N_b\ I_{ab},$$

where $\#\Box_a$ denotes the number of chiral fermions in the fundamental representation. These anomalies vanish in virtue of

$$\sum_b N_b\ [\Pi_b] = 0 \rightarrow \sum_b N_b\ [\Pi_a] \cdot [\Pi_b] = 0.$$

In contrast, the mixed $U(1)_a - SU(N_b)^2$ triangle anomaly is generically non zero

$$A_{ab} = \#\Box_{b,q_a=1} - \#\Box_{b,q_a=-1} = N_a\ I_{ab} \neq 0.$$

This anomaly is cancelled via the Green Schwarz Sagnotti mechanism thanks to the extra couplings

$$\int_{D6_a} C_5 \wedge \mathrm{tr} F_a + \int_{D6_b} C_3 \wedge \mathrm{tr}(F_b \wedge F_b),$$

which reduce in four dimension to

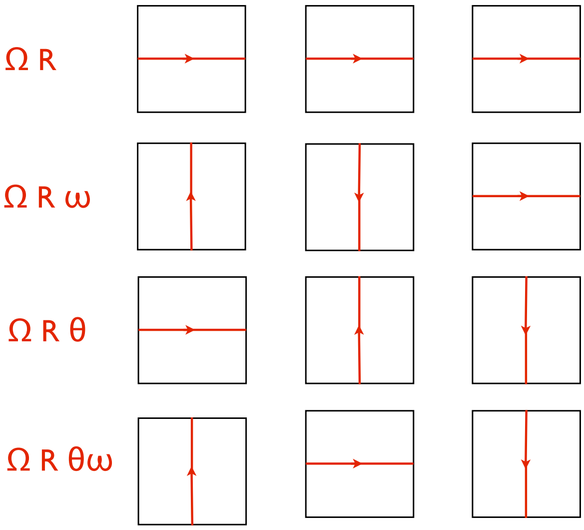
$$\int_{4d} (B_2)a \wedge \mathrm{tr} F_a + \int_{4d} \phi_b \wedge \mathrm{tr}(F_b \wedge F_b),$$

with $(B_2)a = \int_{[\Pi_a]} C_5$, $\phi_b = \int_{[\Pi_b]} C_3$ and $d\phi_b = -\delta_{ab} *_{4d} (B_2)_a$.

Any $U(1)$ coupling to the 2-form B_{2a} will become massive and many $U(1)$'s in the theory receive a mass in this way, which is a welcomed phenomenological feature. We have to make sure though that the hypercharge $U(1)_Y$ stays massless.

2.3 Torodial compactifications with O6-planes

Consider IIA theory compactified on T^6 and mod out by the symmetry $\Omega R(-1)^{f_L}$, with Ω the worldsheet parity, f_L the left moving worldsheet fermion number and R an antiholomorphic involution $z_i \rightarrow \bar{z}_i$. The ΩR symmetry introduces O6-planes in the theory. When introducing a stack of N_a D6$_a$-branes with wrapping numbers (n_a^i, m_a^i), the presence of



The tadpole condition (3) takes the detailled form

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16$$
$$-\sum_a N_a m_a^1 n_a^2 n_a^3 = 16$$
$$-\sum_a N_a n_a^1 m_a^2 n_a^3 = 16$$
$$-\sum_a N_a n_a^1 n_a^2 m_a^3 = 16$$

Figure 3: O-planes in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds.

the O6-planes requires also the introduction of their images in respect with the O-planes, called D6'$_a$-branes, with wrapping numbers $(n_a^i, -m_a^i)$, which characterize the 3-cycle $[\Pi'_a]$.

The RR tadpole cancellation condition is then modified to

$$\sum_a N_a\ [\Pi_a] + \sum_a N_a\ [\Pi'_a] + \sum(-4)[\Pi_{O6}] = 0. \tag{3}$$

The gauge group of the theory is given by $\otimes_a\ U(N_a)$ and the chiral fermions come from sectors

- ab : I_{ab} fermions in the $(\square_a, \bar{\square}_b)$
- ab' : $I_{ab'}$ fermions in the $(\square_a, \square_b)$.

2.4 Toroidal orbifold compactifications

We consider the orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, generated by the elements

$$\begin{aligned}\theta &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)\\ \omega &: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)\\ \theta\omega &: (z_1, z_2, z_3) \rightarrow (-z_1, z_2, -z_3)\end{aligned}$$

and we mod out by ΩR, with $R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3)$. Four types of orientifold planes are present in the theory, generated by the symmetries ΩR, $\Omega R\theta$, $\Omega R\omega$ and $\Omega R\theta\omega$, as depicted in figure 3.

Let's consider the explicit example [7] in table 1. The visible gauge group of this model is $U(3)_a \times U(1)_d \times SU(2)_b \times SU(2)_c$, where we used the fact that N D-branes parallel to the

	N	(n_a^1, m_a^1)	(n_a^2, m_a^2)	(n_a^3, m_a^3)
a, d	$6+2$	$(1,0)$	$(3,1)$	$(3,-1)$
b	2	$(0,1)$	$(1,0)$	$(0,-1)$
c	2	$(0,1)$	$(0,-1)$	$(1,0)$
h_1	2	$(-2,1)$	$(-3,1)$	$(-4,1)$
h_2	2	$(-2,1)$	$(-4,1)$	$(-3,1)$
	24	$(1,0)$	$(1,0)$	$(1,0)$

Table 1: Intersecting D-branes example.

orientifolds generate an $USp(N)$ gauge group and $USp(2) \simeq SU(2)$. Equally, due to the invariance of D-branes a and d under the orbifold, the $U(N)$ gauge symmetry is projected to $U(N/2)$.

The Standard Model particles are obtained as follows

- sectors ad and ab generate 3 representations $(3+1, 2, 1)$ which represent the left handed quarks and leptons Q_L, L
- sectors ac and dc generate 3 representations $(\bar{3}+1, 1, 2)$: right handed particles u_R, d_R, e_R, ν_R
- sector bc generate the Higgs states $(1, 2, 2)$: H_u, H_d

The model represents a L-R extension of the MSSM. It is possible to choose the T^6 generators in such a way to satisfy the supersymmetry condition

$$\tan^{-1}(\chi_1/2) + \tan^{-1}(\chi_2/3) + \tan^{-1}(\chi_3/4) = 0,$$

where $\chi_i = (\frac{R_2}{R_1})_i$ for torus $(T^2)_i$.

The hypercharge is defined as $Q_Y = \frac{1}{3}Q_a - Q_d - \frac{1}{2}Q_c$, while $Q_{B-L} = \frac{1}{3}Q_a - Q_d$.

3 More chiral models

Supersymmetric D-branes on Calabi-Yau compactifications of Type II theories fall in two classes

- IIA : A-branes wrapped on Special Lagrangian 3-cycles, such as the D6-branes in the intersecting D-branes models
- IIB : B-branes wrapped on holomorphic cycles. B-branes carry holomorphic stable gauge bundles.

In the absence of branes, type IIA theory compactified on a Calabi-Yau X is related by mirror symmetry to type IIB theory compactified on the mirror manifold $\tilde{X}$. This duality extends also in the presence of the open string sector relating A-branes to B-branes.

We can describe two tractable regimes in which it is possible to break the 6d parity in IIB theories in order to obtain chirality :

- D3-branes at singularities, for instance at the conical singularity at the origin of an $\mathbb{C}^3/\mathbb{Z}_3$ orbifold : in this case the breaking of the six dimensional parity is achieved through the action of the orbifold, which defines a preferred orientation. For examples of phenomenological model building see [8] - [10].
- Magnetized D-branes : the pseudovector quality of the magnetic fields leads to the breaking of parity. Below we give a brief outline of model building rules with magnetized branes. For further reading see references [11] - [15].

3.1 Magnetized D-branes

Consider N D5-branes wrapped on a T^2 with magnetic field $F \neq 0$. The magnetic field must satisfy the Dirac quantization condition $\frac{1}{2\pi}\int_{T^2} F = n \in \mathbb{Z}$. For a D-brane wrapping m times around the torus we have $m\frac{1}{2\pi}\int_{T^2} F = n \in \mathbb{Z}$. The magnetic field will induce D3 charges on the D5-branes as follows

$$S_{D5} = m\int_{M^4\times T^2} C_6 + \int_{M^4\times T^2} C_4 \wedge \mathrm{tr}F = m\int_{M^4\times T^2} C_6 + n\int_{M^4} C_4$$

The final state is a bound state of m D5 and n D3-branes. If we perform a T-duality along X^4 [4] we obtain m D4-branes along X^5 and n D4-branes along X^4, hence we obtain a D4-brane with wrapping numbers (n, m) on T^2. The angle of the D4-brane with respect to the horizontal axis of the torus is given by tg $\theta = F = \frac{m}{n}$. This is an instance of 1-dimensional mirror symmetry. The intersecting D-brane models described previously are T-dualizable to magnetized branes models. Let's see how chirality arrises in the magnetized branes picture.

Consider stacks of N_a D5$_a$-branes with magnetic fields F_a, characterized by (n_a, m_a), with n_a quantizing the magnetic flux and m_a the wrapping number. The aa sector gives rise to the gauge bosons of $U(N_a)$ and their superpartners. These fields do not see the magnetic field, as they are not charged under the $U(1)$ from $U(N_a)$. In contrast, states arising from sector ab have charges $(1, -1)$ under $U(1)_a \times U(1)_b$ and are sensitive to the difference in the magnetic field $F_a - F_b$ between the different stacks. Kaluza-Klein compactification of these 6d states leads to 4d chiral fermions, whose number is given by the index of the Dirac operator, $\# = q\int F = q\ n$. Hence the number of chiral fermions is given by $I_{ab} = (+1)\int F_a + (-1)\int F_b$. For general wrapping numbers m_a the gauge group generated by stack a is $U(N_a m_a)$ that gets broken to $U(N_a)^{m_a}$ and further to the diagonal $U(N_a)$ of

[4] $X^{4,5}$ are the coordinates of the T^2 torus.

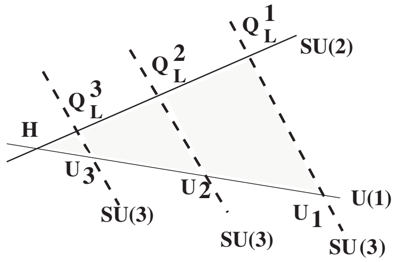


Figure 4: Yukawa couplings in intersecting D-branes models.

the m_a copies. Hence the fundamental representation $(\Box_a, \bar{\Box}_b)$ leads to $m_a \times m_b$ copies of $(N_a, \bar{N}_b)$. If follows that

$$I_{ab} = \int F_a - \int F_b = \left(\frac{n_a}{m_a} - \frac{n_b}{m_b}\right) m_a m_b = n_a m_b - n_b m_a.$$

More realistic cases make use of D9-branes on $T^2 \times T^2 \times T^2$. Notice that so far we have not introduced O9-planes in the theory. Stacks of N_a D9-branes with wrapping numbers (n_a^i, m_a^i), where i labels the i-th torus, characterizing the magnetic fields $F_a^i = \frac{n_a^i}{m_a^i (R_1 R_2)_i} = \tan\theta_a^i$ give rise to the following spectrum

- aa : $U(N_a)$ factors
- ab : $I_{ab} = \prod_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i)$ representations $(N_a, \bar{N}_b)$

The supersymmetry condition $\theta_1 + \theta_2 + \theta_3 = 0$ from intersecting D-branes becomes

$$\tan^{-1} F - 1 + \tan^{-1} F_2 + \tan^{-1} F_3 = 0.$$

In the special case $F_3 = 0$ we obtain $F_1 = -F_2$ and $\mathcal{N} = 2$ supersymmetry.

3.2 Remarks on the phenomenology of particle physics models from D-branes

D-brane models can generate effective field theories that describe the MSSM or some GUT theory, such as $SU(5)$. Gauge coupling unification is not natural in D-brane models, since each Standard Model factor comes from different D-branes stacks

$$\frac{1}{g^2_{YM,a}} = \frac{V_{\Pi_a}}{g_S},$$

unless some symmetry is present for which the volumes are related $V_{\Pi_a} = V_{\Pi_b}$ for $a \neq b$. Another possibilty is to consider some limit in the moduli space in which the volumes align, like large anisotropic volumes with very diluted fluxes.

Yukawa couplings coefficients are computable as function of the moduli $Y_{ijk} \sim e^{-A_{ijk}}$ (see fig. 4). Some couplings can be forbidden in perturbation theory, such as the $10_{+2} 10_{+2} 5_{H_u,+1}$ in $SU(5)$ GUT models. Such couplings can be generated by D-instantons or in F-theory.

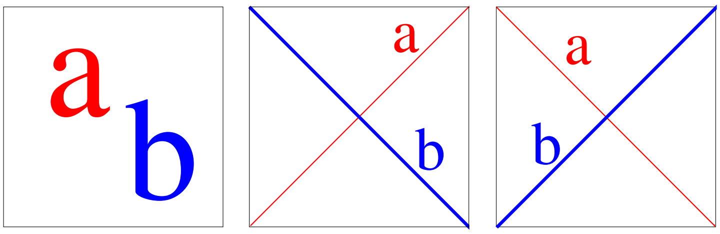


Figure 5: D7-branes fill the first torus and intersect over 2-cycles.

4 F-theory

Consider an orientifold of the type IIB theory compactified on a Calabi-Yau, that contains D7 branes. The D7 branes will wrap 4-cycles, Π_a, in the compact space and will generically intersect over 2-cycles(see figure 5). Gauge interactions are localized on the 4-cycles. Chiral matter is localized at the intersections, hence matter will be 6-dimensional. Yukawa couplings are localized at triple intersections of the matter curves.

F-theory generalizes this picture by englobing non-perturbative effects that generate Yukawa couplings missing in the perturbative picture. Useful reviews and references for this part of the lectures are [16] - [20].

4.1 What is F-theory

Let's recall the relation between M-theory on a T^2 torus and type IIB theory on S^1. Taking the torus to be small and reformulating this as type IIA on a small circle, then T-dualizing along this small circle gives IIB theory on a large circle. In the limit of vanishing area, $A = R_1 R_2$, of the T^2, for fixed $\tau = \frac{R_2}{R_1} e^{i\theta}$ this leads to uncompactified type IIB with complex coupling constant $\tau = 1/g_S + i\ a$, where a is the RR zero form.

A fibration of this duality leads to F-theory. Consider a 7d compactification of M-theory with a T^2 fiber over the complex projective space $\mathbb{P}^1$(elliptically fibered K3). The torus parameter $\tau(z)$ is allowed to depend on the complex coordinate z of $\mathbb{P}^1$. When the T^2 fiber shrinks to zero($A \to 0$, $\tau(z)$-fixed) we obtain a compactification of IIB on $\mathbb{P}^1$ with varying τ. The degenerate fibers correspond to 7-branes in the IIB picture.

The resulting theory is a non perturbative vacuum of IIB. The coupling constant τ suffers $SL(2,\mathbb{Z})$ monodromies at the singularities (7-branes) and, in general, a weak coupling limit cannot be defined. The $\tau \to \tau + 1$ transformation corresponds to a D7-brane.

Non-abelian enhanced gauge symmetries can be obtained for coincident D7-branes or, in the F-theory language, for coincident degenerations of the elliptic fiber. The massless gauge bosons correspond to M2 branes wrapping collapsed 2-cycles : $\int_{\Sigma_2} C_3 = A_1$. If the 2-cycles are blown up, the degenerate elliptic fiber will deform into a chain of spheres(sausage) intersecting according to the Dynkin diagram of the enhanced gauge group.

If the elliptic fibration is described by the equation

$$y^2 = x^3 + f_8(z)x + g_{12}(z),$$

ord(f)	ord(g)	ord(Δ)	fiber-type	singularity-type
0	0	n	I_n	A_{n-1}
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

Table 2: Kodaira classification of singular fibers.

then the enhanced gauge symmetries generated by the singularities of the elliptic fibration are classified according to the vanishing order of the polynomials f, g and of the discriminant $\Delta = 27g^2 + 4f^3$ as depicted in Table 2.

4.2 Model building in F-theory

Consider now M-theory compactified on an elliptically fibered Calabi-Yau fourfold. This leads to IIB on the base B_3 of CY_4, while the complex structure $\tau(z_1, z_2, z_3)$ of the fiber torus encodes the dilaton and the axion. 7-branes are located at the discriminant locus $\Delta(z_1, z_2, z_3) = 0$, where the T^2 degenerates by pinching one of its cycles, hence the 7-branes wrap 4-cycles, S_a, on B_3. The emerging picture is very similar to type IIB models with intersecting D7-branes.

In oder to achieve chirality one needs to introduce magnetic fields. Since gauge bosons arise from the 3-form C_3, the magnetic field is described by the G-flux, $G_4 = dC_3$.

The matter appearing at the intersection of 7-branes is read out through the unfolding procedure [21], which requires to know how the singularity of the elliptic fibration gets enhanced at the intersection. For instance, given the enhancing

$$U(N_a) \times U(N_b) \to U(N_a + N_b),$$

the off-diagonal gauge bosons $\left(\begin{array}{c|c} aa & ab \\ \hline ba & bb \end{array}\right)$ become chiral fields at the intersection

$$Adj_{a+b} \to (Adj_a, 1) + (1, Adj_b) + (N_a, \bar{N}_b) + (\bar{N}_a, N_b).$$

In the case of the enhancement $SO(10) \times U(1) \to E_6$, we obtain chiral matter in the 16 spinorial representation of $SO(10) : 78 \to 45 + 1 + 16 + c.c.$.

So far model building in F-theory has mainly focused on local constructions, due to the complexity of Calabi-Yau fourfolds. These constructions use the bottom-up approach, which has been applied before to models with D3-branes at singularities.

Consider a local base with a single 4-cycle S and 7-branes wrapping S leading to an $SU(5)$ GUT model. Other 7-branes on non-compact cycles S' generate matter. The requirement to have a single small 4-cycle is very restrictive and S must be a del Pezzo surface $dP_n, n = 0...8$. These surfaces are $\mathbb{P}_2$ blown up at n points. One obtains n exceptional

2-cycles E_i, which together with the hyperplane class H satisfy

$$\begin{aligned} H \cdot H &= 1 \\ H \cdot E_i &= 0 \\ E_i \cdot E_j &= -\delta_{ij}. \end{aligned}$$

Since dP_n are rigid one cannot move the branes in order to break the $SU(5)$ GUT group to the Standard Model one. This can be achieved by turning on magnetization along the hypercharge direction

$$F_Y = \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$$24 \to (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-1/3} + (\bar{3},2)_{1/3}.$$

No exotics are obtained through this modified KK reduction.

One must ensure that the hypercharge is massless. This is achieved if class $[F_Y]$ is homologically trivial

$$\int_{8d} C_4 \,\mathrm{tr}(F_Y \wedge F_Y) = \int_{4d} B_2 \wedge \mathrm{tr} F_Y; \quad B_2 = \int_{[F_Y]} C_4 = 0.$$

The matter content is the following $3(\bar{5}+10) + 2_{H_u} + 2_{H_d}$. Please note that while the quarks and leptons fall into $SU(5)$ multiplets, a doublet-triplet splitting operates for the Higgs states. This is obtained in the following manner.

Step 1. The 10, 5, $\bar{5}$ are obtained at interesctions from the local enhancements

$$SU(6) \to SU(5) \times U(1)$$

$$35 \to 24 + 5 + \bar{5} + 1$$

$$S0(10) \to SU(5) \times U(1)$$

$$45 \to 24 + 10 + \bar{10} + 1$$

Step 2. Ensure the appropriate states survive as $SU(5)$ is broken to Standard model gauge group. This can be achieved if the curves Σ_{matter}, Σ_{Higgs} are insensitive, respectively sensitive to the magnetic flux F_Y : $\int_{\Sigma_{\mathrm{matter}}} F_Y = 0$, $\int_{\Sigma_{\mathrm{Higgs}}} F_Y \neq 0$. As Higgs doublets and triplets have different hypercharges, the triplets can be projected out in this way.

As a concrete example let's consider the case where the 4-cycle $S = dP_8$, with $5[F_Y] = E_3 - E_4$, $\int_{E_3} F_Y = -1/2$ and $\int_{E_4} F_Y = 1/5$ (all others =0), where $E_{4,5}$ are exceptional

	Σ	$\int_\Sigma F_Y$	$\int_{\Sigma'} F_Y$	multiplicity
10	$2H - E_1 - E_5$	0	3	3
5	H	0	3	3
$5_{H_u} \to (1,2)_{+3}$	$H - E_1 - E_3$	1/5	2/5	1
$5_{H_d} \to (1,2)_{-3}$	$H - E_2 - E_4$	-1/5	-2/5	1

Table 3: Matter curves

2-cycles from the blown up points of the del Pezzo surface. The curves from which matter is obtained are detailed in Table 3.

Note that the above spectrum could have been obtained from IIB with D7-branes. The novelty in F-theory is the presence of the Yukawa coupling $10 \cdot 10 \cdot 5$.

Yukawa coupligs in F-theory arise from triple intersection of the matter curves. An Yukawa coupling of the form $(N_1, \bar{N}_2)(N_2, \bar{N}_3)(N_3, \bar{N}_1)$ can be obtained from unfolding of the local enhancement

$$U(N_1 + N_2 + N_3) \to U(N_1) + U(N_2) + U(N_3)$$
$$Adj \to Adj_1 + Adj_2 + Adj_3 + (N_1, \bar{N}_2) + (N_2, \bar{N}_3) + (N_3, \bar{N}_1) + c.c.$$

One can engieneer a $10\ \bar{5}\ \bar{5}_{H_d}$ coupling from a local $SO(12)$ enhancement

$$SO(12) \to SO(10) \times U(1) \to SU(5) \times U(1) \times U(1)$$

$$66 = 45 + 1 + 10 + \bar{10} = (24 + 10 + 1) + 1 + \bar{5} + \bar{5} + c.c.$$

Similarly a $10\ 10\ 5_{H_u}$ can be obtained from a local E_6 enhancement

$$E_6 \to SO(10) \times U(1) \to SU(5) \times U(1) \times U(1)$$
$$78 = 45 + 1 + 16 + \bar{16} = (24 + 10 + 1) + 1 + 10 + \bar{5} + c.c.$$

Much progress remains to be done in understanding other phenomenological properties of F-theory, such as gauge coupling unifications, supersymmetry breaking, flavor textures, as well as building global compact models.

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