String Cosmology

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1 Introduction

These notes will provide a brief overview of string inflation. After a motivation for inflation in the context of string theory, we will start out by reviewing inflation. Then we will discuss warped D-brane inflation and conclude with a few comments about DBI-inflation. Throughout the notes we will give only very few references and refer the reader to previous review articles about string cosmology [1, 2, 3, 4, 5, 6, 7] and the references therein. Appendix A summarizes conventions and notation.

2 Motivation

String theory is the best understood candidate for a UV completion of gravity. It therefore should become relevant at energies of the order of the Planck mass M_p. While it is virtually impossible to directly test this energy scale, one might hope that cosmological observation might provide some insight. As we will see in more detail below, inflationary models are UV sensitive. Although it is unlikely that we can make direct tests of string theory through cosmological observations, measurements of the cosmic microwave background (CMB) can exclude certain string inflation models and can rule out part of the landscape.
To further motivate the relevance of string theory for inflation let us look at three examples:

Example 1:
For inflation to last sufficiently long (see below) we need generically that the two slow-roll parameters

$$\epsilon = \frac{1}{2} M_p^2 \left(\frac{V'}{V} \right)^2 , \qquad \eta = M_p^2 \frac{V''}{V} \tag{1}$$

are sufficiently small ($\epsilon, |\eta| \ll 1$). As will be elaborated below, corrections to the inflaton potential up to dimension 6 lead to order 1 corrections to the η parameter since they are of the form $\delta V = \frac{V}{M_p^2} \phi^2$. Therefore, a theory of inflation needs to know about at least dimension 6 terms in the potential. Calculating such Planck suppressed corrections requires a theory of quantum gravity whose prime candidate is string theory.

Example 2:
As we will see below an epoch of inflation will generate scalar and tensor perturbations whose ratio we denote by r. Lyth [8] derived a lower bound on the variation in the inflaton field during inflation known as the Lyth Bound

$$\frac{\Delta\phi}{M_p} = \sqrt{\frac{r}{.01}}\mathcal{O}(1), \tag{2}$$

Future experiment are sensitive to values of r for which $\sqrt{\frac{r}{.01}} \sim 1$. If they would detect tensor fluctuations of this magnitude, then from the expansion of the inflaton potential

$$V(\phi) = V_{\text{renormalizable}} + \phi^4 \sum_{n=1}^{\infty} c_n \left(\frac{\phi}{M_p}\right)^n \tag{3}$$

we see that we not only need to know the renormalizable part of the potential but also infinitely many other terms. To argue for the absence of this terms would require a fine tuning of infinitely many terms. String theory, however, could allow one to calculate these terms or might provide a symmetry that leads to their absence.

Example 3:
Future experiments like the recently started Planck satellite are also searching for non-Gaussianity in the density fluctuations generated by quantum fluctuations in the inflaton field. Those can be generated from higher derivative corrections to the action which we can write as

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2 R + P(X,\phi)\right), \tag{4}$$

where $P(X,\phi)$ is a polynomial in $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and ϕ. Again we need string theory to determine the form of the polynomial P. One particular example of higher derivative corrections that can be nicely summed up to a closed form is the DBI action that can be used for DBI inflation as sketched at the end of these notes.

3 Review of inflation

3.1 Slow-roll inflation

The precise definition of inflation is

$$\text{Inflation} \iff \frac{d}{dt}\frac{1}{aH} < 0. \tag{5}$$

Because $\frac{1}{aH}$ is the comoving Hubble length, the condition for inflation is that the comoving Hubble length is decreasing with time. This means that in coordinates fixed with the expansion, the observable universe actually becomes smaller during inflation.
An alternative definition (for an expanding universe i.e. for $\dot{a} > 0$) is simply an epoch during which the scale factor of the universe is accelerating

$$\text{Inflation} \iff \ddot{a} > 0. \tag{6}$$

If we use the Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p), \tag{7}$$

which can be derived from Einstein's equations for the Robertson-Walker metric (60), we find

$$\text{Inflation} \iff \rho + 3p < 0. \tag{8}$$

Because we always assume that the energy density ρ is positive, it is necessary for the pressure p to be negative.
This condition can be fulfilled by a scalar field with Lagrangian[1]

$$\mathcal{L} = -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{9}$$

From

$$\begin{aligned} T^{\mu\nu} &= \partial^\mu\phi\partial^\nu\phi + g^{\mu\nu}\mathcal{L} \\ &= \partial^\mu\phi\partial^\nu\phi + g^{\mu\nu}(-\tfrac{1}{2}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - V(\phi)), \end{aligned} \tag{10}$$

[1] Although inflation is an intrinsically quantum mechanical process, we are treating the scalar field classically, i.e. consider the expectation value $\langle\phi\rangle$. Quantum effects are negligible if we demand that $V \ll M_p^4$.

we can read off

$$\rho = \quad T^{00} \quad = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi) \quad \text{and} \tag{11}$$

$$p = \frac{a^2}{3}\sum_i T^{ii} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}(\nabla\phi)^2 - V(\phi). \tag{12}$$

If the spatial inhomogeneities in the inflaton field ϕ are small and the potential $V(\phi)$ is much bigger than the square of the time derivative of the inflaton field, i.e.

$$\frac{1}{a^2}(\nabla\phi)^2 \simeq 0 \quad \text{and} \tag{13}$$

$$V(\phi) \gg \dot{\phi}^2, \tag{14}$$

we have the condition

$$\rho = -p \tag{15}$$

and the universe is in accordance with (8) in an inflationary phase.
The standard technique for analyzing inflation is the slow-roll approximation, from which we obtain some restrictions on the potential $V(\phi)$.
From the Euler-Lagrange Equation $\partial_\mu \frac{\partial\sqrt{|\det g|}\mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial\sqrt{|\det g|}\mathcal{L}}{\partial\phi}$ we can derive the equation of motion for ϕ

$$\begin{aligned} -\partial_\mu[\sqrt{|\det g|}g^{\mu\nu}\partial_\nu\phi] &= -\sqrt{|\det g|}V'(\phi) \\ \partial_t[a^3(t)\dot{\phi}] - a(t)\nabla^2\phi &= -a^3(t)V'(\phi) \\ \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \underbrace{\frac{1}{a^2}\nabla^2\phi}_{\simeq 0} &= -V'(\phi) \end{aligned}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{16}$$

We now make the slow-roll approximation that $|\ddot{\phi}|$ is negligible in comparison with $|3H\dot{\phi}|$ and $|V'(\phi)|$. This step is required in order that inflation can happen[2] and leads to the slow-rolling form for the equation of motion

$$3H\dot{\phi} \simeq -V'(\phi). \tag{17}$$

[2]If $|\ddot{\phi}|$ is comparable to $|3H\dot{\phi}|$, $\dot{\phi}$ would change considerably and condition (14) is not satisfied. If we assume that there is a characteristic temporal scale T for the inflaton field, we get from $V(\phi) \gg \dot{\phi}^2 \sim \phi^2/T^2$ that $dV/d\phi \sim V/\phi \gg \phi/T^2 \sim \ddot{\phi}$.

From the Friedman equation that follows from Einstein's equations for the Robertson-Walker metric and using (11) we find

$$H^2 = \frac{\rho}{3M_p^2} - \frac{\kappa}{a^2} \simeq \frac{V}{3M_p^2}. \tag{18}$$

Thus, we can rewrite the condition (14) in form of two dimensionless parameters

$$\epsilon \equiv \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2 \ll 1 \tag{19}$$

$$\eta \equiv M_p^2\left(\frac{V''}{V}\right) \ll 1, \tag{20}$$

where we have differentiated the expression for ϵ

$$M_p|V''| \ll |V'| \ll \frac{1}{M_p}|V| \tag{21}$$

to get η.
These two criteria make perfect intuitive sense: the potential must be flat in the sense of having small derivatives, if the field is to roll slowly enough for inflation to be possible.
Similar arguments could be made for the spacial part. However, they are less critical. Since $a(t)$ increases very rapidly, spacial perturbations are damped away: assuming V is large enough for inflation to start in the first place, inhomogeneities rapidly become negligible.
As we argued above, spatial derivatives of the inflaton field can be neglected. This is not always true for time derivatives. Although they may be negligible initially, the relative importance of time derivatives increases as ϕ rolls down the potential and V approaches zero[3]. Even if the potential does not steepen, sooner or later we will have $\epsilon \simeq 1$ or $|\eta| \simeq 1$ and the inflationary phase will cease. Instead of rolling slowly 'downhill', the field will oscillate about the bottom of the potential. Due to the coupling of the inflaton field to matter fields, which we have neglected so far, the rapid oscillatory phase will produce particles, leading to the reheating of the universe. Thus, even if the

[3]We are leaving aside the subtle question why the potential minimum is so close to zero. Note however that if the minimum would not be close to zero, the universe would continue to inflate without end and not be able to bear life.

minimum of V is at $V = 0$, the universe is left containing roughly the same energy density as it started with, but now in the form of normal matter and radiation - which starts the usual FRW phase, albeit with desired special initial conditions, as we will see now.

3.2 Inflation solves three problems

The very attracting feature of inflation is that a certain amount of inflation solves the most serious problems of the standard cosmology. This leads to a lower bound on the number of e-foldings the universe had to expand during the inflationary phase.

3.2.1 The flatness problem

We can rewrite the Friedmann equation as an equation for the density parameter

$$\Omega - 1 = \frac{\kappa}{a^2 H^2}, \tag{22}$$

where the cosmological constant is included in Ω.
The density parameter therefore evolves. During the matter dominated era we have $a^2H^2 \sim t^{-2/3}$. During the radiation dominated era we have $a^2H^2 \sim 1/t$. From observation we know that, at present, $|\Omega_0 - 1| \sim 10\%$. This requires that for example at nucleosynthesis ($M \simeq 10$ MeV) we had

$$\left| \frac{k}{a^2 H^2} \right| < 10^{-16}. \tag{23}$$

The flatness problem states that such finely tuned initial conditions seem extremely unlikely. Almost all initial conditions lead either to a closed universe that recollapses almost immediately, or to an open universe that very quickly enters the curvature dominated regime and cools below 3 K within the first second of its existence.
Inflation solves the flatness problem because during inflation $a(t)$ increases by a very large factor and the Hubble constant during inflation, H_I, is approximately constant. At the GUT scale $M_{GUT} \sim 10^{15}$ GeV we have the requirement $\left|\frac{k}{a^2H^2}\right| < 10^{-52}$ from which we get $\exp(2H_I t_{inf}) \gtrsim 10^{52}$ and therefore

$$N_{inf} = H_I t_{inf} \gtrsim 60, \tag{24}$$

where t_{inf} denotes the duration of inflation and N_{inf} the number of e-foldings.

3.2.2 The horizon problem

Standard cosmology contains a particle horizon radius

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}, \tag{25}$$

which converges because $a(t) \propto t^{1/2}$ in the early radiation-dominated phase. The particle horizon is the distance over which causal interaction can occur. We want to compare this with the corresponding angular size d_A at the time of last scattering ($t_L = 3 \times 10^5$ yr) when the microwave background is released.

If we make the very good approximation of a matter-dominated epoch back to the Big Bang, we get

$$d_H(t_L) = t_L^{2/3} \int_0^{t_L} dt\, t^{-2/3} = 3t_L = 3 \left(\frac{a(t_L)}{a(t_0)}\right)^{3/2} t_0 = \frac{2}{(1+z_L)^{3/2}} \frac{1}{H_0}. \tag{26}$$

From the definition of d_A

$$\begin{aligned} d_A(t_L) &\equiv \frac{a(t_0) r(t_L)}{1+z_L}, \quad \text{with} \\ r(t_L) &= S\left[\frac{1}{a(t_0)H_0} \int_{\frac{1}{1+z_L}}^{1} \frac{dx}{\sqrt{\Omega_R + \Omega_M x + \Omega_k x^2 + \Omega_\Lambda x^4}}\right], \quad \text{where} \\ S[y] &= \begin{cases} \sin(y) & \text{for } \kappa = +1 \\ y & \text{for } \kappa = 0 \\ \sinh(y) & \text{for } \kappa = -1 \end{cases}, \end{aligned} \tag{27}$$

we get the approximation

$$r(t_L) \simeq \frac{1}{a(t_0)H_0} \quad \Rightarrow \quad d_A(t_L) \simeq \frac{1}{(1+z_L)H_0}, \tag{28}$$

where the error is of order $O(1)$.

Thus the angle subtended at last scattering is

$$\theta = \frac{d_H}{d_A} \simeq \frac{1}{\sqrt{1+z_L}} = \frac{1}{\sqrt{1100}} \simeq 1.7^\circ. \tag{29}$$

The horizon problem is that all the causally disconnected region we see on the microwave sky are at the same temperature and the homogeneity of our universe must form part of the initial conditions.
This problem can again be solved by inflation, which could enlarge a small causally connected part to the size of the angular diameter at last scattering, so that $d_H(t_L) \gtrsim d_A(t_L)$.
From our calculation above we know that this means that the contribution from the inflationary phase to d_H has to be bigger than or equal to $d_A(t_L)$. If we take $t = 0$ to be the beginning of inflation, we get

$$\begin{aligned} d_H(t_L) = a(t_L) \int_0^{t_{inf}} \frac{dt}{a(0) \exp{(H_I t)}} &= \frac{a(t_L)}{a(0)} \left[\frac{1}{H_I} (1 - e^{-H_I t_{inf}}) \right] \\ = \frac{a(t_L)}{a(t_{inf})} \left[\frac{1}{H_I} (e^{H_I t_{inf}} - 1) \right] &\simeq \frac{a(t_L)}{a(t_{inf})} \frac{e^{H_I t_{inf}}}{H_I}. \end{aligned} \quad (30)$$

It follows from the requirement, that the cosmological scale factor is continuous at the transition between the different epochs, that

$$\frac{a(t_L)}{a(t_{inf})} = \left(\frac{t_{eq}}{t_{inf}} \right)^{1/2} \left(\frac{t_L}{t_{eq}} \right)^{2/3}, \quad (31)$$

where $t_{eq} \sim t_L$ [4] is the time of matter-radiation equality. We can now derive the lower limit for the duration of inflation

$$
\begin{aligned}
d_H(t_L) &\gtrsim d_A(t_L) \\
\left(\frac{t_L}{t_{inf}}\right)^{1/2} \frac{e^{H_I t_{inf}}}{H_I} &\gtrsim \frac{1}{(1+z_L)H_0} \\
\underbrace{t_L H_0}_{\simeq \frac{t_L}{t_0} = (1+z_L)^{-3/2}} \frac{t_{inf}^{1/2}}{t_L^{1/2}} \frac{e^{H_I t_{inf}}}{H_I t_{inf}} &\gtrsim \frac{1}{(1+z_L)} \\
\underbrace{\frac{t_{inf}^{1/2}}{t_L^{1/2}}}_{\simeq \frac{10^3 K}{10^{28} K} = 10^{-25}} \frac{e^{H_I t_{inf}}}{H_I t_{inf}} &\gtrsim \sqrt{1+z_L} \simeq 33 \\
\frac{e^{H_I t_{inf}}}{H_I t_{inf}} &\gtrsim 10^{26} \\
N_{inf} = H_I t_{inf} &\gtrsim 64.
\end{aligned}
\tag{32}
$$

This is the most important constrain that should be fulfilled by every inflation model.

3.2.3 Unwanted relict particles

If we have a GUT with a unified group like for example $SO(10)$ or $SU(5)$ which is spontaneously broken to $SU(3) \times SU(2) \times U(1)$, we get topological stable solution like magnetic monopoles. Their number density is of the same order as the baryonic number density, i.e. $\frac{n_\mathrm{m}}{n_\mathrm{b}} \sim 1$. However, we know from observations that $\frac{n_\mathrm{m}}{n_\mathrm{b}} \leq 10^{-30}$. If we assume that no magnetic monopoles were produced after inflation we get an estimation for the duration of inflation by the requirement that the abundance is decreased by a factor of at least 10^{-30}

$$n \sim \frac{1}{a^3} \quad \Rightarrow \quad \exp\left(H_I t_{inf}\right) \gtrsim \sqrt[3]{10^{30}} = 10^{10}$$

[4] $\frac{t_L}{t_{eq}} = \left(\frac{a(t_L)}{a(t_{eq})}\right)^{3/2} \simeq \left(\frac{4}{3}\right)^{3/2}$.

$$N_{inf} = H_I t_{inf} \gtrsim 23. \tag{33}$$

Similarly the problem of the high abundance of gravitinos and some scalar particles, which are predicted in supergravity and superstring theories and would upset nucleosynthesis, could be solved by an inflationary phase.

3.3 Quantum fluctuations

The outstanding feature of inflation is that it not only provides solutions to the problems mentioned above but also predicts that quantum fluctuation can be the seeds for what eventually become galaxies and clusters!
We can see this as follows:
During inflation the universe is in an (almost) de Sitter (dS) phase[5] ($a(t) \propto e^{H_I t}$) and we have an event horizon, in that the comoving distance that particles can travel between a time t_1 and $t = \infty$ is finite,

$$r_{\mathrm{EH}}(t_1) = \int_{t_1}^{\infty} \frac{dt}{a(t)} = \frac{1}{a(t_1) H_I}. \tag{34}$$

Hence, we have an event horizon of proper radius $a(t_1) r_{\mathrm{EH}}(t_1) = H_I^{-1}$.

[5]There are some inflationary models in which the universe is not in a de Sitter phase during inflation. One example is power-law inflation $a(t) \propto t^p$. Because (6) is necessary for inflation, we get $p > 1$ and have therefore also a finite event horizon. This is the only necessary condition for the idea outlined above. However, for simplicity we restrict ourselves to a de Sitter phase.

Waves well inside the horizon ($\lambda \ll H_I^{-1}$) effectively occupy flat space, and so undergo the normal quantum fluctuations for a vacuum state. These modes of fixed comoving wavelength are expanded to sizes $\lambda \gg H_I^{-1}$ during inflation. Because points that are separated by distances larger than H_I^{-1} can never communicate with each other, causality forces the quantum fluctuations to become frozen as classical fluctuations (Figure (1)). During either reheating, radiation-dominated, or matter-dominated phases the modes begin to behave as non-relativistic matter with density fluctuations that can seed large-scale structures like the galaxies. They also lead to the small fluctuations in the cosmic microwave background (CMB). In this subsection we will study small fluctuations in the dilaton and the metric.

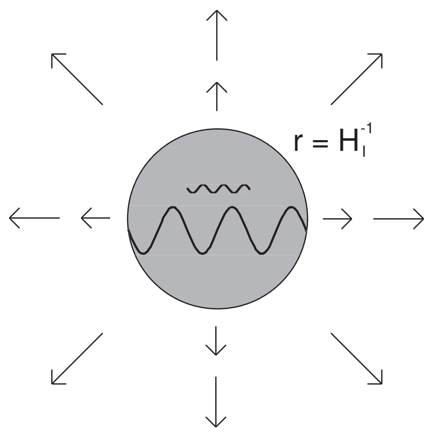

Figure 1: The comoving wavelength of quantum fluctuations are expanded to sizes $\lambda \gg H_I^{-1}$ so that they become frozen.

We expand the metric and the inflaton around their background values

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}(\tau, \vec{x}), \tag{35}$$
$$\phi = \phi^0 + \delta\phi(\tau, \vec{x}), \tag{36}$$

where we used conformal time τ. The equation of motion for the inflaton fluctuations are

$$\delta\phi_k'' + \frac{2a'}{a}\delta\phi_k' + k^2\delta\phi_k = 0, \tag{37}$$

which in terms of $\mu_k = a\phi_k$ becomes

$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0. \tag{38}$$

In the regime where $k^2 \gg \frac{a''}{a}$ the solutions are plane waves $e^{\pm ik\tau}$ while for $k^2 \ll \frac{a''}{a}$ we find $\mu_k \sim a$. If we define the scaling of a with τ in terms of the constant ν by $a \sim \tau^{\frac{1}{2}-\nu}$ then the equation (38) becomes

$$\mu_k'' + \left(k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right)\mu_k = 0 \tag{39}$$

and can be solved in terms of Hankel functions. A solution to the above equation is given by $f_k(\tau)$

$$f_k(\tau) = \frac{\sqrt{-\tau\pi}}{2}\left(c_1(k)H_\nu^{(1)}(-k\tau) + c_2(k)H_\nu^{(2)}(-k\tau)\right). \quad (40)$$

We can now canonically quantize μ_k and its conjugate momentum π_k which satisfy the commutation relation $[\mu_k, \pi_k] = i$ and write

$$\mu_k = \frac{1}{\sqrt{2k}}\left(a_k(\tau) + a^\dagger_{-k}(\tau)\right), \quad (41)$$

$$\pi_k = -i\sqrt{\frac{k}{2}}\left(a_k(\tau) - a^\dagger_{-k}(\tau)\right). \quad (42)$$

Since the solution to the equation (39) is given by $f_k(\tau)$ (cf. (40)) μ_k has to take the form

$$\mu_k = f_k(\tau)a_k(\tau_{in}) + f_k^*(\tau)a^\dagger_{-k}(\tau_{in})), \quad (43)$$

where τ_{in} is an initial time. The so called Bunch-Davies vacuum is defined by $\lim\limits_{\tau_{in}\to-\infty} a_k(\tau_0)|0,\tau_0 >= 0$ but the exact definition of a vacuum is not relevant for us since inflation will wash out the difference between different vacuum states (cf. [9] for more details).

The Hankel functions simplify for $\tau_0 \to -\infty$ so that $H_\nu^{(1)}(-k\tau) \sim \sqrt{\frac{2}{-k\tau\pi}}e^{-ik\tau}$ and $H_\nu^{(2)} = H_\nu^{(1)*}$. Therefore, we can set $|c_1| = 1$ and $c_2 = 0$. Then we only have to consider $H_\nu^{(1)}$ which for $\tau \to 0$ behaves as $H_\nu^{(1)} \sim (-k\tau)^{-\nu}$.

We define the power spectrum and find

$$\mathcal{P}(k) = \frac{4\pi k^3}{(2\pi)^3} < \delta\phi_k\, \delta\phi_{-k} >\sim k^{3-2\nu}. \quad (44)$$

We see that for the special value of $\nu = \frac{3}{2}$ the k dependence drops out an we have a scale invariant power spectrum. For an inflationary phase, we have $a \sim e^{Ht} \sim -\frac{1}{H\tau}$ and therefore $\nu = \frac{3}{2}$. This shows that inflation predicts an (almost) scale invariant power spectrum. Doing the calculation more carefully and including tensor modes arising from the metric[6] one finds the

[6]There are also vector modes arising from fluctuations of the metric. Those are however not sourced during inflation and decay very rapidly in an expanding universe.

following dimensionless power spectrum for scalar and tensor modes

$$\mathcal{P}_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 = \frac{1}{2M_p^2\,\epsilon}\left(\frac{H}{2\pi}\right)^2, \tag{45}$$

$$\mathcal{P}_t(k) = \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2. \tag{46}$$

We see that for $\dot{\phi} \ll 1$ we have $\mathcal{P}_s(k) \gg \mathcal{P}_t(k)$. We also find that the ratio between the two is

$$r = \frac{\mathcal{P}_t(k)}{\mathcal{P}_s(k)} = 16\epsilon. \tag{47}$$

Since the fluctuations in the inflaton field are very weakly coupled to one another, one expects that the late time density fluctuations obey Gaussian statistics. This means that the n-point functions $< \delta\phi_{k_1}\delta\phi_{k_2}\ldots\delta\phi_{k_n} >$ are determined by the 2-point function. So far no non-Gaussian correlations have been observed.

4 Inflation in string theory

Considering inflation in string theory one might ask how natural it is for inflation to arise in string theory. Since we have to demand that for example $\dot{\phi} \ll V(\phi)$ it is likely that there is some fine tuning involved, if one tries to realize inflation in string theory. However, this question is hard to answer since we don't have a very good understanding of the string landscape. An easier question is whether we can realize inflation and the necessary initial conditions in string theory and what the potential problems are.
As we have already seen in the introduction, integrating out very heavy massive fields with $m > M = M_p$ or M_s leads to corrections to the potential $\delta V = \frac{\mathcal{O}_k}{M^{k-4}}$, where $\mathcal{O}_k$ is a dimension k operator. In particular for large field inflation with $\Delta\phi > M$ we need to know correction for all k with $\mathcal{O}_k = (\Delta\phi)^k$. For these models it might still be possible to use an effective field theory (and neglect effects of quantum gravity), if the energy-density remains below M_p. In the next section we will discuss a particular problem that arise from integrating out fields. We will study it in the context of supergravity.

4.1 The η problem in supergravity

In $\mathcal{N} = 1$ supergravity the Lagrangian is

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F, \tag{48}$$

where the scalar potential is

$$V_F = e^{K/M_p^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_p^2}|W|^2\right], \tag{49}$$

and $D_\varphi = \partial_\varphi W + \frac{1}{M_p^2}\partial_\varphi K$. Expanding the Kähler potential around $\varphi = 0$

$$K(\varphi,\bar{\varphi}) = K_0 + K_{\varphi\bar{\varphi}}\varphi\bar{\varphi} + \ldots \tag{50}$$

the Lagrangian becomes

$$\mathcal{L} \approx -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0\left(1 + K_{\varphi\bar{\varphi}}|_{\varphi=0}\frac{\varphi\bar{\varphi}}{M_p^2} + \ldots\right) \tag{51}$$

$$= -\partial\phi\partial\bar{\phi} - V_0\left(1 + \frac{\phi\bar{\phi}}{M_p^2} + \ldots\right), \tag{52}$$

where $\ldots$ denotes model dependent terms that are of the same order as the terms we wrote down and in the second line we have introduced the canonically normalized inflaton fields ϕ. The problem is that we have found a model independent correction to the potential that is $\delta V = V_0\frac{\phi\bar{\phi}}{M_p^2}$ and which leads to

$$\Delta\eta = M_p^2\frac{\Delta V''}{V_0} = 1. \tag{53}$$

This is the so called η problem that one generically faces. One option to deal with it is to fine tune it away by having a model that leads to further corrections that compensate the model independent contribution. Another approach is that the inflation is not in V_F like for example an axion that has no classical potential due to its shift symmetry.
We will now study in detail so called warped D-brane inflation, where the η problem can in principle be avoided by having model dependent corrections to V that cancel the universal contributions.

4.2 Warped D-brane inflation

In string theory one can consider cases where the modulus that corresponds to the distance between a D3 brane and an anti-D3 brane gives rise to inflation. However, in order for the resulting Coulomb potential to be shallow enough for slow-roll inflation, the distance between the two branes needs to be bigger than the size of the compact space they move in. To avoid this generic problem, KKLMMT [10] placed the two branes into a warped throat. The warping had the effect that one can realize slow-roll since the attractive force is effectively reduced by the warping. As was previous shown by KKLT [11] it is also possible in this setup to stabilize all closed string moduli that might spoil inflation if left unstabilized. In this setup it is possible to investigate corrections that arise and check whether it might be possible to avoid the η problem discussed above [12].
The background in which the D3 brane moves is taken to be a warped throat at which tip an anti-D3 brane sits. The throat is connected to a compact Calabi-Yau (CY) space. The metric is

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n, \tag{54}$$

where g_{mn} is the generically unknown CY metric. Since we are only interest in the movement of the D3 brane this is no problem. We take the D3 brane to move in the interval $r_{IR} \leq r \leq r_{UV}$ where r_{IR} is a lower cut-off close to the tip of the throat and r_{UV} is close to the end of the throat were it is connected to the compact CY. For concreteness we take the Klebanov-Strassler (KS) throat

$$g_{mn}dy^m dy^n = dr^2 + r^2 ds^2_{X_5}, \tag{55}$$

with $X_5 = \frac{SU(2)\times SU(2)}{U(1)} = T^{1,1}$. Then we can expand the DBI action for the D3-brane to obtained the normalization of the inflaton field ϕ and find $\phi^2 = T_3 r^2$, where the tension of the D3-brane is $T_3 = \left[(2\pi)^3 g_s \alpha'\right]^{-1}$. Next one can calculate the resulting inflaton potential

$$V(\phi) = V_0(\phi) + H^2\phi^2 + \Delta V(\phi), \tag{56}$$

where $V_0(\phi)$ is the Coulomb potential, $H^2\phi^2$ is the universal contribution we have calculated above and $\Delta V(\phi)$ are model dependent contributions. This leads to and η parameter

$$\eta = \eta_0 + \frac{2}{3} + \Delta\eta. \tag{57}$$

The warping leads to a sufficiently shallow Coulomb potential and therefore to a small η_0. We therefore need to calculate $\Delta\eta$ and hope that it can compensate the large factor of $\frac{2}{3}$.
A systematic approach to study such corrections was made in [12], where the authors made use of the gauge/gravity corresponds and studied corrections of the field theory dual to the KS throat. They were able to provide an explicit mapping between corrections arising on the field theory and gravity side. They found that the corrections are generically of the form

$$\Delta V(\phi) = a_{\frac{3}{2}}\phi^{\frac{3}{2}} + a_2\phi^2. \tag{58}$$

The authors showed that in throat dual to chiral gauge theories $a_{\frac{3}{2}} = 0$ and that even in the case dual to non-chiral gauge theories it is possible that discrete symmetries can lead to $a_{\frac{3}{2}} = 0$. In this case the total potential takes the form

$$V(\phi) = V_0(\phi) + \beta H^2\phi^2, \tag{59}$$

and it is possible to fine tune the parameter β by varying the strength of the UV perturbation arising from coupling the throat to a compact CY. There it is possible to obtain models that do not suffer from the η problem.

4.3 DBI-Inflation

Another inflationary model obtained from string theory is the so called DBI-inflation where one uses the DBI-action of a Dp brane to derive the inflaton potential. This case is different from the previous one since it is not relying on the slow-roll approximation. It therefore also has somewhat different observational signature in the sense that it predicts larger non-Gaussianities. For DBI-inflation one imagines a D3 brane moving down a throat towards and anti-D3 brane at the tip of the throat similar to the case above. This motion, however, is to be taken to be relativistic rather than slow. Surprisingly, this can also lead to an inflationary phase. Since the slow-roll conditions are not applicable there is no η problem.

A Conventions and notation

We will work in units where $c = \hbar = k_B = 1$.
The gravitational constant is denoted by G and defines the Planck mass

squared $M_p^2 = \frac{1}{8\pi G}$.
A dot over a quantity denotes the time-derivative of that quantity, while a prime generally denotes the derivative for functions of a single variable, e.g. $V'(\phi) = \frac{\partial V}{\partial \phi}$.
We will consider the Robertson-Walker metric for which the line element is

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{60}$$

where $a(t)$ is the cosmic scale factor and $\kappa = -1, 0, 1$ depending on whether spacial slices of the universe are hyperbolic surfaces ($\kappa = -1$), flat ($\kappa = 0$) or three spheres ($\kappa = 1$). We will also use the conformal time τ defined by $dt = a(\tau)d\tau$.
The redshift for light emitted at t_1 is $z_1 = \frac{a(t_0)}{a(t_1)} - 1$, where the subscript 0 always refers to the present time.
The density parameter is denoted Ω and is given by the energy density ρ divided by the critical energy density ρ_c i.e. $\Omega = \frac{\rho}{\rho_c}$ subscripts R, M, Λ mean the contribution from radiation, matter or vacuum. We will also use $\Omega_k = -\frac{k}{a^2 H^2}$.

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