

Mass and Spin Measurement with M_{T2} and MAOS Momentum

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arXiv:0810.4853 [hep-ph]

arXiv:0711.4526 [hep-ph]

arXiv:0709.0288 [hep-ph]

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arXiv:0908.0079 [hep-ph]

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Outline

- 1 Motivation: New Physics Events with Missing Transverse Momentum
- 2 M_{T2} -Kink Method for Mass Measurement
- 3 M_{T2} -Assisted On-Shell (MAOS) Momentum and its Applications:
Sparticle Spin and Higgs Mass Determination
- 4 Summary

Motivations for new physics at the TeV scale:

- Hierarchy Problem

$$\delta m_H^2 \sim \frac{g^2}{8\pi^2} \Lambda_{\text{SM}}^2 \sim M_Z^2 \implies \Lambda_{\text{SM}} \sim 1 \text{ TeV}$$

- Dark Matter

$$\text{Thermal WIMP with } \Omega_{\text{DM}} h^2 \sim \frac{0.1}{g^4} \left(\frac{m_{\text{DM}}}{1 \text{ TeV}} \right)^2 \sim 0.1$$

$$\implies m_{\text{DM}} \sim 1 \text{ TeV}$$

Many new physics models solving the hierarchy problem while providing a DM candidate involve a Z_2 -parity symmetry under which the new particles are Z_2 -odd, while the SM particles are Z_2 -even:

SUSY with R -parity, Little Higgs with T -parity, UED with KK -parity, ...

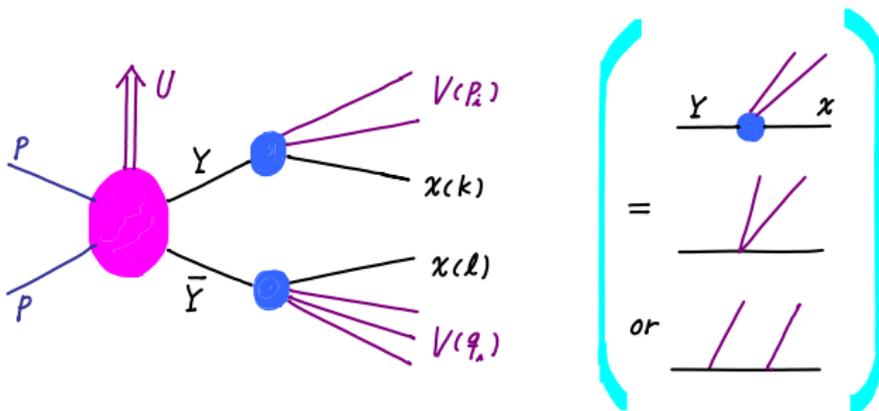
- * At colliders, new particles are produced always in pairs.
- * Lightest new particle is stable, so a good candidate for WIMP DM.

LHC Signal

Pair-produced new particles ($Y + \bar{Y}$) decaying into visible SM particles (V) plus invisible WIMPs (χ):

$$pp \rightarrow U + Y + \bar{Y} \rightarrow U + \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$

(multi-jets + leptons + \cancel{p}_T)



$U \equiv$ Upstream momenta carried by the visible SM particles not from the decay of $Y + \bar{Y}$ (\bar{Y} is not necessarily the antiparticle of Y)

- Mass measurement of these new particles is quite challenging:
 - * Initial parton momenta in the beam-direction are unknown.
 - * Each event involves two invisible WIMPs.

Kinematic methods of mass measurement:

- i) Endpoint Method
 - ii) Mass Relation Method
 - iii) M_{T2} -Kink Method
- Spin measurement appears to be even more challenging:
 - * It often requires a more refined event reconstruction and/or polarized mother particle state.

MAOS momentum provides a systematic approximation to the invisible WIMP momentum, and thus can be useful for spin and mass measurements.

Kinematic Methods of Mass Measurement

i) Endpoint Method Hinchliffe et. al.; Allanach et. al.; Gjelsten et. al.;...

Endpoint value of the invariant mass distribution of visible (SM) decay products depend on the new particle masses.

* 3-step squark cascade decays when $m_{\tilde{q}} > m_{\chi_2} > m_{\tilde{\ell}} > m_{\chi_1}$



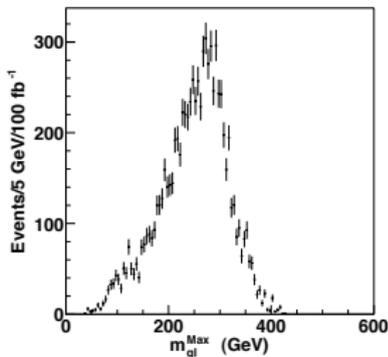
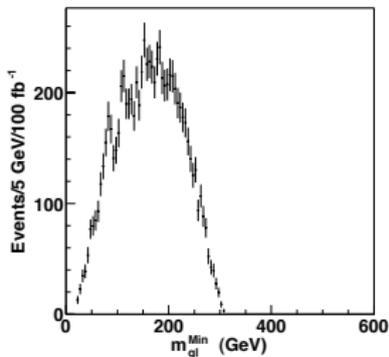
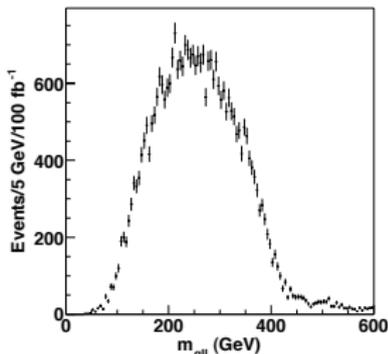
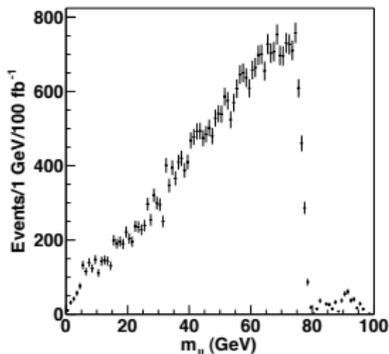
$$m_{\ell\ell}^{\max} = m_{\chi_2} \sqrt{(1 - m_{\tilde{\ell}}^2/m_{\chi_2}^2)(1 - m_{\chi_1}^2/m_{\tilde{\ell}}^2)}$$

$$m_{q\ell\ell}^{\max} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\chi_1}^2/m_{\chi_2}^2)}$$

$$m_{q\ell}^{\max(\text{high})} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\chi_1}^2/m_{\tilde{\ell}}^2)}$$

$$m_{q\ell}^{\max(\text{low})} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\tilde{\ell}}^2/m_{\chi_2}^2)}$$

Result for SUSY SPS1a Point [Weiglein et. al. hep-ph/0410364](#)



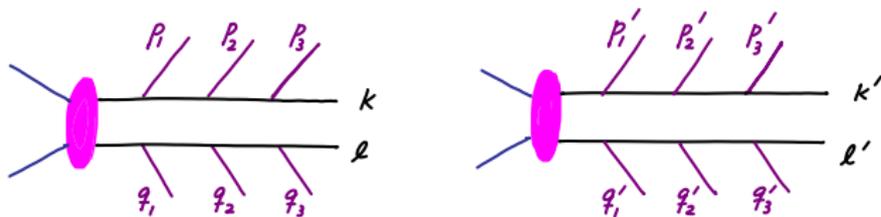
Input masses: $(m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (540, 177, 143, 96)$ GeV

Fitted masses: $(543 \pm 13, 180 \pm 9, 146 \pm 11, 98 \pm 9)$ ($\int \mathcal{L} = 100 \text{ fb}^{-1}$)

ii) Mass Relation Method Nojiri, Polesello, Tovey; Cheng et. al.; ...

Reconstruct the missing momenta using all available constraints.

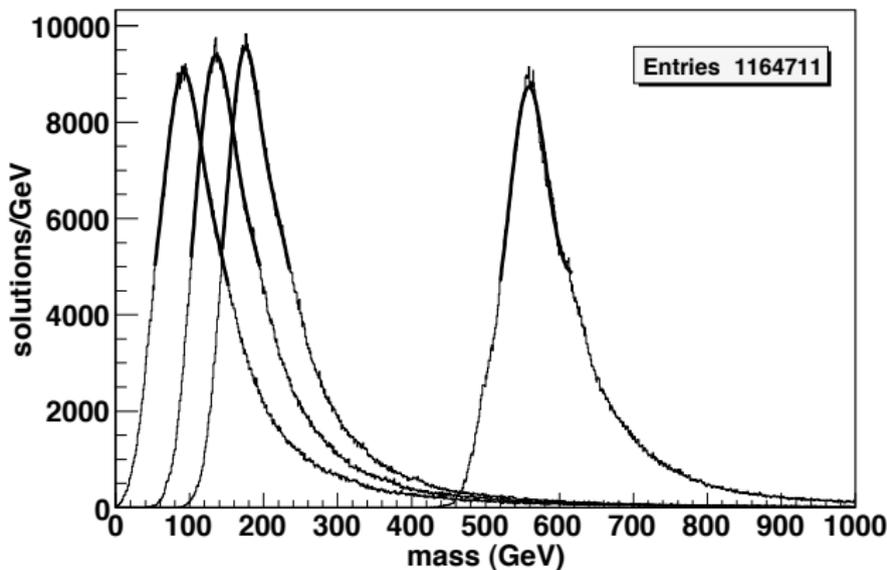
- * A pair of symmetric 3-step cascade decays of squark pair
Cheng, Engelhardt, Gunion, Han, McElrath



- 16 unknowns: $k^\mu, l^\mu, k'^\mu, l'^\mu$
- 12 mass-shell constraints: $k^2 = l^2 = k'^2 = l'^2$,
 $(k + p_3)^2 = (l + q_3)^2 = (k' + p'_3)^2 = (l' + q'_3)^2$,
 $(k + p_2 + p_3)^2 = (l + q_2 + q_3)^2 = (k' + p'_2 + p'_3)^2 = (l' + q'_2 + q'_3)^2$,
 $(k + p_1 + p_2 + p_3)^2 = (l + q_1 + q_2 + q_3)^2 = (k' + p'_1 + p'_2 + p'_3)^2$
 $= (l' + q'_1 + q'_2 + q'_3)^2$,
- 4 \mathbf{p}_T -constraints: $\mathbf{k}_T + \mathbf{l}_T = \mathbf{p}'_T, \quad \mathbf{k}'_T + \mathbf{l}'_T = \mathbf{p}'_T$

- * 8 (complex) solutions for each event-pair, some of which are real.
- * Many wrong solutions from wrong combinatorics.

For given set of event-pairs, number of real solutions shows a peak at the correct masses: [Cheng et. al.](#)



Input masses: $(m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (568, 180, 143, 97)$ GeV

Fitted masses: $(562 \pm 4, 179 \pm 3, 139 \pm 3, 94 \pm 3)$ ($\int \mathcal{L} = 300 \text{ fb}^{-1}$)

Remarks

- Mass relation method and endpoint method require a long decay chain, **at least 3-step chain**, to determine the involved new particle masses.
- However, there are many cases (including a large fraction of popular scenarios) that such a long decay chain is not available:

A simple example: mSUGRA with $m_0^2 > 0.6 M_{1/2}^2 \Rightarrow m_{\tilde{\ell}} > m_{\chi_2}$

- SUSY with $m_{\text{sfermion}} \gg m_{\text{gaugino}}$:
(Focus point scenario, Loop-split SUSY, Some string moduli-mediation, ...)



- * Mass relation method simply can not be applied.
- * Endpoint method determines only the gaugino mass differences.
- * M_{T2} -kink method can determine the full gaugino mass spectrum.

iii) M_{T2} -Kink Method Cho, Choi, Kim, Park; Barr, Gripaios, Lester

M_{T2} is a generalization of the transverse mass to an event producing two invisible particles with the same mass.

Transverse mass of $Y \rightarrow V(p) + \chi(k)$:

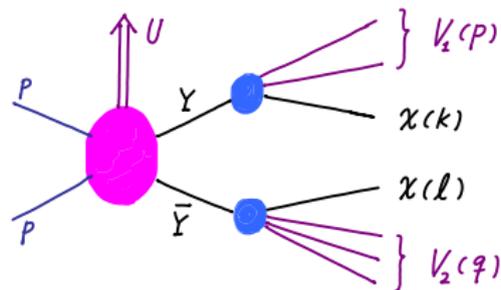
$$M_T^2 = m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |\mathbf{p}_T|^2}\sqrt{m_\chi^2 + |\mathbf{k}_T|^2} - 2\mathbf{p}_T \cdot \mathbf{k}_T$$

* Invariant Mass: $M^2 = m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |\mathbf{p}|^2}\sqrt{m_\chi^2 + |\mathbf{k}|^2} - 2\mathbf{p} \cdot \mathbf{k}$

$$= m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |\mathbf{p}_T|^2}\sqrt{m_\chi^2 + |\mathbf{k}_T|^2} \cosh(\eta_V - \eta_\chi) - 2\mathbf{p}_T \cdot \mathbf{k}_T \geq M_T^2$$

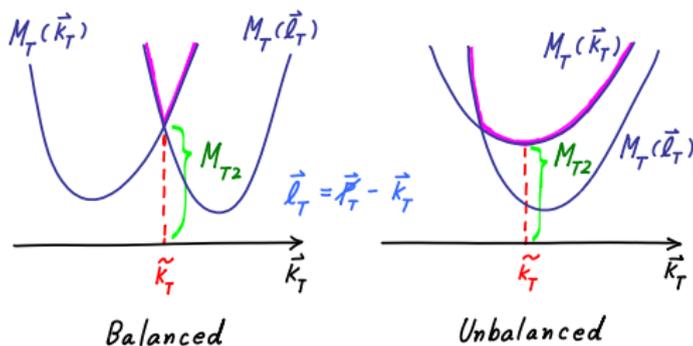
One can use an arbitrary trial WIMP mass m_χ to define M_T .
(True WIMP mass = m_χ^{true}).

- * For each event, M_T is an increasing function of m_χ .
- * For all events, $M_T(m_\chi = m_\chi^{\text{true}}) \leq m_Y^{\text{true}}$ in the zero width limit.



$$M_{T2}(\text{event}; m_\chi) \quad (\{\text{event}\} = \{m_{V_1}, \mathbf{p}_T, m_{V_2}, \mathbf{q}_T, \mathbf{p}'_T\})$$

$$= \min_{\mathbf{k}_T + \mathbf{l}_T = \mathbf{p}'_T} \left[\max \left(M_T(\mathbf{p}_T, m_{V_1}, \mathbf{k}_T, m_\chi), M_T(\mathbf{q}_T, m_{V_2}, \mathbf{l}_T, m_\chi) \right) \right]$$



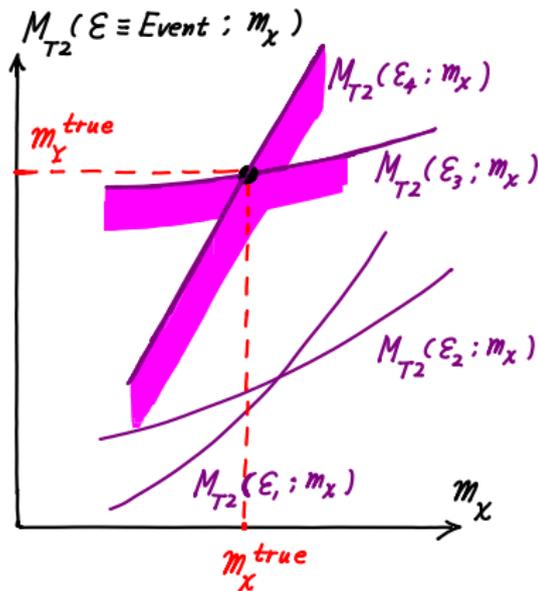
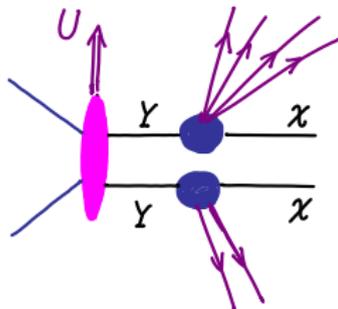
M_{T2} -Kink

If the event set has a **certain variety**, which is in fact quite generic,

$$M_{T2}^{\max}(m_x) = \max_{\{\text{all events}\}} \left[M_{T2}(\text{event}; m_x) \right]$$

has a kink-structure at $m_x = m_x^{\text{true}}$ with $M_{T2}^{\max}(m_x = m_x^{\text{true}}) = m_Y^{\text{true}}$.

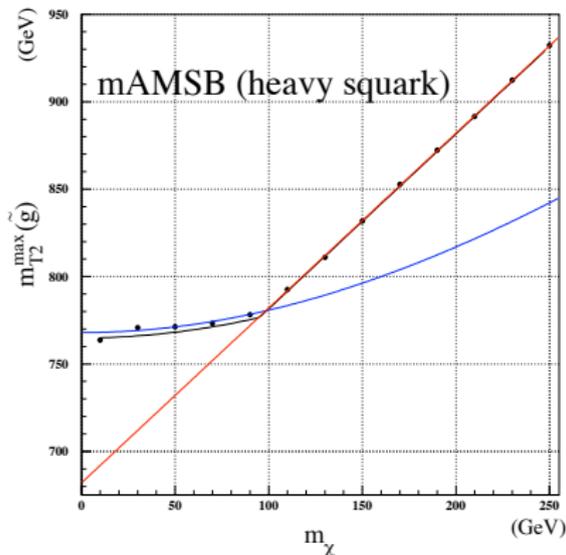
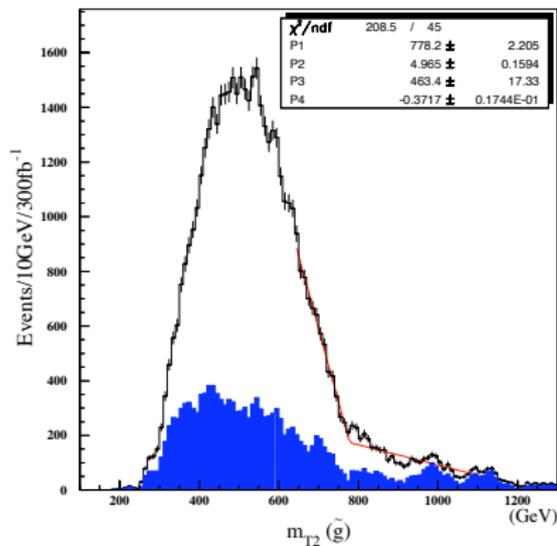
Events 1, 2, 3, 4



Glauino M_{T2} -Kink in heavy sfermion scenario

Cho, Choi, Kim, Park

M_{T2} of hard 4-jets (no b , no ℓ) which are mostly generated by the gluino-pair 3-body decay: $\tilde{g}\tilde{g} \rightarrow q\bar{q}\chi q\bar{q}\chi$, where $m_{\tilde{g}} \lesssim 1$ TeV and $m_{\tilde{q}} \sim$ few TeV.



Input masses: $(m_{\tilde{g}}, m_{\chi_1}) = (780 \text{ GeV}, 98 \text{ GeV})$ (Wino-like χ_1)

Fitted masses: $(776 \pm \text{few}, 97 \pm \text{few})$ ($\int \mathcal{L} = 300 \text{ fb}^{-1}$)

Kink itself is quite generic, but often it might not be sharp enough to be visible in the real analysis.

Related methods which might be useful:

- Number of real solutions for M_{T2} -assisted on-shell (MAOS) momenta, which is expressed as a function of m_χ , might show a sharper kink at $m_\chi = m_\chi^{\text{true}}$. [Cheng and Han; arXiv:0810.5178](#)
- M_{T2} -kink is a fixed point under $\partial/\partial m_V, \partial/\partial u_T$:

$$\implies \left(\frac{\partial}{\partial m_V}, \frac{\partial}{\partial u_T} \right) \left\{ M_{T2}^{\max}(m_\chi^{\text{trial}} = 0), M_{CT}^{\max}, \dots \right\}$$

can provide information which would allow mass determination in the absence of long decay chain. [Torvey; arXiv:0802.2879](#)

[Konar, Kong, Matchev and Park; arXiv:0910.3679](#)

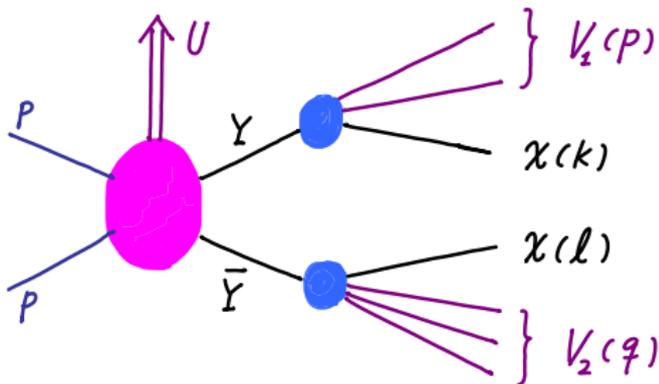
- Algebraic singularity method: [I.W.Kim; arXiv:0910.1149](#)

More general and systematic method to find a variable (= **singularity coordinate**) most sensitive to the singularity structure providing information on the unknown masses in missing energy events.

M_{T2} -Assisted-On-Shell (**MAOS**) Momentum

arXiv:0810.4853[hep-ph]; arXiv:0908.0079[hep-ph]

MAOS momentum is a collider event variable designed to approximate systematically the invisible particle momentum for an event set producing two invisible particles with the same mass.

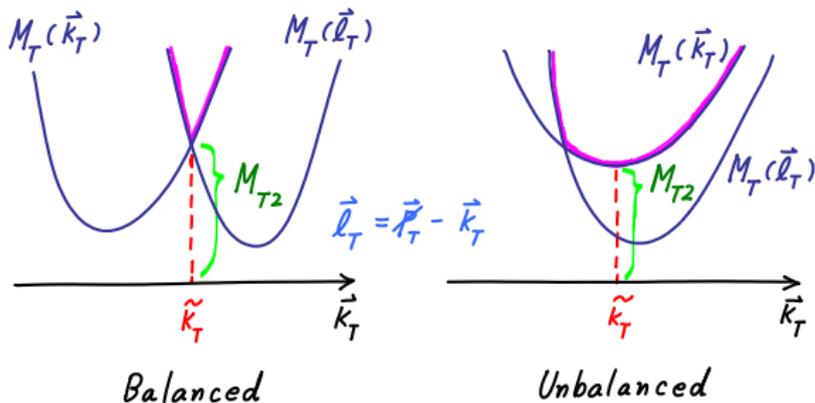


Construction of the MAOS WIMP momenta $\mathbf{k}_\mu^{\text{maos}}$ and $\mathbf{l}_\mu^{\text{maos}}$

- Choose appropriate trial WIMP and mother particle masses: m_χ, m_Y .
- Determine the transverse MAOS momenta with M_{T2} :

$$M_{T2} = M_T(p^2, \mathbf{p}_T, m_\chi, \mathbf{k}_T^{\text{maos}}) \geq M_T(q^2, \mathbf{q}_T, m_\chi, \mathbf{l}_T^{\text{maos}})$$
$$\left(\mathbf{p}_T = \mathbf{k}_T^{\text{maos}} + \mathbf{l}_T^{\text{maos}} \right)$$

* M_{T2} selects unique $\mathbf{k}_T^{\text{maos}}$ and $\mathbf{l}_T^{\text{maos}}$:



iii) Two possible schemes for the longitudinal and energy components:

$$Y(p+k)\bar{Y}(q+l) \rightarrow V_1(p) + \chi(k) + V_2(q) + \chi(l)$$

Scheme 1:

$$k_{\text{maos}}^2 = l_{\text{maos}}^2 = m_\chi^2, \quad (k_{\text{maos}} + p)^2 = (l_{\text{maos}} + q)^2 = m_Y^2$$

Scheme 2:

$$k_{\text{maos}}^2 = l_{\text{maos}}^2 = m_\chi^2, \quad \frac{k_z^{\text{maos}}}{k_0^{\text{maos}}} = \frac{p_z}{p_0}, \quad \frac{l_z^{\text{maos}}}{l_0^{\text{maos}}} = \frac{q_z^{\text{maos}}}{q_0^{\text{maos}}}$$

(Scheme 2 can work even when $Y + \bar{Y}$ are in off-shell.)

The MAOS constructions are designed to have $\mathbf{k}_{\text{maos}}^\mu = \mathbf{k}_{\text{true}}^\mu$ for the M_{T2} endpoint events when $m_\chi = m_\chi^{\text{true}}$ and $m_Y = m_Y^{\text{true}}$.

\implies One can systematically reduce $\Delta\mathbf{k}/\mathbf{k} \equiv (\mathbf{k}_{\text{maos}}^\mu - \mathbf{k}_{\text{true}}^\mu)/\mathbf{k}_{\text{true}}^\mu$ with an M_{T2} -cut selecting the near endpoint events.

- For each event, MAOS momenta obtained in the scheme 1 are real **iff**
 $m_Y \geq M_{T2}(\text{event}; m_\chi)$.

\implies MAOS momenta are real for all events if

$$m_Y \geq M_{T2}^{\max}(m_\chi) \equiv \max_{\{\text{events}\}} \left[M_{T2}(\text{event}; m_\chi) \right] \quad (m_Y^{\text{true}} = M_{T2}^{\max}(m_\chi^{\text{true}}))$$

* If m_χ^{true} and m_Y^{true} are known, use $m_\chi = m_\chi^{\text{true}}$ and $m_Y = m_Y^{\text{true}}$.

* Unless, one can use $m_\chi = 0$ and $m_Y = M_{T2}^{\max}(0)$.

- Precise knowledge of m_χ^{true} and m_Y^{true} might not be essential if
 $(m_\chi^{\text{true}}/m_Y^{\text{true}})^2 \ll 1$:

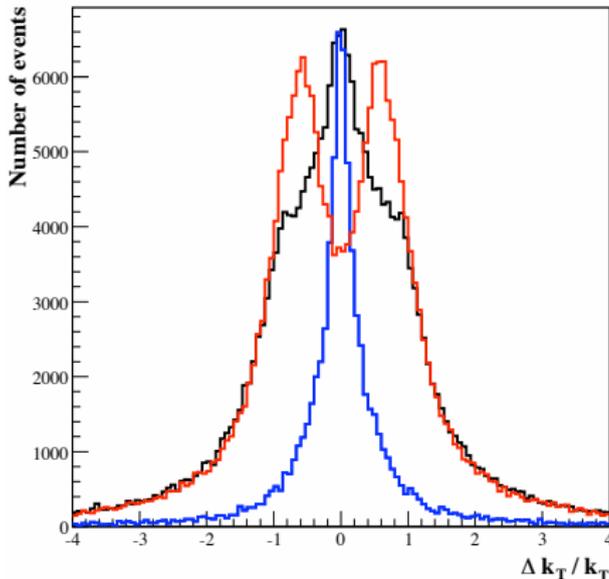
$$\left(\frac{\Delta \mathbf{k}}{\mathbf{k}} \right)_{m_\chi^{\text{true}}, m_Y^{\text{true}}} - \left(\frac{\Delta \mathbf{k}}{\mathbf{k}} \right)_{m_Y = M_{T2}^{\max}(0)} = \mathcal{O} \left(\left(\frac{m_\chi^{\text{true}}}{m_Y^{\text{true}}} \right)^2 \right),$$

$$\frac{\Delta \mathbf{k}_T}{\mathbf{k}_T} = \frac{\tilde{\mathbf{k}}_T - \mathbf{k}_T^{\text{true}}}{\mathbf{k}_T^{\text{true}}} \quad \text{distribution for } \tilde{q}\tilde{q}^* \rightarrow q\chi\bar{q}\chi :$$

$$\tilde{\mathbf{k}}_T = \frac{1}{2}\not{p}_T \quad (\tilde{\mathbf{k}}_T + \tilde{\mathbf{l}}_T = \not{p}_T)$$

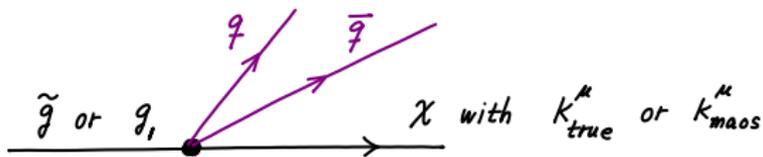
$$\tilde{\mathbf{k}}_T = \mathbf{k}_T^{\text{maos}} \quad \text{for full events}$$

$$\tilde{\mathbf{k}}_T = \mathbf{k}_T^{\text{maos}} \quad \text{for the top 10 \% of near endpoint events}$$



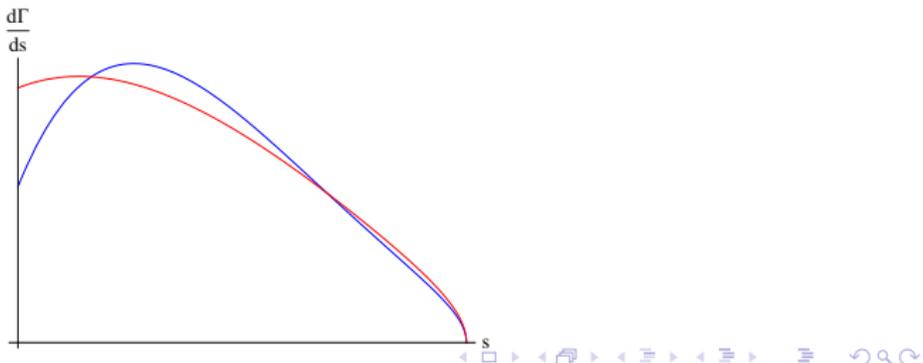
MAOS Momentum and Spin Measurement

Example 1: Gluino/KK-gluon 3-body decay for SPS2 point and its UED equivalent:



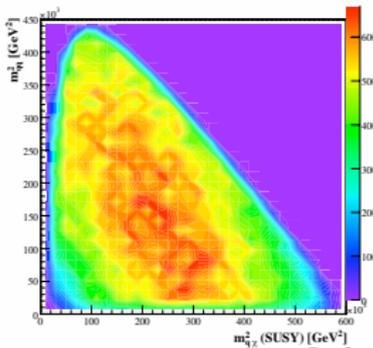
$$\mathbf{s} = (p_q + p_{\bar{q}})^2, \quad \mathbf{t}_{\text{true}} = (p_{q(\bar{q})} + k_{\text{true}})^2, \quad \mathbf{t}_{\text{maos}} = (p_{q(\bar{q})} + k_{\text{maos}}^\pm)^2$$

Without $\mathbf{k}_{\text{maos}}^\mu$, one may consider **the s-distribution** to distinguish **gluino** from **KK-gluon**: Csaki, Heinonen, Perelstein



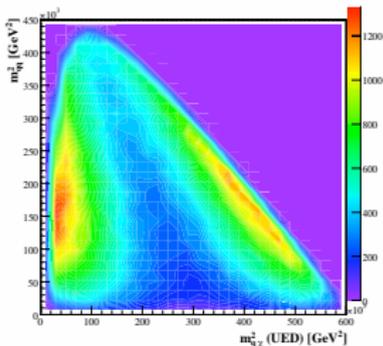
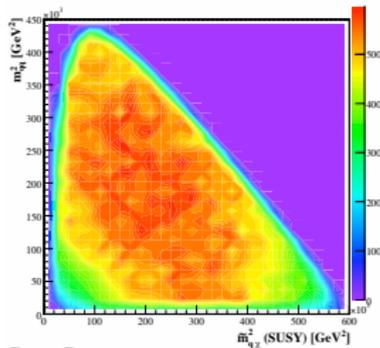
With k_{maos}^μ (scheme 1), one can use **the s-t_{maos} distribution** clearly distinguishing the gluino from the KK-gluon: [arXiv:0810.4853\[hep-ph\]](https://arxiv.org/abs/0810.4853)

$$\frac{d\Gamma}{dsdt_{\text{true}}}$$

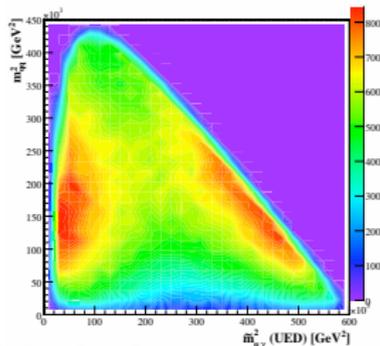


gluino 3-body decay

$$\frac{d\Gamma}{dsdt_{\text{maos}}} \quad (\text{no } M_{T2} \text{ cut})$$



KK-gluon 3-body decay



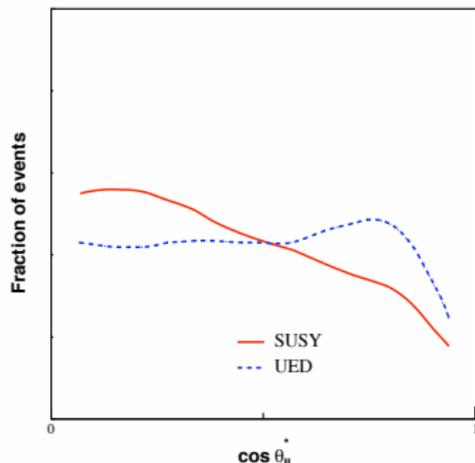
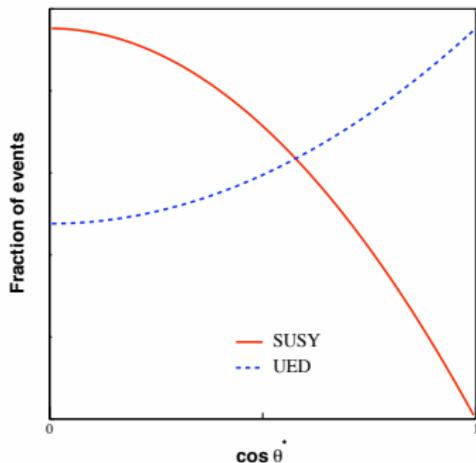
Example 2: Drell-Yan pair production of **slepton** or **KK-lepton** for SUSY SPS1a point and its UED equivalent: Barr

$$\frac{d\Gamma}{d\cos\theta_Y} \text{ and } \frac{d\Gamma}{d\cos\theta_\ell} \text{ of } q\bar{q} \rightarrow Z^0/\gamma \rightarrow Y\bar{Y} \rightarrow \ell\chi\bar{\ell}\chi$$

$Y = \text{slepton or KK-lepton}, \quad \chi = \text{LSP or KK-photon},$

$\cos\theta_Y = \hat{p}_Y \cdot \hat{p}_{\text{beam}}$ in the CM frame of $Y\bar{Y}$,

$\cos\theta_\ell = \hat{p}_\ell \cdot \hat{p}_{\text{beam}}$ in the CR(rapidity) frame of $\ell\bar{\ell}$



Without MAOS, one may look at the lepton angle ($\cos \theta_\ell$) distribution to distinguish the slepton pair production from the KK-lepton pair production: Barr

With MAOS momentum (scheme 1), the mother particle production angle ($\cos \theta_Y$) can be reconstructed: Cho, Choi, Kim, Park

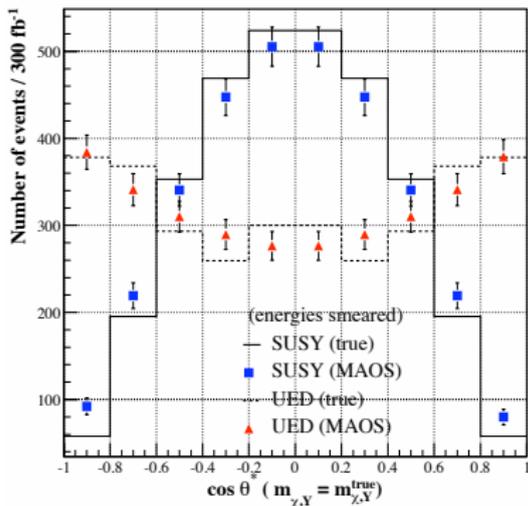
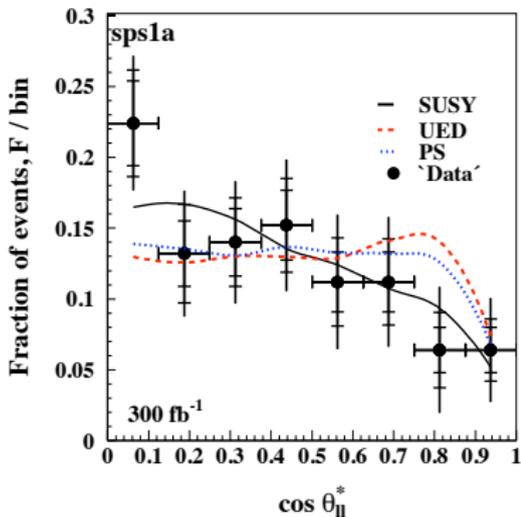
$$Y(p + k_{\text{maos}}^\pm) \bar{Y}(q + l_{\text{maos}}^\pm) \rightarrow \ell(p) \chi(k_{\text{maos}}^\pm) \bar{\ell}(q) \chi(l_{\text{maos}}^\pm)$$

$$\frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}} \equiv \sum_{\alpha=\pm, \beta=\pm} \sum \frac{d\Gamma}{d \cos \theta_{\alpha\beta}}$$

$$(\cos \theta_{\pm\pm} = \hat{p}_Y \cdot \hat{p}_{\text{beam}} \text{ for } k_{\text{maos}}^\pm \text{ and } l_{\text{maos}}^\pm)$$

$$\frac{d\Gamma}{d \cos \theta_\ell} \quad \text{vs} \quad \frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}}$$

with appropriate event cut (\ni the M_{T2} -cut selecting the top 30 %) while including the detector smearing effect for SUSY SPS1a and its UED equivalent: (Knowledge of the mass is not essential.)



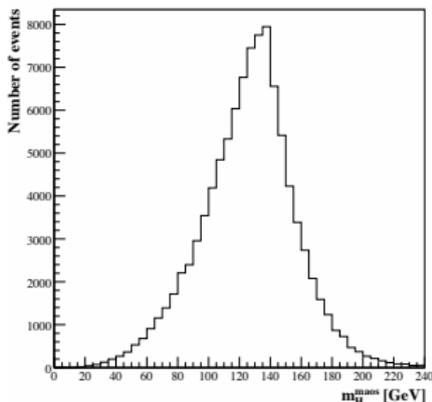
MAOS Momentum and Higgs Mass Measurement

arXiv:0908.0079[hep-ph]

$$H \rightarrow W W \rightarrow \ell(p) \nu(k) \ell(q) \nu(l)$$

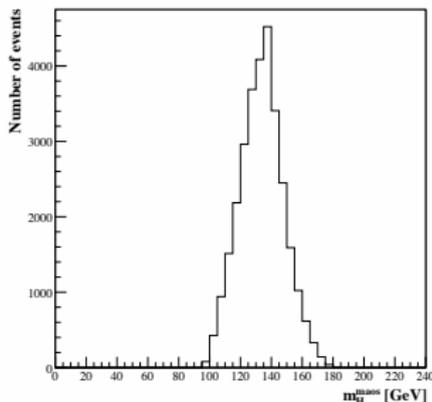
Use the scheme 2 which approximates well the neutrino momenta even when W -bosons are in off-shell.

$$m_H^{\text{maos}} = (p + q + k^{\text{maos}} + l^{\text{maos}})^2$$



full event

$(m_H = 140 \text{ GeV})$

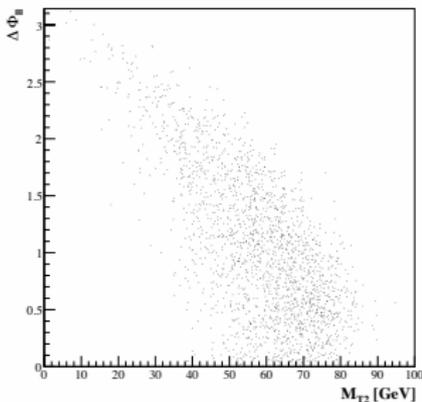


top 30%

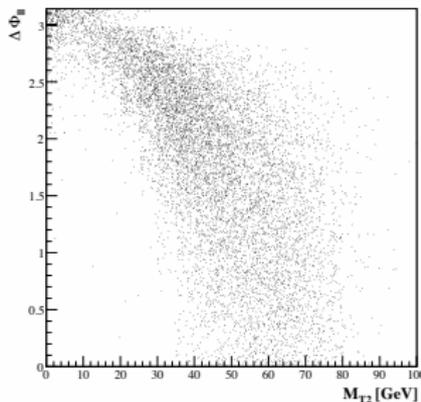
Correlation between $\Delta\Phi_{ll} = \frac{\mathbf{p}_T \cdot \mathbf{q}_T}{|\mathbf{p}_T||\mathbf{q}_T|}$ and M_{T2} :

In the limit of vanishing ISR, $M_{T2}^2 = 2|\mathbf{p}_T||\mathbf{q}_T|(1 + \cos \Delta\Phi_{ll})$

Even with ISR, such correlation persists:



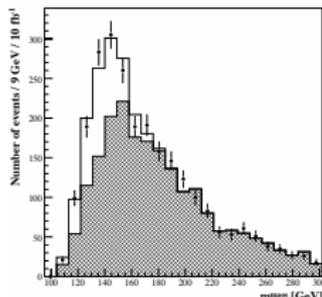
$H \rightarrow WW \rightarrow l\nu l\nu$



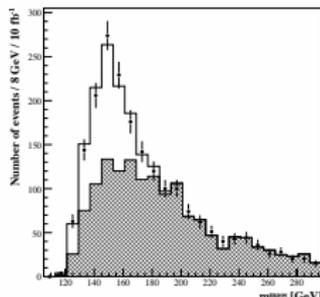
$q\bar{q} \rightarrow WW \rightarrow l\nu l\nu$

Using $\Delta\Phi_{ll}$ and M_{T2} for the event selection, both the signal to background ratio and the efficiency of the MAOS approximation can be enhanced together.

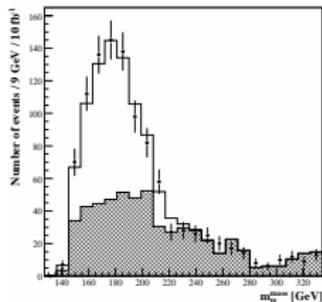
- Event generation with PYTHIA6.4 with $\int L dt = 10 \text{ fb}^{-1}$
- Detector simulation with PGS4
- Include $q\bar{q}, gg \rightarrow WW$ and $t\bar{t}$ backgrounds
- Event selection including the optimal cut of M_{T2} and $\Delta\Phi_{ll}$



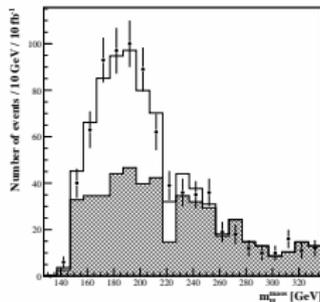
$m_H = 140, M_{T2} > 51$



$m_H = 150, M_{T2} > 57$

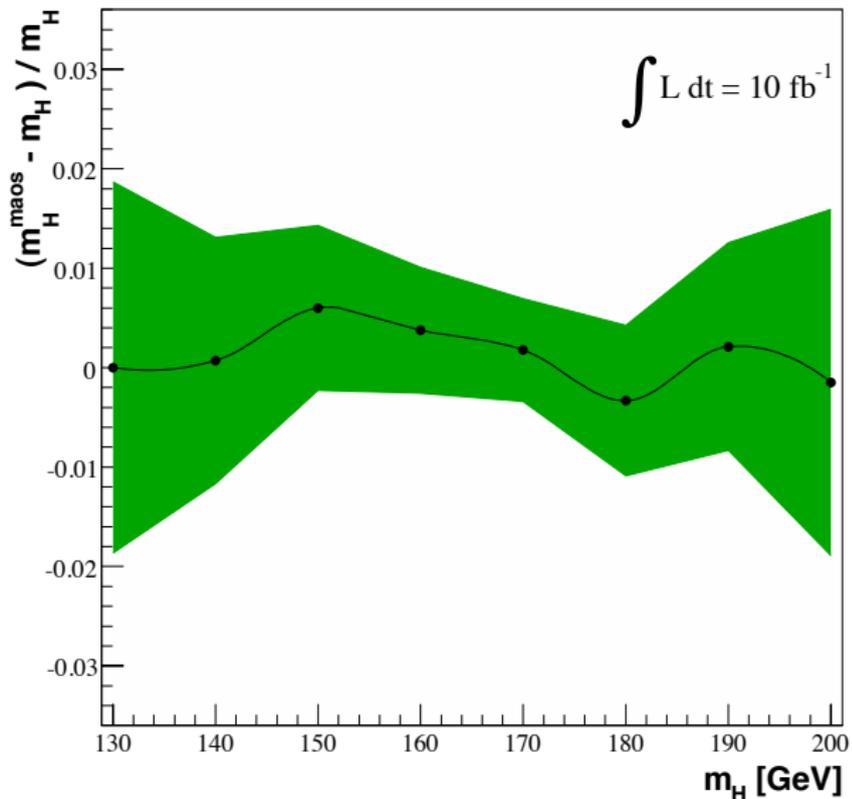


$m_H = 180, M_{T2} > 68$



$m_H = 190, M_{T2} > 70$

$1\text{-}\sigma$ error of m_H from the likelihood fit to the m_H^{maos} distribution



Summary

- **M_{T2} -kink method** (or related methods) might be able to determine new particle masses with missing energy events, even when a long decay chain is not available.
- **MAOS momenta** provide a systematic approximation to the invisible particle momenta in missing energy events, which can be useful for a spin measurement of new particle.
- **MAOS momenta** can be useful also for some SM processes with two missing neutrinos, particularly for probing the properties of the Higgs boson and top quark with
 - * $H \rightarrow W^+W^- \rightarrow \ell^+\nu\ell\bar{\nu}$,
 - * $t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow b\ell^+\nu\bar{b}\ell\bar{\nu}$.